Strategic investment under uncertainty: a synthesis

Benoît Chevalier-Roignant[1,], Christoph M. Flath[1], Arnd Huchzermeier[1], Lenos Trigeorgis[1]

[a] *WHU-Otto Beisheim School of Management, Chair of Production Management, Burgplatz 2, 56179 Vallendar, Germany*
[b] *University of Cyprus, Department of Public and Business Administration 9-11 Larnakas Avenue, 1678 Nicosia, Cyprus*

Abstract

Research contributions providing insights at the intersection of real options analysis and industrial organization have become numerous in the recent decade. In the present paper, we provide an overview aimed at categorizing and relating these research streams. We highlight managerial insights about the type of competitive reaction, the manner in which information is revealed, the nature of the competitive advantage (first vs. second-mover advantage), firm heterogeneity, capital divisibility, and the number of competing firms.
Keywords: Finance, Investment under uncertainty, Real options, Strategic investment, Option games

[*]Corresponding author

Email addresses: benoit.chevalier-roignant@whu.edu (Benoît Chevalier-Roignant), christoph.flath@whu.edu (Christoph M. Flath), ah@whu.edu (Arnd Huchzermeier), lenos@ucy.ac.cy (Lenos Trigeorgis)

Strategic investment under uncertainty: a synthesis

Abstract

Research contributions providing insights at the intersection of real options analysis and industrial organization have become numerous in the recent decade. In the present paper, we provide an overview aimed at categorizing and relating these research streams. We highlight managerial insights about the type of competitive reaction, the manner in which information is revealed, the nature of the competitive advantage (first vs. second-mover advantage), firm heterogeneity, capital divisibility, and the number of competing firms.
Keywords: Finance, Investment under uncertainty, Real options, Strategic investment, Option games

1. Introduction

Research in corporate finance intends to equip managers with quantitative tools useful in assessing investment projects. Brennan and Trigeorgis (2000b) distinguish three stages in the development of valuation models:

1. In *static* models, an investment project is completely described by a specified stream of expected cash flows with no managerial flexibility.
2. In *dynamic* models, projects can be actively managed in response to the resolution of exogenous uncertainty. This approach is embedded in decision-tree analysis, dynamic programming, and real options analysis (ROA).
3. In *real-options game-theoretic models*, firms can presumably condition their decision-making not only on the resolution of exogenous uncertainties but also on the (re)actions of outside parties (e.g., competitors). Future cash flows can be understood as the outcome/ payoffs of a game involving several decision makers and "nature".

The third approach - commonly called option games - was first adopted in the early 1990s by Smets (1991) and Smit and Ankum (1993) and has gained continuous attention in academia since. This paper provides an overview, stressing the new insights from a selection of articles.[1] We classify these research contributions and articulate this paper as follows. Section 2 deals with lumpy investment decisions, e.g., whether to enter a market or not. Section 3 addresses capacity-expansion models where it is recognized that firms may decide on the size of their investment (e.g., capacity choice). Section 4 deals with staged investment problems where an early decision may alter ex post incentive or action alternatives. Section 5 sums up the key managerial insights and outlines future directions of research.

2. Lumpy investment decisions - Entry, improvement and exit

2.1. Exogenous competition and random entry

An early approach modeled competition exogenously and helped identify certain major drivers. Trigeorgis (1991) studies the impact of competition on investment timing using standard ROA based on the geometric Brownian motion (GBM)

$$\mathrm{d}X_t = \mu X_t \,\mathrm{d}t + \sigma X_t \,\mathrm{d}B_t \tag{1}$$

with drift parameter μ, volatility σ and $B = \{B_t\}_{t\geq 0}$ a standard Brownian motion.[2] "Competitive arrivals" may reduce the value of a firm's own investment opportunity by taking away significant market share. Competition in this context can be modeled in one of two ways, depending on whether competitive entry is anticipated or random. The former case is analogous to an increased constant opportunity cost of waiting, while the latter reflects the risk of a sudden drop in profitability (Poisson arrival). Both interpretations suggest earlier investment. This framework is limited in that it does not explain what drives competitors' entry decisions. An endogenous, game-theoretic approach explaining the incentives of firms to enter is more appropriate.

2.2. Discrete-time analysis of new market models

Following Smit and Ankum (1993), option games have often been modeled in discrete time. This approach provides an intuitive introduction to some key insights by use of numerical examples.

[1] Grenadier (2000b) and Huisman (2001) introduce the subject in continuous time, while Smit and Trigeorgis (2004) mainly deal with discrete-time models. Chevalier-Roignant and Trigeorgis (2010) provide an overview over both modeling approaches. Boyer et al. (2004a) review the literature but have a broader sampling than we do, and do not stress the commonalities behind the models. Huisman, Kort, Pawlina, and Thijssen (2004) are restrictive on their scope, focusing on lumpy problems in continuous time.

[2] Throughout, we assume risk-neutrality of the agent. Alternatively, following Cox and Ross (1976) and Harrison and Kreps (1979), we could employ the risk-neutral valuation approach.

2.2.1. Complete-information case

Smit and Ankum (1993) examine a model where two firms share an investment option allowing them to enter the market for a (fixed) investment outlay of I. Market evolution and firms' decisions are concurrently considered by use of a binomial lattice for the market value $X = \{X_t\}_{t\geq 0}$ and a strategic-form game at each stage. If a firm enters first (as "leader"), it grasps a higher market share in case the second firm subsequently enters. Unless one of the firms grasps this first-mover advantage, firms are assumed to be identical, receiving half of the market under simultaneous investment. The option value at any time is determined based on expectation about future market developments. In the symmetric case considered by the authors, the firms optimally choose to defer at the outset. At the next period, they invest simultaneously after an up-move but defer the investment after a down-move. Simultaneous immediate investment is not Pareto-optimal as both firms would be better off if they jointly deferred the investment. For the asymmetric case considered, firm i benefits not only from a payoff advantage but also from a strategic effect: firm i preempts firm j in the up state and secures a leader position.

2.2.2. Incomplete-information case

The previous model considered that the duopolists have perfect information regarding their rivals, e.g., know their production cost. This assumption is relaxed by Zhu and Weyant (2003a,b). Two firms face stochastic linear inverse demand

$$P(X_t, Q) = X_t - bQ \tag{2}$$

where $Q = q_i + q_j$ is the total industry output, $b > 0$, and $X = \{X_t\}_{t\geq 0}$ is an additive stochastic shock (demand intercept). Firms face linear costs, $C_i(q_i) = c_i q_i$. Firm j has perfect information about the game, whereas firm i knows its own cost c_i but not its rival's, believing it to be c_H with probability θ or c_L otherwise, or $\bar{c}_j$ in expectation. If the firms invest simultaneously, firm j optimally sets output $q_j^*(c_j) = \frac{1}{3b}(X_t - 2c_j + c_i)$ in knowledge of both costs. Firm i forms expectations about its rival's quantity, selecting Cournot quantity $q_i^* = \frac{1}{3b}(X_t - 2c_i + \bar{c}_j)$. For sequential investment decisions, the effect of incomplete information crucially depends on the order of the investment decisions. If the best-informed firm j invests first, it may reveal its private cost, c_L or c_H through its quantity choice, affecting the quantity decision of firm i. If the less-informed party, firm

i, moves first, no new information is revealed to firm j. The analysis in Zhu and Weyant (2003a) focuses on single-stage decision making, while Zhu and Weyant (2003b) consider multiple time steps.

2.3. Continuous-time analysis of new market models

While the discrete-time modeling approach is readily implementable, continuous-time models generally provide more clear-cut economic interpretations. We discuss below such models under the premise that no firm is active at the outset.

2.3.1. Complete-information case

Dixit and Pindyck (1994) propose a continuous-time symmetric-duopoly model, simplifying the original work by Smets (1991). An investment opportunity is available to two firms i and j with each incurring a fixed (sunk) investment cost I upon exercise. The stochastic profit function is made up of a deterministic reduced-form profit, denoted π_L in monopoly or π_C in duopoly ($\pi_L > \pi_C$), and a multiplicative shock X following a GBM as per eq. (1) that starts at level X_0 a.s. The common discount factor is r. As profit flows depend on the firms' entry decisions the situation is analogous to a multi-player optimal stopping problem. In case of sequential investment the follower is faced with a single-agent optimal exercise problem. Using traditional optimization tools, the follower's entry threshold X_F is obtained such that $\frac{V_F(X_F)}{I} = \Pi^*$ where $V_F(x) \equiv x\pi_C/\delta$, $\delta \equiv r - \mu$ and the measure

$$\Pi^* \equiv \frac{\beta_1}{\beta_1 - 1} \tag{3}$$

can be interpreted as a required profitability level (profitability index), with

$$\beta_1 = \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}}{\sigma^2} \quad (> 1)\,. \tag{4}$$

The follower's (time-0)-value $F(\cdot)$, as a function of the initial demand level, is

$$F(X_0) = \begin{cases} V_F(X_0) - I & \text{if } X_0 \geq X_F \\ \mathbb{E}_0\left[e^{-k\tau_F}\right]\left[V_F(X_F) - I\right] & \text{if } X_0 < X_F, \end{cases} \tag{5}$$

where $\mathbb{E}_0\left[e^{-k\tau_F}\right] = (X_0/X_F)^{\beta_1}$ is an expected discount factor and $\tau_F = \inf\{t \in \mathbb{R}_+ | X_t \geq X_F\}$ is the first time the barrier X_F is reached from below. After entering the market the leader earns monopoly rents $X_t\pi_L$ as long as the demand shock stays below X_F. After the follower's entry, the firms will form a duopoly; each earn $X_t\pi_C$. Hence, the value of the leader can be expressed as

$$L(X_0) = \begin{cases} V_F(X_0) - I & \text{if } X_0 \geq X_F \\ V_L(X_0) - I + \mathbb{E}_0\left[e^{-k\tau_F}\right]\left[V_F(X_F) - V_L(X_F)\right] & \text{if } X_0 < X_F, \end{cases} \tag{6}$$

with $V_L(x) \equiv x\pi_L/\delta$. Upon the follower's arrival at time τ_F, the leader "exchanges" its perpetuity value as leader $V_L(X_F)$ for the perpetuity value as a duopolist $V_F(X_F)$. Figure 1 depicts the value as leader and follower depending on the state regions considered. At X_P, firms are indifferent between the role and the follower roles.[3]

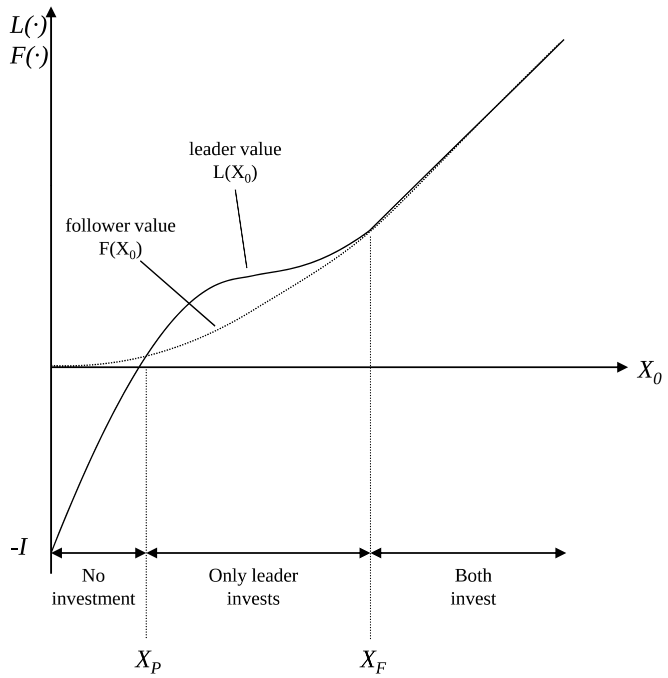

Figure 1: Values as leader and follower in duopoly (Dixit and Pindyck 1994)

For low X_0 ($X_0 < X_P$) the value of the leader is lower than the follower's; no entry will occur in this demand region. In (X_P, X_F), there exists an incentive to invest as leader, since the leader's value exceeds the follower's. This leads to each firm planning to invest an ϵ-increment earlier than its rival, continually dissipating away the first-mover advantage $L(\cdot) > F(\cdot)$. This corresponds to the rent-equalization principle explained by Fudenberg and Tirole (1985) in a deterministic setting. In this region, (at least) one firm invests,[4] while finally for large values $X_0 > X_F$ both firms are operating in the market. Option premia are positive for both firms but the threat of preemption will cause the

[3] Subscript P stands for "preemption".

[4] As shown by Huisman and Kort (1999), an undesired simultaneous investment outcome ("mistake") may result from strategic interaction with positive probability if $X_0 \in (X_P, X_F)$.

leader to invest earlier than in the monopoly case.

Bouis et al. (2009) extend the previous duopoly model by considering a larger number of symmetric firms having the option to (irreversibly) enter a new market. A multiplicative shock X following the GBM of eq. (1) affects the profit streams, where the deterministic profit components π_n are decreasing in the number of incumbents n. To derive their main insights, the authors focus on the three-firm case and provide numerical analysis for larger oligopolies. Two types of investment sequences may arise: sequential equilibria where firms invest at three distinct moments and simultaneous investment where the first two firms invest simultaneously. The authors show that the partitioning of the two equilibria hinges on the duopoly rent π_2. If volatility is high, the likelihood of sequential investment is increased. Investment thresholds in case of sequential equilibria are derived analogously to the above. The investment trigger of the first and the second entrant, X_1 and X_2, are determined by "rent equalization". If the triopoly profit is reduced (and X_3 rises), the second investor has an incentive to invest earlier because it can enjoy duopoly rents longer, thus leading to a decrease in X_2. The first investor then faces earlier entry by the follower and enjoys monopoly profits for a shorter period; its entry threshold is increased. The opposite directions of the change in the wedges between X_1 and X_2 on the one hand, and X_2 and X_3 on the other hand is referred to as the *accordion effect*. In addition, the leader in the three-firm case invests after the leader in duopoly but still before the monopolist. Therefore, increased competition (three firms rather than two) actually delays rather than hastens investment.[5]

Weeds (2002) analyzes a patent race among duopolists and the effect of competitive pressure on firms' research activities. Two (symmetric) firms have the opportunity to launch a R&D project for a cost of I. The first firm to succeed gains an exclusive patent, while the other firm is left with nothing. Firms face two sources of uncertainty: the patent value evolves as the GBM of eq. (1) and the research outcome is random, with mean Poisson arrival rate λ.[6] A firm's R&D strategy implies a profit trigger at which research activity is initiated. The follower's investment threshold X_F is such that

$$\frac{X_F}{I} = \Pi^* \frac{r + 2\lambda - \mu}{\lambda},$$

[5]For larger oligopolies, Bouis et al. argue that the number of expected future entrants is critical (especially whether this number is odd or even) and illustrate that the accordion effect sustains.

[6]Weeds (2002) assumes that the project value at time 0 is below I. By so doing, simultaneous investment is ruled out as equilibrium, as discussed in Huisman and Kort (1999).

while the leader's threshold X_P is obtained from rent equalization. Weeds also derives the optimal behavior imposed by a social planner in two related settings: A) Two decentralized research units have identical launch cost I and rate λ; and B) One common large research institute has doubled launch cost $2I$ and chance of success 2λ. In case A, the social planner would optimally choose to phase research: one firm starts conducting research when the patent value reaches a threshold $X_L \in (X_P, X_F)$; the other firm initiates research later, when the threshold X_F' is first attained. In case B the large research institute commences research activities - at threshold $X_C \in (X_L, X_F')$ - later than a follower would do in a competitive setting. This later result challenges standard antitrust thinking in favor of joint research ventures.

In Mason and Weeds (2009) a firm considering to be the leader might be hurt by the follower's entry or benefit from it (negative vs. positive externality). The type of externalities impacts the investment schedule. The presence of negative externalities can hasten investment compared to the monopoly benchmark (preemption). Two patterns of adoption emerge: sequential vs. simultaneous investment. With no first-mover advantage and no preemption, the leader adopts the new technology at the cooperative trigger point; otherwise, a preemptive sequential investment occurs where the follower adopts earlier than the cooperative solution.

2.3.2. Incomplete-information case

Lambrecht and Perraudin (2003) consider the effect of incomplete information on investment policies. No firm knows the exact realization of its rival's investment cost but has some prior about it in the form of a distribution function $G(I_j)$. As part of a Bayesian equilibrium, there exists a map from firm i's investment cost I_i to its investment threshold. Since firms are ex ante symmetric, the exercise strategies involve a map from the distribution $G(\cdot)$ into a distribution of the rival's entry trigger $F_j(\cdot)$. Firm i can update the beliefs $F_j(\cdot)$ in view of whether its rival invests at new highs. The market is incontestable once a leader has entered (e.g., the market is fully protected by a patent).[7] Given the risk of preemption, firm i's optimal investment threshold, X_P^i, is such that

[7]Under this assumption, we can concentrate solely on the leader's optimal timing decision and the interplay between preemption and information asymmetry.

$\frac{V_L(X_P^i)}{I} = \Pi^{*\prime}$ where $V_L(x) = \frac{x}{\delta}$ and

$$\Pi^{*\prime} \equiv \frac{\beta_1 + h_j(X_P^i)}{\beta_1 - 1 + h_j(X_P^i)}$$

with β_1 as per eq. (4) and $h_j(\cdot)$ being the hazard rate $h_j(x) = \frac{xF_j'(x)}{1-F_j(x)}$. As a benchmark, firm i's myopic threshold X_L^i, i.e., the one selected in ignorance of rival's action, is such that $\frac{V_L(X_L^i)}{I} = \Pi^*$ with Π^* as in eq. (3). In case of information asymmetry, the threshold X_P^i is located between the zero NPV (preemption) threshold and the myopic threshold X_L^i.[8] For certain beliefs (distribution) of the rival's investment cost, there exists a mapping from I to the optimal trigger X_P^i. For a special case of Pareto distribution, closed-form solution obtains.

In Nishihara and Fukushima (2008), duopolist firms face asymmetric investment costs, I_i and I_j, and deterministic profits subject to a multiplicative shock following a GBM as per eq. (1). In this setting, firm j can only enter the market after firm i since $\pi_L^j = 0$ (the entry sequence is exogenous), and earn $\pi_C^j\,(>0)$ in this case. Firm i, receiving $\pi_L^i\,(>0)$ while monopolist, is out of the market after its rival's entry ($\pi_C^i = 0$). Moreover, firm i has incomplete information about its rival's entry cost. Firm j's threshold X_F is obtained by standard single-agent optimization techniques. Here, the leader's optimal strategy involves two thresholds: a lower threshold $\underline{X_L}$ that triggers investment when X reaches it, and an upper threshold $\overline{X_L}$ beyond which firm i will not invest in fear of early follow-up investment by its rival. The leader's lower threshold is myopic since the firm taking the follower role cannot enter as leader. Only the leader's upper threshold involves strategic interactions in the form of expectations concerning the rival's entry cost.[9]. Given the information asymmetry, firm i can only approximate the upper threshold. If this upper trigger is larger than the complete-information one, firm i may invest at high demand levels and be promptly followed suit by its rival. Conversely, if this trigger is smaller than the complete-information one, the firm may, in cases of high initial X level, postpone for too long overestimating the follower's reaction speed. As the lower trigger $\underline{X_L}$ is independent of firm j's investment cost, firm i's decision to invest at intermediate demand levels is not affected by information asymmetry.

[8]Under complete information, the leader's value equals the follower's at the preemption point. The follower's value is here zero (zero NPV threshold). While the updating process raises the value of each firm, it does not alter the firm's investment strategies. When finally one of the firms invest, the rival's value drops to zero.

[9]Rent equalization does not play a role in this setting

Thijssen et al. (2006) consider a duopolistic market and examine a first-mover vs. second-mover advantage (in the form of information spillovers via option exercise). Depending on specific parameters, the first or second-mover advantage may dominate, leading to preemption or a war of attrition. It is shown that more competition does not necessarily lead to higher social welfare.

Grenadier (1999) analyzes a setting where agents formulate option exercise strategies under imperfect information. The payoff received upon entry is not perfectly known to the firms, each receiving an independent private signal about the true underlying value. The firm may infer its rivals' private signals by observing their entry decisions. Grenadier discusses information cascades where firms ignore their private information and jump on the exercise bandwagon.

2.4. Continuous-time analysis of existing market models

In existing market models, the firms are already operating when contemplating making a new investment intended to improve their initial profit flows. Such models typically assume that firms' investments pose a negative externality on their rivals' profit. Again firm i's stochastic profit is made of two components: a multiplicative exogenous shock $X = \{X_t\}_{t\geq 0}$ and a (nonnegative) deterministic reduced-form profit under some industry structure. At the outset, firms receive profit $\bar{\pi}_0^i$ or $\bar{\pi}_0^j$. Upon making a new investment, firm i can possibly make a higher incumbency profit $\bar{\pi}_L^i$ $(> \bar{\pi}_0^i)$. Once the leader has invested, the follower, firm j, experiences a lower profit $\bar{\pi}_F^j (< \bar{\pi}_0^j)$ as long as it has not itself invested. Once both have invested, they make duopoly profits with $\bar{\pi}_C^i > \bar{\pi}_0^i$, $\bar{\pi}_C^i < \bar{\pi}_L^i$, and $\bar{\pi}_C^j > \bar{\pi}_0^j$.[10]

The above difference with respect to profit values drives somewhat different results for *existing market models* compared to *new market models* analyzed in Section **??**.[11]

2.4.1. Symmetric existing market models

Based on the above setting, Huisman and Kort (1999) identify distinct equilibrium scenarios. The first one involves a preemptive investment sequence where the leader invests at a point X_P where $L(X_P) = F(X_P)$ (rent equalization). A second equilibrium scenario involves (timewise) tacit collusion with firms agreeing to invest at the (later)

[10] In addition, we assume that the value increment from leadership $(\bar{\pi}_L^i - \bar{\pi}_C^i)$ is larger than the value increment received by the follower upon investing $(\bar{\pi}_C^j - \bar{\pi}_F^j)$.

[11] In particular, the existence of a drop from $\bar{\pi}_0^j$ to $\bar{\pi}_F^j$ may give rise to tacit collusion equilibria.

point that maximizes joint value.[12] For highly volatile cash flows, firms are reluctant to exercise early, making tacit collusion likely to prevail.

Grenadier (1996) develops a duopoly model involving completion delays that provides insights into the behaviors of property developers. The real-estate market is often characterized by sudden large development efforts, while in other periods smoother development patterns are observed. The model helps determine why such markets sometimes experience building booms in the face of declining demand and property values. Two symmetric real-estate developers have the possibility to refurbish their building for an investment outlay I, increasing their profit stream accordingly.[13] During the D years until completion, the owner cannot receive rents from the building. The follower's threshold X_F is such that $\frac{V_C(X_F)}{I'} = \Pi^* e^{-\delta D}$ where $V_C(x) = x\pi_C/\delta$, $\delta \equiv r - \mu$, and I' is the total cost of exercising the option, i.e., the investment cost I plus the value of the foregone perpetual income stream from the old building, $\frac{\bar{\pi}_F}{\delta}$. For low values of the process (lower than X_P), the follower's value strictly exceeds the leader's and no improvement takes place. For intermediate values $[X_P, X_F]$ only a single firm decides to invest, each firm becoming leader with one-half probability.[14] For large process values $[X_F, \infty)$, both firms renovate their premises; their values are equal. The tacit-collusion scenario may occur for certain parameter values of the underlying process.[15] Grenadier characterizes drivers for certain behaviors observed in real-estate markets, namely investment cascades and recession-induced construction booms: The occurrence of investment cascades is driven by market volatility, while investment lag is the major reason behind recession-induced construction booms.

2.4.2. Asymmetric existing market models

Kort and Pawlina (2006) analyze the effects of firm heterogeneity (asymmetric investment costs) on optimal timing.[16] Investment costs are $I_i = I$ and $I_j = \alpha I$ with $\alpha \geq 1$.

[12]This equilibrium was already identified by Fudenberg and Tirole (1985): if the payoffs from late and joint investment are sufficiently high, there exists a continuum of tacit-collusion equilibria, one of which coincides with the cooperative (Pareto) optimum.

[13]Grenadier's (1996) departs from the classical existing-market set-up as the author assumes that the initial profit level π_0^i and π_0^j are not subject to the multiplicative shock X.

[14]As Dixit and Pindyck (1994), Grenadier (1996) assumes that the leader is selected over the flip of a fair coin. Huisman and Kort (1999) point out that this result only holds if $X_0 < X_P$.

[15]There exists a continuum of tacit-collusion equilibria. As intuited by game theorists, the most likely to occur is the Pareto-efficient one.

[16]Dias and Teixeira (2003) and Joaquin and Butler (2000) discuss new market models with asymmetry in terms of production costs. Joaquin and Butler (2000) take an open-loop, pure-strategy approach. Dias

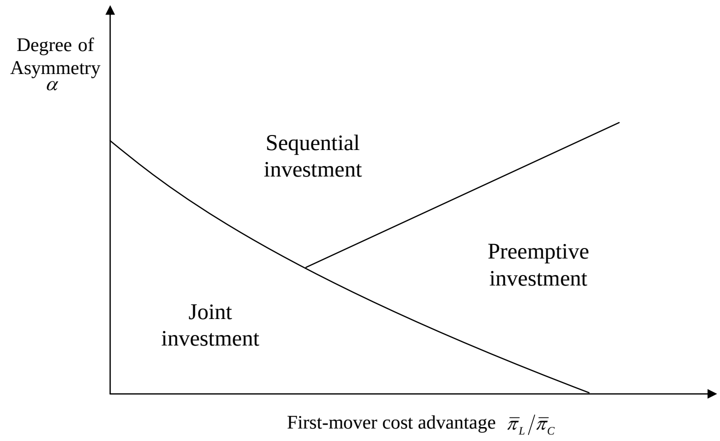

Figure 2: Equilibrium regions (Kort and Pawlina 2006)

The underlying source of uncertainty is a multiplicative stock modeled as per eq. (1). As depicted in figure 2, three types of equilibria may arise depending on the magnitude of the first-mover advantage ($\frac{\bar{\pi}_L}{\bar{\pi}_C}$) and the level of firm asymmetry (α). *Cooperative equilibria* involving simultaneous investments obtains for nearly homogenous firms with no real possibility to gain a first mover advantage. *Preemptive equilibria* characterized by the fear of preemption emerges for mitigate asymmetry. *Sequential (open-loop) equilibria* in which the advantaged firm does not fear preemption and invests as a monopolist would do obtains for large asymmetry (This type of equilibrium does not occur in the symmetric case.) The relationship between firm values and the degree of asymmetry among firms is not clear-cut: a higher degree of homogeneity may give rise to less efficient industry equilibria for both firms.[17]

2.5. Repeated lumpy capacity expansions

Previously, each firm had a single investment opportunity. Murto et al. (2004) consider multiple interacting investment opportunities from a data bandwidth market. Demand function is isoelastic

$$\pi\left(X_t, Q_t\right) = X_t \cdot Q_t^{-\frac{1}{\eta}}, \tag{7}$$

and Teixeira (2003) use a closed-loop, mixed-strategy approach and prove that Joaquin and Butler's open-loop equilibrium obtains for a large cost differential. Kong and Kwok (2007) analyze a new market moel involving a GBM and multiplicative deterministic profit flows where duopolist firms face both asymmetric investment costs and revenues. The authors investigate the effects of these asymmetries as well as the presence of negative vs. positive externalities on the type of investment schedules: open-loop, preemptive, and simultaneous.

[17]These results confirm the observations from the discrete-time case in Smit and Ankum (1993).

with constant elasticity $\eta\,(> 1)$. The multiplicative shock X evolves in a binomial lattice. Starting with a zero initial capacity, firm i (firm j) can decide at any time to invest I_i (I_j) to increase capacity by a lump sum ΔQ_i (ΔQ_i). Investment decisions are sequential with the first mover randomly chosen. Each investment subgame is described by the current firm capacities, the level of market demand, and present time. The optimal (Markov) expansion strategies involves investment-inducing demand thresholds and can be derived using dynamic programming. As these thresholds are increasing in a firm's installed capacity, the smallest firm is more likely to react to small demand shocks by expanding capacity. Symmetric firms can expand capacity by the same increment size and for the same investment cost ($\Delta Q_i = \Delta Q_j$, $I_i = I_j$), while in the asymmetric case analyzed, firm i invests in smaller lumps for a larger unit cash outlay ($\Delta Q_i = a\Delta Q_j$ and $I_i = \alpha I_j$ with $a < \alpha \leq 1$). Despite its higher investment cost, firm i benefits from the situation since it can react quicker to changes in demand.

Boyer et al. (2004b, 2007) consider in continuous time a duopoly where symmetric firms add capacities by lumpy increments.[18] Initially, firms have low capacities. The only possible equilibrium at initial stages of the industry involves preemption.[19] Rent equalization occurs irrespective of the volatility or the speed of market development. When firms already hold substantial capacity, tacit collusion may be sustainable as an industry equilibrium. Such equilibria are more likely in highly volatile or fast growing markets.[20] The possibility of collusion is more attractive to symmetric firms than to asymmetric ones.

Carlson et al. (2009) study the effect of rivals' expansion and contraction options on incumbent firms' risk exposure. Heterogenous duopolists face isoelastic demand of eq. (7) subject to a multiplicative shock following a GBM as per eq. (1). They may expand or contract capacity (by lump sums) in view of market realizations. A rival's investment decisions act as a natural hedge against variation in the exogenous state variable: growing

[18]Boyer et al. (2004b) assume that reduced-form stage profits are the outcome of Bertrand price competition, whereas Boyer et al. (2007) consider Cournot quantity competition.

[19]When firms do not hold any existing capacity, tacit-collusion equilibria are ruled out as firms are not threatened with the loss of any existing rents. Due to preemption, the first industry-wide investment occurs earlier than what would be socially optimal (from the viewpoint of the industry participants). This distortion implies riskier entry and lower expected returns.

[20]In such a context, the conventional real options result that high volatility leads to investment postponement gets reinforced by the fact that higher volatility may result in a switch from the preemption equilibrium to a tacit-collusion equilibrium involving later investment and higher values.

demand may induce rival's capacity expansion, limiting its one's upside potential, while a bearish market increases the likelihood of rival's capacity contraction, reducing one's downside risk. This phenomenon affects the appropriateness of using peer betas to proxy a firm's risk.

2.6. Industry exit models

Fine and Li (1989) supplement deterministic models of exit (e.g., Ghemawat and Nalebuff 1985; Fudenberg and Tirole 1986; Ghemawat and Nalebuff 1990) by allowing for the stochastic decline of a duopoly market. The authors show (in discrete time) that the sequence of exit is not unique due to "jumps" in the demand process.

Sparla (2004) analyzes in continuous time closure/ exit options for a duopoly where firms face a stochastically declining market. A multiplicative shock as per eq. (1) affects the profit value received by the firms. The follower's exit problem is of a decision-theoretic nature; the divestment threshold X_F (to be reached from above) satisfies[21]

$$\frac{V_F\left(X_F\right)}{S} = \Pi_*, \tag{8}$$

with $V_F\left(x\right) = x\pi_F/\delta$, $\delta \equiv r - \mu$, and Π_* being a required profitability level, given by

$$\Pi_* \equiv \frac{\beta_2}{\beta_2 - 1}, \tag{9}$$

where

$$\beta_2 \equiv \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) - \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2}}{\sigma^2} \quad (< 0)\,.$$

In this a *war of attrition* or *chicken game*, both have an incentive to wait until the rival exits first or until market conditions deteriorate so much that both firms are better off leaving the market irrespective of their rival's action. In equilibrium, both firms exit the first time X_F as in eq. (8) is hit. Duopolist firms disinvest later than a monopolist. This symmetric equilibrium profile is the best achievable outcome from the viewpoint of the whole industry.[22]

Murto (2004) considers a similar problem in which firms differ in terms of production scale and face isoelastic demand as per eq. (7). The resulting exit thresholds are X_L^i for

---

[21] For enhanced comparability, we simplify Sparla's (2004) model by assuming exit rather than partial closure and using a single aggregated salvage value S. Sparla (2004) and Murto (2004) (discussed later) decompose the salvage value into cost savings made upon exiting the market and costs incurred to make exit effective (e.g., lay-off costs).

[22] The author also discusses the case where firms are asymmetric in terms of production cost and size respectively.

the first exiting firm (leader) and X_F^j for the last one (follower), where X_L^i and X_F^j are such that

$$\frac{V_L^i(X_L^i)}{S} = \Pi_*; \frac{V_F^j(X_F^j)}{S} = \Pi_*,$$

where $V_L^i(x) = x\pi_C^i/\delta$, $V_F^j(x) = x\pi_L^j/\delta$, $\delta \equiv r - \mu$, and Π_* as in eq. (9). The leader's willingness to stay in the market increases with the underlying volatility as its (put) option value is increased. For low levels of volatility, there is a unique exit sequence where the smaller firm (i) exits as leader when threshold X_L^i is first reached, and the largest firm (j) exits when X_F^j is attained for the first time. For high volatility levels, however, this equilibrium is no longer unique and the reverse ordering with the largest firm exiting first may also obtain.

3. Incremental capacity expansion

Another stream of research focuses on capacity problems where firms can increase their capacity (or, generally speaking, their capital stock) incrementally.[23] Section 3.1 deals with duopolistic situations. Section 3.2 discusses oligopolies and capacity utilization. Section 3.3 elaborates on investment behaviors in perfect competition and the connection with the social optimum.

3.1. Duopoly

Huisman and Kort (2009) allow firms to chose optimally their capacity/ production scale at the time they enter the market. The multiplicative shock follows a GBM as per eq. (1). Demand is linear. For low volatility, the follower chooses a higher capacity than the leader and for high volatility the leader chooses a higher capacity. Compared to the model without capacity choice, the monopolist and the follower invest later in more capacity for high volatility. Conversely, the leader will invest earlier in a higher capacity for higher volatility.

Novy-Marx (2009) derives Markov perfect equilibrium outcomes for duopolistic capacity competition. Two firms with differing initial capacity face isoelastic demand as per eq. (7) and an exogenous shock following a GBM. They have negligible operating costs. The author identifies and evaluates three distinct equilibria: Cournot, shared monopoly and 'preemptive preemption'. He stresses that Spence's (1979) notion of Stackelberg

[23]As Pindyck (1988) suggests, this approach is extreme as most investments are lumpy, but it offers convenient mathematical tractability.

leaders where the larger firm profitably forecloses the market hinges on "static market" assumptions of zero-growth, no uncertainty, no depreciation and does not obtain in more general industry settings under uncertainty.

3.2. Oligopoly

In the following, we summarize some of the oligopoly models and distinguish them depending on whether firms have to decide on their capacity utilization level.

3.2.1. Constant return to scale

Symmetric firms. Grenadier (2002) describes an oligopoly with n symmetric firms producing a single, homogeneous, non-storable good. Profits are affected by a fairly general aggregate shock modeled as a diffusion

$$\mathrm{d}X_t = \mu(X_t)\,\mathrm{d}t + \sigma(X_t)\,\mathrm{d}B_t \tag{10}$$

where $\mu(X_t)$ and $\sigma(X_t)$ are the drift and diffusion terms and $B = \{B_t\}_{t\geq 0}$ is a standard Brownian motion. Each firm possesses q_t^i units of capacity as of time t and can increase capacity incrementally at any time at a cost of I per capacity unit. Building new capacity takes D years until completion. Given constant returns to scale, firms produce at full capacity and sell at the market-clearing price $P_t \equiv P(X_t; Q_t)$, with $Q_t = \sum_i q_t^i$ being the total industry output. Assuming symmetric investment strategies, the firms expand simultaneously and in the same proportion, i.e., $\mathrm{d}q_t^i = \mathrm{d}q_t = \frac{\mathrm{d}Q}{n}$. The equilibrium implies an upper trigger for the price level $\bar{P}(Q)$ at which the firms will increase their aggregate capacity by $\mathrm{d}Q$. For $D > 0$ the firms will not trigger investment at time t based on the current price level P_t but on their expectation concerning the price at time $t + D$.[24] Grenadier (2002) formulates the problem in a very tractable way and derives closed-form solutions for specialized cases: arithmetic, geometric Brownian motion and square-root process together with the linear (eq. 2) or the isoelasticity demand function (eq. 7). When the market approaches perfect competition, option values are getting less valuable.

Asymmetric firms. Novy-Marx (2007) allows for (a continuum of) firms differing in their initial capacities (with the logarithms of the capacities being uniformly distributed between the largest and the smallest firm). The assets produce a constant non-storable

[24] Grenadier (2002) notes that with time-to-build the important aspect is to consider committed capacity expansions encompassing all capacity units available in D years, that is Q_{t+D}.

output stream Q_t which is sold at the market-clearing price P_t, each firm realizing a revenue $\pi(X_t; q_t^i, Q_t) = q_t^i P_t$. Inverse demand is isoelastic as in eq. (7) with the shock following GBM as in eq. (1). Firms can repeatedly redevelop their assets-in-place q^i at a cost of $q^{i\gamma}$ ($\gamma > 1$), facing an increasing cost-to-scale ratio. Redevelopment requires abandoning the profit stream from the current assets-in-place.[25] In equilibrium a firm with capacity q^i expand to κq^i when the market-clearing price hits a certain threshold $\bar{P}(q^i, Q)$. Both the intensity and the timing of redevelopment depend on this parameter $\kappa(> 1)$. Remarkably, κ is industry-specific and stationary, which ensures that the distribution of capacities is preserved over time. At any point in time the next firm to exercise its expansion option will always be the smallest firm. Simultaneous investment does not occur and firm heterogeneity results in a natural ordering of firm investments. Comparative statics with respect to κ are therefore key to understanding the equilibrium outcome. Novy-Marx (2007) also challenges Grenadier's (2002) assertion that for an increasing number of firms, option values are continually eroded.[26]

3.2.2. Optimal capacity utilization choice

Both Grenadier (2002) and Novy-Marx (2007) assume that output generation is costless with capacity being fully utilized (constant returns to scale). Aguerrevere (2003) relaxes this assumption and obtains a mean-reverting price evolution exhibiting volatility spikes, although the additive demand shock $X = \{X_t\}_{t\geq 0}$ follows a GBM. A number of identical firms sell a non-storable good and face the inverse demand of eq. (2). At each instant, firms choose their level of capacity utilization optimally and incurs increasing production costs. They can expand capacity incrementally for a cost of I (per capacity unit), with expansion taking D years to complete.[27] The option to build a new capacity unit is analogous to an American call on a set of European calls.[28] For (n-firm) oligopolies, a symmetric equilibrium results such that firms add capacity at the same time as a monopolist would.[29] Capacity utilization turns out to be independent of the number of firms. Without time-to-build delays, aggregate committed capacity is strictly

[25]This assumption ensures that investments are not incremental but lumpy.

[26]Novy-Marx (2007) notes that the option values in Grenadier (2002) are merely super-normal rents from oligopolistic competition.

[27]As in Grenadier (2002), the state space is reduced by introducing committed capacity.

[28]Since at any time, a unit of capacity can be shut down at no cost, a capacity unit under construction (at time t) can effectively be considered as a set of European call options (with maturities greater $t + D$), the underlying being the net profit from an extra unit of capacity.

[29]The oligopoly quantities can be expressed as the corresponding monopoly capacity times a constant.

decreasing in volatility. With time-to-build, the effect of uncertainty is, however, ambiguous: the committed capacity is decreasing with the volatility for low current demand, but increasing for high demand levels. Thus firms may provide more capacity if faced with greater uncertainty. This ambiguity arises from the trade-off between the increased risk of capacity under-utilization and the higher uncertainty increasing the value of capacity under construction.[30] The output price paths exhibit mean reversion and significant spikes in times of full capacity utilization. Due to time-to-build delays, completion of capacity expansion is preceded by a phase of high utilization and high prices.

Aguerrevere (2009) examines a firm's systematic risk (beta) under competition. The firm's beta is determined as the weighted average of the beta of assets in place and the beta of the firm's growth options.[31] In line with Grenadier (2002), the value of growth options decreases with the number of firms and approaches zero when n tends to infinity (as in perfect competition). Under intensified competition, the capacity held in the industry is utilized more in response to demand increases. Irrespective of the number of firms, assets in place are generally less risky when demand is high as capacity utilization is increased.[32] In case of GBM, the growth option's beta obtains to be constant (independent of industry capacity, demand level and the number of the firms). For high demand the firm beta decreases with the number of rivals, while it increases for low demand.

3.3. Perfect competition

Leahy (1993) sets the benchmark case for continuous-time analysis of (infinitely divisible) capacity expansion and scrapping under perfect competition. Firms face an uncertain demand subject to an exogenous shock modeled as per eq. (10) and the total capacity held in the industry. The firms' optimal exercise strategies exhibit some form of myopia in that firms invest at the same time as a firm ignoring potential future capacity expansions. The author subsequently compares this investment strategy with the one formulated by a social planner and imposed to decentralized firms. Myopic investment policies obtain to be socially optimal.[33] Grenadier (2000a) extends Leahy's (1993) model

[30]Capacity under construction is analogous to a set of European call options whose value is strictly increasing in volatility.

[31]Weights are determined based on the present values of assets in place and of growth options.

[32]The beta of assets in place will be zero at the time demand reaches the capacity-expansion threshold; at this point, the firm's riskiness corresponds to the riskiness of added capacity units.

[33]Leahy (1993) also points out that the impossibility to increase one's capital by (infinitely) small amounts might explain the emergence of excess returns for firms behaving myopically (in lumpy investment models). Dixit (1991) discusses a perfect-competition model in lines with Leahy (1993).

to allow for completion delays.

Baldursson and Karatzas (1996) consider capacity expansion and allow for non-Markovian stochastic processes. The correspondence between perfect competition, myopic strategies, and social optimality is established based on the probabilistic approach to stochastic control theory. Baldursson (1998) discusses both expansion and downsizing. The result obtained in the Nash equilibrium are compared with the choice of a social planner. A general model for the inverse demand function (price as a function of shock and capacity) is given. Some special cases admit closed-form solutions.

Back and Paulsen (2009) discuss the appropriateness of the Nash or open-loop equilibrium concept employed in most models of oligopoly and perfect competition (e.g., Grenadier, 2002; Baldursson, 1998; Baldursson and Karatzas, 1996). Open-loop strategies allow firms to respond to the resolution of uncertainty with respect to the exogenous shock but not to the observed actions by rivals. Optimal open-loop strategies have to form a Nash equilibrium, as part of the "open-loop equilibrium". Back and Paulsen (2009) discuss the fact that if firms could in effect respond to their rivals' actions, i.e., formulate "closed-loop strategies", the equilibrium strategies derived by Grenadier (2002) would fail perfectness.[34] Formulating the dynamic capacity-expansion problem in closed-loop strategies is difficult, but Back and Paulsen manage to show that in the limit, the perfect competition outcome derived in Leahy (1993) is part of a (perfect) closed-loop equilibrium.

4. Staged investment appraisal under competition

Firms may also move in stages and exercise early strategic investments meant to alter later stages for the better by, e.g., opening up new market opportunities or enhancing the value of their investment options.

4.1. Commitment vs. flexibility

Smit and Trigeorgis (2001) analyze in discrete time the trade-offs between managerial flexibility and commitment in a dynamic competitive setting under uncertainty.[35] Firm i can make a first-stage strategic investment K_i possibly altering the later equilibrium

[34]If a firm were to pursue such equilibrium open-loop strategies even though they observe their rivals' actions and may revise their strategies accordingly, they would face the risk of preemption.

[35]Smit and Trigeorgis (2001) extend the framework developed in Smit and Ankum (1993) by explaining the source of firm heterogeneity and quantifying the trade-off between commitment and flexibility.

strategy choices $\alpha_i^*(K_i)$ and $\alpha_j^*(K_i)$. Firms are initially assumed on an equal footing in the second competition stage but firm i may introduce some asymmetry by making this first-stage investment. Hence, the initial investment decision requires firm i to weigh the commitment cost against the expected future strategic benefits of commitment. For the different possible investment orderings (simultaneous, sequential, singular) they define corresponding market outcomes (Cournot, Stackelberg, monopoly) and use these to calculate the final payoffs, V_i and V_j. Following Fudenberg and Tirole (1984), the strategic effect of the committing first-stage investment depends on the type of competitive reaction and the nature of the commitment, as represented in Figure 3. Firm i's investment

	Strategic substitutes $(\partial\alpha_j^*/\partial\alpha_i < 0)$	**Strategic complements** $(\partial\alpha_j^*/\partial\alpha_i > 0)$
Tough investment $(dV_j/dK_i < 0)$	Positive strategic effect	Negative strategic effect
Soft investment $(dV_j/dK_i > 0)$	Negative strategic effect	Positive strategic effect

Figure 3: Sign of the strategic effect (Smit and Trigeorgis 2001)

is either *tough* (if $\mathrm{d}V_j/\mathrm{d}K_i < 0$) or *soft* (if $\mathrm{d}V_j/\mathrm{d}K_i > 0$). If firm (re)actions are *strategic substitutes* (as under Cournot quantity competition), firm j will engage "less" for an aggressive action by firm i ($\partial\alpha_j^*/\partial\alpha_i < 0$). Conversely, firms' (re)actions can be *strategic complements* (as under differentiated Bertrand price competition) with $\partial\alpha_j^*/\partial\alpha_i > 0$. The authors construct and solve four numerical examples illustrating all possible combinations of competitive reaction and the investment type. Upfront investment is only optimal for firm i in the two cases where the strategic effect is positive. For the cases with negative strategic effect, firm i should not invest. Firm i benefits from increased uncertainty as its stage-two investment option becomes more valuable. But at the same time uncertainty erodes the value of committing as the upfront investment becomes riskier. Smit and Trigeorgis (2007, 2009) utilize this framework to assess R&D strategies and infrastructure investment decisions.

4.2. R&D investment models

The existence of completion delays affect the (optimal) investment strategies of firms. For R&D projects (or in bio-tech, IT and oil exploration), such delay is rarely known in advance due to uncertain innovation success. Weeds (2000) examines this source of uncertainty and its impact on the duopolists' technology adoption. The firm invests in research with the aim to acquire a patent giving it exclusive access to a new market. If the innovation is successful, the firm has the option to make an additional sunk investment to adopt the new technology. The entire R&D investment opportunity is a compound option where the value of the (first-stage) research option partly derives from the (second-stage) commercial investment option. The framework provides a rational explanation for the existence of sleeping patents, i.e., patents granted but kept in a stand-by or "sleep" mode. Policy-makers typically regard sleeping patents as anti-competitive devices employed by dominant firms to erect entry barriers (blockaded entry). However, in this context sleeping patents may arise when options co-exist with completion uncertainty.[36]

Lambrecht (2000) derives optimal investment strategies for two symmetric firms sharing the option to make a two-stage sequential investment under incomplete information about the rival's profit. In the first stage, each firm is competing to acquire a patent enabling it to proceed to the second commercialization stage. Lambrecht derives conditions under which inventions are likely to be patented without being put to immediate commercial use. Sleeping patents are more likely to exist in an R&D portfolio when interest rates are low, volatility is high and when the second-stage cost is high relative to the first-stage.

Miltersen and Schwartz (2004) analyze patent-protected R&D investment projects when there is imperfect competition in the development and commercialization of the product. Competition in R&D not only may increase production and reduce prices, but it may also shorten the time of developing the product and increase the probability of a successful development. These benefits to society are offset by increased R&D investment costs in oligopolistic markets and lower aggregate value of the R&D investment projects.

5. Summary of key managerial insights

Porter (1980) already noted the importance of the industry structure in determining optimal investment strategies. He identified economic and technological uncertainty, the nature of the investment (lumpy vs. incremental), first-mover advantages, competition

[36]By restricting a firm's ability to let patents sleep, antitrust authorities may actually reduce - via compulsory licensing - option values and weaken firms' incentive to conduct research in the first place.

intensity (number of incumbents) and the type of competitive reactions as major drivers. Option games provide an economic foundation to such findings and suggest to look at the following.

First-mover advantage vs. second-mover advantage The existence of a first-mover advantage generally give rise to preemption. The presence of second-mover advantages on the other hand may mitigate the risk of preemption as shown by Mason and Weeds (2002). Cottrell and Sick (2002) point out that managers often tend to overestimate first-mover advantages and consequently invest too early.

Firm homogeneity vs. heterogeneity Kort and Pawlina (2006) demonstrate that in duopolies, a firm with a large comparative advantage (e.g., lower investment cost) may enter the market or expand production with limited fear of preemption. Firm heterogeneity can thus explain a natural market-entry sequencing where each firm formulates its investment strategy myopically and select the best entry time as would a monopolist. This sequence is also socially optimal from the viewpoint of the firms.

Complete vs. incomplete information As underlined by Lambrecht and Perraudin (2003), information asymmetry is not necessarily detrimental to firms as the risk of preemption is reduced under certain configurations. Uncertainty about research outcomes may affect investment behaviors as well.

Divisibility of capacity The size of the lump sums by which firms can invest affect the reactivity to economic changes. As underlined by Murto, Näsäkkälä, and Keppo (2004), the possibility to invest in smaller capacity increments may justify incurring higher (unit) investment cost.[37]

Capacity utilization and return to scale Aguerrevere (2003) examines operating costs and their connection with capacity utilization. As demand declines, firms will reduce capacity utilization, explaining output prices are often mean reverting.

Number of competitors The number of option-holding firms may determine the risk of preemption and the likely option value erosion. When the number of firms is large, myopic (open-loop) behaviors can be optimal (see Back and Paulsen, 2009). Such a property eases the quantitative analysis of markets with a large number of firms as discussed in Grenadier (2002).

[37] Kort et al. (2010) analyze the effect of uncertainty on the value of divisibility in a non-strategic context.

Competitor's reaction Smit and Trigeorgis (2001) demonstrate that the nature of the competitive reaction influences the incentive to commit to certain investments. Whether strategic actions are complements or substitutes and investment makes tough or soft are key factors to consider in multi-stage investment settings. Even if a commitment is optimal in a steady-state market, the expected benefit obtained from committing must be traded off under uncertainty against the present cost of killing one's flexibility.

In the above we offered an overview over a number of option games papers, stressing the insights one can derive from considering real-options problems in a competitive setting. Some selected contributions are summarized in Table 1. Trigeorgis (1996) identified this theme as one of the prevalent research gaps in the then-emerging literature on real options analysis. Even though some gaps have been bridged, many roads are still open as to account for the way information is revealed over time (open-loop vs. closed-loop approach) in more general settings. Especially, the dynamic approach to how many firms build up competitive advantage and erect (endogenous) market-entry barriers have not yet been systematically researched on. Besides the research so far has been mainly technical and theoretical. More applied discussions are also desirable, increasing the relevance of option games for management practice.[38]

[38] An enhanced applicability of these ideas could be potentially achieved by increasing research efforts on discrete-time modeling approaches. Ferreira et al. (2009) underline the relevance of option games for strategic management formulation.

Author and year	Option		Industry				Stochastics			Investment type			Demand shock			Key notions
	Entry/ expansion	Exit/ disposal	Duopoly	Oligopoly	Perfect comp.	Heterogeneity[a]	Discrete-time	Continuous-time	Non-GBM	Lumpy	Repeated lumpy	Incremental	Linear[b]	Multiplicative[c]	General	
Aguerrevere (2003, 2009)	✓	(✓)		✓				✓				✓		✓		Utilization discretion creates put options
Baldursson (1998)	✓	✓		✓				✓	✓			✓			✓	Optimality of myopic strategies
Baldursson and Karatzas (1996)	✓			✓				✓				✓			✓	Itô process; optimality of myopic strategies
Bouis et al. (2009)	✓			✓				✓		✓				✓		Preemption in oligopoly
Boyer et al. (2007)	✓		✓					✓			✓			✓		Duopoly evolution over time
Grenadier (1996)	✓		✓					✓		✓				✓		Application to real estate market
Grenadier (1999)	✓			✓		✓		✓		✓				✓		Private signals are revealed through option exercises
Grenadier (2002)	✓			✓	✓			✓	✓			✓			✓	Formulates simple investment policies in oligopolies
Huisman and Kort (1999)	✓		✓					✓		✓				✓		Formal discussion of new vs. existing market
Joaquin and Butler (2000)	✓		✓			✓		✓		✓				✓		Open-loop approach
Kort and Pawlina (2006)	✓		✓			✓		✓		✓				✓		Closed-loop approach
Lambrecht (2000)	✓		✓					✓		✓				✓		Application to R&D investments
Lambrecht and Perraudin (2003)	✓		✓			✓		✓		✓				✓		Information asymmetry
Leahy (1993)	✓	✓				✓		✓	✓			✓			✓	Optimality of myopic strategies
Mason and Weeds (2002)	✓		✓					✓		✓				✓		Application to R&D investments
Murto (2004)		✓	✓					✓		✓				✓		War of attrition in stochastic environment
Murto et al. (2004)	✓		✓			✓	✓				✓			✓		Application to telecommunications
Nishihara and Fukushima (2008)	✓		✓			✓		✓		✓				✓		Exogenous entry ordering
Novy-Marx (2007)	✓		✓			✓		✓			✓			✓		Logarithmic distribution of firms' capacities
Smit and Ankum (1993)	✓		✓			✓	✓			✓			✓			Early paper on discrete-time option games
Smit and Trigeorgis (2001)	✓		✓			✓	✓			✓			✓			Trade-off between flexibility and commitment
Sparla (2004)		✓	✓					✓		✓				✓		War of attrition in stochastic environment
Thijssen et al. (2006)	✓		✓					✓	✓	✓					✓	Càdlàg process; timing games in stochastic environment
Trigeorgis (1991)	✓			(✓)	(✓)	(✓)		✓		✓					✓	Exogenous modeling of competitive entries
Weeds (2002)	✓		✓					✓	✓	✓				✓		Poisson process for technological uncertainty in R&D
Zhu and Weyant (2003b,a)	✓		✓			✓	✓			✓			✓			Discrete-time option games with information asymmetry

Table 1: Summary of selected contributions

[a] Heterogeneity refers to competitive advantages or other differing characteristics between firms (including information asymmetries)

[b] Linear demand corresponds to equation above

[c] Multiplicative demand shock encompasses isoelastic demand as per equation

References

Aguerrevere, F. L., 2003. Equilibrium investment strategies and output price behavior: A real-options approach. Review of Financial Studies 16 (4), 1239–1272.

Aguerrevere, F. L., 2009. Real options, product market competition, and asset returns. Journal of Finance 64 (2), 957–983.

Back, K., Paulsen, D., 2009. Open-loop equilibria and perfect competition in option exercise games. Review of Financial Studies 22 (11), 4531–4552.

Baldursson, F. M., 1998. Irreversible investment under uncertainty in oligopoly. Journal of Economic Dynamics and Control 22, 627–644.

Baldursson, F. M., Karatzas, I., 1996. Irreversible investment and industry equilibrium. Finance and Stochastics 1 (1), 69–89.

Bouis, R., Huisman, K. J. M., Kort, P. M., 2009. Investment in oligopoly under uncertainty: The accordion effect. International Journal of Industrial Organization 27 (2), 320 – 331.

Boyer, M., Gravel, E., Lasserre, P., 2004a. Real options and strategic competition: a survey. Working paper, University of Montreal.

Boyer, M., Lasserre, P., Mariotti, T., Moreaux, M., 2004b. Preemption and rent dissipation under price competition. International Journal of Industrial Organization 22 (3), 309–328.

Boyer, M., Lasserre, P., Moreaux, M., 2007. The Dynamics of Industry Investments. Working paper, University of Montreal.

Brennan, M. J., Trigeorgis, L. (Eds.), 2000a. Project Flexibility, Agency, and Competition: New Developments in the Theory and Application of Real Options. Oxford University Press, USA.

Brennan, M. J., Trigeorgis, L., 2000b. Real options: Development and new contributions. In: Brennan and Trigeorgis (2000a).

Carlson, M. D., Dockner, E. J., Fisher, A. J., Giammarino, R., 2009. Leaders, Followers, and Risk Dynamics in Industry Equilibrium. SSRN eLibrary.

Chevalier-Roignant, B., Trigeorgis, L., 2010. Competitive Strategy: Options and Games. The MIT Press, forthcoming.

Cottrell, T., Sick, G., 2002. Real options and follower strategies: The loss of real option value to first-mover advantage. Engineering Economist 47, 232 – 263.

Cox, J. C., Ross, S. A., 1976. The valuation of options for alternative stochastic processes. Journal of Financial Economics 3 (1-2), 145–166.

Dias, M. A. G., Teixeira, J. P., 2003. Continuous-time option games: review of models and extensions. Part 1: Duopoly under uncertainty. In: Proceedings of the 2003 International Real Options Conference, George Town University, USA.

Dixit, A. K., 1991. Irreversible investment with price ceilings. Journal of Political Economy 99 (3), 541–557.

Dixit, A. K., Pindyck, R. S., 1994. Investment under uncertainty. Princeton Univ. Press.

Ferreira, N., Kar, J., Trigeorgis, L., 2009. Option Games: The Key to Competing in Capital-Intensive Industries. Harvard Business Review 87 (3).

Fine, C. H., Li, L., 1989. Equilibrium exit in stochastically declining industries. Games and Economic Behavior 1 (1), 40–59.

Fudenberg, D., Tirole, J., 1984. The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. American Economic Review 74 (2), 361–366.

Fudenberg, D., Tirole, J., 1985. Preemption and rent equilization in the adoption of new technology. Review of Economic Studies 52 (3), 383–401.

Fudenberg, D., Tirole, J., 1986. A theory of exit in duopoly. Econometrica 54 (4), 943–960.

Ghemawat, P., Nalebuff, B., 1985. Exit. RAND Journal of Economics 16 (2), 184–194.

Ghemawat, P., Nalebuff, B., 1990. The devolution of declining industries. Quarterly Journal of Economics 105 (1), 167–186.

Grenadier, S. R., 1996. The strategic exercise of options: Development cascades and overbuilding in real estate markets. Journal of Finance 51 (5), 1653–79.

Grenadier, S. R., 1999. Information revelation through option exercise. Review of Financial Studies 12 (1), 95–129.

Grenadier, S. R., 2000a. Equilibrium with time-to-build: A real options approach. In: Brennan and Trigeorgis (2000a), pp. 275–296.

Grenadier, S. R. (Ed.), 2000b. Game Choices: The Intersection of Real Options and Game Theory. Risk Books.

Grenadier, S. R., 2002. Option exercise games: An application to the equilibrium investment strategies of firms. Review of Financial Studies 15 (3), 691–721.

Harrison, J. M., Kreps, D. M., 1979. Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory 20 (3), 381–408.

Huisman, K. J. M., 2001. Technology Investment: A Game Theoretic Real Options Approach. Springer.

Huisman, K. J. M., Kort, P. M., 1999. Effects of strategic interactions on the option value of waiting. Working paper, Tilburg University.

Huisman, K. J. M., Kort, P. M., 2009. Strategic capacity investment under uncertainty. In: 13th Annual International Real Options Conference, June 17-20, 2009.

Huisman, K. J. M., Kort, P. M., Pawlina, G., Thijssen, J. J. J., 2004. Strategic investment under uncertainty: Merging real options with game theory. Zeitschrift für Betriebswirtschaft 74, 97–123.

Joaquin, D. C., Butler, K. C., 2000. Competitive investment decisions - a synthesis. In: Brennan and Trigeorgis (2000a), pp. 324–339.

Kong, J. J., Kwok, Y. K., 2007. Real options in strategic investment games between two asymmetric firms. European Journal of Operational Research 181 (2), 967–985.

Kort, P. M., Murto, P., Pawlina, G., 2010. Uncertainty and stepwise investment. European Journal of Operational Research 202 (1), 196–203.

Kort, P. M., Pawlina, G., 2006. Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage? Journal of Economics & Management Strategy 15, 1–35.

Lambrecht, B., 2000. Strategic sequential investments and sleeping patents. In: Brennan and Trigeorgis (2000a), pp. 297–323.

Lambrecht, B., Perraudin, W., 2003. Real options and preemption under incomplete information. Journal of Economic Dynamics and Control 27, 619–643.

Leahy, J. V., 1993. Investment in competitive equilibrium: The optimality of myopic behavior. Quarterly Journal of Economics 108 (4), 1105–1133.

Mason, R., Weeds, H., 2002. Reverse Hysteresis: R&D Investment and Stochastic Innovation. Tech. rep., University of Cambridge Working Paper.

Mason, R., Weeds, H., 2009. Investment, uncertainty and pre-emption. International Journal of Industrial Organization.

Miltersen, K., Schwartz, E. S., 2004. R&D investments with competitive interactions. Review of Finance 8 (3), 355–401.

Murto, P., 2004. Exit in duopoly under uncertainty. RAND Journal of Economics 35 (1), 111–127.

Murto, P., Näsäkkälä, E., Keppo, J., 2004. Timing of investments in oligopoly under uncertainty: A framework for numerical analysis. European Journal of Operational Research 157, 486–500.

Nishihara, M., Fukushima, M., 2008. Evaluation of firm's loss due to incomplete information in real investment decision. European Journal of Operational Research 188, 569–585.

Novy-Marx, R., 2007. An equilibrium model of investment under uncertainty. Review of Financial Studies 20, 1461–1502.

Novy-Marx, R., 2009. Preempting Preemptive Investment. Working paper, University of Chicago.

Pindyck, R. S., 1988. Irreversible investment, capacity choice, and the value of the firm. American Economic Review 78 (5), 969–985.

Porter, M. E., 1980. Competitive Strategy. Free Press.

Smets, F., 1991. Exporting versus foreign direct investment: The effect of uncertainty, irreversibilities and strategic interaction. Ph.D. thesis, Yale University, New Haven, CT.

Smit, H. T. J., Ankum, L. A., 1993. A real options and game-theoretic approach to corporate investment strategy under competition. Financial Management 22 (3), 241–250.

Smit, H. T. J., Trigeorgis, L., 2001. Flexibility and commitment in strategic investment. In: Schwartz, E. S., Trigeorgis, L. (Eds.), Real options and investment under uncertainty: Classical readings and recent contributions. MIT Press.

Smit, H. T. J., Trigeorgis, L., 2004. Strategic Investment: Real Options and Games. Princeton University Press.

Smit, H. T. J., Trigeorgis, L., 2007. Strategic options and games in analysing dynamic technology investments. Long Range Planning 40 (1), 84 – 114.

Smit, H. T. J., Trigeorgis, L., 2009. Valuing infrastructure investment: an option games approach. California Management Review 51(2), 79–90.

Sparla, T., 2004. Closure Options in a Duopoly with Strong Strategic Externalities. Zeitschrift für Betriebswirtschaft 67, 125–155.

Spence, A. M., 1979. Investment strategy and growth in a new market. Bell Journal of Economics 10 (1), 1–19.

Thijssen, J. J. J., Huisman, K. J. M., Kort, P. M., 2006. The effects of information on strategic investment and welfare. Economic Theory 28 (2), 399–424.

Trigeorgis, L., 1991. Anticipated competitive entry and early preemptive investment in deferrable projects. Journal of Economics and Business 43 (2), 143–156.

Trigeorgis, L., 1996. Real Options: Managerial Flexibility and Strategy in Resource Allocation. MIT Press.

Weeds, H., 2000. Sleeping patents and compulsory licensing: an options analysis. Working paper, University of Warwick.

Weeds, H., 2002. Strategic delay in a real options model of R&D competition. Review of Economic Studies 69, 729–747.

Zhu, K., Weyant, J., 2003a. Strategic exercise of real options: investment decisions in technological systems. Decision Sciences 34 (4), 643–675.

Zhu, K., Weyant, J., 2003b. Strategic exercise of real options: investment decisions in technological systems. Journal of Systems Science and Systems Engineering 12 (3), 257–278.