The Two Factor Process for Forecasting Carbon Prices

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Abstract

Most carbon capture and storage projects do not pass the economic feasibility studies. Often this is because their benefits are small due to unattractive value of carbon emission mitigation measures. We believe one reason that such measures are currently unattractive is due to a lack of informed understanding of the future of this commodity. To address this issue, we conducted a study to develop forecasts of carbon allowance prices based on the two-factor stochastic model of Schwartz and Smith (2000). We implemented the analytical framework described in Jafarizadeh (2022a, 2022b) using the Distribution of Sum Discounted Prices technique. The analysis led to informed forecasts of pessimistic, expected, and optimistic carbon prices. We further used these forecasts in economic analysis of a carbon storage project.

INTRODUCTION

Commodities such as farm produce (wheat, barley, cocoa, sugar cane, etc.), metals (gold, silver, bronze, copper, etc.), and other natural resources have been traded for goods or services for centuries. Uncertain prices and the need for project valuations have led to the development of a technique for price forecasting. These techniques often forecast future prices based on fundamental of the business—factors such as supply and demand, industry cycles, seasonality, and trends.

The markets instruments provide significant insight into the expected evolution of prices. Almost two decades ago, the European CO_2 allowance market was introduced. Trading of carbon allowances began on platforms such as the EU-ETS, where carbon emitters trade these allowances to avoid paying penalties for excess carbon emissions. Since then, the price of these allowances has fluctuated over time, displaying several characteristics like the physical traded commodities.

Carbon trading, also known as Emission Trading System (ETS) or cap-and-trade system, is part of the Carbon Pricing Scheme that aims to provide a market-based approach to reducing greenhouse gas emissions. These greenhouse gas emissions are believed to be the major contributor to climate change. The goal of carbon trading is to create a financial incentive for companies and countries to reduce their greenhouse gas emissions and transition to cleaner forms of energy. The other measure of emission mitigation is carbon taxation which involves setting a tax on each unit of emitted carbon dioxide. The schemes aim to reduce greenhouse gas emissions by providing financial incentives for companies to adopt cleaner forms of energy and reduce their carbon footprint.

Common carbon trading systems include the European Union Emissions Trading System (EU ETS), which was established in 2005 and covers approximately 11,000 power stations and industrial plants across the European Union. The United Kingdom exiting the EU led to the creation of a new trading scheme called the UK-ETS. In other parts of the world such as the United States, different forms of carbon trading schemes exist at state and regional levels, examples are the Regional Greenhouse Gas Initiative (RGGI) and the California Carbon Trading Market.

In a carbon trading system, the total amount of carbon to be emitted is capped by the regulating body responsible for greenhouse gas emissions reduction. Companies that emit more carbon than their allotted allowance must acquire additional allowances to compensate for their excess emissions or face penalties. While those with fewer emissions have the choice to bank their excess allowances or sell them on a Carbon trading platform. Carbon allowances are issued by regulating bodies to companies either for free or sold through auctions annually. The dynamic of the price of these allowances is affected by different factors, among which are the market supply and demand, allowances allocated by the regulating body, the quantity demanded, regulatory policies such as banking and the overall level of greenhouse gas emissions.

To support valuations and decision making, in this paper we aim to develop forecasts for carbon allowance prices. We generate low (pessimistic), expected, and high (optimistic) forecasts using the characteristic of a price model calibrated with market information.

Commodity prices are known to fluctuate over time, and these fluctuations often exhibit various patterns. Most price fluctuations have a random and probabilistic element while general trends point to mean-reversion as a result of supply and demand forces. Several models have been proposed to reflect mean reversion, random walk, and Geometric Brownian Motion (GBM), either individually or in combination, to forecast future prices. For example, Pindyck, (1999), Schwartz & Smith (2000), and Geman (2007) discuss mean reversion towards an evolving and variable equilibrium level. A two-factor model reflects this combination of mean-reverting short term moves along with random walk moves of the long-term equilibrium.

All models are wrong, but some are useful.[1] The trade-off between a price model's simplicity and verisimilitude should be based on its usefulness. In this paper, we use the two-factor price model of Schwartz and Smith (2000) as we believe it is simple enough to be understood and used and sophisticated enough to reflect the key features of the carbon prices. Some examples earlier implementations of this model include Jafarizadeh & Bratvold (2012), Bakker et al. (2021), and Jafarizadeh (2022a, 2022b).

This study aims to forecast the future spot prices of EU-ETS carbon allowances using the Short-Term-Long-Term (STLT) Two-factor price model, which has not been previously utilized for this purpose. To accomplish this, we first estimate the parameters for price forecasting using the SLTLT model. Our approach involves utilizing futures and options of EU-ETS carbon allowance obtained from the Inter-Continental Exchange (ICE) and conducting a curve-fitting analysis to calibrate the initial guessed parameters to fit the futures prices and implied volatilities.

Once we calibrate the model, we simulate several price paths and generate a distribution for sum of discounted price paths. We use this distribution to establish confidence band for forecasted prices. We show the forecast in the context of economic valuation of a project involving the use of carbon. The forecasted EU-ETS carbon allowances serve as a proxy price of Carbon in our analysis. Integrating carbon price into the economic valuation of a CO2-emitting company aids management in making informed decisions on project executions and investments.

This paper is structured into five sections. Section 2 covers price modelling and parameter estimations, while Section 3 discusses the informed sensitivity analysis on the forecasted price. Section 4 presents the integrated valuation, and Section 5 concludes the study.

2. PRICE MODELING

2.1 The TWO-FACTOR PRICE MODEL

Schwartz & Smith, (2000) proposed a model for describing the evolution of commodity prices. The model considers both short and long-term factors. The short-term factor χ_t is represented by Ornstein-Uhlenbeck process and the long-term factor by ξ_t, a Brownian motion. Equation 1 shows that the spot price S_t at time t is as the sum of these two factors.

[1] This phrase is attributed to George Box.

$$\ln(S_t) = \chi_t + \xi_t \tag{1}$$

The short-term factor, χ_t tends to revert to a the equilibrium, as shown in equation 2.

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \tag{2}$$

On the other hand, the long-term factor, ξ_t, follows Brownian motion[2], and has random moves, as shown in equation 3.

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \tag{3}$$

The mean reversion coefficient for the short-term factor is denoted by κ, while μ_ξ is the drift for the long-term factor. The standard deviations for the short and long-term factors are σ_χ and σ_ξ, respectively, and dz_χ and dz_ξ are correlated increments of the standard Brownian motion, with $dz_\chi dz_\xi = \rho_{\chi\xi} dt$.

The log of future spot prices yields a normal distribution with an expectation and variance, which can be expressed using equations 4 and 5, respectively.

$$E(lnS_t) = e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t \tag{4}$$

$$Var(lnS_t) = (1 - e^{-\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t = 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \tag{5}$$

Equation 6 shows the re-written form of the expectation and variance using the Ito lemma principle, $\ln\ E(S_t) = E(\ln S_t) + \frac{1}{2}Var(lnS_t)$

$$\begin{aligned}\ln E(S_t) &= e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t \\ &+ \frac{1}{2}((1 - e^{-\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa})\end{aligned} \tag{6}$$

For the risk-neutral prices, the short and long-term risk premiums represented by λ_χ and λ_ξ can be deducted from the expectation. Hence, the expected future spot price of Carbon allowance equals the futures prices at the same delivery. Thus, the present-day value of a futures contract $F_{0,T}$ for delivery at maturity time T can be expressed using equation 7;

$$\begin{aligned}\ln F_{0,T} &= e^{-\kappa T}\chi_0 + \xi_0 + (\mu_\xi - \lambda_\xi)T - (1 - e^{-\kappa T})\frac{\lambda_\chi}{\kappa} \\ &+ \frac{1}{2}((1 - e^{-\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1 - e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa})\end{aligned} \tag{7}$$

Equation 8 shows the instantaneous variance that represents the volatility of the futures prices of carbon, independent of risk premiums. The instantaneous variance is a function of the short and long-term factors' standard deviations and correlation, as well as the mean reversion coefficient.

$$Var\ (\ln F_{0,T}) = e^{-2\kappa T}\sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T}\rho_{\chi\xi}\sigma_\chi\sigma_\xi \tag{8}$$

[2] Brownian motion, also known as Wiener process, is a stochastic process that describes the random movement of particles in a fluid or gas due to continuous collision with molecules in the medium. It is named after the botanist Robert Brown who observed the random motion of pollen particles suspended in water. The mathematical formulation of Brownian motion is described by a continuous-time stochastic process that has independent and identically distributed Gaussian increments with zero mean and variance proportional to the time interval. The path of a Brownian motion is continuous but nowhere differentiable, and it has several important properties such as scaling invariance, self-similarity, and the Markov property. Brownian motion has applications in various fields such as physics, finance, biology, and engineering

In summary, the Schwartz and Smith model provides a framework for understanding the factors that drive the spot price of carbon allowance. By considering both short and long-term factors, the model can provide insights into the behavior of the carbon market and can help forecast future prices.

2.2 TWO-FACTOR PARAMETER ESTIMATION FOR CARBON

To forecast carbon prices, it is necessary to estimate various parameters that govern the behavior of the carbon market. In this study, we use market information on carbon, such as the EUA futures and Call and Put options on EUA futures, to estimate these parameters. The data used in this study were obtained from the Intercontinental Exchange (ICE) on the 8th of August 2022. To estimate the parameters, we use a curve fitting[3] technique that iteratively estimates the parameters from the futures prices and the implied volatility of the options. We start by developing a forward curve using equation 7, and then assign initial guess values to each parameter to predict the observed futures prices at different maturities.

Next, we compare the market-implied volatility values with the volatility term curves constructed using equation 8. Unlike the futures, the implied volatilities are not observed directly in the market but can be obtained using either the Black & Scholes (1973) model or the Schwartz & Smith (2000) approach from European options on carbon futures. Here, we use the Schwartz-Smith approach, which suggests that the value for a given European Call and Put options, denoted as c_T and p_T respectively, can be evaluated using equations 9 and 10.

$$c_T = e^{-rT}\left(F_{0,T}N(d) - KN\left(d - \sigma_\varphi(T)\right)\right) \qquad 9$$

$$p_T = e^{-rT}\left(KN\big(\sigma_\varphi(T) - d\big) - F_{0,T}N(d)\right) \qquad 10$$

Where r is the risk-free rate, $N(d)$ is the cumulative probability for the standard normal distribution, K is the strike price, T is the delivery date (in the analysis, we are assuming that the underlying futures and the option expire at the same time) and $\sigma_\varphi(T)$ is the volatility of the futures contract. In Addition to that, d is a function of the futures, strike price and volatility which is expressed as

$$d = \frac{ln(F/K)}{\sigma_\varphi(T)} + \frac{\sigma_\varphi(T)}{2} \qquad 11$$

With the aid of a simple computer code, a loop iteration was designed to optimize the parameters such that both the estimated futures and option values obtained using equations 7 and 8, and the market futures value and the implied volatility values matched with a low sum of square errors. Plots showing a good fit between both estimated and market data are presented in Figures 1 and 2, while the estimated parameter values are shown in Table 2.

[3] Curve fitting is a statistical method used to find the best fit line or curve to a set of data points. It involves selecting a mathematical function that best represents the data and finding the parameters of the function that best describe the relationship between the variables. The curve or line that is fitted to the data can be used to make predictions or to estimate unknown values. The choice of the technique depends on the nature of the data and the relationship between the variables. Iterative curve fitting is a common approach where the parameters of the function are estimated using an optimization algorithm that minimizes the difference between the observed data and the fitted curve.

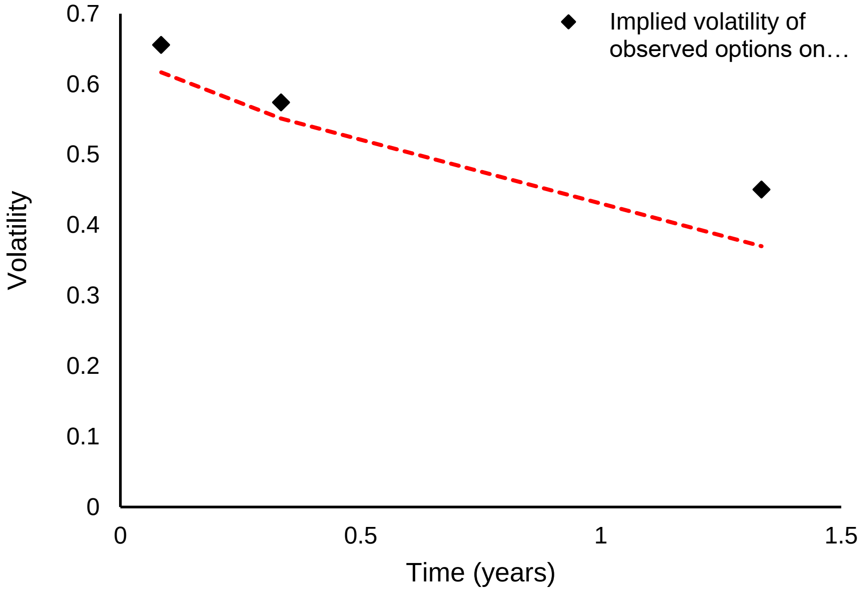

Figure 1: Curve Fitting on Market-Observed Options on Carbon Futures

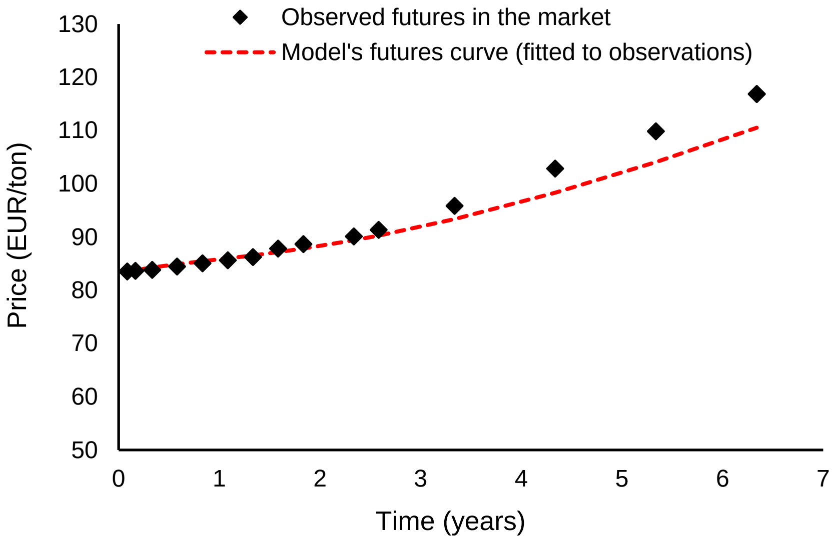

Figure 2: Curve Fitting on Market-Observed Carbon Futures

The carbon trading market is a relatively new market, and as a result, there is still a lack of comprehensive data on carbon futures allowances. Thus, we were only able to obtain a limited number of options on carbon futures from the Intercontinental Exchange (ICE) website (ICE, 2022), which resulted in an analysis period of about a year and a half. Despite this limitation, we were able to obtain a good fit for both the futures prices and the implied volatility of the options, which gives us confidence in the reliability of the parameters estimated using our curve fitting technique.

With the estimated parameters in hand, we can now move forward in developing a carbon price forecast. The estimation of these parameters allows us to better understand the behaviour of the carbon market and its underlying factors, including supply and demand, regulatory policies, and technological advancements. By analysing the trends and patterns in the market and incorporating these factors, we can develop a model that will predict future carbon prices with a reasonable level of accuracy.

The carbon price forecast can be a valuable tool for various stakeholders, including governments, corporations, and investors, as it provides them with insights into the potential future costs of carbon emissions. Governments can use this information to design and implement effective carbon pricing policies that align with their climate goals. Corporations can use the forecast to plan their investments and operations, and investors can use it to inform their investment decisions and manage their risks.

Overall, the estimation of the parameters from market information on carbon using a curve fitting technique has proven to be a reliable approach for forecasting carbon prices. With this method, we can derive meaningful insights into the carbon market and its drivers, which can help us to mitigate climate change and build a more sustainable future.

Table 2: Two-factor Carbon price parameters

Parameters	**Values**
χ_0	0.342
ξ_0	4.079
μ	0.051
σ_χ	0.450
κ	0.612
σ_ξ	0.162
$\rho_{\chi\xi}$	0.930

2.3 THE TWO-FACTOR CARBON PRICE FORECAST

The carbon spot prices forecasted in this study are based on the estimated parameters obtained from the options and futures price data, along with the curve fitting technique employed. It is important to note that the accuracy of these forecasts is dependent on the reliability of the estimated parameters. The carbon market is highly complex, and various factors influence its price, including changes in regulations, technological advancements, and geopolitical events. Therefore, any forecast is inherently uncertain, and the actual future carbon spot prices may differ from the predictions presented in this study.

To simplify the analysis and reduce the computational power required, the assumption was made that the price of carbon only changes once a month. With this assumption, a monthly value is used to represent Δt when necessary. Utilizing equation 6 and the estimated parameters shown in Table 2, the expected future carbon spot prices were derived. These prices are represented in Figure 3, along with their corresponding 90% confidence band.

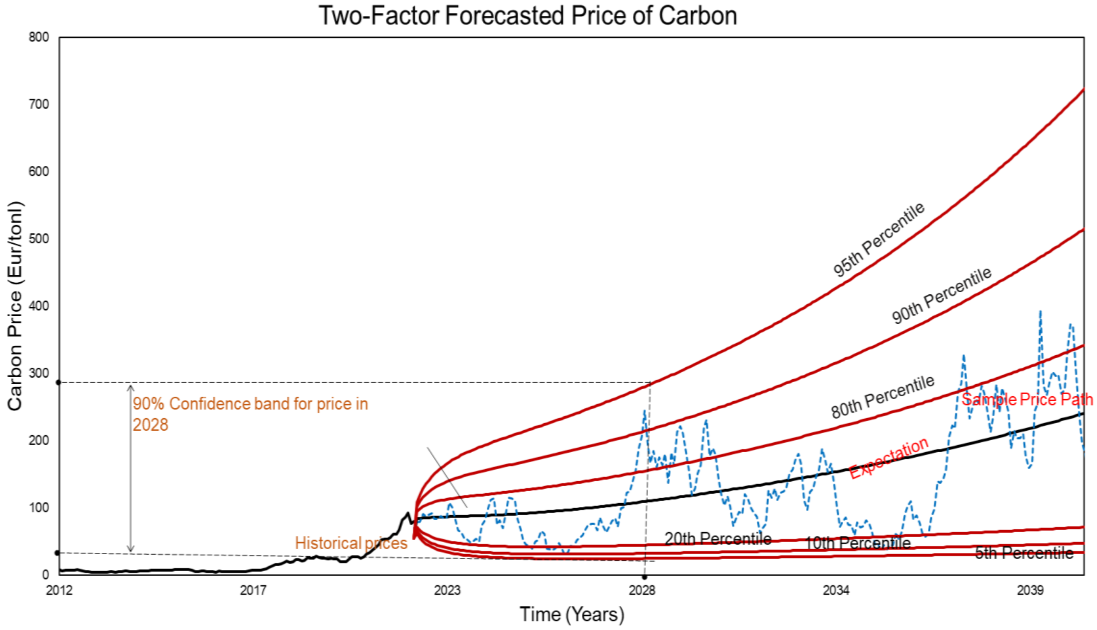

Figure 3: Two-factor Carbon price model indicating different percentage confidence bands

Figure 3 provides compelling evidence that both the simulated and expected future spot prices of carbon are following a contango curve throughout the forecast years. This suggests that the prices of a unit of carbon will likely continue to rise. This trend is supported by several global movements aimed at reducing carbon emissions, such as the consistent review of allowances issued to emitters, the establishment of deadlines for the importation and manufacture of hydrocarbon cars by European countries, and countries proposing to increase their carbon tax, are just a few examples of events that could lead to a surge in the demand for carbon allowances in the future. It is conventionally understood that an increase in the demand for a commodity leads to undersupply and consequently a rise in price.

From the historical spot price point of view, it can be observed that the price of carbon has risen by about 400% within a span of two years, between 2020 to 2022. These and many other indicators strongly suggest that the era of lower carbon prices is long gone. For instance, the confidence bands in Figure 3 show that the price of carbon emitted in 2028 could be anywhere between $40 to $290, with only about a 5% chance both ways of the price going beyond or less than the prices indicated within the confidence band. It is important to note that the actual price of carbon could be influenced by various unpredictable factors, but the upward trend of carbon prices appears to be consistent with the global push towards carbon neutrality and decarbonization.

3. INFORMED SENSITIVITY ANALYSIS

Informed sensitivity analysis is a type of sensitivity analysis that involves adjusting input parameters or assumptions in a model to assess their impact on the output of the model. Unlike traditional sensitivity analysis, informed sensitivity analysis considers the knowledge and expertise of the modelers, as well as available data and information on the inputs being tested. In other words, informed sensitivity analysis aims to identify the most important and influential factors in a model by testing a range of plausible scenarios that reflect real-world conditions and expert judgment. This helps to provide a more robust understanding of the model and its outputs, as well as identify areas where additional research or data may be needed.

Sensitivity analysis is a critical tool used in various fields to evaluate how changes in input variables can impact output or outcome. These enable one to see and quantify how a deviation from certain variables could affect the entire process[4]. In this study, we employ the use of sensitivity analysis to assess the impact of price changes on the outcome of a carbon emission project. Using a range of prices from pessimistic to optimistic, we evaluate how uncertainties in price could significantly affect project outcomes. Given that most carbon emission mitigation projects are capital-intensive investments, it is paramount to conduct a comprehensive sensitivity analysis before making any decision. Sensitive input variables such as the cost of CO_2, incentives (where applicable), and capital and operating expenditures need to be rigorously analyzed to check the impact of each on the outcome.

In a traditional sensitivity analysis such as the tornado diagram, input variables are often evaluated based on extreme values, without considering the shape of their distribution. For example, the P10 and P90 values used to quantify geological reservoirs do not reflect the overall distribution of oil in place, which could take any form. Consider two distinct input variables to be used in a sensitivity analysis, one having a normal while the other a log-normal distribution. If the range of the two extremes happens to be the same, these two distinct variables will be considered the same in such evaluations., but in reality, these extremes are differently distributed and hence should be considered differently. When deciding on a range of values to be used in sensitivity analysis, the pessimistic and optimistic values should not be mere minimum and maximum values rather they should be a meaningful range of values that truly represent the input variable[5].

The limitations of traditional sensitivity analysis, such as the inability to test more than one variable at a time, have led some researchers to question the efficacy of tools like the tornado diagram. In contrast, the approach used in this study is more robust and superior to traditional techniques. By taking into account the shape of the distribution of input variables and using a more meaningful range of values, this method provides a more accurate and useful assessment of the impact of uncertain carbon prices on a carbon emission mitigation project. In short, this approach goes beyond the usual sensitivity analysis and represents an improvement in the field.

3.1 Sum-Discounted Prices

The uncertainty of future spot prices is described by stochastic models, which provide both an expected value and a probability distribution for future spot prices. However, traditional sensitivity analyses do not align with such descriptions. Instead, they typically define a range of price forecasts from the most optimistic to the most pessimistic, which are entirely disconnected from the stochastic models. As a result, these analyses cannot indicate whether the value of a project at the optimistic forecast is the maximum possible project value, the highest likely value, or even a rational value at all. To address this limitation, this section discusses a method for generating optimistic and pessimistic price forecasts that are consistent with stochastic price models

In order to accurately reflect the uncertainty of future spot prices, it is important to have a range of price forecasts that are consistent with the stochastic properties of spot prices, at the same time be useful in cash flow models. To address this, we utilize the distribution of sum of discounted prices as a means of measuring the impact of prices on project value. This approach, which was introduced by Dixit in 1993, allows for the estimation of price forecasts that are consistent with the stochastic process. By using the expected forecast, referred to $S_t^*, 0 < t < T$ as, we can generate a price scenario that accurately reflects the uncertainty of future spot prices.

$$\int_0^T S_t^* e^{-rt} dt = E\left(\int_0^T S_t e^{-rt} dt\right) \qquad 12$$

Here, T represents the forecast time, r the discount rate and e^{-rt} the discounting factor.

[4] Sensitivity analysis can also be used to check the robustness of models.

[5] Historically, the lowest price of oil ever was -$5 (negative five dollar) and the highest is in the axis of $150, this doesn't mean that when deciding for a range of oil price the pessimistic and optimistic values should be -$5 and $150.

In most cases, the overall impact of a series of prices over time as well as the decision maker's preferred time frame are captured by the sum of discounted prices. Since there is no single price for a lengthy project, decision makers instead adopt a series of prices over time. To compare price series, it is simpler to consider the summation and/or averages of the prices in the series. The discount factor can be used to show the significance of the prices on project years that are sooner versus later. Therefore, sum-discounted prices may be an effective way to quantify the effect of a series of prices on project value.

To evaluate both the expected price and any *n*th percentile of the forecast of prices, we implement the use of numerical approximation on equation 6. With then simulate the stochastic spot prices within the forecast limit and determine the distribution values of the sum of the discounted prices. Hence, with the aid of an optimization model and solver (MS Excel add-in), we developed a forecast that is a replica of the distribution of the sum of discounted prices.

3.2 NUMERICAL PROCESS

To simulate prices, we discretized equations (2) and (3) to calculate the log of spot price which can be re-written as;

$$\ln S_{t+\Delta t} = \xi_{t+\Delta t} + \chi_{t+\Delta t} \tag{13}$$

$$\xi_{t+\Delta t} = \xi_t + \mu_\xi \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t} \tag{14}$$

$$\chi_{t+\Delta t} = e^{-\kappa \Delta t}\chi_t - \left(1 - e^{-\kappa\Delta t}\right)\frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}} \tag{15}$$

Here all variables have their earlier defined meaning, in addition to Δt representing the change in time. Also, assuming that ε_ξ is a standard normal distribution, and ε_χ a function of ε_ξ and ε (an independent normal distribution), a correlation between the simulated factors can be accounted for using;

$$\varepsilon_\chi = \varepsilon_\xi \rho_{\chi\xi} + \varepsilon\sqrt{1 - \rho_{\chi\xi}^2} \tag{16}$$

For a forecast period $0 < t < T$, we employ the use of equations 13 through 16 to numerically simulate the price path for the two-factor forecast. In addition to that, we implement the Riemann approximation on equation 12 to solve for the sum of the discounted prices as shown in equation 17.

$$\int_0^T S_t e^{-rt} dt \cong \sum_{t=0}^{T} S_t e^{-rt} \Delta t \tag{17}$$

While simulating the prices paths, we use a monthly time steps (an average tonnage price for a month) for the 20 years forecast horizon and calculate the sum-discounted values. A histogram of the distribution of sum-discounted prices (DSDP) of carbon is shown in Figure 4. The figure also indicates the 10th, expected and the 90th percentile of the distribution.

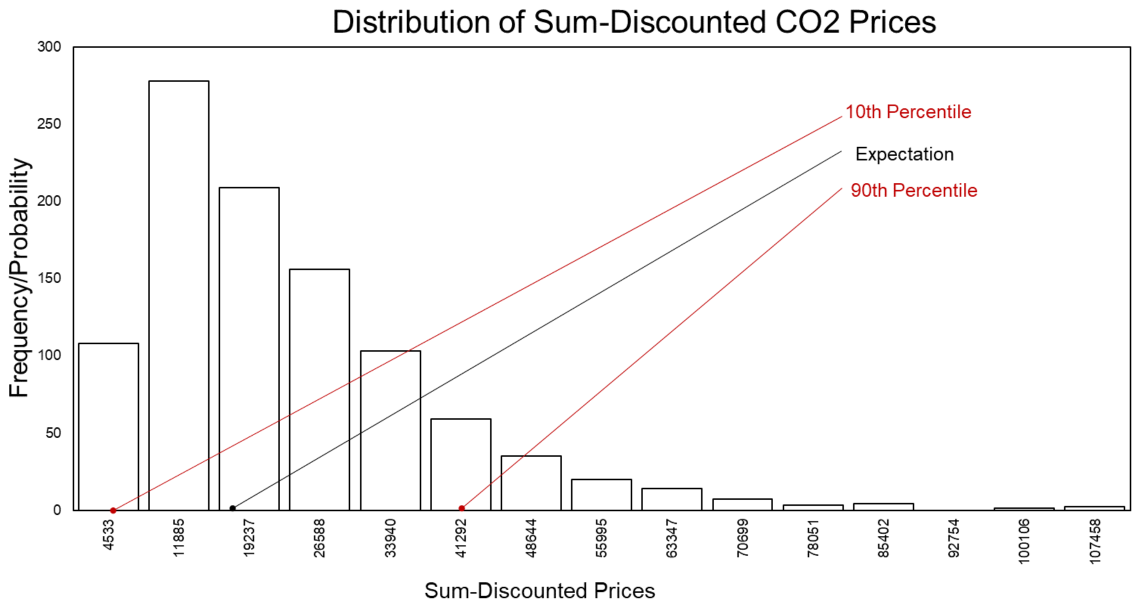

Figure 4: Distribution of Sum-discounted CO_2 Prices for 1,000 sSmulated Price Paths

3.2 OPTIMIZATION

Now that we have obtained a distribution for sum-discounted prices for the two-factor price forecasts of carbon, we assign the 90^{th} and 10^{th} percentile of the distribution to represent the high and low cases. Subsequently, we went ahead to simulate a higher and lower cost price forecast to achieve the same sum-discounted values as the high and low cases. The optimization equations used to calculate the forecast of prices S_t^* matching the given percentile of the discounted prices are shown below.

$$\min_{\lambda_\xi} \left(\sum_{t=0}^{T} S_t^* e^{-rt} - X \right)^2$$

s.t

$$P\left(\sum_{t=0}^{T} S_t^* e^{-rt} < X \right) = \frac{p}{100}$$

Here, t represents the discretized time steps from 0, $\Delta t, \dots . . T$, whereas X is an internal variable defined to calculate the percentiles.

We employed Microsoft Excel solver to determine the short-term premium λ_χ in equation 7, which optimizes the sum-discounted simulated prices of the expected, and the 90th and 10th percentiles to match the sum-discounted monthly prices generated earlier. The results are displayed in Figure 5, which shows the 90th, expected, and 10th percentile prices derived from the distribution of the sum-discounted technique.

It's worth noting that the forecast obtained from the distribution of sum-discounted technique is distinct from the conventional percentiles that are obtained usually by multiplying the expected price with a certain positive and negative fraction. Instead, the forecast obtained from DSDP is a representation of all the thousand sample price paths simulated instead of just one. Furthermore, the optimized sum-discounted simulated prices allow for a more accurate representation of the possible price scenarios.

This means that decision-makers can have a better understanding of the range of potential outcomes and the associated risks.

In summary, by employing numerical approximations, simulation, and optimization models, we were able to obtain an accurate representation of the expected price and percentile forecasts for carbon prices. The DSDP method provides an alternative approach that incorporates all the possible outcomes into the forecast, thus making it a more realistic representation of future price scenarios.

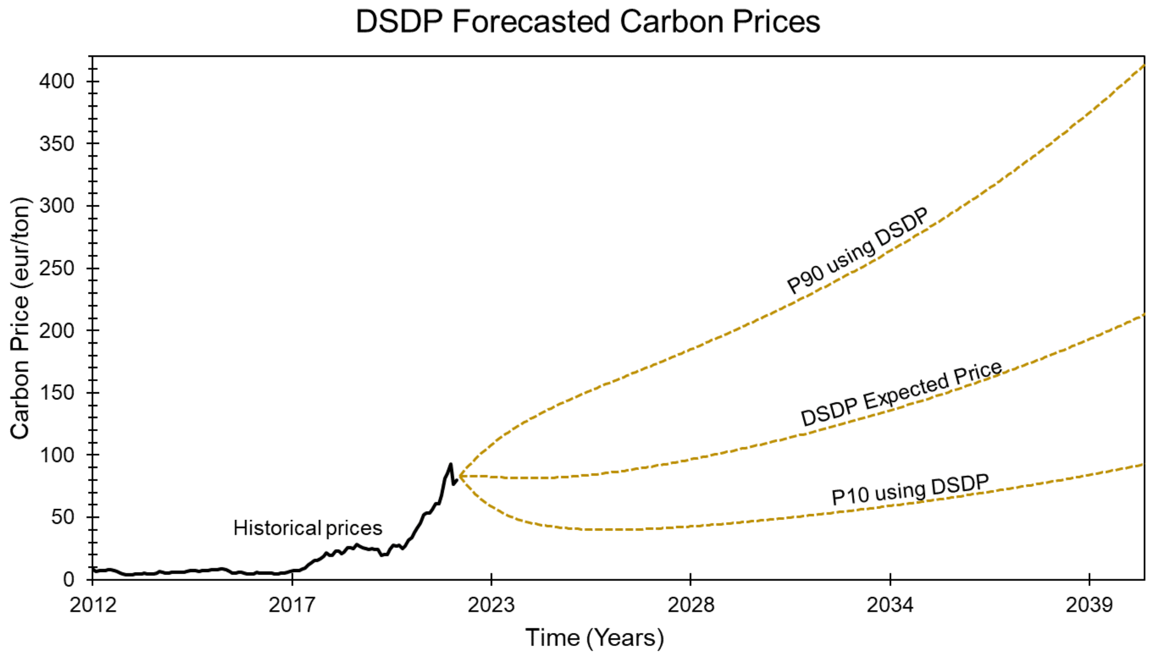

Figure 5: Expected and pessimistic price forecasts of Carbon

To better understand the distinction between the two forecasted prices - the conventional forecast in Figure 3 and the DSDP forecast in Figure 5- we conducted a detailed comparison. This comparison is discussed in the conventional vs DSDP forecast section.

4. CONVENTIONAL Vs DSDP FORECAST

Figure 6 is a comparison between two different techniques used for forecasting prices. One technique is the conventional method, while the other technique is DSDP (Distribution of Sum-Discounted Prices). The figure also shows the confidence bands of prices for both techniques. The comparison between the conventional and DSDP techniques in forecasting the expected and percentiles of prices for carbon has revealed a clear distinction between the two methods. The results indicate that the conventional method has overestimated the 90th and expected price values while underestimating the P10 values when compared to the DSDP method.

The conventional method relies on the multiplication of the expected price by a certain positive and negative fraction to obtain the high and low cases, respectively. This approach ignores the potential variability and uncertainty of the underlying factors affecting the prices, resulting in an overestimation of the expected and 90th percentile prices. Conversely, the P10 value is often underestimated since the conventional method assumes that the distribution of prices follows a symmetrical pattern, which is not always the case in reality.

The DSDP technique overcomes the limitations of the conventional method by using a distribution of simulated prices to calculate the expected and percentile prices. The distribution accounts for the variability and uncertainty of the underlying factors affecting the prices, resulting in a more accurate representation of the prices. The DSDP approach is particularly useful in highly volatile and uncertain markets, such as the carbon market. The significance of these findings extends beyond the carbon market as it highlights the limitations of the conventional approach in accurately predicting prices in uncertain markets. The DSDP approach can be applied to other markets such as energy, commodities, and financial markets, where volatility and uncertainty are prevalent.

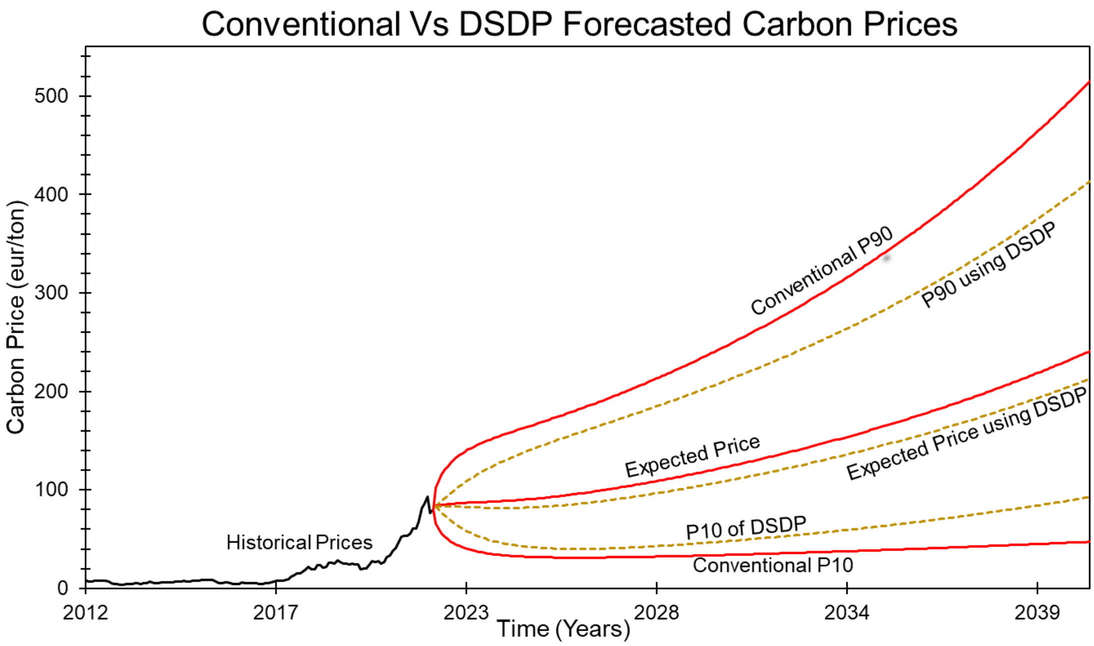

Figure 6: Comparison between Percentile of Expected prices and that of Distribution of Sum-Discounted Prices

The 90th and 10th percentile values obtained from the DSDP forecast are more reliable, as they are based on the actual distribution of simulated prices, rather than being derived from a simple multiplication of the expected price. Overall, our comparison showed that the DSDP forecast provides a more comprehensive and reliable representation of the range of possible outcomes and is therefore a more suitable approach for forecasting future prices. Hence, in our next section - the integrated economic valuation- we will be utilizing the DSDP forecasted values of P90, expected, and P10 to evaluate a project that involves the use of CO_2 to enhance hydrocarbon production from a depleting reservoir.

5.0 INTEGRATED ECONOMIC VALUATION

In this section, we demonstrate how to use the forecasted carbon price to construct a reliable cash flow. However, it is important to note that conducting a detailed and complex analysis may require significant computational time and power, at the same time analyzing the significance of each input variable on the project's profitability is paramount. Researchers must reflect on the implications of conducting an overly complex analysis. Before making an analysis too cumbersome, it is always a good practice to compare the distinctions between the results of the complex and simplified analysis. If the results show little to no substantial variations between the two, it is preferred to keep the analysis simple, informed, and useful.

To achieve the desired goal of having a realistic cash flow analysis while considering the constraints mentioned above, we run a simple sensitivity analysis to assess the impact of the input variables on the result. Our finding suggests that the production rates of Oil, Water and CO_2 as well as, oil price, CO_2 cost, and CAPEX are crucial to the outcome of the cash flow analysis. For other input variables that are less sensitive, we use fixed rates. We also use yearly oil and CO_2 rate obtained by averaging the monthly prices. This is on the assumption that the prices of oil and carbon remains constant throughout the year. By employing this simplified approach, we construct a cash flow that provides a reliable representation of the project's profitability. However, it is important to note that even though this approach may not capture all the complexities and risks associated with the project, the findings are dependable when making decision.

Example:

This example presents a case study on the valuation of an oil-producing field owned by Alpha Company Limited. The field is expected to have a lifespan of 20 years, as estimated by the Reservoir Engineering department using simulation techniques. Production data for oil, water, and CO_2 are illustrated in Figure 7. Alpha must decide whether to invest in a Carbon Capture and Storage (CCS) project or pay a fine for emitting CO_2, given that the produced oil contains a high percentage of CO_2. Investing in CCS will require an additional expenditure of $120 million on a carbon capture facility, CO_2 injector well drilling and completion, compressor pump, and other equipment. Alternatively, if Alpha decides to continue with business as usual and pay the emission fine, the forecasted Carbon price will be used to evaluate the emission cost.

Information obtained from nearby fields suggests that the Unit operating cost of the region is approximately $13 per barrel of oil produced without CCS, and $18 per barrel oil produced with carbon storage in a nearby aquifer. In addition, Alpha must pay a 30% royalty and a 40% corporation tax to the government. To discount the cash flow, Alpha uses a risk-free rate of 4% since both CO_2 cost and oil prices were obtained from a forecast, and market price uncertainties are hedged. The prices of oil used in this analysis (shown in Table A1 of the appendix) were obtained from (Jafarizadeh, 2022b), with minor adjustments made to extend the forecast duration.

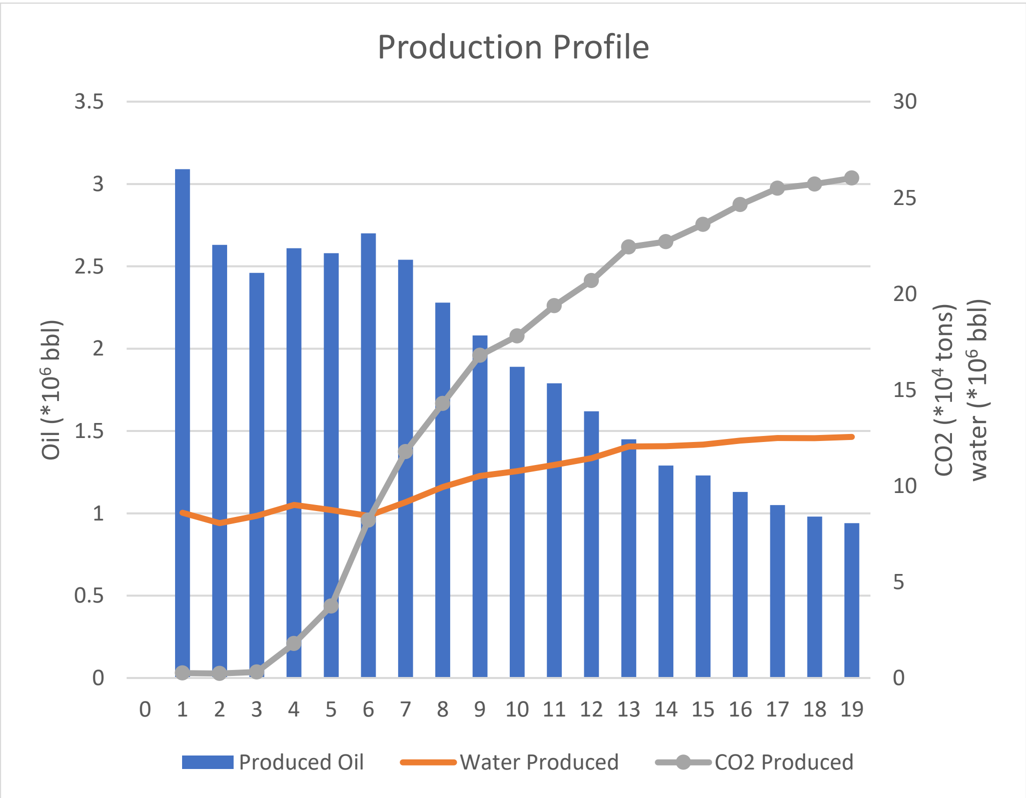

Figure 7: Reservoir Simulation Showing Oil, Water and CO2 production data

The primary variables used for the cash flow analysis can be found in the Appendix. These include the annual rates of oil, water, and CO2, the selling price of a barrel of oil, and the cost of a ton of CO2. Since there are three forecasted oil prices and three carbon costs, the cash flow analysis will comprise of more than a single Net Present Values (NPVs). First, If Alpha decides not to invest in CCS, there will be nine different NPVs based on price and cost combinations. Alternatively, if Alpha decides to invest in CCS, there will only be three outcomes purely depending on the range of oil prices. Table 3 below shows the NPV outcomes of the scenario where Alpha decides not to invest in CCS. It should be noted that none of the outcomes indicate a loss (negative NPV); however, the best outcome is approximately two orders of magnitude better than the least case. This is because the best outcome was obtained using an optimistic oil price and a lower cost of carbon, while the least case resulted from a pessimistic oil price and a higher cost of carbon.

Table 3: Different NPVs for Project without CCS having different Oil and CO_2 Prices

	High CO2 Cost forecast	***Expected CO_2 Cost forecast***	***Low CO_2 Cost forecast***
Optimistic Oil forecast	$690.00 x$10^6$	$862.70 x$10^6$	$953.16 x$10^6$
Expected Oil forecast	$298.60 x$10^6$	$497.60 x$10^6$	$593.10 x$10^6$
Pessimistic Oil forecast	$-75.5 x$10^6$	$148.60 x$10^6$	$266.4 x$10^6$

In the scenario where Alpha invests in CCS, the cost of producing a barrel of oil is assumed to be $8 to account for the additional expenses related to CO2 storage. The results of the cash flow analysis for this scenario are shown in Table 4 below

Table 4: Different NPVs for Project involving CCS having different oil prices

	NPV (Million $)
Optimistic Oil Price	*$860.51 x$10^6$*
Expected Oil Price	*$500.44 x$10^6$*
Pessimistic Oil Price	*$174.42 x$10^6$*

To effectively compare the two scenarios where Alpha invests in CCS or not, it is important to estimate the Expected Monetary Value (EMV) for each decision outcome. The EMV is a valuable tool used in decision-making, particularly when assessing multiple decision outcomes, by assigning probability values to each potential outcome. To illustrate this, a decision tree can be used to visually represent the potential outcomes and their associated probabilities. The outcome with the highest EMV is typically the preferred option. It is worth noting that if the EMV is negative, it is usually recommended to avoid that decision, , but if other desired non-monetary values are to derived (e.g valuable information) from executing the project, it may be worthwhile even if the EMV is negative.

In our analysis, we use Swanson's Rule[6] to assign probability values to the oil prices and the cost of Carbon. This rule assumes a 30% chance of the best-case scenario, a 40% chance of the most likely outcome, and a 30% chance of the worst-case scenario. Figure 8 below presents a decision tree that

[6] Swanson's rule is a commonly used method to assign probability values in decision-making situations where there is a lack of historical data or other sources of information. The rule suggests assigning a probability of 30% to the worst-case scenario, 40% to the most likely scenario, and 30% to the best-case scenario. This method is often used in risk management and decision analysis to estimate probabilities when there is uncertainty about future events. However, it is important to note that Swanson's rule is a heuristic and should not be used as a substitute for more rigorous statistical methods when such methods are available and appropriate.

combines the results from Tables 3 and 4. It illustrates the potential outcomes of both scenarios along with their associated probabilities. This allows us to determine the EMV before making decision. By considering the decision tree and the EMV, Alpha can make an informed decision on whether to invest in CCS or not based on the potential outcomes and their likelihoods.

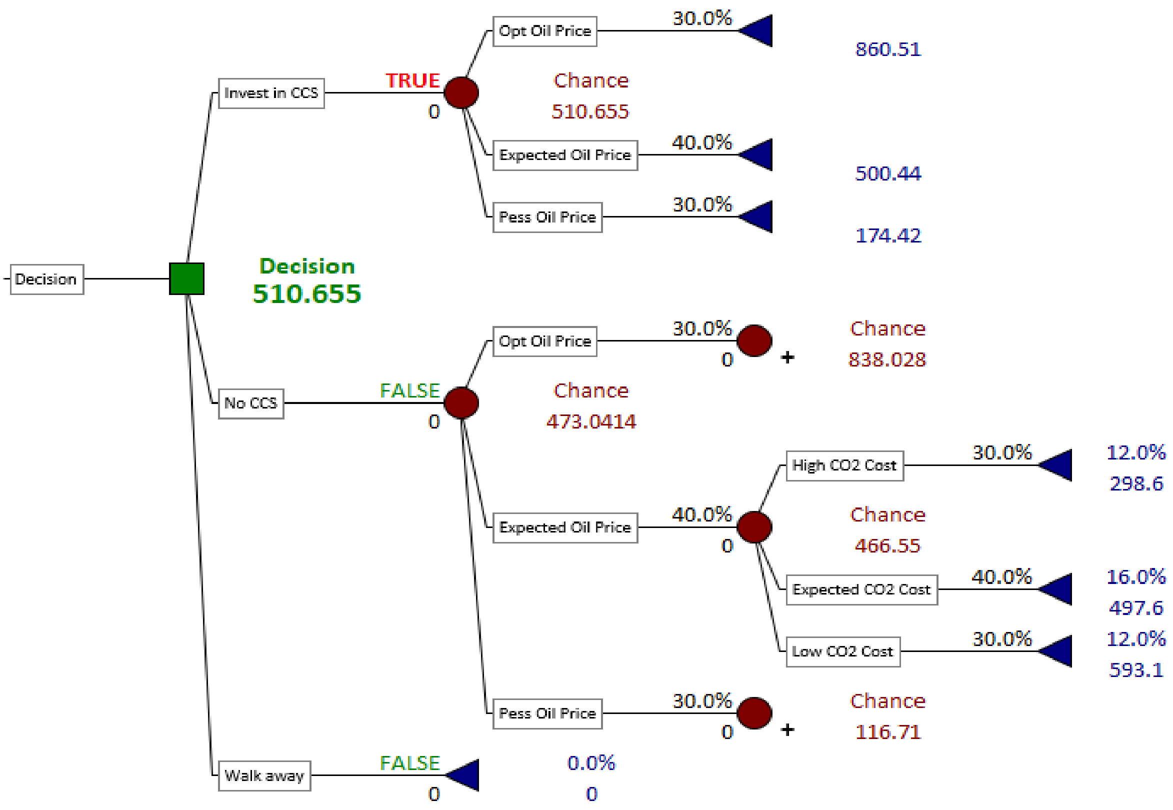

Figure 8: Decision Tree showing the NPVs, assign probabilities and EMV

The decision tree provides a clear representation of the financial outcomes of different investment options and highlights the value of investing in Carbon Capture and Storage (CCS). It shows that by investing in CCS, Alpha can potentially generate $510.65 million, which is significantly more than the other options available. This figure is arrived at by calculating the Expected Monetary Value (EMV) of each option based on assigned probability values. The result also demonstrates that not investing in CCS not only yields lower financial returns (EMV=$473.04 million) but also contributes to the ongoing challenge of global warming. Given the increasing global concern about climate change and the need for more sustainable practices in various industries, investing in CCS can be seen as a responsible and ethical decision by Alpha. By reducing the amount of CO_2 emissions, they Alpha can help mitigate the impact of climate change and contribute to a more sustainable future. In the event that the management of Alpha decides to walk away and not pursue either of the two options, they will not only end up with nothing but also miss out on the opportunity to create greater value for their shareholders. Therefore, it is crucial to carefully evaluate the potential benefits and risks associated with both investment options and make a well-informed decision that aligns with the company's long-term goals and values. By choosing to invest in CCS, the company can not only generate significant financial returns but also demonstrate its commitment to sustainable development and reducing its environmental impact.

In addition to evaluating the expected monetary values of the two scenarios, we conducted a sensitivity analysis using goal seeks to determine the capital expenditure cost at which the two scenarios would have similar values. The results indicate that at the same operating expenditure of $18/bbl for the case involving CCS and $13/bbl for the case without CCS, the EMV of the two scenarios will only be equal when the CAPEX of the project with CCS is above $175 million or when the unit operating cost increases to $20.3/bbl. This means that until the CAPEX for installing CCS facilities costs $50 million more than the current value used in the analysis, investing in CCS will always be the most suitable course of action.

Moreover, investing in CCS offers the potential to improve the project revenue stream. This can be achieved through the utilization and storage of captured CO_2 via techniques used in the oil and gas industry, such as CO_2 Enhanced Oil Recovery (CO_2-EOR) or CO2 Water Alternating Gas Recovery (CO_2-WAGR), which can result in an increase in cumulative oil production. When CO_2 is injected into a hydrocarbon-bearing reservoir, it increases the mobility of hydrocarbons, either by pushing them away from the injector well to the producer well or by dissolving in hydrocarbons, making them lighter and easier to flow.

Overall, it is evident that investing in CCS is the best decision to improve shareholder value and mitigate the impact of CO2 emissions on global warming. This example demonstrates how accounting for the cost of emissions in a company's economic valuation can make it easier for management to decide on the best course of action. By considering the long-term benefits of investing in CCS, companies can not only create greater value for their shareholders but also contribute to a sustainable future.

Conclusion

In conclusion, this research work has used advanced forecasting techniques, such as the two-factor price model and the Distribution of sum Discounted price approach (DSDP), to predict the future spot prices of carbon. Our analysis suggests a contango situation for the future spot prices of carbon, and we have shown that the DSDP approach provides more accurate price forecasts than the conventional approach.

Furthermore, this research has explored the potential benefits of investing in Carbon Capture and Storage (CCS) technology for an oil and gas company, Alpha. Through the use of financial analysis techniques such as Net Present Value (NPV), Expected Monetary Value (EMV), and decision tree analysis, we have shown that investing in CCS can create significant value for the company and its shareholders. Our sensitivity analysis also indicates that until the CAPEX of investing in CCS increases by approximately $50 million, the preferred decision will always be to invest in CCS.

Overall, this research highlights the importance of considering the cost of emissions in a company's economic valuation and exploring the potential benefits of investing in CCS technology. By making use of advanced financial analysis and forecasting techniques, companies can make informed decisions that create value for their shareholders while also reducing their impact on the environment.

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Nomenclature

EU-ETS	= European Union Emission Trading Scheme
UK-ETS	= United Kingdom Emission Trading Scheme
RGGI	=Reginal Greenhouse Gas Initiative
STLT	= Short-Term-Long-Term
EUA	= European Union Allowance
CO_2	=Carbon dioxide
CCS	= Carbon Capture and Storage
CO_2-EOR	= Carbon dioxide Enhanced Oil Recovery
CO_2-WAGR	= Carbon dioxide Water Alternating Gas Recovery
NPV	= Net Present Value
CAPEX	= Capital Expenditures
EMV	= Expected Monitory Value
DSDP	= Distribution of Sum Discounted Price
GBM	= Geometric Brownian Motion
MA	= Moving Average
AR	= Arithmetic Mean
ARIMA	= Autoregressive Integrated Moving Average
ARCH	= Autoregressive Conditional Heteroskedasticity
GARCH	= Generalized Autoregressive Conditional Heteroskedasticity

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Appendix

Table A1: Simulation data for CO_2-WAG EOR and waterflooding along with price forecast data.

	YEARS																				
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
OIL, WATER AND CO2 DATA FROM SIMULATION																					
Oil Prod (bbl) x10^6		3.09	2.63	2.46	2.61	2.58	2.70	2.54	2.28	2.08	1.89	1.79	1.62	1.45	1.29	1.23	1.13	1.05	0.98	0.94	0.90
Water Prd (bbl) x10^6		8.6	8.06	8.44	9.01	8.74	8.44	9.15	9.94	10.52	10.77	11.09	11.44	12.05	12.07	12.15	12.36	12.49	12.48	12.55	12.58
CO_2 (tons) x10^3		2.56	2.32	3.13	17.9	37.43	82.15	117.80	142.85	167.90	178.0	193.76	206.83	224.28	227.09	236.06	246.40	254.92	257.14	260.21	265.21
CAPEX (Million$)	-200																				
FORECAST DATA																					
Exp Oil forecast $/bbl		69.31	71.23	72.62	73.72	76.46	75.53	76.35	77.15	77.95	78.74	79.54	80.34	81.15	81.97	82.8	83.63	84.47	85.32	86.10	86.75
Optimistic Oil forecast $/bbl		92.55	98.18	101.9	104.4	106.27	107.8	109.15	110.39	111.58	112.75	113.91	115.07	116.23	117.4	118.59	119.78	120.99	122.2	123.38	124.26
Pessimistic Oil forecast $/bbl		46.85	46.25	46.13	46.26	46.54	46.91	47.32	47.77	48.23	48.71	49.19	49.68	50.18	50.68	51.19	51.71	52.23	52.75	53.26	53.26
[7]High CO_2 Cost forecast $/ton		92.62	113.91	131.98	145.90	158.23	170.14	182.23	194.76	207.92	221.84	236.65	252.39	269.14	286.98	306.05	326.34	347.96	371.02	395.66	421.88
[8]Expected CO_2 Cost forecast $/ton		83.19	82.13	81.57	82.95	85.97	90.20	95.33	101.15	107.56	114.52	122.03	130.06	138.64	147.81	157.62	168.06	179.19	191.06	203.74	217.25
[9]Low CO_2 Cost forecast $/ton		73.15	54.80	44.72	40.91	40.06	40.75	42.35	44.53	47.13	50.04	53.24	56.71	60.42	64.40	68.67	73.21	78.06	83.22	88.75	94.63

[7,8,9] The price of carbon estimated is in Euros/ton which needs to be converted to $/ton. As of the time of writing this report, $1 = £1 (Xe.com, 2022)