THE TWO-FACTOR Process for Forecasts of CARBON Prices

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Abstract

According to projections the number of emission mitigation facilities that should be up and running presently should be an order of magnitude of what we have today. Many researchers and experts have attributed this failure to the unattractive value. Often carbon emission mitigation measures fall short of their effectiveness. We think this is due to a lack of informed understanding of the future of this commodity. In this study, we develop forecasts of carbon allowance prices based on the two-factor stochastic model of (Schwartz & Smith, 2000).

We implement the analytical framework in Jafarizadeh (2022) using the Distribution of Sum Discounted Prices technique to determine a pessimistic, an expected, and an optimistic forecast. We use the forecasted prices in an example of economic valuation of a project incorporating decision analysis to determine the best course of action for the project.

INTRODUCTION

Commodities such as farm produce (wheat, barley, cocoa, sugar cane etc.), metals (gold, silver, bronze, copper etc.) and other natural resources have been traded for other goods or services for centuries. Later, because of fluctuations and inconsistencies in the price of these products, prediction studies on the likely future prices gave birth to a technique called price forecasting. This day-by-day variation in the price of traded goods is caused by different factors such as the market supply and demand, seasonality, technological advancement etc. Price forecasting has been practised for decades with the sole aim of predicting the future using the pieces of information we have today.

Several forecasting techniques exist in the literature, these techniques are categories based on the number of factors considered in the model. A one-factor model is that which studies a single factor affecting price stability and predicts the future base on the observed characteristics of the factor under study. Examples of a one-factor price model are the Mean-Reversion (MR), Geometric Brownian Motion (GBM), Moving Averages (MA) and Auto-Regression (AR) models. Two-factor model studies two factors affecting the price dynamics at a time to predict the future, examples include the Short-

Term-Long-Term (STLT) model, ARIMA, ARCH, GARCH etc. While a Three-factor is a combination of three and the list goes on.

Quoting from a famous saying that says "all models are wrong, but some are useful" insinuating that a correct model doesn't exist. The question of how many factors are good enough for a study is a trade between simplicity and realism. A one-factor model is usually simple but not realistic, a two-factor model is typically more complex and realistic enough than a one-factor model. A three-factor model is usually very complex and slightly more realistic than a two-factor. The choice of technique to use boils down to the degree of precision that the researcher aims to achieve. Several research works have suggested the use of the two-factor price model, especially the STLT model because of its simplicity, ease of understanding and above all genuine and useful enough. Some examples can be found in the work of (Schwartz & Smith, 2000, Jafarizadeh & Bratvold, 2012, Bakker et al., 2021). In the next section, we will briefly introduce carbon trading, price forecasting and carbon price forecasting.

CARBON TRADING

Nearly two decades ago, a new commodity called CO_2 allowance was introduced in the trading marking. Trading began on platforms such as the EU-ETS where CO_2 allowances are traded between Carbon emitters to avoid paying penalties for excess carbon emission. Since then, the price of this allowance has been observed to fluctuate from time to time displaying some common features of physical commodities that are being traded. Some factors affecting the price of carbon allowances include the total allowances allocated by the regulating body, the quantity demanded, regulatory policies such as banking and the overall level of greenhouse gas emissions. Even though Carbon is being traded today the question that many still ask is "is carbon a commodity?". Economists have argued whether CO_2 allowance should be considered a commodity or not. An example of such arguments can be found in (Parsons et al., 2009), an article published by market intel on March 24, 2021, titled "Sustainability Market, Part 4, Is carbon a commodity?", this and many more can be found in the literature, unfortunately, these discussions are outside the scope of this work.

Carbon trading, also known as Emission Trading System (ETS) or cap-and-trade system is a branch of the Carbon Pricing Scheme that aims at providing a market-based approach to reducing greenhouse gas emissions. Greenhouse gas emissions are believed to be the major contributor to climate change. The goal of carbon trading is to create a financial incentive for companies and countries to reduce their greenhouse gas emissions and transition to cleaner forms of energy. Other branches of carbon pricing schemes include carbon taxation and internal carbon pricing. The focus of the research is on carbon trading as such the paper will not discuss further the other techniques.

There are two main types of carbon trading systems: voluntary and mandatory. Voluntary carbon trading involves companies or individuals choosing to offset their greenhouse gas emissions by purchasing carbon credits from a project that reduces or removes CO_2 from the atmosphere, such as a wind farm or reforestation project. Mandatory carbon trading on the other hand is established by governments or regional bodies mandating companies or countries to reduce their emissions to a certain level or face penalties.

One of the prevalent carbon trading systems is the European Union Emissions Trading System (EU ETS), which was established in 2005 and covers approximately 11,000 power stations and industrial plants across the European Union. With the United Kingdom exiting the EU, a new trading scheme called the UK-ETS was born. Outside of Europe in other parts of the world such as the United States, different forms of carbon trading schemes exist at state and regional levels, examples are the Regional Greenhouse Gas Initiative (RGGI) and the California Carbon Trading Market.

How does carbon trading work?

In a carbon trading system, the total amount of CO_2 that can be emitted is capped at a certain level. This cap is typically determined by governments or regional organizations, and it is designed to reduce greenhouse gas emissions over time. Companies that emit more CO_2 than their allotted amount must acquire additional allowances to compensate for their excess emission or be penalized for their actions. While those companies that emit less have the option of banking their excess allowances or selling them on a Carbon trading platform. Thus, the platform provides an avenue that allows entities (individuals or cooperate bodies) to buy and/or sell CO_2 allowances.

Every year, a certain number of allowances are issued by regulating bodies to companies for free or sold through auctions. CO_2 allowances are commonly traded in EUR/ton, the dynamic of this price is affected by different factors among which is the market supply and demand. Meaning that the price of an allowance upsurge when its demand increases either because of a lower supply of allowances or because of a higher demand. Consequently, as the price of allowances becomes more and more expensive, it translates to how expensive emitting a ton of CO_2 becomes. Therefore, for a company emitting large tons of CO_2 to remain in business, it must look for an alternative way to dispose of the produced CO_2 rather than emitting it to the atmosphere and hence tackle the greenhouse gas emission problem.

In addition to the statement above, to swiftly decrease greenhouse gas emissions through carbon pricing, the price of allowances and/or taxes needs to be high enough to encourage emitters to seek alternative ways of minimizing if not stopping the emissions.

PRICE FORECASTING

From time immemorial, commodities' prices usually fluctuate over a given period. These prices tend to exhibit different patterns on a time scale graph, due to consistent variations. Some commodity prices vary every second while others tend to have a longer time frame, this may be hours or days. Most fluctuations have a random and probabilistic element. Authors and practitioners have suggested models that reflect mean reversion, random walk, and Geometric Brownian Motion (GBM) individually or a combination of these factors e.g., (Pindyck, 1999; Schwartz & Smith, 2000, Geman, 2007, Jafarizadeh & Bratvold, 2012).

A mean-reversion process reflects the market forces of supply and demand. The idea is simple, whenever the market supply of a given commodity is high, competition is created among producers, which leads to a price drop. When prices drop, marginal producers exit the market allowing only a hand full of producers in the market. With the departure of some producers in the supply chain, the supply drops, and this now creates a demand in the market. Because of the scarcity of the commodity, the price goes up and whenever a substantial rise in the price of the commodity is attended, exited producers return, and this leads to another oversupply, followed by a price drop, this circle continues to repeat itself. During the cause of price movement, there exists an arbitrary common price (more of an average) where the price of the commodity keeps revolving around. This arbitrary price is called the mean-reverting price.

Knowing fully that there is no such thing as a fixed commodity price. It is wrong to assume a constant price throughout the lifespan of a project, nor does it make any sense to assume an annual increment of one, two or three percent in commodity price every year. It is true that while studying historic prices, one might notice that for some commodities, the spot price revolves around a certain value. This only proves the existence of mean-reversion but not a justification for use of a constant price in economic valuations. For this reason, some organizations use a plus or minus certain percentage to a deterministic value to work out the project valuation. Certainly, the latter approach is better than just assuming a constant value; still, there is room for improvement.

To minimize risk and uncertainties in prices, price forecasting is a better way to estimate the future spot prices of a given commodity. Although, forecasting in itself is not an exact science but rather an informed estimation technique that enables one to predict the future based on the present and/or past with a certain level of accuracy, thus minimizing risk. Price forecasting employs the use of historical spots or futures prices, and Call and Put options to forecast the future by looking out for some features such as trend, seasonality, stationarity etc., within the data which are likely to re-appear in the future. Researchers have studied and forecasted the future prices of different commodities using the one, two, and three-factor price models. Some prominent studies in this field are the work of (Pindyck, 1999; Schwartz & Smith, 2000).

CARBON PRICE FORECASTING

Carbon price forecasting is a process of predicting the future price of carbon allowances using market information. This is carried out using economic models that consider a wide range of factors, including the expected level of economic growth, volatility, mean reversion, trend etc. These models are designed to provide a range of possible carbon prices, rather than a single, precise prediction. Integrating the forecasted future spot price of carbon into an economic valuation of a CO_2-emitting company aids management in making informed decisions on project executions and investments in cleaner forms of energy.

Based on the findings of our extensive literature review, for the first time, this paper aims to use the Short-Term-Long-Term (STLT) Two-factor price model to forecast the future spot prices of EU-ETS carbon allowances as described in (Schwartz & Smith, 2000). The forecasted EU-ETS carbon allowances will serve as a proxy price of Carbon in our analysis. The study will also in a form of an exercise shows how to incorporate forecasted carbon prices into an integrated economic valuation of a CO_2 emitting company.

The primary goal of this work is to use the market information of EUA futures and the Call and Put option on the EUA futures to forecast the price of carbon using the STLT two-factor price model. To the best of our knowledge, this research is the first of its kind on carbon. This paper can be categorized into the following sections. Section 2 deals with the price modelling and parameter estimations, section 3 is the informed sensitivity analysis on forecasted price, section 4 is the integrated valuation and finally, section 5 concludes the study.

2.0 PRICE MODELING

Before subjecting time series data to a forecast analysis, it is important to expose them to a test that will confirm the stationarity and linearity status of the data. These tests are aimed at categorizing the time series to determine the most suitable technique to be used in estimating the future. To distinguish whether a time-series data is stationary or not, the use of Unit root test e.g. the Augmented Dickey-Fuller (ADF), Phillips-Perron etc can be employed. In ADF, whenever the null hypothesis is not rejected (i.e. implying the presence of a unit root), the series is considered to be nonstationary and vice-versa for a rejected null hypothesis. On the other hand, tests like the Keenan and Brock-Decher-Scheikman can be adapted to test for nonlinearity in time series data. Studies by (Arouri et al., 2012;

Chai et al., 2020; Chevallier, 2011) have shown that EUA prices of carbon are both nonstationary and nonlinear.

In light of the above, we adopt the outcomes of these studies and consider the price of carbon as nonstationary and nonlinear. We went ahead to work on determining the suitable technique to be implemented in forecasting the future spot prices of carbon. Employing the use of statistical tools such as Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC), we rank different techniques used in forecasting today. Our analysis uses the historic spot price of carbon, we first consider the entire historic price from July 2008 to April 2022 and name it as Historic Spot Price A. We then consider another time frame of the historic price (from April 2018 to April 2022) to see if reducing the time frame will affect the outcome. And last but not least we tested the futures price of the 8th of August 2022 for September 2022 to Dec 2028. The results of our analysis are presented in Table 1 below.

Table 1: Model Fit RankingBased on Historicspot and Futures Prices of Carbon

		Historic Spot Price A		**Historic Spot Price B**		**Futures Price**	
	Technique	**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
1	**MR with GBM**	***<u>698.94</u>***	***<u>717.72</u>***	***<u>272.77</u>***	***<u>284.13</u>***	***<u>83.29</u>***	87.93
2	**MR**	788.48	797.87	289.23	294.90	93.71	96.04
3	**ARMA**	786.69	799.21	290.33	297.89	83.30	***<u>86.53</u>***
4	**GARCH**	1,203.13	1,215.57	400.88	408.37	117.45	120.28
5	**ARCH**	1,217.20	1,243.48	405.26	410.88	115.45	117.57
6	**GBM**	N/A	N/A	N/A	N/A	N/A	N/A

In statistics, AIC and BIC are used in comparing different models by fitting data sets to determine a model that fits best. The lower the AIC and BIC values the better the fit between the data and the model. From Table 1, values highlighted in bold, italic and underlined are the lowest of each category. With the exception of the results of BIC of the futures prices where the preferred model came second, the two-factor MR-GBM has the least AIC and BIC values in all three scenarios. This means that the two-factor MR-GBM model provides the best fit for both the historic spot price and the futures price of carbon. Therefore, we considered it as the preferred model for forecasting the future spot price of carbon. Contrary to what other researchers believed, the GBM model was discovered to be the worst model for such data. This is because the statistical tools could not establish any reasonable fit between GBM and the data. These results have thus proven that it is wrong to use GBM alone in forecasting the future spot price of carbon.

2.1 TWO-FACTOR PRICE MODEL

In the study by (Schwartz & Smith, 2000), it was assumed that the spot price of a commodity in this case carbon allowance at time t denoted by S_t have both short and long-term factors represented as χ_t and ξ_t respectively, these can be expressed as;

$$\ln(S_t) = \chi_t + \xi_t \qquad 1$$

Here the short-time factor χ_t also referred to as the short-term deviation in prices is assumed to follow the Ornstein-Uhlenbeck process where the price reverts toward a common mean;

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \qquad 2$$

Whereas the long-term factor ξ_t also known as the equilibrium price level is assumed to represent the Brownian motion;

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \qquad 3$$

Here κ represents the short-term factor mean reversion coefficient, μ_ξ is the long-term factors' drift, σ_χ and σ_ξ are the respective standard deviations for the short and long-term factors, while dz_χ and dz_ξ are correlated increments of the standard Brownian motion with $dz_\chi dz_\xi = \rho_{\chi\xi} dt$.

Depending on the values of χ_0 and ξ_0, the log of future spot prices will yield a normal distribution with the following expectation and variance.

$$E(lnS_t) = e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t \qquad 4$$

$$Var(lnS_t) = (1 - e^{-\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t = 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \qquad 5$$

According to the Ito lemma principle, $\ln E(S_t) = E(\ln S_t) + \frac{1}{2}Var(lnS_t)$ therefore, the above expectation and variation equation can be re-written as;

$$\begin{aligned}\ln E(S_t) &= e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t \\ &+\frac{1}{2}((1 - e^{-\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa})\end{aligned} \qquad 6$$

Considering the risk-neutral valuation, the short and long-term risk premiums represented by λ_χ and λ_ξ can be deducted from the expectation. Here, the expected future spot price of Carbon allowance equals the futures prices having the same T until maturity

$$\begin{aligned}\ln F_{0,T} &= e^{-\kappa T}\chi_0 + \xi_0 + (\mu_\xi - \lambda_\xi)T - (1 - e^{-\kappa T})\frac{\lambda_\chi}{\kappa} \\ &+\frac{1}{2}((1 - e^{-\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1 - e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa})\end{aligned} \qquad 7$$

Whereas the instantaneous variance depicting the volatility of the futures prices of Carbon which is independent of risk premiums can be expressed as

$$Var\ (\ln F_{0,T}) = e^{-2\kappa T}\sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T}\rho_{\chi\xi}\sigma_\chi\sigma_\xi \quad (8)$$

2.2 TWO-FACTOR PARAMETER ESTIMATION FOR CARBON

To estimate the above parameters needed for the carbon price forecast analysis, we employ the use of market information on carbon i.e. the EUA futures and Call and Put options on EUA futures. The values used in this analysis were obtained from (ICE, 2022) the Intercontinental Exchange (ICE) market on the 8th of August 2022. Using a curve fitting technique, we iteratively estimate these parameters from the futures prices and the implied volatility of the options. We start by developing a forward curve using equation 7. Initial guess values were assigned to each parameter to estimate the futures prices at different maturities, this is then compared with the observed futures price from the market.

On the other hand, the market-implied volatility values were compared with the volatility term curves constructed using equation 8. Unlike the futures, the implied volatilities are not observed directly in the market but can be obtained using either the (Black & Scholes, 1973) model or the (Schwartz & Smith, 2000) approach from European options on carbon futures. The (Schwartz & Smith, 2000) approach was used here and it suggested that the value for a given European Call and Put options denoted as c_T and p_T respectively, can be evaluated using the;

$$c_T = e^{-rT}\left(F_{0,T}N(d) - KN\left(d - \sigma_\varphi(T)\right)\right) \quad (9)$$

$$p_T = e^{-rT}\left(KN\big(\sigma_\varphi(T) - d\big) - F_{0,T}N(d)\right) \quad (10)$$

Where r is the risk-free rate, $N(d)$ is the cumulative probability for the standard normal distribution, K is the strike price, T is the delivery date (in the analysis, we are assuming that the underlying futures and the option expire at the same time) and $\sigma_\varphi(T)$ is the volatility of the futures contract. In Addition to that, d is also a function of the futures, strike price and volatility which is expressed as

$$d = \frac{ln(F/K)}{\sigma_\varphi(T)} + \frac{\sigma_\varphi(T)}{2} \quad (11)$$

With the aid of a simple computer code, a loop iteration was designed to optimize the parameters such that both the estimated futures and option values obtained using equations 7 and 8, and the market futures value and the implied volatility values matched with a low sum of the square of error values. A plot indicating a good fit between both estimated and market data are presented in Figures 2 and 3, while the estimated parameter values are shown in Table 2.

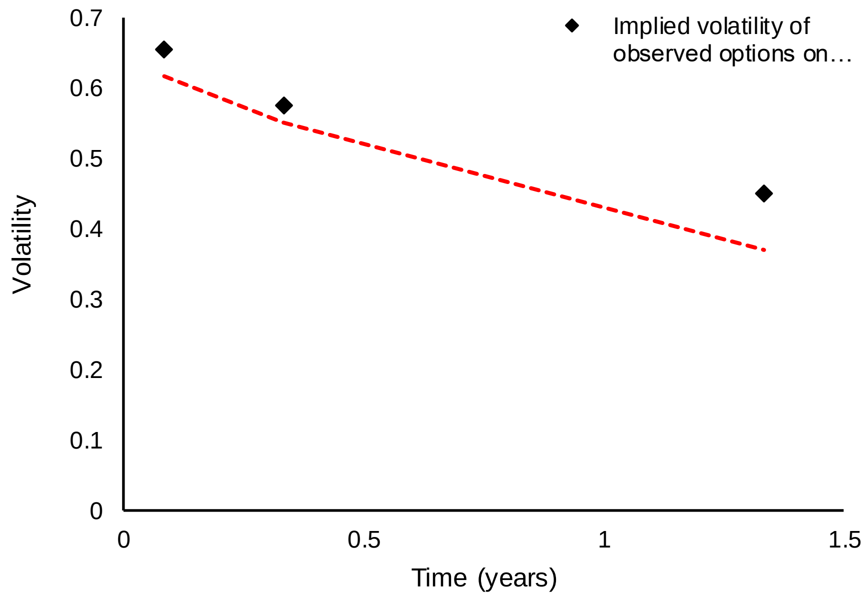

Figure 1: Curve Fitting on Market-Observed Options on Carbon Futures

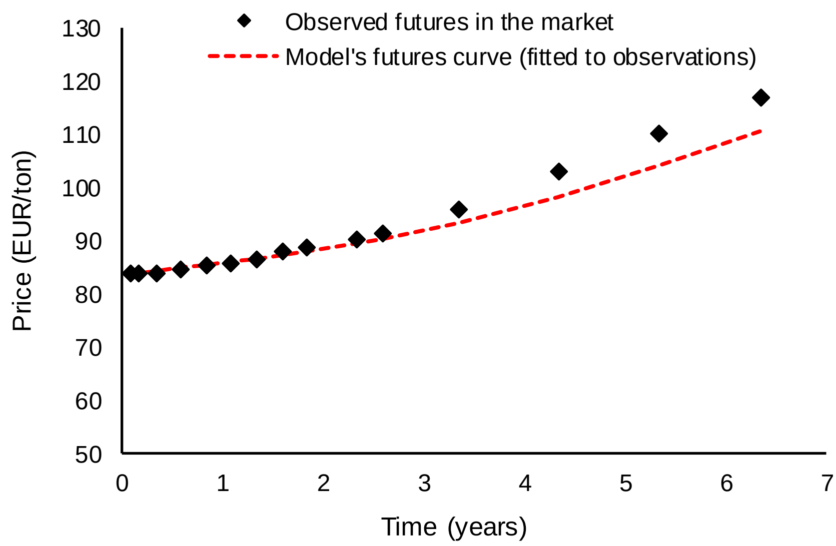

Figure 2: Curve Fitting on Market-Observed Carbon Futures

Due to the immaturity of carbon trading data, only a few options on carbon futures allowance were obtained from the ICE website (ICE, 2022), as such the options analysis was only for about a year and a half. Nevertheless, a good fit was obtained for the two analyses and therefore, an indication of the reliability of the parameter estimated. We will now use the estimated parameters in developing a carbon price forecast.

Table 2: Two-factor Carbon price parameters

Parameters	Values
χ_0	0.342

ξ_0	4.079
μ	0.051
σ_χ	0.450
κ	0.612
σ_ξ	0.162
$\rho_{\chi\xi}$	0.930

2.3 THE TWO-FACTOR CARBON PRICE FORECAST

For ease of analysis and to avoid the need for high computational power, we assumed that the price of carbon swings once a month. With this assumption, we use a monthly value to represent Δt when required. Using equation 6 along with the estimated parameters shown in Table 2, we developed the expected future carbon spot prices as shown in Figure 3 with their corresponding 90% confidence band.

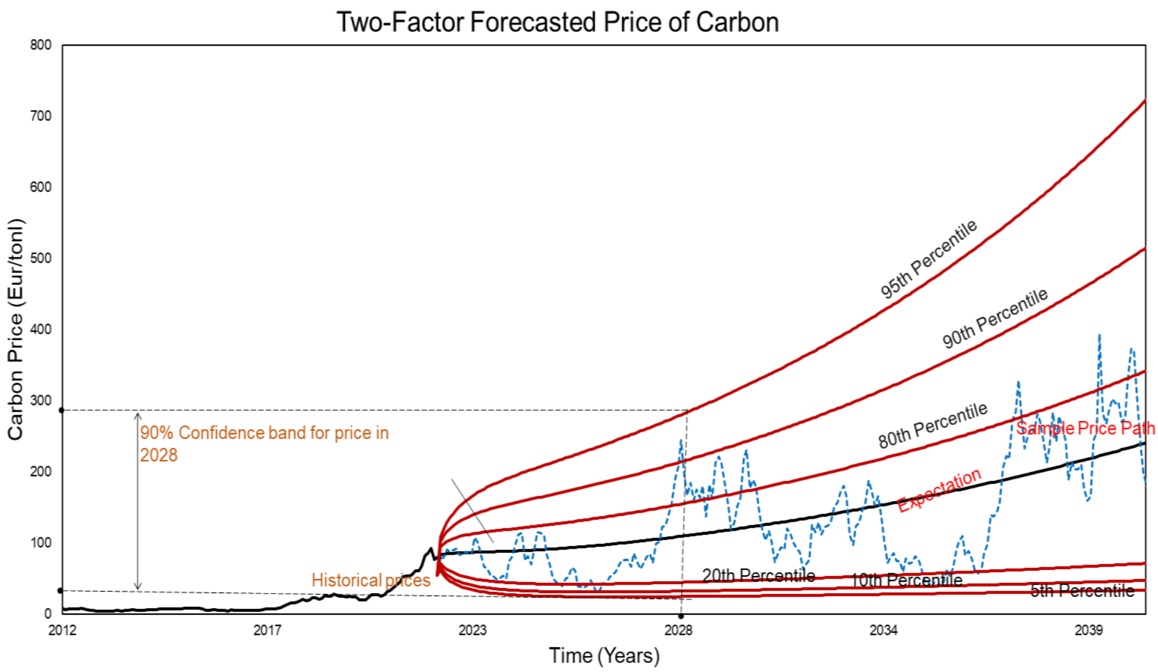

Figure 3: Two-factor Carbon price model indicating different percentage confidence bands

We could observe that the expected future spot prices of carbon are in contango i.e. an upward-sloping curve, suggesting that the prices of a unit of carbon will continue to rise. Some justification for the future price rise includes the ongoing global movements on reducing Carbon emissions, the consistent review of the volume of allowances issued to emitters, some European countries setting up datelines on the importation and manufacture of hydrocarbon cars, and some countries around the world proposing to increase their carbon tax, etc. Even so from the Figure, it can be observed that just within the span of 2 years i.e. 2020 to 2022, the historic prices of carbon have risen 400% more. These and many more are clear indications that the era of lower carbon prices is gone. From the forecast, it could be observed that in 2028 the price of carbon emitted can be anywhere between $40 - 290 based on the 90% confidence bands. With only about a 5% chance both ways, the price could go beyond or less than the prices indicated within the confidence band.

3.0 INFORMED SENSITIVITY ANALYSIS

Just like in any other field of study, sensitivity analysis is carried out to check for changes in the value of output or outcome based on the changes made to the input variables. These enable one to see and quantify how a deviation from certain variables could affect the entire process[1]. In this study, we employ the use of sensitivity analysis to check how changes in prices could affect the outcome of a given project. Using a range of prices from pessimistic to optimistic, we evaluate how uncertainties in price could significantly affect project outcomes. Most carbon emission mitigation projects are capital-intensive investments, as such, it is of paramount importance that a detailed sensitivity analysis is conducted before a decision is made. Sensitive input variables such as the cost of CO_2, incentives (where applicable), and capital and operating expenditures need to be rigorously analyzed to check the impact of each on the outcome.

In a traditional sensitivity analysis such as the tornado diagram, considerations are not given to the shape of the distribution of the input variable, rather only the extreme values are used. These extreme values just like the P10 and P90 used in quantifying geological reservoirs don't reflect on the overall distributions because the distribution of reservoir oil in place could be in any form. Consider two distinct input variables to be used in a sensitivity analysis, one having a normal while the other a log-normal distribution. If the range of the two extremes (pessimistic and optimistic) happens to be the same, these two distinct variables will be considered the same in such evaluations. When deciding on a range of values to be used in sensitivity analysis, the pessimistic and optimistic values should not be mere minimum and maximum values rather they should be a meaningful range of values that truly represent the input variable[2]. It is because of this and other lapses such as the inability to test more than one variable at a time that some researchers argued that the tornado diagram is not sufficient to be used as a sensitivity analysis tool. Contrary to the traditional approach discussed above, the

[1] Sensitivity analysis can also be used to check the robustness of models.

[2] Historically, the lowest price of oil ever was -$5 (negative five dollar) and the highest is in the axis of $150, this doesn't mean that when deciding for a range of oil price the pessimistic and optimistic values should be -$5 and $150.

approach used in this study is robust and better than the customary sensitivity analysis techniques. Therefore, employing such a technique to check on the impact of the uncertain carbon price in a carbon emission mitigation project is beyond the usual day-to-day sensitivity analysis.

3.1 Sum-Discounted Prices

Conventional economic sensitivity analysis often defines and uses a range of forecast prices from the most optimistic to the most pessimistic prices during a cash flow analysis. Unfortunately, the price forecasts generated using the stochastic model are not fit for such analyses. This is because, stochastic price model forecasting focuses on describing the uncertainties, probability distribution and expected future spot prices. For instance, when an optimistic value from a stochastic forecast is used as the value of a project, the outcome will not indicate if the result obtained is the extreme possible project valuation or the utmost likely value. For these reasons, the stochastic price model forecast cannot be used in a such sensitive analysis. To solve this problem, the use of the distribution of sum-discounted prices approach which could be found in the work of (Dixit, 1993) can be adapted to generate forecasts of prices consistent with the stochastic approach. Assuming that the expected forecast equals the price scenario we call; $S_t^*, 0 < t < T$, therefore

$$\int_0^T S_t^* e^{-rt} dt = E\left(\int_0^T S_t e^{-rt} dt\right) \qquad 12$$

Here, T represents the forecast time, r the discount rate and e^{-rt} the discounting factor.

For any given percentile, we can estimate the expectation of the forecast of prices by implementing the numerical approximation technique on equation 6. With that, we can now simulate the stochastic spot prices within the forecast limit and consequently the distribution values for the sum of the discounted prices. Hence, with the aid of an optimization model and solver (MS Excel add-in), we were able to develop a forecast that is a replica of the distribution of the sum of discounted prices.

3.1 NUMERICAL PROCESS

Suppose we discretize the spot price equations mentioned earlier, the two-factor spot price equation could be re-written as;

$$\ln S_{t+\Delta t} = \xi_{t+\Delta t} + \chi_{t+\Delta t} \qquad 13$$

$$\xi_{t+\Delta t} = \xi_t + \mu_\xi \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t} \qquad 14$$

$$\chi_{t+\Delta t} = e^{-\kappa \Delta t}\chi_t - \left(1 - e^{-\kappa \Delta t}\right)\frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \qquad 15$$

Here all variables have their earlier defined meaning, in addition to Δt representing the change in time. Also, assuming that ε_ξ is a standard normal distribution, and ε_χ a function of ε_ξ and ε (an independent normal distribution), a correlation between the simulated factors can be accounted for using;

$$\varepsilon_\chi = \varepsilon_\xi \rho_{\chi\xi} + \varepsilon\sqrt{1-\rho_{\chi\xi}^2} \qquad 16$$

For a forecast period $0 < t < T$, we employ the use of equations 13 through 16 to numerically simulate the price path for the two-factor forecast. In addition to that, we implement the Riemann approximation on equation 12 to solve for the sum of the discounted prices as shown in equation 17.

$$\int_0^T S_t e^{-rt} dt \cong \sum_{t=0}^{T} S_t e^{-rt} \Delta t \qquad 17$$

Finally, we generate 1,000 uncertain prices using the model for monthly time steps for the entire forecast period and calculate the sum-discounted values. Figure 5 shows the distribution of sum-discounted prices (DSDP) of carbon.

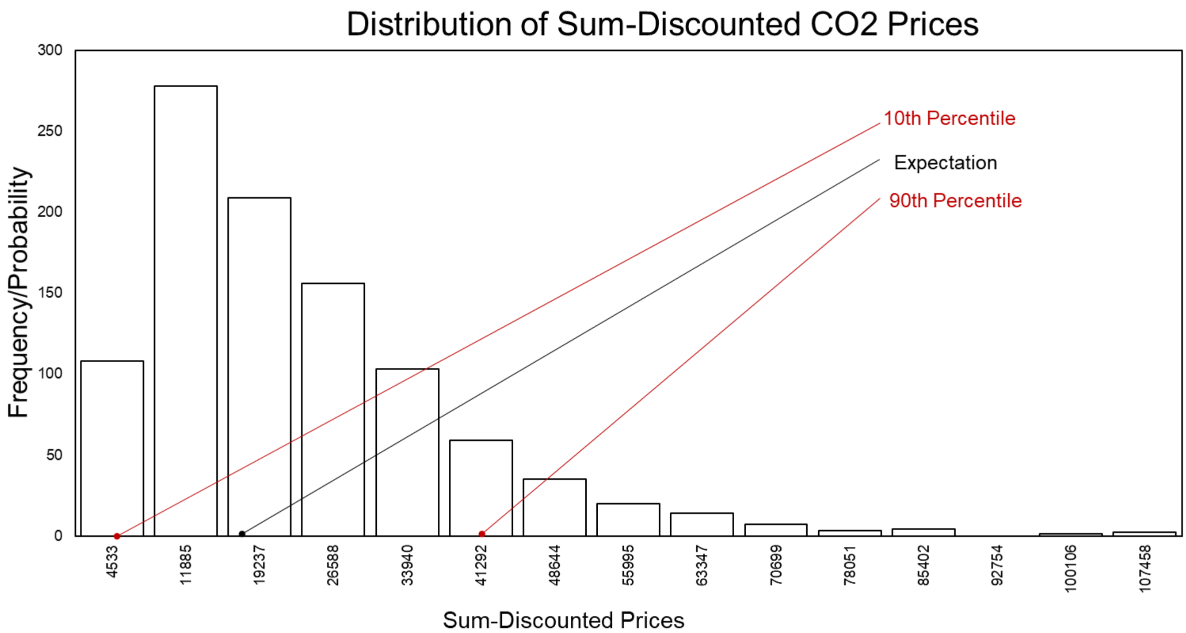

Figure 4: Distribution of Sum-discounted CO_2 Prices

3.2 OPTIMIZATION

Now that we have obtained a distribution for sum-discounted prices for the two-factor price forecasts of carbon, we assign the 90th and 10th percentile of the distribution to represent the high and low cases. Subsequently, we went ahead to simulate an optimistic and a pessimistic price forecast to have equal sum-discounted values as the high and low cases. The optimization equations used in calculating the forecast of prices S_t^* matching the given percentile of the discounted prices is shown below.

$$\min_{\lambda_\xi} \left(\sum_{t=0}^{T} S_t^* e^{-rt} - X \right)^2$$

s.t

$$P\left(\sum_{t=0}^{T} S_t^* e^{-rt} < X \right) = \frac{p}{100}$$

Here, t represents the discretized time steps from 0, $\Delta t, \ldots..T$, whereas X is an internal variable defined to calculate the percentiles.

Using Microsoft excel solver, we solve for the short-term premium λ_χ in eqn 7 such that the optimized sum-discounted simulated prices of the expected, and the 90th and 10th percentile equals the sum-discounted monthly prices generated earlier. Figure 5 below shows the 90th, 50th (representing the expected price) and 10th percentile determined from the distribution of sum-discounted technique. Unlike the conventional percentiles obtained usually by multiplying the expected price with a certain positive and negative fraction, the forecast obtained from DSDP is a representation of all the sample price paths simulated instead of just one.

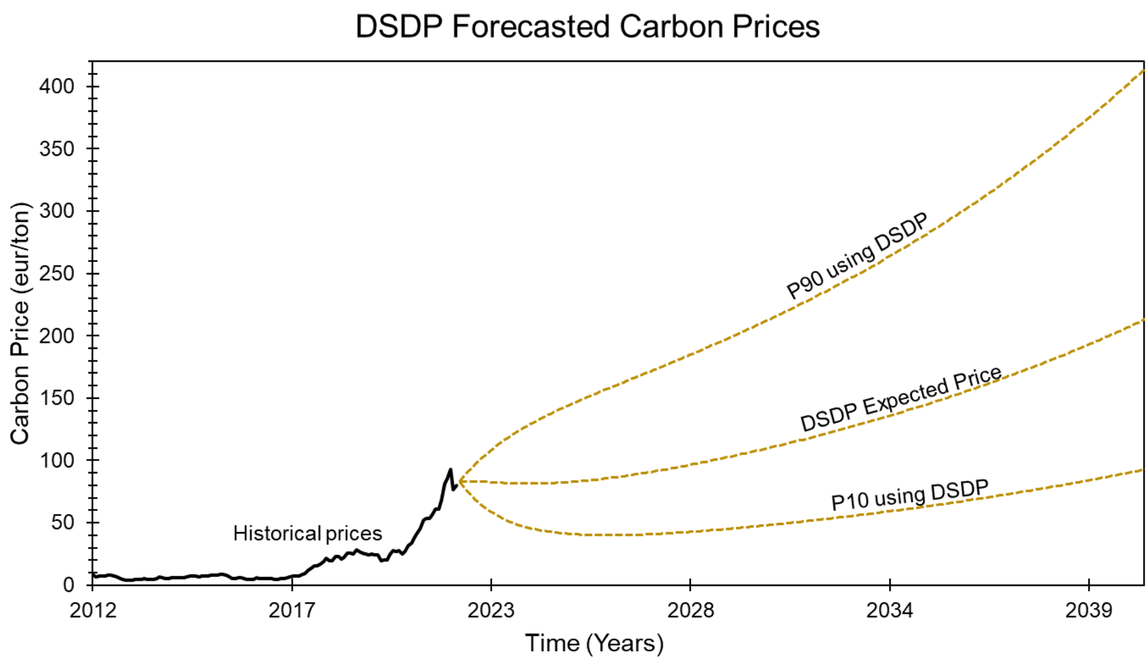

Figure 5: Expected and pessimistic price forecasts of Carbon

To understand the distinction between the two forecasted prices, we bring the two forecasts together in the next discussion.

CONVENTIONAL Vs DSDP CONFIDENCE BANDS

Figure 6 compares the two forecasted prices along with the confidence bands for prices (i.e. . the conventional and the DSDP technique). The figure shows a clear distinction between the two results. It can be observed that the conventional technique has overestimated the 90th and the expected price in comparison to what was obtained using the DSDP technique. Whereas the conventional P10 was an underestimation compared to the P10 of DSDP. Because of our confidence in the forecasted prices, we will be employing the use of the DSDPs' P90, expected and P10 forecasted values in an integrated economic valuation analysis.

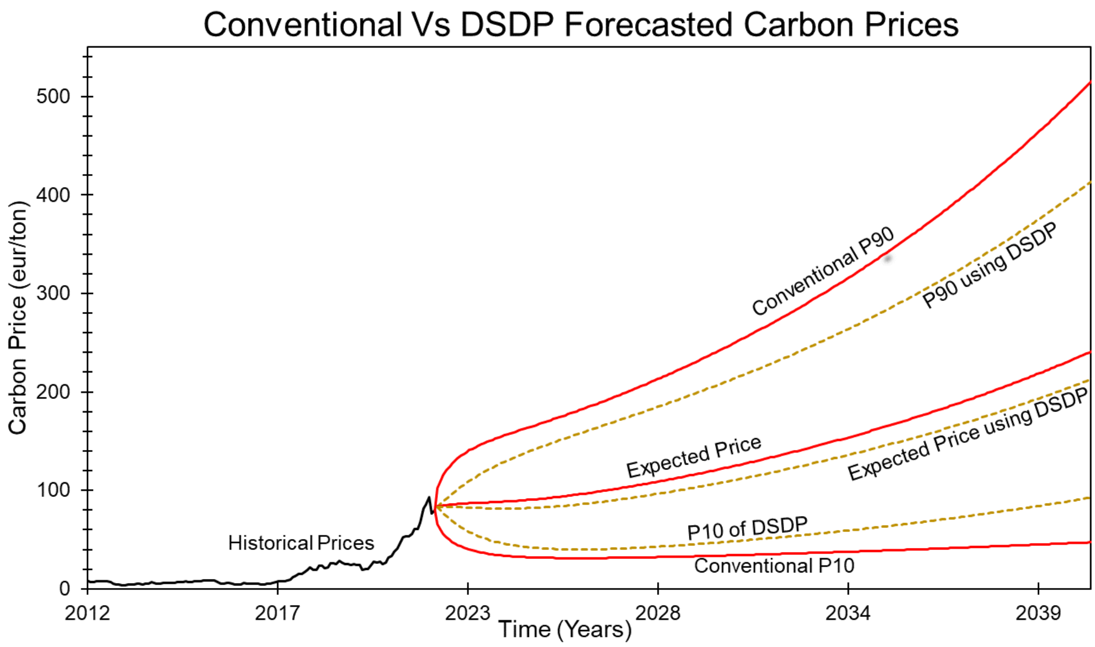

Figure 6: Comparison between Percentile of Expected prices and that of Distribution of Sum-Discounted Prices

5.0 INTEGRATED ECONOMIC VALUATION

In this section, we demonstrate how the forecasted carbon price can be used in constructing a reliable cash flow. Bearing in mind the computational time and power that will be required, so also the significance of each variable to the analysis. A question one should ask himself before carrying out an analysis is, does making my analysis detailed and complex makes it stand out from simplified versions? If the answer is no, then it is always a better option to make analysis simple, informed and most importantly useful. Therefore, to attain the desired goal of having a realistic cash flow analysis while at the same considering the constraints above, we use fixed rates for some variables such as water treatment and recycling costs. We also use a yearly oil and CO_2 rate (by averaging the monthly prices) on the assumption that both prices of oil and carbon remain constant throughout a given year i.e. prices changes once a year.

Example:

The example discussed here shows the valuation of a decision to be made on an oil-producing field. The field belongs to Alpha company limited; it has an expectancy life of 20 years based on the simulation carried out by the Reservoir Engineering department to estimate the reserve. The oil, water and CO_2 production data obtained from the simulation are shown in Figure 2 below. Because the produced oil contains a high percentage of CO_2, Alpha needs to decide on either paying a fine for emitting the produced CO_2 or investing in a Carbon Capture and Storage (CCS) project. If the company is to invest in CCS, the sum of $200 Million is expected to be spent on the installation of a carbon capture facility, drilling and completion of a CO_2 injector, compressor pump and other equipment. On the other hand, if the company decides to continue with business as usual and pay the emission fine, the forecasted Carbon price will be used to evaluate the emission cost.

It is expected to cost Alpha about $6 to produce a barrel of oil without CCS and $8 per barrel when considering the option of including CCS. A royalty of 30% and a corporation tax of 40% is to be paid to the government. Alpha decided to use a risk-free rate of 4% to discount the cash flow having hedged the market price uncertainties. The forecasted prices of oil used in this analysis as shown in Table A1 of the appendix were obtained from (Jafarizadeh, 2022) with slight adjustments to extend the duration of the forecast.

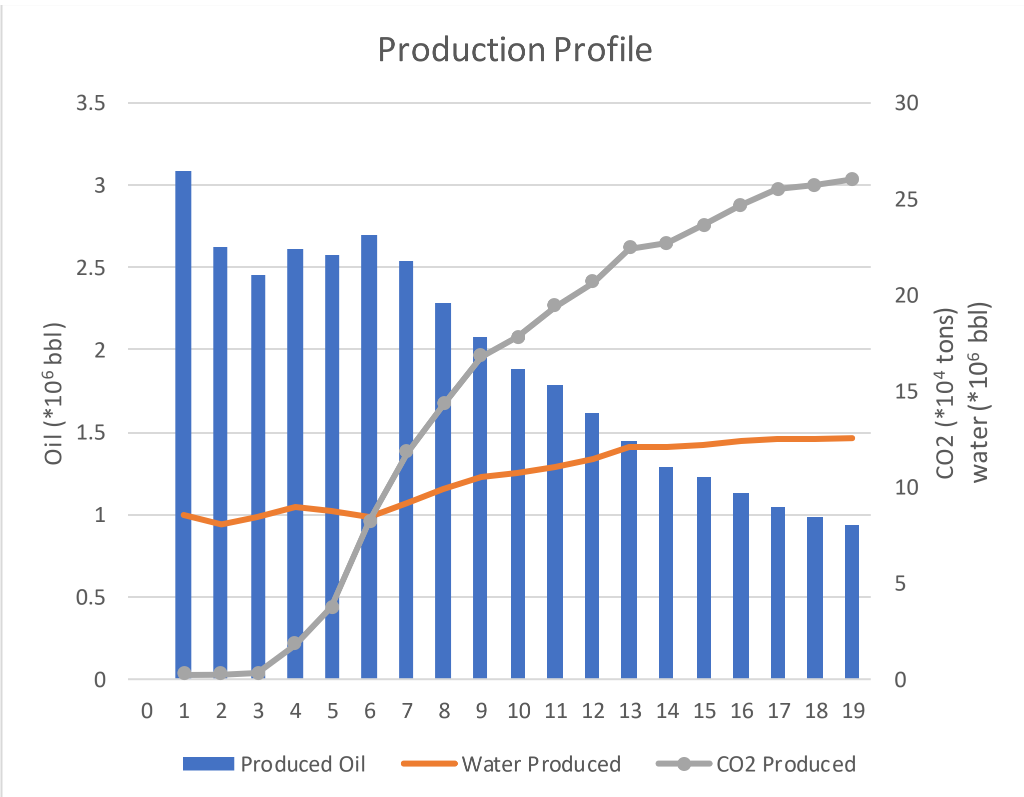

Figure 4: Reservoir simulation showing Oil, water and CO2 production data

From Table A1 in the appendix section, the primary variables for the cash flow analysis are the annual oil, water, CO_2 rates, the selling price of a barrel of oil and the cost of a ton of CO_2. Having three forecasted oil prices and three costs of carbon, the outcome of the cash flow will have different NPV values. If Alpha decides not to invest in CCS, the study outcome will have nine (9) different Net Present Values (NPV). On the other hand, if Alpha decides to invest in CCS, there will be only three outcomes depending on the price of oil. Table 3 below reveals the NPV outcomes of the scenario where Alpha decides not to invest in CCS. Even though none of the outcomes indicates a loss i.e. a negative NPV, it can be seen that the best outcome is approximately two orders of magnitude better than the least case. This is because the best outcome was obtained using a combination of an optimistic oil price and a lower cost of carbon, at the same time the least case was due to having a pessimistic oil price and a higher cost of carbon.

Table 3: Different NPVs for due to the differences in Oil and CO_2 Prices

	High CO2 Cost forecast	***Expected CO_2 Cost forecast***	***Low CO_2 Cost forecast***
Optimistic Oil forecast	$770.64 x$10^6$	$944.50 x$10^6$	$1,034.94 x$10^6$
Expected Oil forecast	$380.17 x$10^6$	$578.46 x$10^6$	$674.86 x$10^6$
Pessimistic Oil forecast	$10.43 x$10^6$	$230.20 x$10^6$	$347.49 x$10^6$

In the case of investment in CCS, we assume the cost of producing a barrel of oil to be $8 per barrel to account for the operating cost associated with the process of CO_2 storage. In Table 4 below, we present the outcome of the cash flow analysis for Alpha having invested in CCS.

Table 4: Different NPVs for Water flooding due to having different oil prices

	NPV (Million $)
Optimistic Oil Price	*$938.45 x$10^6$*
Expected Oil Price	*$577.38 x$10^6$*
Pessimistic Oil Price	*$251.37 x$10^6$*

The NPV outcomes for the two scenarios in which Alpha invest in CCS or not can be presented on a decision tree along with assigned probability values to estimate the Expected Monetary Value (EMV). EMV is used in assessing two or more decision outcomes, the outcome with a higher positive value is usually the preferred choice. It is recommended to walk away whenever the EMV is negative except if other values aside from monetary are to be derived by executing the project. Figure 7 below, shows how we could represent the results in Tables 3 and 4 on a decision tree to determine the EMV before we make decisions. Also, in the analysis, we use Swanson's 30-40-30 Rule to assign probability values to the oil prices and the cost of Carbon.

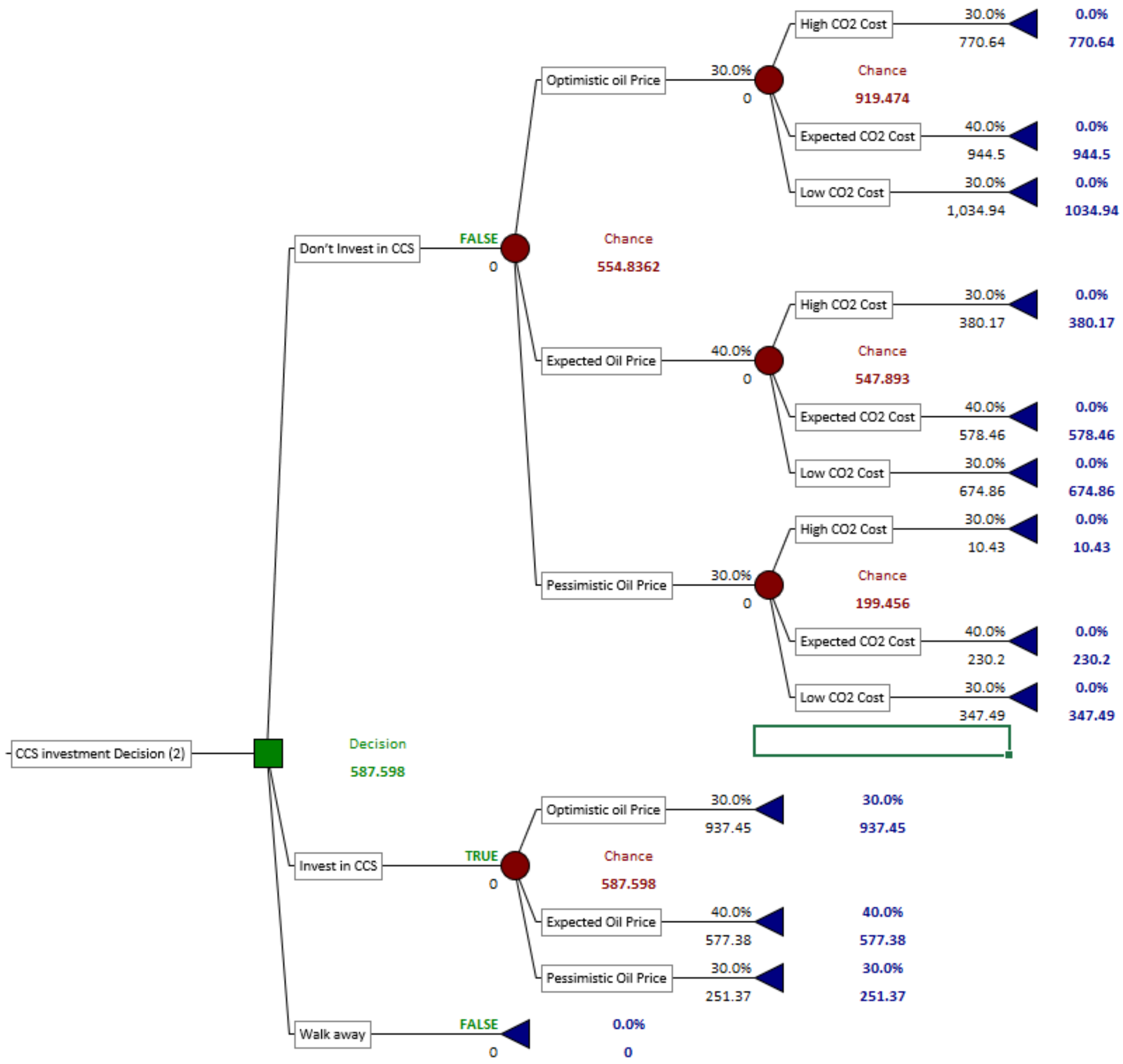

Figure 7: Decision Tree showing the NPVs, assign probabilities and EMV

The decision tree above shows that out of the three options (i.e., invest in CCS, Don't invest in CCS or Walk away) to decide from, the option of investing in CCS creates more value than the rest. As shown in the tree, investing in CCS has the potential to return $587.60 million which is approximately $34 million more valuable than the case without CCS. The Not investing in CCS case doesn't only return a low valuation compared to the case of investing in CCS but also contributes more to the current devastating challenge of global warming affecting the whole world. Finally, should the management decided not to execute the options and walks away, they aren't only walking away with nothing but also losing the opportunity to create greater value for their shareholders.

Furthermore, in a sensitivity analysis using goal seeks, we try to evaluate the Capital Expenditure cost at which the two scenarios will have similar expected monetary values. The result of the analysis reveals that at the same operating expenditure of $8/bbl for the case involving CCS and $6/bbl for the case without CCS, the EMV of the two scenarios will be equal when the CAPEX of the project with CCS is approximately $250 million. This means until the CAPEX for installing CCS facilities cost $50 million more than the current value used in the analysis, the suitable course of action will always be to invest in CCS. In addition to that, investing in CCS has the tendency of improving the project revenue stream. This can be achieved when the captured CO_2 is utilized and stored at the same time via a technique used in the oil and gas called CO_2 Enhanced Oil Recovery (CO_2-EOR) or CO_2 Water Alternating Gas Recovery (CO_2-WAGR), which leads to an improvement in the cumulative oil produced. Whenever CO_2 is injected into a hydrocarbon-bearing reservoir, the mobility of the hydrocarbon increases either due to the pistol-like displacement effect that pushes the hydrocarbon away from the injector well to the producer well or due to the ability of CO_2 to dissolve in hydrocarbon as such makes it lighter and hence easier to flow.

It is crystal clear that investing in CCS in the example above is the best decision to be made to improve shareholder value and reduce the impact of CO_2 emission on global warming. As shown in the example, it was easier for the management to decide on the best course of action by accounting for the cost of emissions in the company's economic valuation.

Conclusion

In this paper, we use the (Schwartz & Smith, 2000) two-factor price model to forecast the spot prices of carbon. Employing the technique used in (Jafarizadeh, 2022) we estimate the forecast parameters from carbon futures and implied volatility using the call and put options on carbon futures. The result of the forecast indicates a contango behaviour for the future spot prices of carbon.

We then carry out a sensitivity analysis on the forecasted price obtained using the conventional technique and another technique known as the Distribution of sum Discounted price approach. Contrary to the conventional approach, the DSDP approach takes into cognizance of all the simulated price paths in this case a thousand (1000) sample prices against the conventional method that multiples the expected (mean) by a negative and positive fraction to construct its confidence band. Our finding reveals that because the conventional approach is built based on one realization, it is not efficiently reliable compared to the DSDP approach whose outcome is carved out of a distribution of all the simulated sample price paths. It was found in our study that both the P90 and expected price obtained from the conventional approach overestimates the forecasted prices of carbon while the P10 underestimates the carbon price compared to the DSDP approach.

Finally, we use the DSDP forecasted price of Carbon in the cash flow analysis of Alpha to assist its managers in deciding whether to invest in the CCS project or not. Because we use different uncertain oil prices and costs of carbon in the analysis, we came up with twelve (12) different NPVs for not investing in the CCS project and three (3) for the option to invest in CCS. Incorporating those NPVs into a decision tree, we estimate the EMVs. The results reveal that investing in CCS in this scenario is preferred, this is because the EMV of investing in CCS overshadow the rest. In a sensitivity analysis we also show that until the CAPEX of investing in CCS increases by about $50 million, the preferred decision will always be to invest in CCS.

Acknowledgement

I would like to acknowledge Dr, Babak Jafarizadeh for his comments and contributions. I will also like to acknowledge the Petroleum Technology Development Fund (PTDF) Nigeria for the sponsorship. Finally, I wish to thank Computer Modelling Group CMG.

Nomenclature

EU-ETS	= European Union Emission Trading Scheme
UK-ETS	= United Kingdom Emission Trading Scheme
RGGI	=Reginal Greenhouse Gas Initiative
STLT	= Short-Term-Long-Term
EUA	= European Union Allowance
CO_2	=Carbon dioxide
CCS	= Carbon Capture and Storage
CO_2-EOR	= Carbon dioxide Enhanced Oil Recovery
CO_2-WAGR	= Carbon dioxide Water Alternating Gas Recovery
NPV	= Net Present Value
CAPEX	= Capital Expenditures
EMV	= Expected Monitory Value
DSDP	= Distribution of Sum Discounted Price
GBM	= Geometric Brownian Motion
MA	= Moving Average
AR	= Arithmetic Mean
ARIMA	= Autoregressive Integrated Moving Average
ARCH	= Autoregressive Conditional Heteroskedasticity
GARCH	= Generalized Autoregressive Conditional Heteroskedasticity

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Appendix

Table A1: Simulation data for CO_2-WAG EOR and waterflooding along with price forecast data.

	YEARS																				
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
OIL, WATER AND CO2 DATA FROM SIMULATION																					
Oil Prod (bbl) x10⁶		3.09	2.63	2.46	2.61	2.58	2.70	2.54	2.28	2.08	1.89	1.79	1.62	1.45	1.29	1.23	1.13	1.05	0.98	0.94	0.90
Water Prd (bbl) x10⁶		8.6	8.06	8.44	9.01	8.74	8.44	9.15	9.94	10.52	10.77	11.09	11.44	12.05	12.07	12.15	12.36	12.49	12.48	12.55	12.58
CO_2 (tons) x10³		2.56	2.32	3.13	17.9	37.43	82.15	117.80	142.85	167.90	178.0	193.76	206.83	224.28	227.09	236.06	246.40	254.92	257.14	260.21	265.21
CAPEX (Million$)	-200																				
FORECAST DATA																					
Exp Oil forecast $/bbl		69.31	71.23	72.62	73.72	76.46	75.53	76.35	77.15	77.95	78.74	79.54	80.34	81.15	81.97	82.8	83.63	84.47	85.32	86.10	86.75
Optimistic Oil forecast $/bbl		92.55	98.18	101.9	104.4	106.27	107.8	109.15	110.39	111.58	112.75	113.91	115.07	116.23	117.4	118.59	119.78	120.99	122.2	123.38	124.26
Pessimistic Oil forecast $/bbl		46.85	46.25	46.13	46.26	46.54	46.91	47.32	47.77	48.23	48.71	49.19	49.68	50.18	50.68	51.19	51.71	52.23	52.75	53.26	53.26
[3]High CO_2 Cost forecast $/ton		92.62	113.91	131.98	145.90	158.23	170.14	182.23	194.76	207.92	221.84	236.65	252.39	269.14	286.98	306.05	326.34	347.96	371.02	395.66	421.88
[4]Expected CO_2 Cost forecast $/ton		83.19	82.13	81.57	82.95	85.97	90.20	95.33	101.15	107.56	114.52	122.03	130.06	138.64	147.81	157.62	168.06	179.19	191.06	203.74	217.25
[5]Low CO_2 Cost forecast $/ton		73.15	54.80	44.72	40.91	40.06	40.75	42.35	44.53	47.13	50.04	53.24	56.71	60.42	64.40	68.67	73.21	78.06	83.22	88.75	94.63

[3,4,5] The price of carbon estimated is in Euros/ton which needs to be converted to \$/ton. As of the time of writing this report, \$1 = £1 (Xe.com, 2022)