Strategic investment and subsidies within an asymmetric duopoly under uncertainty

Luciana Barbosa[a], Artur Rodrigues[b], and Alberto Sardinha[c]

[a]Business Research Unit (BRU-IUL) and Iscte - Instituto Universitário de Lisboa
[b]NIPE and School of Economics and Management, University of Minho
[c]INESC-ID and Instituto Superior Técnico, Universidade de Lisboa

February 17, 2023

Abstract

This paper analyzes the effect of revenue and investment subsidies on strategic investment and optimal investment timing within an asymmetric duopoly, whereby two heterogeneous firms have different maximum capacities, marginal costs, and investment costs. Within this market, a firm optimally decides whether to be a leader or a follower and the optimal quantity to produce. In addition, firms can be active or idle after investment. An interesting finding from our analysis is that the subsidies accelerate investment when a market has an incumbent firm and the other firm has the option to invest. When both firms have the option to invest, we observe different equilibria and investment triggers' slopes as the revenue and investment subsidies change.

Keywords: Investment Analysis; Real Options; Option Game; Government Subsidies.
JEL Classification: D81; L94; Q42; C72.

1 Introduction

Over the years, policymakers have devised many different subsidies, such as cash subsidies and tax concessions, to accelerate investment projects that promote growth and increase welfare. For instance, feed-in tariffs have been widely used to boost renewable energy generation (Barbosa et al., 2018), and regional airport subsidies have been employed to increase airport activity (Wu et al., 2023). Other examples include demand-based subsidies to encourage private firms to participate in Build-Operate-Transfer projects (Wang et al., 2022), and a subsidy to cover a fraction of the initial investment in public-private partnerships contracts (Silaghi and Sarkar, 2021). However, subsidies have to be carefully designed and analyzed, so that policymakers can shed some light on what to expect from investors' decisions.

Many research works have analyzed subsidies and their impact on investment projects under uncertainty. For example, Bigerna et al. (2019) analyze a feed-in premium and the impact on the investment decision of a renewable energy project. This work considers a monopolistic firm under market uncertainty and derives the optimal investment timing and capacity as a function of the subsidy level. An interesting finding is that higher subsidy levels can accelerate a firm's investment decision at the expense of the capacity level. Barbosa et al. (2020) analyze four feed-in tariffs under market and policy uncertainties. This work assumes a price-taker scenario, derives the optimal investment thresholds, and models the policy uncertainty as a jump event that can reduce the subsidy level. The results show that policy uncertainty accelerates the investment decision for all four feed-in tariffs because investing earlier increases the chance of obtaining a higher subsidy level. Barbosa et al. (2022) derive the optimal price subsidies for a monopolistic firm under market uncertainty. This optimal subsidy induces the monopolistic firm to invest at the social planner's optimal investment timing, thus attaining optimal social welfare.

However, few works (if any) have analyzed the impact of subsidies on a firm's optimal investment timing within an asymmetric duopoly under market uncertainty. We present a novel model that derives the optimal investment triggers of two firms that act strategically, and one of them receives subsidies. In fact, this work extends the work from Pawlina and Kort (2006) and Kong and Kwok (2007) to include revenue and investment subsidies. We also include one more extension, whereby we derive the optimal operational decisions (i.e., optimal quantities to produce) when the two firms are limited by production capacity.

Our main findings are twofold. First, we show that the revenue and investment subsidies accelerate a firm's investment decision in a market with an incumbent firm. We also compare both subsidies and identify the subsidy that induces an earlier investment for a given subsidy level. Second, when both firms have the option to invest, we observe a different pattern, whereby the equilibria and investment triggers' slopes change with different revenue and subsidy levels.

The remaining sections of this paper are organized as follows. Section 2 derives the optimal operational decisions with limited capacity, the optimal investment triggers with subsidies, and the equilibria from the strategic interaction of both firms. Section 3 presents the comparative statics of our model to analyze the impact of key parameters on the optimal investment triggers. Finally, Section 4 presents our conclusions and directions for future work.

2 The investment and operational decisions

Our model builds on the work from Pawlina and Kort (2006) and Kong and Kwok (2007), where two asymmetric firms strategically choose the timing to invest. We extend these previous works by including revenue and investment subsidies. In addition, we also include a strategic operational decision, whereby firms decide the quantity to produce and are limited by a maximum capacity.

We assume two heterogeneous firms where each firm has the option to invest. In particular, the firms have different marginal and investment costs. The firms also face a linear demand function with a demand shock, as shown below.

Assumption 1. *The firms operate in a market with the following demand function:*

$$P(a_t, Q_T) = a_t - bQ_T \tag{1}$$

where $b > 0$ is the slope and Q_T is the total annual market output. Note that Q_T is the sum of firm 1's annual output and firm 2's annual output.

Assumption 2. *The demand shock at time t is $a = \{a_t, t \geqslant 0\}$, where the process a_t follows a geometric Brownian motion:*

$$da_t = \alpha a_t dt + \sigma a_t dB_t \tag{2}$$

where $B = \{B_t, t \geqslant 0\}$ is the Brownian motion, $\alpha < r$ is the risk-neutral drift, r is the risk-free interest rate, and σ is the volatility.

Assumption 3. *The profit function for firms 1 and 2 are the following:*

$$\Pi_1 = (P(a_t, Q_T) - c)q_1 \tag{3}$$

$$\Pi_2 = (P(a_t, Q_T) - \epsilon c)q_2 \tag{4}$$

where c is the marginal cost for firm 1 and ϵ is a constant, $0 \leqslant \epsilon \leqslant 1$.

Note that the profit functions are different due to the marginal costs. We also assume that the marginal cost of firm 2 is lower than the marginal cost of firm 1. Hence, firm 2 has a competitive advantage. In addition, both firms have a maximum capacity Q_i. The subscript i denotes a particular firm in our model, where subscript 1 is for firm 1 and subscript 2 is for firm 2.

We include revenue and investment subsidies in our model and compare them with the plain (no-subsidy) case. Moreover, we assume that only firm 2 receives the subsidy. This situation may arise when firm 1 has an older technology, and firm 2 wants to invest in a new and more efficient technology with a lower marginal cost. However, firm 2 has a higher investment cost to deploy the project.

Firm 1's investment cost is $I_1 = \delta Q_1$, where δ is the marginal investment cost. Firm 2's investment cost is $I_2 = \kappa \delta Q_2$, where κ is a constant ($\kappa > 1$). The investment subsidy equals $\lambda \delta Q_2$ and the revenue subsidy is equal to ηq_2, which would change the profit function to $\Pi_2 = (P(a_t, Q_T) + \eta - \epsilon c) q_2$.

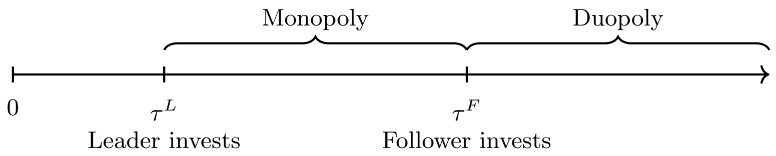


Figure 1: Investment timing of firms

Our model has two sequential stages, as depicted in Figure 1. In the first stage, the leader rationally chooses the optimal time to invest in the project and benefits from being in a monopoly until it becomes optimal for the follower to invest. Hence, the second stage starts when the follower invests, and both firms compete à la Cournot. Note that both firms must decide whether to be inactive or active in the two stages.

We thus have two optimization problems: (i) an investment decision, where firms rationally choose the optimal time to invest, and (ii) an operational decision, where firms decide the optimal quantity to produce. Therefore, a firm must rationally choose if it is better to be the leader or the follower and then decide to be idle or active. When active, this firm has to also decide whether to produce the maximum capacity or not.

Using backward induction, we start by finding the optimal operational decision for each firm. Next, we analyze the follower's optimal investment timing. Finally, we determine the leader's optimal investment timing.

2.1 The operational decision

This section presents the firms' operational decisions without subsidies (i.e., plain case), where each firm decides the quantity to produce. We start deriving in Section 2.1.1 the quantities in several static states and then present the corresponding profit flows. In Section 2.1.2, we derive the conditions that influence the firms to switch between the static states.

2.1.1 Static model

In this section, we derive all the possible static scenarios when firms have already invested in the project. The firms must rationally decide whether to be idle or active and how much to produce. Recall that each firm cannot produce more than a maximum capacity. These assumptions lead to the following states $s \in S$: (i) $s = 00$: both firms are idle; (ii) $s = 01$: firm 2 is active and produces a quantity q_2 lower than its maximum capacity, and firm 1 is idle; (iii) $s = 11$: both firms are active and produce a quantity q_1 and q_2, which are lower than their maximum capacities; (iv) $s = f1$: both firms are active and firm 1 produces its maximum capacity Q_1 while firm 2 produces a quantity q_2, lower than its maximum capacity; (v) $s = 0f$: firm 2 is active and produces the maximum capacity Q_2, and firm 1 is idle; (vi) $s = 1f$: both firms are active and firm 1 produces a quantity q_1 lower than its maximum capacity, while firm 2 produces its maximum capacity Q_2; and (vii) $s = ff$: firms 1 and 2 are active and produce their maximum capacities Q_1 and Q_2, respectively. Note that the states $s = 10$ and $s = f0$ are not considered because the marginal cost of firm 1 (i.e., c) is greater than the marginal cost of firm 2 (i.e., ϵc).

Proposition 1. *The quantity that maximizes the profit of firm $i \in \{1, 2\}$ in state $s \in S$:*

s	q_1^s	q_2^s
00	0	0
01	0	$\frac{a_t - \epsilon c}{2b}$
11	$\frac{a_t - (2-\epsilon)c}{3b}$	$\frac{a_t - (2\epsilon - 1)c}{3b}$
$f1$	Q_1	$\frac{a_t - bQ_1 - \epsilon c}{2b}$
$0f$	0	Q_2
$1f$	$\frac{a_t - c - bQ_2}{2b}$	Q_2
ff	Q_1	Q_2

which yields firm i's profit flow in each state $s \in S$:

$$\pi_i(a) = d_{0i}^s + d_{1i}^s \ a_t + d_{2i}^s \ a_t^2 \tag{5}$$

where d_{0i}^s, d_{1i}^s, d_{2i}^s are the following constants:

s	d^s_{01}	d^s_{11}	d^s_{21}	d^s_{02}	d^s_{12}	d^s_{22}
00	0	0	0	0	0	0
01	0	0	0	$\frac{(\epsilon c)^2}{4b}$	$-\frac{2\epsilon c}{4b}$	$\frac{1}{4b}$
11	$\frac{((\epsilon-2)c)^2}{9b}$	$\frac{2(\epsilon-2)c}{9b}$	$\frac{1}{9b}$	$\frac{((2\epsilon-1)c)^2}{9b}$	$-\frac{2(2\epsilon-1)c}{9b}$	$\frac{1}{9b}$
$f1$	$-\frac{Q_1(bQ_1+(2-\epsilon)c)}{2}$	$\frac{Q_1}{2}$	0	$\frac{(bQ_1+\epsilon c)^2}{4b}$	$-\frac{2(bQ_1+\epsilon c)}{4b}$	$\frac{1}{4b}$
$0f$	0	0	0	$-Q_2(bQ_2+\epsilon c)$	Q_2	0
$1f$	$\frac{(c+bQ_2)^2}{4b}$	$-\frac{2(c+bQ_2)}{4b}$	$\frac{1}{4b}$	$-\frac{Q_2((2\epsilon-1)c+bQ_2)}{2}$	$\frac{Q_2}{2}$	0
ff	$-Q_1(b(Q_1+Q_2)+c)$	Q_1	0	$-Q_2(b(Q_1+Q_2)+\epsilon c)$	Q_2	0

We calculate the quantities and profit flows in Proposition 1 by either modeling a static state as a Cournot game or fixing the quantity (i.e., $q_i = 0$ or $q_i = Q_i$) for a firm i and calculating the first-order condition (i.e., $\frac{\partial \Pi_{3-i}}{\partial q_{3-i}} = 0$) for the other firm, thus best responding to firm i's decision.

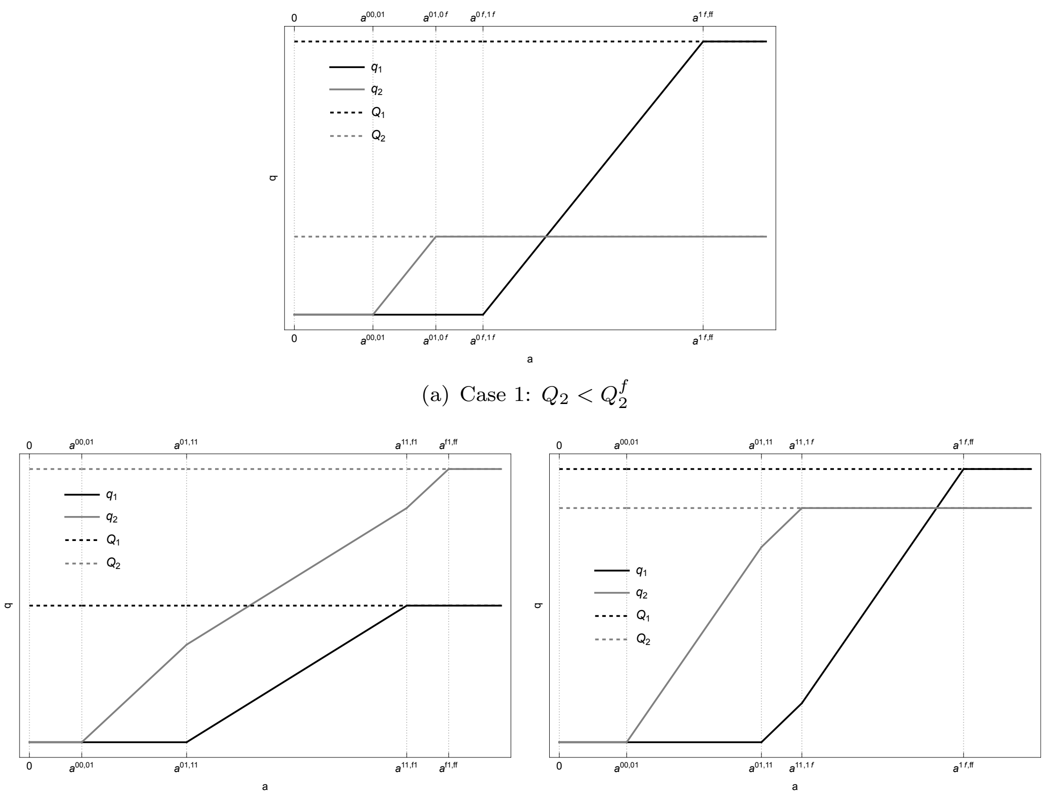
2.1.2 The dynamic model

Due to the stochastic component of the market price, firms can switch between the static states in Section 2.1.1. In addition, the full capacities Q_1 and Q_2 create three distinct cases: (i) firm 2 produces its full capacity Q_2 before firm 1 becomes active (Figure 2(a)); (ii) firm 2 produces full capacity Q_2 when firm 1 is already active (Figure 2(b)). Moreover, firm 1 reaches its full capacity before firm 2; and (iii) firm 1 produces its full capacity Q_1 after firm 2 (Figure 2(c)).

Table 1: Transition conditions

	01	11	$f1$	$0f$	$1f$	ff
00	ϵc					
01		$(2-\epsilon)c$		$2bQ_2+\epsilon c$		
11			$3bQ_1+(2-\epsilon)c$		$3bQ_2+(2\epsilon-1)c$	
$f1$						$c+b(2Q_1+Q_2)$
$0f$					bQ_2+c	
$1f$						$b(Q_1+2Q_2)+\epsilon c$

Within each case of Figure 2, note that the firms decide to transition between the static states at specific values of the demand shock a_t. In other words, there are values of a_t that trigger this transition. Table 1 presents all the transition values in Figure 2, which



(a) Case 1: $Q_2 < Q_2^f$

(b) Case 2: $Q_2 \geqslant Q_2^f \wedge Q_1 < Q_1^f$

(c) Case 3: $Q_2 \geqslant Q_2^f \wedge Q_1 \geqslant Q_1^f$

Figure 2: Three cases that depend on Q_1 and Q_2

are calculated as follows:

- $a^{00,01}$ is the transition value when the state changes from $s = 00$ to $s = 01$ (i.e., firm 2 becomes active). This transition occurs when the market price is equal to ϵc, leading to $a^{00,01} = \epsilon c$.

- $a^{01,11}$ corresponds to the transition value when the state changes from $s = 01$ to $s = 11$ (i.e., firm 1 also becomes active). The transition happens when the market price is equal to c, thus yielding $a^{01,11} = (2 - \epsilon)c$.

- $a^{01,0f}$ is the transition value when $s = 01$ changes to $s = 0f$ (i.e., firm 2 reaches the maximum capacity), which occurs when $q_2^{01} = q_2^{0f}$ (i.e., $q_2^{01} = Q_2$). This yields an $a^{01,0f} = 2bQ_2 + \epsilon c$.

- $a^{11,f1}$ is the transition value when the state changes from $s = 11$ to $s = f1$ (i.e., firm 1 reaches the maximum capacity). This occurs when $q_1^{11} = q_1^{f1}$ (i.e., $q_1^{11} = Q_1$), yielding $a^{11,f1} = 3bQ_1 + (2 - \epsilon)c$.

- $a^{11,1f}$ is the transition value when $s = 11$ changes to $s = 1f$ (i.e., firm 2 reaches the maximum capacity), which occurs when $q_2^{11} = q_2^{1f}$ (i.e., $q_2^{11} = Q_2$). Thus, $a^{11,1f} = 3bQ_2 + (2\epsilon - 1)c$.

- $a^{f1,ff}$ corresponds to the transition value when $s = f1$ changes to $s = ff$ (i.e., firm 2 reaches the maximum capacity). The transition happens when $q_2^{f1} = q_2^{ff}$ (i.e., $q_2^{f1} = Q_2$), thus yielding $a^{f1,ff} = c + b(2Q_1 + Q_2)$.

- $a^{0f,1f}$ is the transition value when $s = 0f$ changes to $s = 1f$ (i.e., firm 1 becomes active), which occurs when $q_1^{0f} = q_1^{1f}$. Thus, $a^{0f,1f} = bQ_2 + c$.

- $a^{1f,ff}$ corresponds to the transition value when $s = 1f$ changes to $s = ff$ (i.e., firm 1 reaches the maximum capacity). The transition happens when $q_1^{1f} = q_1^{ff}$ (i.e., $q_1^{1f} = Q_1$), thus yielding $a^{1f,ff} = b(Q_1 + 2Q_2) + \epsilon c$.

Cases

Depending on the values of the maximum capacities Q_1 and Q_2, the firms end up in one of the three cases of Figure 2. We now derive the values of the maximum capacities Q_1^f and Q_2^f that enable us to know the exact case based on the values of Q_1 and Q_2.

Recall that Figure 2(a) depicts a scenario where firm 2 produces its full capacity Q_2 before firm 1 becomes active. This occurs when the transition $a^{01,0f}$ is smaller than the transition $a^{01,11}$ (i.e., $a^{01,0f} < a^{01,11}$), which yields $Q_2 < \frac{(1-\epsilon)c}{b}$ and thus Q_2^f is the following:

$$Q_2^f = \frac{(1-\epsilon)c}{b} \tag{6}$$

In Figure 2(b), we present a scenario where firm 2 produces its full capacity Q_2 when firm 1 is already active. Moreover, firm 1 reaches its full capacity before firm 2. Such a scenario occurs when $Q_2 \geqslant Q_2^f$ and the transition $a^{11,f1}$ is smaller than the transition $a^{11,1f}$ (i.e., $a^{11,f1} < a^{11,1f}$). With the latter inequality, we derive $Q_1 < Q_2 - \frac{(1-\epsilon)c}{b}$ and the following result:

$$Q_1^f = Q_2 - \frac{(1-\epsilon)c}{b} \tag{7}$$

Figure 2(c) depicts the scenario where firm 1 produces its full capacity Q_1 after firm 2. This only occurs when $Q_2 \geqslant Q_2^f$ and $Q_1 \geqslant Q_1^f$. Finally, we derive the value of the projects for these 3 cases.

Proposition 2. *The value of firm $i \in \{1,2\}$ in state $s \in S$, valid for $a^{s_-,s} \leqslant a < a^{s,s_+}$, is given by:*

$$V_i^s(a) = A_{1i}^s\, a^{\beta_1} + A_{2i}^s\, a^{\beta_2} + d_{0i}^s \frac{1}{r} + d_{1i}^s \frac{a}{r-\alpha} + d_{2i}^s \frac{a^2}{r-(2\alpha+\sigma^2)} \tag{8}$$

where the state sets are:

$$S = \begin{cases} \{00, 01, 0f, 1f, ff\} & \textit{for } Q_2 < Q_2^f \\ \\ \{00, 01, 11, f1, ff\} & \textit{for } Q_2 \geqslant Q_2^f \wedge Q_1 < Q_1^f \\ \\ \{00, 01, 11, 1f, ff\} & \textit{for } Q_2 \geqslant Q_2^f \wedge Q_1 \geqslant Q_1^f. \end{cases} \tag{9}$$

In addition, constants A_{1i}^s and A_{2i}^s are:

$$\begin{aligned} A_{1i}^s = A_{1i}^{s_+} + \frac{(a^{s,s_+})^{-\beta_1}}{\beta_1 - \beta_2} \Bigg(& \beta_2 (d_{0i}^s - d_{0i}^{s_+}) \frac{1}{r} + (\beta_2 - 1)(d_{1i}^s - d_{1i}^{s_+}) \frac{a^{s,s_+}}{r - \alpha} \\ & + (\beta_2 - 2)(d_{2i}^s - d_{2i}^{s_+}) \frac{(a^{s,s_+})^2}{r - (2\alpha + \sigma^2)} \Bigg) \end{aligned} \tag{10}$$

$$\begin{aligned} A_{2i}^s = A_{2i}^{s_-} + \frac{(a^{s_-,s})^{-\beta_2}}{\beta_1 - \beta_2} \Bigg(& \beta_1 (d_{0i}^{s_-} - d_{0i}^{s}) \frac{1}{r} + (\beta_1 - 1)(d_{1i}^{s_-} - d_{1i}^{s}) \frac{a^{s_-,s}}{r - \alpha} \\ & + (\beta_1 - 2)(d_{2i}^{s_-} - d_{2i}^{s}) \frac{(a^{s_-,s})^2}{r - (2\alpha + \sigma^2)} \Bigg). \end{aligned} \tag{11}$$

s_- and s_+ indicate the preceding and following states, respectively. Furthermore, constants d_{0i}^s, d_{1i}^s, and d_{2i}^s are in Proposition 2, and $a^{s,s}$ as in Table 1.

We calculate the constants A_{1i}^s and A_{2i}^s by equating the values and derivatives because $V_i^s(a)$ must be continuously differentiable in a. In addition, β_1 is a positive root and β_2 is a negative root of the fundamental quadratic equation:

$$Q(\beta) = \frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta + r = 0, \tag{12}$$

and the solution is:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left(\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2} \right)^{\frac{1}{2}} > 1 \tag{13}$$

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \left(\left(-\frac{1}{2} + \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2} \right)^{\frac{1}{2}} < 0. \tag{14}$$

2.2 The investment decision

In this section, we derive the value of both firms' investment options and their investment triggers without subsidies (i.e., plain case). In this investment decision, each firm has to strategically choose whether to be a leader or a follower. Note that the leader benefits from being a monopolist until it becomes optimal for the second firm, the follower, to invest.

We assume that the firms have a zero payoff before investment. Recall that firm 1's investment is $I_1 = \delta Q_1$, and firm 2's investment cost is $I_2 = \kappa\delta Q_2$ ($\kappa > 1$). To derive the investment model, we first derive the optimal investment timing for the follower, then the optimal investment timing for the leader with and without preemption.

2.2.1 The follower's optimal investment timing

The follower, in a scenario with or without preemption, is the only firm with the option to invest because the other firm has already invested. The follower receives the amount in (8) after investing at time τ. Consequently, we present the follower's optimization problem in (15), whereby it selects an optimal time that maximizes $V_i(a)$ net of the investment cost I_i ($i \in \{1, 2\}$).

$$\begin{aligned} F_i^F(a) &= \sup_{\tau^F} E\left[\int_{\tau^F}^{+\infty} (d_{0i}^s e^{-rt} + d_{1i}^s\ a_t e^{-(r-\alpha)t} + d_{2i}^s\ a_t^2 e^{-(r-(2\alpha+\sigma^2))t})dt - I_i e^{-r\tau^F} | a_0 = a\right] \\ &= \sup_{\tau^F} E\left[V_i^s(a_{\tau^F}) - I_i e^{-r\tau^F} | a_0 = a\right] \end{aligned} \tag{15}$$

where s is the state for a_t.

The investment is not yet optimal when a is smaller than the investment threshold a_i^F. Therefore, the value function in this region is the solution of the following differential equation:

$$0.5\,\sigma^2\,a^2\,\frac{\partial^2 F_i^F(a)}{\partial a^2} + \alpha\,a\frac{\partial F_i^F(a)}{\partial a} - rF_i^F(a) = 0 \tag{16}$$

In general, the solution of (16) is the following (Dixit and Pindyck, 1994):

$$F_i^F(a) = A_i^F a^{\beta_1} \tag{17}$$

Hence, the solution of the optimization problem in (15), known as the value of the investment opportunity, takes the following (general) form:

$$F_i^F(a) = \begin{cases} A_i^F a^{\beta_1} & a < a_i^F \\ \\ V_i^s(a) - I_i & a \geqslant a_i^F \end{cases} \tag{18}$$

where β_1 and β_2 were already defined in (13) and (14), respectively. In addition, A_i^F and a_i^F are found solving $A_i^F {a_i^F}^{\beta_1} = V_i^s(a) - I_i$ and $\beta_1 A_i^F {a_i^F}^{\beta_1 - 1} = \frac{\partial V_i^s(a_i^F)}{\partial a}$ (i.e., the value matching and smooth pasting conditions).

Proposition 3. *The value of the follower's investment option is given by:*

$$F_i^F(a) = \begin{cases} \left(V_i^s(a_i^F) - I_i\right)\left(\dfrac{a}{a_i^F}\right)^{\beta_1} & a < a_i^F \\ V_i^s(a) - I_i & a \geqslant a_i^F \end{cases} \tag{19}$$

where a_i^F is the follower's optimal investment threshold.

2.2.2 The leader's optimal investment timing

The leader's investment strategy differs when considering a scenario with or without preemption. We start deriving the leader's value and investment threshold with preemption. Recall that the leader benefits from being a monopolist after the investment decision, as shown in Figure 1. The value of the firms as a monopolist $V_i^M(a)$ is the same as 8, where the state sets for firms 1 and 2 are $M_1 = \{00, 10, f0\}$ and $M_2 = \{00, 01, 0f\}$, respectively. Note that we do not consider states 10 and $f0$ in Proposition 2 because these states do not exist when both firms have invested. Therefore, firm 1's profit flows as a monopolist in states sets 10 and $f0$ are given by:

$$\pi(a) = d_{01}^{10} + d_{11}^{10}\ a_t + d_{21}^{10}\ a_t^2 \tag{20}$$

$$\pi(a) = d_{01}^{f0} + d_{11}^{f0}\ a_t \tag{21}$$

where $d_{01}^{10} = \frac{c^2}{4b}$, $d_{11}^{10} = -\frac{2c}{4b}$, $d_{21}^{10} = \frac{1}{4b}$, $d_{01}^{f0} = -Q_1(bQ_1 + c)$, $d_{11}^{f0} = Q_1$, and $d_{21}^{f0} = 0$. In addition, the transition conditions are $a^{00,10} = c$ and $a^{10,f0} = 2b\text{Q}1 + c$.

We present the preemptive leader's expected value $L_i^P(a)$ in (23) when it takes the preemptive action. Note that the first integral inside the expected value is due to the revenue flow when the leader is a monopolist. In addition, the second integral is due to the revenue flow when the leader and follower have invested.

$$\begin{aligned} L_i^P(a) = E\Bigg[\int_0^{\tau^F} &(d_{0i}^m e^{-rt} + d_{1i}^m\ a_t e^{-(r-\alpha)t} + d_{2i}^m\ a_t^2 e^{-(r-(2\alpha+\sigma^2))t})dt + \\ &\int_{\tau^F}^{+\infty}(d_{0i}^s e^{-rt} + d_{1i}^s\ a_t e^{-(r-\alpha)t} + d_{2i}^s\ a_t^2 e^{-(r-(2\alpha+\sigma^2))t})dt - I_i | a_0 = a\Bigg] \end{aligned} \tag{22}$$

$$= V_i^m(a) - I_i + E\left[(V_i^s(a_{\tau^F}) - V_i^m(a_{\tau^F}))e^{-r\tau^F} | a_0 = a\right] \tag{23}$$

where $m \in M_i$ and $s \in S$ are the states for a_t.

The investment trigger of the leader with preemption is the point where the firm is indifferent between being the leader or the follower. However, the firm with the lowest preemptive stopping time invests in the preemptive stopping time of the other firm because this leads to a higher value.

Proposition 4. *The value of the leader with preemption is given by:*

$$L_i^P(a) = \begin{cases} V_i^m(a) - I_i + (V_i^s(a_{3-i}^F) - V_i^M(a_{3-i}^F)) \left(\dfrac{a}{a_{3-i}^F}\right)^{\beta_1} & \text{for } a < a_{3-i}^F \\ \\ V_i^s(a) - I_i & \text{for } a \geqslant a_{3-i}^F \end{cases} \tag{24}$$

where $m \in M_i$ and $s \in S$ are the states for a_t.

The investment trigger of the leader with preemption a_i^P is the point where the firm is indifferent between being the leader or the follower (i.e., the solution of $L_i^P(a) = F_i^F(a)$).

Next, we derive the value of the designated leader (the leader without preemption). Note that in this case, the investment time of the designated leader does not depend on the follower's investment trigger. The designated leader knows that the other firm prefers to be the follower because its value as a leader is always lower than its value as a follower and, consequently, will never invest before it. In this case, it acts as a monopolist.

The value function for the leader without preemption, $V_i^L(a)$, is the solution of the following differential equation:

$$\begin{aligned} F_i^L(a) = \sup_{\tau^L} E \Bigg[& \int_{\tau^L}^{\tau^F} (d_{0i}^m e^{-rt} + d_{1i}^m \, a_t e^{-(r-\alpha)t} + d_{2i}^m \, a_t^2 e^{-(r-(2\alpha+\sigma^2))t}) dt - I_i e^{-r\tau^L} \\ & + \int_{\tau^F}^{+\infty} (d_{0i}^s e^{-rt} + d_{1i}^s \, a_t e^{-(r-\alpha)t} + d_{2i}^s \, a_t^2 e^{-(r-(2\alpha+\sigma^2))t}) dt | a_0 = a \Bigg] \end{aligned} \tag{25}$$

$$= \sup_{\tau^L} E \left[(V_i^m(a_{\tau^L}) - I_i) e^{-r\tau^L} + (V_i^s(a_{\tau^F}) - V_i^m(a_{\tau^F})) e^{-r\tau^F} | a_0 = a \right] \tag{26}$$

Proposition 5. *The value of the option for the leader without preemption is:*

$$F_i^L(a) = \begin{cases} (V_i^m(a_i^L) - I_i) \left(\dfrac{a}{a_i^L}\right)^{\beta_1} - V_i^s(a_i^F) \left(\dfrac{a}{a_i^F}\right)^{\beta_1} & \text{for } a < a_i^L \\ \\ L_i^P(a) & \text{for } a \geqslant a_i^L \end{cases} \tag{27}$$

2.2.3 Equilibria

Similar to Pawlina and Kort (2006), we can identify two different equilibria due to the strategic interaction of the firms, namely, a preemptive equilibrium and a sequential equilibrium.

The preemptive equilibrium occurs when both firms want to invest as the leader. In this scenario, each firm has to take into account that the other firm might also invest. Without loss of generality, if firm i has the lowest preemptive trigger a_i^P (i.e., $a_i^P < a_{3-i}^P$) then it invests when $a = a_{3-i}^P$. Note that firm i invests when $a = a_{3-i}^P$ because this leads to the highest value it can obtain before the other firm would invest. Once the leader has invested, the other firm invests as a follower when $a = a^F_{3-i}$.

In the sequential equilibrium, one of the firms does not have the incentive to be a leader and always invests as a follower. This scenario occurs when the value function of the firm as a leader is always lower than its follower value function. Hence, without loss of generality, if firm $3 - i$ is always a follower, then it invests when $a = a^F_{3-i}$. This makes firm i as the leader, which invests when $a = a^L_i$.

2.3 Operational and investment decisions with subsidies

We consider two different subsidies in our model, namely revenue and investment subsidies, which are only given to firm 2. Recall that the investment subsidy is $\lambda\delta Q_2$ and the revenue subsidy is ηq_2.

Consequently, firm 2's profit function changes to:

$$\Pi_2 = (P(a_t, Q_T) + \eta - \epsilon c)q_2 = \left(P(a_t, Q_T) - \left(\epsilon - \frac{\eta}{c}\right) c\right) q_2 \tag{28}$$

We can thus substitute $\left(\epsilon - \frac{\eta}{c}\right)$ for ϵ in the equations of the plain case to include the revenue subsidy. Regarding the investment subsidy, we substitute $\kappa - \lambda$ for κ in the equations of the plain case to include the investment subsidy.

3 Comparative statics

In this section, we perform a comparative static analysis of the main drivers of our model. In particular, we study the influence of some parameters on the optimal investment triggers and strategic investments. For the numerical study, we use the base-case parameters in Table 2.

We first analyze the impact of the parameters on the investment trigger of firm 2 when firm 1 is already in place. Hence, firm 1 is the incumbent firm, and firm 2 is the follower.

3.1 Firm 2 can invest as a follower when firm 1 is already in the market

Figure 3 presents the investment thresholds for three different schemes, namely without subsidy, with an investment subsidy, or with a revenue subsidy as a function of η. As expected, the plots show that the investment threshold of firm 2 with revenue subsidy decreases as the revenue subsidy η increases. In other words, the decision to invest is accelerated when η increases. Note that the investment and revenue triggers are the same when $\eta \approx 0.55$. Hence, firm 2 invests earlier with the investment subsidy when η is smaller than this point. On the side, the revenue subsidy makes firm 2 invest earlier for η greater than this point.

Table 2: Base-case parameters.

Parameter	Description	Value
σ	Volatility	0.2
r	Risk-free rate	0.08
α	Risk-neutral drift rate	0.01
c	Firm 1's marginal cost	2.0
ϵ	Reduction factor of firm 2's marginal cost	0.5
a	Current level of the demand shock	5
b	Slope of the linear demand function	2
Q_1	Maximum capacity firm 1	0.7
Q_2	Maximum capacity firm 2	0.6
δ	Firm 1's investment per unit of capacity	20
κ	Increase factor of firm 2's investment cost	2
η	Revenue subsidy	0.5
λ	Investment subsidy $= \eta/(\delta r)$	0.3125

Figure 4 presents the investment thresholds for the three different schemes, namely without subsidy, with an investment subsidy, and with a revenue subsidy as a function of λ. As expected, the plots show that the investment threshold of firm 2 with revenue subsidy decreases as the revenue subsidy λ increases. An interesting result is that for values of λ lower than the point where both triggers with subsidy meet, the firm's 2 investment trigger with a revenue subsidy is lower than the investment trigger with an investment subsidy. Hence, for values of λ lower than this point, a revenue subsidy policy is a better policy.

3.2 Behavior of firm 1

Next, we analyze the impact of the parameters in the investment trigger and investment strategy when both firms have have not invest yet. The goal is no analyze how firm 1 changes its behavior in anticipation of subsidies granted to firm 2.

3.2.1 Without subsidies

Figure 5 presents the optimal investment timing for the follower a_i^F, the leader with preemption a_i^P, the leader without preemption a_i^L, and the optimal investment strategy a_i^* decision as a function of volatility. Considering firm 1 with a high capacity (i.e., $Q_1 = 0.6$), the firms act strategically in a sequential equilibrium where firm 1 is a designated leader, and firm 2 is the follower. In contrast, when firm 1 has a low capacity (i.e., $Q_1 = 0.2$), we observe a preemptive equilibrium. However, for lower values of volatility, firm 2 invests in the preemptive trigger of firm 1, and firm 1 invests in its trigger as a follower. For higher volatilities, firm 1 invests in the preemptive trigger of firm 2, and firm 2 invests

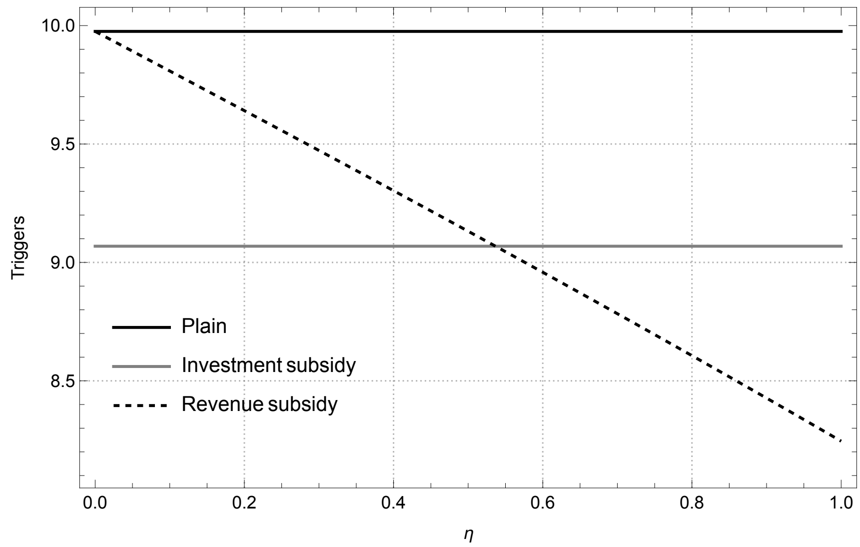


Figure 3: Sensitivity to η

in its trigger as a follower. The investment triggers increase as the volatility increases. In addition, the investment strategies do not change when firm 1 has a high capacity. In contrast, for a low capacity of firm 1, the investment strategies change as volatility increases.

Figure 6 presents the optimal investment timing for the follower a_i^F, the leader with preemption a_i^P, the leader without preemption a_i^L, and the optimal investment strategy a_i^* decision as a function of the maximum capacity of firm 1 and firm 2. We can see that the equilibria change as the maximum capacity Q_1 increases. For lower values of Q_1, firm 1 invests as a designated leader and firm 2 as a follower. As the maximum capacity of firm 1 increases, the investment strategy switches to a preemptive strategy where firm 1 invests in the preemptive trigger of firm 2 and firm 2 as a follower. In addition, for higher values of Q_1, firm 2 invests in the preemptive trigger of firm 1 and firm 1 as a follower. In contrast, the optimal investment strategy does not change as the maximum capacity of firm 2 increases. In this scenario, the optimal investment strategy is a preemptive equilibrium where firm 1 invests in the preemptive trigger of firm 2, and firm 2 invests in its follower's trigger.

3.2.2 Impact of the subsidies

Now we analyze the impact of the subsidies on firm 1 and firm 2 investment triggers and the investment equilibria.

Figure 7 presents the optimal investment triggers of firms 1 and 2 without subsidy, with revenue subsidy, and with an investment subsidy as a function of the revenue subsidy η. We can see that the investment triggers of both firms decrease until a certain level

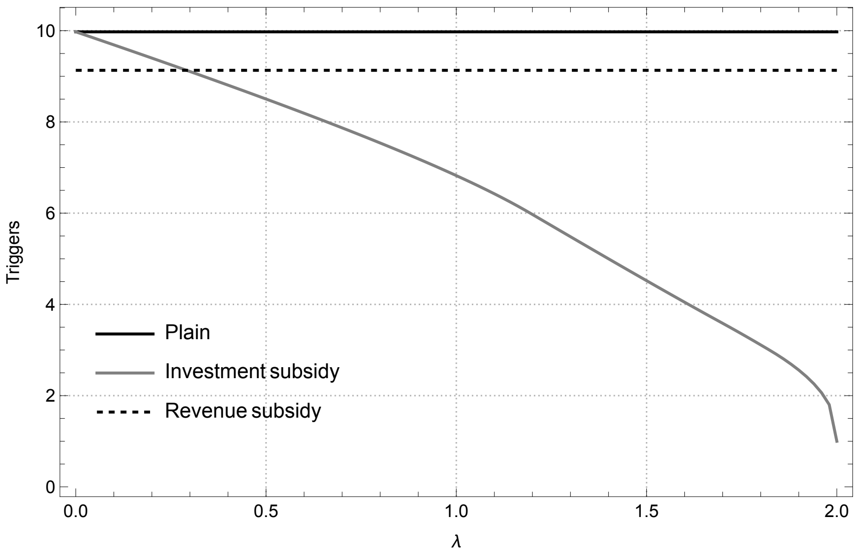
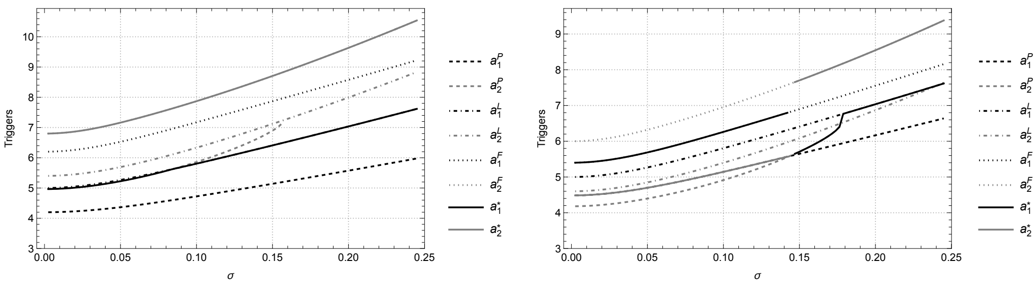


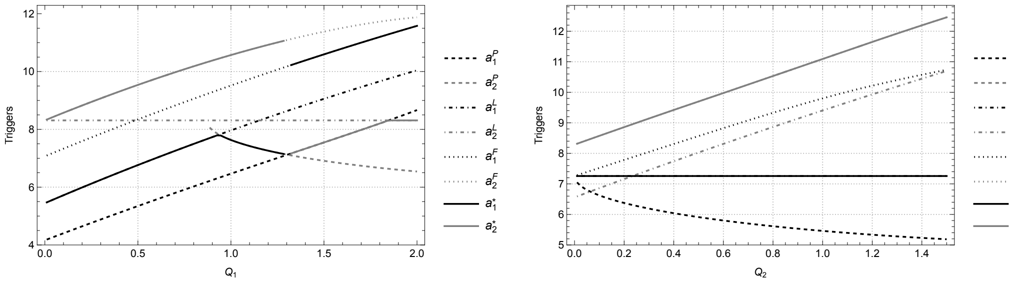
Figure 4: Sensitivity to λ



(a) High capacity ($Q_1 = 0.6$) (b) Low capacity ($Q_1 = 0.2$)

Figure 5: Sensitivity of the triggers to σ

as the revenue subsidy increases. This point represents the optimal investment strategy switching from firm 1 as a leader and firm 2 as a follower to firm 2 as a leader and firm 1 as a follower. If the revenue subsidy is too large, firm 1 waits for firm 2 to invest as follower. Moreover, after that point, firm 2's optimal investment trigger increases with the revenue subsidy because firm 1 prefers always to the the follower. Hence, policymakers may not offer a subsidy greater than this value. Another interesting result from a policymaker's perspective is that the plots show the point where the firms are indifferent between a revenue or an investment subsidy (for $\eta \approx 0.55$). In summary, as the revenue subsidy increases, the investment triggers of firm 1 as a leader and firm 2 as a follower decrease until a certain level. For values greater than this subsidy level, the investment strategy changes to firm 2 as a leader and firm 1 as a follower, and firm 2's investment trigger increases.



(a) Sensitivity of the triggers to Q_1 (b) Sensitivity of the triggers to Q_2

Figure 6: Sensitivity of the triggers to maximum capacity

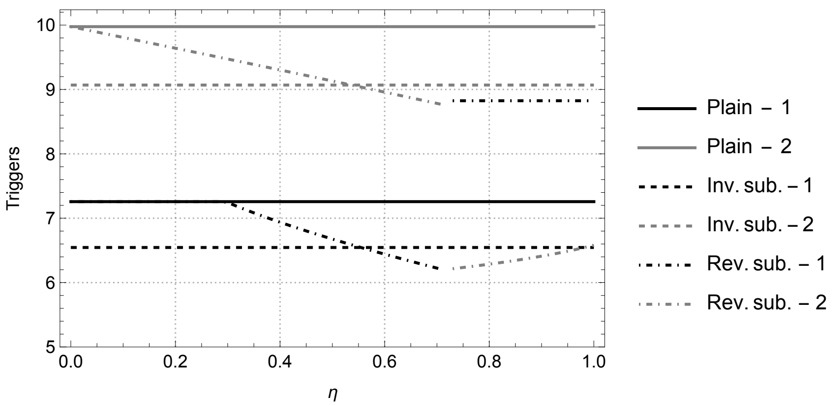


Figure 7: Sensitivity to η

Figure 8 presents the optimal investment triggers of firms 1 and 2 without subsidy, with revenue subsidy, and with investment subsidy as a function of the investment subsidy λ. For lower values of λ (i.e., values lower than $\lambda \approx 0.4$), firm 1 is the leader, firm 2 is the follower, and both investment triggers decrease. For λ above this point, firm 2 becomes the leader and firm 1 turns into the follower, i.e. a too large subsidy deters entry of firm 1. In this region, firm 2's investment trigger first increases and then decreases as a leader. Hence, policymakers may not offer a subsidy when the investment trigger of firm 2 is increasing. In summary, as the investment subsidy increases, the investment triggers of firm 1 as a leader and firm 2 as a follower decrease for lower subsidy values. For higher subsidy values, firm 2's investment trigger first increases and then decreases as a leader.

4 Concluding remarks

This work presents a novel model of a duopoly whereby firms act strategically to decide the optimal timing of an investment project under market uncertainty. The model also adds an incentive to one of the firms in the form of revenue and investment subsidies. In

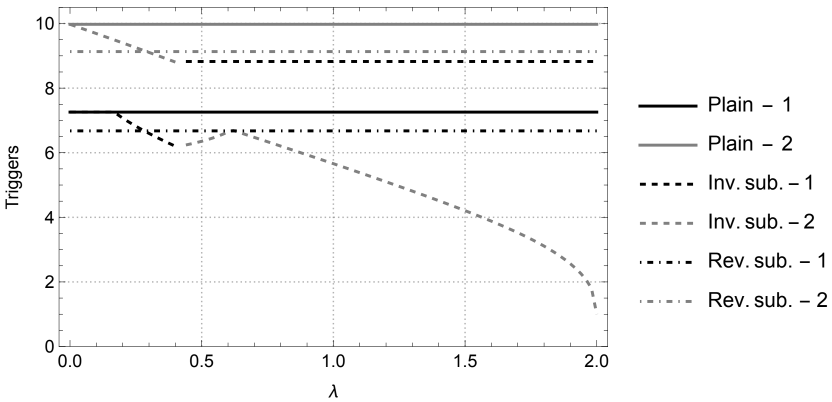


Figure 8: Sensitivity to λ

addition, after investing, the firms must decide the optimal quantities to produce while maximum capacities limit both firms.

We derive the optimal investment thresholds of both firms and present the equilibria of the investment strategies. The derivation also includes investment and revenue subsidies. In particular, we can observe two investment equilibria, namely, a preemptive equilibrium and a sequential equilibrium.We then derive the firms' strategic operational decisions under uncertainty, when both firms are limited by a maximum capacity.

The comparative statics analysis presents two key findings. The first finding is that revenue and investment subsidies accelerate the investment decision when only one firm has the option to invest and the other firm is already in the market. When both firms have the option to invest, the second finding is that more than one equilibrium and investment trigger slope may occur with different investment and revenue subsidy values.

There are a few possible directions for future research. One could include other subsidies, such as fixed-price and collar subsidies, in this analysis. In addition, the subsidy contracts could have a finite duration, and the analysis could include social welfare.

Acknowledgements

This work was supported by national funds through FCT, Fundação para a Ciência e a Tecnologia, under projects UIDB/50021/2020, UID/ECO/03182/2020, and UIDB/00315/2020. In addition, this work was supported by FCT under the HOTSPOT project with reference PTDC/CCI-COM/7203/2020 and the RELEvaNT project with reference PTDC/CCI-COM/5060/2021.

References

Barbosa, Luciana, Artur Rodrigues and Alberto Sardinha (2022), 'Optimal price subsidies under uncertainty', *European Journal of Operational Research* **303**(1), 471–479.

Barbosa, Luciana, Cláudia Nunes, Artur Rodrigues and Alberto Sardinha (2020), 'Feed-in tariff contract schemes and regulatory uncertainty', *European Journal of Operational Research* **287**(1), 331–347.

Barbosa, Luciana, Paulo Ferrão, Artur Rodrigues and Alberto Sardinha (2018), 'Feed-in tariffs with minimum price guarantees and regulatory uncertainty', *Energy Economics* **72**, 517–541.

Bigerna, Simona, Xingang Wen, Verena Hagspiel and Peter M. Kort (2019), 'Green electricity investments: Environmental target and the optimal subsidy', *European Journal of Operational Research* **279**(2), 635–644.

Dixit, Avinash K. and Robert S. Pindyck (1994), *Investment Under Uncertainty*, Princeton University Press, Princeton, New Jersey.

Kong, Jean J. and Yue Kuen Kwok (2007), 'Real options in strategic investment games between two asymmetric firms', *European Journal of Operational Research* **181**(2), 967–985.

Pawlina, Grzegorz and Peter M. Kort (2006), 'Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage?', *Journal of Economics & Management Strategy* **15**(1), 1–35.

Silaghi, Florina and Sudipto Sarkar (2021), 'Agency problems in public-private partnerships investment projects', *European Journal of Operational Research* **290**(3), 1174–1191.

Wang, Wei (Walker), Xi Jin, Zhijia Tan, Huijun Sun and Jianjun Wu (2022), 'Modeling the effects of government subsidy and regulation on bot transport project contract design within contractible service quality', *Transportation Research Part E: Logistics and Transportation Review* **164**, 102820.

Wu, Hanjun, Kan Wai Hong Tsui, Thanh Ngo and Yi-Hsin Lin (2023), 'Airport subsidies impact on wellbeing of smaller regions: A systemic examination in new zealand', *Transport Policy* **130**, 26–36.