The Option to Switch from Oil to Natural Gas in Active Offshore Petroleum Fields

Authors:
Øystein Dahl HEM
Alexander SVENDSEN
Stein-Erik FLETEN
Vidar GUNNERUD

Abstract

An important decision in the development of a mixed oil and natural gas field is whether to produce or re-inject the natural gas. There is a trade-off between the income from the sale of natural gas and the higher oil production obtained from re-injection. We consider the optimal timing problem of when to stop natural gas injection for a set of offshore petroleum fields using a real options approach. The real options valuation prices the option to switch significantly higher than a net present value approach. A two-factor price model is implemented for both the oil price and the gas price. The option valuation is based on the Least-Squares Monte Carlo simulation algorithm.

Keywords: Investment uncertainty, petroleum production, field development, energy commodities

January 15, 2011

1 Introduction

We study the flexibility related to extraction timing in offshore petroleum production and its effect on the total value of the reservoir. As fewer new fields are discovered, decisions related to fields already in production become more important. An important choice in this respect is whether to re-inject the produced natural gas in order to produce more oil or to sell the produced natural gas. On the Norwegian Continental Shelf, injected water is the most common pressure support, but several fields employ natural gas injection (Norwegian Petroleum Directorate (2009)). Examples include the fields Oseberg, where natural gas export has been delayed several times in order to extract more oil, and Ula, where natural gas is purchased from a nearby field for injection. Oseberg contained 2.3 billion barrels of oil when it went into production, making even a small increase in producible reserves a project worth millions, if not billions, of dollars. At the Prudhoe Bay field, the largest one in North America, operators have increased the recovery factor substantially due to gas injection together with other techniques (Szabo and Meyers (1993)). As oil production from the field falls, a gas pipeline to export the gas is being discussed; necessary infrastructure for large-scale gas export is not currently present. Gas is also used for enhanced oil production in the Middle East. In 2008 around 16% of Iran's gas production was re-injected to increase oil production[1]. We intend to value the flexibility related to stopping injection, and exporting the produced natural gas. Since 2000, oil and gas prices have been increasingly volatile, thereby making it more difficult to determine an optimal switching time using static valuation approaches such as deterministic net present value (NPV) calculations. The problem of optimal extraction timing may be interesting for both practitioners assessing field strategies as well as analysts and researchers forecasting future petroleum supply.

Real option valuation (ROV) has been applied to petroleum projects for a long time as they have attributes such as a large irreversible initial investment, a risky and easily traded output commodity, reservoir uncertainty which is resolvable only through drilling, and strategic flexibility over choices related to timing, technology and capacity. Siegel, Smith, and Paddock (1988) assess investing in offshore petroleum leases and compare ROV with both NPV approaches and observed bid prices. In contrast, Cortazar and Schwartz (1998) use a Monte Carlo model to find the optimal timing of investing in a field with a set oil rate that declines exponentially and with varying but deterministic operating costs. With this predetermined production rate the value of the field becomes a function of the oil price, which is modeled as a two-factor model where the spot price follows a geometric Brownian motion and the convenience yield follows a mean-reverting process. Similarly, McCardle and Smith (1998) consider the timing of investment, the option to abandon, and the option to tie in surrounding fields. Both prices and production rates are modeled as stochastic processes, where the price follows a geometric Brownian motion process. Ekern (1988) also values the development of satellite fields and adding incremental capacity using a binomial ROV model. Lund (1999) considers an offshore field development by using a case from a North Sea field, Heidrun, employing a dynamic programming model that takes into account the uncertainty regarding both reservoir size and well rates, in addition to the oil price. The paper assumes the price follows a geometric Brownian motion process, and uses a binomial lattice valuation model to find the optimal size of the rig and investment timing. Dias, Lazo, Pacheco, and Vellasco (2003) utilize Monte Carlo simulations together with non-linear optimization in order to find an optimal development strategy for oil fields when considering three mutually excluding alternatives. Chorn and Shokhor (2006) combine dynamic programming and real options valuation to value investment opportunities related to petroleum exploration. Dias (2004) provides a more thorough review of real option valuation related to the petroleum industry, and Suslick and Schiozer (2004) give an overview of risk analysis for petroleum exploration and production.

[1] EIA Iran Country Analysis Briefs, `www.eia.doe.gov/cabs/Iran/pdf.pdf`, retrieved 28.05.2010

In this work we take into account price risk and reservoir risk in a ROV model based on the Least Squares Monte Carlo simulation algorithm presented in Longstaff and Schwartz (2001). We do not consider the problem of initial investment, as it has been applied to petroleum projects and a large range of other industries before. Also, in their comparison of a option valuation and a discounted cash flow approach, Siegel et al. (1988) could not prove that the option valuation gave a different result for the price of a set of real-world oil tracts. Lund (1999) found that the value of deferring an investment is generally low in petroleum production as opportunity costs associated with delaying are high and new information rarely change the investment decision. Instead, we focus on decisions being made as the field is in production, specifically when to begin exporting the gas.

In Section 2, we present the data. In Section 3, we introduce the price models, reservoir model and valuation framework used in this work. We apply these in Section 4 to three real-life cases, and present our conclusions in Section 5.

2 Data

To study the long-term behavior of the oil price and to find a suitable time-series model, we have used forward prices of crude oil with siix expiration maturities from 0.25 years to 2.5 years[2]. The forward series have weekly observations. Using these data, we estimate the long- and short-term volatility, as well as risk-neutral price growth. An equivalent forward series for gas forwards have been used for estimating the gas price process. The forwards for natural gas are from NYMEX[2], and are converted into USD per standard cubic meter oil equivalent, $USD/Sm^3 o.e.$

There are several methods for estimating the risk-free rate. Bruner, Eades, Harris, and Higgins (1998) argue that on the one hand, 90-day T-bills are more consistent compared to long-term bonds and truly reflect risk-free returns. On the other hand, short-term interest rates fluctuate over time, thereby introducing reinvestment risk when considering long periods. Long-term bonds reflect the default-free holding period returns closer than short-term T-bills for long investment periods and avoid the reinvestment problem. The empirical study performed by Bruner et al. (1998) shows that practitioners have a strong preference for using long-term bonds to estimate long-term risk-free interest rates. Koller, Goedhart, and Wessels (2005) recommend using liquid long-term zero-coupon government securities. To estimate the USD-denominated risk free rate, we have used 20-year US Treasury bonds[2]. The risk-free rate is estimated to be 4.3% per year.

The reservoir data used in the case study are from the Norwegian Petroleum Directorate (2009), where reserves and production data for all Norwegian fields can be found. Observed production rates for Oseberg are also retrieved[3].

3 Methodology

Several key choices needs to be made when constructing a ROV model. These include which uncertainties to take into account, how to model these, and which simplifications to make. Bickel and Bratvold (2008) make a survey of which uncertainties oil and gas practitioners find most relevant. The most important uncertainties from the survey are subsurface risk followed closely by hydrocarbon prices. Subsurface risk cover a wide range of uncertainties, from reservoir properties to the well-flow response to new production or injection wells. In our model, we take account of subsurface risks by modeling the extractable reserves as uncertain. We also include price risk. The valuation procedure is presented in Section 3.3.

[2] EcoWin Reuter Database, retrived 03.02.2010
[3] Norwegian Petroleum Directorate Web page, fact section, `www.npd.no`, retrieved 07.05.2010

3.1 Time-series analysis of oil and gas prices When considering the decision to switch from oil to natural gas production, two critical factors that must be included are the oil and natural gas prices. Time series of these are shown in Figure 3. The two variables are thought to be cointegrated to a certain extent. In the United States, oil power plants have been able to switch between using oil and natural gas as fuel. This connects the two prices as producers choose the most economical fuel. Today, there are a smaller share of power plants than before that can switch between the two fuels (Brown 2005). The crude oil and natural gas prices are still to some degree linked since they are substitutes in other areas as well, i.e. heating. Crude oil and natural gas are used as substitutable inputs in refineries, with the result that the two petroleum prices are more cointegrated than they otherwise would be. Another factor linking the prices of oil and natural gas is that natural gas is often a co-product of oil production, meaning that if one increases oil production, then the gas production will also often increase. An increased demand in crude oil following a price increase may also lead to increased costs related to petroleum extraction (Villar and Joutz 2006). This will increase the marginal cost of natural gas as well, thereby creating an upwards pressure on gas prices.

Villar and Joutz (2006) show that natural gas and crude oil prices have had a stable relationship historically, even though there are periods where they have appeared to decouple. Brown and Yücel (2008) also document a long-term relationship between the two prices. Brown (2005) found that U.S natural gas prices are related to crude oil prices, with natural gas prices adjusting to changes in the crude oil prices. Most Liquefied Natural Gas (LNG) contract are indexed to oil prices. This creates a direct link between crude oil and natural gas prices (Foss 2005).

The geometric Brownian motion (GBM) and the Ornstein-Uhlenbeck process has often been used to describe the movement of oil prices. Pindyck (1999) argues that oil price can be forecasted by incorporating mean reversion to a stochastically fluctuating trend line. Even though the mean-reversion model often is used, Pindyck (1999) finds that the rate of mean reversion is low for oil, coal, and natural gas prices. This suggest that the GBM process may be a fair approximation for many applications. Schwartz and Smith (2000) demonstrate that the two-factor model outperforms the simpler one-factor models. For that reason, we use this two-factor model to estimate the oil and gas price processes. We proceed by testing the two time series obtained from the two-factor model and analyze if they have the characteristics assumed in the model description. This provides an indication on whether the two-factor model renders a good explanation of observed price behavior.

3.1.1 A two-factor model for commodity prices by Schwartz and Smith (2000) Schwartz and Smith (2000) develop a two-factor model of commodity prices that allows mean reversion in the short-term price while the long-term price follows an arithmetic Brownian motion process. The two-factor model combines the Brownian motion and mean-reverting effects seen in commodity prices, and gives a more realistic representation of commodity price movement than any of the two one-factor models on their own. Forward prices are used to separate the long-term and short-term components. The changes in the long-term prices reflect the changes in demand and supply following political situations, exhaustions and discovery of commodities and improving technology, while changes in short-term price reflects changes in demand resulting from variations in weather or intermittent problems (Schwartz and Smith 2000).

The two factor risk-neutral processes are given by:

$$ln(S_t) = \chi_t + \xi_t \tag{1}$$

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi \quad (2)$$

$$d\xi_t = \mu_\xi^* dt + \sigma_\xi dz_\xi \quad (3)$$

S_t denotes the modeled spot price, dz_χ and dz_ξ are increments of risk-neutral Brownian motion processes with $dz_\chi dz_\xi = \rho_{\chi\xi} dt$. The short-term deviation, χ_t, is an Ornstein-Uhlenbeck process reverting to $-\lambda_\chi/\kappa$ rather than zero as assumed in the true process, while the equilibrium prices or the long-run price, ξ_t, is a Brownian motion process with drift μ_ξ^*. κ denotes the mean-reversion coefficient, λ_χ the short-term risk premium, and $\rho_{\chi\xi}$ the correlation in increments while σ_χ and σ_ξ denote the volatility for the short-term deviation and the equilibrium price level, respectively (Schwartz and Smith 2000).

3.1.2 Implementation of the two-factor model To estimate the model parameters, we fit the constructed forward curve from the model to the observed forward curve for a number of observations, and change the parameters in order to minimize the sum of squared errors. We use equations (4) and (5) in order to estimate a forward curve, $F_{t,T}$, which is the price of a forward contract with maturity at T observed at time t.

$$ln(F_{T,t}) = ln(E^*[S_T]) = e^{-\kappa T}\chi_t + \xi_t + A(T) \quad (4)$$

$$A(T) = \mu_\xi^* T - (1-e^{-\kappa T})\frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left((1-e^{-2\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right) \quad (5)$$

The estimation is done in three steps following Lucia and Schwartz (2002). In the first step, while estimating a set of reasonable parameters, a linear least-squares regression is performed to estimate the long-term and short-term components that give the smallest error between the models forecast and market forward prices. Each time-step is estimated independently from the others. In the second step, the variance and correlation estimate is updated, and the regression is repeated until the short-term and long-term estimated variance and correlation is close to the observed values. The correlation between the oil and gas prices is estimated from the modeled short-term and long-term time-series. In the third step, long-term and short-term prices and variances are fixed, and a non-linear optimization is performed in order to find the values of κ, μ_ξ^*, and λ_χ that minimize the sum of squared errors in the forward curve fit. For the non-linear optimization, we have used the interior-point solver Ipopt developed by Wachter and Biegler (2006). The three steps are then repeated until the parameters converges.

3.1.3 Oil model The forward curve is obtained from NYMEX crude oil futures maturing at 3, 6, 9, 12, 18, 24, and 30 months, with observations from week 36 in 1995 to week 5 in 2010, a total of 749 observations for each of the seven futures. Figure 1 compares the spot price to the modeled spot price and the long-term price. We see that the modeled price is close to the historical spot price. The mean absolute deviation between the spot price and the estimated spot price from the model is 1.66% of the average spot price.

Statistical analysis of the oil price data In order to decide if the model is suitable for describing the price movements, then one needs to examine if different characteristics can be found in the series. Schwartz and Smith (2000)

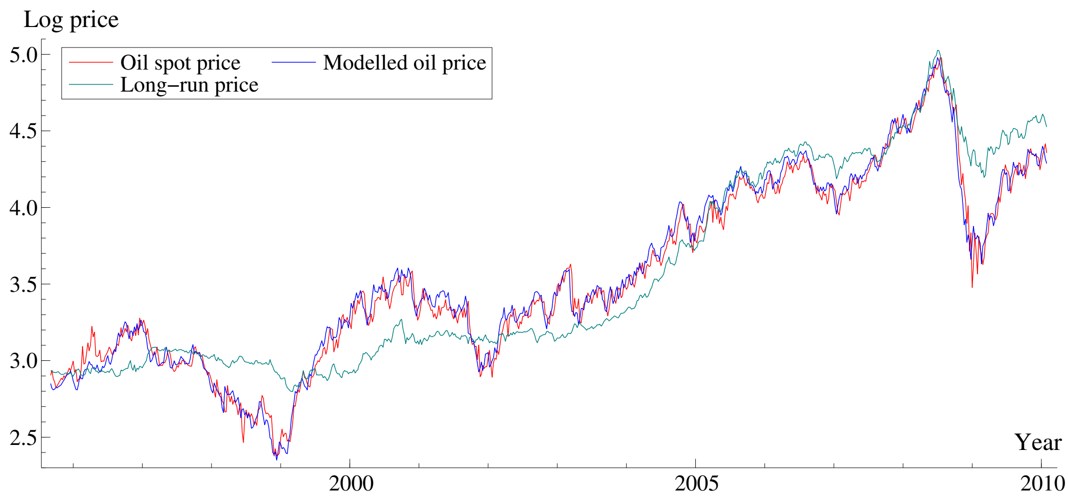


Figure 1: Comparing log-values of spot price, modeled spot price and long-term price denominated in log USD per bbl

Table 1: Testing oil two-factor model for unit root

Time series	DF	Result DF	KPSS	Result KPSS
χ	-2.73	Reject H_0^*	0.577	Reject H_0^{**}
ξ	-0.075	Do not reject H_0	3.00	Reject H_0^{***}
$\Delta\xi$	-22.5	Reject H_0^{***}	0.17	Do not reject H_0

model the long-term price as an arithmetic Brownian motion process in the two-factor model. In order to examine if our model is in pursuant with the theory, we test for a unit root. In a random walk, a unit root and no serial correlation should be present.

In this analysis, the logarithmic values of the short-term and long-term prices are used. We find evidence of some positive autocorrelation in the long-term oil price residuals. When autocorrelation is detected the estimates will still be unbiased, but they will be inefficient and have larger standard errors.

Unit root test results The Dickey-Fuller (DF) test presented in Dickey and Fuller (1979) investigates if a series is stationary or not by trying to prove that it does not contain a unit root. If there are no unit roots in a series, then it is stationary. We also test for a unit root using a Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski, Phillips, Schmidt, and Shin 1992). The null hypothesis for a KPSS-test is the reverse of a DF-test. By using both, one tests the robustness of the result, which is shown in Table 1. H_0^* indicates a rejection of H_0 on a 10% significance level, 5% level for H_0^{**}, and 1% level for H_0^{***}.

The DF test indicates that a unit root is present; hence the series does not seem to be stationary. This is confirmed by the KPSS test, which rejects the stationarity of the long-term price on a 1% significance level. The long-term returns are, however, deemed to be stationary by the DF test, also on a 1% significance level, which is corroborated by the KPSS-test. Both results support the assumption that the long-term price can be modeled as a Brownian motion

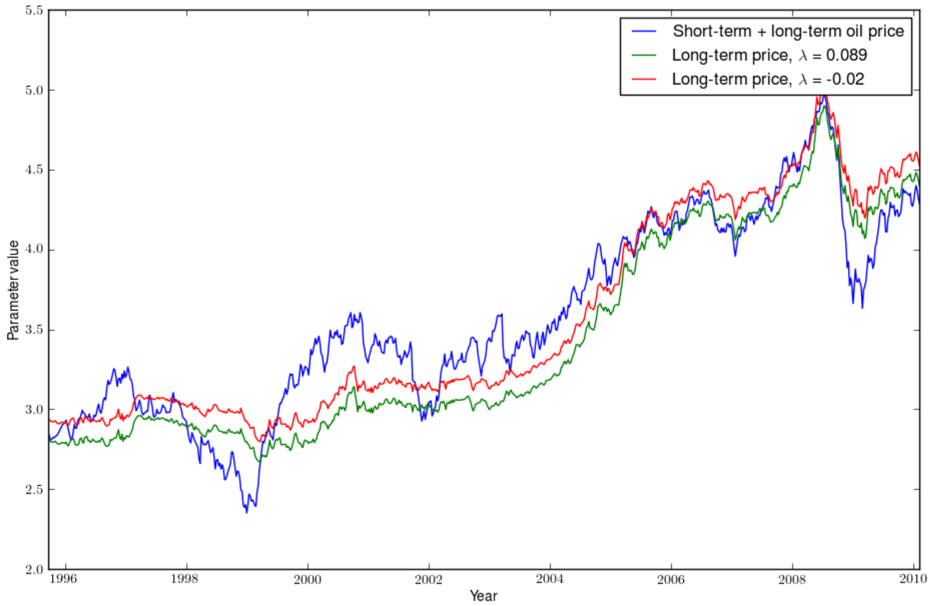


Figure 2: Price estimation sensitivity to short-term risk premium parameter denominated in log USD per bbl

process.

The DF test statistic for the short-term price rejects the null hypothesis at a 10% significance level. According to a KPSS test, stationarity is rejected on a 5% level. The results from the DF and the KPSS tests conflict, which implies that the conclusion is not robust. Since we cannot prove that the short-run price series is stationary, it could signify that the two-factor model is too simple for this commodity. We still use the two-factor model as our results are inconclusive, and since it captures main properties of interest in capital budgeting analysis, including a (nonstationary) long-term uncertainty. Furthermore, Schwartz and Smith (2000) have found it to be a good model for commodity prices including crude oil.

Convergence and sensitivity The parameter estimation was performed from several different starting points, in order to discover possible sub-optimal local minima and to assess the convergence of the algorithm. When attempting different starting points for all parameters, the procedure converges to the same values for all parameters, except the short-term risk premium, λ_χ. Two local minima were discovered, one with a sum of squared residuals of 0.5599 and a λ of -0.012. The best result had a sum of squares for the residuals of 0.5598, and with a λ of 0.086. The difference between the two minima is very modest. The other parameters did not change significantly. This large difference in the short-term risk-premium has a large impact on the absolute values of the short-term and long-term prices. The shapes of the long-term and short-term curves are, however, not affected, as can be seen in Figure 2. The estimation procedure compensates for changes in the short-term risk premium by changing the level of the short-term price. This makes it difficult to assess which of the models is more correct, as the difference in squared residuals is very small.

Table 2: Comparing oil model with results from Schwartz and Smith (2000)

Parameter	Description	Schwartz and Smith estimates	Oil model
κ	Short-term mean reversion rate	1.49	0.789
σ_χ	Short-term volatility	0.286	0.267
λ_χ	Short-term risk premium	0.157	0.086
μ_ξ^*	Equilibrium drift rate	-0.0125	-0.0137
σ_ξ	Equilibrium volatility	0.145	0.161
$\rho_{\xi\chi}$	Correlation in increments	0.300	-0.124

Results The two-factor model results are given in Table 2. We see that the results are similar to those given in Schwartz and Smith (2000). They use data from NYMEX futures from 1/2/90 to 2/17/95. The parameters are similar to the estimates from Schwartz and Smith (2000) and have the same signs, except the correlation in increments, which has an opposite sign, and the short-term mean reversion rate, which differs substantially. One obvious reason for the difference in the estimates is that the data-set used by Schwartz and Smith (2000) is older than the data used in our model.
Negative correlation in increments indicates that when there is a positive shock in the equilibrium price one can to some extent expect the short-term price to fall. This suggests that there is some inertia in the spot price, as a long-term change is to some degree offset by a short-term price adjustment in the opposite direction.

3.1.4 Gas model Gas prices are more regional than the oil price. There are several regional gas markets around the world, and, in contrast to the oil market, the different gas markets does not have as much influence on each other. One important link between the different gas markets is the LNG shipments, which transport natural gas over long distances. We use data from the Henry Hub gas market traded on NYMEX. Henry Hub is one of the largest gas markets in the world and should be more liquid and complete than smaller markets. This is discussed further in Section 3.4. Villar and Joutz (2006) show that while the changes in oil price have an impact on gas prices, the natural gas price does not influence oil price considerably. And though the prices of crude oil and natural gas at times have appeared to drift away from each other, oil and gas have still had a stable relationship (Villar and Joutz 2006). There are several rules of thumb regarding the relationship between oil and gas prices, but none of these rules of thumb has been consistently accurate over time. We see in Figure 3 that the crude oil price and natural gas price have followed each other and have the same characteristics.

To estimate the model parameters, we follow the procedure in Section 3.1.3 and fit a forward curve from the model to the observed forward curve for a number of observations. We then change the parameters in order to minimize the sum of squared errors. The forward curve is obtained from Henry Hub spot price and futures maturing at 3, 6, 9, 12, 18, 24, and 30 months, with observations from week 15 in 1996 to week 5 in 2010, a total of 719 observations. The estimated forward curve is found using equations (4) and (5).

Analysis of the gas model We model the gas price by using a two-factor process. In order to analyze the data, we do the same unit root tests as for the oil price model. We analyze the logarithmic values of the short-term and long-term prices for natural gas in the period from week 15 in 1996 to week 5 in 2010 (a total of 710 observations). The results of the DF-test and the KPSS-test are shown in Table 3.

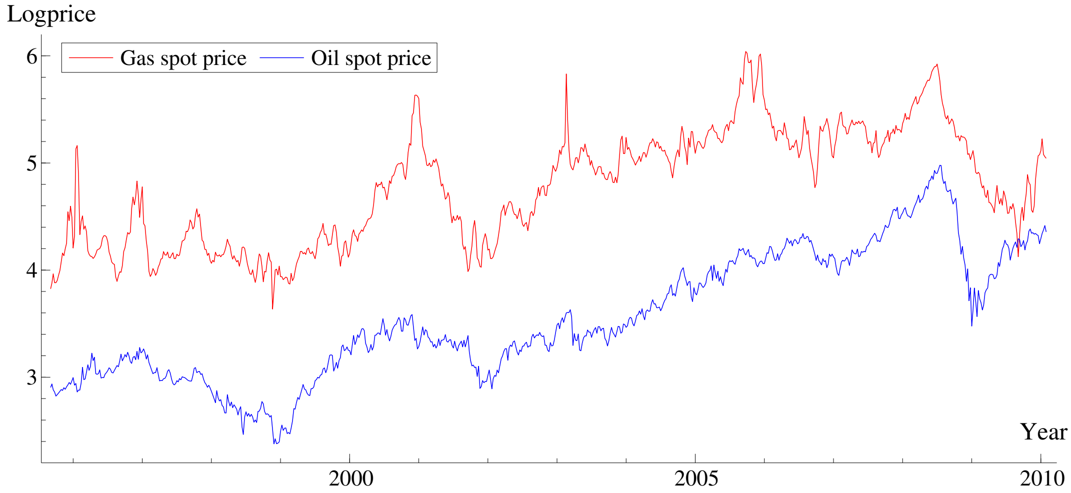


Figure 3: Comparing natural gas log-prices denominated in log USD per $Sm^3o.e$ and oil log-prices denominated in log USD per bbl using weekly observations from 1995 to 2010

Table 3: Testing gas two-factor model for unit root. H_0^* indicates a 10% significance level, H_0^{**} a 5% level and H_0^{***} a 1% level.

Time-series	DF	Result DF	KPSS	Result KPSS
χ	-4.71	Reject H_0^{***}	0.90	Reject H_0^{***}
ξ	-1.34	Do not reject H_0	2.86	Reject H_0^{***}
$\Delta\xi$	-28.43	Reject H_0^{***}	0.16	Do not reject H_0

The DF test statistic for the long-term price series implies that the null hypothesis cannot be rejected and there can be a unit root present. This result is confirmed by the KPSS test, which rejects stationarity. The long-term returns DF test indicates that the return is stationary on a 1% confidence interval, while the KPSS test cannot reject stationarity. This is consistent with the assumption that the long-term price can be modeled as an arithmetic Brownian motion process.

The test statistic for the short-term natural gas price is significant at a 1% confidence interval, thereby suggesting that the series does not contain a unit root. The KPSS test rejects stationarity, however. The results for the short-run price series are not robust, and it is difficult to say if the stationarity assumptions for the two-factor model should be rejected. This is the same conclusion as for the crude oil price. The reason for the inconclusive result may be that the time period in the analysis is too short. We find no evidence of autocorrelation in the long-term natural gas price residuals. This is according to the two-factor assumptions.

Results gas model Figure 4 compares the modeled gas price against the time series of the natural gas spot price. The model fits the time series very well, as the mean absolute deviation between the time series of the spot price and the modeled gas price is 0.19% of the average gas spot price. The reason why this value is lower than for the oil price is mainly because we have included the spot price in the estimation of the natural gas price model. When excluding the spot price, the accuracy of the model was low when encountering price spikes. We find that the short-term price

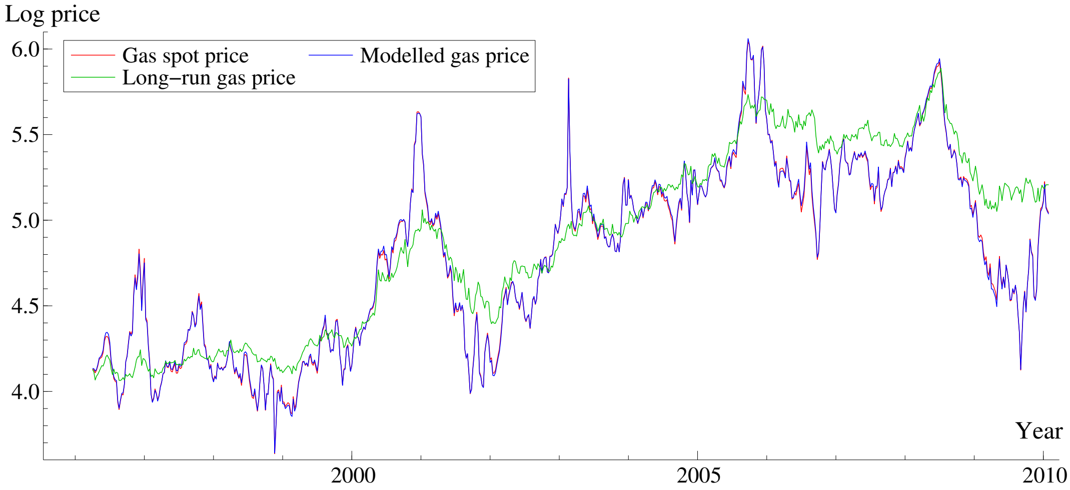


Figure 4: Comparing log values of natural gas spot price, modeled spot price and long-term price denominated in denominated in log USD per $Sm^3 o.e$

on average is close to $-\lambda_\chi/\kappa$ even though this was not a specified restriction in our gas model. This is in accordance with the risk-neutral two-factor model assumptions, where the short-term deviations are expected to have a mean of $-\lambda_\chi/\kappa$.

The two-factor model results are given in Table 4. The results from the model developed in this paper are compared

Table 4: Comparing natural gas model with futures data found in Cartea and Williams (2008)

Parameter	Description	Cartea and Williams estimate	Gas model
κ	Short-term mean reversion rate	10.18	7.0
σ_χ	Short-term volatility	1.38	0.5
λ_χ	Short-term risk premium	1.29	0.12
μ_ξ^*	Equilibrium drift rate	0.15	-0.04
σ_ξ	Equilibrium volatility	0.24	0.24
$\rho_{\xi\chi}$	Correlation in increments	-0.33	-0.05

to the futures data given in Cartea and Williams (2008). The data set in Cartea and Williams (2008) use data from IPE Natural Gas Futures from Aug 2003 - Jan 2006. Our parameters seem reasonable compared to the estimates from Cartea and Williams (2008). Their values are estimated for the European gas market, and one would expect to see some differences. They have also removed seasonality in their model. This will also cause some deviation from our results. A difference is that we found the short-term volatility and the correlation in increments to be lower. Both models have high short-term mean-reversion rates, suggesting that the price reverts quickly to a short-term mean value. This is explained by price spikes in natural gas, which last for a short period before reverting back to a more normal level. As in the estimates from Cartea and Williams (2008), we have a negative correlation in increments, but it is almost zero. This means that for this gas market, increments in the short-term price and increments in the equilibrium level seem to be almost uncorrelated with each other.

The results given in Cartea and Williams (2008) are estimated using the London-based IPE futures. That our results

Table 5: Correlation matrix for two-factor components in natural gas and oil. (*) indicates a 10% significance level, (**) a 5% level and (***) a 1% level against a two-tailed test.

	Short-run gas	Long-run gas	Short-run oil	Long-run oil
Short-run gas	1.00	-0.06^{*}	0.14^{***}	0.05
Long-run gas	-0.06^{*}	1.00	0.11^{**}	0.21^{***}
Short-run oil	0.14^{***}	0.11^{**}	1.00	-0.12^{**}
Long-run oil	0.05	0.21^{***}	-0.12^{**}	1.00

are fairly close to their estimates, can be an indication that the behavior of the two gas markets is similar. We assume that our estimates are satisfactory to apply in the case studies from the North Sea.

3.1.5 Estimating oil and natural gas correlation Finally, we need to determine how the oil and natural gas time series are correlated. Villar and Joutz (2006) found that the natural gas price depends on the oil price, and Brown and Yücel (2008) found the natural gas price to be anchored in a long-term relationship with crude oil prices but with the short-term dynamics of natural gas prices driven by exogenous factors. After estimating a two-factor model for the crude oil price and one for the natural gas price, we have a total of four time series. In Table 5, we show the correlation between the first differences of the time series. We see that the strongest correlation is between the two long-run time series, and the second strongest between the two short-run time series. In each case, we also have negative correlation between the short-run and long-run processes for both the oil and the natural gas prices. We note that all correlations are significant except the correlation between long-run oil price and short-run gas price.

3.2 Reservoir model To determine the effect from the switching decision, the consequences the switch will have on oil and gas rates need to be estimated as well as the price behavior. Hence, a reservoir model is necessary. There are many ways of modeling a petroleum reservoir, from simple zero-dimensional tank models, which only model the reservoir as a scalar to full compositional models that considers the amount of different hydrocarbons and H_2S in a spatial three-dimensional cell grid system. Wallace, Helgesen, and Nystad (1987) argue that simpler models might be preferable for general studies or for ranking many different project opportunities and that the more complex models are preferable for finding optimal production patterns in specific fields. For our analysis, we have chosen to use a tank model in order to keep the analysis transparent. However, the valuation method is possible to implement employing commercial reservoir simulators, as the only outputs required are production profiles of the different production modes.

3.2.1 Model specifications To estimate the option value of producing natural gas, the production volumes and the time of production are required. In this work, a zero-dimensional model based on the models in Wallace et al. (1987) is modified to include gas injection. The model treats the reservoir as a tank with a uniform fluid and with uniform properties in the whole reservoir. Thus, it does not account for differences in permeability in different areas or local differences in pressure caused by the well flow as the areas surrounding the producing wells empty. It is a simple model that has great computational advantages compared to more complex reservoir models, and it creates the shape of reservoir production profiles of a wide range of petroleum fields (Lund 1999).

Table 6: Reservoir Parameters

$P_{w,0}$	-	Initial reservoir pressure
$P_{w,t}$	-	Reservoir pressure at time t
P_{min}	-	Abandonment pressure
R_0	-	Initial reservoir volume
R_t	-	Reservoir volume at time t
$q_{r,t}$	-	Maximum reservoir depletion rate at time t
q_w	-	Maximum well rate
q_{max}	-	Maximum capacity, or plateau production
$q_{ramp-up,t}$	-	Maximum production during field development
N_t	-	Number of wells producing at time t

The reservoir pressure and volume are related as follows:

$$P_{w,t} = P_{w,0} - \frac{R_0 - R_t}{R_0} * (P_{w,0} - P_{min}) \qquad (6)$$

The reservoir pressure provides the maximum well flow, which decays exponentially with time with continuous production if there are no other constraints on the well flow. The maximum well rate is based on the capacity of the wells installed.

$$q_{r,t} = N_t * q_w * \frac{P_{w,t} - P_{min}}{P_{w,0} - P_{min}} \qquad (7)$$

Together, equations (6) and (7) become the simple equation

$$q_{r,t} = N_t * q_w * \frac{R_t}{R_0} \qquad (8)$$

This is the maximum production from the field given the pressure in the reservoir. It is rarely optimal to construct the production unit so that it can produce at the maximum rate, $q_{r,t}$, because this requires high investment costs. When the platform maximum processing capacity is lower than the potential production from the wells installed at the field, the production profile will have a flat region where the production is equal to the platform maximum capacity. This level is called the plateau production. The optimal plateau level is mainly a function of investment cost, production and required rate of return, since it is a trade-off between investment cost and the ability to get the petroleum quickly out of the ground. There might also be technical reasons to limit the capacity. We have included a ramp-up period of three years, which is similar to the case found in Robinson (2009). During this ramp-up period, we have assumed that the production grows linearly to the capacity maximum over a two year period. The reason for including such a ramp-up period is related to well drilling. It will not be possible to drill all wells at the same time, and connecting the streams to the platform will also require some time. The actual production is the minimum of $q_{r,t}$, q_{max} and $q_{ramp-up,t}$.

3.2.2 Oil and gas production When modeling production of both oil and gas, an important assumption is the ratio between the two fluids in the production. This will be influenced by a large number of factors, where both individual well placement and flow will be important. In a zero-dimensional model, we are not able to capture these effects and, thus, have to make simplifying assumptions. We assume that the composition of the well flow is the same as the reservoir composition, so that:

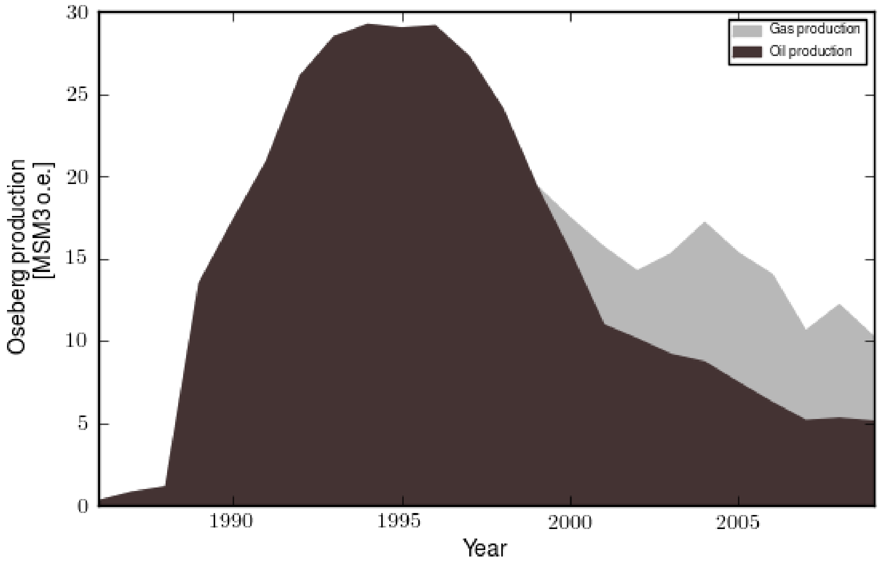


Figure 5: Historical Oseberg oil and gas production

$$q_o = \frac{V_o}{V_o + V_g} * q \tag{9}$$

$$q_g = \frac{V_g}{V_o + V_g} * q \tag{10}$$

V_o and V_g are denominated in $Sm^3 o.e.$ q_o and q_g represents the flow of oil and gas, respectively, denominated in $Sm^3 o.e.$ per year.

3.2.3 The effects of gas production Another complex question related to the joint production of oil and gas is the interactions between the two fluid regions. In the case of the Troll field, the gas stabilizes the oil region so the oil does not move upwards into the gas domain. This makes it possible to extract the oil. Producing the gas too early or too fast can cause large amounts of oil to become impossible to extract[4]. This is one reason to re-inject the gas in the beginning of the field's lifetime. Another reason to inject gas is to push the oil towards the wells. The effect of gas injection is dependent on the reservoir properties of each field, and placement of injecting and producing wells. We have assumed that the oil production drops to zero when the gas is produced to capture the effect of reduced oil production. This might be a fair approximation if the oil layer is thin, where many wells can move below the oil-water contact if this shifts slightly upwards. In fields where gas is mostly used for moving the oil towards the wells this might be a poor approximation, and more complex reservoir models will be necessary. In Figure 5, the production data from the field Oseberg is shown. Gas production was started in 2000, fourteen years after the field started producing oil. The oil production does not drop as quickly as we have modeled it to do, suggesting that in order to get realistic results a more complex reservoir model must be used. For initial analysis and to identify key parameters in the ROV, the proposed model should be sufficient, however.

[4]Norwegian Petroleum Directorate, Petroleum resources on the Norwegian Continental Shelf, 2009, http://www.npd.no/en/

3.2.4 Uncertainty in production Production volumes are uncertain, and they remain uncertain during the entire field lifetime. There are several elements contributing to this uncertainty. One of them is that the operator bases his/her predictions and actions on imperfect information about the reservoir. As the field is producing, some of this uncertainty will be resolved. Another important source of uncertainty is the development of technology (Haugen 1996). The development of new technologies can cause jumps in the extractable reserves, where the effect, cost, and timing of these innovations are uncertain.

Lund (1999) considers the uncertainty related to varying well capacity. The well capacities depend on a number of factors including the location of the water/oil and oil/gas contact and the permeability and rock-fractures near the well. These change over time and can be difficult to predict. Lund (1999) models the well capacity as a simple stochastic function where the well can either have a high or a low oil rate. Each well capacity will be highly random, but with a large number of wells the process resembles a mean-reverting stochastic process. The variance of the field production will be very dependent on the number of wells connected to the field. McCardle and Smith (1998) take a different approach by modeling the decline rate as a geometric Brownian motion process. This might be appropriate when the field is in decline, but it does not take into account the effect of the production capacity limit and it does not clarify which fundamental property that varies. Salomao and Grell (2001) estimate probability functions for the oil volume and recovery factor in order to obtain the distribution of the recoverable oil. They provide numerical probabilities for their test case, where the uncertainty in recoverable volume is 10% of the mean. If we assume that the process follows an arithmetic Brownian motion process, then this leads to annual volatility of 2.2 % for a twenty-year lifetime. We use the probability distribution from Salomao and Grell (2001) and assume that the recoverable resources vary according to equation (11). We have assumed that the expected increase in reserves is zero. This leads to similar results as the model by McCardle and Smith (1998) but includes the possibility of plateau production. To include the possibility of future technological advances, one can include a growth term in the reserve function. We have chosen not to do this due to the difficulty of assessing the expected growth in reservoir producible reserves.

$$dR_t = \sigma dZ \qquad (11)$$

3.3 Least-Squares Monte Carlo Simulation An important decision is the choice of valuation procedure. When analytical solutions are not possible due to option complexity, numerical techniques can provide an answer. There are several numerical methods that can be used to value American options. Of these, the most common are the finite differences approach, lattice-based methods, and simulation methods. Geske and Shastri (1985) compare the binomial model and a finite differences valuation of American calls and puts. They find that the binomial model is more intuitive and easy to implement, while the finite differences method is more efficient. They conclude that a binomial approach might be best for researchers evaluating relatively few options, and the finite differences model might be more suitable for practitioners evaluating a large number of options. The binomial lattice method is developed by Cox, Ross, and Rubinstein (1979). An issue with lattice-based methods is the size of the lattices. Due to the exponential increase in size with the number of stochastic processes, they become computationally expensive to solve for more than one stochastic element (Stentoft 2004a). This is especially an issue in real options valuation that often depends on several stochastic factors.

Simulation procedures based on Monte Carlo simulations provide an alternative to the lattice-based methods. These methods have the advantage of not growing significantly in size and computational demand when increasing the number of stochastic elements compared to alternative methods. They are also considered to be flexible, easy to implement and modify (Boyle, Broadie, and Glasserman 1997). One issue with the Monte Carlo method is that many simulation-paths may be required to obtain robust results, especially in complex problems. Boyle et al. (1997) list several techniques to reduce variance, where the most common are antithetic draws and the control variate method. In pricing American options using Monte Carlo simulation, the main challenge is to analyze the conditional expectation and the value of early exercise. One of the more popular approaches is presented in Longstaff and Schwartz (2001), and is known as Least-Squares Monte Carlo (LSM) simulation. The paper proposes a least-squares regression on a set of basis functions to estimate the continuation value, including only in-the-money paths in the regression. The convergence properties of the LSM method have been examined, and it is found that the algorithm converges to the true price[5]. Moreno and Navas (2003) studied the robustness of the LSM to the number and choice of basis functions. They found that for simple American options, the method produces similar valuation results with different choices of basis functions, but that robustness is not guarantied when applied to more complex options. Stentoft (2004b) shows that the LSM algorithm is more efficient than finite differences and the binomial model when considering options on multiple assets, and that simple monomials are preferable to Laguerre polynomials when choosing basis functions. Cortazar, Gravet, and Urzua (2008) have extended the approach to cover multi-factor risk models more suitable to long-term commodity real options.

The LSM algorithm starts at the last possible exercise time, T, by calculating the exercise value, $V_{n,T} = max(S-K,0)$. The option is exercised if and only if the estimated value of exercising is positive, which provides input into the continuation value regression in the previous period. The value of keeping the option alive is calculated by approximating the continuation value, $F_{n,T-1}$, by $F_{n,T-1} = a_{T-1} * X_{n,T-1}$, where X is the set of basis functions and a_t represent the regression coefficients from Equations 14. The basis function coefficients are found solving the following least squares problem:

$$\vec{X_t} = [X^1_{n,t}, X^2_{n,t}, .., X^j_{n,t}] \tag{12}$$

$$\vec{Y_{t+1}} = \vec{V_{n,t+1}} \tag{13}$$

$$a_t = (\vec{X_t}^T * \vec{X_t})^{-1} \vec{X_t}^T \vec{Y_{t+1}} \tag{14}$$

By only including the realizations where the option exercise is positive in the regression, the accuracy of the algorithm is increased significantly (Longstaff and Schwartz 2001). The value in each node, $V_{n,t}$, is the largest of the exercise value and the discounted continuation value, se Equation 16. A higher exercise value will trigger early exercise in that node for all t less than T.

$$F_{n,t} = a_t * X_{n,t} \tag{15}$$

$$V_{n,t} = max(F_{n,t} * e^{r_f * \Delta T}, S_{n,t} - K) \tag{16}$$

3.4 Risk-neutral pricing Determining an asset's price can be done by assessing the risk and return profile of the asset, and estimating how much investors are willing to pay to own it. The capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) is a well-known example of this class of models. They argue that since individual risk can be hedged by investing in other companies and industries there will be a risk premium only for

[5] See, for example, Clement, Lamberton, and Protter (2002) and Stentoft (2004a).

the systematic risk, i.e. related to the asset's correlation with the market. This model is widely used, but it has a poor empirical track record (Fama and French 2003). Risk-neutral pricing is based on a different principle and avoids having to estimate investors preference towards risk and reward. It is based on the assumption of no arbitrage and prices derivatives by replicating their payoff using other traded securities. Since both the derivative and the portfolio have the same payoff in all states, their prices need to be the same to avoid arbitrage. This treats risk in a consistent manner compared to market prices and avoid biases that can occur otherwise (Laughton, Guerrero, and Lessard 2008). The pricing formula for European options developed by Black and Scholes (1973) is based on this principle, and so is the contingent claims approach as presented by Dixit and Pindyck (1994) for pricing real options.

In our case, the market for crude oil is a well-developed and global market. This allows us to use risk-neutral pricing by treating the oil production as a portfolio of future oil sales. A similar argument is made for the natural gas price risk. The markets for natural gas are to a high degree regional, however, and the price behavior might be different in the different markets. Furthermore, according to Juris (1998), there have been few liquid and mature spot markets for natural gas, and the only well-developed financial gas market is located in the United States. Due to this shortcoming, we have used natural gas price data from Henry Hub, USA, to obtain risk-neutral parameters for the gas price model. The reservoir risk is difficult to price in a risk-neutral framework as there is no traded asset that has the exact risk profile of an individual petroleum reservoir. To price the reservoir risk, we use an argument similar to the CAPM argument based on the assumption that the reserve risk is not correlated with the market. Thus, including reserve risk in the valuation does not change the risk premium of the project.

3.5 Outline of the model The model solves the optimal timing problem of when to stop gas injection and start producing natural gas. There can be significant costs related to switching, e.g., if gas processing equipment or pipe lines need to be built. Once gas production is started, we assume that decision is irreversible. These elements make the decision to switch a ROV candidate. The decision to switch is modeled as the option to buy a derivative worth the risk-neutral expected income from future gas production less the expected income from the lost oil production and the investment cost.

$$V(S,t_0) = max\mathscr{E}[(S_t - K) * e^{-r_f * \Delta T * (t-t_0)}, 0] \quad (17)$$

V_t denotes the option value of switching, S_t the NPV from switching, and K the investment needed to switch. K is in options terminology also known as the strike price. We see from equation (17), that the operator will only switch if it is profitable at that time. He/she must also take into account that the decision to switch is irreversible. In equation (17) $V(S)$ is written as a conditional expectation that is a function of the observed S, the stochastic behavior of S and time. In our models the cost of switching is assumed to be deterministic and not time dependent. This is a simplification as the cost of pipelines are highly dependent on steel prices and laying costs that both are uncertain.

$$S_t = \sum_{i=t}^{i=T} (F_{g,t,i} * E(P_{g,t,i}) - F_{o,t,i} * E(P_{o,t,i})) * e^{-r_f * \Delta T * (i-n)} \quad (18)$$

T denotes the last period. $F_{g,t,i}$ and $F_{o,t,i}$ denotes the forward price at time t of gas and oil delivered at time i, $E(P_{g,t,i})$ and $E(P_{o,t,i})$ the expected production at time t of oil and gas in time i and $e^{-r_f * \Delta T * (i-n)}$ is the risk-neutral discount factor with continuous compounding. K represents the investment needed to switch production. The present value of switching production, S, is the difference in value between oil and gas production based on the knowledge at that time. In each time-step, a forward curve is observed, which combined with the expectation of the oil and gas production generates a risk-neutral expectation of the value from the oil and gas production. This allows us to discount the cash-flows with a risk-free rate of return. The option value is dependent on the stochastic processes that the reservoir production and petroleum prices follow, as these determine the form of the forward curves and the expected production. Hence, the choice of price model and the description of the stochastic behavior impacts the option price. We also compare the results from the ROV to a static net present value (NPV) maximization. The optimal NPV is found by switching at the point giving the highest value:

$$NPV(S_0) = max\mathscr{E}[(S_t - K) * e^{-r_f * \Delta T * t}, 0] \quad (19)$$

The difference between the NPV approach as implemented in equation (19) and the ROV in equation (17) is that the NPV is based on the expected value seen from t=0, while the ROV uses updated conditional expectations at each time step that take into account the development of prices and reservoir conditions. Updating the conditional expectations will for example lead to a earlier switch decision if gas prices are higher than expected and oil prices lower than expected, and vice versa. This is most likely a more accurate description of the decision process of the operator than the static NPV and will, thus, lead to a more accurate valuation of the switching option.

We have implemented the ROV model using the LSM method using monomial polynomials as recommended by Stentoft (2004b). 100 time steps and 100 000 price paths are simulated using antithetic draws, and the continuation value is regressed on monomials up to the second order and cross terms of the five different stochastic factors. We also calculate an optimal static NPV as a benchmark for the option value. The optimal static NPV is found by switching at the time that gives the highest NPV when taking the mean of all simulations.

Long- and short-term oil and gas prices are simulated by Monte Carlo simulations of the two-factor model. We obtain correlated normally distributed increments using a Cholesky decomposition. When the Cholesky decomposition is applied to the correlation matrix and multiplied with a vector of uncorrelated samples, a vector with correlated normal random variables is obtained. The method is described further in Lurie and Goldberg (1998). The simulation is done by drawing four sets of correlated standard normally distributed increments. From the long- and short-term prices, a set of forward prices are produced using equation (4) to form a risk-neutral expectation of future oil and gas prices. The petroleum production is predicted based on the level of production in that node by using equation (11).

4 Case Studies

In this section, we apply the model to data sets based on the fields Norne, Oseberg, and Troll on the Norwegian Continental Shelf. In the three cases, we have assumed a lifetime of the production unit of twenty years, and an annual maximum production of one tenth of the initial total reserves. Furthermore, we model the cost of switching to be equal to the cost of a connecting pipeline, estimated at 200 MUSD. This was the cost of the gas pipeline connecting Norne in 2001[6]. We estimate an initial long-run oil-price of 60 USD per barrel and an initial long-run gas price of 155 USD per $Sm^3 o.e.$. The short-run prices are assumed to be zero at the start of production. As the lifetime of the field is fixed, we are not considering operating costs or initial investment costs as these will be incurred regardless of which decision is made. Hence, both net present values and option values will reflect only the change in income from the switch and not the overall value of the field.

[6]Norwegian Petroleum Directorate, Facts 2009 - The Norwegian petroleum sector, 2009

When it began production in 1997, the Norne field contained about 110 $MSm^3o.e.$. About 85% of this was oil, the rest was natural gas and natural gas liquids. Using parameters from Section 2 and 3.1, we obtain an optimal static NPV of 0, indicating that with a NPV approach switching production would not take place. Valuing the possibility to switch as an option, we obtain a option value of 35.5 MUSD.

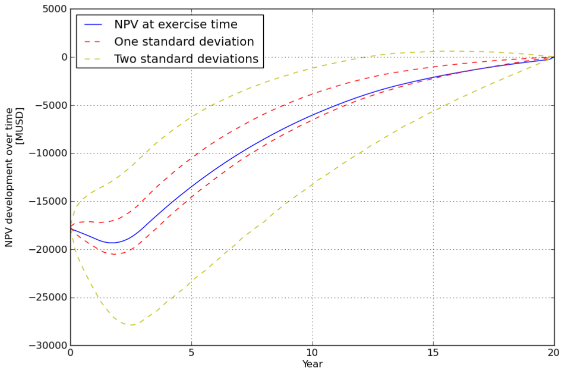
Oseberg is a larger field than Norne, containing 490 $MSm^3o.e.$ extractable reserves when brought on stream. Of these 75% was oil. In this case the static NPV is still 0, indicating the value of the oil lost still outweighs the value of the gas. The option value of switching is 1170 MUSD. If we adjust for the size difference between Oseberg and Norne, we obtain an option value of 225 MUSD. This is significantly higher than the 35 MUSD from the Norne case, and shows that the relative oil and gas amount is an important parameter.

We also consider the case of Troll. Troll is primarily known for its large gas production, but it also have substantial amounts of oil in the reservoir. In the western oil-producing part of the reservoir much of the produced gas is re-injected in order to continue the oil production, while gas production is taking place at the Troll I platform. We simplify by assuming one single production unit. The field initially contained 1600 $MSm^3o.e.$, of which around 15% was oil. Switching to gas-production gives a NPV of 80 billion USD, and an option value of 84 billion USD. We also note that when treating Troll like a single reservoir the optimal time to switch from a NPV point of view is after 6.5 years.

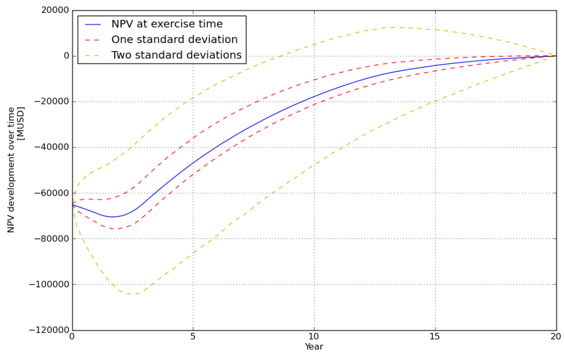
In Figure 6 we present the development of the NPV for the three fields. Troll is the only field where switching is profitable in a static NPV framework. The shape of the NPV-curve is fairly flat around the maximum, indicating that little value is lost if the operator switches at a sub-optimal time. This indicate that simpler, albeit sub-optimal, valuation methods might be a viable alternative. The decision to change production at Oseberg have a significant option value, even though the NPV-approach advices not to switch at any time. Hence, a ROV approach might be warranted when considering this issue in fields where the switch is not very valuable at current prices. Figure 6 also display the standard deviation. The future value of switching is uncertain, supporting the adoption of a ROV approach as new information can have an impact on the optimal timing.

4.1 Sensitivity analysis Many of the parameters in the model are uncertain, and in order to investigate which parameters that have a significant impact on the option value and the NPV we perform a sensitivity analysis. We consider a model field with 200 $MSm^3o.e.$ in reserves, and of which 50% is oil and 50% is gas. The NPV-development of the model field is shown in Figure 7.

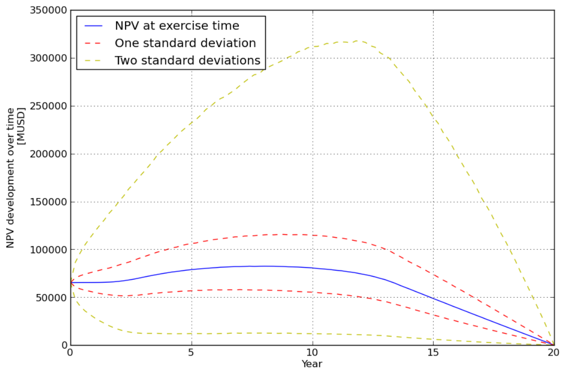
4.1.1 Initial long-term prices An important parameter in our study is the initial oil and gas long-term prices. Since both the oil and gas long-term price is modeled as a Brownian motion, the initial long-term price will have a large impact on both the NPV and the option value. Figure 8a shows the development of the real option value and the optimal static NPV when changing the initial long-term oil price. The option behaves like a put with respect to the oil price, and having the option to switch can be seen as insurance against falling oil prices. We observe that the value of switching is larger at all times than the optimal static rule. From this we conclude that arranging to adjust development plans when facing changing prices is a better alternative than the best static policy found when starting extraction. The long-term gas price is also expected to have a significant impact on the value of switching. In Figure 8b the NPV and option value is shown with varying initial gas price. The option acts as a call with respect to the gas price, increasing



(a) Norne



(b) Oseberg



(c) Troll

Figure 6: Value of switching when varying the time of switching while considering the cost of switching, the value from the gain in gas production and the lost income from the forfeited oil

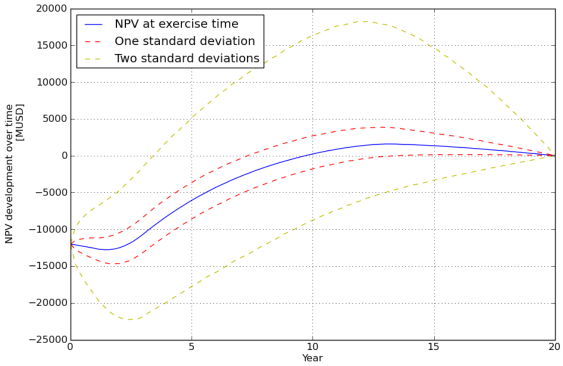
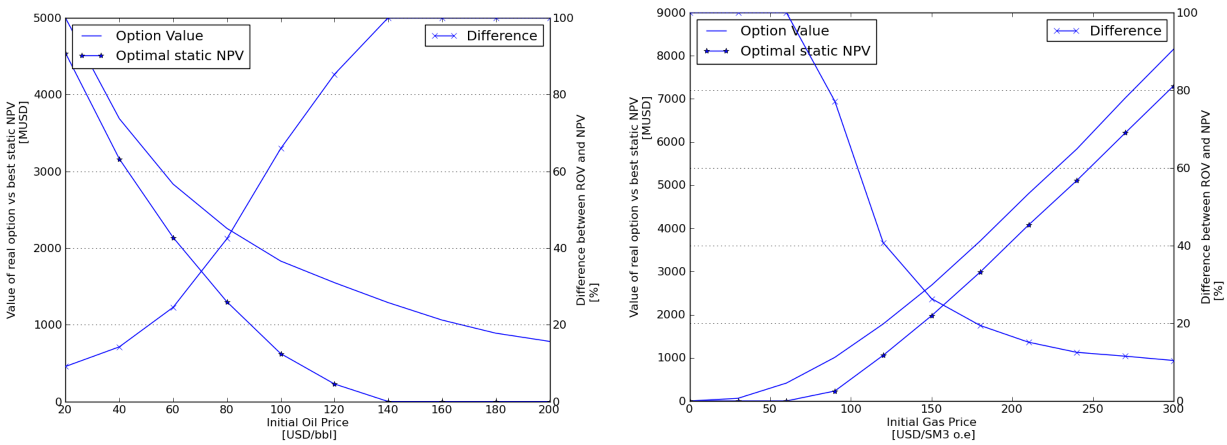


Figure 7: NPV development of the decision to switch considering the model field

in value as the gas price rise.



(a) Initial oil price

(b) Initial gas price

Figure 8: Value sensitivity to initial long-term prices

4.1.2 Investment cost In the base case the investment cost taken into account is the cost of the pipeline. This is a valid approximation, given that the production unit already has gas processing equipment and the platform has not already been connected to a gas grid. The unit will need to separate the gas from the oil when re-injecting the gas, so the gas processing equipment will in most cases be built before starting production. The gas pipe line might be built as the platform starts producing, leading to a very low switching cost. In any case the cost of connecting the field will be very dependent on the distance to existing infrastructure. We have investigated the effect of the investment cost on the option value and static NPV. As expected, the NPV decreases linearly with the added switching cost. The option value does not have a linear decrease, and is less sensitive to the switching cost. This implies that it is worthwhile to consider the possibility of extracting natural gas from fields currently too far from existing infrastructure, since increased gas prices, new infrastructure in surrounding areas or falling steel prices can make switching attractive.

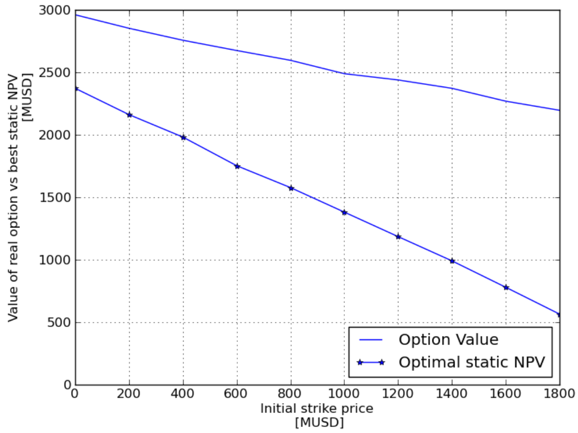


Figure 9: Value sensitivity to investment cost

4.1.3 Long-term volatility An important characteristic of price models like the geometric Brownian motion and the Ornstein-Uhlenbeck process is that the expected value of the price is not a function of the variance. In the two-factor model by Schwartz and Smith (2000) the expected value of the logarithm of the long-term price is $\xi + \mu t$, and will not vary with changing volatility. The real long-term price will however be affected by the volatility. This can be explained by Ito's lemma in equation (20). The long-term price can be written as e^{ξ} and ξ follows $d\xi = (\mu - \lambda)dt + \sigma dZ^*$ where dZ^* is increments of a standard Brownian motion. Ito's lemma gives us:

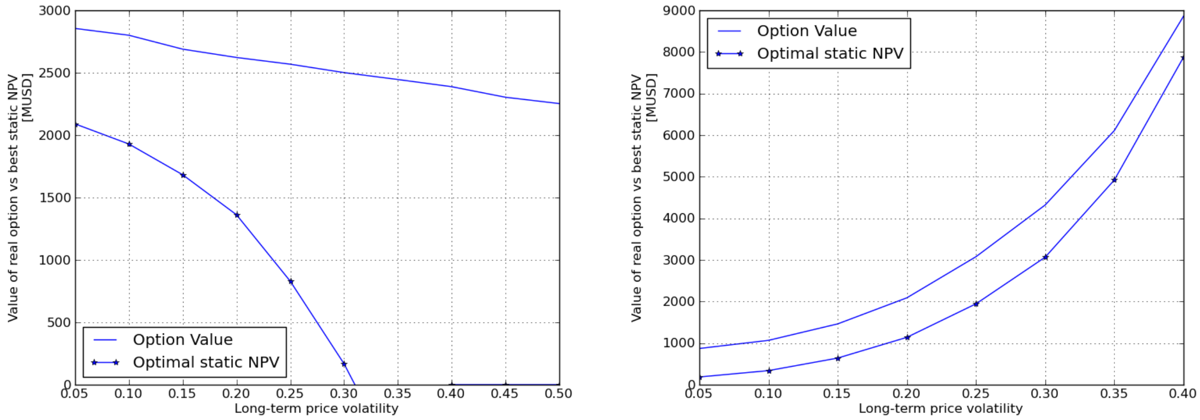
$$de^{\xi} = \frac{\delta e^{\xi}}{\delta \xi} * d\xi + \frac{1}{2}\frac{\delta^2 e^{\xi}}{(\delta \xi)^2} * (d\xi)^2 + \frac{\delta e^{\xi}}{\delta t} * dt \qquad (20)$$

$$de^{\xi} = e^{\xi}((\mu + \frac{1}{2}\sigma_{\xi}^2)dt + \sigma_{\xi} dZ^*) \qquad (21)$$

As the volatility of the log-price increases, the expected increase with time of the long-term price will increase with a factor of $\frac{1}{2}\sigma$. This is the reason for the negative impact on the NPV from the increased volatility in the log-price of oil, and the positive impact from the gas volatility seen in Figure 10.

4.1.4 Short-term volatility Option values are known to increase with volatility, but this is not necessarily true for the short-term volatility when considering long-term decisions. It will be difficult to take advantage of positive short-term price shocks, as it will quickly revert back to normal levels. The sensitivity to short-term price volatiltiy is seen in Figure 11. As illustrated, the sensitivity of both the option value and for the NPV to the short-term price volatility parameter is very low. An increase in the oil price volatility has a slight tendency to decrease the value, and similarly, an increase in the gas price volatility tend to increase value. This is explained by a similar argument as in Section 4.1.3. We have calculated the increment in the short-term price, and we note that increased σ_{χ} causes an increase in the mean-reversion level of short-term price. Unlike the increase in the long-term drift, the mean-reversion level does not compound and the effect from the increased volatility should thus be smaller. This also explain the slight negative slope from an increase in the short-term volatility of oil in Figure 10a, and the slight positive slope in Figure 10b.

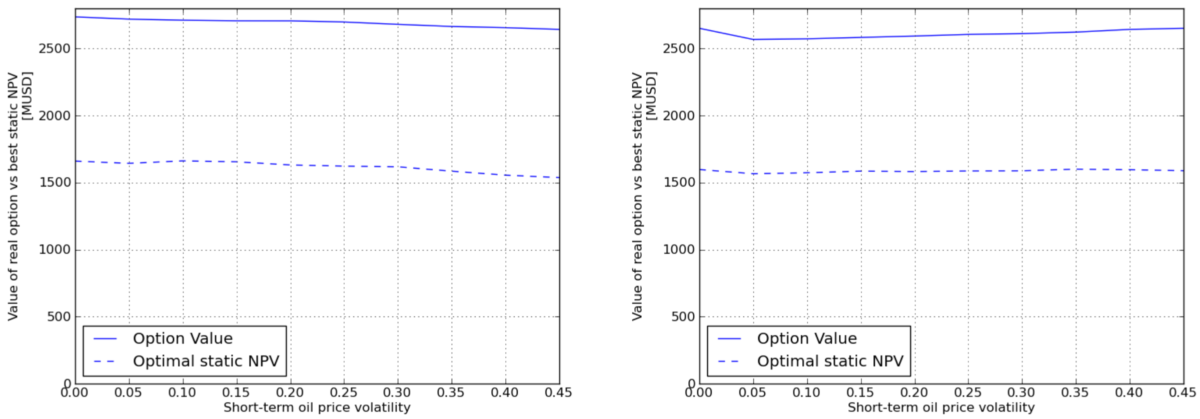
$$de^{\chi} = e^{\chi}((-\kappa\chi - \lambda + \frac{1}{2}\sigma_{\chi}^2)dt + \sigma_{\chi} dZ^*) \qquad (22)$$



(a) Oil

(b) Gas

Figure 10: Value sensitivity to long-term price volatility



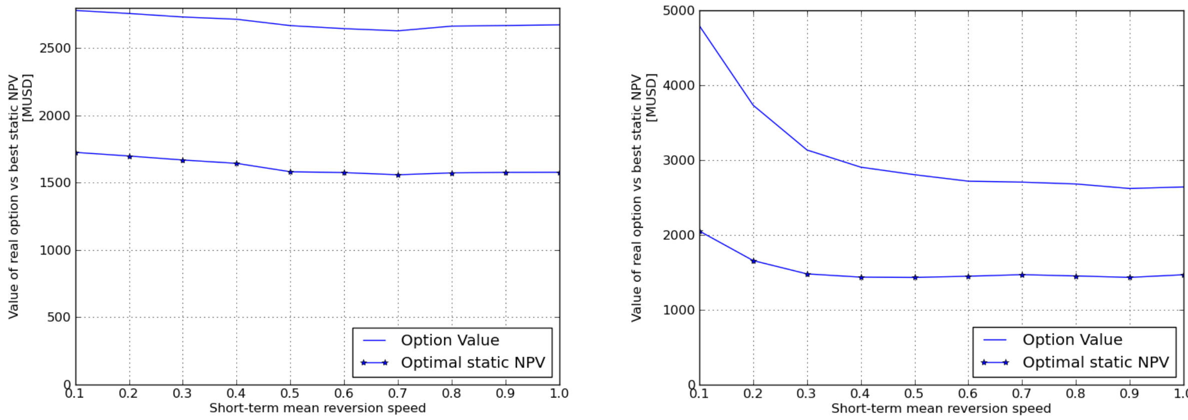
(a) Oil

(b) Gas

Figure 11: Value sensitivity to short-term price volatility

4.1.5 Short-term mean-reversion speed The mean-reversion speed is expected to have an impact on the option value. A slower mean-reversion speed should increase the volatility, as short-term shocks diminish more slowly. This should in turn increase the option price. Figure 12 shows that when the gas mean-reversion speed becomes very low, the option value increases. The short-term volatility is much higher for the gas price compared to the oil price, and hence the effect from the lower mean reversion speed is greater in the gas price case. However, in the relevant area around to the modeled parameters of oil and gas at 0.8 and 7.0 the sensitivity is very low. Slight changes in the mean-reversion speed over time should thus not make an impact on the switch-decision.

4.1.6 Short-term risk premium In the estimation procedure in Section 3.1 estimating the short-term risk premium proved to be challenging and important, as it determined the mean level of the short-term price. It is interesting to see how a mistake in the short-term risk premium influence the option value, since there is some uncertainty around the



(a) Oil

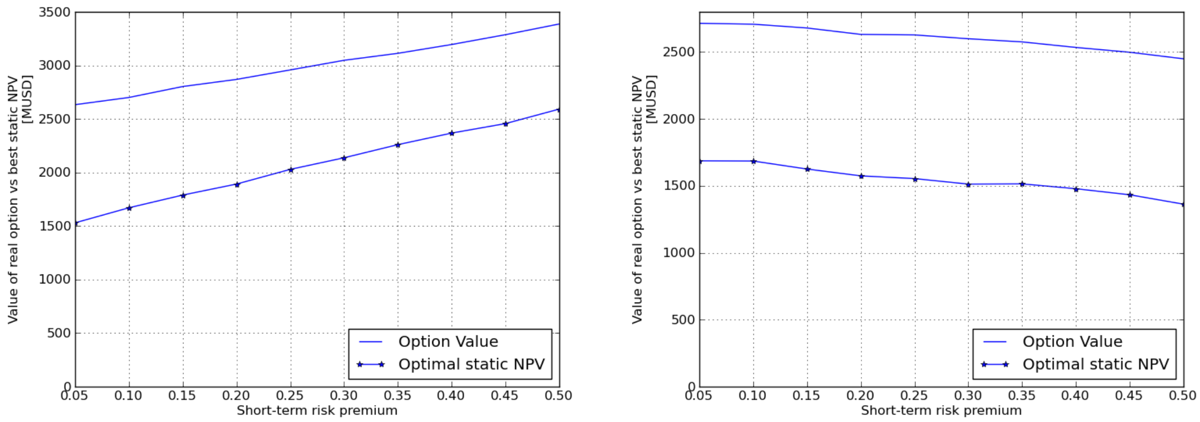
(b) Gas

Figure 12: Value sensitivity to short-term mean-reversion

true value of the parameter. An increase in short-term risk-premium will reduce the mean level of the short term price, explained by equation (22). This explains why an increase in the short-term risk premium of oil (gas) has a positive (negative) impact on the value of switching. Figure 13 show the option value and NPV as a function of the short-term risk premium of oil and gas. The value is more sensitive to the risk premium of oil than that of gas, attributable to the lower mean-reversion speed of oil. The log of the short-term mean price revert to $-\frac{\lambda}{\kappa}$, causing the fast-reverting short-term gas price to be less sensitive to the short-term risk premium. The short-term risk premium also seem to affect the NPV more than it affects the option value in both cases. Decreasing oil prices make the decision to produce the gas more attractive. The reduced uncertainty concerning whether the gas can be produced profitably should reduce the difference between the two valuation methods. Similarly, as the gas price decrease the uncertainty increase, amplifying the difference between the two approaches.

4.1.7 Sensitivity with respect to oil-percentage The amount of oil initially in the reservoir is likely to have a large impact on the decision to switch. For a low oil-percentage, we expect the switch to be more valuable and to occur earlier. This is confirmed in Figure 14, with a highly valuable switch when the oil-percentage is low. The value of the switch is very sensitive to the oil content in the field. We also note that the option to switch has value even in cases where the NPV of switching is zero. The advantage of the ROV-approach is higher in fields with a low gas content, when the NPV of switching is close or equal to zero and the ROV-approach still has a value greater than zero.

4.1.8 Reservoir risk As illustrated in Figure 15 the reservoir-risk seem to have little effect on the option value. The reservoir risk is small compared to other sources of risk, and the small effect on the option value indicate that reservoir risk could be left out of the formulation. If reservoir-risk can be disregarded, several benefits can be realized. First, leaving out reservoir risk let us simulate only one reservoir-simulation compared to 100 000 simulations needed in a Monte Carlo simulation. The run-time can be significantly reduced by this as the reservoir model is one of the most time-consuming parts of the algorithm. Second, as fewer simulations are needed, more complex and more realistic simulation models can be utilized.



(a) Oil

(b) Gas

Figure 13: Value sensitivity to short-term risk premimum

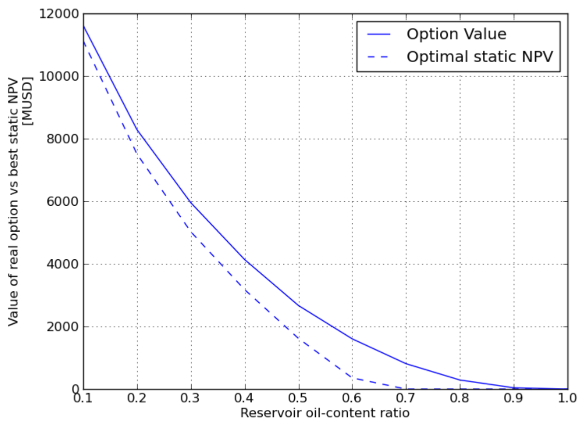


Figure 14: Value sensitivity to reservoir oil-content ratio

5 Conclusion

In this paper, we investigate the flexibility related to the option to switch from oil production to gas production in a petroleum reservoir containing both hydrocarbons. We find that option valuation is superior to a static NPV approach for such real options and that taking into account the development of prices and reservoir behavior is important for optimal management of a petroleum field. The main source of risk that influence the switch value is found to be the long-term oil and gas prices. The short-term component of the price model seem to have little effect on the switch decision. This can be explained by the fact that the decision to switch is a long-term decision, and short-term changes will not affect the decision to a large degree. A simpler one-factor long-term price model might thus be a sufficient alternative when considering this class of strategic decisions. Reservoir uncertainty as formulated in this paper have a minor impact on the option value, and could be left out of the formulation.

For further work, we recommend incorporating a more realistic reservoir model. The reservoir model used in this

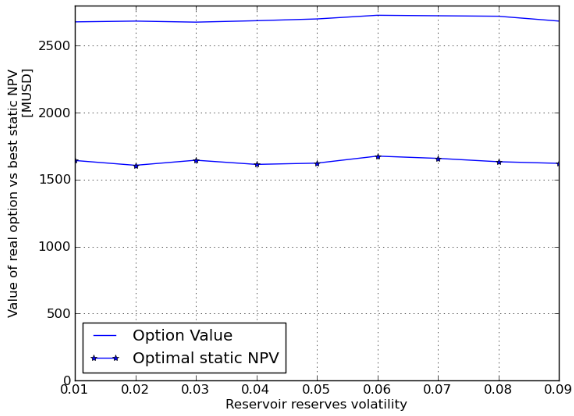


Figure 15: Value sensitivity to reservoir risk

paper is a simple model that captures the main features of petroleum production profiles, but it does not capture the complex interactions related to joint oil and gas production. Uncertainty in the development of the well flow is shown to have little impact, and the production can be modeled as deterministic. Another interesting addition would be to include a stochastic investment cost, which will likely be important in areas where large infrastructure investments are needed.

References

J. E. Bickel and R. B. Bratvold. From uncertainty quantification to decision making in the oil and gas industry. *Energy Exploration & Exploitation*, 26(5):311–325, 2008.

F. Black and M. S. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–54, 1973.

P. Boyle, M. Broadie, and P. Glasserman. Monte Carlo methods for security pricing. *Journal of Economic Dynamics and Control*, 21(8-9):1267 – 1321, 1997.

S. P. A. Brown. Natural gas pricing: Do oil prices still matter? *Southwest Economy*, 4, 2005.

S. P. A. Brown and M. K. Yücel. What drives natural gas prices? *The Energy Journal*, 29(2), 2008.

R. F. Bruner, K. M. Eades, R. Harris, and R. C. Higgins. Best practices in estimating the cost of capital: Survey and synthesis. *Financial Management*, 27:13–28, 1998.

A. Cartea and T. Williams. UK gas markets: The market price of risk and applications to multiple interruptible supply contracts. *Energy Economics*, 30(3):829, 2008.

L.G. Chorn and S. Shokhor. Real options for risk management in petroleum development investments. *Energy Economics*, 28(4): 489 – 505, 2006.

E. Clement, D. Lamberton, and P. Protter. An analysis of a least squares regression method for American option pricing. *Finance and Stochastics*, 6:449–471, 2002.

G. Cortazar and E. S. Schwartz. Monte Carlo evaluation model of an undeveloped oil field. *Journal of Energy Finance & Development*, 3(1):73–84, 1998.

G. Cortazar, M. Gravet, and J Urzua. The valuation of multidimensional American real options using the LSM simulation method. *Computers & Operations Research*, 35(1):113 – 129, 2008.

J. C. Cox, S. A. Ross, and M. Rubinstein. Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3):229–263, 1979.

M. A. G. Dias. Valuation of exploration and production assets: An overview of real options models. *Journal of Petroleum Science and Engineering*, 44(1-2):93 – 114, 2004.

M. A. G. Dias, J. G. L. Lazo, M. A. C. Pacheco, and M. M. B. R. Vellasco. Real option decision rules for oil field development under market uncertainty using genetic algorithms and Monte Carlo simulation. *7th Annual Real Options Conference, Washington DC*, 2003.

D. A. Dickey and W. A. Fuller. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association*, 74(366):427–431, 1979.

A. K. Dixit and R. S. Pindyck. *Investment under uncertainty*. Princeton University Press, 1994.

S. Ekern. An option pricing approach to evaluating petroleum projects. *Energy Economics*, 10(2):91–99, 1988.

E. F. Fama and K. R. French. The capital asset pricing model: Theory and evidence. CRSP Working Paper No. 550; Tuck Business School Working Paper No. 03-26, 2003.

M. M. Foss. Global natural gas issues and challenges: A commentary. *The Energy Journal*, 26(2):111–128, 2005.

R. Geske and K. Shastri. Valuation by approximation: A comparison of alternative option valuation techniques. *The Journal of Financial and Quantitative Analysis*, 20(1):45–71, 1985.

K. K. Haugen. A stochastic dynamic programming model for scheduling of offshore petroleum fields with resource uncertainty. *European Journal of Operational Research*, 88(1):88 – 100, 1996.

A. Juris. The emergence of markets in the natural gas industry. World Bank Policy Research Working Paper No. 1895, 1998.

T. Koller, M. Goedhart, and D. Wessels. *Valuation: measuring and managing the value of companies*. McKinsey & Co, Wiley, 2005.

D. Kwiatkowski, P. C. B. Phillips, P. Schmidt, and Y. Shin. Testing the null hypothesis of stationarity against the alternative of a unit root : How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54(1-3):159–178, 1992.

D. Laughton, R. Guerrero, and D. R. Lessard. Real asset valuation: A back-to-basics approach. *Journal of Applied Corporate Finance*, 20(2):46 – 65, 2008.

J. Lintner. The valuation of risk assets and selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*, 47:13–37, 1965.

F. A. Longstaff and E. S. Schwartz. Valuing American options by simulation: A simple least-squares approach. *Review of Financial Studies*, 14(1):113–147, 2001.

J. J. Lucia and E. S. Schwartz. Electricity prices and power derivatives: Evidence from the Nordic power exchange. *Review of Derivatives Research*, (5):5–50, 2002.

M. W. Lund. Real options in offshore oil field development projects. *3rd Annual Real Options Conference, Leiden*, 1999.

P. M. Lurie and M. S. Goldberg. An approximate method for sampling correlated random variables from partially-specified distributions. *Management Science*, 44(2):203–218, 1998.

K. F. McCardle and J. E. Smith. Valuing oil properties: Integrating option pricing and decision analysis approaches. *Operations Research*, 46(2):198 – 217, 1998.

M. Moreno and J. R. Navas. On the robustness of least-squares Monte Carlo (LSM) for pricing American derivatives. *Review of Derivatives Research*, 6(2):107–128, 2003.

The Norwegian Petroleum Directorate. Petroleum resources on the Norwegian Continental Shelf, 2009.

R. S. Pindyck. The long-run evolution of energy prices. *The Energy Journal*, 20(2):1–27, 1999.

R. Robinson. The economic impact of early production planning (EPP) on offshore frontier developments. *Offshore Mediterranean Conference and Exhibition, Ravenna*, March 2009.

M. C. Salomao and A. P. Grell. Uncertainty in production profiles on the basis of geostatistic characterization and flow simulation. *SPE Latin American and Caribbean Petroleum Engineering Conference, Buenos Aires*, 2001. SPE 69477.

E. S. Schwartz and J. E. Smith. The short-term variations and long-term dynamics in commodity prices. *Management Science*, 46 (7):893–911, July 2000.

W. F. Sharpe. Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3):425–442, 1964.

D. R. Siegel, J. L. Smith, and J. L. Paddock. Valuing offshore oil with option pricing models. *Midland Corporate Finance Journal*, 5:22–30, 1988.

L. Stentoft. Convergence of the Least Squares Monte Carlo approach to American option valuation. *Management Science*, 50(9): 1193–1203, 2004a.

L. Stentoft. Assessing the Least Squares Monte-Carlo approach to American option valuation. *Review of Derivatives Research*, 7 (2):129–168, 2004b.

S. B. Suslick and D. J. Schiozer. Risk analysis applied to petroleum exploration and production: An overview. *Journal of Petroleum Science and Engineering*, 44(1-2):1 – 9, 2004.

D.J. Szabo and K.O. Meyers. Prudhoe bay: Development history and future potential. *SPE Western Regional Meeting, 26-28 May 1993*, 1993.

J. A. Villar and F. L. Joutz. The relationship between crude oil and natural gas prices. Technical report, Energy Information Administration, October 2006.

A. Wachter and L. T. Biegler. On the implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, 2006.

S. W. Wallace, C. Helgesen, and A. N. Nystad. Generating production profiles for an oil-field. *Mathematical Modelling*, 8:681 – 686, 1987.