The effects of costly exploration on optimal investment timing[*]

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Abstract

This paper investigates a principal-agent model in which an owner (principal) optimizes a contract with a manager (agent) delegated to undertake an investment project. In the model, we explore the effects of costly exploration by which the manager learns the real value of development cost. We show that high exploration cost can lead to a pooling policy not contingent on project type. Further, and more notably, we show that, in the presence of asymmetric information, higher exploration cost leads to wealth transfer from owner to manager and can then play a positive role in preventing a greedy contract by the owner and improving social welfare.

JEL Classifications Code: D86, G13, G31.

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1 Introduction

The real options approach has become an increasingly standard framework for investment timing decisions in corporate finance (e.g., (Dixit and Pindyck 1994)). Although the early literature on real options (e.g., (Dixit 1989, McDonald and Siegel 1986)) considers investment decisions by a monopolist, more recent studies have investigated the problem of several firms competing in the same market from a game theoretic approach. For instance, (Grenadier 1996, Weeds 2002) derive equilibrium in real options timing games, and (Grenadier 2002) investigates equilibrium investment strategies of firms in a Cournot–Nash framework.

While these studies have focused on strategic interactions among rival firms, (Grenadier and Wang 2005) investigated investment timing in a decentralized firm where the owner (principal) delegates the investment decision to a manager (agent) who holds private information by combining real options theory and incentive theory. In most modern corporations, for example, shareholders delegate investment decisions to managers, thereby taking advantage of their special skills and expertise. In their model, asymmetric information changes the investment policy from the first-best case because the owner designs a contract to provide a bonus-incentive for the manager to truthfully reveal private information.[1]

Previous studies assume that a manager observes real project value with no costs immediately after a contract is made; however, in most real investing, a manager gathers information about the type of project by exploratory investment which has a high cost.[2] This exploration cost is especially high in the following cases. The first example is a resource extraction project, which has been the main application of the real options analysis (e.g., (Paddock, Siegel, and Smith 1988) among others). The exploration stage involves seismic and drilling activity to obtain information on the quantities of natural resource reserves, as well as the costs of extracting them. It depends on the exploration results whether the manager proceeds to development investment such as construction of platforms and drilling of production wells.

Another example is an R&D program with learning (e.g., (Childs, Ott, and Triantis 1998, Perlitz, Peske, and Schrank 1999)). An R&D project frequently comprises multiple phases.

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[1](Nishihara and Shibata 2008, Shibata 2009) extend (Grenadier and Wang 2005) to a case with an audit mechanism. (Shibata and Nishihara 2010) extends (Grenadier and Wang 2005) to a context of the dynamic investment and capital structure. On the other hand, (Morellec and Schürhoff 2009) investigates a real options signaling model rather than the screening model. (Grenadier and Malenko 2010) investigates the real options signaling game in a more general framework.

[2]In incentive theory, (Crémer and Khalil 1992, Crémer, Khalil, and Rochet 1998a, Crémer, Khalil, and Rochet 1998b, Kessler 1998, Laffont and Martimort 2002) investigate how cost of gathering information affects the optimal incentive mechanism. For example, (Crémer and Khalil 1992, Crémer, Khalil, and Rochet 1998a, Crémer, Khalil, and Rochet 1998b) show that high cost of information leads to a contract which provides no incentive for an agent to accumulate information.

In each phase, the manager gains information on the R&D project, such as the probability of success, the costs required to complete, and the profits expected from the successfully developed product. The manager decides whether to proceed to the next phase while taking account of the preliminary results. In addition, several empirical works (e.g., (Aboody and Lev 2000, Barth, Kasznik, and Mcnichols 2001)) have shown a relatively large information asymmetry associated with R&D.

This paper extends the previous real options model with asymmetric information to a setting in which the manager learns project type by costly exploration. We consider a principal-agent model in which the owner (principal) optimizes a contract with a manager (agent) delegated to undertake an investment project. The investment consists of exploratory and development stages. Development cost can take one of two possible values. Only the manager observes a realized development cost by costly exploration. Specifically, we investigate how the ratio of exploration cost to total cost affects the investment policy. The results are summarized as follows.

When exploration cost is relatively low, the firm, whether under symmetric or asymmetric information, adopts a separating policy. The firm immediately proceeds to development investment for a favorable result in the exploration stage, while it delays the investment with an unfavorable result. If the project turns out to be a bad type, exploration investment leads to inefficiency ex post. However, because of low cost, the firm attempts earlier to acquire information on project type. On the other hand, the firm takes a pooling policy when exploration cost is relatively high. In this case, the firm invests in the development phase immediately after the exploratory phase whether or not the exploration result is favorable, because high cost reduces the incentive to gather information by earlier investment in the exploratory stage. Under asymmetric information, the contract becomes useless because the owner pays the manager the maximal rent for information. The pooling solution never appears in (Grenadier and Wang 2005), where the manager observes project type at no cost. Our result is similar to a finding by (Crémer, Khalil, and Rochet 1998a) in incentive theory. They showed that high cost of information acquisition leads to a pooling contract not contingent on project type, although their model does not treat a dynamic investment timing problem but a static problem.

A key role of asymmetric information is to provide an additional incentive for the owner to separate investment timing of good- and bad-type projects. Consistent with the previous studies, the owner can decrease the bonus to the manager by deferring investment in the bad-type project. Thus, the firm chooses a pooling policy under symmetric information but chooses a separating contract under asymmetric information when exploration cost is intermediate. If this is the case, the investment timing differs between symmetric and asymmetric information cases, not only for the bad-type project but also for the good-type project. In addition, unlike in

the previous studies, the contract by which the owner cuts the information rent to the manager can yield less social welfare than the case with no contract. This implies that wealth transfer from manager to owner in the contract is not always efficient from the social viewpoint.

Most notably, we show that unlike the case with symmetric information, a higher proportion of exploration cost can increase social welfare in the asymmetric information case. When a higher proportion of exploration cost and asymmetric information are separately present, both lead to inefficiency; however, combined with asymmetric information, costly learning can play a positive role in mitigating excessive wealth transfer from manager to owner. The intuition behind the key result is as follows. A higher exploration cost decreases the owner's value and increases the manager's value because the owner finds it difficult to decrease the bonus to the manager by a separating contract. Accordingly, high exploration cost leads to wealth transfer from owner to manager.[3] This may mitigate the social loss stemming from the owner's greedy contract. If information about exploration cost is also partially unknown to the owner, the manager pretends to be ignorant of project type in order to increase his/her information rent. This resembles the argument by (Kessler 1998), who showed that an agent's ignorance may generate strategic benefits in a static model. The manager's moral hazard decreases the owner's value but does not necessarily reduce social welfare. The manager's moral hazard may enhance social welfare by mitigating inefficient asset substitution by the owner's greedy contract.

The key result resembles several findings known as "two incentive problems are better than one" in corporate finance (e.g., (Hirshleifer and Thakor 1992, Mookherjee and Png 1995, Noe and Rebello 1996)). Similar results are reported in recent papers in the real options context. For example, (Hackbarth 2009) showed that managerial optimism and overconfidence, which distort the investment and financing policy, can increase welfare in the presence of bondholder-shareholder conflicts. (Nishihara and Shibata 2010) showed that a financing constraint, that decreases firm value in a monopoly, can play a positive role in mitigating preemptive competition and improving firm value in equilibrium.

This paper contributes to the literature of both investment timing and incentive theory as follows. First, we complement the real options literature by pointing out the possibility that costly learning greatly distorts the investment policy, leading especially to a pooling policy. We also demonstrate the possibility that the owner's greedy contract yields less social welfare than the case with no contract. The main contribution is our demonstration that costly learning can play a positive role through interactions with asymmetric information. In addition, this paper complements several works in incentive theory by extending their findings in the static models to those of the dynamic investment timing model.

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[3]This is also consistent with an empirical finding by (Aboody and Lev 2000).

The paper is organized as follows. Section 2 introduces the setup and presents the investment policies under symmetric and asymmetric information. Section 3 investigates social welfare and social loss. Section 4 explores more detailed properties of the solutions in numerical examples. Section 5 concludes the paper. All proofs appear in the appendix.

2   Model solutions

2.1   Setup

Consider an owner (principal) with an option to invest in a single project. We assume that the owner delegates the investment decision to a manager (agent). Throughout our analysis, all agents are assumed to be risk neutral and to maximize their expected payoff. For simplicity, we assume that project value follows the geometric Brownian motion

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \quad (t > 0), \quad X(0) = x, \tag{1}$$

where $\mu, \sigma > 0$ and $x > 0$ are constants, and $B(t)$ denotes the one-dimensional standard Brownian motion. Throughout the paper, we assume that the initial value x is sufficiently low so that the firm has to wait for its investment condition to be met. For convergence, we assume that $r > \mu$ where r is a constant interest rate.

The investment comprises two phases: exploratory and development investment. Exploratory investment requires irreversible cost aI and reveals development cost, which can take one of two possible values $(1-a)I$ (good type) or $(1-a)I + \Delta I$ (bad type). The probabilities of drawing $(1-a)I$ and $(1-a)I + \Delta I$ are equal to q and $1-q$, respectively. Assume that $I > 0, a \in [0, 1)$, and $q \in (0, 1)$ are constants. We also assume that project value $X(t)$ and exploration cost aI is observed by both the owner and the manager while development cost $(1-a)I$ or $(1-a)I + \Delta I$ is privately observed only by the manager.[4] Total investment cost becomes I for the good-type project and $I + \Delta I$ for the bad-type project.

This model is an extension of the simplified model of (Grenadier and Wang 2005). Indeed, in the case of $a = 0$, the model is equal to the hidden information case of (Grenadier and Wang 2005).[5] As mentioned in Section 1, the setting with a positive a is suitable for real investment

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[4]The assumption that a portion of the project value is privately observed only by one person (here, the manager) and not observed by the other (here, the owner) is quite common in the asymmetric information literature (e.g., (Myers and Majluf 1984)). The model is not essentially changed when the privately observable component corresponds to part of project value, as in (Grenadier and Wang 2005, Nishihara and Shibata 2008), rather than part of investment cost.

[5]Although the manager's one-time effort, which cannot be observed by the owner, changes the likelihood q in the original model of (Grenadier and Wang 2005), we exclude the effect of this hidden action.

such as a resource extraction and an R&D program. To preserve tractability, we assume that investment takes place instantaneously, but this is not a severe restriction. The conclusions and insights provided in this paper will be unchanged even if the period of investment is considered.

2.2 Symmetric information

As a benchmark, we investigate the case where there is no delegation of the exercise decision and the owner observes the true value of development cost by exploratory investment. The owner's exploratory investment at $\tau_P = \inf\{t > 0 \mid X(t) \geq x_P\}$ and development investment at $\tau_i = \inf\{t > 0 \mid X(t) \geq x_i\}$, where $i = 1$ and 2 denote the investment policies for the good- and bad-type projects, yield the expected profit

$$
\mathbb{E}[e^{-r\tau_P}(-aI)] + q\mathbb{E}[e^{-r\tau_1}(X(\tau_1) - (1-a)I)] + (1-q)\mathbb{E}[e^{-r\tau_2}(X(\tau_2) - ((1-a)I + \Delta I))]
$$

$$
= \left(\frac{x}{x_P}\right)^{\beta}(-aI) + q\left(\frac{x}{x_1}\right)^{\beta}(x_1 - (1-a)I) + (1-q)\left(\frac{x}{x_2}\right)^{\beta}(x_2 - ((1-a)I + \Delta I)),
$$

where $\beta = 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2}(> 1)$ is a positive characteristic root. Then, the owner's optimal policy (x_P^*, x_1^*, x_2^*) solves the following problem:

$$
\pi_o^*(x) := \max_{x_P, x_1, x_2} \quad \left(\frac{x}{x_P}\right)^{\beta}(-aI) + q\left(\frac{x}{x_1}\right)^{\beta}(x_1 - (1-a)I)
$$
$$
+ (1-q)\left(\frac{x}{x_2}\right)^{\beta}(x_2 - ((1-a)I + \Delta I)) \tag{2}
$$
$$
\text{s.t.} \quad x_i \geq x_P \quad (i = 1, 2)
$$

Throughout the paper, the superscript $*$ refers to the solution under symmetric information. Recall that the initial value $x = X(0)$ is sufficiently low. Clearly, in the optimum of problem (2), $x_P^* = \min(x_1^*, x_2^*)$ and $x_1^* \leq x_2^*$ are satisfied. Then, problem (2) can be reduced to

$$
\pi_o^*(x) = \max_{x_1, x_2} \quad q\left(\frac{x}{x_1}\right)^{\beta}(x_1 - (a/q + 1 - a)I) + (1-q)\left(\frac{x}{x_2}\right)^{\beta}(x_2 - ((1-a)I + \Delta I)) \tag{3}
$$
$$
\text{s.t.} \quad x_2 \geq x_1.
$$

The following proposition presents the solution to problem (3).

Proposition 1 The optimal policy (x_P^*, x_1^*, x_2^*) and the owner's value $\pi_o^*(x)$ are given as follows:
Case (i-S): $a < q\Delta I/I$ (separating)

$$
x_P^* = x_1^* = \frac{\beta}{\beta - 1}(a/q + 1 - a)I, \quad x_2^* = \frac{\beta}{\beta - 1}((1-a)I + \Delta I) \tag{4}
$$

$$
\pi_o^*(x) = \frac{qx^{\beta}}{\beta x_1^{*\beta - 1}} + \frac{(1-q)x^{\beta}}{\beta x_2^{*\beta - 1}} \tag{5}
$$

Case (ii-S): $a \geq q\Delta I/I$ (pooling)

$$x_P^* = x_1^* = x_2^* = \frac{\beta}{\beta - 1}(I + (1 - q)\Delta I) \tag{6}$$

$$\pi_o^*(x) = \frac{x^\beta}{\beta x_1^{*\beta-1}} \tag{7}$$

First, we explain Case (i-S). In this case, the owner invests in the exploratory phase when project value $X(t)$ hits the threshold x_P^*. If exploratory investment reveals the good-type project, the owner immediately proceeds to development investment. Otherwise, the owner delays the investment until $X(t)$ hits the higher level x_2^*. When costly exploration is unnecessary, i.e., $a = 0$ as in (Grenadier and Wang 2005), the owner can choose the best investment timing, $x_1^* = \beta I/(\beta - 1)$ and $x_2^* = \beta(I + \Delta I)/(\beta - 1)$ for the good- and bad-type projects, respectively. However, when the exploratory phase requires a portion of investment cost, $a > 0$, the optimal investment policy changes. The threshold x_1^* for the good-type project increases and x_2^* for the bad-type project decreases with a. The cost of early exploration leads to ex-post inefficiency when the project turns out to be the bad type. Then, a higher a and a lower q delays exploratory investment. On the other hand, a higher a decreases remaining investment cost. This decrease in development cost shortens the waiting time for the bad-type project.

When exploratory investment requires a relatively large portion of total cost, the owner does not gather information on project type by an earlier exploratory investment. This is Case (ii-S). The owner proceeds to the development stage immediately after exploratory investment regardless whether the exploration result is favorable. The owner does not have to delay the investment because of low development cost even if the project turns out to be the bad type. As a result of high exploration cost, information acquisition by exploratory investment is useless. Thus, the owner makes the entire investment at the threshold $\beta(I + (1 - q)\Delta I)/(\beta - 1)$ as if investment cost were equal to the average $qI + (1 - q)(I + \Delta I)$.

For fixed x_i satisfying $x_1 < x_2$, the objective function in problem (3) decreases with a. This leads to the following corollary.

Corollary 1 In Case (i-S), $\partial \pi_o^*(x)/\partial a < 0$ holds.

Corollary 1 is intuitive because a high a means difficulty in judging the profitability of the project. This difficulty distorts the owner's investment policy (i.e., delays investment in the good-type project and hastens investment in the bad-type project). Then, a higher a increases loss due to the investment distortion. Note that in Case (ii-S), $\pi_o^*(x)$ is a constant because the optimal policy does not depend on a (cf. (6) and (7)).

2.3 Asymmetric information

The owner delegates the investment decision to the manager who will acquire private information with exploratory investment. Before contracting, both the owner and the manager know that the probability of drawing $(1-a)I$ (good type) equals q. Immediately after exploratory investment, only the manager knows the true development cost. As in (Grenadier and Wang 2005, Nishihara and Shibata 2008, Shibata and Nishihara 2010), we focus on the owner's optimal contract at the initial time. At time 0, the owner offers the manager a contract that commits the owner to pay the manager at the time of exercise. We assume that no opportunity for renegotiation exists. Although the commitment may lead to ex post inefficiency in investment timing, it increases ex-ante value of the project. In fact, if the owner makes no contract with the manager, the owner's value becomes $\pi_o^*(x)$ with $q = 0$ (see (7)). This is because the manager requires the owner the higher cost $(1-a)I + \Delta I$ and makes ΔI his/her own even if the true development cost is $(1-a)I$.

As in (Grenadier and Wang 2005), the optimal contract is included in a mechanism represented by $(x_P, x_1, x_2, w_1, w_2)$. In the contract, the owner forces the manager to make exploratory investment at the threshold x_P and report project type. For a report of the good-type (bad-type) project, the owner specifies the investment threshold x_1 (x_2) and pays the manager w_1 (w_2) at the development investment time. Since the revelation principle (cf. (Laffont and Martimort 2002, Bolton and Dewatripont 2005)) ensures that the manager reveals the true development cost in the optimum, the owner's optimal contract $(x_P^{**}, x_1^{**}, x_2^{**}, w_1^{**}, w_2^{**})$ solves the following problem:

$$
\begin{aligned}
\pi_o^{**}(x) := \max_{x_P, x_1, x_2, w_1, w_2} \quad & \left(\frac{x}{x_P}\right)^\beta (-aI) + q\left(\frac{x}{x_1}\right)^\beta (x_1 - (1-a)I - w_1) \\
& + (1-q)\left(\frac{x}{x_2}\right)^\beta (x_2 - ((1-a)I + \Delta I) - w_2) \\
\text{s.t.} \quad & x_i \geq x_P \quad (i = 1, 2) \\
& w_i \geq 0 \quad (i = 1, 2) \\
& \left(\frac{x}{x_1}\right)^\beta w_1 - \left(\frac{x}{x_2}\right)^\beta (w_2 + \Delta I) \geq 0 \\
& \left(\frac{x}{x_2}\right)^\beta w_2 - \left(\frac{x}{x_1}\right)^\beta (w_1 - \Delta I) \geq 0,
\end{aligned}
\tag{8}
$$

Throughout the paper, the superscript $**$ refers to the solution under asymmetric information. In the constraints of problem (8), the second inequalities correspond to the ex-post limited-liability constraints, while the last two inequalities are the ex-post incentive-compatibility constraints. The incentive-compatibility constraint means that with a truthful report, the manager who observes $(1-a)I$ $((1-a)I + \Delta I)$ obtains the expected payoff $(x/x_1)^\beta w_1$ $((x/x_2)^\beta w_2)$, which is larger than the expected payoff with a false report, $(x/x_2)^\beta (w_2 + \Delta I)$ $((x/x_1)^\beta (w_1 - \Delta I))$.

Clearly, in the optimum of problem (2) $x_P^{**} = x_1^{**} \leq x_2^{**}$ and $w_2^{**} = 0$ are satisfied. Then, problem (8) can be reduced to

$$\pi_o^{**}(x) = \max_{x_1, x_2, w_1} \quad q\left(\frac{x}{x_1}\right)^\beta (x_1 - (a/q + 1 - a)I - w_1) + (1 - q)\left(\frac{x}{x_2}\right)^\beta (x_2 - ((1 - a)I + \Delta I))$$

$$\text{s.t.} \quad x_2 \geq x_1$$

$$w_1 \geq 0$$

$$\left(\frac{x}{x_1}\right)^\beta w_1 - \left(\frac{x}{x_2}\right)^\beta \Delta I = 0. \tag{9}$$

The following proposition shows the solution to problem (9).

Proposition 2 The optimal contract $(x_P^{**}, x_1^{**}, x_2^{**}, w_1^{**}, w_2^{**})$, the owner's value $\pi_o^{**}(x)$, and the manager's value $\pi_m^{**}(x)$ are given as follows:

Case (i-A): $a < q\Delta I/(1 - q)I$ (separating)

$$x_P^{**} = x_1^{**} = x_1^*, \; x_2^{**} = \frac{\beta}{\beta - 1}((1 - a)I + \Delta I/(1 - q)), \; w_1^{**} = \left(\frac{x_1^*}{x_2^{**}}\right)^\beta \Delta I, \; w_2^{**} = 0 \tag{10}$$

$$\pi_o^{**}(x) = \frac{qx^\beta}{\beta x_1^{*\beta - 1}} + \frac{(1 - q)x^\beta}{\beta x_2^{**\beta - 1}} \tag{11}$$

$$\pi_m^{**}(x) = q\Delta I \left(\frac{x}{x_2^{**}}\right)^\beta \tag{12}$$

Case (ii-A): $a \geq q\Delta I/(1 - q)I$ (pooling)

$$x_P^{**} = x_1^{**} = x_2^{**} = \frac{\beta}{\beta - 1}(I + \Delta I), \; w_1^{**} = \Delta I, \; w_2^{**} = 0 \tag{13}$$

$$\pi_o^{**}(x) = \frac{x^\beta}{\beta x_1^{**\beta - 1}} \tag{14}$$

$$\pi_m^{**}(x) = q\Delta I \left(\frac{x}{x_1^{**}}\right)^\beta \tag{15}$$

Proposition 2 can be interpreted similarly to Proposition 1. Indeed, exploration cost plays a role in deferring the good-type project and accelerating the bad-type project for the same reason as Proposition 1. However, under asymmetric information, the owner widens the gap between the investment timing for the good- and bad-type projects to decrease the bonus to the manager. Actually, $x_2^{**}/x_1^{**} \geq x_2^*/x_1^* \geq 1$ is satisfied for any a. Note that, by (10), the bonus to the manager, w_1^{**}, decreases as x_2^{**}/x_1^{**} increases. This spread increases the critical value between the separating and pooling solutions from $a = q\Delta I/I$ in the symmetric information case to $a = q\Delta I/(1 - q)I$ in the asymmetric information case. Accordingly, Case (i-S) is included in Case (i-A).

In the region $a < q\Delta I/I$, i.e., Case (i-S), the thresholds x_P^{**} and x_1^{**} remain unchanged from x_P^* and x_1^*, while x_2^{**} becomes larger than x_2^*. The manager's private information distorts the investment timing for the bad-type project rather than the good-type project. This result is the same as that of previous studies such as (Grenadier and Wang 2005, Nishihara and Shibata 2008, Shibata and Nishihara 2010). Because of low exploration cost, the owner designs the contract contingent on information obtained with exploratory investment.

In the region $q\Delta I/I \le a < q\Delta I/(1-q)I$, i.e., Case (ii-S) and Case (i-A), the investment policy is quite different from that of the symmetric information case. To decrease the information rent to the manager, the owner chooses a separating contract instead of a pooling contract. Although exploration cost is not very low, the owner forces the manager to gather information on project type earlier for the purpose of saving the bonus to the manager. Unlike the previous studies, the investment timing for both bad- and good-type projects is delayed from that of the symmetric information case. This complements the previous works by enforcing the results of underinvestment caused by agency costs.

When a increases above $q\Delta I/(1-q)I$, i.e., Case (ii-A), the owner makes a pooling contract independent of the exploration result. Because exploration cost is very high, the owner gives up utilizing information regarding project type. Naturally, the owner must pay the entire rent ΔI of the manager's private information. This means that the owner cannot offer any effective contract to the manager. Indeed, the owner's value $\pi_o^{**}(x)$ is equal to $\pi_o^*(x)$ with $q = 0$ (cf. (7) and (14)). This type of pooling contract never appears in the previous studies with no exploration cost. However, a similar result is found in a static model by (Crémer, Khalil, and Rochet 1998a). They showed that a high cost of information acquisition leads to a pooling contract independent of project type, although their model cannot account for dynamic investment timing.

For fixed w_1 and x_i satisfying $x_1 < x_2$, the objective function in problem (9) decreases with a. In Case (i-A), we have $\partial x_2^{**}/\partial a < 0$. These immediately lead to the following corollary.

Corollary 2 In Case (i-A), $\partial \pi_o^{**}(x)/\partial a < 0$ and $\partial \pi_m^{**}(x)/\partial a > 0$ hold.

Note that in Case (ii-A) neither $\pi_o^{**}(x)$ nor $\pi_m^{**}(x)$ depends on a. Corollary 2 shows that an increase in exploration cost transfers a portion of the owner's value to the manager. A higher a decreases the investment threshold x_2^{**} and, hence, increases the manager's value $\pi_m^{**}(x)$. In addition to the increased rent to the manager, distortion due to exploration cost decreases the owner's value $\pi_o^{**}(x)$ with a. The wealth transfer is consistent with empirical evidence by (Aboody and Lev 2000). They found insider gains in R&D-intensive firms substantially larger than insider gains in firms without R&D, and then identified R&D as a major contributor to information asymmetry. Our result provides a complementary explanation for their finding.

Large insider gains in R&D-intensive firms originate not only from large information asymmetry but also from high exploration cost for R&D activities.

Now, consider a case in which the proportion of exploration cost is also privately observed by the manager. For example, assume that only the manager observes the realized value of $a \in [a_L, a_R]$ at time 0 while the owner knows only a range $[a_L, a_R]$. In this case, the manager always reports a_R with no additional mechanism, because a higher a increases the manager's value $\pi_m^{**}(x)$. In other words, the manager pretends to be ignorant of project type. This result is similar to that of (Kessler 1998) in incentive theory. He showed that, in a static model an agent may be better off if he/she can commit to remain ignorant with some probability.

3 Social welfare

In the previous subsection, we have focused on the owner's optimal contract with the manager who will acquire information on project type with exploratory investment. In this subsection, we explore how the agency problem affects social welfare. We define the social welfare and the social loss by $V^{**}(x) := \pi_o^{**}(x) + \pi_m^{**}(x)$ and $L^{**}(x) := \pi_o^*(x) - V^{**}(x)$, respectively. As documented by (Grenadier and Wang 2005), this social loss stemming from agency costs can be indicative of a corporate structure. With potentially large social loss, a firm will be forced to be privately held or to be organized in a manner that provides the closest alignment between the owner and the manager. The following proposition shows how the ratio of exploration cost to total investment cost, a, affects the social welfare and loss.

Proposition 3 Assume that $q\Delta I/(1-q)I \leq 1$. There exists a unique $\tilde{a} \in (q\Delta I/I, q\Delta I/(1-q)I)$ such that

$$\frac{\partial V^{**}(x)}{\partial a} \begin{cases} < 0 & (0 \leq a < \tilde{a}) \\ = 0 & (a = \tilde{a}) \\ > 0 & (\tilde{a} < a < q\Delta I/(1-q)I), \end{cases} \tag{16}$$

which means that the social welfare $V^{**}(x)$ is minimized at $a = \tilde{a}$. The social loss $L^{**}(x)$ satisfies

$$\frac{\partial L^{**}(x)}{\partial a} \begin{cases} > 0 & (0 \leq a < \tilde{a}) \\ = 0 & (a = \tilde{a}) \\ < 0 & (\tilde{a} < a < q\Delta I/(1-q)I), \end{cases} \tag{17}$$

which means that the social loss $L^{**}(x)$ is maximized at $a = \tilde{a}$.

Note that in the region $a \geq q\Delta I/(1-q)I$ neither $V^{**}(x)$ nor $L^{**}(x)$ depends on a. Proposition 3 contrasts asymmetric and symmetric information cases. In the absence of asymmetric information, a higher a leads to less efficiency because exploration cost distorts the in-

11

vestment policy; however, the outcome changes with asymmetric information. In the region $q\Delta I/I \le a < q\Delta I/(1-q)I$, the owner, unlike in the symmetric information case, chooses a separating contract to decrease the information rent to the manager. For $a \approx \tilde{a}$, the owner's contract maximizing his/her interest becomes far from efficient from the social viewpoint. The owner's contract may lead to less social welfare than the case with no contract. Indeed, the social welfare with no contract agrees with (14) + (15), which is larger than $V^{**}(x)|_{a=\tilde{a}}$ by (16). This can also be verified from the opposite side, i.e., the social loss $L^{**}(x)$.

In the region $a \in (\tilde{a}, q\Delta I/(1-q)I)$, $V^{**}(x)$ increases with a. When costly learning and asymmetric information are separately present, both lead to inefficiency. However, higher exploration cost can lead to less inefficiency due to asymmetric information by preventing the owner's greedy contract. This complements the literature by revealing interactions between costly learning and asymmetric information. Similar to this result, several corporate finance studies have shown that "two incentive problems are better than one" (e.g., (Hirshleifer and Thakor 1992, Mookherjee and Png 1995, Noe and Rebello 1996)). Related results are seen in the context of real options. In (Hackbarth 2009), managerial optimism and overconfidence, which distort the investment and financing policy, play a potentially positive role in ameliorating bondholder-shareholder conflicts such as debt overhang, asset stripping, and risk-shifting. (Nishihara and Shibata 2010) developed a model that reveals complex interactions between preemptive competition and a financing constraint and showed that a financing constraint can mitigate preemptive competition.

As discussed after Corollary 2, let us look at the situation where the manager has private information about $a \in [a_L, a_R]$. The manager always reports the maximal value $a = a_R$ because a higher a increases the manager's value (cf. Corollary 2). This moral hazard decreases the owner's value but does not necessarily reduce the social welfare. For example, consider a case of $a_L = \tilde{a}$. By Proposition 3, we have $V^{**}(x)|_{a=\tilde{a}} < V^{**}(x)|_{a=a_R}$ and $L^{**}(x)|_{a=\tilde{a}} > L^{**}(x)|_{a=a_R}$. This means that the manager's moral hazard may enhance the social welfare by mitigating inefficient asset substitution by the owner's greedy contract.

4   Numerical examples

We investigate more detailed properties of the solutions in numerical examples. Section 4.1 focuses on the effects of the ratio of exploration cost, a. Section 4.2 explores the comparative statics with respect to the volatility of project value, σ.

4.1 The effects of exploration cost

For comparison, we use the same base parameter values as (Shibata 2009, Shibata and Nishihara 2010):[6]

$$q = 0.5, \sigma = 0.2, r = 0.07, \mu = 0.03, I = 50, \Delta I = 30, X(0) = x = 100 \qquad (18)$$

For the base parameter values (18) the critical value between Case (i-S) (separating) and Case (ii-S) (pooling) in the symmetric information case becomes $q\Delta I/I = 0.3$ (cf. Proposition 1), while the critical value between Case (i-A) (separating) and Case (ii-A) (pooling) in the asymmetric information case becomes $q\Delta I/(1-q)I = 0.6$ (cf. Proposition 2).

The upper-left and upper-right panels in Figure 1 depict the investment thresholds in the symmetric and asymmetric information cases, respectively, with varying levels of a. In the absence of exploration cost, i.e., $a = 0$, the solution is equal to that of the benchmark case by (Grenadier and Wang 2005). Indeed, as in (Shibata 2009), the investment thresholds are $x_1^* = 128.43, x_2^* = 205.49, x_1^{**} = 128.43, x_2^{**} = 282.56$. In the previous studies, the agency problem does not distort the investment threshold for the good-type project, because the owner can effectively decrease the bonus to the manager by changing the investment threshold for the bad-type project. However, as discussed after Proposition 2, together with exploration cost $a > 0.3$, the investment is delayed whether the project is the good- or bad-type. The upper panels also show that distortion due to high exploration cost increases with asymmetric information. For example, we compare the cases of $a = 0$ (separating) and $a = 0.6$ (pooling). For $a = 0.6$ the investment thresholds are $x_P^* = x_1^* = x_2^* = 166.96, x_P^{**} = x_1^{**} = x_2^{**} = 205.49$. The distortion due to $a = 0.6$ is $166.96 - 128.43 = 38.53$ under symmetric information and $205.49 - 128.43 = 77.06$ under asymmetric information.

Next, let us turn to the owner's and manager's values. The lower-left and lower-right panels in Figure 1 plot the owner's value under symmetric information and the owner's and manager's values under asymmetric information, respectively, with varying levels of a. As shown in Corollaries 1 and 2, the owner suffers from inefficiency due to exploration cost while the manager benefits from increased information rent. Comparing $\pi_o^*(x)$ in the lower-left panel and $\pi_o^{**}(x)$ in the lower-right panel, we find that the decrease in the owner's value is more serious in the asymmetric information case than in the symmetric information case. Asymmetric information reinforces inefficiency caused by costly exploration from the owner's viewpoint.

Finally, we explore the effects of a on the social welfare and loss. The left and right panels in Figure 2 plot $V^{**}(x) = \pi_o^{**}(x) + \pi_m^{**}(x)$ and $L^{**}(x) = \pi_o^*(x) - V^{**}(x)$, respectively, with varying levels of a. As shown in Proposition 3, $V^{**}(x)$ is U-shaped and $L^{**}(x)$ is unimodal. The worst

---

[6]We carried out a lot of computations with varying parameter values and distilled robust results into this section.

a is $\tilde{a} = 0.3894$. We see from the right panel that the agency cost is very high for $a \approx 0.3894$. As mentioned above (cf. the lower-right panel in Figure 2), an increase in a plays a role in transferring a portion of the value from the owner to the manager. In the region $a < 0.3894$, the decrease in the owner's value dominates the increase in the manager's value. Then, $V^{**}(x)$ decreases with a. This is straightforward and the same as the symmetric information case. On the other hand, in the region $a \in (0.3894, 0.6)$, the increase in the manager's value dominates the decrease in the owner's value. This leads to a counter-intuitive result that $V^{**}(x)$ increases with a. A higher a moderates inefficient asset substitution from owner to manager under asymmetric information.

4.2 The effects of uncertainty

The effect of the volatility σ on the investment thresholds are straightforward; hence, we omit a figure illustrating it. Actually, an increase in σ increases the investment thresholds, as well as most real options models. Figure 3 illustrates $\pi_{\mathrm{o}}^{**}(x), \pi_{\mathrm{m}}^{**}(x), V^{**}(x)$, and $L^{**}(x)$ for $\sigma = 0.1, 0.2, 0.3$, and 0.4. Note that the critical value between Case (i-A) (separating) and Case (ii-A) (pooling) in the asymmetric information case, $q\Delta I/(1-q)I = 0.6$, does not depend on σ.

The upper panels show that a higher volatility increases the owner's value and decreases the manager's value. This asset substitution is consistent with previous findings by (Shibata 2009, Shibata and Nishihara 2010). In the lower panels, we verify that asset substitution is efficient from the social viewpoint. The lower panels demonstrate that $V^{**}(x)$ increases with σ and $L^{**}(x)$ decreases with σ. The impact of σ is especially great for $L^{**}(x)$. Indeed, an increase in σ from 0.2 to 0.4 reduces $L^{**}(x)$ by half. We also see from the figure that the effects of σ are relatively robust with respect to a.

5 Conclusion

This paper has investigated a principal-agent model in which the owner (principal) optimizes the contract with the manager (agent) delegated to undertake the investment project. The model assumes that the manager learns the real value of development cost by exploratory investment. For low cost in the exploration phase, the firm invests in the exploratory stage early and separates the investment timing for good- and bad-type projects. On the other hand, for high cost in the exploration phase, the firm proceeds to the development stage immediately after exploratory investment whether or not the exploration result is favorable. High exploration cost leads to a pooling solution not contingent on the exploration result. Asymmetric information increases the owner's incentive to take a separating contract rather than a pooling contract

because the owner can decrease the information rent to the manager with a separating contract. However, the owner's greedy contract may seriously reduce the social welfare especially when exploration cost is intermediate. Most notably, costly learning, or equivalently, the manager's pretense of ignorance, could improve social welfare by preventing inefficient wealth transfer by the owner's greedy contract.

A   Proof of Proposition 1

We can ignore x^β. Define the Lagrangian

$$L(x_1, x_2, \lambda) := qx_1^{-\beta}(x_1 - (a/q + 1 - a)I) + (1 - q)x_2^{-\beta}(x_2 - ((1 - a)I + \Delta I)) + \lambda(x_2 - x_1),$$

where λ denotes the Lagrangian multiplier. The Karush-Kuhn-Tucker conditions are

$$qx_1^{*-\beta-1}((-\beta + 1)x_1^* + \beta(a/q + 1 - a)I) - \lambda^* = 0 \tag{19}$$

$$(1 - q)x_2^{*-\beta-1}((-\beta + 1)x_2^* + \beta((1 - a)I + \Delta I)) + \lambda^* = 0 \tag{20}$$

$$x_2^* \geq x_1^*, \ \lambda^* \geq 0, \ \lambda^*(x_2^* - x_1^*) = 0 \tag{21}$$

First, consider the case of $x_1^* < x_2^*$. We have $\lambda^* = 0$ by (21). Substituting $\lambda^* = 0$ in (19) and (20), we have (4). Taking account of

$$
\begin{aligned}
x_1^* < x_2^* \ &\Leftrightarrow \ \frac{\beta}{\beta - 1}(a/q + 1 - a)I < \frac{\beta}{\beta - 1}((1 - a)I + \Delta I) \\
&\Leftrightarrow \ a < q\Delta I/I,
\end{aligned}
$$

we have the optimal policy in Case (i-S). The maximum value can be easily calculated as (5).

Next, consider the case of $x_1^* = x_2^*$. By (19) + (20), we have (6). By (19) we have

$$
\begin{aligned}
\lambda^* \geq 0 \ &\Leftrightarrow \ qx_1^{*-\beta}((-\beta + 1)x_1^* + \beta(a/q + 1 - a)I) \geq 0 \\
&\Leftrightarrow \ -(I + (1 - q)\Delta I) + (a/q + 1 - a)I \geq 0 \\
&\Leftrightarrow \ a \geq q\Delta I/I,
\end{aligned}
$$

which leads to the optimal policy in Case (ii-S). The maximum value becomes (7).   □

B   Proof of Proposition 2

We can ignore x^β. Define the Lagrangian

$$
\begin{aligned}
L(x_1, x_2, w_1, \lambda_1, \lambda_2, \lambda_3) \ := \ &qx_1^{-\beta}(x_1 - (a/q + 1 - a)I - w_1) + (1 - q)x_2^{-\beta}(x_2 - ((1 - a)I + \Delta I)) \\
&+ \lambda_1(x_2 - x_1) + \lambda_2 w_1 + \lambda_3(x_1^{-\beta}w_1 - x_2^{-\beta}\Delta I),
\end{aligned}
$$

15

where λ_i denotes the Lagrangian multiplier. The Karush-Kuhn-Tucker conditions are

$$qx_1^{**-\beta-1}((-\beta+1)x_1^{**} + \beta((a/q+1-a)I + w_1^{**})) - \lambda_1^{**} - \beta\lambda_3^{**}x_1^{**-\beta-1}w_1^{**} = 0 \quad (22)$$

$$(1-q)x_2^{**-\beta-1}((-\beta+1)x_2^{**} + \beta((1-a)I + \Delta I)) + \lambda_1^{**} + \beta\lambda_3^{**}x_2^{**-\beta-1}\Delta I = 0 \quad (23)$$

$$-qx_1^{**-\beta} + \lambda_2^{**} + \lambda_3^{**}x_1^{**-\beta} = 0 \quad (24)$$

$$x_2^{**} \geq x_1^{**}, \ \lambda_1^{**} \geq 0, \ \lambda_1^{**}(x_2^{**} - x_1^{**}) = 0 \quad (25)$$

$$w_1^{**} \geq 0, \ \lambda_2^{**} \geq 0, \ \lambda_2^{**}w_1^{**} = 0 \quad (26)$$

$$x_1^{**-\beta}w_1^{**} - x_2^{**-\beta}\Delta I = 0 \quad (27)$$

By (27) we have $w_1^{**} = (x_1^{**}/x_2^{**})^\beta \Delta I > 0$. Then, by (26) and (24) we have $\lambda_2^{**} = 0$ and $\lambda_3^{**} = q$. We can rewrite (22) and (23) as follows:

$$qx_1^{**-\beta-1}((-\beta+1)x_1^{**} + \beta(a/q+1-a)I) - \lambda_1^{**} = 0 \quad (28)$$

$$(1-q)x_2^{**-\beta-1}((-\beta+1)x_2^{**} + \beta((1-a)I + \Delta I/(1-q))) + \lambda_1^{**} = 0 \quad (29)$$

First, suppose $x_1^{**} < x_2^{**}$ in (25). Substituting $\lambda_1^{**} = 0$ in (28) and (29), we have (10) in Case (i-A). Note that

$$
\begin{aligned}
x_1^{**} < x_2^{**} \ &\Leftrightarrow \ \frac{\beta}{\beta-1}(a/q+1-a)I < \frac{\beta}{\beta-1}((1-a)I + \Delta I/(1-q)) \\
&\Leftrightarrow \ a < q\Delta I/(1-q)I.
\end{aligned}
$$

We can easily show (14) and (15).

Next, suppose $x_1^{**} = x_2^{**}$ in (25). By (28) + (29), we have (13). We have

$$
\begin{aligned}
\lambda_1^{**} \geq 0 \ &\Leftrightarrow \ qx_1^{**-\beta-1}((-\beta+1)x_1^{**} + \beta(a/q+1-a)I) \geq 0 \\
&\Leftrightarrow \ -(I + \Delta I) + (a/q+1-a)I \geq 0 \\
&\Leftrightarrow \ a \geq q\Delta I/(1-q)I,
\end{aligned}
$$

which leads to the solution in Case (ii-A). $\qquad\square$

C Proof of Proposition 3

In Case (i-A), by (11) and (12), we have

$$\frac{\partial V^{**}(x)}{\partial a} = I\left(\frac{x}{x_2^{**}}\right)^\beta \left((1-q)\left(1 - \left(\frac{x_2^{**}}{x_1^*}\right)^\beta\right) + \frac{\beta^2 q\Delta I}{(\beta-1)x_2^{**}}\right). \quad (30)$$

Because of $\partial x_1^*/\partial a > 0$ and $\partial x_2^{**}/\partial a < 0$, we have $\partial^2 V^{**}(x)/\partial^2 a > 0$. Further, we have

$$
\begin{aligned}
\left.\frac{\partial V^{**}(x)}{\partial a}\right|_{a=q\Delta I/I} \\
= \quad I\left(\frac{x}{x_2^{**}}\right)^\beta \left((1-q)\left(1-\left(1+\frac{q\Delta I}{(1-q)(I+(1-q)\Delta I)}\right)^\beta\right)+\frac{\beta q\Delta I}{I-q\Delta I+\Delta I/(1-q)}\right) \\
\leq \quad I\left(\frac{x}{x_2^{**}}\right)^\beta \left(-\frac{\beta q\Delta I}{I+(1-q)\Delta I}+\frac{\beta q\Delta I}{I-q\Delta I+\Delta I/(1-q)}\right) \qquad (31) \\
< \quad 0,
\end{aligned}
$$

where (31) follows from the inequality $(1+y)^\beta \geq 1+\beta y$ $(y \geq 0)$, and we also have

$$
\left.\frac{\partial V^{**}(x)}{\partial a}\right|_{a=q\Delta I/(1-q)I} = I\left(\frac{x}{x_2^{**}}\right)^\beta \frac{\beta^2 q\Delta I}{(\beta-1)x_2^{**}} > 0.
$$

Then, there exists a unique $\tilde{a} \in (q\Delta I/I, q\Delta I/(1-q)I)$ satisfying (16).

Considering $\partial \pi_o^*(x)/\partial a = 0$ for $a \geq q\Delta I/I$, we have only to show that $\partial L^{**}(x)/\partial a > 0$ for $a \in (0, q\Delta I/I)$. For $a \in (0, q\Delta I/I)$, by (5), (11), and (12) we have

$$
\begin{aligned}
\frac{\partial L^{**}(x)}{\partial a} \\
= \quad I\left(\frac{x}{x_2^{**}}\right)^\beta \left((1-q)\left(\left(\frac{x_2^{**}}{x_2^*}\right)^\beta-1\right)-\frac{\beta^2 q\Delta I}{(\beta-1)x_2^{**}}\right) \\
= \quad I\left(\frac{x}{x_2^{**}}\right)^\beta \left((1-q)\left(\left(1+\frac{q\Delta I}{(1-q)((1-a)I+\Delta I)}\right)^\beta-1\right)-\frac{\beta q\Delta I}{(1-a)I+\Delta I/(1-q)}\right) \\
\geq \quad I\left(\frac{x}{x_2^{**}}\right)^\beta \left(\frac{\beta q\Delta I}{(1-a)I+\Delta I}-\frac{\beta q\Delta I}{(1-a)I+\Delta I/(1-q)}\right) \qquad (32) \\
> \quad 0,
\end{aligned}
$$

where (32) follows from the inequality $(1+y)^\beta \geq 1+\beta y$ $(y \geq 0)$. This leads to (17). $\qquad \square$

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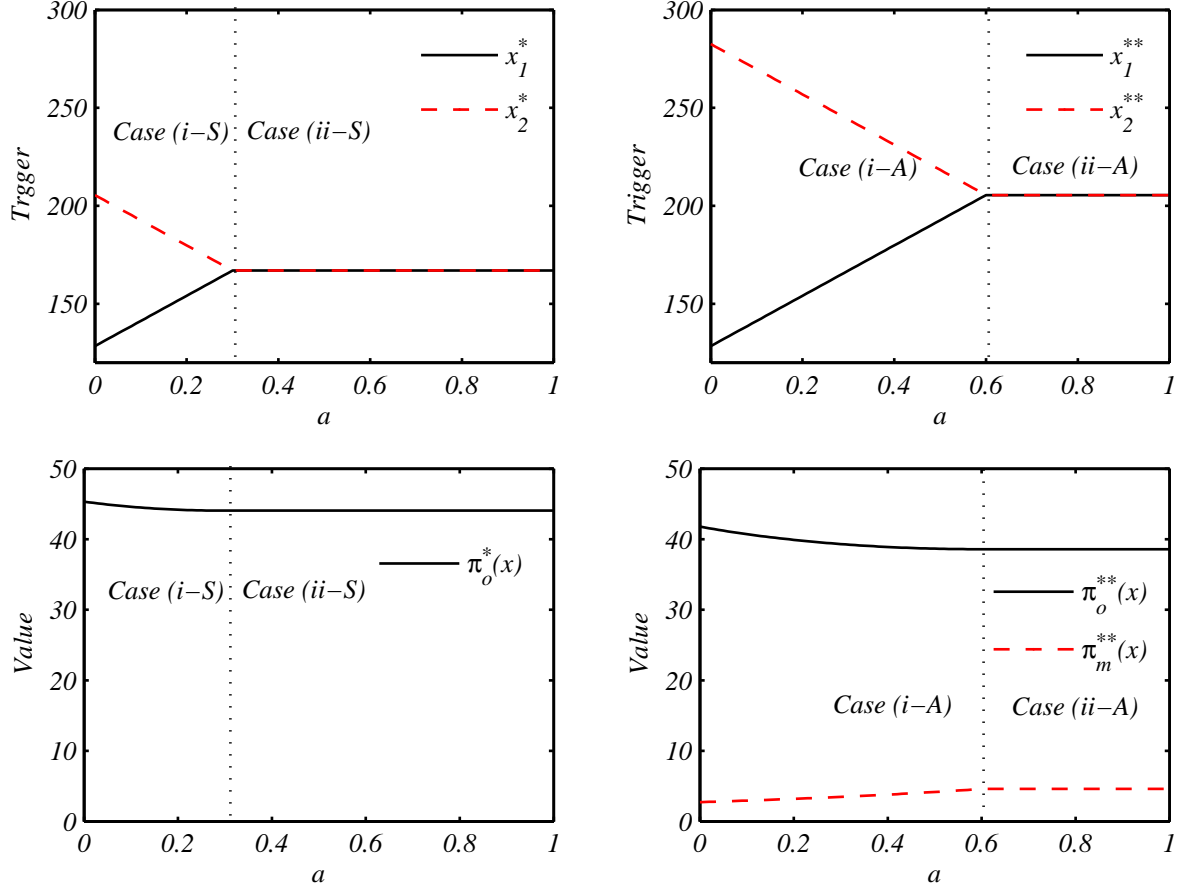
Figure 1: The investment thresholds and values under symmetric and asymmetric information. The upper-left panel plots the investment thresholds for good- and bad-type projects, x_1^* and x_2^* in the symmetric information case. The critical value between Case (i-S) (separating) and Case (ii-S) (pooling) is $a = 0.3$. The upper-right panel plots the investment thresholds for good- and bad-type projects, x_1^{**} and x_2^{**} in the asymmetric information case. The critical value between Case (i-A) (separating) and Case (ii-A) (pooling) is $a = 0.6$. The lower-left panel plots the owner's value under symmetric information, $\pi_o^*(x)$, while the lower-right panel plots the owner's and manager's values under asymmetric information, $\pi_o^{**}(x)$ and $\pi_m^{**}(x)$. The parameter values are set at the base case (18).
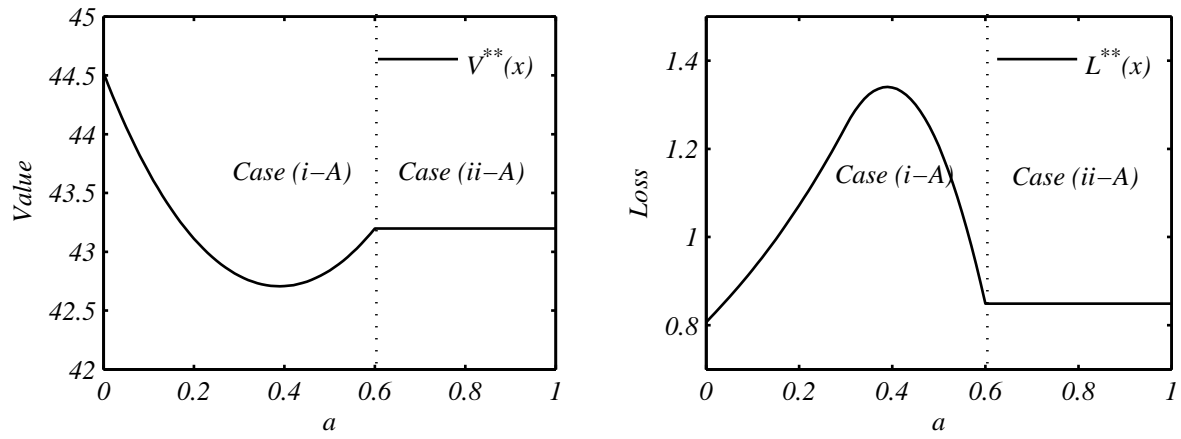
Figure 2: The social welfare and loss. The left panel plots the social welfare $V^{**}(x)$, while the right panel plots the social loss $L^{**}(x)$. At $a = \tilde{a} = 0.3894$, the social welfare (loss) is minimized (maximized). The parameter values are set at the base case (18).
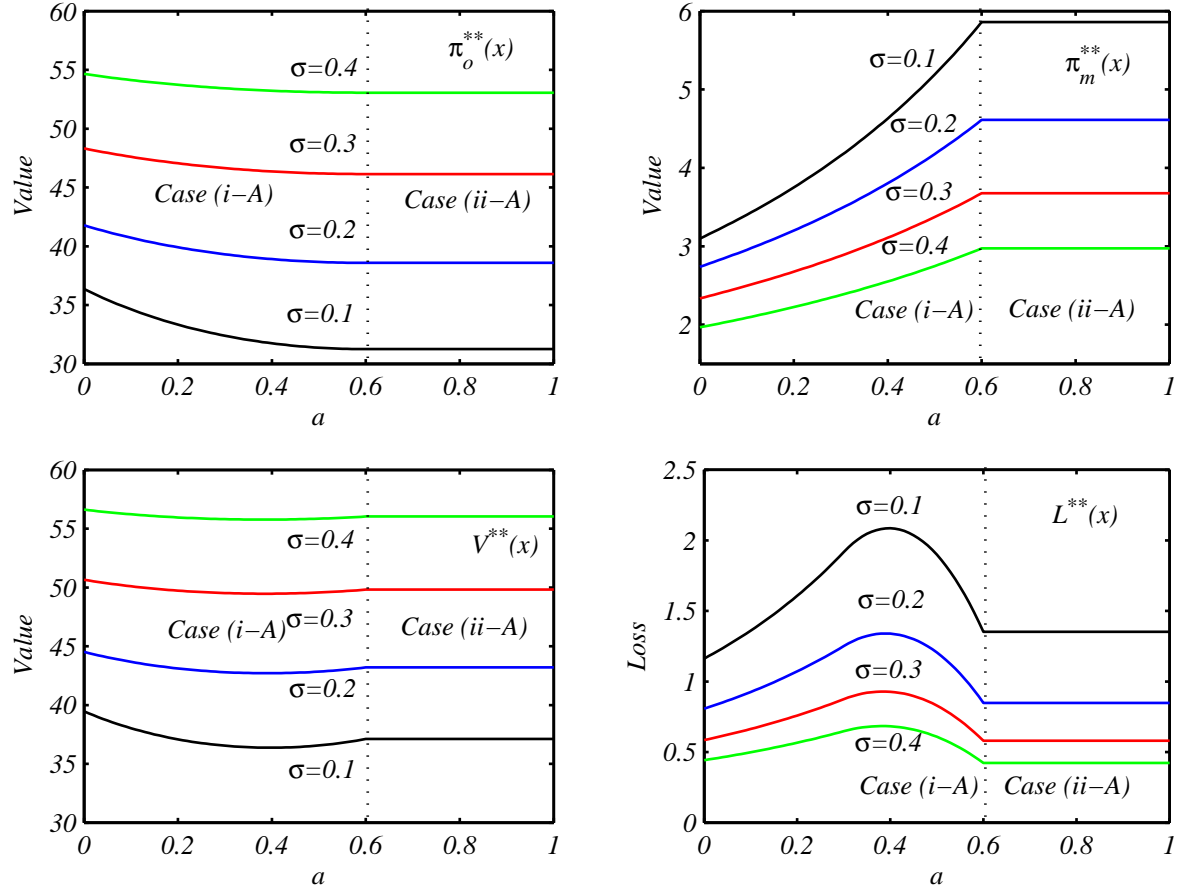
Figure 3: The comparative statics with respect to the volatility σ. The upper-left and upper-right panels plot the owner's and manager's values under asymmetric information, $\pi_o^{**}(x)$ and $\pi_m^{**}(x)$, respectively. The lower-left and lower-right panels plot the social welfare and loss, $V^{**}(x)$ and $L^{**}(x)$, respectively. The parameter values other than σ are set at the base case (18).