Investment and leverage with caps and floors*

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Abstract

This paper studies investment timing and leverage decisions of firms under caps and floors. Caps and floors have significant effects, not only on investment timing and firm value, but also on leverage ratios and credit spreads. With leverage, a floor has a moderate effect if below a critical level. Above that level, the firm tends to issue less risky debt, being even able to issue risk-free debt, and the floor has a more significant effect, accelerating investment. A lower cap deters investment, increases leverage, but reduces credit spreads. When combined with a floor, in a collar regime, the effects of the cap become non-monotonic, as the firm is able to issue risk-free debt for intermediate levels of the cap. Uncertainty always deters investment, but the effects on leverage and credit spreads are non-monotonic.

Keywords: Floors; Caps; Capital structure; Investment decisions; Real options
JEL codes: G31; G32; D81

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1 Introduction

Caps and floors can be used as government policies to induce investment and regulate firms. Price or revenue floors offer protection against the downside risk. Caps can be used to mitigate excessive firm market power or, when combined with floors, in a collar regime, to reimburse part of the protection offered by floors. Regarding investment incentives, these arrangements are alternatives to investment subsidies and taxation with significant differences, as they modify the cash-flows risk and do not require an upfront payment.

Collars can be seen as generic arrangements, encompassing pure floor (for an infinite cap) and pure cap (for a null floor) regimes, fixed subsidies (when the cap equals the floor), an even the market regime (for an infinite cap and a null floor). Under uncertainty, collars, caps and floors are better evaluated within a real options framework, given their option-like characteristics.

The study of the effects of caps and floors on investment timing decisions using a real options approach dates back to Dixit (1991), who studies price caps. Several authors extended his analysis of using caps for regulation purposes. Dobbs (2004) show that even if the price cap is optimally chosen, a monopolist under-invests and imposes quantity rationing to the customers. Roques and Savva (2009) show that price cap regulation can accelerate investment, but a low price cap can deter investment. The regulation of both prices and quantities with caps is studied by Evans and Guthrie (2012). Literature on the combination of price floors and caps (collars) is more sparse. Dixit and Pindyck (1994) extend the analysis of Dixit (1991) for both caps and floors. Collar arrangements have recently been studied by Adkins and Paxson (2017, 2019), for the case of perpetual collars, and Adkins et al. (2019), for the case of finite-lived and retractable collars.

In the previous literature, the impact of caps and floors is restricted to investment timing decisions. The use of leverage is largely ignored. Sarkar (2016) is one of the few exceptions. However, while he studies the effect of caps on leverage decisions and capacity choice, investment timing decisions are ignored.

The aim of this paper is to analyze the effects of caps and floors on both investment timing and leverage decisions. For that purpose it extends previous collar valuation models that only considered equity financing, to include debt financing. Floors and caps have significant effects. Floors accelerate investment and reduce credit spreads but only significantly after a certain level. The effect on leverage in non-monotonic, with smaller leverage ratios for intermediate floors. A lower cap deters investment, increases leverage, but reduces credit spreads. When combined with a floor (in a collar regime) the effects of the cap become non-monotonic. A higher uncertainty always deters investment, but the effects on leverage and credit spreads are non-monotonic. A higher expected earnings growth rate accelerates investment, increases leverage and reduces credit spreads, except for low growth rates that induce risk-free debt issuance.

The remainder of the paper is organized as follows: Section 2 presents the benchmark model of the plain investment opportunity without caps and floors; Section 3 extends the model to consider caps and floors for both the unlevered and levered firm; A comparative statics analysis is performed in Section 4, and Section 5 concludes.

2 Investment and leverage decisions for a plain project

Consider a firm with a proprietary investment opportunity that generates a stream of cash flows (EBIT) subject to a shock modeled by means of a geometric Brownian motion, i.e.:

$$dX(t) = \alpha X(t)dt + \sigma X(t)dW \tag{1}$$

where $X(0) = X > 0$, α (with $\alpha < r$) is the risk-neutral drift, r is the risk-free interest rate, σ is the instantaneous volatility, $dW(t)$ is the increment of a Wiener process. Alternatively, for price controls regimes, the stochastic variable could be the market price.

Unlevered firm

For an unlevered firm, the contingent investment opportunity was first valued by McDonald and Siegel (1986). Following a similar approach, let us assume that the instantaneous after-tax profit of the firm is:

$$\pi(t) = X(t)(1-\tau) \tag{2}$$

where τ is the corporate tax rate.

The unlevered firm's investment of a sunk cost K occurs optimally at the following threshold:

$$X_u = \frac{\beta_1}{\beta_1 - 1}\left(\frac{r-\alpha}{1-\tau}\right)K \tag{3}$$

where:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{4}$$

The value of the idle firm (the investment opportunity) when the EBIT is below the investment threshold ($X < X_u$) is:

$$F_u(X) = \left(\frac{X_u(1-\tau)}{r-\alpha} - K\right)\left(\frac{X}{X_u}\right)^{\beta_1} \tag{5}$$

Levered firm

For a firm with leverage, there are multiple models that prescribe an optimal capital structure in a dynamic real-options setting. The seminal contribution of Leland (1994) extended the static trade-off theory using the contingent claims approach, that set the stage for a (still) growing literature on the application of contingent claims analysis to corporate finance issues.[1] Although later extensions of the model made it more realistic, abstracting from those more complex features in order to focus on the effects of leverage when a firm is under caps and floors.

The instantaneous profit of the active firm is:

$$\pi(t) = (X(t) - c)(1 - \tau) \tag{6}$$

were c is the perpetual coupon payment.

Upon investment the firm chooses its optimal leverage, issuing perpetual debt with coupon c^*. Shareholders decide only once and jointly on investment and debt issuance. This is equivalent to assume that equityholders have "deep pockets", and that there are no emission costs, with all profits distributed as dividends and losses covered by new equity issues. Upon default, debtholders receive a fraction of the unlevered firm, bearing a default cost ϕ.

Under these assumptions, investment occurs when a threshold X_l is reached from below, and default occurs when a threshold X_d is reached from above:

$$X_l = \left(1 + \frac{1}{h}\left(\frac{\tau}{1-\tau}\right)\right)^{-1} \frac{\beta_1}{\beta_1 - 1}\left(\frac{r-\alpha}{1-\tau}\right) K = \left(1 + \frac{1}{h}\left(\frac{\tau}{1-\tau}\right)\right)^{-1} X_u < X_u \tag{7}$$

$$X_d = hX_l \tag{8}$$

where

$$h = \left(1 - \beta_2\left(1 - \phi + \frac{\phi}{\tau}\right)\right)^{1/\beta_2} < 1 \tag{9}$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{10}$$

The optimal coupon that maximizes total firm value, decided by equityholders that internalize future debtholders' claims is:

$$c^* = \frac{r}{1-\tau}\left(\frac{\beta_2 - 1}{\beta_2}\right)\left(\frac{\beta_1}{\beta_1 - 1}\right)\left(h + \frac{\tau}{1-\tau}\right)^{-1} K \tag{11}$$

The value of securities before default ($X > X_d$) is for equity:

$$E(X) = \left(\frac{X}{r-\alpha} - \frac{c^*}{r}\right)(1-\tau) - \left(\frac{X_d}{r-\alpha} - \frac{c^*}{r}\right)(1-\tau)\left(\frac{X}{X_d}\right)^{\beta_2}, \tag{12}$$

[1]Please refer to Strebulaev et al. (2012) for a review.

for debt:

$$D(X) = \frac{c^*}{r} - \left(\frac{c^*}{r} - (1-\phi)\frac{X_d(1-\tau)}{r-\alpha}\right)\left(\frac{X}{X_d}\right)^{\beta_2}, \tag{13}$$

and the firm:

$$V_l(X) = E(X) + D(X) = \frac{X(1-\tau)}{r-\alpha} + \frac{c^*}{r}\tau - \left(\phi\frac{X_d(1-\tau)}{r-\alpha} + \frac{c^*\tau}{r}\right)\left(\frac{X}{X_d}\right)^{\beta_2}. \tag{14}$$

The value of the idle levered firm ($X < X_l$) for the equityholders is:

$$F_l(X) = (E(X_l) - K + D(X_l))\left(\frac{X}{X_l}\right)^{\beta_1} \tag{15}$$

3 Investment and leverage decisions with caps and floors

This section presents the model for the valuation of investment opportunities for unlevered and levered firms under caps and floors, i.e. a collar regime. For the sake of simplicity, it is assumed that the collar regime has limits for the EBIT X. Specifically, the EBIT can fluctuate freely between a floor X_L and a cap X_H ($\geqslant X_L$). This generic regime encompasses particular cases, namely pure floors ($X_H = \infty$), pure caps ($X_L = 0$), fixed regimes ($X_H = X_L$), including the free market regime ($X_H = \infty$ and $X_L = 0$). The firm receives the instantaneous $EBIT(X, X_L, X_H) = \min\{\max\{X_L, X\}, X_H\}$.

After presenting the case of the unlevered firm, previously studied by Adkins and Paxson (2017) and Adkins et al. (2019), their models are extended to include the effects of leverage.

Unlevered firm

The value of an investment opportunity without leverage when a firm is under perpetual collars is modeled by Adkins and Paxson (2017) and Adkins et al. (2019), who also study finite-live collars.

The value of an active unlevered firm is given by:

$$V_u^c(X) = \begin{cases} A_{11}X^{\beta_1} + \dfrac{X_L(1-\tau)}{r} & X < X_L \\ A_{21}X^{\beta_1} + A_{22}X^{\beta_2} + \dfrac{X(1-\tau)}{r-\alpha} & X_L \leqslant X < X_H \\ A_{32}X^{\beta_2} + \dfrac{X_H(1-\tau)}{r} & X \geqslant X_H \end{cases} \tag{16}$$

The constants $A_{11}, A_{21}, A_{22}, A_{32}$ are found by ensuring that $V_u(X)$ is continuous and continuously differentiable along X. The solutions for the constants are as follows:

$$A_{11} = \frac{X_H^{1-\beta_1} - X_L^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r}\right)(1-\tau) \tag{17}$$

$$A_{21} = \frac{X_H^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{r - \alpha} - \frac{\beta_2}{r}\right)(1-\tau) \tag{18}$$

$$A_{22} = -\frac{{X_L}^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r}\right)(1-\tau) \tag{19}$$

$$A_{32} = \frac{{X_H}^{1-\beta_2} - {X_L}^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r}\right)(1-\tau) \tag{20}$$

The EBIT floor must be lower than $Kr/(1-\tau)$, otherwise the investment will never return a value less than K. Notice that the trigger X_u^c can be either below or above X_H (but above X_L). Considering this domain from Equation (16), the investment threshold is:

$$X_u^c = \left(\frac{\beta_1}{(\beta_1 - \beta_2)A_{32}}\left(K - \frac{X_H}{r}\right)\right)^{\frac{1}{\beta_2}} > X_H \qquad \text{for } K \geqslant K_u^H, \tag{21}$$

where[2]

$$K_u^H = \frac{{X_H}^{\beta_2}}{\beta_1}\left({X_H}^{1-\beta_2} - {X_L}^{1-\beta_2}\right)\left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r}\right)(1-\tau) + \frac{X_H(1-\tau)}{r}, \tag{22}$$

and it is found by solving numerically the following equation for the remaining cases:

$$(\beta_1 - \beta_2)A_{22}{X_u^c}^{\beta_2} + (\beta_1 - 1)\frac{X_u^c(1-\tau)}{r - \alpha} - \beta_1 K = 0, \qquad \text{for } X_L(1-\tau)/r < K < K_u^H. \tag{23}$$

The value of the idle firm, before the investment threshold is reached ($X < X_u^c$), is:

$$F_u^c(X) = (V_u^c(X_u^c) - K)\left(\frac{X}{X_u^c}\right)^{\beta_1} \tag{24}$$

Figure 1 depicts the results of this model, showing that the investment threshold X_l^c is always above X_L.

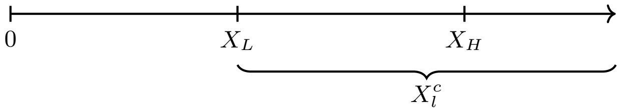


Figure 1: Unlevered firm

[2] K_u^H is found solving $X_u^c(K_u^H) = X_H$, using Equation (21).

Levered firm

In this section the previous model is extended for the case of a levered firm. For the sake of simplicity, and in order to focus on the effects of caps and floors, the Leland (1994) setting is used, whereby a firm decides only once on debt issuance.

When a firm benefits from a EBIT floor subsidy, the risk for the stakeholders changes significantly, particularly for the debtholders. Specifically, there are circumstances under which risk-free debt can be issued. Additionally, when debt is risky, the floor may or may not be binding.

Risk-free debt

When the coupon payment is smaller than the EBIT floor ($c < X_L$), the firm is always able to pay it and, therefore, never defaults. Debt becomes risk-free and because there are no default costs, the optimal leverage is the maximum possible, i.e. $c^{*c} = X_L$, maximizing the tax shield benefit provided by debt.

Proposition 1. *The investment threshold X_l^c for a firm that issues risk-free debt ($c^{*c} = X_L$) is:*

$$X_l^c = \left(\frac{\beta_1}{(\beta_1 - \beta_2) A_{32}} \left(K - \frac{X_H(1-\tau)}{r} - \frac{X_L \tau}{r} \right) \right)^{\frac{1}{\beta_2}} > X_H \qquad \text{for } K \geqslant K_l^H, \tag{25}$$

where

$$K_l^H = \frac{{X_H}^{\beta_2}}{\beta_1} \left({X_H}^{1-\beta_2} - {X_L}^{1-\beta_2} \right) \left(\frac{\beta_1 - 1}{r - \alpha} - \frac{\beta_1}{r} \right) (1-\tau) + \frac{X_H(1-\tau)}{r} + \frac{X_L \tau}{r}, \tag{26}$$

and it solves the following equation:

$$(\beta_1 - \beta_2) A_{22} X_l^{c\beta_2} + (\beta_1 - 1) \frac{X_l^c (1-\tau)}{r - \alpha} - \beta_1 \left(K - \frac{X_L \tau}{r} \right) = 0 \qquad \text{for } K < K_l^H. \tag{27}$$

The value of equity is given by:

$$E^c(X) = \begin{cases} A_{11} X^{\beta_1} & 0 < X < X_L \\ A_{21} X^{\beta_1} + A_{22} X^{\beta_2} \\ \qquad + \left(\frac{X}{r-\alpha} - \frac{X_L}{r} \right)(1-\tau) & X_L \leqslant X < X_H \\ A_{32} X^{\beta_2} \\ \qquad + \left(\frac{X_H}{r-\alpha} - \frac{X_L}{r} \right)(1-\tau) & X \geqslant X_H, \end{cases} \tag{28}$$

and the value of debt is:

$$D^c = \frac{X_L}{r} \tag{29}$$

Figure 2 depicts the results of this proposition.

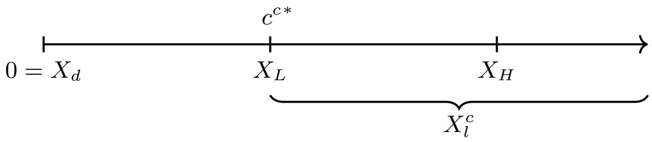


Figure 2: Case with risk-free debt

Risky debt with binding floor

When risky debt is issued (default is possible), two cases arise, depending when whether defaults occurs below or above the EBIT floor. When the default threshold is below the floor $X_d^c < X_L$ (the floor is binding), the firm benefits from it before equityholders choose optimally to default. Upon default, it is assumed that debtholders claim the assets in place and the collar prevails. The model can easily accommodate the case of the collar ending with default.

As before, equitylholders decide jointly on the investment timing and the coupon level, and the value of the firm securities is obtained knowing the optimal default policy by the equityholders. It is possible to show that, as for the unlevered firm, the investment threshold in never below X_L.

Proposition 2. *The value of a firm under an EBIT cap and a binding EBIT floor for the equityholders is:*

$$E^c(X,c) = \begin{cases} A_{11}X^{\beta_1} + B_{12}X^{\beta_2} \\ \qquad + \left(\dfrac{X_L}{r} - \dfrac{c}{r}\right)(1-\tau) \qquad X_d^c \leqslant X < X_L \\ \\ A_{21}X^{\beta_1} + B_{22}X^{\beta_2} \\ \qquad + \left(\dfrac{X}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) \quad X_L \leqslant X < X_H \\ \\ B_{32}X^{\beta_2} \\ \qquad + \left(\dfrac{X_H}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) \quad X \geqslant X_H \end{cases} \tag{30}$$

where

$$B_{12}(c) = -\frac{\beta_1}{\beta_1 - \beta_2}\left(\frac{X_L}{r} - \frac{c}{r}\right)(1-\tau)\left(\frac{1}{X_d^c(c)}\right)^{\beta_2} \tag{31}$$

$$B_{22}(c) = B_{12}(c) + A_{22} \tag{32}$$

$$B_{32}(c) = B_{12}(c) + A_{32} \tag{33}$$

$$X_d^c(c) = \left(\frac{\beta_2}{(\beta_1 - \beta_2)A_{11}}\left(\frac{X_L}{r} - \frac{c}{r}\right)(1-\tau)\right)^{1/\beta_1}, \tag{34}$$

and for the debtholders is:

$$D^c(X,c) = \frac{c}{r} - M_2(c)X^{\beta_2} \tag{35}$$

where

$$M_2(c) = \left(\frac{c}{r} - (1-\phi)\left(A_{11}X_d^c(c)^{\beta_1} + \frac{X_L}{r}(1-\tau)\right)\right)\left(\frac{1}{X_d^c(c)}\right)^{\beta_2} \tag{36}$$

The value of firm is $V_l^c(X_l^c, c) = E^c(X,c) + D^c(X,c)$.

Equityholders invest at the optimal threshold X_l^c *and choose the the optimal coupon* c^{*c}*, by maximizing the firm value. They are found solving simultaneously:*

$$\frac{\tau}{r} - \frac{\tau}{r}\left(1 - \beta_2\left(1 - \phi + \frac{\phi}{\tau}\right)\frac{c^{*c}}{\beta_1(c^{*c} - X_L)}\right)\left(\frac{X_l^c}{X_d^c(c^{*c})}\right)^{\beta_2} = 0 \tag{37}$$

and

$$X_l^c = \left(\frac{\beta_1}{(\beta_1 - \beta_2)\left(B_{32}(c^{*c}) - M_2(c^{*c})\right)}\left(K - \frac{X_H(1-\tau)}{r} - \frac{c^{*c}\tau}{r}\right)\right)^{1/\beta_2} \quad \text{for } X_l^c \geqslant X_H, \tag{38}$$

or

$$(\beta_1 - \beta_2)(B_{22}(c^{*c}) - M_2(c^{*c})) + (\beta_1 - 1)\frac{X_l^c(1-\tau)}{r-\alpha} - \beta_1\left(K - \frac{c^{*c}\tau}{r}\right) = 0 \quad \text{for } X_L < X_l^c < X_H. \tag{39}$$

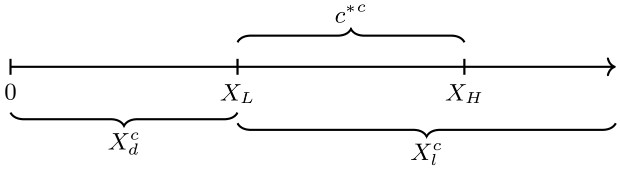


Figure 3: Case with risky debt with binding floor

The case of risky debt issuance with a binding floor is depicted in Figure 3. The optimal coupon can never be above the EBIT cap, otherwise the firm would be unable to pay for it. Investment occurs above the floor, but, for some parameter values, can be optimal only above the cap, while the default threshold for a binding floor lies always below the floor.

Risky debt with non-binding floor

It may be optimal for equityholders to choose a higher coupon, inducing optimal default above the EBIT floor ($X_L \leqslant X_d^c$), making the floor non-binding, i.e. the firm never benefits from it, as default occurs before it is reached. Therefore, the default policy by equityholders and their value are the same as without the floor (i.e. with a pure cap). However, since debtholders are entitled with the floor upon default, the value of debt depends on the floor.

Proposition 3. *The value of a firm under an EBIT cap and a non-binding EBIT floor for the equityholders is:*

$$E^c(X,c) = E^{cap}(X,c) = \begin{cases} A_{21}X^{\beta_1} + H_{22}X^{\beta_2} \\ \qquad + \left(\dfrac{X}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) & X_d^c \leqslant X < X_H \\ G_{22}X^{\beta_2} + \left(\dfrac{X_H}{r-\alpha} - \dfrac{c}{r}\right)(1-\tau) & X \geqslant X_H \end{cases} \tag{40}$$

where

$$G_{22} = \frac{{X_H}^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r}\right)(1-\tau) \tag{41}$$

$$H_{12}(c) = -\frac{1}{\beta_2}\left(\beta_1 A_{21} X_d^{cap}(c)^{\beta_1} + \frac{X_d^{cap}(c)(1-\tau)}{r-\alpha}\right)\left(\frac{1}{X_d^{cap}(c)}\right)^{\beta_2} \tag{42}$$

$$H_{22}(c) = H_{12}(c) + G_{22} \tag{43}$$

and $X_d^{cap}(c)$ solves the following equation

$$(\beta_1 - \beta_2)A_{21}X_d^{cap}(c)^{\beta_1} - (\beta_2 - 1)\frac{X_d^{cap}(c)(1-\tau)}{r-\alpha} + \beta_2\frac{c(1-\tau)}{r} = 0. \tag{44}$$

The value of the debt, considering that debtholders get $(1-\phi)V_u^c$ upon default, is:

$$D^c(X,c) = \frac{c}{r} - N_2(c)X^{\beta_2} \tag{45}$$

where

$$N_2(c) = \left(\frac{c}{r} - (1-\phi)\left(A_{21}X_d^{cap}(c)^{\beta_1} + A_{22}X_d^{cap}(c)^{\beta_2} + \frac{X(1-\tau)}{r-\alpha}\right)\right)\left(\frac{1}{X_d^{cap}(c)}\right)^{\beta_2} \quad (46)$$

Equityholders invest at the optimal threshold X_l^c *and choose the the optimal coupon* c^{*c}*, by maximizing the firm value* $V_l^c(X_l^c, c) = E^{cap}(X, c) + D^c(X, c)$*. Since the solution for the default threshold is not closed-form, the model solution (*X_l^c*,* c^{*c}*, and* X_d^c*) is obtained by solving simultaneously Equations* (44) *and*

$$\frac{\tau}{r} - \frac{\tau}{r}\left(1 - \beta_2\left(1 - \phi + \frac{\phi}{\tau}\right)\theta\right)\left(\frac{X_l^c}{X_d^c(c^{*c})}\right)^{\beta_2} = 0 \quad (47)$$

where

$$\theta = \frac{(\beta_1-\beta_2)A_{21}X_d^c(c^{*c})^{\beta_1} - (\beta_2-1)\dfrac{X_l^c(1-\tau)}{r-\alpha}}{\beta_1(\beta_1-\beta_2)A_{21}X_d^c(c^{*c})^{\beta_1} - (\beta_2-1)\dfrac{X_l^c(1-\tau)}{r-\alpha}} < 1, \quad (48)$$

and

$$X_l^c = \left(\frac{\beta_1}{(\beta_1-\beta_2)(H_{22}(c^{*c}) - N_2(c^{*c}))}\left(K - \frac{X_H}{r}(1-\tau) - \frac{c^{*c}}{r}\tau\right)\right)^{1/\beta_2} \qquad \textit{for } X_l^c \geqslant X_H, \quad (49)$$

or

$$(\beta_1-\beta_2)(H_{12}(c^{*c}) - N_2(c^{*c}))X_l^{c\beta_2} + (\beta_1-1)\frac{X_l^c(1-\tau)}{r-\alpha} - \beta_1\left(K - \frac{c^{*c}}{r}\tau\right) = 0, \qquad \textit{for } X_d \leqslant X_l^c < X_H. \quad (50)$$

The results of this proposition are depicted in Figure 4.

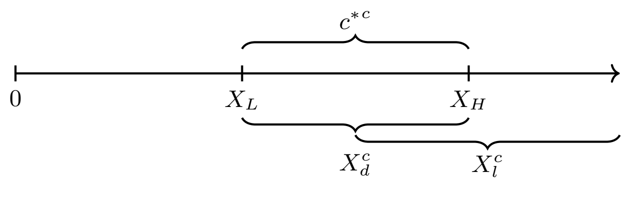


Figure 4: Case with risky debt and non-binding floor

Optimal leverage and the value of the investment opportunity

When the investment threshold is reached, equityholders decide how to finance it, considering that different levels of leverage induce one of the three cases above. They have do decide between issuing less debt but risk-free or choose a higher riskier leverage (more

Parameter	Description	Value
X_L	EBIT floor	2
X_H	EBIT cap	8
σ	Volatility	0.2
r	Risk-free interest rate	0.04
α	Risk-neutral drift rate	0.015
τ	Corporate tax rate	0.15
K	Investment cost	100
ϕ	Bankruptcy cost	0.5

Table 1: The base case parameter values.

costly). Obviously they will decide on their best interest, by maximizing equity value $E^c(X_l^c, c^{*c})$ and, theoretically, all cases are possible. Indeed, the next section shows that, depending on the parameter values, low or high leverage may be optimal, including cases for which the firm is indifferent.

The value of the idle levered firm is:

$$F_l^c(X) = (E^c(X_l^c, c^{*c}) - K + D^c(X_l^c, c^{*c})) \left(\frac{X}{X_l^c}\right)^{\beta_1} \tag{51}$$

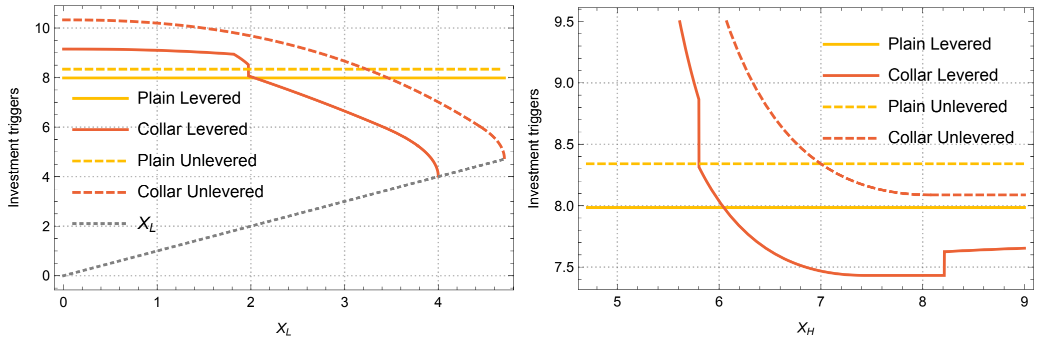
The results in this section suggest that leverage changes significantly the behavior of equityholders. The value of the investment opportunities and investment timing are different when firms use leverage to finance investment. The next section studies how leverage affects investment timing and the effects of caps and floors on leverage and investment decisions.

4 Comparative statics

This section presents a numerical comparative statics of the effects of leverage on investment timing under caps and floors, and their effects on investment timing, default policy, leverage ratios and credit spreads. For that purpose, the base-case parameter values in Table 1 are assumed.

4.1 The effect of leverage

Figure 5 shows the investment triggers for different levels of the floor (Figure 5(a)) and cap (Figure 5(b)), for both the levered and unlevered firm. As shown previously by Adkins and Paxson (2017) and Adkins et al. (2019), without leverage, a higher floor accelerates investment (reduces the investment trigger), while a lower cap deters investment. Leverage enables the firm to invest earlier for the plain project, without caps and floors. When they are in place, leverage still induces earlier investment and the acceleration seems to be more



(a) Floor (X_L) (b) Cap (X_H)

Parameter values as in Table 1.

Figure 5: The effect of leverage (X_L)

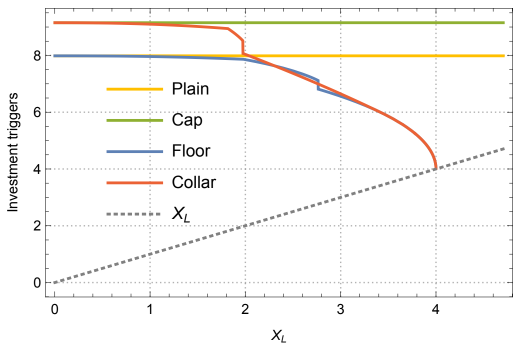
significant. However, the effects of caps and floors on investment timing are significantly different from the unlevered case. Floors have now a limited effect when set too low, and caps have a non-monotonic effect on investment timing: as the cap initially decreases, investment may be accelerated, and, for lower caps, the effect is an investment delay. As shown next, these effects result from the choice of different levels of leverage, changing the type of debt issued and making the floor binding or not.

4.2 The effect of caps and floors

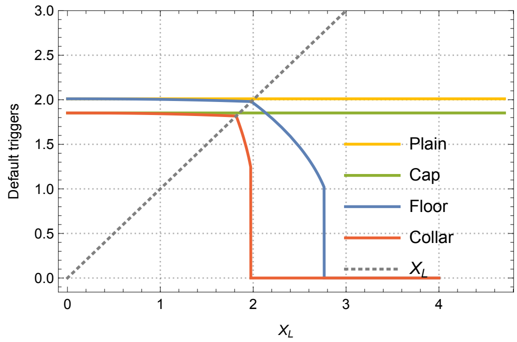
Let us study in more detail the effects of the caps and floors on investment and leverage decisions.

Figure 6 shows the effect of the EBIT floor on the investment trigger (Figure 6(a)), the default trigger (Figure 6(b)), the optimal coupon level (Figure 6(d)), the leverage ratio measured at the investment trigger (Figure 6(c)), and the credit spread (Figure 6(e)). For comparison purposes, the plain, the collar, the pure floor and the pure cap regimes are included. Notice that the solutions for the pure floor and the pure cap regimes can be obtained following the same steps in the previous section, letting $X_H \to \infty$ and $X_L \to 0$, respectively.

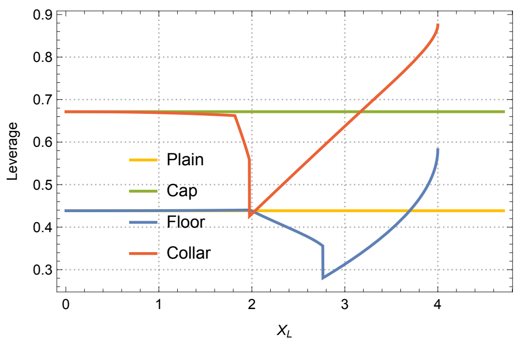
Figure 6(a) shows the above mentioned moderate effect of low floors on investment timing, that seems to be even more present for the pure floor case. Furthermore, the leverage decision of firms are not significantly affected by a low floor (Figure 6(d)), as it is non-binding, i.e. the default trigger is above the floor (Figure 6(b)). The floor starts producing a more significant effect, even while inducing the issuance of risky debt, when it becomes binding. Equityholders choose lower debt levels and invest sooner as the floor increases. At a certain point, equityholders choose significantly lower coupons that allow the issuance of risk-free debt. The comparison of the collar and pure floor cases in Figures 6(a) and 6(d) suggest that the cap reduces the possibility of issuing risky debt with a binding floor, accelerating the transition to risk-free debt. Furthermore, and because of that, the presence of a cap may paradoxically accelerate investment for levels of pure floors that do not induce the issuance of risk-free debt (Figure 6(a)). As expected, credit spreads reduce as the floor increases, as the likelihood of default decreases up to the point of becoming null (Figures 6(e) and 6(b). While a higher floor initially decreases leverage, when risk-free debt starts to be issued, leverage starts increasing (Figures 6(c) and 6(e)). The effect of the floor on leverage is, therefore, non-monotonic.



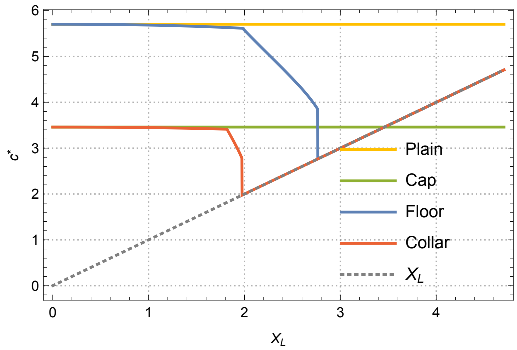
(a) Investment triggers (X_I)



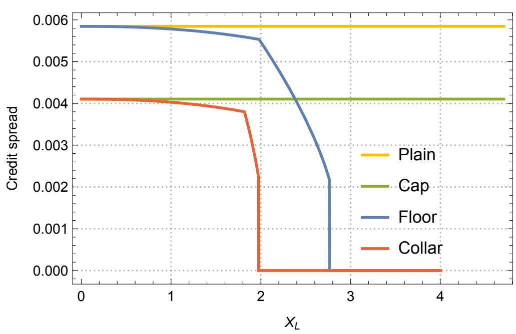
(b) Default triggers (X_d)



(c) Leverage



(d) Optimal coupon (c^*)



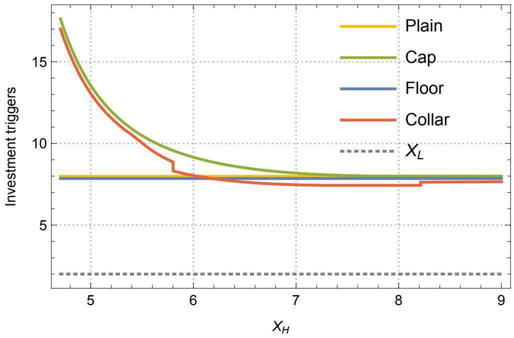
(e) Credit spread

Parameter values as in Table 1.

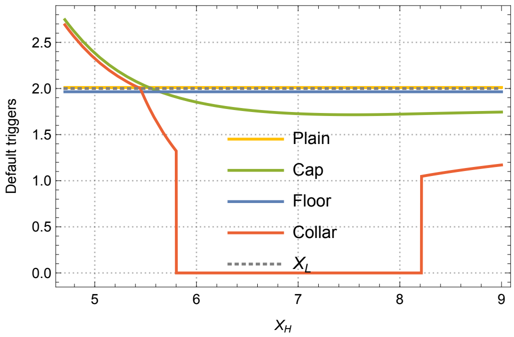
Figure 6: The effect of the floor (X_L)

The effects of the cap are shown in Figure 7. As previously suggested by Figure 5(b), several non-monotonic effects are observed. Let us first focus on the effect of a cap in a pure cap regime. While a higher cap accelerates investment up to the level of a plain project (Figure 7(a)), the effects are non-monotonic for the default trigger (Figure 7(b)) and the coupon level (Figure 7(d)). The introduction of a cap induces lower coupons and credit spreads when compared with the plain case (Figures 7(d) and 7(e)) and, because debt value increases, higher leverage ratios (Figure 7(c)). Higher caps reduce leverage ratios and increase credit spreads.

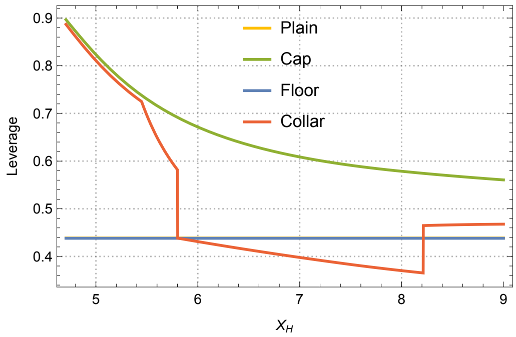
When combined with a floor, in a collar regime, all the effects of the cap become non-monotonic. For the lowest levels of the cap (closer to the floor), debt is risky and the floor is non-binding, and the most significant impacts of the caps are observed, while the floor has a limited effect, as suggested before. As the cap increases, the floor becomes binding, leverage and credit spreads decrease, and after a certain level debt becomes risk-free. This allows the firm to invest earlier than it would without the floor. Not so expected is the fact that for a sufficiently large cap, risk-free debt is no longer the best choice, as equityholders prefer higher riskier leverage, but more valuable, reducing the investment cost.



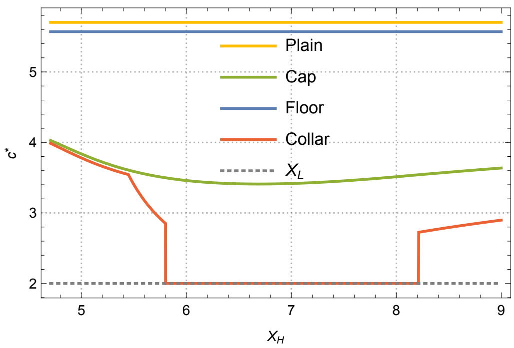
(a) Investment triggers (X_I)



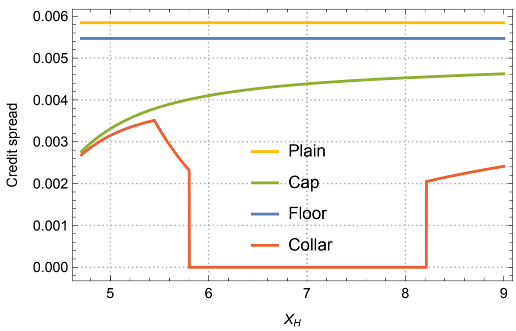
(b) Default triggers (X_d)



(c) Leverage



(d) Optimal coupon (c^*)



(e) Credit spread

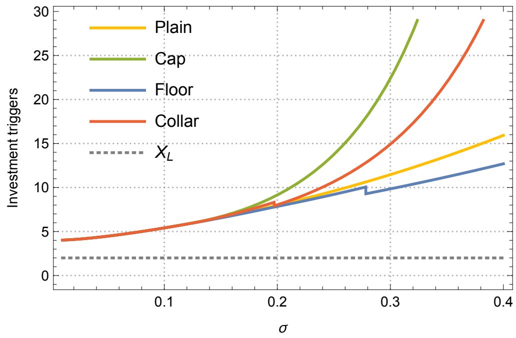
Parameter values as in Table 1.

Figure 7: The effect of the floor (X_H)

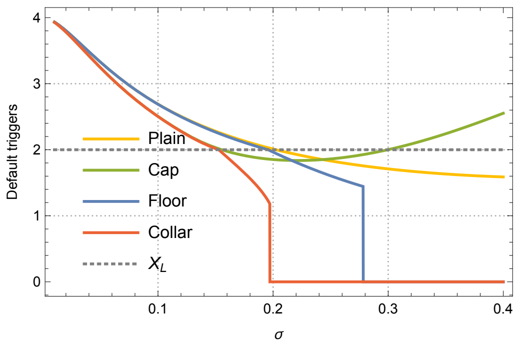
4.3 The effect of EBIT uncertainty and expected growth

Finally, the effects of the EBIT stochastic process parameters are shown in Figure 8 for uncertainty, and Figure 9 for the expected growth rate.

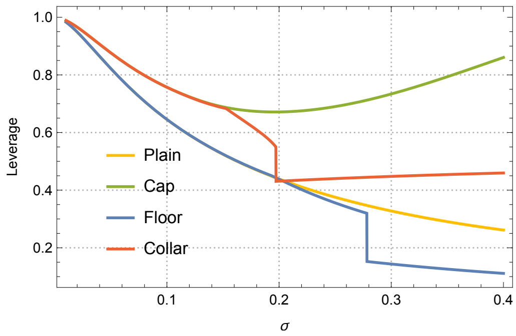
As is common in real options models, a higher uncertainty always deters investment (Figure 8(a)). The effect is more pronounced the for pure cap regimes, and less pronounced for pure floor regimes, even less than for the plain case. The introduction of the floor reduces earnings uncertainty, and mitigates the effect of uncertainty. Additionally, as it enables the issuance of both risky debt with a binding floor and risk-free debt, the non-monotonic effects of uncertainty on the coupon level (Figure 8(d)), default trigger (Figure 8(b)), and leverage ratios (Figure 8(c)) are mitigated. The effect of uncertainty on credits spreads is also non-monotonic when a floor is in place (Figure 8(e)). These non-monotonic effects result from the fact that a higher uncertainty increases both the likelihood of benefiting from the floor and being subject to the cap. Under low uncertainty, the floor has no effect, i.e. the pure cap and the collar regimes are identical, as well as the pure floor and plain regimes. On the other hand, a high uncertainty, makes the coupon level, default triggers and credit spreads insensitive to the cap (it becomes the risk-free floor), but has still an effect on debt and equity values, changing the investment timing.



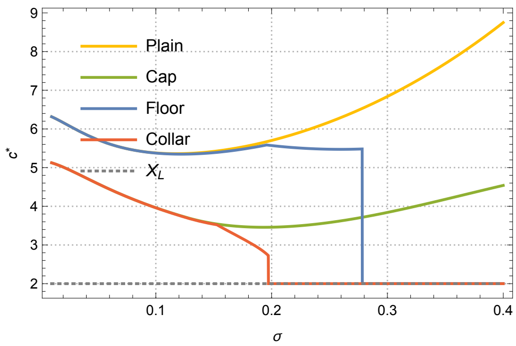
(a) Investment triggers (X_l)



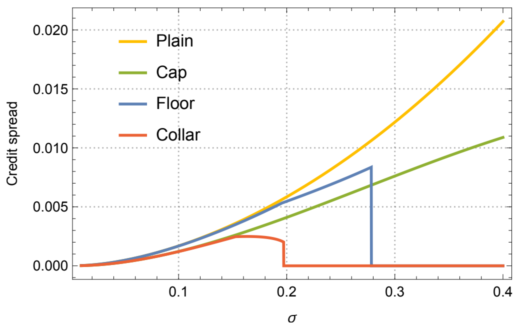
(b) Default triggers (X_d)



(c) Leverage



(d) Optimal coupon (c^*)



(e) Credit spread

Parameter values as in Table 1.

Figure 8: The effect of EBIT uncertainty (σ)

A higher earnings growth rate accelerates investment (Figure 9(a)), and, as for uncertainty, the effect is more pronounced the for pure cap regimes, and less pronounced for pure floor regimes. As the growth rate increases, the effect of caps and floors on investment timing are mitigated. Additionally, the pure floor regime become identical to the plain case, and the pure cap regime becomes similar to the collar regime (eliminating the effect of the floor), for all the variables analyzed. This is because for a high expected growth the floor is lees likely to be reached. For the same reason, when a floor is in place, a higher growth rate increases leverage and reduces credit spreads, except for low growth rates that induce risk-free debt issuance.

5 Conclusion

This paper extends the previous literature on the effects of caps and floors on investment decisions, by studying the effect of leverage. Compared with the uneleverd firm, the effects of caps and floors are significantly different. Floors have only moderate effects if below a critical level. Above that level, the firm tends to issue less risky debt, being even able to issue risk-free debt. A higher floor accelerates investment and has a non-monotonic effect on the leverage ratio. A lower cap deters investment, increases leverage, but reduces credit spreads. When combined with a floor, in a collar regime, the effects of the cap becomes non-monotonic, as the firm is able to issue risk-free debt for intermediate levels of the cap. A higher uncertainty always deters investment, but the effects on leverage and credit spreads are non-monotonic when a floor is in place. A higher expected earnings growth accelerates investment, increases leverage and reduces credit spreads, except for low growth rates that induce risk-free debt issuance.

This paper studies the case of perpetual caps and floors. Extending it for the frequently more realistic case of finite-lived collars as in Adkins et al. (2019), poses some challenges, as the default decision becomes time-dependent, requiring a numerical valuation method. For the case of finite-lived with a random termination (retractable collars), analytical solutions would be possible to obtain. The simple setting of Leland (1994) dynamic version of the static capital structure model used in this paper, can be extended to include dynamic refinancing (Goldstein et al., 2001) and debt renegotiation (Hackbarth et al., 2007). The effect of caps and floors on the firms' payout policy is also worth exploring.

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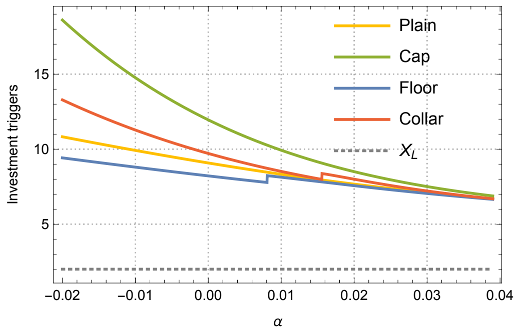
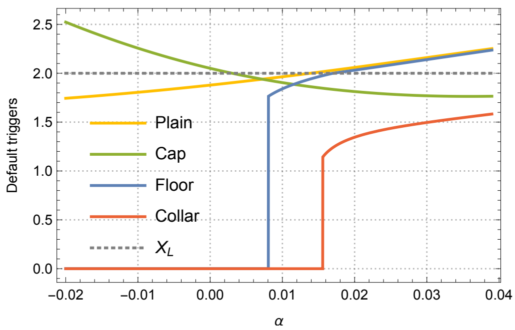
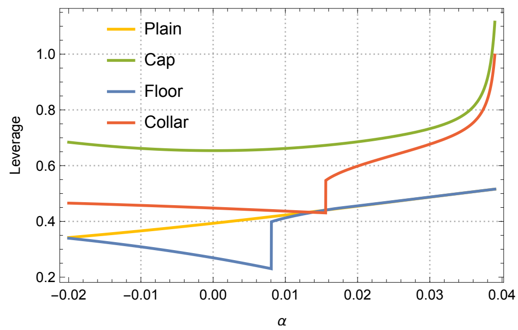


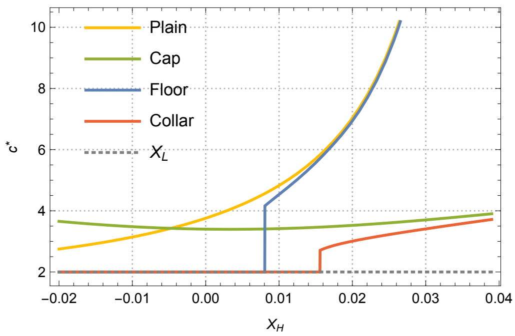
(a) Investment triggers (X_I)



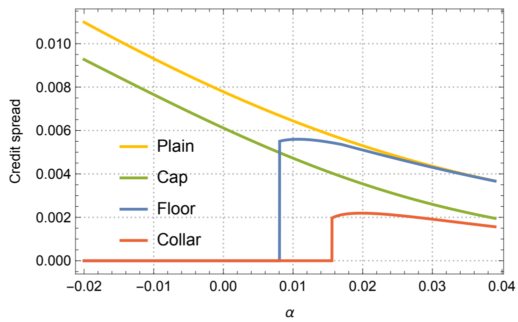
(b) Default triggers (X_d)



(c) Leverage



(d) Optimal coupon (c^*)



(e) Credit spread

Parameter values as in Table 1.

Figure 9: The effect of the EBIT drift rate (α)

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Appendices

A Proofs

To be added.