Duopolistic Competition under Risk Aversion and Uncertainty

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Abstract

A monopolist typically defers entry into an industry as both price uncertainty and the level of relative risk aversion increase. The former attribute may be present in most deregulated industries, while the latter may be relevant for reasons of market incompleteness or the presence of technical uncertainty. By contrast, it has been shown that the presence of a rival hastens entry under risk neutrality in certain frameworks. Here, we examine how duopolistic competition affects the entry decisions of risk-averse investors. We also explore how the impact of competition on the value of a firm under two different oligopolistic frameworks varies with risk aversion and uncertainty.

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1 Introduction

Due to the deregulation of many sectors of the economy, decision rules for managing capital projects should consider not only uncertainty in the underlying variables but also competition in the output market. For example, in Europe, ever since the euro was introduced, there has been an increase in competition in sectors such as transport, energy, and telecommunications, which only a decade ago were the preserve of state monopolies. Furthermore, the ongoing process of mergers and takeovers as well as legislation against monopolies justifies the existence of and development toward more competitive markets. Indicative of this situation is the new partnership between Nokia and Microsoft. This alliance was the result of tough competition due to which Nokia lost its leadership in the area of smartphone operating system shipments to Android (and, in turn, market share to rivals such as Google and Apple) and risk aversion due to costs of financial distress (The Wall Street Journal, 2011). Another example is from the energy sector where the natural gas industry is undergoing significant changes as European legislation regarding competition is forcing gas companies to restructure their business and make room for new entrants, thus leading to increased competition (Independent Energy Review, 2010).

Canonical real options theory finds particular application in such sectors as it facilitates the analysis of capital budgeting decisions by accounting for the flexibility embedded in them. However, treatment of such decision-making problems via canonical real options theory has mainly been under monopoly or perfect competition. Moreover, recent work that considers a duopolistic setting where two firms have the option to invest in the same market has assumed risk neutrality. In this paper, we extend the traditional real options approach to strategic decision making under uncertainty by examining how duopolistic competition affects the entry of a risk-averse firm. We consider two identical firms that are risk averse and hold an option each to invest in a project that yields stochastic revenues. The firms face the same output market, and, as a result, investment decisions of one firm impact the revenues of both firms. We begin by analysing the monopolistic case and then extend this framework by adding one more firm assuming either a pre-emptive or a non-pre-emptive setting. In the pre-emptive duopoly, both firms have the incentive to invest in order to obtain the leader's advantage, while in the non-pre-emptive duopoly, the role of the leader is assigned exogenously. For each setting, we analyse the impact of uncertainty and risk aversion on the optimal investment timing decisions of the two competing firms and examine the degree to which the presence of a competitor impacts the entry of a risk-averse firm. Hence, the contribution of this paper is threefold. First, we develop a theoretical framework for analysing investment under uncertainty and risk aversion for a monopoly as well as pre-emptive and non-pre-emptive duopolies in order to derive closed-form expressions where possible for the optimal investment thresholds. Second, we quantify the degree to which competition impacts the strategic investment decisions of a risk-averse rival. Finally, we provide managerial insights for investment decisions and relative firm values under each setting based on analytical and numerical results.

We proceed by discussing some related work in Section 2 and formulate the problems in Section 3. In Section 4, we solve the problems and analyse the impact of uncertainty and risk aversion on the optimal investment timing decisions of the two competing firms in each setting. In Section 5, we provide numerical examples for each case in order to examine the effects of volatility and risk aversion on the optimal investment timing decisions and quantify the degree to which the entry of the risk-averse firm is affected by the presence of a rival. We also illustrate the interaction between risk aversion and uncertainty and present managerial insights to enable more informed investment decisions. Section 6 concludes by summarising the results and offering directions for future research.

2 Related Work

The majority of real options models account for the problem of optimal investment timing without considering competition (McDonald and Siegel, 1985 and 1986), while the ones that do, assume risk neutrality (Dixit and Pindyck, 1994). In the area of competition, Huisman and Kort (1999) examine how the deterministic duopoly framework of Fudenberg and Tirole (1985) is affected when uncertainty is introduced. According to Fudenberg and Tirole (1985), under large first-mover advantages, a pre-emption equilibrium occurs with dispersed adoption timings since it is essential for each firm to move quickly and pre-empt investment by its rivals. The introduction of uncertainty creates an opposing force since now there is a positive option value of waiting that becomes larger with higher uncertainty, thereby delaying investment. In the simultaneous investment and pre-emptive equilibrium cases, the results of Huisman and Kort (1999) agree with those of Fudenberg and Tirole (1985); however, in the stochastic case, uncertainty raises the required entry threshold for both firms as it increases the value of waiting. Finally, if first-mover advantages are lower but sufficiently large for the pre-emptive equilibrium to result in the deterministic model, then Huisman and Kort (1999) show that sufficiently high uncertainty results in simultaneous investment equilibrium, thereby reducing the number of scenarios where the pre-emptive equilibrium is optimal.

Paxson and Pinto (2005) extend the traditional real options approach that treats the number of units sold and the price per unit as an aggregate variable by presenting a rivalry model in which the profits per unit and the number of units sold are both stochastic variables. They examine a pre-emptive setting (where both firms fight for the leader's position) and a non-pre-emptive setting (where the role of the leader is defined exogenously). Their results indicate that the triggers of both the leader and the follower increase in both settings as the correlation between the profits per unit and the quantity of units increases since then the aggregate volatility involving the number of units and the profits per unit also increases. Furthermore, they illustrate how the value of the active leader increases by more than the value of her investment opportunity when the number of units sold while being alone in the market increases. This, in turn, increases the non-pre-emptive leader's incentive to invest, thereby reducing the discrepancy between the pre-emptive leader's and non-pre-emptive leader's entry thresholds. Finally, they illustrate how increasing first-mover advantages create an incentive for the pre-emptive leader to enter the market sooner since then the entry of the follower is less damaging.

Unlike earlier studies concerning investment strategies in the electricity market, Takashima *et al.* (2008) show the effect of competition on market entry and the strategies of firms with different types of power plants. They analyse the entry strategies into the electricity market of two firms that have power plants under price uncertainty and competition and consider firms with either a thermal power plant or a nuclear power plant. Among other results, they show that for a nuclear power plant the entry threshold of the leader is higher compared to a liquified natural gas thermal power plant, since the latter has mothballing options that facilitate investment. Also, compared to the firm with a coal power plant or an oil thermal power plant, a firm with a nuclear power plant tends to be the leader because variable and construction costs for a nuclear power plant are lower compared to those of a coal power plant, while the oil thermal power plant may have lower construction cost but has variable cost that is twice as much as that of the nuclear power plant.

Huisman and Kort (2009) model not only the timing but also the size of the investment. They consider a monopoly setting as well as a duopoly setting and compare the results with the standard models in which the firms do not have capacity choice. They identify the region of demand where the leader can choose either to deter temporarily or to accommodate the entry of the follower and find that the leader can choose the deterrence strategy only up to a certain high level of demand. If the demand is higher than that level, then it is optimal for the follower to enter at the same time as the leader. Similarly, if the demand is low, then it is not optimal for the leader to choose the deterrence strategy as this would result in negative profits. Also, at high levels of demand, the leader's optimal strategy is either to deter or to accommodate the entry of the follower. However, the region in which the leader can choose either one of the two strategies decreases with uncertainty, thereby increasing the range of demand where the leader chooses the deterrence strategy.

Extending the traditional approach that considers only two competing firms, Bouis *et al.* (2009) analyse investments in new markets where more than two identical competitors are present. In the setting including three firms, then they find that if entry of the third firm is delayed, then the second firm has an incentive to invest earlier because this firm can enjoy the duopoly market structure for a longer time. This reduces the investment incentive for the first firm, which now faces a shorter period in which it can enjoy monopoly profits, and, thus, it invests later. This effect is denoted as the accordion effect and is also observed when the number of competing firms is greater. Indeed, with more than three firms competing, exogenous demand changes affect the timing of entry of the first, third, fifth, etc., investor in the same qualitative way, while the entry of the second, fourth, sixth, etc., investor is affected in exactly the opposite qualitative way. In other words, if a delay is observed for the "odd" investors, then the "even" investors will invest sooner.

Each of these papers assumes a risk-neutral decision maker, and, as a result, the implications of risk aversion are not addressed. We contribute to this line of work by developing a utility-based framework in order to examine how optimal investment decisions under uncertainty are affected by competition and risk aversion. This is relevant to a knowledge-based sector in which firms compete to launch a new product while simultaneously facing costs of financial distress or shareholder pressure. In order to describe the preferences of the two firms, we apply a CRRA utility function and determine the optimal strategies that maximise the expected utility of their future profits in both pre-emptive and non-pre-emptive settings.

We confirm the results of Hugonnier and Morellec (2007) and Chronopoulos *et al.* (2011) by showing that risk aversion lowers the expected utility of the project, thereby delaying the entry of the leader and the follower in both pre-emtpive and non-pre-emptive settings. We also find that, relative to the monopolist, the non-pre-emptive leader is hurt less from the follower's entry than the pre-emptive leader since the former has the flexibility to delay entry into the market. Interestingly, risk aversion does not impact the relative loss in the pre-emptive leader's value due to the follower's entry, but makes the non-pre-emptive leader relatively better off. Furthermore, we show that higher uncertainty reduces the loss in value of the pre-emptive leader relative to the monopolist by delaying the entry of the follower, thereby allowing the pre-emptive leader to enjoy monopoly profits for longer time. Finally, we show that if the discrepancy between the market share of the leader and the follower is small, then the impact of uncertainty on the leader's option value is more profound and offsets the loss in value due to the follower's entry. By contrast, a large discrepancy in market share makes the increase in option value less profound as it increases the first-mover advantage and, at the same time, increases the impact of the follower's entry, thereby making the non-pre-emptive leader worse off.

3 Problem Formulation

3.1 Assumptions and Notation

Assume that each firm i, $i = 1, 2$, can incur an investment cost, K, in order to start a project that produces output forever. Time is continuous and denoted by t, and the revenue received from the project at time $t \geq 0$ is $R_t = P_t D(Q_t)$ (\$/annum). Here, Q_t denotes the number of firms in the industry, i.e., $Q_t = 0, 1, 2$, and $D(Q_t)$ is a strictly decreasing function reflecting the quantity demanded from each firm per annum. We assume that the price per unit of the project's output, P_t, follows a geometric Brownian motion (GBM) process:

$$dP_t = \mu P_t dt + \sigma P_t dZ_t, \; P_0 > 0 \tag{1}$$

where $\mu \geq 0$ is the growth rate of P_t, $\sigma \geq 0$ is the volatility of P_t, and dZ_t is the increment of the standard Brownian motion process. Also, we denote by $r \geq 0$ the risk-free discount rate and by $\rho \geq \mu$ the subjective discount rate. Let τ_i^j be the time at which firm j, $j = \ell, f$ (denoting leader or follower, respectively), enters the industry given market structure $i = m, p, n$ (denoting monopoly, pre-emptive duopoly, or non-pre-emptive duopoly, respectively), i.e.,

$$\tau_i^j \equiv \min\left\{t \geq 0 : P_t \geq P_{\tau_i^j}\right\} \tag{2}$$

where $P_{\tau_i^j}$ is the corresponding output price. Finally, we denote by $F_{\tau_i^j}(P_0)$ the expected value of firm j's investment opportunity under market structure i that is exercised at time τ_i^j and by $V_i^j(P_0)$ the expected NPV of firm j given the initial output price, P_0.

In order to account for risk aversion, we assume that the preferences of both firms are described by an identical increasing and concave utility function, $U(\cdot)$. As a result, our analysis can accommodate a wide range of utility functions, such as hyperbolic absolute risk aversion (HARA), constant absolute risk aversion (CARA), and CRRA utility functions. In our analysis, we apply a CRRA utility function as in Hugonnier and Morellec (2007) defined as follows:

$$U(P_t) = \begin{cases} \frac{P_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \geq 0 \;\&\; \gamma \neq 1 \\ \ln(P_t) & \text{if } \gamma = 1 \end{cases} \tag{3}$$

The relative risk aversion parameter is γ, which, for the purposes of this analysis, is restricted to $[0, 1)$ and reflects greater risk aversion as it increases.

3.2 Monopoly

We begin by formulating the problem for the case of monopoly, where a single firm starts a perpetually operating project at a random time τ_m^j. Up to time τ_m^j, the monopolist invests K in a risk-free bond and earns an instantaneous cash flow of rK per time unit with utility $U(rK)$ discounted at her subjective rate of time preference, $\rho > \mu$. At τ_m^j, when the output price is $P_{\tau_m^j}$, the monopolist swaps this risk-free cash flow for a risky one, $P_t D(1)$, with utility $U(P_t D(1))$ as illustrated in Figure 1.

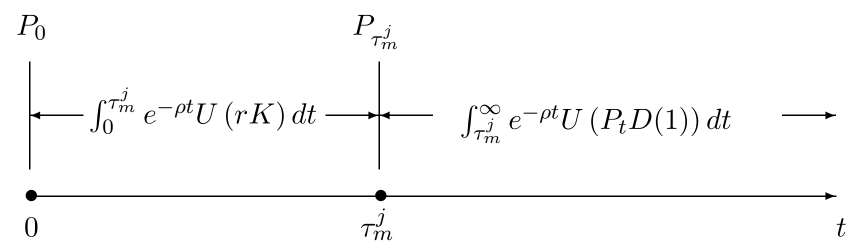

Figure 1: Investment under risk aversion for a monopoly

The conditional expected utility of the cash flows discounted to time $t = 0$ is:

$$\int_0^{\tau_m^j} e^{-\rho t} U(rK)\, dt + \mathbb{E}_{P_0}\left[\int_{\tau_m^j}^{\infty} e^{-\rho t} U(P_t D(1))\, dt\right] = \int_0^{\infty} e^{-\rho t} U(rK)\, dt + \mathbb{E}_{P_0}\left[e^{-\rho \tau_m^j}\right] V_m^j\left(P_{\tau_m^j}\right) \quad (4)$$

where,

$$V_m^j\left(P_{\tau_m^j}\right) = \mathbb{E}_{P_{\tau_m^j}}\left[\int_0^{\infty} e^{-\rho t}\left[U(P_t D(1)) - U(rK)\right] dt\right] \quad (5)$$

is the expected utility of the project's cash flows discounted to τ_m^j, and the monopolist's objective is to maximise the discounted expected utility of the project's cash flows, i.e., $\mathbb{E}_{P_0}\left[e^{-\rho \tau_m^j}\right] V_m^j\left(P_{\tau_m^j}\right)$. Here, $\mathbb{E}_{P_0}$ denotes the expectation operator, which is conditional on the initial value of the price process.

3.3 Duopoly

3.3.1 Pre-Emptive Duopoly

We extend the previous framework by adding one more firm to the industry. As there are two firms in the industry fighting for the leader's position, each one of them runs the risk of pre-emption, and, as a result, there is no value in waiting. The firm that enters the market first is the leader, and the firm that enters second the follower as shown in Figure 2.

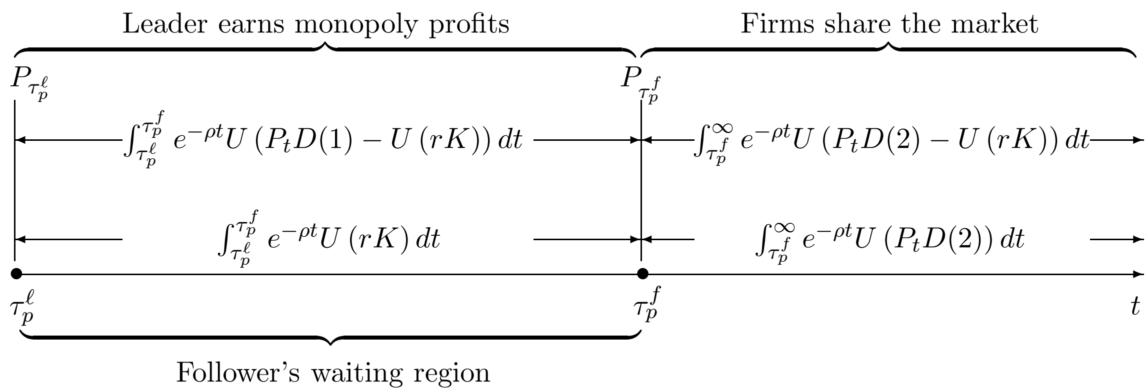

Figure 2: Investment under risk aversion for a pre-emptive duopoly

Consequently, the conditional expected utility of all future cash flows of the follower discounted to $t = \tau_p^\ell$ is:

$$\begin{aligned}
\int_{\tau_p^\ell}^{\tau_p^f} e^{-\rho t} U\left(rK\right) dt + \mathbb{E}_{P_{\tau_p^\ell}}\left[\int_{\tau_p^f}^{\infty} e^{-\rho t} U\left(P_t D(2)\right) dt\right] &= \int_{\tau_p^\ell}^{\infty} e^{-\rho t} U\left(rK\right) dt \\
&\quad + \mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f - \tau_p^\ell\right)}\right] V_p^f\left(P_{\tau_p^f}\right) \qquad (6)
\end{aligned}$$

where,

$$V_p^f\left(P_{\tau_p^f}\right) = \mathbb{E}_{P_{\tau_p^f}}\left[\int_0^{\infty} e^{-\rho t}\left[U\left(P_t D(2)\right) - U\left(rK\right)\right] dt\right] \qquad (7)$$

is the expected utility of the project's cash flows discounted to τ_p^f, and, like the monopoly case, the scope of the pre-emptive follower is to maximise the discounted to τ_p^ℓ expected utility of the project's cash flows, i.e., $\mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f - \tau_p^\ell\right)}\right] V_p^f\left(P_{\tau_p^f}\right)$.

Next, the conditional expected utility of all future cash flows of the leader discounted to $t = \tau_p^\ell$ is:

$$\begin{aligned}
V_p^\ell\left(P_{\tau_p^\ell}\right) &= \mathbb{E}_{P_{\tau_p^\ell}}\left[\int_0^{\tau_p^f} e^{-\rho t}\left[U(P_t D(1)) - U\left(rK\right)\right] dt + \int_{\tau_p^f}^{\infty} e^{-\rho t}\left[U(P_t D(2)) - U\left(rK\right)\right] dt\right] \\
&= V_m^j\left(P_{\tau_p^\ell}\right) + \mathbb{E}_{P_{\tau_p^\ell}}\left[e^{-\rho\left(\tau_p^f - \tau_p^\ell\right)}\right] \mathbb{E}_{P_{\tau_p^f}}\left[\int_0^{\infty} e^{-\rho t}\left[U\left(P_t D(2)\right) - U\left(P_t D(1)\right)\right] dt\right] \qquad (8)
\end{aligned}$$

Notice that up to time τ_p^f, the leader enjoys monopolistic profits as in (5), while after the entry of the follower the two firms share the market, as illustrated in Figure 2. This implies that, although up to time τ_p^f the leader is alone in the market, her value function does not correspond to that of a monopolist since the future entry of the follower reduces the expected utility of the leader's profits. This reduction is reflected by the second term on the right-hand side of (8), which is negative since $D(2) < D(1)$.

3.3.2 Non-Pre-Emptive Duopoly

Here, the roles of the leader and the follower are defined exogenously. Consequently, the future cash flows of both the leader and the follower are discounted to time $t = 0$ as illustrated in Figure 3.

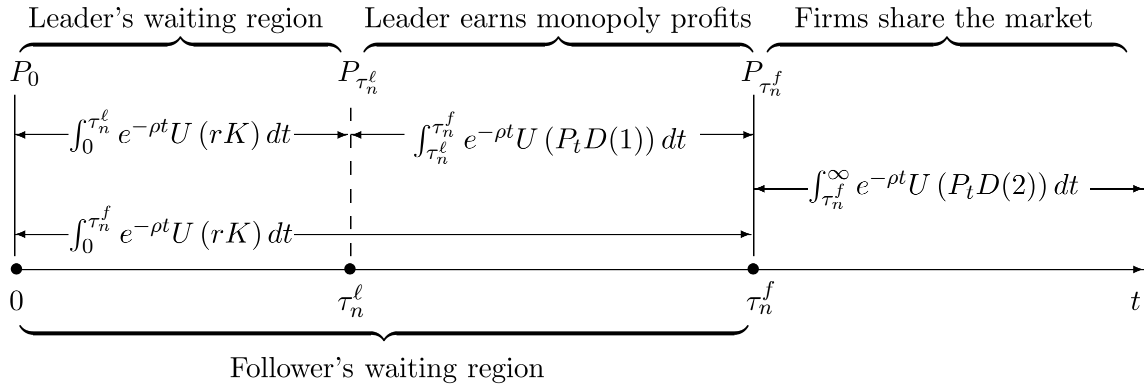

Figure 3: Investment under risk aversion for a non-pre-emptive duopoly

The conditional expected utility of the follower's cash flows is the same as in the pre-emptive case but discounted to $t = 0$, i.e.,

$$\int_0^\infty e^{-\rho t} U(rK)\, dt + \mathbb{E}_{P_0}\left[e^{-\rho \tau_n^f}\right] V_n^f\left(P_{\tau_n^f}\right) \tag{9}$$

where $V_n^f(\cdot) = V_p^f(\cdot)$ and the objective of the follower is to maximise $\mathbb{E}_{P_0}\left[e^{-\rho \tau_n^f}\right] V_n^f\left(P_{\tau_n^f}\right)$.

The leader now knows that she has the right to enter the market first and, therefore, does not run the risk of pre-emption. As a result, the expected utility of the leader's future cash flows discounted to $t = 0$ is:

$$\int_0^{\tau_n^\ell} e^{-\rho t} U(rK)\, dt + \mathbb{E}_{P_0}\left[\int_{\tau_n^\ell}^{\tau_n^f} e^{-\rho t} U(P_t D(1))\, dt\right] + \mathbb{E}_{P_0}\left[\int_{\tau_n^f}^{\infty} e^{-\rho t} U(P_t D(2))\, dt\right] =$$
$$\int_0^\infty e^{-\rho t} U(rK)\, dt + \mathbb{E}_{P_0}\left[e^{-\rho \tau_n^\ell}\right] V_p^\ell\left(P_{\tau_n^\ell}\right) \tag{10}$$

where $V_p^\ell(\cdot)$ is defined as in (8). Here, the objective of the leader is to maximise $\mathbb{E}_{P_0}\left[e^{-\rho \tau_n^\ell}\right] V_p^\ell\left(P_{\tau_n^\ell}\right)$.

4 Analytical Results

4.1 Monopoly

In this case, there is a single firm in the market that contemplates investment without the fear of pre-emption from the entry of a competitor. Consequently, the firm has the option to delay investment until the output price hits the optimal threshold, $P_{\tau_m^{j*}}$, that will trigger investment. Hence, for $P_0 \leq P_{\tau_m^{j*}}$, (11) indicates the value of the monopolist's investment opportunity:

$$
\begin{aligned}
F_{\tau_m^j}(P_0) &= \sup_{\tau_m^j \in \mathcal{S}} \mathbb{E}_{P_0} \left[\int_{\tau_m^j}^{\infty} e^{-\rho t} \left[U\left(P_t D(1)\right) - U\left(rK\right) \right] dt \right] \\
&= \sup_{\tau_m^j \in \mathcal{S}} \mathbb{E}_{P_0} \left[e^{-\rho \tau_m^j} \right] V_m^j \left(P_{\tau_m^j} \right) \qquad (11)
\end{aligned}
$$

Here, $\mathcal{S}$ denotes the collection of admissible stopping times of the filtration generated by the price process. Using Theorem 9.18 of Karatzas and Shreve (1999) for the CRRA utility function in (3), we find that the expression in (5) can be simplified using the following:

$$
\mathbb{E}_{P_0} \int_0^{\infty} e^{-\rho t} U\left(P_t\right) dt = \mathcal{A} U\left(P_0\right) \qquad (12)
$$

where $\mathcal{A} = \frac{\beta_1 \beta_2}{\rho(1-\beta_1-\gamma)(1-\beta_2-\gamma)} > 0$, and $\beta_1 > 1$, $\beta_2 < 0$ are the solutions for x to the following quadratic equation:

$$
\frac{1}{2}\sigma^2 x(x-1) + \mu x - \rho = 0 \qquad (13)
$$

By using the fact that the expected discount factor is $\mathbb{E}_{P_0}\left[e^{-\rho \tau_m^j}\right] = \left(\frac{P_0}{P_{\tau_m^j}}\right)^{\beta_1}$ (Karatzas and Shreve, 1999) and applying the strong Markov property along with the law of iterated expectations, (11) can be written as follows:

$$
F_{\tau_m^j}(P_0) = \max_{P_{\tau_m^j} \geq P_0} \left(\frac{P_0}{P_{\tau_m^j}} \right)^{\beta_1} V_m^j \left(P_{\tau_m^j} \right) \qquad (14)
$$

Solving the unconstrained optimisation problem (14), we obtain the optimal investment threshold, $P_{\tau_m^{j*}}$, for the monopolist:

$$
P_{\tau_m^{j*}} = \frac{rK}{D(1)} \left(\frac{\beta_2 + \gamma - 1}{\beta_2} \right)^{\frac{1}{1-\gamma}} \qquad (15)
$$

According to (15), uncertainty and risk aversion drive a wedge between the optimal investment threshold and the amortised investment cost. Indeed, it can be shown that higher risk aversion increases the required investment threshold by decreasing the expected utility of the investment's payoff, while increased uncertainty delays investment by increasing the value of waiting. All proofs can be found in the appendix.

Proposition 4.1 *Uncertainty and risk aversion increase the optimal investment threshold.*

4.2 Symmetric Pre-Emptive Duopoly

We solve this dynamic game backward by first assuming that the leader has just entered the market. The value of the follower at $\tau_p^\ell < \tau_p^f$ is indicated in (16):

$$\begin{aligned} F_{\tau_p^f}(P_{\tau_p^\ell}) &= \sup_{\tau_p^f \geq \tau_p^\ell} \mathbb{E}_{P_{\tau_p^\ell}} \left[e^{-\rho \tau_p^f} \right] V_p^f \left(P_{\tau_p^f} \right) \\ &= \max_{P_{\tau_p^f} \geq P_{\tau_p^\ell}} \left(\frac{P_{\tau_p^\ell}}{P_{\tau_p^f}} \right)^{\beta_1} V_p^f \left(P_{\tau_p^f} \right) \end{aligned} \tag{16}$$

Solving the unconstrained optimisation problem described by (16), we obtain the optimal threshold, $P_{\tau_p^{f*}}$, that triggers the entry of the follower:

$$P_{\tau_p^{f*}} = \frac{rK}{D(2)} \left(\frac{\beta_2 + \gamma - 1}{\beta_2} \right)^{\frac{1}{1-\gamma}} \tag{17}$$

Notice that since $D(2) < D(1)$, we have $P_{\tau_p^{f*}} > P_{\tau_m^{j*}}$, i.e., the optimal entry threshold of the pre-emptive follower is higher than that of the monopolist. Intuitively, this happens because the follower requires compensation for losing the first-mover advantage. After the critical threshold, $P_{\tau_p^{f*}}$, is hit, the value of the follower is the discounted expected utility of the project's cash flows, as indicated by (7).

Assuming that the follower chooses the optimal policy, the value function of the leader for $P_{\tau_p^\ell} \leq P_t < P_{\tau_p^{f*}}$, i.e., when the leader is alone in the market, is:

$$\begin{aligned} V_p^\ell(P_t) &= \mathbb{E}_{P_t} \left[\int_0^{\tau_p^{f*}} e^{-\rho t} \left(U(P_t D(1)) - U(rK) \right) dt + \int_{\tau_p^{f*}}^{\infty} e^{-\rho t} \left(U(P_t D(2)) - U(rK) \right) dt \right] \\ &= \mathcal{A}U\left(P_t D(1)\right) - \frac{U\left(rK\right)}{\rho} + \left(\frac{P_t}{P_{\tau_p^{f*}}} \right)^{\beta_1} \mathcal{A}U\left(P_{\tau_p^{f*}}\right) \left[D(2)^{1-\gamma} - D(1)^{1-\gamma} \right] \end{aligned} \tag{18}$$

For $P_t \geq P_{\tau_p^{f*}}$, the two firms share the market and, as a result, the value function of the leader is the same as the follower's.

As we show in Proposition 4.2, under a large discrepancy in market share, there exists a finite output price at which the pre-emptive leader's value function is maximised. Otherwise, the pre-emptive leader's value function is strictly increasing. Intuitively, a higher output price simultaneously increases the expected discounted utility of cash flows and facilitates the follower's entry. With a higher loss in market share, the impact of the latter effect dominates.

Proposition 4.2 *The value function of the pre-emptive leader is concave, and its maximum value is obtained prior to the entry of the pre-emptive follower provided that:*

$$D(2) < D(1) \left(\frac{\beta_1 + \gamma - 1}{\beta_1} \right)^{\frac{1}{1-\gamma}} \tag{19}$$

In order to determine the leader's optimal investment threshold, we need to consider the strategic interactions between the leader and the follower. Let $P_{\tau_p^{\ell*}}$ denote the threshold price at which a firm is indifferent between becoming a leader or a follower. Recall that in the pre-emptive setting both firms want to enter first in order to obtain the leader's advantage. However, for $P_t < P_{\tau_p^{\ell*}}$, the follower has not entered the market, and a firm would be better off being the follower since then $V_p^{\ell}(P_t) < F_{\tau_p^f}(P_t)$, while for $P_t > P_{\tau_p^{\ell*}}$, a firm is better off being a leader since then $V_p^{\ell}(P_t) > F_{\tau_p^f}(P_t)$. Hence, it must be the case that $V_p^{\ell}\left(P_{\tau_p^{\ell*}}\right) = F_{\tau_p^f}\left(P_{\tau_p^{\ell*}}\right)$ for entry, a condition that is found numerically by solving the following equation:

$$\mathcal{A}U\left(P_{\tau_p^{\ell*}}D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1} \mathcal{A}U\left(P_{\tau_p^{f*}}\right)\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}}D(2)\right) - \frac{U(rK)}{\rho}\right] \tag{20}$$

Solving (20) for $P_{\tau_p^{\ell*}}$, we obtain the entry threshold of the leader that denotes the output price at which a firm is indifferent between becoming a leader or a follower. Indeed, as we show in Proposition 4.3, the optimal entry threshold of the pre-emptive leader is lower than that of the monopolist. This happens because the risk of pre-emption deprives the leader of the option to postpone investment, thereby lowering the required investment threshold.

Proposition 4.3 *The pre-emptive leader's optimal entry threshold is lower than that of the monopolist.*

Although increased risk aversion raises the required investment threshold by decreasing the expected utility of the investment's payoff, the loss in the value of the leader due to the entry of the follower, evaluated at $P_{\tau_p^{\ell*}}$, relative to that of the monopolist is not affected by risk aversion. Intuitively, the value of the leader at $P_{\tau_p^{\ell*}}$ equals the value of the follower's investment opportunity. Since both the follower and the monopolist hold a single option each to enter the market, increased risk aversion poses a proportional decrease in the option value of the follower relative to the monopolist.

Proposition 4.4 *The loss in the pre-emptive leader's value relative to the monopolist's value of investment opportunity at the pre-emptive leader's optimal entry threshold price is unaffected by risk aversion.*

We next investigate how this ratio changes with uncertainty. In Figure 4, the horizontal lines represent the utility of the instantaneous revenues the leader receives over time under low uncertainty, σ, and under high uncertainty, σ'. As we will illustrate numerically, increased uncertainty raises the required entry threshold of the follower by more than that of the leader. This results in the increase of the expected utility of the leader's profits, represented by the shaded area of Figure 4, since, under higher uncertainty, she enjoys monopoly profits for longer time and the loss in the leader's expected utility due to the entry of the follower is not significant enough to offset it. In fact, this result is enhanced when the discrepancy in market share is large, since the greater $D(1)$ is, the greater the pre-emptive leader's incentive to invest will be as then the first-mover advantages are greater. Notice also that as greater uncertainty raises the required entry threshold of the follower, the leader's instantaneous revenues cannot drop below the level corresponding to σ' for $t \geq \tau_p^{f'}$.

Proposition 4.5 *The relative discrepancy between the value of the pre-emptive leader and the monopolist at the pre-emptive leader's optimal entry threshold price diminishes with increasing uncertainty.*

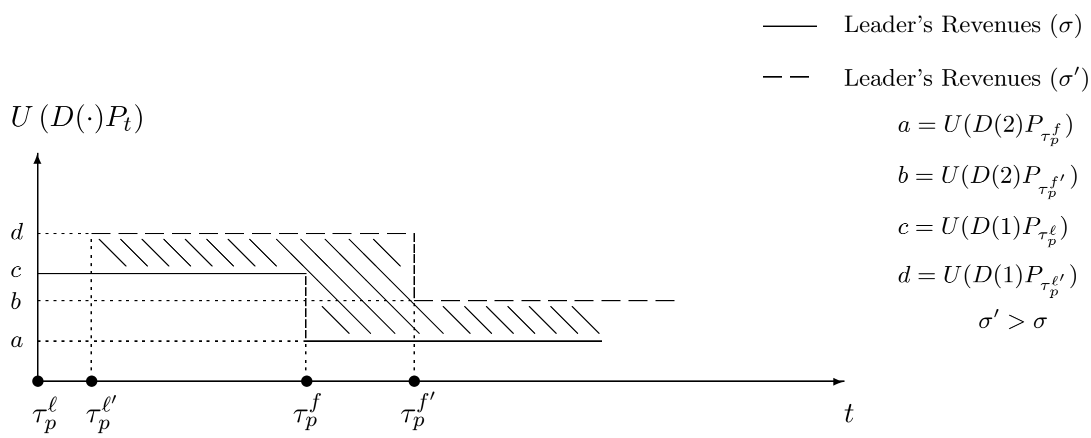

Figure 4: Incremental change in pre-emptive leader's instantaneous revenues due to increased uncertainty

4.3 Symmetric Non-Pre-Emptive Duopoly

In the non-pre-emptive setting, the roles of the leader and the follower are defined exogenously, and, as a result, both firms have the option to delay their entry into the market as the risk of pre-emption is eliminated. The follower's value function and entry threshold are unchanged from the pre-emptive case since she will still enter the market considering that the leader is already there. Hence, the follower's value of investment opportunity at τ_n^ℓ is:

$$F_{\tau_n^f}\left(P_{\tau_n^\ell}\right) = \max_{P_{\tau_n^f} \geq P_{\tau_n^\ell}} \left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^f}}\right)^{\beta_1} V_n^f\left(P_{\tau_n^f}\right) \tag{21}$$

Since the non-pre-emptive leader has discretion over investment timing, her value of investment opportunity is described by:

$$F_{\tau_n^\ell}(P_0) = \max_{P_{\tau_n^\ell} \geq P_0} \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left[\mathcal{A}U\left(P_{\tau_n^\ell} D(1)\right) - \frac{U(rK)}{\rho} + \right.$$
$$\left. + \left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f*}}}\right)^{\beta_1} \mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right]\right] \tag{22}$$

The solution to the optimisation problem (22) yields the optimal entry threshold of the non-pre-emptive leader:

$$P_{\tau_n^{\ell*}} = \frac{rK}{D(1)}\left(\frac{\beta_2 + \gamma - 1}{\beta_2}\right)^{\frac{1}{1-\gamma}} \tag{23}$$

Notice that by delaying entry, the leader suffers from forgoing cash flows but benefits from temporarily delaying the entry of the follower. At the same time, allowing the project to start at a higher output price yields a higher NPV but then the leader enjoys monopoly revenues for less time. As it is shown in the appendix, the marginal benefit and marginal cost corresponding to the entry of the follower cancel.

Proposition 4.6 *The optimal entry threshold of the non-pre-emptive leader is the same as that of the monopolist.*

Notice that the leader's option to invest consists of the expected utility of the immediate payoff reduced by an amount corresponding to the expected loss in utility due to the entry of the follower. After the leader has entered the market and prior to the entry of the follower, i.e., for $P_{\tau_n^\ell} \leq P_t < P_{\tau_n^{f*}}$, the leader receives monopolistic profits with expected utility described by (24):

$$\mathcal{A}U\left(P_t D(1)\right) - \frac{U(rK)}{\rho} + \left(\frac{P_t}{P_{\tau_n^{f*}}}\right)^{\beta_1} \mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{24}$$

According to (24), although the leader is alone in the industry, the expected utility of her profits do not correspond to those of a monopolist since the potential entry of a rival reduces the expected utility of the leader's profits. Finally, after the follower's entry, i.e., for $t \geq \tau_n^f$, the two firms share the industry, thereby making equal profits, and their value is simply the discounted expected utility of the projects cash flows.

In the non-pre-emptive framework, the value of the leader would be the same as the monopolist's if it were not for the potential entry of the follower that reduces the expected utility of the leader's profits. However, the reduction in the leader's value of investment opportunity due to the potential entry of the follower decreases with risk aversion. This happens because risk aversion delays the entry of the follower, thereby reducing the expected loss in the option value of the leader.

Consequently, the relative discrepancy between the leader's value of investment opportunity and the monopolist's diminishes with increasing risk aversion, thereby reducing the relative loss in the value of the non-pre-emptive leader.

Proposition 4.7 *The loss in the value of the investment opportunity for the non-pre-emptive leader relative to that of a monopolist at the pre-emptive leader's optimal entry threshold price decreases with risk aversion.*

According to Proposition 4.8, depending on the discrepancy in market share, uncertainty may increase or decrease the relative loss in the value of the investment opportunity for the non-pre-emptive leader relative to that of a monopolist. Notice that the value of the non-pre-emptive leader consists of the value of the monopolistic investment opportunity and the expected loss in project value due to the entry of the follower. Both of these components increase with uncertainty; however, for the latter, the impact of uncertainty becomes less profound as the discrepancy in market share diminishes. As a result, under low discrepancy in market share, the impact of uncertainty on the non-pre-emptive leader's value of monopolistic investment opportunity dominates, thereby making her better off. By contrast, under large discrepancy in market share, increased uncertainty causes the loss in project value to increase faster than the value of the investment opportunity, thereby making the non-pre-emptive leader worse off.

Proposition 4.8 *The discrepancy between the non-pre-emptive leader's value of investment opportunity and the monopolist's at the pre-emptive leader's optimal entry threshold price increases with uncertainty if:*

$$\left(\frac{D(1)}{D(2)}\right)^{\beta_1} > e, \ e \simeq 2.718 \tag{25}$$

In Figure 5, the instantaneous revenues of the leader are represented by the solid line for low uncertainty, σ, and by the broken line for high uncertainty, σ'. Here, unlike the pre-emptive setting, the leader has the option to delay entry into the market. Notice that a large discrepancy in market share implies a greater first-mover advantage but also leads to a greater loss in the value of the leader upon the entry of the follower, which becomes more profound with higher uncertainty. However, increased uncertainty also raises the value of the leader's investment opportunity, thereby creating an opposing effect. According to Proposition 4.8, under small discrepancy in market share, the increase in option value due to increased uncertainty, represented by the shaded area between τ_n^{ℓ} and $\tau_n^{\ell'}$ in Figure 5, offsets the loss in the leader's revenues due to the entry of the follower, thereby reducing the discrepancy between the value of the monopolist and the leader. The opposite result is observed if the discrepancy in market share is large, since then the loss in the leader's revenues is more profound than the increase in the value of her investment opportunity. This happens because a higher first-mover advantage reduces the required entry threshold of the leader. Consequently, the increase in the value of the investment opportunity is less profound, and as higher uncertainty impacts the loss in project value by more, the non-pre-emptive leader becomes worse off.

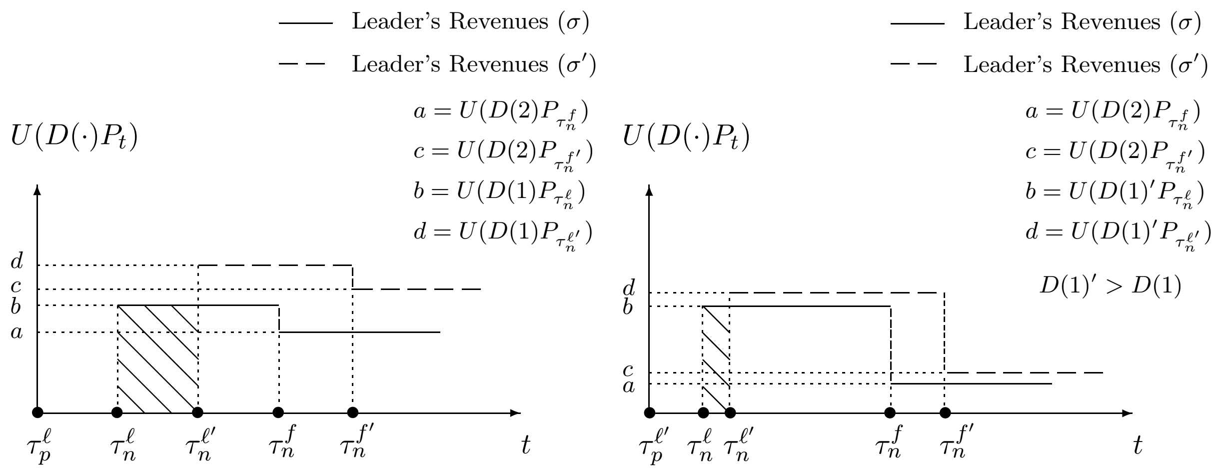

Figure 5: Incremental change in non-pre-emptive leader's instantaneous revenues due to increased uncertainty under low discrepancy in market share (left) and large discrepancy (right)

5 Numerical Results

5.1 Pre-Emptive Duopoly

In order to examine the impact of risk aversion and uncertainty on the entry of the pre-emptive leader and follower, we assume the following parameter values: $\gamma \in [0, 1)$, $\sigma \in [0.1, 0.5]$, $\mu = 0.01$, $r = \rho = 0.05$, $K = \$100$, $D(0) = 0$, $D(1) = 1.5$ or 3, and $D(2) = 1$. Figure 6 illustrates the impact of uncertainty on the value of the pre-emptive leader and follower under risk aversion. First, we observe that the leader's entry threshold is lower than the monopolist's. This happens due to pre-emption since the leader does not have the option to defer investment and, as a result, the risk of pre-emption reduces the required investment threshold. On the other hand, the required investment threshold of the pre-emptive follower is higher than that of the monopolist since the former requires compensation for losing the first-mover advantage. According to the graph on the right, uncertainty increases the value of waiting, thereby raising the required investment threshold and delaying the entry of the follower. This, in turn, increases the time interval in which the leader enjoys monopoly profits and diminishes the relative discrepancy between the value of the pre-emptive leader and that of the monopolist.

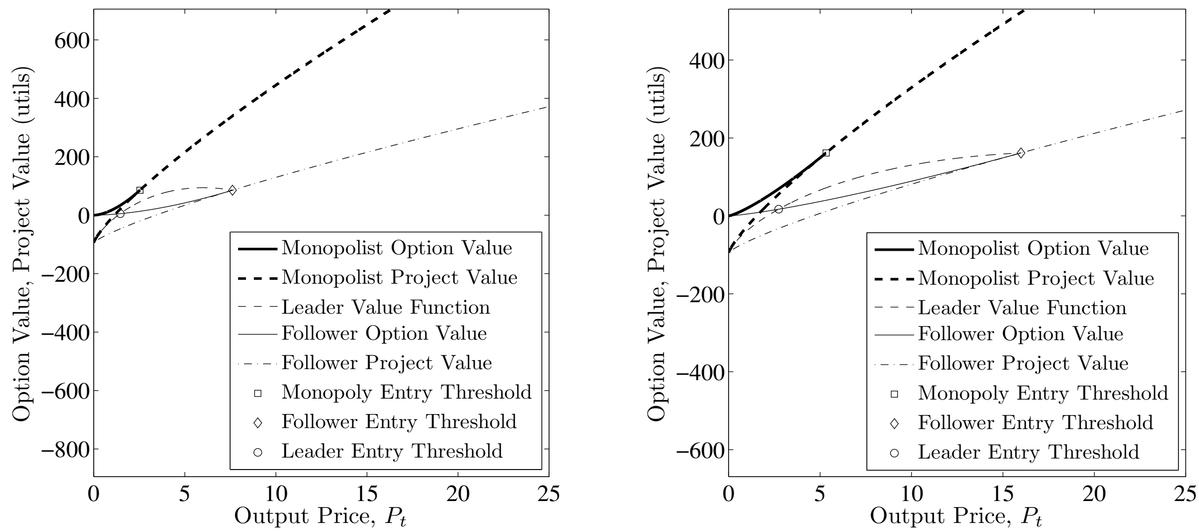

Figure 6: Project and investment opportunity value of monopolist, pre-emptive leader, and follower for $\sigma = 0.2$ (left) and $\sigma = 0.4$ (right) under risk aversion ($\gamma = 0.2$) for $D(1) = 3$

Figure 7 illustrates the impact of risk aversion on the value of the pre-emptive leader and follower. According to the graph on the right, increased risk aversion reduces the expected utility of the investment's payoff for both the leader and the monopolist, thereby raising their required investment thresholds. Furthermore, it seems that the impact of risk aversion on the pre-emptive leader's value is greater than on the follower's value. Consequently, the two curves intersect at a higher output price, thereby indicating that the output price at which a firm is indifferent between becoming a leader or a follower increases with higher risk aversion.

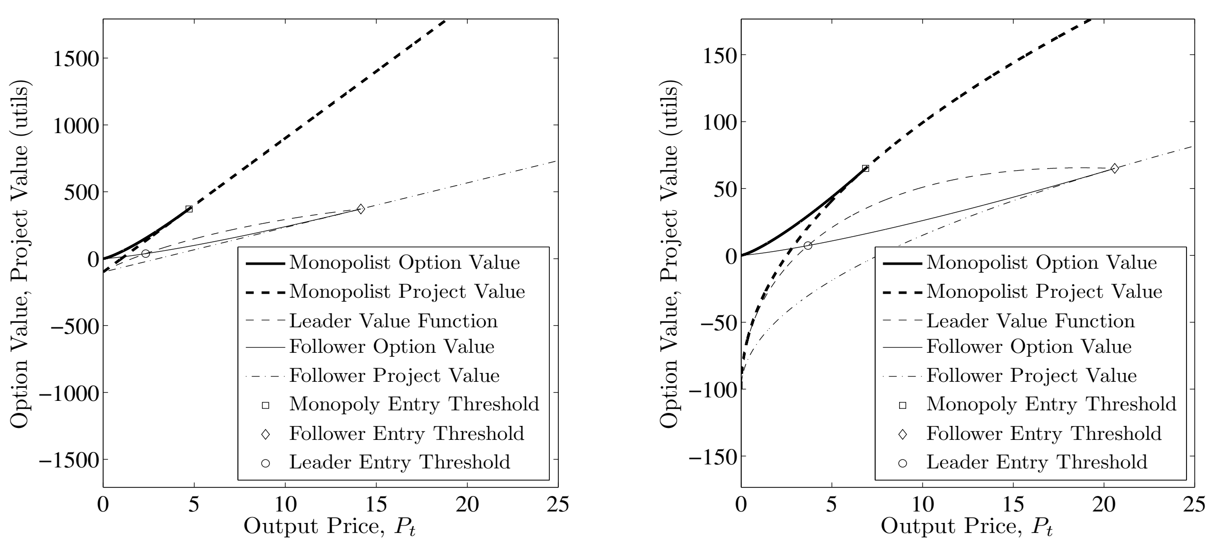

Figure 7: Investment opportunity and project value of monopolist, pre-emptive leader, and follower under risk neutrality (left) and risk aversion ($\gamma = 0.5$) (right) for $\sigma = 0.4$ and $D(1) = 3$

5.2 Non-Pre-Emptive Duopoly

In the non-pre-emptive duopoly, the roles of the leader and the follower are pre-assigned, and, as a result, both firms have the option to postpone their entry into the market. According to Figure 8, the optimal entry threshold of the non-pre-emptive follower is the same as in the pre-emptive case since the follower will still enter the market considering that the leader has already invested. Notice also that, the optimal entry threshold of the non-pre-emptive leader is the same as the monopolist's, and, as a result, the required investment threshold of the non-pre-emptive leader is higher than that in the pre-emptive scenario. Although the optimal entry threshold is the same for the monopolist and non-pre-emptive leader, the investment opportunity value of the latter is lower than that of the former since the potential entry of the follower reduces the expected utility of the leader's profits. As the graph on the right illustrates, increased uncertainty raises the value of waiting, which, in turn, postpones investment in all cases, thereby increasing the required investment thresholds.

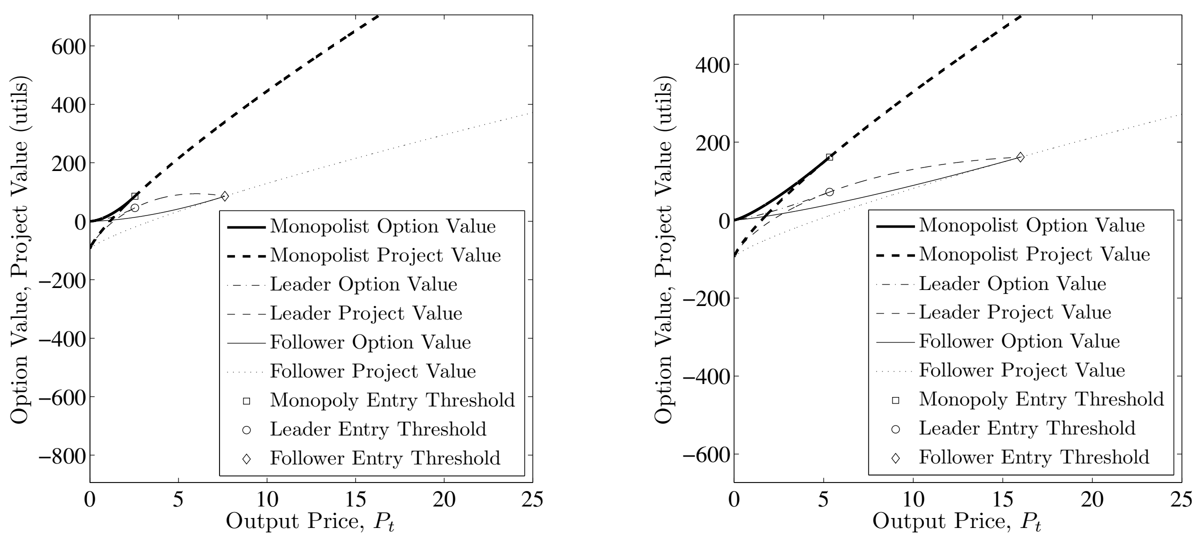

Figure 8: Project and investment opportunity value for non-pre-emptive leader and follower for $\sigma = 0.2$ (left) and $\sigma = 0.4$ (right) under risk aversion ($\gamma = 0.2$) for $D(1) = 3$

Figure 9 illustrates the impact of risk aversion on the optimal entry thresholds of the monopolist and the non-pre-emptive leader and follower. As indicated in the graphs, higher risk aversion reduces the expected utility of the investment's payoff in all cases, thereby raising the required investment thresholds.

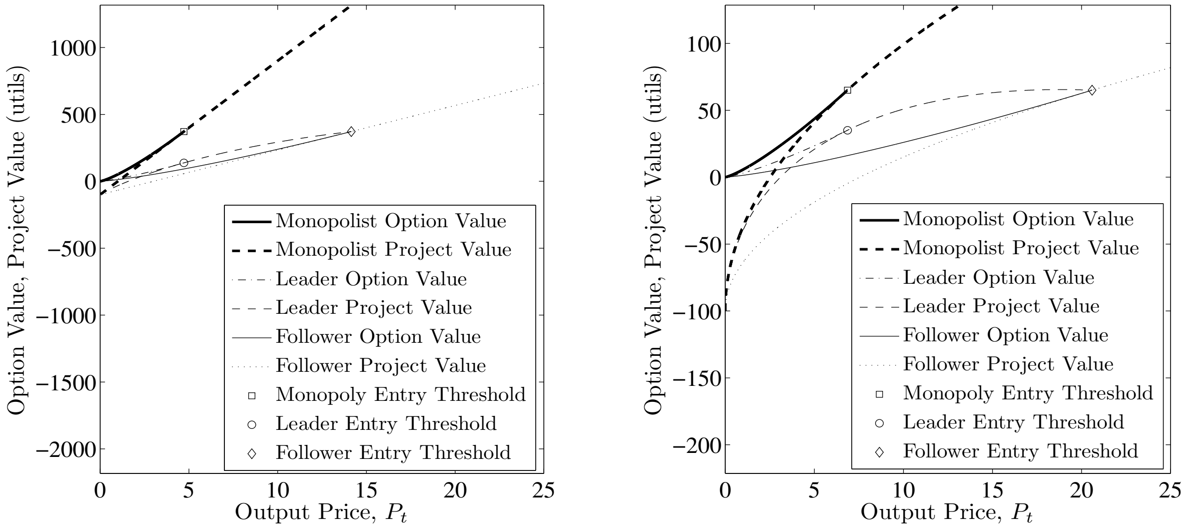

Figure 9: Project and investment opportunity value for non-pre-emptive leader and follower under risk neutrality (left) and risk aversion ($\gamma = 0.5$) (right) for $\sigma = 0.4$ and $D(1) = 3$

5.3 Sensitivity Analysis

As the left panel in Figure 10 illustrates, all entry thresholds increase with volatility as greater uncertainty implies greater value of waiting and are higher with risk aversion as it delays investment both for the leader and the follower by decreasing the expected utility of the project's cash flows. Proposition 4.6 is illustrated by the fact that the leader's optimal investment threshold is the same as the monopolist's. Also, higher first-mover advantages represented by greater $D(1)$ result in the decrease of the required entry thresholds of the pre-emptive and non-pre-emptive leader as illustrated in the graph on the right.

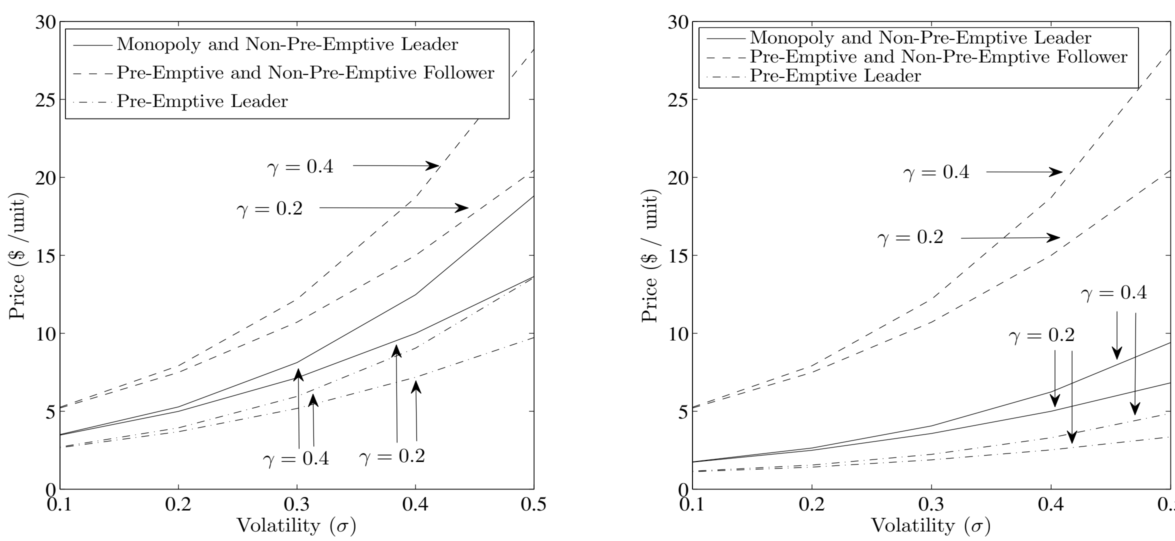

Figure 10: Optimal entry thresholds for $D(1) = 1.5$ (left) and $D(1) = 3$ (right)

In order to compare the pre-emptive and non-pre-emptive leader's values to the monopolist's, we evaluate both at the pre-emptive leader's optimal entry threshold, i.e., at $P_{\tau_p^{\ell*}}$. According to the graph on the left in Figure 11, increased uncertainty diminishes the relative loss in the pre-emptive leader's value function, i.e.,

$$\frac{F_{\tau_m^j}(P_{\tau_p^{\ell*}}) - V_p^{\ell}(P_{\tau_p^{\ell*}})}{F_{\tau_m^j}(P_{\tau_p^{\ell*}})} \tag{26}$$

thereby reducing the discrepancy between the pre-emptive leader's value and the monopolist's value of investment opportunity. This happens because uncertainty postpones the entry of the follower, thus allowing the pre-emptive leader to enjoy monopoly profits longer. Notice that the impact of uncertainty is more profound when the discrepancy in market share is low since then the expected loss due to the follower's entry is smaller.

Uncertainty increases the discrepancy in the non-pre-emptive leader's value of investment opportunity, i.e.,

$$\frac{F_{\tau_m^j}(P_{\tau_p^{\ell*}}) - F_{\tau_n^{\ell}}(P_{\tau_p^{\ell*}})}{F_{\tau_m^j}(P_{\tau_p^{\ell*}})} \tag{27}$$

if the discrepancy in market share is small, i.e., $\left(\frac{D(1)}{D(2)}\right)^{\beta_1} < e$, as in the graph on the left. Intuitively, this happens because under low discrepancy in market share, the increase in the non-pre-emtpive leader's value of investment opportunity due to increased uncertainty is greater than the expected loss due to the entry of the follower. However, if the discrepancy is large, then the increase in option value is less profound with higher uncertainty due to higher first-mover advantages and, as a result, cannot offset the expected loss from the follower's entry, which is now greater.

Furthermore, risk aversion does not affect the relative loss in the value of the leader for the pre-emptive duopoly setting, but it makes the loss in value relatively less for the leader in a non-pre-emptive duopoly setting due to delayed entry of the follower. Notice that at $P_{\tau_p^{\ell*}}$, the value function of the pre-emptive leader is the same as the option value of the pre-emptive follower. As a result, the impact of risk aversion on the value of the pre-emptive leader at $P_{\tau_p^{\ell*}}$ is the same as that on the value of the follower's investment opportunity at the same output price. Since the follower's investment opportunity value differs from the monopolist's only with respect to the market share, risk aversion impacts the values of the follower and the monopolist proportionally.

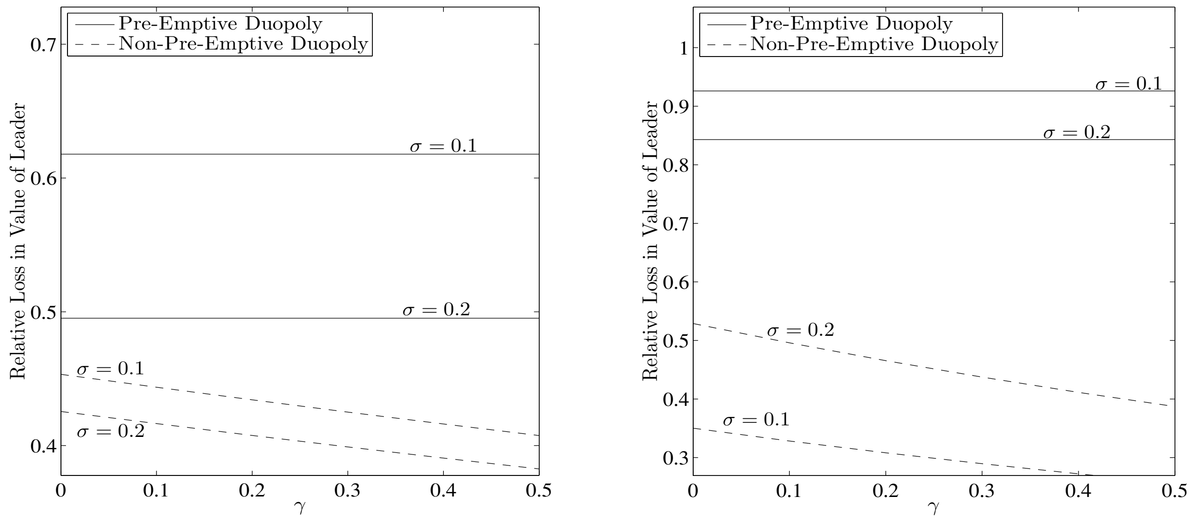

Figure 11: Relative loss in value of the pre-emptive and non-pre-emptive leader for $D(1) = 1.5$ (left) and $D(1) = 3$ (right)

The numerical results of Sections 5.1 and 5.2 are also illustrated in Table 1, under small ($D(1) = 1.5$) and large ($D(1)' = 3$) discrepancy in market share. Notice that, as $D(1)$ increases, the pre-emptive and non-pre-emptive leader's entry thresholds diminish as the first-mover advantages are higher, thereby increasing the incentive to invest.

		Optimal Entry Thresholds					Relative Loss in Value of Leader			
σ	γ	$P_{\tau_m^{j*}} = P_{\tau_n^{\ell*}}$		$P_{\tau_p^{\ell*}}$		$P_{\tau_n^{f*}} = P_{\tau_p^{f*}}$	Pre-Emptive		Non-Pre-Emptive	
		$D(1)$	$D(1)'$	$D(1)$	$D(1)'$	$D(2) = 1$	$D(1)$	$D(1)'$	$D(1)$	$D(1)'$
0.1	0	3.4574	1.7287	2.6450	1.1074	5.1861	0.6178	0.9262	0.4533	0.3502
	0.1	3.4683	1.7342	2.6600	1.1200	5.2025	0.6178	0.9262	0.4436	0.3284
	0.2	3.4796	1.7398	2.6753	1.1325	5.2194	0.6178	0.9262	0.4342	0.3082
	0.3	3.4913	1.7457	2.6908	1.1449	5.2370	0.6178	0.9262	0.4251	0.2896
	0.4	3.5034	1.7517	2.7066	1.1574	5.2552	0.6178	0.9262	0.4162	0.2723
	0.5	3.5160	1.7580	2.7226	1.1698	5.2741	0.6178	0.9262	0.4075	0.2564
0.2	0	4.9149	2.4574	3.6171	1.3759	7.3723	0.4952	0.8431	0.4255	0.5290
	0.1	4.9937	2.4968	3.6868	1.4157	7.4905	0.4952	0.8431	0.4165	0.4960
	0.2	5.0794	2.5397	3.7619	1.4576	7.6191	0.4952	0.8431	0.4076	0.4656
	0.3	5.1729	2.5865	3.8430	1.5018	7.7594	0.4952	0.8431	0.3990	0.4374
	0.4	5.2756	2.6378	3.9310	1.5489	7.9133	0.4952	0.8431	0.3907	0.4114
	0.5	5.3887	2.6943	4.0272	1.5992	8.0830	0.4952	0.8431	0.3826	0.3872
0.3	0	6.8929	3.4465	4.9787	1.7887	10.3394	0.4352	0.7873	0.3979	0.5994
	0.1	7.1400	3.5700	5.1755	1.8818	10.7100	0.4352	0.7873	0.3894	0.5621
	0.2	7.4215	3.7108	5.3984	1.9853	11.1323	0.4352	0.7873	0.3811	0.5276
	0.3	7.7455	3.8728	5.6533	2.1017	11.6183	0.4352	0.7873	0.3731	0.4957
	0.4	8.1226	4.0613	5.9485	2.2342	12.1840	0.4352	0.7873	0.3653	0.4661
	0.5	8.5674	4.2837	6.2948	2.3876	12.8511	0.4352	0.7873	0.3577	0.4388
0.4	0	9.4347	4.7174	6.7449	2.3403	14.1521	0.4022	0.7520	0.3793	0.6295
	0.1	9.9950	4.9975	7.1724	2.5227	14.9925	0.4022	0.7520	0.3712	0.5903
	0.2	10.6622	5.3311	7.6795	2.7362	15.9933	0.4022	0.7520	0.3633	0.5541
	0.3	11.4700	5.7350	8.2912	2.9906	17.2049	0.4022	0.7520	0.3557	0.5206
	0.4	12.4674	6.2337	9.0442	3.3004	18.7011	0.4022	0.7520	0.3482	0.4896
	0.5	13.7291	6.8645	9.9940	3.6877	20.5936	0.4022	0.7520	0.3410	0.4609
0.5	0	12.5759	6.2880	8.9354	3.0324	18.8639	0.3825	0.7292	0.3671	0.6440
	0.1	13.6388	6.8194	9.7284	3.3503	20.4582	0.3825	0.7292	0.3593	0.6039
	0.2	14.9567	7.4784	10.7093	3.7399	22.4351	0.3825	0.7292	0.3517	0.5668
	0.3	16.6299	8.3149	11.9519	4.2293	24.9448	0.3825	0.7292	0.3442	0.5325
	0.4	18.8168	9.4084	13.5732	4.8637	28.2252	0.3825	0.7292	0.3370	0.5008
	0.5	21.7815	10.8908	15.7681	5.7179	32.6723	0.3825	0.7292	0.3300	0.4715

6 Conclusions

In this paper, we develop a utility-based framework in order to examine the impact of risk aversion and uncertainty on the optimal investment timing decisions of a firm that faces competition. The analysis is motivated both by the increasing competition resulting from the deregulation of many sectors of the economy such as energy, telecommunications, transport, etc, and the fact that attitudes towards the risk arising from the potential entry of a rival may impact investment decisions of a firm. The combination of these two factors creates the need to incorporate risk aversion into the real options framework, in order to analyse strategic aspects of decision making under uncertainty.

We find that, under the fear of pre-emption, higher uncertainty reduces the relative loss in the value of the leader due to competition by delaying the entry of the follower. However, in the non-pre-emptive setting, the impact of uncertainty is ambiguous and depends on the discrepancy in market share. If the discrepancy is large, the non-pre-emptive leader's relative loss in value increases with uncertainty since then the impact of the follower's entry is more profound and offsets the increase in the leader's value of investment opportunity. By contrast, under low discrepancy in market share, higher uncertainty makes the non-pre-emptive leader better off as the increase in the value of investment opportunity is greater than the expected loss in value due to competition. Interestingly, the relative loss in the pre-emptive leader's value is not affected by risk aversion, while the non-pre-emptive leader becomes better off with greater risk aversion as it delays the entry of the follower.

This work considers the case where the two competing firms exhibit the same level of risk aversion and, as a result, a potential extension is to relax this assumption and assume different levels of risk aversion for each firm. Directions for future research may also include the application of a different stochastic process, i.e., arithmetic Brownian motion, or the study of other aspects of the real options literature, such as the time to built or capacity sizing, under the same framework.

7 Appendix

Proposition 4.1: *Uncertainty and risk aversion increase the optimal investment threshold.*

Proof: See Propositions 4.2 and 4.3 in Chronopoulos *et al.* (2010). ■

Proposition 4.2: *The value function of the pre-emptive leader is concave and its maximum value* *is obtained prior to the entry of the pre-emptive follower provided that:*

$$D(2) < D(1)\left(\frac{\beta_1+\gamma-1}{\beta_1}\right)^{\frac{1}{1-\gamma}} \tag{28}$$

Proof: The value of the pre-emtpive leader is:

$$V_p^{\ell}(P_t) \;=\; \mathcal{A}U\left(P_t D(1)\right) - \frac{U\left(rK\right)}{\rho} + \left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1} \mathcal{A}U(P_{\tau_p^{f*}})\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{29}$$

Differentiating (29) with respect to P_t we have:

$$\frac{\partial V_p^{\ell}(P_t)}{\partial P_t} = \mathcal{A}D(1)^{1-\gamma}{P_t}^{-\gamma} + \beta_1\left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1}\frac{1}{P_t}\mathcal{A}U(P_{\tau_p^{f*}})\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right] \tag{30}$$

Hence,

$$\frac{\partial V_p^{\ell}(P_t)}{\partial P_t} = 0 \Rightarrow P_t = P_{\tau_p^{f*}}\left\{\frac{\beta_1}{1-\gamma}\left[1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma}\right]\right\}^{\frac{1}{1-\beta_1-\gamma}} \tag{31}$$

Notice that $\frac{1}{1-\beta_1-\gamma} < 0$. Hence, for (31) to be valid we must have:

$$1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma} > 0 \Leftrightarrow \left(\frac{D(2)}{D(1)}\right)^{1-\gamma} < 1 \Leftrightarrow D(2) < D(1) \tag{32}$$

which is true. In order to show that the value of the pre-emptive leader obtains a maximum, we partially differentiate (30) with respect to P_t.

$$\begin{aligned}\frac{\partial^2 V_p^{\ell}(P_t)}{\partial {P_t}^2} \;&=\; \mathcal{A}D(1)^{1-\gamma}(-\gamma){P_t}^{-\gamma-1} \\ &\quad+\beta_1(\beta_1-1){P_t}^{\beta_1-2}\left(\frac{1}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U(P_{\tau_p^{f*}})\left[D(2)^{1-\gamma} - D(1)^{1-\gamma}\right]\end{aligned} \tag{33}$$

As both terms in (33) are negative, we have $\frac{\partial^2 V_p^{\ell}(P_t)}{\partial {P_t}^2} < 0$ for all $P_t \in \left[P_{\tau_p^{\ell}}, P_{\tau_p^{f*}}\right)$. Finally, we will derive the condition under which the output price at which $V_p^{\ell}(P_t)$ becomes maximised is lower than the optimal entry threshold of the follower:

$$\begin{aligned}\left\{\frac{\beta_1}{1-\gamma}\left[1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma}\right]\right\}^{\frac{1}{\beta_1+\gamma-1}} \;&>\; 1 \\ \Leftrightarrow \frac{\beta_1}{1-\gamma}\left[1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma}\right] \;&>\; 1 \\ \Leftrightarrow 1-\left(\frac{D(2)}{D(1)}\right)^{1-\gamma} \;&>\; \frac{1-\gamma}{\beta_1} \\ \Leftrightarrow D(2) \;&<\; \left(\frac{\beta_1+\gamma-1}{\beta_1}\right)^{\frac{1}{1-\gamma}} D(1)\end{aligned} \tag{34}$$

Notice that $\frac{\beta_1+\gamma-1}{\beta_1} < 1$. This implies that in order for the value function of the pre-emptive leader to decrease prior to the entry of the follower, the discrepancy in market share must be significantly large. ■

Proposition 4.3: *The pre-emptive leader's entry threshold is lower than that of the monopolist.*

Proof: First, notice that the follower's value of investment opportunity is:

$$\begin{aligned}
F_{\tau_p^f}(P_t) &= \left(\frac{P_t}{P_{\tau_p^{f*}}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^{f*}}\right) \\
\Rightarrow \frac{\partial F_{\tau_p^f}(P_t)}{\partial P_t} &= \beta_1 P_t^{\beta_1-1}\left(\frac{1}{P_{\tau_p^{f*}}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^{f*}}\right) > 0,\ \forall P_t \in \left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right) \\
\Rightarrow \frac{\partial^2 F_{\tau_p^f}(P_t)}{\partial P_t^2} &= \beta_1(\beta_1-1) P_t^{\beta_1-2}\left(\frac{1}{P_{\tau_p^{f*}}}\right)^{\beta_1} V_p^f\left(P_{\tau_p^{f*}}\right) > 0,\ \forall P_t \in \left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right)
\end{aligned} \tag{35}$$

Thus, the value of the follower's investment opportunity is convex and strictly increasing from zero. Second, from Proposition 4.2, we know that the pre-emptive leader's value function is strictly concave in P_t starting from a negative value. Consequently, for $P_t < P_{\tau_p^{f*}}$ the two value functions intersect at most once. In order to show that the pre-emptive leader's entry threshold is lower than that of the monopolist, we will evaluate the pre-emptive leader's value and the pre-emptive follower's value of investment opportunity at the monopolist's entry threshold. The objective is to prove that at the monopolist's optimal entry threshold, the value of the pre-emptive leader is greater than the value of the pre-emptive follower's investment opportunity, i.e.,

$$\begin{aligned}
\mathcal{A}U\left(P_{\tau_m^{j*}} D(1)\right) - \frac{U(rK)}{\rho} &+ \left(\frac{P_{\tau_m^{j*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}} D(2)\right) - \mathcal{A}U\left(P_{\tau_p^{f*}} D(1)\right)\right] > \\
& \left(\frac{P_{\tau_m^{j*}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}} D(2)\right) - \frac{U(rK)}{\rho}\right]
\end{aligned} \tag{36}$$

Substituting for $P_{\tau_m^{j*}}$ and $P_{\tau_p^{f*}}$ we have:

$$\begin{aligned}
& \frac{\beta_1\beta_2}{\rho(\beta_1+\gamma-1)(\beta_2+\gamma-1)}\left(\frac{rK}{D(1)}\right)^{1-\gamma}\left(\frac{\beta_2+\gamma-1}{\beta_2}\right)\frac{D(1)^{1-\gamma}}{1-\gamma} - \frac{(rK)^{1-\gamma}}{\rho(1-\gamma)} \\
& + \left(\frac{D(2)}{D(1)}\right)^{\beta_1}\frac{\beta_1\beta_2}{\rho(\beta_1+\gamma-1)(\beta_2+\gamma-1)}\left(\frac{rK}{D(2)}\right)^{1-\gamma}\left(\frac{\beta_2+\gamma-1}{\beta_2}\right)\left[\frac{D(2)^{1-\gamma}}{1-\gamma} - \frac{D(1)^{1-\gamma}}{1-\gamma}\right] > \\
& \left(\frac{D(2)}{D(1)}\right)^{\beta_1}\left[\frac{\beta_1\beta_2}{\rho(\beta_1+\gamma-1)(\beta_2+\gamma-1)}\left(\frac{rK}{D(2)}\right)^{1-\gamma}\left(\frac{\beta_2+\gamma-1}{\beta_2}\right)\frac{D(2)^{1-\gamma}}{1-\gamma} - \frac{(rK)^{1-\gamma}}{\rho(1-\gamma)}\right] \\
\Leftrightarrow\ & 1-\gamma+\beta_1\left(\frac{D(2)}{D(1)}\right)^{\beta_1}\left[1-\left(\frac{D(1)}{D(2)}\right)^{1-\gamma}\right] > \left(\frac{D(2)}{D(1)}\right)^{\beta_1}(1-\gamma) \\
\Leftrightarrow\ & (1-\gamma)\left(\frac{D(1)}{D(2)}\right)^{\beta_1} - \beta_1\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} > 1-\beta_1-\gamma
\end{aligned} \tag{37}$$

The last inequality can be written as follows:

$$b - a + ax^b - bx^a > 0 \tag{38}$$

where $a = 1 - \gamma < 1$, $b = \beta_1 > 1$, and $x = \left(\frac{D(1)}{D(2)}\right) > 1$. Since $b - a = \beta_1 + \gamma - 1 > 0$, in order to show (38), we need to show that $ax^b - bx^a > 0$. For this reason, let:

$$f(x) = ax^b - bx^a \tag{39}$$

Notice that:

$$f'(x) = ab\left(x^{b-1} - x^{a-1}\right) \tag{40}$$

Since $b > a \Rightarrow f'(x) > 0$. Notice also that:

$$f''(x) = ab\left((b-1)x^{b-2} - (a-1)x^{a-2}\right) > 0 \tag{41}$$

which implies that $f(x)$ is increasing and convex. Also:

$$f'(x) = 0 \Rightarrow x = 1 \ \ and \ \ f(1) = 0 \tag{42}$$

As a result, the minimum value of $f(x)$ is at $x = 1$ and is equal to $f(1) = 0$. Thus,

$$f(x) > f(1) = 0 \Rightarrow ax^b - bx^a > 0, \ \ \forall\, x > 1 \tag{43}$$

Therefore, at the entry threshold of the monopolist, the value function of the pre-emptive leader is greater than the follower's value of investment opportunity. Notice also that:

$$P_t \to 0 \Rightarrow V_p^\ell(P_t) < F_{\tau_p^f}(P_t) \tag{44}$$

Since, according to Proposition 4.2, the maximum that the value of the pre-emptive leader can obtain in $\left[P_{\tau_p^\ell}, P_{\tau_p^{f*}}\right)$ is global, this implies that, $\forall P_t : P_t < P_{\tau_m^{j*}}$, $\exists$ at most one price $P_{\tau_p^{\ell*}} : F_{\tau_p^f} = V_p^\ell$. Hence, from (43) and (44), we conclude that $P_{\tau_p^{\ell*}} < P_{\tau_m^{j*}}$. ■

Proposition 4.4: *The loss in the pre-emptive leader's value relative to the monopolist's value of investment opportunity at the pre-emptive leader's optimal entry threshold price is unaffected by risk aversion.*

Proof: In order to show that the relative loss in value is unaffected by risk aversion, we consider the following ratio:

$$\frac{V_p^\ell\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} \tag{45}$$

Notice that $F_{\tau_m^j}(\cdot)$ is given by (11), which we re-write here for $P_0 = P_{\tau_p^{\ell *}}$:

$$F_{\tau_m^j}\left(P_{\tau_p^{\ell *}}\right) = \left(\frac{P_{\tau_p^{\ell *}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_m^{j*}}D(1)\right) - \frac{U(rK)}{\rho}\right] \tag{46}$$

Similarly, the expression for $V_p^{\ell}(\cdot)$ evaluated at $P_{\tau_p^{\ell *}}$ is given by:

$$\begin{aligned} V_p^{\ell}\left(P_{\tau_p^{\ell *}}\right) = \mathcal{A}U\left(P_{\tau_p^{\ell *}}D(1)\right) &- \frac{U(rK)}{\rho} \\ &+ \left(\frac{P_{\tau_p^{\ell *}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_p^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right) \end{aligned} \tag{47}$$

Notice also that for $P_{\tau_p^{\ell *}}$, the equality $V_p^{\ell}(P_{\tau_p^{\ell *}}) = F_{\tau_p^f}(P_{\tau_p^{\ell *}})$ holds, i.e.:

$$\begin{aligned} \mathcal{A}U\left(P_{\tau_p^{\ell *}}D(1)\right) - \frac{U(rK)}{\rho} &+ \left(\frac{P_{\tau_p^{\ell *}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\mathcal{A}U\left(P_{\tau_p^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right) = \\ &\left(\frac{P_{\tau_p^{\ell *}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_p^{f*}}D(2)\right) - \frac{U(rK)}{\rho}\right] \end{aligned} \tag{48}$$

Substituting the expressions for $P_{\tau_p^{\ell *}}$ and $P_{\tau_m^{j*}}$ from (15) and (17) into (46) and (48), respectively, we have:

$$F_{\tau_m^j}\left(P_{\tau_p^{\ell *}}\right) = \left(\frac{P_{\tau_p^{\ell *}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\frac{1-\gamma}{\beta_1+\gamma-1}\right]\frac{U(rK)}{\rho} \tag{49}$$

and

$$V_p^{\ell}\left(P_{\tau_p^{\ell *}}\right) = \left(\frac{P_{\tau_p^{\ell *}}}{P_{\tau_p^{f*}}}\right)^{\beta_1}\left[\frac{1-\gamma}{\beta_1+\gamma-1}\right]\frac{U(rK)}{\rho} \tag{50}$$

By cancelling the $P_{\tau_p^{\ell *}}$ term and substituting for $P_{\tau_m^{j*}}$ and $P_{\tau_p^{f*}}$, we have:

$$\frac{V_p^{\ell}\left(P_{\tau_p^{\ell *}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell *}}\right)} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \tag{51}$$

As a result, the relative loss in the value of the pre-emptive leader is constant and, for this reason, is unaffected by risk aversion. ■

Proposition 4.5: *The relative discrepancy between the value of the pre-emptive leader and the monopolist at the pre-emptive leader's optimal entry threshold price diminishes with increasing uncertainty.*

Proof: According to (51), the relative value of the pre-emptive leader compared to that of a monopolist is:

$$\frac{V_p^{\ell}\left(P_{\tau_p^{\ell *}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell *}}\right)} = \frac{V_p^{\ell}\left(P_{\tau_p^{\ell *}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell *}}\right)} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \tag{52}$$

Partially differentiating (52) with respect to σ, we have:

$$\frac{\partial}{\partial\sigma}\left\{\left(\frac{D(2)}{D(1)}\right)^{\beta_1}\right\} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \ln\left(\frac{D(2)}{D(1)}\right)\frac{\partial\beta_1}{\partial\sigma} \tag{53}$$

Notice that since $\frac{\partial\beta_1}{\partial\sigma} < 0$ and $\ln\left(\frac{D(2)}{D(1)}\right) < 0$, we have:

$$\frac{\partial}{\partial\sigma}\left[\frac{V_p^{\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)}\right] > 0 \tag{54}$$

This implies that with increasing uncertainty, the loss in the value of the pre-emptive leader relative to the monopolist's diminishes. ■

Proposition 4.6: *The optimal entry threshold of the non-pre-emptive leader is the same as that of the monopolist.*

Proof: Given the initial output price, P_0, and assuming that the follower has chosen the optimal policy, the non-pre-emptive leader's entry problem is described by (55):

$$\begin{aligned} F_{\tau_n^{\ell}}(P_0) \quad = \quad & \max_{P_{\tau_n^{\ell}} \geq P_0} \left\{\left(\frac{P_0}{P_{\tau_n^{\ell}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_n^{\ell}}D(1)\right) - \frac{U(rK)}{\rho}\right.\right. \\ & \left.\left. + \left(\frac{P_{\tau_n^{\ell}}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right]\right]\right\} \end{aligned} \tag{55}$$

Partially differentiating (55) with respect to $P_{\tau_n^{\ell}}$ yields:

$$\begin{aligned} \frac{\partial F_{\tau_n^{\ell}}}{\partial P_{\tau_n^{\ell}}} \quad = \quad & \beta_1\left(\frac{P_0}{P_{\tau_n^{\ell}}}\right)^{\beta_1}\left(-\frac{1}{P_{\tau_n^{\ell}}}\right)\left[\mathcal{A}U\left(P_{\tau_n^{\ell}}D(1)\right) - \frac{U(rK)}{\rho}\right] + \\ + \quad & \left(\frac{P_0}{P_{\tau_n^{\ell}}}\right)^{\beta_1}\mathcal{A}D(1)^{1-\gamma}P_{\tau_n^{\ell}}^{-\gamma} \\ + \quad & \beta_1\left(\frac{P_0}{P_{\tau_n^{\ell}}}\right)^{\beta_1}\left(-\frac{1}{P_{\tau_n^{\ell}}}\right)\left(\frac{P_{\tau_n^{\ell}}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right] \\ + \quad & \beta_1\left(\frac{P_0}{P_{\tau_n^{\ell}}}\right)^{\beta_1}\left(\frac{P_{\tau_n^{\ell}}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\left(\frac{1}{P_{\tau_n^{\ell}}}\right)\left[\mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right] \end{aligned} \tag{56}$$

Rearranging (56) in order to equate the marginal benefit of delaying investment to the marginal cost yields (57). The first term on the left-hand side of (57) corresponds to the reduction in marginal cost due to saved investment cost, while the second term is the marginal benefit from starting the project at a higher output price. The third term reflects the marginal benefit from delaying investment, which postpones the entry of the follower. The first term on the right-hand side of

(57) corresponds to the marginal cost of forgone cash flows due to postponed investment, while the second term reflect the marginal cost from enjoying monopoly profits for less time.

$$\begin{aligned}
&\beta_1 \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left(\frac{1}{P_{\tau_n^\ell}}\right) \frac{U(rK)}{\rho} + \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \mathcal{A}D(1)^{1-\gamma} P_{\tau_n^\ell}^{-\gamma} + \\
&\qquad \beta_1 \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left(-\frac{1}{P_{\tau_n^\ell}}\right) \left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f*}}}\right)^{\beta_1} \left[\mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right] \\
= \;& \beta_1 \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left(\frac{1}{P_{\tau_n^\ell}}\right) \mathcal{A}U\left(P_{\tau_n^\ell} D(1)\right) - \\
&\qquad \beta_1 \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left(\frac{P_{\tau_n^\ell}}{P_{\tau_n^{f*}}}\right)^{\beta_1} \left(\frac{1}{P_{\tau_n^\ell}}\right) \left[\mathcal{A}U\left(P_{\tau_n^{f*}}\right)\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right] \qquad (57)
\end{aligned}$$

Notice that the marginal benefit from postponing investment cancels with the marginal cost from enjoying monopoly profits for less time, and, thus, we obtain (58):

$$\begin{aligned}
\frac{\partial F_{\tau_n^\ell}}{\partial P_{\tau_n^\ell}} = 0 \;\Leftrightarrow\;& \beta_1 \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \left(-\frac{1}{P_{\tau_n^\ell}}\right) \left[\mathcal{A}U\left(P_{\tau_n^\ell} D(1)\right) - \frac{U(rK)}{\rho}\right] + \\
& + \left(\frac{P_0}{P_{\tau_n^\ell}}\right)^{\beta_1} \mathcal{A}D(1)^{1-\gamma} P_{\tau_n^\ell}^{-\gamma} = 0 \\
\Leftrightarrow\;& P_{\tau_n^{\ell*}} = \frac{rK}{D(1)} \left(\frac{\beta_2 + \gamma - 1}{\beta_2}\right)^{\frac{1}{1-\gamma}} \qquad (58)
\end{aligned}$$

From (58), we see that the optimal investment threshold for the non-pre-emptive leader is the same as the monopolist's. ■

Proposition 4.7: *The loss in the value of the investment opportunity for the non-pre-emptive leader relative to that of a monopolist at the pre-emptive leader's optimal entry threshold price decreases with risk aversion.*

Proof: The relative loss in the non-pre-emptive leader's value is:

$$\frac{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) - F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = 1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} \qquad (59)$$

Recall that prior to investment, the non-pre-emptive leader's value of investment opportunity at $P_{\tau_p^{\ell*}}$ is:

$$\begin{aligned}
F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right) \;=\;& \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_n^{\ell*}}}\right)^{\beta_1} \left[\mathcal{A}U\left(P_{\tau_n^{\ell*}} D(1)\right) - \frac{U(rK)}{\rho}\right. \\
& \left. + \left(\frac{P_{\tau_n^{\ell*}}}{P_{\tau_n^{f*}}}\right)^{\beta_1} \left[\mathcal{A}\frac{P_{\tau_n^{f*}}^{1-\gamma}}{1-\gamma}\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right]\right] \qquad (60)
\end{aligned}$$

Notice that the expression of the monopolist's value of investment opportunity, $F_{\tau_m^j}(\cdot)$, evaluated at $P_{\tau_p^{\ell*}}$ is given by (61):

$$F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right) = \left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\mathcal{A}U\left(P_{\tau_m^{j*}}D(1)\right) - \frac{U(rK)}{\rho}\right] \tag{61}$$

Hence,

$$\begin{aligned}
1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} &= -\frac{\left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_n^{\ell*}}}\right)^{\beta_1}\left(\frac{P_{\tau_n^{\ell*}}}{P_{\tau_n^{f*}}}\right)^{\beta_1}\left[\mathcal{A}P_{\tau_n^{f*}}^{1-\gamma}\left(D(2)^{1-\gamma} - D(1)^{1-\gamma}\right)\right]}{\left(\frac{P_{\tau_p^{\ell*}}}{P_{\tau_m^{j*}}}\right)^{\beta_1}\left[\mathcal{A}\left(P_{\tau_m^{j*}}D(1)\right)^{1-\gamma} - \frac{(rK)^{1-\gamma}}{\rho}\right]} \\
&= \left(\frac{D(2)}{D(1)}\right)^{\beta_1}\frac{\mathcal{A}\left(\frac{rK}{D(2)}\right)^{1-\gamma}\left(\frac{\beta_2+\gamma-1}{\beta_2}\right)\left(D(1)^{1-\gamma} - D(2)^{1-\gamma}\right)}{\mathcal{A}\left(\frac{rK}{D(1)}\right)^{1-\gamma}\left(\frac{\beta_2+\gamma-1}{\beta_2}\right)D(1)^{1-\gamma} - \frac{(rK)^{1-\gamma}}{\rho}} \\
&= \left(\frac{D(2)}{D(1)}\right)^{\beta_1}\frac{\beta_1}{1-\gamma}\left[\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1\right]
\end{aligned} \tag{62}$$

Partially differentiating (59) with respect to γ yields:

$$\frac{\partial}{\partial\gamma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_n^{\ell*}}\right)}\right] = \left(\frac{D(2)}{D(1)}\right)^{\beta_1}\frac{\beta_1}{1-\gamma}\left\{\frac{\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1}{1-\gamma} - \left(\frac{D(1)}{D(2)}\right)^{1-\gamma}\ln\frac{D(1)}{D(2)}\right\} \tag{63}$$

According to (63),

$$\begin{aligned}
\frac{\partial}{\partial\gamma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_n^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_n^{\ell*}}\right)}\right] \leq 0 \;&\Leftrightarrow\; \frac{\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1}{1-\gamma} - \left(\frac{D(1)}{D(2)}\right)^{1-\gamma}\ln\frac{D(1)}{D(2)} \leq 0 \\
&\Leftrightarrow\; \left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1 - \left(\frac{D(1)}{D(2)}\right)^{1-\gamma}\ln\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} \leq 0
\end{aligned} \tag{64}$$

Setting $x = \left(\frac{D(1)}{D(2)}\right)^{1-\gamma} > 0$ we have:

$$\begin{aligned}
x - 1 - x\ln x \leq 0 \;&\Leftrightarrow\; x(1-\ln x) \leq 1 \\
&\Leftrightarrow\; 1 - \ln x \leq \frac{1}{x} \\
&\Leftrightarrow\; 1 + \ln\frac{1}{x} \leq \frac{1}{x} \\
&\Leftrightarrow\; -\ln\frac{1}{x} \geq 1 - \frac{1}{x}
\end{aligned} \tag{65}$$

Setting $\frac{1}{x} = y$ we have:

$$-\ln y \geq 1 - y \Leftrightarrow \ln y \leq y - 1 \tag{66}$$

which is true. ■

Proposition 4.8: *The discrepancy between the non-pre-emptive leader's value of investment opportunity and the monopolist's at the pre-emptive leader's optimal entry threshold price increases with uncertainty if:*

$$\left(\frac{D(1)}{D(2)}\right)^{\beta_1} > e$$

Proof: According to (62), the relative loss in option value of the non-pre-emptive leader is:

$$1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)} = \left(\frac{D(2)}{D(1)}\right)^{\beta_1} \frac{\beta_1}{1-\gamma}\left[\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1\right] \tag{67}$$

Partially differentiating with respect to σ we have:

$$\frac{\partial}{\partial\sigma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)}\right] = \left[\left(\frac{D(1)}{D(2)}\right)^{1-\gamma} - 1\right]\left(\frac{D(2)}{D(1)}\right)^{\beta_1}\left\{\frac{\frac{\partial\beta_1}{\partial\sigma}}{1-\gamma} + \frac{\partial\beta_1}{\partial\sigma}\ln\left(\frac{D(2)}{D(1)}\right)\frac{\beta_1}{1-\gamma}\right\} \tag{68}$$

Hence,

$$\begin{aligned}
\frac{\partial}{\partial\sigma}\left[1 - \frac{F_{\tau_n^\ell}\left(P_{\tau_p^{\ell*}}\right)}{F_{\tau_m^j}\left(P_{\tau_p^{\ell*}}\right)}\right] > 0 \quad &\Leftrightarrow \quad \frac{\frac{\partial\beta_1}{\partial\sigma}}{1-\gamma} + \frac{\partial\beta_1}{\partial\sigma}\ln\frac{D(2)}{D(1)}\frac{\beta_1}{1-\gamma} > 0 \\
&\Leftrightarrow \quad \ln\left(\frac{D(2)}{D(1)}\right)^{\beta_1} < -1 \\
&\Leftrightarrow \quad \left(\frac{D(2)}{D(1)}\right)^{\beta_1} < e^{-1} \\
&\Leftrightarrow \quad \left(\frac{D(1)}{D(2)}\right)^{\beta_1} > e
\end{aligned} \tag{69}$$

■

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