Evaluation of Optional Cancellation Contracts using Quantitative Finance Techniques

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Abstract

We consider the problem of evaluating the cost of the optionality to cancel a future delivery of a commodity when the seller has a number of markets to choose from. The technique has potential applications to contracts of Liquefied Natural Gas loads and requires solving certain diffusion problems in a multi-variable context.

1 Introduction

Energy commodities form an important part of most markets. Several mercantile exchanges around the world trade energy commodities in different forms.

The possibility of cancelling the delivery of a commodity at specific times before the delivery gives a substantial flexibility for the buyer. The purpose of this work is to price the value of such optionality. More specifically we are interested on the following question: Given a delivery contract of a commodity, how much should the buyer pay for the optionality of canceling such delivery at different times in the future?

An instance of commodity where such contracts might be interesting is liquid natural gas. Natural gas is an important source of electricity generation, as fuel for vehicles, and in households for heating and cooking. The global demand for natural gas has been growing steadly in the last years.

Because of its nature, transportation is a crucial issue for the natural gas market. There are two ways of transporting natural gas: pipelines and *liquefied natural gas* (LNG). If there is a pipeline available, then it is the cheapest transportation option. LNG is the choice when no pipeline is available. LNG takes up to 1/600th the volume of natural gas. It requires special ships, known as LNG carriers.

In what follows, we concentrate in the example of LNG. However, the ideas and techniques are readily extendable to other contexts.

One of the main differences between commodities and financial derivatives is the presence of the *cost of carry*. This factor consists of the cost of holding a position in a commodity. To enter in a long position in a commodity like rice, soy or oil there is the cost of storage, the cost

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of financing the position and the cash flow for owing the asset. The important implication of the cost of carry is that we do not have a non arbitrage price for futures prices of commodities, what we do have is a non-arbitrage interval. .
In this study we will focus on LNG market, we will study a spot price model for LNG and introduce some derivatives over LNG. LNG has received a lot of attention in recent years.
LNG "*per se*" is not directly considered a commodity, since it is not traded in any mercantile exchange. LNG trades are over-the-counter (OTC) and in general between two countries. LNG is not a local market, we can not try to find a proxy for the price in any specific markets, basically because the owner of the cargo is arbitrating among different markets.
In the Asian-Pacific market, LNG prices are indexed to crude oil prices. In Europe there are different indices, such as crude oil (note that crude oil prices are different in Asia and Europe), or a basket of indexes (like oil products, coal, inflation, among others). In the USA, LNG is usually indexed by Henry Hub gas prices.
For a literature on natural gas see [VJ06, Jen03, SLNvH05].
One of the main advantages of our model is that no data for LNG is used. We find the spot price based on the prices of the local market, which makes it possible to calibrate the model using the huge amount of data for local markets.
Once the spot price was modeled we are able to treat derivatives and introduce some contracts for LNG. The two contracts studied were futures for LNG and cancellation options. This second contract is uncommon in the literature. It is a contract that gives the owner the right to buy LNG in a given date and the right to cancel the contract paying some fee on certain dates. Some uses and motivation of this contract are shown.

2 Spot Price

Oftentimes, spot prices of energy commodities are not directly traded or available. The first step toward modeling commodities is to find its spot price. As it turns out, LNG trades are done over-the-counter (OTC), and in general take place between two countries. Furthermore we do not have any database regarding these trades. LNG is a global market, therefore we cannot find a proxy for the price in any specific markets, basically because the owner of the cargo is arbitrating among different markets.
Natural gas is a commodity in most countries. Several mercantile exchanges have derivatives over natural prices. Just to give one example, let us consider USA. The most used reference pricing point in the USA is Henry Hub which is the pricing point for natural gas contracts traded on the New York Mercantile Exchange (NYMEX). The NYMEX trades futures contracts for natural gas (called Henry Hub Futures), they also have options over Henry Hub futures. The natural gas prices are very well modeled in each market. We will use the fact that there is a model for natural gas in every country.
In order to model the market let us consider a specific producer like Nigeria, for example. Nigeria can sell LNG to any country, see figure 1. For instance, if it sells in the USA it will receive for the gas the spot price for natural gas in the USA, which in general is related to NYMEX Henry Hub prices. To sell in the USA the producer pays the netback costs (transport,liquefaction,re-gasification,...). The profit of selling to USA is then the Henry Hub price subtracted by the netback cost.

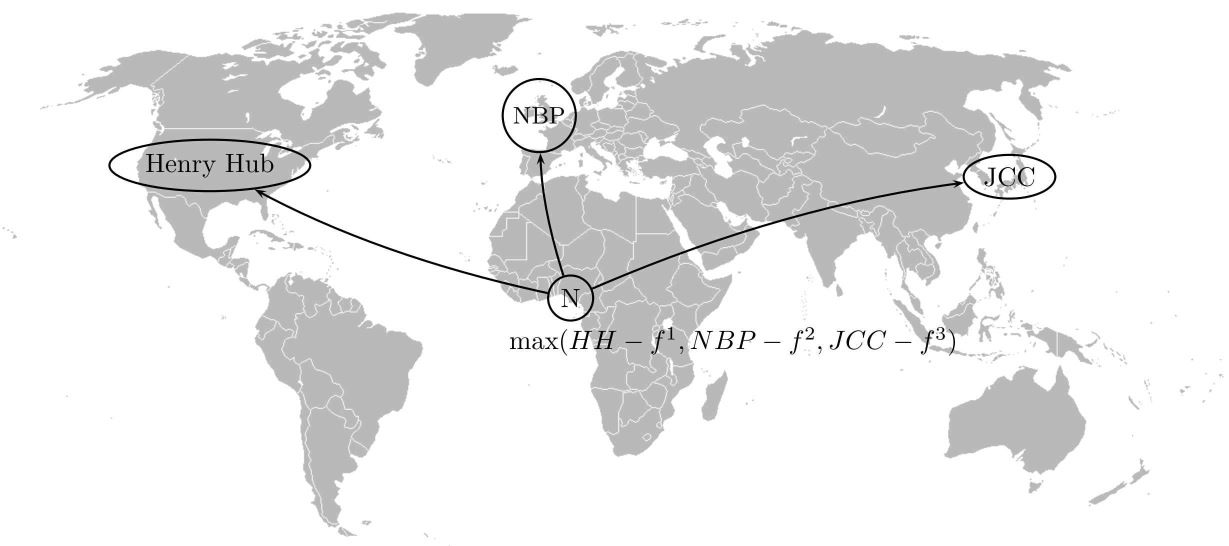

Figure 1: Spot Model Conceptual Map

3 The Model

Given a specific seller we can model the market. The price will be given by a profit maximization of this seller. Some basic market hypotheses will be needed:

- The seller has access to K markets, and pays $f^k \in \mathbb{R}$ of netback cost in order to sell to market k.
- The prices of natural gas are given by $S_t \in \left(\mathbb{R}^+\right)^K$, a K-dimensional stochastic process defined on on the filtered, probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$. In the k-th market natural gas is a commodity and the price will be given by the stochastic process S_t^k.

If the producer sells to market k the producer receives a profit of $S_t^k - f^k$. He tries to maximize profit, and the maximum profit is given by

$$G(S_t) = \max_{k=1,\ldots,K} \left(S_t^k - f^k\right) \tag{1}$$

If a buyer from market k wants to buy from this producer he will have to pay no less than $G(S_t)$ if he pays the netback costs or $G(S_t) - f^k$ if the cost is payed by the seller. In both cases the seller makes a profit of no less than $G(S_t)$.

This price rule implies that buying LNG is no better than buying in the local market. That is a simple consequence of the law of one price in each market. Each market has its own price, so the law of one price is not true globally, and the seller of LNG is doing arbitrage in global market. Once LNG is re-gasified and is inside a local market, it becomes natural gas, and so has the price given by local market.

In this model the amount of natural gas traded by LNG is small, so it does not affect local market prices. This is clearly a simplification.

This model reflects several aspects of the LNG market. In which every seller and buyer have different netback costs. It is common for a country to concentrate the demand for LNG,

which occurs when a market has a spike in prices for natural gas. In this case, this same country is the natural destiny for every free cargo.
Future prices of LNG are straightfoward once we have the spot price model:

$$F^G(t, T, S_t) = \mathbb{E}\left[G(S_T)|\, \mathcal{F}_t\right] \tag{2}$$

This is the most basic contract possible. It is a useful hedge alternative when you are sure of your demand in a specific time.

4 Cancellation Options

Futures contracts are not flexible enough to cover all the hedge needs which we addressed at the begining of this chapter. Cancellation options are an attempt to deal with wider hedging needs.
Its is natural to have a contract that gives the owner the right, but not the obligation to buy. This makes it possible to hedge against uncertainty in a future time.
We can generalize this flexibility. During the validity of the contract, the owner may receive some information that alows him to know beforehand that the cargo will not be needed. In this case he may want to cancel the cargo in advance.
We now define a contract that has these properties.

Definition: The cancellation option is a contract that gives the holder the following rights:

- At times $t_1, \ldots, t_N \leq T$ the holder has the right to cancel the contract paying fees $c_1, \ldots, c_N$ respectively.
- If the holder does not cancel the contract, at T he will buy the LNG cargo paying $A \cdot S_T^1 + B$, where A is a proportion of some benchmark market, and B is a fixed cost.

The value of a cancellation option at time t, for market prices S_t will be denoted by $V(t, S_t)$. To value this contract, first note that at delivery the value of the contract is:

$$V(T, S_T) = G_T(S_T) - \left(AS_T^1 + B\right). \tag{3}$$

Or if we can cancel at deliver $t_N = T$

$$V(T, S_T) = \max\left(G_T(S_T) - \left(AS_T^1 + B\right), -c_N\right). \tag{4}$$

To help fix the notation, consider the example of the cancellation option for which it is possible to cancel for $t = T$ as in equation (4). The payoff is shown in Figure 2.
We can find the value of the cancellation option backwards. To do this, we will first define auxiliar functions $V^n(t, S)$ defined on $[t_{n-1}, t_n] \times \mathbb{R}^K$. Given $t \in [t_{n-1}, t_n)$, then V^n is given by

$$V^n(t, S) = \mathbb{E}\left[V^n(S_{t_n}, t_n)|\, \mathcal{F}_t\right] \tag{5}$$

For t_n, with $n < N$ the value of V^n is given by

$$V^n(t_n, S_{t_n}) = \max\left(V^{n+1}(t_n, S_{t_n}), -c_n\right) \ . \tag{6}$$

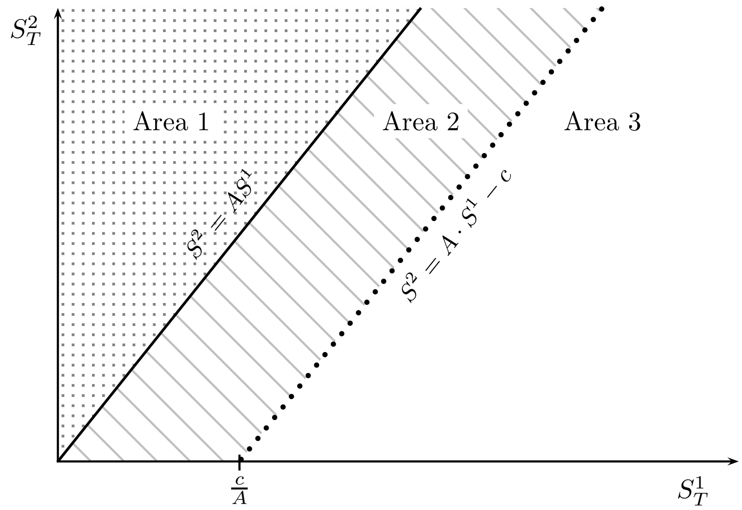

Figure 2: Payoff when it is possible to cancel at delivery time (4). The fee to cancel is c, and simplifying assume $B = 0$. In area 1, the payoff is positive, so it is not optimal to cancel. In area 2 the payoff is negative but better than the fee. In Area 3 the payoff is smaller than the cancellation fee, so it is optimal to cancel.

If the contract does not give the right to cancel at T, the final payoff is then given by (3) so

$$V^{N+1}(T, S_T) = G_T(S_T) - \left(AS_T^1 + B\right) \ , \tag{7}$$

where the $N+1$ interval is $[t_N, T]$, the time after the last cancelation date. When the contract gives the right to cancel at T then the final payoff is given by (4), then we have

$$V^N(T, S_T) = \max\left(G_T(S_T) - \left(AS_T^1 + B\right), -c_N\right) \ . \tag{8}$$

By means of such construction we are able to find the price for every $t \in [0, T]$. Where we have

$$V(t, S)|_{(t_{n-1}, t_n]} = V^n(t, S) \tag{9}$$

5 Facts

Theorem 5.1 *For a cancellation contract we have the following properties:*

1. *The contract value is nonincreasing in A*
2. *The contract value is nonincreasing in B*
3. *If for some $j < i$, we have that $c_j < c_i e^{-r(t_i - t_j)}$, then removing the cancellation date t_j does not affect the contract value.*

The results (*1*) and (*2*) of Theorem 5.1 show that if you increase the deliver price for LNG the price decrease. In other words, if you need to pay more money, the value of this contract is smaller. The result (*3*) establishes restrictions on the cancellation fee, which proves that we need increasing fees in order to make the cancellation dates meaningful.

We can also study the value of the cancellation optionality. In this case we must define the value of the contract without any cancellation possibility. The contract without cancellation possibility is simply a future given by

$$F^V(t,S_t) = \mathbb{E}\left[G_T - \left(AS_T^1 + B\right) \middle| \mathcal{F}_t \right] \ . \tag{10}$$

Using (10) we can define the optionality value as

$$O(t,S_t) = V(t,S_t) - F^V(t,S_t). \tag{11}$$

Some properties of the optionality value are given in the next theorem.

Corollary 5.2 *The optionality value has several properties:*

1. *The optionality value is always positive;*
2. *The optionality value is non decreasing in A;*
3. *The optionality value is non decreasing in B;*
4. *If for some $j < i$ we have that $c_j < c_i e^{-r(t_i - t_j)}$, then removing the cancellation date t_j does not affect the optionality value.*

The above result has an intuitive explanation. Property (*1*) one follows from the fact that the cancellation contract is an option over a future, so its value is always above the future value. Properties (1) and (2) of Theorem 5.1 show that the price of the contract drops with A and B, so the probability of a negative payoff increases, and therefore the value of the cancellation right is higher. This explains (2) and (3) of Corollary 5.2.

Proposition 5.3 *If $A \leq 1$ and $B \leq \min_{k=1,\dots,K}(-f_k)$ it is never optimal to cancel.*

Proof: At T the owner of the contract can receive a LNG cargo, paying $AS_T^1 + B < G_T(S)$. ∎

The above result shows a clearly case were the optionality has null value. When we have A and B as in Proposition 5.3 it is never optimal to cancel, so the value of the optionality is zero.

6 Least-Squares Method and Monte Carlo Simulations

We describe the method used to solve numerically the model. The model we choose was the regression-based Monte Carlo proposed by [LS01].
The convergence of the method, under fairly general conditions, was proved by [CLP02]. For exemple, considering only markovian processes S_t such that $S_t \in L^2(\Omega, d\mathbb{P})\ \forall t \in [0,T]$, then convergency of least-squeres Monte Carlo method for cancelation options follows from [CLP02].
The main idea of the method is to use the fact that

$$\mathbb{E}\left[V\left(t_{n+1}, X(t_{n+1})\right) \middle| \mathcal{F}_{t_n} \right] \tag{12}$$

is $\mathcal{F}_{t_n}$-mensurable, so it may be represented as

$$\mathbb{E}\left[e^{-r(t_{n+1}-t_n)}V(t_{n+1}, X(t_{n+1})) \mid X(t_n) = x\right] = \sum \beta_r \gamma_r(x) \ , \tag{13}$$

for some base $\{\gamma_r\}$ of the function space $L^2(\Omega, d\mathbb{P}, \mathcal{F}_{t_n})$. We can approximate (13) by

$$V(t_n^+, X(t_n)) \approx \sum_{n=1}^{N} \beta_r \gamma_r(X(t_n)). \tag{14}$$

Thus compute β we solve the following problem by least-squares

$$\begin{bmatrix} \gamma_1(X^1(t_n)) & \cdots & \gamma_R(X^1(t_n)) \\ \vdots & \ddots & \vdots \\ \gamma_1(X^J(t_n)) & \cdots & \gamma_R(X^J(t_n)) \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_R \end{bmatrix} = e^{-r(t_{n+1}-t_n)} \begin{bmatrix} V(t_{n+1}, X^1(t_{n+1})) \\ \vdots \\ V(t_{n+1}, X^J(t_{n+1})) \end{bmatrix} \tag{15}$$

Using equation (6) we have

$$V(t_n, X(t_n)) \approx \max\left(\sum \beta_r \gamma_r(X(t_n)), -c_n\right) \tag{16}$$

7 An Example

The goal of this section is to present some numerical examples. The model chosen was the mean reverting example which is very popular in commodities. The dynamic of the prices in this model is given by

$$\begin{aligned} S_t^i &= e^{X_t^i} \\ dX_t^i &= \kappa_i \left(\theta_i - X_t^i\right) dt + \sum_j A_{i,j} dW_j(t) \end{aligned} \tag{17}$$

The solutions is then

$$X_i(t) = e^{-\kappa_i(t-s)} X_i(s) + \theta_i \left(1 - e^{-\kappa_i(t-s)}\right) + \int_s^t e^{-\kappa_i(t-u)} \sum_j A_{i,j} dW_j(u) \tag{18}$$

It is easy to see that

$$\begin{aligned} \mathbb{E}\left[X_i\right] = \mu_i &= e^{-\kappa_i(t-s)} X_i(s) + \theta_i \left(1 - e^{-\kappa_i(t-s)}\right) \\ \operatorname{Cov}\left[X_i(t), X_j(t) \mid \mathcal{F}_s\right] &= \frac{1}{(\kappa_i + \kappa_j)} \left(1 - e^{-(\kappa_i+\kappa_j)(t-s)}\right) (AA^t)_{(i,j)} \end{aligned} \tag{19}$$

Taking $t \to \infty$ in the cova In this section we will present the results of the algorithm for the following parameters:

- Maximum degree of the base: 5
- $A = 1.0$, $B = 2.0$, $r = 0$
- fee=$(1, 1.25)$

- Cancellation dates $(0.5, 1.0)$ and $T = 1.5$
- $\kappa = (3, 3)$
- $X_0 = (3, 3)$
- $\theta = (3, 1.5)$
- Ergodic covariance: $\begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}$

The results in this section were produced by the $C++$ program of the previous section. This section focuses on numerical results.
We measured the computational time in order to calculate the option value. The results are expressed in Figure 3. The computational time indicates a linear increase of the computational time with the number of simulations. The time seems to increase polynomially with the number of elements in the base.

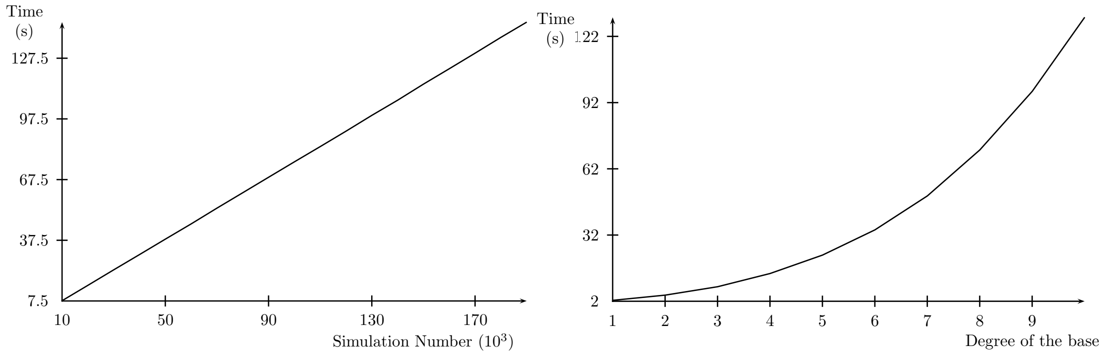

Figure 3: Computational time. In order to calculate the time for different base degrees we made 30.000 simulations. The algorithm seems to increase linearly with the number of simulations and polynomially in the base degree.

The algorithm converges for a large number of simulations. The option value calculated from 10.000 to 200.000 had a difference of no more than 5.12%. And if we only consider the results for more than 50.000 simulations the difference was no bigger than 1.5%. Prices also change with maximum base degree but again the result is robust, and no bigger than 6.3%.
The results can be seen in figure. 4

The final result is the numerical standard error of the estimator. To calculate this, a total of 20 samples of the algorithm were used, and the standard error was calculated with the usual estimator. As expected the standard error of the estimator decays with the number of simulations. The standard error for large bases seems to be bigger. The result can be seen in figure 5.

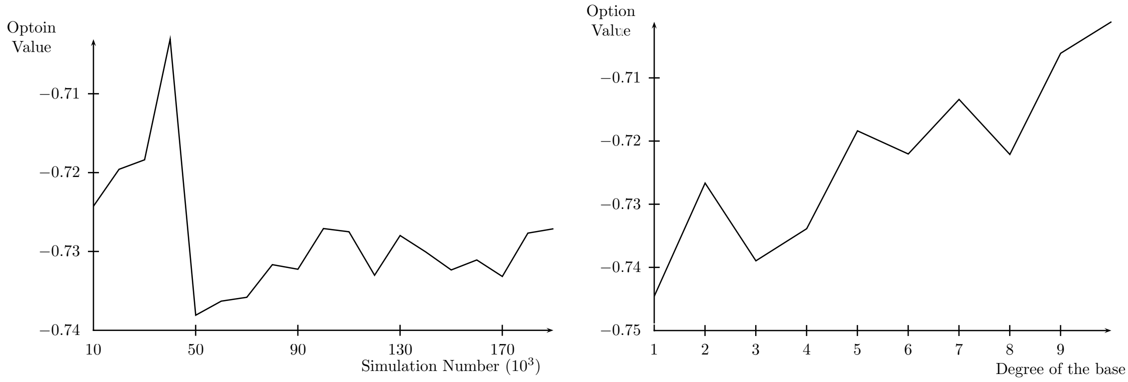

Figure 4: Accuracy. The change of the option price is not very sensitive to change in the number of simulations, and in the number of elements in the base. For the different bases we made 30.000 simulations.

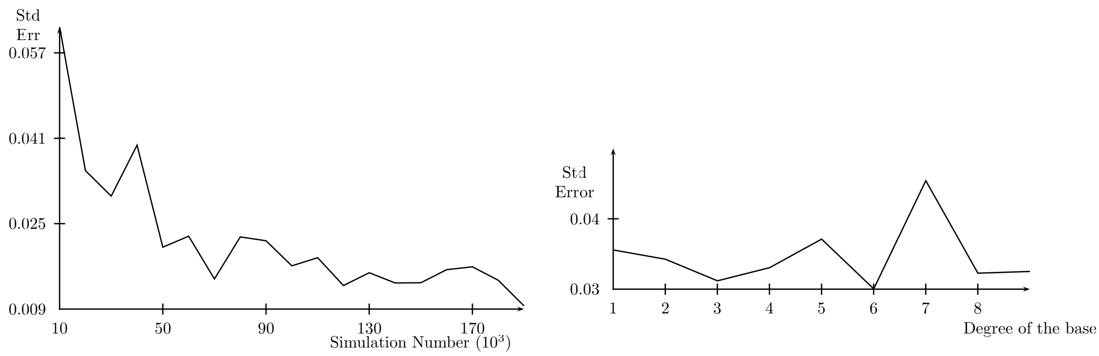

Figure 5: Standar error of the estimator as a function of the number of simulations, with a fixed degree basis, and of the degree of the basis, for a fixed nunber of simulations.

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