Strategic Delinquency Options in US Residential Mortgages

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Abstract

Mortgage option theoretic research generally focuses on the valuation of the default and prepayment options. Exercise of the delinquency option is often regarded as an interim step before terminal states of mortgage default or cure and of limited option value in its own right. However, in the current US housing crisis, borrowers may choose to defer a mortgage payment in order to mitigate a cash flow problem or improve their equity position. This option is available when the lender does not automatically foreclose on the delinquent borrower. Invariably, a negotiation commences between lender and borrower, each motivated by different sets of objectives. We model those negotiations by considering future unavoidable foreclosure costs. We derive closed form solutions for the optimal *ex ante* mortgage loan terms, such as LTV and coupon payment, offered by a lender to a borrower with a strategic delinquency option. We then compare the optimal *ex post* exercise, in terms of the borrower's book LTV, of the delinquency option to the exercise of a default option for borrowers with heterogeneous expectations. We show that the ability to negotiate a larger share of unavoidable foreclosure costs in one's favour has a significant influence on the optimal *ex ante* financing and *ex post* delinquency decisions.

1. Introduction

In light of the current US economic and social situation causing widespread mortgage payment difficulties, we develop a "strategic" delinquency option and investigate the conditions of its optimal exercise. The borrower may threaten the lender with the exercise of this option, triggered by an external event (such as a major purchase, divorce or unemployment) or from the desire to maximise their current equity worth. Our presentation is a unique contribution related to current US empirical papers (Piskorski, Seru and Vig (PSV) 2010 or Adelino, Gerardi and Willen (AGW) 2009) which investigate, on behalf of US policymakers, the (re)negotiation of (non performing) mortgage loans and the so called phenomenon of strategic default and its mitigation.

Our treatment is directed at US owner occupied residential mortgages where delinquency does not automatically result in immediate foreclosure. In this case, the credible threat of default or actual delinquency may be of value in negotiating concessions from the lender. We assume that the lender offers no concessions with respect to the loan principal but is prepared to negotiate the mortgage coupon conditional on the immitigable costs of foreclosure[1]. We assume that with declining house prices, the borrower's other options of prepayment, new credit or sale of their illiquid housing asset are of little value.

We recognise that the borrower's exercise of a strategic delinquency option may be a temporary or transient reaction to trigger events whereby the borrower believes that the lender will not or cannot immediately foreclose. Hence, this option is seen as an interim option on a timeline which follows the exercise of the investment option (t=0) but precedes default, cure or permanent loan term modification. The borrower might eventually decide that (continuing) negative changes in house prices or other adverse trigger events (after exercising the delinquency option) would make exercise of the default option optimal. On the other hand, positive changes in property prices or positive trigger events might make it optimal for the borrower to cure the mortgage to prevent the lender exercising his foreclosure option. The delinquency option, even though of a temporary nature, must have some value to the borrower whereby its valuation and optimal exercise should be of interest to lender, borrower and policymaker.

[1] Our model is more characteristic of current US "forbearance" programs, where the borrower agrees a reduction in the monthly mortgage payment, rather than "loan modification" programs where the terms of the loan such as principal or maturity are permanently adjusted to the advantage of the borrower.

2. Model Outline and Assumptions

To demonstrate this delinquency option, we firstly model the optimal *ex ante* investment decisions of the borrower (equity) and lender (debt) at t=0. This is felt necessary, as the optimal *ex post* delinquency or default decision should, ideally, be examined conditional on the lender and borrower agreeing the optimal debt/equity (LTV) ratio at loan origination. It is of course relatively simpler to model the optimal *ex post* delinquency or default decision where the mortgage is already in situ with a given (non-optimal) LTV and coupon.

The borrower's "spot" property price (V) follows a random gBm process with drift μ. The difference between the drift μ and the risk free rate r is treated as a convenience yield (or market imputed rent[2]) which the borrower "collects" by living in their preferred accommodation. This market imputed rent will vary proportionally with the "spot" value of the property. In a declining housing market, considerations of maintenance and depreciation are of minor importance.

When the local property market is performing well, borrowers will see a notional increase in their housing asset and equity value. Borrowers will therefore continue to supply the needed funds to service the debt when and if it is in their interest to do so -- an example being the property still having positive net equity or the imputed market rent being of sufficient value or convenience.

The situation is different if the property market is not performing well as default is not costless to the lender or borrower. No new equity or debt is available to the borrower due to declining house prices or credit restrictions while no mortgage debt forgiveness occurs on the part of the lender. We model this by assuming that the mortgage is of a perpetual interest only nature. On default, the borrower will lose all equity as well as the ability to collect the imputed market rent while the lender will only receive the house value less foreclosure costs to cover any outstanding debt.

Consequently, rational lenders and borrowers will try to avoid costly foreclosure and in many cases be morally (or legally) obliged to attempt to negotiate and agree a forbearance program. We introduce a parameter ϕ $(0 \leq \phi \leq 1)$ at the optimal strategic delinquency exercise point to model the effect and strength of this inevitable (re)negotiation over the sharing of foreclosure and

[2] During the recent US housing boom this market imputed rent was negative due to average yearly housing capital gains of 5-6% triggering prepayment options with lower exercise of default or delinquency options.. However our focus is on housing markets with low or negative capital appreciation where borrowers might be more likely to exercise their default or delinquency options.

forbearance costs. For ease of exposition, we refer to a borrower who negotiates a smaller notional share of the unavoidable foreclosure costs as a weak borrower ($\phi \to 0$) and one who negotiates a larger share ($\phi \to 1$) as a strong borrower. We treat ϕ as a heterogeneous variable determined by each party's knowledge of the other's share of the potential foreclosure costs. Both the lender and the borrower take a view on how much of the unavoidable foreclosure costs the other would be liable for and condition their *ex ante* loan negotiation on this view.

We believe that the actual identification of these borrowers' characteristics is facilitated by consideration of measurements such as their FICO credit score or residency in a US recourse/non-recourse state. It is reasonable to assume that those borrowers with strong credit scores may be able to negotiate and extract different and better concessions and terms from lenders than borrowers with weaker scores. Recent empirical papers (Pence 2006, PSV 2010, AGW 2009[3]) make comparisons between borrowers and lenders with heterogeneous characteristics and we uniquely attempt to link an important and well documented *ex ante* characteristic FICO score to the negotiated mortgage contract outcome both *ex* and *post* any delinquency or default event.

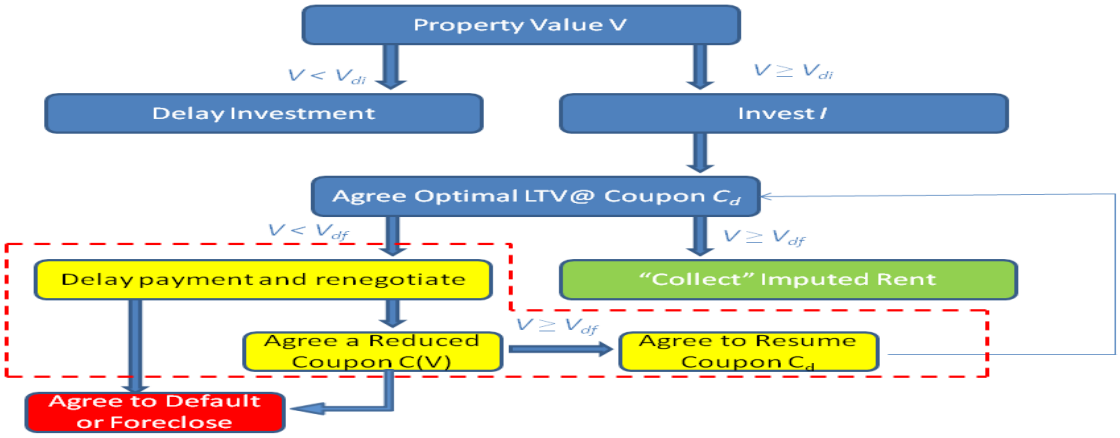
In contrast to the traditional option theoretic approach, as described by Kau and Keenan (1995), we arrive at our solution by extending Leland (1994) as well as Fan and Sundaresan (2000) to cover the borrower's irreversible delinquency, default and investment options. This methodology is thus similar to the endogenous default approach to corporate debt found in finance literature, whereby the management chooses the timing of default to maximise equity value. In general, traditional option theoretic models proceed, using numerical solutions methodology, to optimally calculate the value of the default and prepayment options using two stochastic factors (property prices and interest rates) and a finite mortgage term. To ensure tractability and obtain closed form solutions we employ just one stochastic factor with a perpetual mortgage term. We believe this approach is justified as the stochastic interest rate factor is mainly of influence on the prepayment option (which we assume is valueless) and where new credit is readily available (which we again assume is unlikely with declining house prices).

[3] Piskorski, Seru and Vig (2010) or Adelino, Gerardi and Willen (2009) examine whether default and cure rates are different for securitised and non-securitised loans from borrowers with high and low FICO scores after mortgage (re)negotiation.

Pence (2006) examines whether borrowers in non-recourse states make larger down payments than borrowers with similar individual and property characteristics in recourse states.

The lender and borrower (Diagram 1) play a generalised Nash cooperative game to avoid foreclosure costs. They have perfect knowledge of each other's options and costs whereby, having *ex ante* negotiated the initial mortgage contract (LTV and mortgage payment) conditional on anticipated delinquency, may *ex post* renegotiate the contract should a credible or actual threat of default arise due to a unfavourable shock to spot property prices. We then allow the mortgage to be (temporarily) reinstated at a lower agreed mortgage on a successful negotiation. Should property prices recover, we assume that the lender and borrower would contractually revert to the original or higher mortgage payment to cure the mortgage or the borrower would risk premature foreclosure. Otherwise, on an extended timeline, with continuing negative property price shocks or trigger events, the borrower will default. This presupposes that the lender (as in the current US property market) has not immediately exercised his option to foreclose and recover arrears.

Diagram 1 Delinquency Model Flow Diagram



The initial mortgage contract agrees a relationship, whereby the borrower has limited liability and can default on the mortgage contract at any time with no short-term consequences to a subsequent credit rating[4]. We do not consider options where the borrower voluntarily sells the property, being less costly than liquidating through repossession due to administration costs. Both parties analyse the local property market and each other's circumstances and instantaneously agree a (re)negotiated mortgage coupon, based on an anticipated share of the unavoidable foreclosure costs. The lender will (*ex ante)* express the probable foreclosure costs as a % α of the initial investment *I* (which is at least 90 days in all US states). Both lender and borrower will *ex ante* anticipate the same negotiated outcome whether the borrower *ex post* actually becomes delinquent or merely threatens delinquency.

[4] We note the resulting moral hazard issue that many US lenders and servicers (almost) consider a 90 day delinquency period as a given - an attitude also shared by some borrowers who may well ignore any long term credit rating effect.

We present the optimal delinquency trigger points by transforming the stochastic property price to book LTV for each type of borrower. It is common for policymakers and lenders to measure the likelihood that a borrower will default in terms of their book LTV whereby a book LTV greater than 100% indicates negative equity i.e. the property value is less than the outstanding loan. Using typical US mortgage data we show that optimal delinquency option exercise should occur earlier than default option exercise for all borrowers but strong borrowers (e.g. with high FICO credit scores) should exercise their delinquency option earlier than weak borrowers (e.g. with low FICO credit scores). We show that the lenders *ex ante* mortgage yield spread should increase to pay for the borrower's *ex post* strategic delinquency option. We show that the optimal equity down payment or deposit is conditional on the borrower's negotiation ability (ϕ) in the case of a delinquency but not a default option and that a lender could offer a larger mortgage to a weaker borrower. Finally, we show that while increasing property price volatility should motivate borrowers to *delay* exercising the default option it will on the other hand *accelerate* exercise of their delinquency option.

In Section 3 we model the investment/strategic delinquency options as well as the investment/default options. Detailed derivation of the closed form solutions are largely omitted for clarity but can be obtained from the corresponding author upon request.

In Section 4 using stylised US mortgage data, we examine in detail the fundamental differences between the *ex post* behaviour of both options and highlight the effect of heterogeneous delinquency negotiation on the endogenous delinquency threshold expressed in terms of negative equity, mortgage yield spread and LTV ratios.

A summary and conclusions are drawn in Section 5.

3. Default and Delinquency Options Model Derivation

The property price process is exogenous and the borrower and lender have rational expectations and are sufficiently small to have no effect on local property prices. The borrower will make the mortgage payment to the lender c^* (for the default option) and c_d (for the strategic delinquency option). The mortgage payment c^* or c_d is tax deductible. The borrower thus chooses a mixture of equity and (risky) debt to finance the property investment I at an endogenously chosen time T.

We assume that the borrower has only one property with a property price process given by a gBm (geometric Brownian motion).

$$dV = \mu V dt + \sigma V dW \qquad [1]$$

where W is a standard Brownian motion, μ the net price drift and σ is the property price volatility.

Let $r > 0$ denote the risk free interest rate. Assume $r > \mu$ for convergence. We view the difference $(r - \mu)$ as a convenience yield or the flow of benefits that ownership of the property provides in addition to the expected capital gain μ per unit change of V. This is then treated as a form of imputed or implied housing rent which is proportional to the current value of the property V and equal to $(r - \mu)V$.

Let the tax rate be $0 \leq \tau < 1$. Property asset value is given by $F(V) = E(V) + D(V)$ where $E(V)$ is the value of equity and $D(V)$ the value of debt. The borrower decides when to exercise the investment option by purchasing the property for a fixed cost I and then benefits from the net stochastic property price increase/decrease of V $(V \geq 0)$ as well as collecting the convenience yield or market imputed rent by occupying the property.

After purchasing the property and taking on the mortgage liability, if the equity value $E(V)$ is sufficiently or consistently lower than the value of debt $D(V)$, the borrower may consider defaulting on the mortgage payments, forcing the lender to consider repossession or foreclosure.

In this case, following Leland (1994), the liquidation value to the lender is given by $(1-\alpha)F(V)$ while the borrower will retain zero equity.

Alternatively, due to so-called "trigger" events the borrower may delay one or several mortgage payments initiating a negotiation with the lender. In this case, the lender may not always wish to repossess but instead agree to renegotiate the mortgage contract resulting in a new lower and more affordable mortgage payment for the borrower. The new mortgage payment is conditional on the "surplus" equity generated by avoiding foreclosure being "notionally" divided between the borrower and lender based on their relative negotiating strength, (ϕ and $1-\phi$ whereby $\phi = 1$ implies that the borrower has the greater share). The preservation of this "surplus" equity is the only potential "asset" of value, where over both a lender and borrower may want to negotiate[5]. We model the process as a cooperative Nash bargaining game (Fan and Sundaresan, 2000).

The methodological approach to solving the problem is similar to a perpetual American (scale) option entry/exit problem and a solution is found for the different ODEs in terms of the critical entry and exit thresholds for the default or delinquency options, respectively, V_i or V_{di} and V_f or V_{df}. Solutions are of the form $F(V) = A_0 + A_1 V^{\gamma} + A_2 V^{\beta}$ with the appropriate boundary conditions leading to different specific solutions.

Conventionally modelled default results in the lender acquiring the property. However, with exercise of the delinquency option, borrower and lender (re)negotiate a new mortgage, conditional on the optimal sharing of the avoidable foreclosure costs, at the delinquency trigger point V_{df} with both willing to temporarily change or adapt the contract terms. The lender would agree a renegotiated mortgage coupon *C(V)* based on the current property price, lower than the initial mortgage c_d (agreed at the investment threshold V_{di}) and the borrower would continue to own the property and collect the market imputed rent.

[5] This of course does not preclude the existence of other separate motivating factors e.g. imputed rent or policymaker regulations, which may affect the negotiation stance of both parties. However, ultimately these factors must be reflected in how the common "asset" is divided.

Let $F(V,c)$ be the property asset value before investment. The borrower chooses the optimal investment threshold V_{di} and the optimal mortgage repayment c_d to maximise his equity position $E(V,c)$. As the property price V approaches infinity, the mortgage becomes riskless and hence the property value must satisfy an upper boundary condition whereby

$$\lim_{V\to\infty} F(V,c) = V + \frac{\tau c}{r} \qquad [2]$$

Lower boundary conditions for the strategic delinquency option differ from the default option as lender/borrower are prepared to vary the contract terms at the lower threshold, where the total value of the property $F(V_{df}, c_d)$ includes the value of future tax benefits. The borrower and lender thus bargain over a larger amount (when $V \leq V_{df}$) resulting in a property asset value *F(V)* of

$$F(V) = V + \frac{\tau c_d}{r}\left[1 - \left(\frac{\beta}{\beta-\gamma}\right)\left(\frac{V}{V_{df}}\right)^{\gamma}\right] \quad when\ V \geq V_{df} \qquad [3]$$

$$F(V) = V + \frac{\tau c_d}{r}\left(\frac{-\gamma}{\beta-\gamma}\right)\left(\frac{V}{V_{df}}\right)^{\beta} \quad when\ V < V_{df} \qquad [4]$$

where $\beta > 1, \gamma < 0$ are the roots of $\frac{\sigma^2 V^2}{2} + (\mu - \sigma^2/2)V - r = 0$

The equity equation *E(V)* ($V < V_{df}$) is also adjusted to account for the new mortgage payment $C(V)$ which is now a function of the current property value and $(r-\mu)V$ the market imputed rent.

$$\frac{1}{2}\sigma^2 V^2 \mathrm{E}_{VV}(V) + \mu V \mathrm{E}_V(V) - rE(V) + (r-\mu)V - C(V) = 0 \quad when\ V < V_{df} \qquad [5]$$

With upper boundary conditions the same for both the delinquency and default options, we obtain revised lower boundary conditions from the "extra" value of $F(V)$ using equation [4] and the Nash negotiation sharing rule to get

$$\lim_{V\downarrow V_{df}} E(V) = \phi\left(\alpha V_{df} - \frac{\tau c}{r}\frac{\gamma}{\beta-\gamma}\right) \qquad [6]$$

Differentiating [6] gives

$$\lim_{V\downarrow V_{df}} E_V(V) = \phi\left(\alpha - \frac{\tau c}{V_{df} r}\frac{\gamma\beta}{\beta-\gamma}\right) \qquad [7]$$

Further development (which can be obtained from the corresponding author) leads to closed form expressions for the key outputs for the strategic delinquency option and the comparable outputs for the default option.

a) The borrower's investment threshold for the delinquency option V_{di} is given by

$$V_{di} = \frac{\beta}{\beta - 1}\left[1 + \frac{\tau}{gL}\right]^{-1} I \qquad [8]$$

where $g = \left[\frac{\beta}{\beta-\gamma}(1-\gamma)\right]^{-\frac{1}{\gamma}} = \frac{V_{di}}{V_{df}}$ and $L = \frac{1-\tau(1-\phi)}{1-\phi\alpha}$

The investment threshold for the default option V_i is given by

$$V_i = \frac{\beta}{\beta - 1}\left[1 + \frac{\tau}{h}\right]^{-1} I \qquad [9]$$

where $h = \left[1 - \frac{\gamma(\tau+\alpha)}{\tau}\right]^{-1/\gamma} = \frac{V_i}{V_f}$

b) The mortgage coupon for the delinquency option c_d (for $V \geq V_{df}$) is given by

$$c_d = r\frac{\gamma - 1}{\gamma}\frac{\beta}{\beta - 1}(gL + \tau)^{-1} I \qquad [10]$$

The mortgage coupon for the default option c^* (for $V \geq V_f$) is given by

$$c^* = r\frac{\gamma - 1}{\gamma}\frac{\beta}{\beta - 1}[h + \tau]^{-1} I \qquad [11]$$

We show in Section 5 that the consequence of these different results for the default and delinquency option is that lenders *ex ante* mortgage yield spread should increase significantly to pay for the borrower's *ex post* delinquency option.

c) Borrowers attempt to renegotiate with lenders when $V(t) < V_{df}$, where V_{df} is the endogenously determined delinquency threshold given by

$$V_{df} = \frac{\beta}{\beta - 1}\left[g + \frac{\tau}{L}\right]^{-1} I \qquad [12]$$

Borrowers default/foreclose with lenders when $V(t) < V_f$, where V_f is the endogenously determined default threshold given by

$$V_f = \frac{\beta}{\beta - 1}[h + \tau]^{-1} I \quad [13]$$

We show in Section 5 that the implications of these equations are that delinquency option exercise will occur earlier than the default option exercise for all borrowers but strong borrowers will exercise their delinquency option earlier than weak borrowers will.

The borrower renegotiates a new coupon

$$C(V) = (1 - \alpha\phi)[(r - \mu)V] \quad [14]$$

In other words the renegotiated mortgage coupon is the current notional market imputed rent $[(r - \mu)V]$ times the factor $(1 - \alpha\phi)$ which is either equal to or less than 1 depending on the borrower/lender heterogeneous bargaining power and the probable foreclosure costs.

We define the optimal *risk adjusted or market* LTV_{di}/LTV_i at mortgage origination V_{di} *or* V_i as the contemporaneous market value of debt divided by the property value at mortgage origination and is defined for the delinquency option as

$$LTV_{di} = \frac{D(V_{di}, c_d)}{V(V_{di}, c_d)} \quad [15]$$

This can be shown to be equivalent to

$$LTV_{di} = \frac{\gamma - [(1 - g^{\gamma})(1 + \tau(\phi - 1))]}{\gamma(gL + \tau)}$$

The ex post yield spread at origination is defined as

$$YS_{di} = \frac{c_d}{D(V_{di}, c_d)} - r \quad [16]$$

and

$$YS_i = \frac{c_*}{D(V_i, c^*)} - r \quad [17]$$

for both options respectively where D(.) is the value of debt at the investment threshold V_i or V_{di}.

4. Delinquency and Default Option Analysis –A Stylised Example

The strategic delinquency option represents the relationship between the investment and financing decisions, where the initial *ex ante* investment decision is dependent on the (potential) strategic delinquency renegotiation between lender and borrower. On the other hand, the default (non-bargaining) option represents the relationship where the borrower makes the investment decision knowing that non-payment of the mortgage will certainly result in the forfeiture of all equity.

This section will demonstrate the effects of a strategic delinquency option in a graphical manner and compare the fundamentally different quantitative results that arise from the two options using stylised US mortgage data and the equations derived in the preceding section. Where appropriate we transform the stochastic property price V ($) to a book BLTV (%) where a book BLTV greater than 100% represents so-called negative equity. The parameter ϕ represents heterogeneous characteristics of the borrower in relation to the lender impacting on their ability to (re)negotiate. Recognising this impreciseness, we only observe how *the delinquency region*, delineated by the extreme corner values of ϕ = (0,1), in the various graphs, compares to the single *default point*.

The borrower decides to invest in a new build property financed partly with debt paying the optimal coupon to a willing lender. The analysis proceeds as follows:

a) Calculate the mortgage (book) loan and payment at the optimal investment point.
b) Establish the critical delinquency region and default point as a function of BLTV.
c) Calculate the lender's risk spread (over the riskless rate) implicit in the mortgage payment.
d) Illustrate some model sensitivities to foreclosure costs and volatility.

Figure 1 (overleaf) demonstrates one conclusion of this paper. The original loan size and (perpetual) mortgage coupon is conditional on ϕ or the *ex ante* consideration of how avoidable foreclosure costs might be shared (Pence 2006). It demonstrates that any borrower should attempt to negotiate a new lower coupon *C(V)* on choosing delinquency which is the product of the current market imputed rent $(r - \mu)V$ and a combination of the unavoidable foreclosure costs and their own negotiation ability $(1 - \alpha\phi)$. It demonstrates that a stronger borrower (e.g. higher FICO score) should threaten delinquency earlier than the weaker borrower. If successful, the borrower pays a

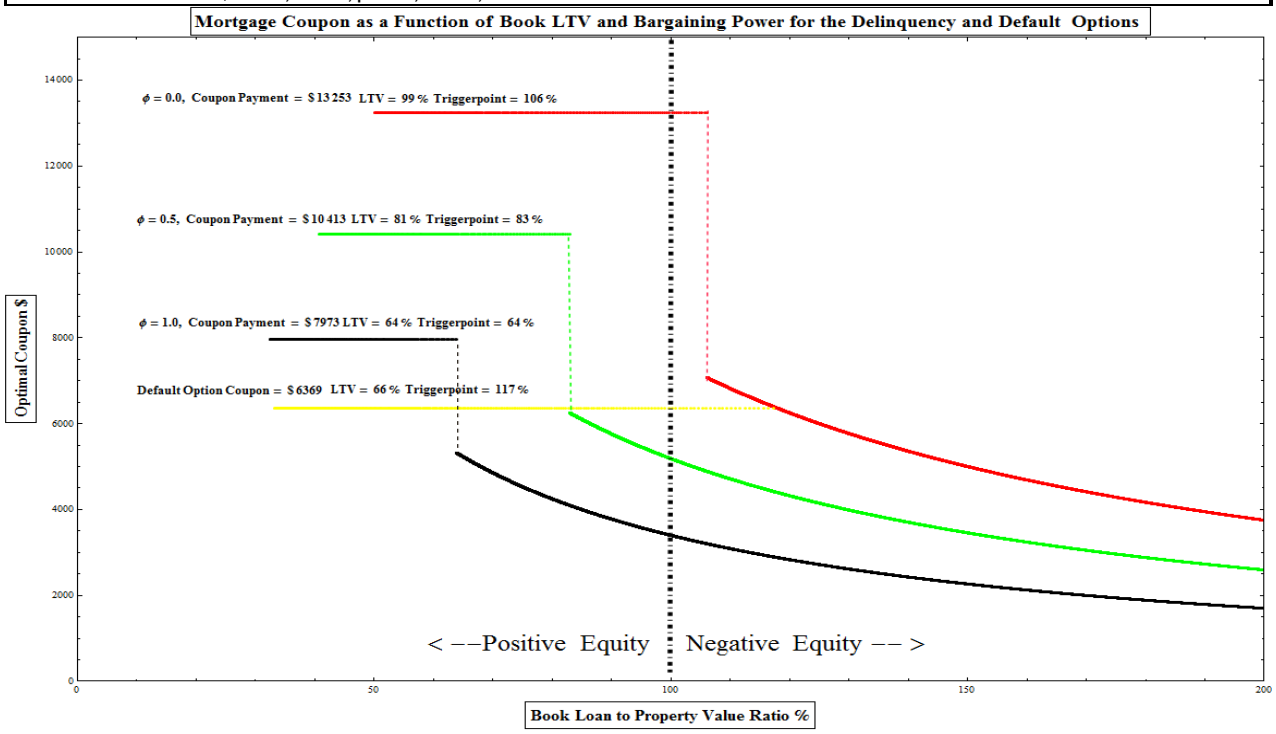
reduced coupon, retains ownership of the property, still collects the lower market imputed rent but retains the "hope" that property values may bounce back recovering some of their lost equity.

It can be seen that in a limit or corner case where the borrower is a weak negotiator ($\phi = 0$) the lender offers the highest mortgage loan (LTV_{di} = 99%) and the borrower pays the highest coupon. In the other limit case where the borrower is strong ($\phi = 1$) the lender offers the lowest mortgage loan (LTV_{di} = 64%) and the borrower pays the lowest coupon. This contrasts with the default option only where the mortgage ($LTV_d = 66\%$) and coupon is not dependent on negotiation ability or heterogeneous characteristics of the borrower and is constant until default.

Finally, the weaker the borrower (i.e. lower FICO score) the closer the delinquency trigger point is to the optimal default point and the more likely, given unfavourable property shocks, that a borrower may very quickly move from exercising their delinquency option to exercising a default option. The stronger the borrower, the earlier that the borrower, who may not yet be in negative equity, exercises his delinquency option but the less likely that default will eventually result.

Figure 1 Optimal Coupon Payment $ as a Function of Book LTV and Bargaining Power φ for the Delinquency and Default Options

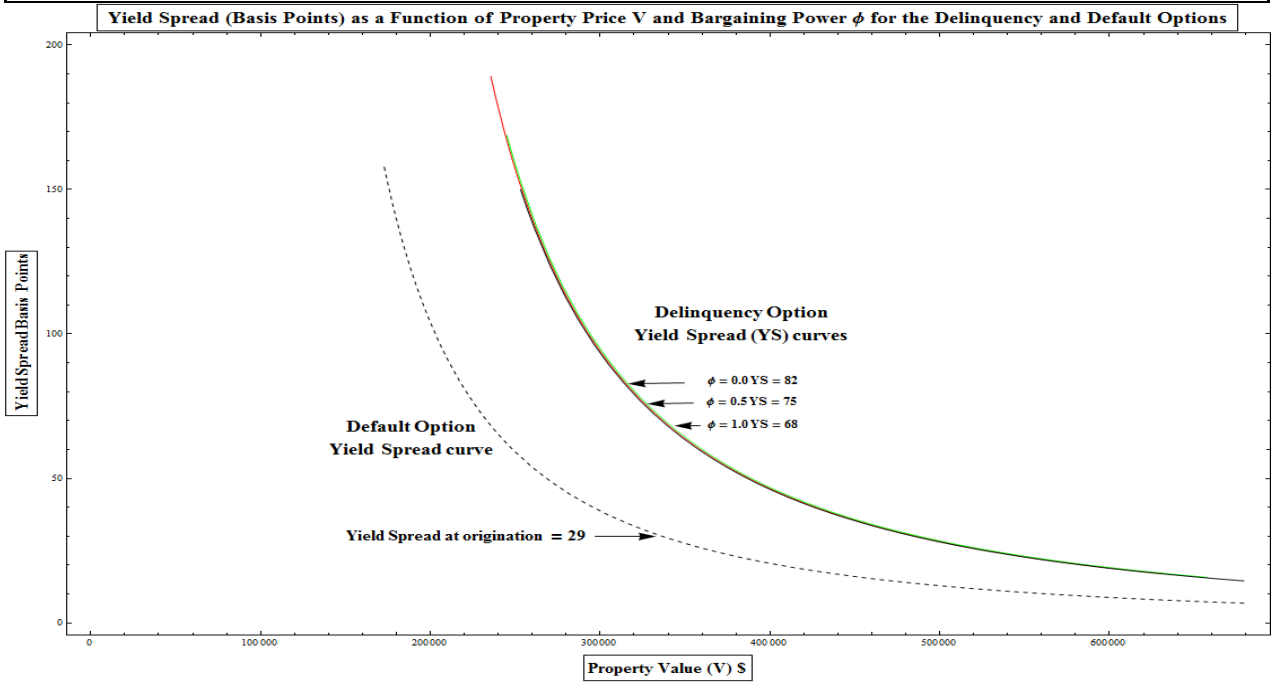
The three discontinuous curves labelled φ=0.0, 0.5 and 1.0 are the coupon payment curves
For φ=0.0 a coupon c_d of $13253 is paid up to the delinquency exercise point and thereafter a decreasing coupon depending on the Book LTV
Coupon payments decrease as φ increases reflecting the lower mortgage (debt capacity) offered by the lender. The LTV quoted is at origination
The lower (heavy yellow) straight line is the constant coupon for the default option and terminates at the default exercise point (117%)
A coupon is always paid after exercising the delinquency option which becomes more affordable with increasing BLTV or negative equity
Parameter values : I = $ 250000, r = 0.03, μ = 0.00, τ = 0.20, α = 0.3 and σ = 0.10



It is clear that, with decreasing property values (i.e. increasing BLTV), the economic consequences of the delinquency option are that the borrower should endogenously choose to enter delinquency earlier and start paying a more affordable mortgage earlier than with a default option where the borrower will default (and lender will foreclose) as soon as the critical threshold (BLTV=117%) is reached. The more power the borrower is perceived to have, the earlier that delinquency will occur because the more financial concessions that may be extracted. The overall direction of these results are consistent with PSV(2010) who claim that significant differences exist between the delinquency and default behaviour of securitised and non-securitised loans and that these effects are larger for borrowers with a high FICO credit rating.

We compare the lender's yield spread over the risk free rate for a delinquency and default option in Figure 2. The granting of a delinquency option increases the lender's required risk spread compared to a default option. Variation between yield curves for the three values of parameter ϕ in the delinquency case is relatively small compared to that of the default option. Differences in yield spreads are more dependent on the optimal investment entry points with the weaker borrower paying a higher yield because they optimally make the investment earlier than a strong borrower does. The existence of *any* measure of bargaining or sharing introduces a fundamental change to the contract whereby the lender charges a higher yield spread on the risk free rate.

Figure 2 Yield Spread (Basis Points) as a Function of Property Value V and Bargaining Power φ for the Delinquency and Default Options
The three convex curves labelled φ=0.0, 0.5 and 1.0 are the yield spread curves
For φ=0.0 the yield spread at the entry threshold is 82 basis points and decreases as φ increases reflecting greater bargaining power.
The lower dashed line is the yield spread curve for the default option with a value of 29 basis points at the investment entry threshold
Yield curves for the delinquency option coincide closely (in this example) but all differ significantly from that of the default option
Parameter values : I = $ 250000, r = 0.03, μ = 0.00, τ = 0.20, α = 0.3 and σ = 0.10



We conclude by summarising key graphical data from Figures 1 and 2 in Table 1, showing the effect of increasing foreclosure costs α and changes in property volatility σ. Increases in property price volatility σ with no changes in other parameters behaves as expected delaying investment, increasing yield spreads and reducing debt capacity (LTV) at origination. The higher the foreclosure costs, the lower the LTV the lender should agree with the very strong borrower while continuing to offer the same LTV to the very weak borrower. A large decrease in (LTV) lending capacity from 89% to 72% can be observed for the average value of ϕ=0.5 as foreclosure costs increase from 10% to 50%. Whether a strong lender might lend to a (very) weak borrower with probable large foreclosure costs is perhaps best left to a reflection on lending practises and the effects of securitisation in the recent US sub prime crises.

Increasing volatility has a surprising effect resulting in an earlier exercise of the delinquency option but later exercise of the default option. This might indicate that a borrower facing certain foreclosure "sits tighter" longer during periods of high volatility while a borrower with a delinquency option will initiate negotiation for a more affordable mortgage coupon earlier.

Table 1 Table of Results for a Range of Different Parameters Illustrating Delinquency Option Sensitivity

The table summarises the range of results for different values of α and σ illustrating the sensitivity of the output to different input parameters.
The second sub-table from the top down are the results for the base parameter case used for Figures 1 and 2
The first three sub-tables demonstrate that a lender offers a smaller loan (LTV@origination) as foreclosure costs α increase.
Note the general effect of volatllity σ increasing required Yield Spread, lowering the delinquency trigger point but increasing the default trigger point
Unless otherwise stated parameter values : I = $ 250000, r = 0.03, μ = 0.00, τ = 0.20, α = 0.3 and σ = 0.10

φ	Mortgage Coupon $	Book LTV % @ Default or Delinquency	Yield Spread @ Origination Basis Points	LTV @ Origination %
Foreclosure % α = 10% ,σ=0.10,μ=0.00				
0	13253	106	82	99
0.5	11472	93	75	89
1	9981	82	68	81
Default	7670	113	36	77
Foreclosure % α = 30% ,σ=0.10,μ=0.00				
0	13253	106	82	99
0.5	10413	83	75	81
1	7973	64	68	64
Default	6369	117	31	66
Foreclosure % α =50% , σ=0.10,μ=0.00				
0	13253	106	82	99
0.5	9323	73	75	72
1	5853	45	68	47
Default	5572	119	27	59
Foreclosure % α = 30% ,σ=0.20,μ=0.00				
0	29321	61	255	85
0.5	23051	46	243	68
1	17658	35	233	52
Default	9002	130	91	71

4. Summary and Conclusions

We have combined two different aspects of real options that of irreversible investment and debt pricing/capital structure, to develop closed form solutions by which the borrower can choose the optimal *ex ante* mortgage terms (LTV and mortgage coupon) and *ex post* timing to exercise their delinquency option. We achieve this by applying methodological aspects of strategic endogenous default developed for corporate bond valuation to the *ex ante* valuation of delinquent mortgages.

Even though the model has been developed within an equity maximising option theoretic framework, in the real world, exercise of the delinquency option may be initiated not only by a desire to optimise the borrower's equity but also by a sub optimal trigger event. In both cases, a prudent lender (or policymaker) who has *ex ante* priced their mortgages based on a potential optimal delinquency may be financially better able to (re)negotiate, assuming that a sub optimal delinquency will cost the borrower but not the lender. From a borrower's viewpoint both motives (optimise equity or improve ability to pay) are strategic. From a lender's viewpoint, the first motivation may be less deserving (and more strategic) than the second as the model suggests that in a declining house market those stronger negotiators, whether deserving or not, will initiate a negotiation earlier. Consequently, lenders may need to screen these applicants (negotiators) more closely with consequent higher screening and monitoring costs.

Policymakers and lenders should also be aware that with increasing property price volatility the model suggests borrowers may accelerate the moment of delinquency while trying paradoxically to delay the moment of default. This makes perfect sense from a (lack of) moral hazard viewpoint. If default results in certain foreclosure then borrowers will not be anxious to default, however if delinquency results in a more affordable mortgage coupon then borrowers in contrast will accelerate the exercise of the option. Whether this is desirable or not and actually costs the lender is an interesting discussion as to the question as to whether this effect is observable in the current housing market.

We emphasise that the option to renegotiate the mortgage payment, by choosing delinquency, is not a "free ride" for the borrower. The lender charges *ex ante* higher yield spreads for this right compared to the default option. We have shown that the lender is no worse off in whatever bargaining position he finds himself and in most cases will be better off. Ultimately, if the lender

cannot agree a new mortgage payment with a delinquent borrower then he can always foreclose with inevitable costs. We do not introduce "extra" moral hazard issues as no permanent loan modification occurs but do highlight the possible moral hazard issues present in temporary payment modification. The borrower remains responsible for paying off the full mortgage principal.

Implicit in our modelling is that the lender and borrower should always agree new (sliding) mortgage payments conditional on the current property value. This is surely an abstraction from reality where in practise, due to the same aforementioned monitoring and screening costs, only one new lower affordable mortgage payment may be agreed, whereupon non-performance might lead to irrevocable foreclosure.

We have introduced an additional bargaining parameter ϕ (related to future unavoidable foreclosure costs) compared to the traditional option theoretic mortgage default literature. This parameter ϕ is a convenient construct to easily divide the benefits of avoiding foreclosure costs between lender and borrower. The parameter is heterogeneous in that, two borrowers with the same lender (or servicing agent) may have different values resulting in different outcomes of the (re)negotiated mortgage payment. In any case, we are less interested in the exact value of ϕ and more interested in delineating the maximum and minimum boundaries of the critical region where delinquency or renegotiation of the mortgage coupon may occur as a result of both parties wishing to avoid foreclosure costs. Better understanding of this region, compared to the traditional default region, may help lenders better screen (weak) borrowers who apply later from those (strong) borrowers who apply earlier and may also try to take advantage of lender weakness.

The strategic delinquency option has been demonstrated to have *ex post* distinct economic and financial consequences. It remains to empirically investigate whether this idea of borrowers strategically delaying payments actually occurs within an option theoretic equity optimising framework or rather within some other "affordability optimising" framework.

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