1. Introduction

Negative environmental phenomena, such as global warming, pollution, or depletion of natural resources are often associated with production externalities. Environmental regulation may then be needed in order to cope with this market failure. However, the regulation itself may inflict welfare losses by changing the market structure of regulated industries.

A significant strand of the literature on environmental policy has focused on how regulation may affect the industry equilibrium under different endogenous market structure assumptions. The main finding in these studies is that the internalization of the external damages generated by production externalities depends on the degree of market competition. This is because the regulator must take into account the welfare losses that under imperfect competition may be due to output distortions and suboptimal market entries.[1] Yet, so far, in this literature the analysis has been developed only using static models, abstracting thus from the consideration of the dynamic nature of the process leading to the industry equilibrium and of the role played by market uncertainty and irreversibility.

In order to start filling this void, in this paper, we set up a model analyzing the industry equilibrium under perfect competition in a dynamic setup where market demand is stochastic and entry is irreversible. Production entails pollution, generating an external damage for society which is assumed increasing and convex in the market output. We include regulation by considering the following two polar policy instruments for emission control: (i) a quantity control exerted by introducing a cap on market output; (ii) a price control exerted by imposing an emission tax. We determine the optimal entry strategy set by private firms acting in a decentralized setting under both policies and then the cap level and the tax rate maximizing welfare. We compare the two policies from a welfare-maximizing perspective and find that: (i) the tax policy strictly dominates the cap policy; (ii) the optimal tax secures the complete internalization of the external damage associated with production.

Our analysis finds a solid basis in the literature on irreversible investment under uncertainty (see Dixit and Pindyck, 1994, for a thorough illustration of theory and

[1]See, for example, Spulber (1985) Katsoulacos and Xepapadeas (1995, 1996), Requate (1997), Lee (1999), Lahiri and Ono (2007), and the survey by (Millimet et al., 2009).

applications). In one of the more important studies in this literature, Leahy (1993) has shown, in a decentralized competitive setting, that firms invest (in order to enter the market) sequentially. In addition, he also shows that, due to the uncertainty characterizing future profits and the irreversibility of the investment decision, they invest only when the output price is above the sum of user cost of capital and uncertainty premium. In a similar model set-up, the same result is obtained by Bartolini (1995) which considers the implications of a cap on the market output for firms' market entry and welfare However, the welfare analysis therein is not conclusive since the level of the cap is taken as exogenous and no external damages associated with firms' investment and production are explicitly considered. This gap has been filled by Di Corato and Maoz (2019) where an external damage linearly increasing in the market output is included in the welfare objective and the cap is set endogenously. They show that a welfare-maximizing cap policy can only take one of the following two forms: (i) immediately banning further market entries by setting the cap at the current level of market output; or (ii) to have no cap at all. The choice depends on the level of uncertainty charactering the firm's profits. In fact, when the output price triggering firms' entry is, due to the consideration of the uncertainty, above the social marginal cost of production, no cap should be introduced as further entries raise welfare. In contrast, when the uncertainty premium is too small to counterbalance the external damage, banning further market entries is optimal because these entries would occur at a price below the social marginal cost. The linearity of the external damage in the market output is of course crucial for this bang-bang result. In contrast, in our model we take the alternative but rather standard assumption that the external damage is an increasing and strictly convex function of the market output. Consequently, in our model, an optimal finite cap may also emerge.

As in Bartolini (1995), we find that the presence of a cap does not affect the optimal entry strategy set at firm level, which remains identical to the one set in the absence of regulation. We then identify the circumstances in which a finite cap is optimal from a welfare-maximizing perspective. We show that the key element is the gap between the marginal external damage and the marginal surplus gain associated with a new entry. If at the current level of market output, in response to new market entries, the external damage grows more than the surplus, a ban on further entries should be imposed. Otherwise, a finite cap above the current level of market output should be set. We find that the cap is increasing in the level of market uncertainty. This confirms, once again, the counterbalancing effect that, in the presence of an external damage, the uncertainty premium may have.

The analysis of the tax policy is straightforward. In this case, the model by Leahy (1993) applies perfectly since the introduction of an emission tax is increasing the cost of production. Market entries are sequential but occur later (in expected terms), compared to the case where the industry is not regulated. This is because the price threshold triggering market entries is increasing in the cost of production. We then determine the tax rate maximizing welfare and find that at each time period it must be set equal to the marginal external damage associated with the market output in that time period. This implies that at each new entry, as the market output increases, the tax rate increases as well. This in turn implies that, as the market output increases, new entries become less likely since the price threshold triggering them increases as well.

When comparing a cap policy with a tax policy, an evident trade-off emerges. With quantity control, we have a temporal evolution of the market output which is bounded by the cap but the policy does not affect the timing of market entries. In contrast, with price control, there is no limit to market entries but the policy affects the timing of market entries by delaying them in expected terms.

For a regulator maximizing welfare, a market entry is desirable as far as the associated surplus gain covers its marginal social cost. In our set-up, there is always a time point where this condition is met. Therefore, the ideal policy should be one able to delay market entries so that they occur at the "right" time from the regulator's perspective. We show that this is feasible only via price control by equating at each time point the tax rate to the marginal external damage. This allows a complete internalization of the external damage when setting the entry strategy at firm level and, consequently, firms enters the market following a first-best time trajectory. In contrast, a cap policy, even though it can limit the external damages without delaying market entries, causes society the loss of potential welfare gains since no further entries are allowed once the cap has been reached. We show that this loss is higher than the one due to delayed market entries under the tax policy and then conclude that the tax policy dominates the cap from a welfare-maximizing perspective one.

The paper remainder is as follows. In Section 2, we present our model set-up. In Section 3, we determine the industry equilibrium under no policy intervention. In Section 4, we introduce the two instruments for emission control and determine the optimal entry strategy under each policy. We determine the optimal cap and the optimal tax rate, compare the associated welfare levels and discuss our findings. Section 5 offers some additional remarks on the results, and Section 6 concludes.

2. The basic model

Within a continuous time setting, we consider a competitive industry comprised of a large number of identical firms that producing a certain good. Their individual size, *dn*, is infinitesimally small with respect to the market and they are all price takers.[2]

At each time point $t \geq 0$, the demand for this good is given by:

$$P_t = X_t \cdot \phi(Q_t), \quad (1)$$

where P_t and Q_t are the price and quantity of the good, respectively, $\phi(Q_t)$ is a deterministic component of the market demand with $\phi(Q_t) > 0$ and $\phi'(Q_t) < 0$ for any $Q_t > 0$, and $\lim_{Q_t \to \infty} \phi(Q_t) = 0$. The term X_t, is a demand shift factor that evolves stochastically over time according to the following Geometric Brownian Motion:

$$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dZ_t, \quad (2)$$

where μ is the drift parameter, σ is the instantaneous volatility, and dZ_t is the increment of a standard Wiener process uncorrelated across time and satisfying $E(dZ_t) = 0, \quad E(dZ_t)^2 = dt$ at each time point.

Each firm rationally forecasts the future evolution of the whole market. Market entry is free and an idle firm can enter the market at any time. By entering the market, the firm commits to permanently offer one unit of the good at each *t*. This implies that the market output, Q_t, equals the number of active firms in the industry. Producing one unit of the good has a cost equal to $M > 0$.

Production entails a negative externality, i.e. pollution, generating an external damage for society. For simplicity, we assume one unit of emissions for each unit of the produced good. This implies that aggregate emissions equal the market output Q_t.

[2] Firms of infinitesimally small size is a standard assumption in models investigating the competitive equilibrium in a dynamic setting. See for instance Jovanovic (1982), Hopenhayn (1992), Lambson (1992), Leahy (1993), Dixit and Pindyck (1994, Ch. 8), Bartolini (1993, 1995) and Moretto (2008).

Further, no abatement technology exists. The external damage caused by these emissions is a function of their aggregate level, denoted by $D(Q_t)$. We take the standard assumptions that $D(0)=0$, $D(Q)$, $D'(Q_t)>0$ and $D''(Q_t)>0$ for any $Q_t>0$, implying that the external damage is positive, increasing and convex in the market output.

Last, firms are risk-neutral profit maximizers and discount future payoffs using the interest rate r.[3] As standard in the literature, we assume that $r > \mu$ to secure that the firm's value is finite.

3. Industry equilibrium under no policy intervention

Let start by considering a scenario where no emission control policies pollution are present. Under our model setup, a firm contemplating market entry is facing the same situation as the investors in Leahy (1993). Therefore, in the following, we use Leahy's analysis in order to determine the optimal entry strategy.[4]

At each time point, an idle firm has to decide whether to enter the market or not. By assumption, a firm entering the market commits to permanently produce one unit of the good at a cost equal to M. The present value of the associated flow of production costs, i.e. M/r, can be viewed as the irreversible investment that a firm must undertake in order to enter the market. As future revenues are uncertain, market entry will occur when the expected profitability of such investment is sufficiently high.

Let $V(X,Q)$ be the value of an active firm given the current levels of X and Q. The standard no-arbitrage analysis in Appendix A shows that

(3) $$V(X,Q)=Y(Q)\cdot X^{\beta}+\frac{P(X,Q)}{r-\mu}-\frac{M}{r},$$

where $\beta>1$ is the positive root of the quadratic equation

(3.1) $$\tfrac{1}{2}\cdot\sigma^2\cdot x^2+\left(\mu-\tfrac{1}{2}\cdot\sigma^2\right)\cdot x-r=0.$$

[3] Note that introducing risk aversion would not change our results, but merely require the development of the analysis under a risk-neutral probability measure. See Cox and Ross (1976) for further details.
[4] In the following, we will drop the time subscript for notational convenience.

In (3), the term $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$ represents the expected present value of the flow of the firm's future profits conditional on Q remaining forever at its current level. Therefore, the first term, $Y(Q)\cdot X^{\beta}$, accounts for how future market entries reduce the value of the firm. This is because the market output Q increases as new firms enter the market and, consequently, the firm's profit lowers.

Two boundary conditions are required for finding the threshold function $X^*(Q)$ triggering market entry. The first one is the *Value Matching Condition*:

(5) $$V\left[X^*(Q), Q\right] = 0 \ ,$$

and the second one is:

(6) $$V_X\left[X^*(Q), Q\right] = 0 \, .$$

Condition (5) is a standard zero-profit condition at the entry, that is, the value of an idle firm, which is null under free entry, must equal the value of an active one. Condition (6) concerns instead the evolution of the demand shift, X_t, over time (see Dixit and Pindyck, 1994, Ch. 8, pp. 252-260). Each time the process $\{X_t, t \geq 0\}$ hits the threshold $X^*(Q)$ a new firm enters the market and the price of the good, $P(Q)$, lowers since the supplied market output has increased. Thus, $X^*(Q)$ is an upper reflecting barrier regulating the process $\{X_t, t \geq 0\}$ by keeping its level over time below $X^*(Q)$.

Proposition 1*:* Entry in a perfectly competitive market occurs every time the process $\{X_t, t \geq 0\}$ hits the barrier:

(7) $$X^*(Q) = \frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r}}{\phi(Q)} \ ,$$

where $\hat{\beta} \equiv 1 + \frac{1}{\beta - 1} > 1$.

Proof: Follows from applying (3) in (5) and (6). □

The threshold $X^*(Q)$ is an increasing function of Q since the larger the market output, the stronger the competition due to the higher the number of active firms and, ceteris paribus, the higher the expected profitability required for entering the market. Note that, by the Marshallian rule, a firm should enter the market as long as $P(Q) = X \cdot \phi(Q) \geq (r - \mu) \cdot \frac{M}{r}$. Hence, the term $\frac{1}{\beta - 1} > 0$ is the wedge by which the entry price should be adjusted in order to take the uncertainty and irreversibility into account (see Dixit and Pindyck, 1994, Ch. 5, Section 2).

Last, note that $dX^*(Q)/d\sigma^2 < 0$ since $dX^*(Q)/d\beta < 0$ and $d\beta / d\sigma < 0$. This means that the higher the demand volatility, the higher the threshold triggering firm's entry, which implies that market entry is delayed in expected terms.

4. Industry equilibrium under policy intervention

The optimal entry strategy based on Eq. (6) does not account for the external damage associated with the flow of emissions that production entails once the firm has entered the market. In this section, we consider two policies for the reduction of the external damage: i) a cap on aggregate emissions and ii) an emission tax on each unit of emissions. We first determine the industry equilibrium under each policy and then the level of the cap and the tax rate, respectively, maximizing welfare.

4.1 Industry equilibrium and welfare under a cap on aggregate emissions

Assume that the government sets a cap on aggregate emissions. In our model, this is equivalent to setting a cap, $\overline{Q} \geq Q$, on the market output and, consequently, the number of firms active in the industry since, by assumption, we have one unit of emissions for each unit of production. Further, assume that entry/emission licenses are distributed when the cap is announced. Each license allows producing one unit of output and their number is equal to difference between the cap, $\overline{Q}$, and the current level of market outputmarket quantity, Q. We abstract from how the licenses are distributed since for our purposes their distribution has no other implications than providing to each firm owning a license the right to enter the market.[5]

[5] Note that, as shown by Bartolini (1995), the government may fully extract producer's surplus through a competitive auction of the licenses.

4.1.1 The optimal entry strategy

The analysis of the firm's optimal entry under rationing is technically similar to the analysis in Section 3. The relevant difference between the two cases is that under licensing the option to enter is an asset having a positive value that the firm gives up by entering the market. Thus, alongside the function $V(X, Q, \overline{Q})$ which represents the value of an active firm, we define the function $F(X, Q, \overline{Q})$ which stands for the value of the option to enter the market. A standard no-arbitrage analysis, similar to the one conducted in Appendix A for determining $V(X, Q, \overline{Q})$, yields:

(8) $$F(X, Q, \overline{Q}) = H(Q, \overline{Q}) \cdot X^{\beta},$$

(9) $$V(X, Q, \overline{Q}) = Y(Q, \overline{Q}) \cdot X^{\beta} + \frac{P(X, Q)}{r - \mu} - \frac{M}{r},$$

where $H(Q, \overline{Q})$ is to be found alongside the threshold $X^{*}(Q, \overline{Q})$ by imposing the following *Value Matching Condition*:

(10) $$V\left[X^{*}(Q, \overline{Q}), Q, \overline{Q}\right] = F\left[X^{*}(Q, \overline{Q}), Q, \overline{Q}\right],$$

and *Smooth Pasting condition*:

(11) $$V_X\left[X^{*}(Q, \overline{Q}), Q, \overline{Q}\right] = F_X\left[X^{*}(Q, \overline{Q}), Q, \overline{Q}\right].$$

By Condition (10), we require that the value of the option to enter, that is, the implicit cost of market entry, equals the value of an active firm, that is, the implicit return associated with market entry. Condition (11) secures optimality by imposing that the marginal cost of market entry equals its marginal return.[1] Let recall that, as shown by Dixit (1993), Condition (10) holds for any entry threshold and merely reflects a no arbitrage assumption, while Condition (11) is an optimality condition which holds only at the optimal threshold.

Proposition 2: Entry in a perfectly competitive market under a cap on aggregate emissions $\overline{Q} \geq Q$ occurs every time the process $\{X_t, t \geq 0\}$ hits the barrier:

(12) $$X^*(Q, \overline{Q}) = X^*(Q) = \frac{\hat{\beta} \cdot (r - \mu) \cdot \frac{M}{r}}{\phi(Q)}.$$

Proof: Follows from applying (8) and (9) in (10) and (11). □

Notably, the threshold function (12) does not depend on $\overline{Q}$ and is equal to the threshold function (6) determined under no policy intervention. The relevant difference here is that $X^*(Q)$ applies only until the cap $\overline{Q}$ is reached. The same result is found by Bartolini (1995) which explains it by highlighting the crucial role played by the presence of entry licenses. In fact, since there is no threat of being preempted by others, firms holding a license may optimally exercise their option to invest. Otherwise, if entries are not rationed, firms will gradually enter the market by (12) only up to a certain time point, then a competitive run will start and the cap will be instantly reached (see Bartolini, 1995, Section 5).

4.1.2 Welfare and the optimal cap

Once determined the industry equilibrium, in this section we determine the cap level maximizing welfare. This optimal level will trade off the welfare gains associated with lower emissions and the corresponding losses, in terms of surplus, due to a lower quantity of the good available on the market.

Following a procedure similar to the one conducted in Appendix A for determining the value of an active firm, the expected discounted social welfare, given the current levels of X, Q and $\overline{Q}$, is:

(13) $$W(X, Q, \overline{Q}) = C(Q, \overline{Q}) \cdot X^{\beta} + \int_0^Q \left[\frac{P(X, q)}{r - \mu} - \frac{M}{r} \right] \cdot dq - \frac{D(Q)}{r},$$

where the expected present value of the impact on welfare associated with future market entries, the second term represents the expected present value of the net surplus flow associated with the current level of market output Q, that is, the surplus resulting from the supply of those units minus their production cost, while the third term is the present value of the flow of external damages associated with the current level of market output *Q*.

At $X^*(Q)$ the following *Value Matching Condition* must hold:

(14) $$W_Q\left[X^*(Q), Q, \overline{Q}\right] = 0 .$$

Condition (14) is a standard boundary condition stating that at each market entry the marginal welfare gain must equal the marginal welfare loss.

Further, at $Q = \overline{Q}$ we must impose that:

(15) $$C\left(\overline{Q}, \overline{Q}\right) = 0 .$$

The intuition behind Condition (15) is immediate. In (12), the term $C\left(Q, \overline{Q}\right) \cdot X^{\beta}$ captures the welfare associated with future increases of the market output. No such changes are possible if *Q* has reached the cap $\overline{Q}$ and then the term $C\left(Q, \overline{Q}\right)$ must be null at $Q = \overline{Q}$.

Based on (13), (14) and (15) we show in Appendix B that:

(16) $$C\left(Q, \overline{Q}\right) = \int_{Q}^{\overline{Q}} \frac{(\hat{\beta} - 1) \cdot M - D'(q)}{r} \cdot X^*(q)^{-\beta} \cdot dq .$$

Differentiating $C\left(Q, \overline{Q}\right)$ with respect to $\overline{Q}$ yields:

(17) $$C_{\overline{Q}}\left(Q, \overline{Q}\right) = \frac{(\hat{\beta} - 1) \cdot M - D'\left(\overline{Q}\right)}{r} \cdot X^*\left(\overline{Q}\right)^{-\beta} .$$

We denote the price at which entry occurs by P^*. From (1) and (12) it follows that:

(18) $$P^* = X^*(Q) \cdot \phi(Q) = \hat{\beta} \cdot (r - \mu) \cdot \tfrac{M}{r}$$

Applying (18) in (17) and rearranging terms, yields:

(19) $$C_{\overline{Q}}\left(Q,\overline{Q}\right)=\left[\frac{P^*}{r-\mu}-\frac{M+D'\left(\overline{Q}\right)}{r}\right]\cdot X^*\left(\overline{Q}\right)^{-\beta}.$$

Proposition 3:

(a) If the current level of market output, *Q*, is sufficiently large so that $\frac{P^*}{r-\mu}\leq\frac{M+D'(Q)}{r}$ then it is optimal to set the cap at the current *Q*, i.e., to immediately ban any further market entry.

(b) otherwise, if the current level of market output, *Q*, is sufficiently small so that $\frac{P^*}{r-\mu}>\frac{M+D'(Q)}{r}$ then the optimal level of the cap, denoted by $\overline{Q}^*$, is the root of the following equation:

(20) $$\frac{P^*}{r-\mu}=\frac{M+D'\left(\overline{Q}^*\right)}{r},$$

Proof: Follows from (19) taken together with *D*”(*Q*) > 0 for any *Q* > 0. □

Based on *Proposition 3* and (13), in the case where the optimal cap is at the current *Q*, the expected discounted social welfare is equal to:

(21) $$W^{cap}\left(X,Q\right)=\int_0^Q\left[\frac{P(q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq,$$

otherwise, when the optimal cap is $\overline{Q}^*$, the expected discounted social welfare is:

(22) $$W^{cap}\left(X,Q\right)=\int_Q^{\overline{Q}^*}\frac{(\hat{\beta}-1)\cdot M-D'(q)}{r}\cdot\left[\frac{X}{X^*(q)}\right]^{\beta}\cdot dq$$
$$+\int_0^Q\left[\frac{P(q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq.$$

By Propositions (2), a new firm enters the market every time the process $\{X_t, t \geq 0\}$ hits the threshold $X^*(Q)$ until the cap $\overline{Q}$ is reached. Thus, the reflecting barrier $X^*(Q)$ regulates the process $\{X_t, t \geq 0\}$ only up to $\overline{Q}$. This implies that, until $\overline{Q}$ is reached, the price of the good, $P(Q)$, remains below P^*. This happens since when a new firm enters the market, the supplied market output increases and the price of the good lowers. Once $\overline{Q}$ has been reached, the process $\{X_t, t \geq 0\}$ is unregulated and the price of the good moves freely over time and may, depending on the circumstances, also exceed the barrier level P^*. Figure 1 shows these dynamics.

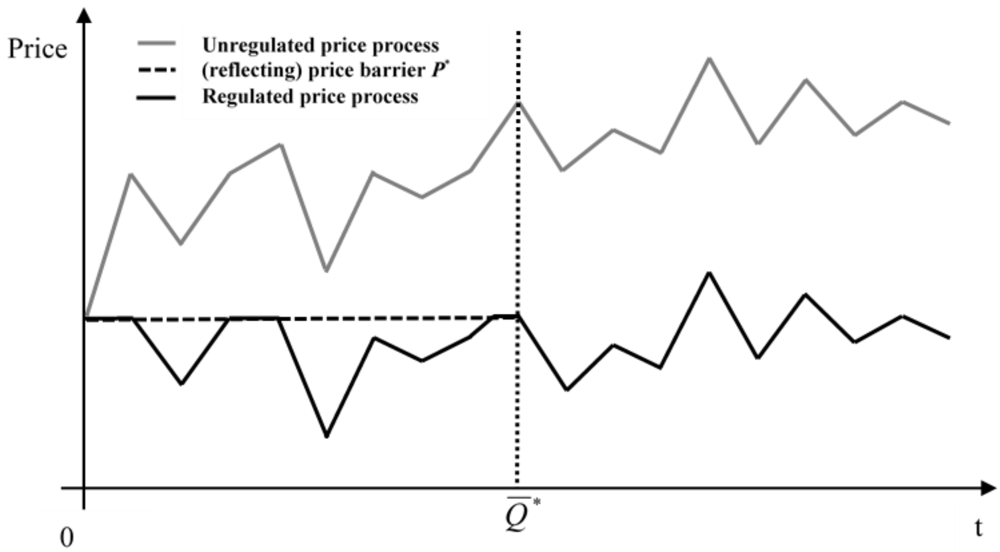

Figure 1: Price dynamics under a cap on aggregate emissions

By Proposition (3), if $\frac{P^*}{r-\mu} \leq \frac{M + D'(Q)}{r}$, a ban deterring any further market entry is optimal. This is because the expected present value of the flow of surplus added by the firm entering the market, i.e. $\frac{P^*}{r-\mu}$, does not cover the present value of the flow of social costs, i.e. $\frac{M + D'(Q)}{r}$, associated with the production of one more unit of the good. Otherwise, having a cap at a level higher than the current market output, i.e. $Q < \overline{Q}^*$, is optimal. In fact, note that in this case, since $D'(Q)$ is increasing in Q, $\frac{P^*}{r-\mu} > \frac{M + D'(Q)}{r}$ for any $Q < \overline{Q}^*$. Therefore, market entries are beneficial for welfare. Firms will then be allowed to enter the market until the market output $\overline{Q}^*$ is reached and where $\frac{P^*}{r-\mu} = \frac{M + D'\left(\overline{Q}^*\right)}{r}$.

Implicit differentiation of (20) yields that:

(23) $$\frac{d\overline{Q}^*}{dM} = \frac{1}{D''\left(\overline{Q}^*\right)} \cdot \frac{1}{\beta - 1} > 0,$$

where the inequality follows from $D''(Q) > 0$ and $\beta > 1$. Thus, the higher the production cost the larger the optimal cap and therefore the larger the market size that the regulator is going to allow for. The reason for that is that the larger *M*, the higher the price that triggers entry for any given *Q* and the higher the expected welfare gains in terms of surplus, gains that are high enough to compensate for the external damage generated.

Implicit differentiation of (20), also yields:

(24) $$\frac{d\overline{Q}^*}{d\sigma^2} = -\frac{1}{D''\left(\overline{Q}^*\right)} \cdot \frac{M}{(\beta - 1)^2} \cdot \frac{d\beta}{d\sigma^2} > 0,$$

where the inequality follows from $D''(Q) > 0$, $\beta > 1$ and $\frac{d\beta}{d\sigma^2} < 0$. Thus, the higher the demand uncertainty the larger the optimal cap and the larger the market output that the regulator is going to allow for. The reason is that a higher σ^2 leads, via its effect on the option wedge $\hat{\beta}$, to a higher entry threshold for any given *Q* and therefore the expected welfare gains associated with a market entry are high enough compensate for the external damage generated.

Last, note that

(25) $$\lim_{\sigma^2 \to \infty} \overline{Q}^* = \infty,$$

which means that for a sufficiently high level of uncertainty, the expected welfare gains are so high that setting a finite cap, $\overline{Q}^*$, is not optimal.

4.2 Industry equilibrium and welfare under an emission tax

Assume that the government levies a tax $\tau > 0$ per unit of emissions. As above, in our model, we have one unit of emissions for each unit of production. Hence, taxing emissions is equivalent to taxing output.

4.2.1 Optimal entry strategy

The analysis of the industry equilibrium under emission taxation is technically identical to the one conducted in Section 3. The only difference is that here the cost for producing one unit of output is equal to $M + \tau$. Hence:

Proposition 4*:* Entry in a perfectly competitive market under an emission tax occurs every time the process $\{X_t, t \geq 0\}$ reaches the barrier:

$$(26) \qquad X^{**}(Q,\tau) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \dfrac{M+\tau}{r}}{\phi(Q)} > X^*(Q).$$

Proof: Follows from repeating the proof of *Proposition 1*, this time with a private marginal cost equal to $M + \tau$. □

By (26), the entry threshold is increasing in the tax rate τ, which implies, in expected terms, that market entries are slower than in the case where a cap is introduced.

Following the discussion above, the introduction of an emission tax $\tau > 0$ raises the price triggering entry at a level equal to:

$$(27) \qquad P^{**} = \hat{\beta} \cdot (r-\mu) \cdot \frac{M+\tau}{r} > P^*,$$

for any Q.

By Proposition 4, Equation (26) implies that with an emission tax the expected welfare gains at the entry are higher than the ones under a cap policy. However, since $X^{**}(Q,\tau) > X^{*}(Q)$, these higher gains occur later in expected terms and would be lower once discounted back to $\{t \geq 0 : X = X^{*}(Q)\}$. Further, under a tax policy there is no limit to market entries. Therefore, there is a level of the market output $\tilde{Q} > \overline{Q}^{*}$, where

$$(28) \qquad \frac{P^{**}}{r-\mu} < \frac{M + D'(Q)}{r},$$

for any $Q > \tilde{Q}$ with $\tilde{Q}$ solving the equation

$$(29) \qquad \frac{P^{**}}{r-\mu} = \frac{M + D'(\tilde{Q})}{r}.$$

This means that, when compared to the cap policy, the higher gains in terms of expected welfare accruing when $Q \leq \tilde{Q}$ will be followed by losses when $Q > \tilde{Q}$, losses that are not incurred under the cap policy as in that case entries stop at $\overline{Q}^{*}$.

In the light of these considerations, let proceed to the next section where the optimal tax rate is determined.

4.2.2 Welfare and the optimal tax rate

The expected discounted social welfare, given the current levels of *X* and *Q*, is:

$$(30) \qquad W(X,Q,\tau) = C(Q,\tau)\cdot X^{\beta} + \int_0^Q \left[\frac{P(q,X)}{r-\mu} - \frac{M}{r}\right]\cdot dq - \frac{D(Q)}{r}.$$

By setting the tax rate τ the government affects the timing of market entry because of the direct effect that the tax rate has on the entry threshold. The two boundary conditions for finding the optimal tax policy are the following *Value Matching Condition*:

(31) $$W_Q\left[X^{**}(Q,\tau),Q\right] = 0\,,$$

and *Smooth Pasting condition*:

(32) $$W_{QX}\left[X^{**}(Q,\tau),Q\right] = 0\,.$$

As in Section (3), Condition (31) is not an optimality condition but merely a no-arbitrage condition that holds for any entry threshold, not necessarily the optimal one. In contrast, Condition (32) is an optimality condition that leads to the entry threshold which is optimal from the regulator's perspective and to the tax rate that leads to this optimal threshold.

Applying (30) in (31) and (32) yields that the socially optimal threshold satisfies:

(33) $$X^{**}\left(Q,\tau^*\right) = \frac{\hat{\beta}\cdot(r-\mu)\cdot\dfrac{M + D'(Q)}{r}}{\phi(Q)}\,,$$

which leads to the following proposition regarding the socially optimal tax:

Proposition 5*:* The welfare maximizing tax rate is:

(35) $$\tau^* = D'(Q),$$

Proof: Follows directly from comparing (26) and (33). □

Eqs. (30) in (31) and (32) also yield that under the regulator's optimal policy:

(36) $$C_Q\left(Q,\tau^*\right) = -(\hat{\beta}-1)\frac{M + D'(Q)}{r}\cdot X^{**}\left(Q,\tau^*\right)^{-\beta}$$

A boundary condition in the regulator's problem is that at $Q = \overline{Q}$ the following limit holds:

(37) $$\lim_{Q\to\infty} C(Q,\tau) = 0.$$

The intuition behind Condition (37) is immediate. In (30), the term $C(Q,\tau)\cdot X^{\beta}$ captures the welfare associated with future increases of the market output Q. No such changes are expected when $Q \to \infty$ because in that case the entry threshold (26) goes to infinity since by assumption $\lim_{Q\to\infty}\phi(Q) = 0$.

Integrating (36) and applying (37) yields:

(38) $$C(Q,\tau^*) = \int_Q^{\infty} (\hat{\beta}-1)\cdot\frac{M + D'(q)}{r}\cdot X^{**}(q,\tau^*)^{-\beta}\cdot dq\,.$$

Applying (380 in (30) yields that the expected discounted social welfare when the tax rate is optimally set is equal to:

(39) $$W^{tax}(X,Q;\tau^*) = \int_Q^{\infty}\left[(\hat{\beta}-1)\cdot\frac{M + D'(q)}{r}\right]\cdot\left[\frac{X}{X^{**}(q,\tau^*)}\right]^{\beta}\cdot dq$$

$$+\int_0^{Q}\left(\frac{P(q)}{r-\mu} - \frac{M + D'(q)}{r}\right)\cdot dq$$

Notably, by rearranging (26), one may easily show that

(40) $$\frac{P^{**}}{r-\mu} = \hat{\beta}\cdot\frac{M + D'(Q)}{r} > \frac{M + D'(Q)}{r},$$

for any given Q.

By (40), market entries are always beneficial since the expected present value of the flow of surplus added by a firm entering the market, i.e. $\frac{P^{**}}{r-\mu}$, covers always the present value of the flow of social costs, i.e. $\frac{M+D'(Q)}{r}$, associated with the production of one more unit of the good. This is because at each entry the barrier level P^{**} is adjusted upward by taxing at a tax rate τ^{*} increasing in *Q* (see Figure 2). Therefore, market entries occur always at a price which is sufficiently high to secure a positive contribution to welfare.

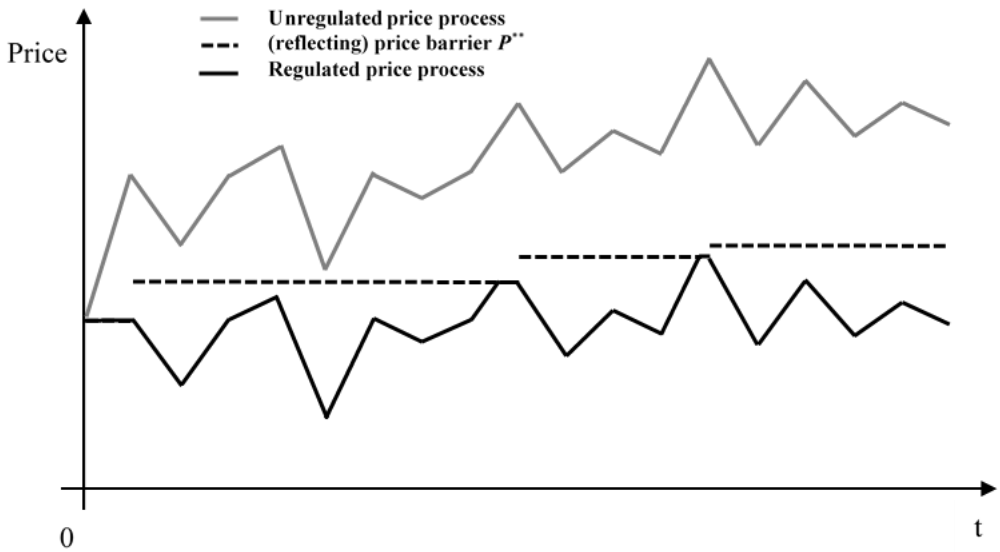

Figure 2: Price dynamics under emission taxation

Summing up, when comparing payoffs at the entry for any given *Q*, an emission tax secures always higher gains in terms of expected welfare with respect to the cap policy. However, one should keep into account that the entry process under emission taxation is slower in expected terms since $X^{**}(Q,\tau^{*}) > X^{*}(Q,\tau^{*})$. This means that these higher gains, once discounted back, may be significantly lowered.

Having this is in mind, let compare the expected discounted social welfare under the two policies. In Appendix C, we show that

Proposition 6*:* From a welfare maximizing perspective, taxing emissions at a rate $\tau^{*} = D'(Q)$ is strictly preferred to setting a cap $\overline{Q}^{*}$ on aggregate emissions since $W^{tax}(X,Q;\tau^{*}) > W^{cap}(X,Q;\overline{Q}^{*})$ for any $\overline{Q}^{*} \geq Q$.

Last, it is worth checking the optimality of the entry strategy under the two policies from a first-best perspective. In this respect, a natural benchmark is represented by the optimal entry strategy that would be set by a welfare-maximizing planner. In this case, the expected discounted social welfare, given the current levels of *X* and *Q*, is:

(41) $$W(X,Q)=C(Q)\cdot X^{\beta}+\int_0^Q\left(\frac{P(q,X)}{r-\mu}-\frac{M}{r}\right)\cdot dq-\frac{D(Q)}{r}$$

Denoting by $X^{CP}(Q)$ the threshold for market entry and maximizing (27) subject to:

(42) $W_Q\left[X^{CP}(Q),Q\right]=0$ (Value Matching Condition),

(43) $W_{QX}\left[X^{CP}(Q),Q\right]=0$ (Smooth Pasting Condition),

(44) $\lim_{Q\to\infty}C(Q)=0$,

Yields:

(45) $$X^{CP}(Q)=\frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M+D'(Q)}{r}}{\phi(Q)}=X^{**}(Q,\tau^*)>X^*(Q),$$

This allows us concluding that:

Proposition 7: Taxing emissions at a rate $\tau^*=D'(Q)$ is a first-best policy instrument while setting a cap $\overline{Q}^*$ on aggregate emissions is a second-best one.

5. Final remarks

Some final remarks are in order:

1) assume that each firm has a productive capacity allowing to produce up to a certain amount of output. For simplicity, let us normalize that maximum amount to 1. Assume that the regulator announces a cap $\overline{Q}$ on aggregate emissions and impose that each firm may produce not more than $0<\lambda<1$ units. As one may immediately see, introducing this variation in our model set-up would have no impact on our results. The

only thing that one should keep in mind is that in this case i) the number of active firms in the industry is equal to Q/λ and ii) the maximum number of firms entering the market is equal to $\overline{Q}/\lambda$;

2) Taxing emissions at the optimal rate implies that firms are taxed at a rate increasing in the market output. The principle of optimality of myopic behavior by Leahy (1993) applies also in this case. In fact, a competitive firm would keep entering the market as a myopic firm i) ignoring the truncation of the price process due to future market entries to the increase in the tax rate associated with those entries, and ii) assuming that they only hold an option to enter the market. As well-known, the first mistake implies an overestimation of the profitability of the market entry which should induce an earlier entry while the second one gives value to delaying entry. The two mistakes offset each other and the decision taken by a myopic firm is optimal.

6. Conclusions

In this paper, we have presented a model of endogenous market structure under uncertainty, with production externalities regulated by introducing a cap on market output or an emission tax. The main result is that the tax policy dominates the cap policy when aiming at the maximization of the welfare. Further, we show that the emission tax completely internalizes the external damage associated with pollution. We are aware that, concerning the complete internalization of the external damage, the assumption of perfectly competitive firms is crucial. It becomes then of interest, as potential lead for future research, extending the analysis in order to consider the impact that market power the ability to distort the output have on the degree of internalization and, potentially, on how the two policies should be ranked from a welfare-maximizing perspective.

APPENDIX

Appendix A – The value of an active firm

In this Appendix, we present the derivation of the value function in (3), i.e. $V(X,Q)$. By a standard no-arbitrage argument (see e.g. Dixit, 1989), $V(X,Q)$ is the solution of the following Bellman equation:

(A.1) $$r \cdot V(Q, X) \cdot dt = P(X,Q) - M + E[dV(X,Q)],$$

which states that the instantaneous profit, $P(X,Q) - M$, along with the expected instantaneous capital gain, $E[V(X,Q)]$, from a change in *X*, must be equal to the instantaneous normal return, $r \cdot V(X,Q) \cdot dt$.

Applying Itô's lemma and rearranging, we can restate (A.1) as follows:

(A.2) $$\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X,Q) + \mu \cdot X \cdot V_X(X,Q) - r \cdot V(X,Q) + P(X,Q - M = 0.$$

As standard, a solution of (A.2) takes the form:

(A.3) $$V(X,Q) = Z(Q) \cdot X^{\alpha} + Y(Q) \cdot X^{\beta} + \frac{P(X,Q)}{r - \mu} - \frac{M}{r},$$

where $\alpha < 0$ and $\beta > 1$ are the roots of the quadratic equation:

(A.4) $$\tfrac{1}{2} \cdot \sigma^2 \cdot x \cdot (x - 1) + \mu \cdot x - r = 0.$$

The term $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$ represents the expected present value of the flow of profits conditional on *Q* remaining forever at its current level. Therefore, the first and second term on the RHS of (A.3) should capture the impact that changes in *Q* over time have on the value of the firm in expected terms.

By the properties of the Geometric Brownian Motion, when *X* goes to 0 the probability of ever hitting the barrier triggering a new entry, i.e., X^*(Q), and, consequently, an increase in *Q*, tends to 0. This leads to the following limit condition:

(A.5) $$\lim_{X \to 0} \left[Z(Q) \cdot X^{\alpha} + Y(Q) \cdot X^{\beta} \right] = 0.$$

Note that as α < 0, (A.5) holds only if $Z(Q) = 0$ for any $Q > 0$. Hence, substituting $Z(Q) = 0$ into (A.3) gives (3).

Appendix B – Welfare maximization under a cap on aggregate emissions - Substituting the derivative of (13) with respect to Q in (14), applying (12), and rearranging terms, yields:

(B.1) $$C_Q\left(Q,\overline{Q}\right)=-\frac{(\hat{\beta}-1)\cdot M-D'(Q)}{r}\cdot X^*(Q)^{-\beta},$$

Integrating (B.1) yields:

(B.2) $$C\left(\overline{Q},\overline{Q}\right)-C\left(Q,\overline{Q}\right)=-\int_Q^{\overline{Q}}\frac{(\hat{\beta}-1)\cdot M-D'(q)}{r}\cdot X^*(q)^{-\beta}\cdot dq.$$

The term $C\left(Q,\overline{Q}\right)\cdot X^{\beta}$ in (13) captures the welfare associated with future increases of the market output. No such changes are possible if Q has reached the cap $\overline{Q}$. Therefore, the following boundary condition holds at $Q=\overline{Q}$:

(B.3) $$C\left(\overline{Q},\overline{Q}\right)=0,$$

Substituting (B.3) in (B.2) yields:

(B.4) $$C\left(Q,\overline{Q}\right)=\int_Q^{\overline{Q}}\frac{(\hat{\beta}-1)\cdot M-D'(q)}{r}\cdot X^*(q)^{-\beta}\cdot dq.$$

Appendix C – Proof of Proposition 6

In this appendix, we prove *Proposition 6* which states that welfare under emission taxation is larger than welfare under a cap on aggregate emissions. The proof is as follows:

(C.1) $$W^{tax}\left(X,Q^*\right)-W^{cap}\left(X,Q\right)$$

$$=\int_Q^{\infty}(\hat{\beta}-1)\cdot\frac{M+D'(q)}{r}\left[\cdot\frac{X}{X^{**}\left(q,\tau^*\right)}\right]^{\beta}\cdot dq$$

$$- \int_{Q}^{\overline{Q}^*} \frac{(\hat{\beta}-1)\cdot M - D'(q)}{r} \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq$$

$$> \int_{Q}^{\overline{Q}^*} (\hat{\beta}-1) \frac{M + D'(q)}{r} \cdot \left[\frac{M}{M + D'(q)} \right]^{\beta} \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq$$

$$- \int_{Q}^{\overline{Q}^*} \frac{(\hat{\beta}-1)\cdot M - D'(q)}{r} \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq$$

$$= \int_{Q}^{\overline{Q}^*} \frac{M + D'(q)}{r \cdot (\beta - 1)} \cdot g(q) \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq \geq 0 ,$$

where the first equality follows from (22) and (39), the first inequality follows from narrowing the range over which the first integral goes (as the integrand is positive), and from applying:

(C.2) $$X^{**}(Q, \tau^*) = \frac{M + D'(Q)}{M} \cdot X^*(Q)$$

which follows from (12) and (33). The second equality in (C.1) follows from applying $\hat{\beta} - 1 = \frac{1}{\beta - 1}$, rearranging terms, and defining:

(C.3) $$g(q) \equiv \left[\frac{M}{M + D'(q)} \right]^{\beta} - \beta \cdot \frac{M}{M + D'(q)} + \beta - 1.$$

The second inequality in (C.1) follows from $g(q)$ being positive for any $q > 0$ because $0 < \frac{M}{M + D'(q)} < 1$ which implies:

(C.3) $$g(q) > \frac{M}{M + D'(q)} - \beta \cdot \frac{M}{M + D'(q)} + \beta - 1 = (\beta - 1) \cdot \left[1 - \frac{M}{M + D'(q)} \right] > 0$$

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