The Dynamics of Outsourcing and Integration*

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Abstract

This paper models the choice between outsourcing and integration for a firm that faces price uncertainty in both the upstream and downstream market. The firm can outsource the production of some input at an exogenous, stochastic price or it can produce the input internally at an average cost of production that is U-shaped. Up to three different production regimes can arise: pure outsourcing, pure integration, or a mixed regime where the firm produces the input internally up to some threshold quantity, and outsources all production in excess of this threshold. Investment in costly capacity narrows the range of output prices over which internal production is optimal because integrated production is more capacity intensive than outsourcing. The amount of capacity installed is determined – among other factors – by the cost of capital, the unit cost of capacity and the fraction of the time that the marginal unit of capacity is expected to be utilized. Switching back and forth between outsourcing and integration in response to demand shocks can be an efficient way for the firm to make optimal use of its capacity and to minimize production costs.

Keywords: outsourcing, vertical integration, real options, capacity, uncertainty

JEL: D24, D81, L22, L23, L24

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1 Introduction

Outsourcing is an important part of today's economic environment. Yet, it is a phenomenon that is still puzzling in a number of ways. First, there seems to be clear evidence that aggregate outsourcing activity within an economy is not constant over time, but occurs in waves and that these waves are to some extent linked to the state of the economy.[1] Second, Fan and Goyal (2006) shows that an increase in outsourcing can coincide with an increase in vertical merger activity. Several studies report evidence of a positive effect on firm performance of both outsourcing and vertical integration (e.g. Fixler and Siegel (1999), Ten Raa and Wolff (2001) and Kurz (2004)). Third, the extent to which an individual firm relies on outsourcing can vary dramatically over time, with firms switching back and forth between outsourcing and integration (in-house production). The above points are vividly illustrated by the following quotes:

"Businesses are using the fear of a downturn to press outsourcers to cut prices by up to 23 per cent... Demands for substantial discounts come as the outsourcing industry is facing something of a midlife crisis. High-profile outsourcing deals have been taken back in-house as some employers have begun to question the long-term savings." (Financial Times, March 13, 2008, page 30)

"In spite of the troubled global economy - or more likely, because of it - one of the few business sectors that continues to thrive is outsourcing. No wonder: Companies looking to cut expenses in the face of soft demand are keener than ever to hand off parts of their operations to lower-cost providers." (Business Week, May 4, 2009)

While both quotes stress the effect of the recent economic downturn for out-

[1]Although outsourcing has been used for more than a century, the first wave of outsourcing started in the 1970s and 1980s in the manufacturing sector. The reader is referred to Domberger (1999) for further details on past outsourcing activity.

sourcing activity, they are at odds about the direction of the effect. The first quote argues that firms respond to the downturn by taking outsourcing deals back in house, whereas the second argues exactly the opposite by saying that firms switch to outsourcing. We return to this apparent paradox later and provide a possible explanation. For now we merely point out that the quotes refer to different types of firms: the first quote refers to large firms only, whereas the second quote refers to outsourcing in aggregate, general terms.[2]

These empirical findings and quotes, while interesting raise many questions. Why is it the case that some firms respond to the economic downturn by switching to outsourcing, while other firms respond by taking outsourcing deals back in house? How is it possible that outsourcing as well as vertical integration can both create value in the same economic environment? Why would a firm first choose to outsource some of its production, later integrate its production process (or even acquire its supplier), and then further down the road decide to outsource again (or even divest this supply facility)? This paper develops a model that provides a rational explanation for some of the above observed behavior. The existing literature has not examined the dynamics of outsourcing and vertical integration. In a literature review chapter of vertical integration, Joskow (2005) highlights the need to understand better why organizations' modes of production change over time and how firms adapt to changing demand and supply conditions.

Our model is, however, not a "Theory of Everything" and some important aspects of outsourcing discussed elsewhere in the literature will not be considered. In particular, we ignore product market competition and its effect on outsourcing. We

[2]The first quote refers to a study by Compass Management Consulting which focuses exclusively on 100 outsourcing deals each worth more than 30 million pounds a year. The sample comprises outsourcing deals by large corporations that would be considered leaders in their industry. The second quote is based on data provided by the International Association of Outsourcing Professionals and refers to the overall demand for outsourcing activity by small, medium-sized and large firms.

neither consider the contractual nature of the relationship between a buyer and its supplier. Instead we assume that the firm is a price-taker in both the upstream and downstream markets. This allows us to focus on production costs and on a firm's investment in costly capacity under economic uncertainty, which are first order determinants of a firm's outsourcing decision.

The two main new features or contributions of our model are as follows. First, in our setting firms have the flexibility to produce part of the production in house, while simultaneously outsourcing the remainder. For example, firms may use outsourcing as an "overflow mechanism" once the firm's marginal cost of internal production exceeds the supplier's outsourcing price. This generates a number of new insights compared to the existing literature which traditionally assumes that firms can either outsource or produce in-house, but not both at the same time. Second, our paper studies the role of "strategic capacity reallocation" when firms operate in an uncertain economic environment. Since outsourcing is less capacity intensive then integration, uncertainty and sunk investment in capacity play an important role in the choice of production mode. This point is well known. What has not been recognized, however, is that firms, in response to economic shocks, can strategically reallocate capacity by switching back and forth between outsourcing and integration. For example, when idle capacity arises as a result of reduced demand this allows a firm to cut some of its outsourcing activities and move them back in house.

We now briefly sketch the model and its main results. We consider a price taking firm that produces a product A that can be sold in the market at an exogenously given unit price that varies stochastically over time according to the state of the economy. At any given time the economy can be in a boom or a recession. Switching between these 2 states is governed by a Poisson arrival process. For each unit of A the firm requires one unit of a product (or service) B as input. B can be produced internally, or it can be bought at an exogenously given price that varies according to the state of the economy. With outsourcing the average unit cost of product B is constant[3], whereas with vertical integration the average production cost of B is the standard U-shaped function of the quantity produced.[4]

If the purchase price for product B is below the minimum average cost of internal production, then outsourcing always dominates vertical integration. If the opposite is true then integration dominates outsourcing for levels of production (assuming investment in capacity is not too costly) that minimize the average cost of production. Interestingly, the firm does not adopt *pure* outsourcing for very high production levels. Instead of outsourcing *all* production, the firm produces part of the production in house, and outsources the remaining units for which the marginal cost of internal production exceeds the outsourcing price.

There can be up to three alternative production regimes. For low production levels the firm may outsource *all* production of product B (i.e. *pure* outsourcing). For intermediate output levels that minimize the average cost of internal production, the firm produces *all* units of B in house. Finally, for high production levels the firm produces a fixed threshold quantity of B internally and outsources any quantity in excess of the threshold. Outsourcing is therefore used as an "overflow" facility to deal with excess production. We believe this to be an important and original result.

As capacity becomes more costly, integration becomes more expensive relative to outsourcing and the output price range over which the firm prefers vertical integration shrinks. We show that the cost of a marginal unit of capacity is determined by the opportunity cost of capital, the unit cost of capacity and the fraction of the

[3]The constant price assumption is not strictly necessary. The results would still hold if the outsourcing price is a decreasing function of the quantity ordered, provided that the downward slope is not too steep.

[4]In our paper the U-shape is the result of a fixed cost of production combined with a technology that is subject to increasing marginal costs of production.

time that this marginal unit of capacity is utilized. If capacity is cheap or if it is in use most of the time (i.e. recessions are short relative to booms) then the firm does not mind having idle capacity during recessions. If, however, capacity is expensive or if downturns last relatively long then the firm may limit its investment in capacity to ensure capacity is fully utilized in booms and in recessions. But, how can the firm keep operating at full capacity if the output in recession is less than in booms? In response to a negative demand shock, the firm can keep operating at full capacity by switching production regime and by reallocating capacity. In particular when demand is very high in booms, the firm can limit its capacity needs by using outsourcing as an overflow mechanism. When output then drops in recession, the firm can fill the resulting excess capacity by cutting back on some or all of its outsourcing. This type of capacity reallocation is available to firms that are operating above their ideal firm size (i.e. the average cost of production is increasing) during booms because they can switch to a more capital intensive production regime in recession and in the process of doing so decrease the average production cost. The situation is more problematic for an integrated firm that operates around the ideal firm size in booms. For this firm a large cut in output increases the average cost of internal production, which could make outsourcing preferable to vertical integration. However, a switch to outsourcing could then lead to substantial overcapacity: the firm not only faces a cut in output but it also stops producing product B internally. This illustrates an inherent vulnerability of integrated firms that operate at the ideal firm size during booms, compared to larger firms that operate above the ideal firms size during booms.

Returning to the two quotes at the start of this introduction, we now have a possible explanation why, in response to a downturn, the largest firms switch to integration, whereas other firms may switch to outsourcing. In our model large firms respond to a downturn by cutting outsourcing and moving activities in house.

A substantial drop in output means that these firms also experience a sharp fall in their marginal cost of internal production. Consequently, for these firms to stick to outsourcing during recession, a substantial discount would have to be offered by the supplier. This may explain the request of large firms for the quoted discounts up to 23%. On the other hand, medium sized firms suspend in-house production and switch to pure outsourcing in our model if their reduced output level in recession no longer justifies the fixed cost of internal production. Finally, small firms outsource during booms because they lack the scale to produce in house. In recession they become even smaller and, as a result, cut some of their outsourcing deals.

The previous discussion focuses on the effect of a sharp drop in the downstream market's output price. Note that the effect of a price drop in the upstream market can be just as dramatic. For example, a sharp drop in the price of the supplier's market can trigger a wave of outsourcing. A clear example of this is the outsourcing wave to low cost countries such as China and India.

We conclude with a brief review of the literature and contrast it with our paper. Models from the *incomplete contract* literature, as Grossman and Helpman (2002), McLaren (2000), Antras and Helpman (2004) and Grossman and Helpman (2005) assume that vertically integrated firms face higher marginal and fixed cost of production than specialized suppliers because vertically integrated firms "have many divisions to manage" and do not benefit from the learning that comes with specialization in a single activity. In these models this cost advantage from outsourcing is balanced against the costs that arise from incomplete contracting. In our model, we abstract from incomplete contract considerations and neither assume that internal production always involves higher marginal costs of production.

On the other hand, the *strategic outsourcing* literature (or models that have highlighted the role of strategic competition for a firm's decision to choose a particular production mode) does not always support the idea that suppliers have lower marginal cost of production. While Chen (2001), Chen (2005) and Chen, Ishikawa, and Zhihao (2004) recognize that specialization provides suppliers with lower marginal costs of production, Shy and Stenbacka (2003) and Buehler and Haucap (2006) assume that there is a mark-up in the suppliers' market due to imperfect competition. The production decision in both sets of papers involves a trade-off between incurring a fixed cost under vertical integration and the option to avoid this cost under outsourcing.[5] We also assume that by outsourcing the firm avoids the fixed cost of internal production, but we do not explicitly model product market competition as a balancing force. Instead, we focus on the role of economic shocks and costly capacity for the firm's choice of production mode.

Finally, our work is also related to the real options literature that studies a firm's optimal capacity choice. Seminal papers on this topic are Abel (1983, 1984) and Pindyck (1988) (see also Dixit and Pindyck (1994) for a review). Van Mieghem (1999) uses a real options approach to calculate the option value of subcontracting when investment in capacity for in-house production is costly. He analyzes outsourcing conditions for three different types of contracts between the manufacturer and the subcontractor. We do not consider contractual aspects of outsourcing.

The remainder of the chapter is organized as follows. Section 2 presents the basic assumptions of the model and examines the optimal production decision assuming that capacity is costless. Section 3 analyzes the optimal production and capacity decisions assuming costly capacity. In this section we discuss the optimal production choice for two cases: one where there is idle capacity in recessions and another where there is no idle capacity, and therefore the firm always operates at full capacity.

[5] A report by CAPS and A. T. Kearney (2005) documents that important cost-related reasons to outsource include the reduction in operating costs and in capital investment and the possibility of turning fixed costs into variable costs.

Section 4 concludes and summarizes some empirical predictions of our theory.

2 Outsourcing and integration with costless capacity

Consider a firm that sells product A. The production of A requires product (or service) B as input. Without loss of generality, we assume that one unit of B is required to produce one unit of A. The firm can either buy units of B (outsourcing) or make product B in house (integration). The firm can also combine outsourcing with integration by producing some units of B in house, and by outsourcing the remainder (mixed integration-outsourcing).

We assume that the firm's profit function under *pure* integration (i.e. when *all* units of B are produced internally) is given by:

$$\pi = p_A q - c_A q^2 - f_A - c_B q^2 - f_B \tag{1}$$

where p_A is the price per unit of product A and q is the quantity of unit A that is produced. f_A and f_B are fixed costs associated with the production of product A and B, respectively. $c_A q^2 + c_B q^2$ is the total variable cost associated with producing q units of A in house. The average cost function associated with the integrated production mode is then:

$$AC(q) = (c_A + c_B) q + \frac{f_A + f_B}{q} \tag{2}$$

The average cost is the traditional U-shaped function of quantity.

We assume that the firm's profit function under *pure* outsourcing (i.e. when *all* units of B are bought) is given by:

$$\pi = p_A q - c_A q^2 - f_A - p_B q \tag{3}$$

where p_B is the price per unit of product B. Since the production of q units of A requires q units of B as input, the total cost associated with producing q units in house is now $c_A q^2 + f_A + p_B q$. Note that under outsourcing the firm avoids not only the variable cost $c_B q^2$, but also the fixed cost f_B. The average cost of production is again a U-shaped function in q. The presence of the fixed production cost f_A means that there are initially economies of scale, because the fixed cost can be spread over a larger amount of units produced. However, since the marginal cost of production is increasing, at some point diseconomies of scale kick in. Our assumptions essentially boil down to the idea that there is an 'ideal' firm size.

Our cost assumptions allow us immediately to derive the profit function when the firm combines outsourcing with integration. If the firm produces Q^{**} units of B in house and buys the remaining $(q - Q^{**})$ then the profit function under the mixed production strategy is given by:

$$\pi = p_A q - c_A q^2 - f_A - c_B {Q^{**}}^2 - f_B - p_B \left(q - Q^{**}\right) \tag{4}$$

The general profit function can therefore be written as:

$$\pi(q; \varphi_2, \varphi_3) = p_A q - c_A q^2 - f_A - p_B q \left(1 - \varphi_2\right) - \varphi_2 \left(c_B q^2 + f_B\right) - \varphi_3 \left(c_B {Q^{**}}^2 - p_B Q^{**} + f_B\right) \tag{5}$$

where

$$\begin{aligned} \varphi_2 &= 1 \text{ under pure integration and zero otherwise} \\ \varphi_3 &= 1 \text{ under mixed outsourcing/integration and zero otherwise.} \end{aligned} \tag{6}$$

The problem to be solved can now be formulated as follows. What output level q will the firm produce and what production mode will be adopted (pure outsourcing, pure integration or mixed outsourcing/integration) in order to maximize the profit function $\pi(q; \varphi_2, \varphi_3)$? We solve the problem assuming that the firm is a price taker (i.e. p_A and p_B are exogenously given). For the moment we assume that p_A and

p_B are fixed, but we consider later the more interesting case where p_A and p_B can be stochastic. In what follows we refer to pure outsourcing ($\varphi_2 = \varphi_3 = 0$), pure integration ($\varphi_2 = 1$; $\varphi_3 = 0$) and combined outsourcing/integration ($\varphi_2 = 0$ and $\varphi_3 = 1$) as regimes 1, 2, and 3, respectively. π_i and q_i denote the profit and output level under regime i (with $i = 1, 2$ or 3). The solution to the above optimization problem is given in the following proposition (the proof is in the appendix):

Proposition 1 *Production under pure outsourcing (regime 1), pure integration (regime 2), or integration combined with outsourcing (regime 3) can only be viable if the output price p_A exceeds a minimum threshold, which is given respectively by:*

$$p_{A1min} \equiv p_B + 2\sqrt{c_A f_A} \tag{7}$$

$$p_{A2min} \equiv 2\sqrt{(c_A + c_B)(f_A + f_B)} \tag{8}$$

$$p_{A3min} \equiv p_B + 2\sqrt{c_A\left(f_A + f_B - \frac{p_B^2}{4c_B}\right)} \tag{9}$$

If $p_B \leq 2\sqrt{c_B f_B} \equiv \widehat{p}_B$ then pure outsourcing is optimal for all price levels p_A at which production is viable. If $\widehat{p}_B < p_B$ then :

regime 1 is optimal for $p_A \in [p_{A1min}, p_A^*]$.

regime 2 is optimal for $p_A \in]\max[p_A^*, p_{A2min}], p_A^{**}]$.

regime 3 is optimal for $p_A \in]\max[p_A^{**}, p_{A3min}], +\infty[$.

The price level p_A^* (p_A^{**}) at which it is optimal to switch from regime 1 to regime 2 (from regime 2 to regime 3) is given by:

$$p_A^* = p_B\left(1 + \frac{c_A}{c_B}\right) - \frac{\sqrt{(p_B^2 - 4c_B f_B)(c_A + c_B)c_A}}{c_B} \tag{10}$$

$$p_A^{**} = p_B\left(1 + \frac{c_A}{c_B}\right) \tag{11}$$

The optimal output level is given by: $q^o = \frac{p_A - p_B(1-\varphi_2)}{2(c_A + \varphi_2 c_B)}$. The optimal output levels *at p_A^* and at p_A^{**} are:*

$$\begin{aligned} Q^* &\equiv q_1^o\left(p_A^*\right) = \frac{p_B}{2c_B} - \frac{\sqrt{\left(p_B^2 - 4c_B f_B\right)\left(c_A + c_B\right)c_A}}{2c_A c_B} \\ &< q_2^o\left(p_A^*\right) = \frac{p_B}{2c_B} - \frac{\sqrt{\left(p_B^2 - 4c_B f_B\right)\left(c_A + c_B\right)c_A}}{2c_B\left(c_A + c_B\right)} \quad (12) \\ Q^{**} &\equiv q_2^o\left(p_A^{**}\right) = q_3^o\left(p_A^{**}\right) = \frac{p_B}{2c_B} \quad (13) \end{aligned}$$

Proposition 1 conveys a number of important insights about the economics of outsourcing and vertical integration.

[Please insert Figure 1 about here.]

A key determinant in the make or buy decision is the price p_B at which the product B can be bought versus the cost at which it can be produced. This comparison is reflected in the condition $p_B \gtrless 2\sqrt{c_B f_B} \equiv \widehat{p}$, where the right hand of the inequality is determined by c_B and f_B, the parameters of the cost function for product B. The proposition states that if $p_B \leq \widehat{p}$ then outsourcing always dominates vertical integration because it is cheaper to buy than to make for all possible output levels q. The intuition behind this result is illustrated in Figure 1 which compares the cost of making versus buying q units of product B. The straight lines represent the cost $p_B q$ of outsourcing the production of q units of B for different levels of p_B $(p_B = p'_B, p''_B, p'''_B)$. The convex curve is the cost of producing q units of B in house. Since $p'''_B < 2\sqrt{c_B f_B}$ it is cheaper for all levels of q to outsource rather than to produce in house. In fact one can show that $p_B \leq \hat{p}_B \iff AC(q)_2 - AC_1(q) \geq 0$ for all q. In other words, $p_B \leq \hat{p}_B$ if and only if the average production cost under pure integration exceeds the average cost under outsourcing for all output levels.

For $p'_B > 2\sqrt{c_B f_B}$ it is cheaper to produce q units through pure integration than through pure outsourcing for $q \in \left]\widetilde{Q}\,,\widehat{Q}\right[$. It is important to point out that

$\widetilde{Q}$ is not equal to the output level Q^* at which the firm optimally switches from pure outsourcing to pure integration. The switching happens optimally at the price p_A^* where the optimal (i.e. maximum attainable) profit under regime 1 equals the optimal profits under regime 2. This is not equivalent to imposing an equality between the cost of outsourcing and the cost of pure integration because at the optimal switching point p_A^* there is a discrete jump in the output level (i.e. $q_1^o(p_A^*) = Q^* < q_2(p_A^*)$). Consequently, at p_A^* the optimal profits from outsourcing and from integration relate to two different output levels (see condition (14) below). In fact, it is easy to show that $Q^* < \widetilde{Q}$. Note that at $\widetilde{Q}$, the marginal cost from outsourcing equals p'_B, which is strictly higher than the marginal cost from integration. Since $Q^* < \widetilde{Q}$, the convexity of the cost curve under integration implies that the marginal cost of integration at Q^* is strictly less than p'_B the marginal cost from outsourcing, causing the discrete jump in output at Q^*.

The output level at which the firm optimally switches from regime 2 (pure integration) to regime 3 (integration combined with outsourcing) is given by $Q^{**}(p_B)$. At this point the marginal cost of outsourcing (p_B) equals the marginal cost of integration ($2c_BQ^{**}$). Therefore at Q^{**} the gradients of the cost curves associated with outsourcing and integration must be the same, as is illustrated in Figure 1 for $Q^{**}(p'_B)$ and $Q^{**}(p''_B)$. Since there is no jump in the optimal production level at Q^{**} (i.e. $q_2^o(p_A^{**}) = q_3^o(p_A^{**})$), the profit functions value match and smooth paste at p_A^{**}, and Q^{**} is therefore the optimal switching quantity. Note that Q^{**} is increasing in p_B: the higher p_B, the longer the firm sticks to pure integration. The convexity of the costs curve associated with in house production of product B, ensure however, that for sufficiently high quantity levels the firm will always want to combine integration with outsourcing.

Another important determinant in the make or buy decision is the output price

p_A. The price at which the firm can sell the finished production determines the marginal revenue and therefore the optimal output level. The optimal output level is monotonically increasing in p_A, creating a link between p_A and the optimal production regime.

The price level p_A^* at which the firm optimally switches from pure outsourcing to pure integration satisfies the following value-matching condition:

$$\pi_1(q_1^o(p_A^*) \ = \ \pi_2(q_2^o(p_A^*)) \tag{14}$$

As pointed out before, this switch entails a *discrete* upward jump in output since $q_1^o(p_A^*) \ = \ Q^* \ < \ q_2^o(p_A^*)$.

The price level p_A^{**} at which the firm switches from regime 2 to regime 3 satisfies the following value-matching and smooth-pasting conditions:

$$\pi_2(q_2^o(p_A^{**})) \ = \ \pi_3(q_3^o(p_A^{**})) \quad \text{and} \quad \frac{\partial \pi_2\left(q_2^o\left(p_A\right)\right)}{\partial p_A}\Big|_{p_A=p_A^{**}} = \frac{\partial \pi_3\left(q_3^o\left(p_A\right)\right)}{\partial p_A}\Big|_{p_A=p_A^{**}}$$

The optimal profit function is therefore not only continuous but also differentiable at p_A^{**}. One can show (see appendix) that the smooth-pasting condition is equivalent to the condition $q_2^o(p_A^{**}) \ = \ q_3^o(p_A^{**})$. For the switch from regime 2 to regime 3 to be optimal at p_A^{**} there should not be a jump in the optimal output level at the switching price p_A^{**}. This result is in sharp contrast with the behavior at p_A^* where a switch from regime 1 to regime 2 coincides with a jump in the optimal output level.

Figure 2 plots the optimal output level q^o as a function of the output price p_A. For $p_A < p_{A1min}$ the firm does not produce at all (i.e. $q^o = 0$). For $p_{A1min} \leq p_A < p_A^*$ the firm outsources the production of B. The optimal output level increases linearly in p_A with $\frac{\partial q_1^o}{\partial p_A} = \frac{1}{2c_A}$. For $p_A^* \leq p_A \leq p_A^{**}$ product demand is sufficiently high to make it economically worthwhile to produce product B in house. Note the discrete increase in output at p_A^*. In regime 2, the optimal output level increases linearly in p_A, but at a slower rate ($\frac{\partial q_2^o}{\partial p_A} = \frac{1}{2(c_A+c_B)}$), reflecting the increasing marginal cost of in house production. For $p_A^{**} < p_A$ the firm combines integration with outsourcing. A quantity Q^{**} of product B is produced in house, and the residual quantity $q - Q^{**}$ is outsourced. As a result the optimal output level increases linearly in p_A at the same rate as in the pure outsourcing regime (i.e. $\frac{\partial q_3^o}{\partial p_A} = \frac{\partial q_1^o}{\partial p_A} = \frac{1}{2c_A}$).

[Please insert Figure 2 about here.]

It should be pointed out that all three regimes do not necessarily exist. The lower bound for each interval for which a particular regime i ($i = 1, 2, 3$) is optimal depends on the minimum viable threshold p_{Aimin}. Consequently, if the lower bound exceeds the upper bound of the interval then the interval is empty and the regime does not occur. For example, if $p_{A1min} > p_A^*$ then regime 1 does not occur. Furthermore, the interval $[p_A^*, p_A^{**}]$ converges to an empty set as p_B drops below $2\sqrt{c_B f_B}$. In other words, if p_B is sufficiently low then producing input B in house is never optimal.

The results lend support to the notion of an "ideal" firm size. The firm can only efficiently produce B for output levels in the range $[q_2^o(p_A^*), q_2^o(p_A^{**})]$. It is more efficient to buy quantities below $q_2^o(p_A^*)$, and to outsource any production in excess of $q_2^o(p_A^{**})$.

One might argue that it is always optimal for the firm to produce Q^{**} of quantity B, even if the demand for product A is lower. The argument would be that the firm could simply sell any surplus production of product B in the market at p_B. This scenario is, however, unlikely in practice. First, the firm would incur costs associated with selling product B, and may therefore be unable to achieve p_B net of all costs. Second, it is neither obvious that the firm could generate the required demand for product B. In what follows we therefore ignore the possibility of the firm actually selling product B. Furthermore, as pointed out before, the problem becomes trivial in that case.

So far we have ignored the capacity dimension of the production decision. This was possible because we implicitly assumed that capacity is costless and available without limits. This assumption will be relaxed in next section, but before we can do so we need to specify what the firm's capacity needs are to produce a unit of output through outsourcing versus integration.

Assumption 1 *The firm needs* 1 *(*2*) unit(s) of capacity per unit of output produced through outsourcing (integration).*

The assumption implies that the firm needs one unit of capacity to produce one unit of product B in house. As a result vertical integration is twice as capital intensive as pure outsourcing.[6] In general the capacity required to produce an output level q is therefore given by:

$$K = q(1+\varphi_2) + \varphi_3 Q^{**} \quad (15)$$

where φ_2, φ_3 and Q^{**} are as previously defined. The required capacity levels in regimes 1, 2 and 3 are therefore respectively q, $2q$ and $q + Q^{**}$. For simplicity we implicitly assume that all capacity units are homogenous in that they can be used both towards the production of the input and the final product.[7] Figure 2 plots the behavior of the optimal capacity level as a function of p_A. Notice the enormous jump in capacity required when the firm switches from pure outsourcing to pure integration. The increase in capacity equals $K_2(p_A^*) - K_1(p_A^*) = 2q_2 - q_1 = q_2 + (q_2 - q_1)$. The increase in capacity equals the output level q_2 plus the increase in

[6]The assumption is without loss of generality, as we could easily reformulate the problem to allow the production of B to be more or less capacity intensive.

[7]In reality the degree of transferability may be less than 100%. While warehouse and office space, for example, can easily be reallocated, some machines and equipment may have a more specific use and be more difficult to reallocate across the production process. A more complete model that distinguishes between capacity that can and cannot be reallocated is left as a topic for future research.

output $(q_2 - q_1)$ at the switching point. This massive increase in capacity will have important implications in next section where we discuss the case of costly capacity.

So far we have assumed that the prices p_A and p_B are fixed and known with certainty. We now generalize the model to allow prices to vary with the state of nature. For simplicity we restrict the model to two states: booms and recessions. When the industry is in a boom (recession), recession (boom) arrives according to a Poisson process with parameter $\overline{\lambda}$ ($\underline{\lambda}$). In booms the prices for product A and B are given by $\overline{p}_A$ and $\overline{p}_B$, whereas in recessions the prices are respectively $\underline{p}_A$ and $\underline{p}_B$.

Define $\overline{V}_{ij}$ $\left(\underline{V}_{ij}\right)$ as the firm value in booms (recessions) when the firm adopts production regime i in booms and regime j in recessions, when i, j can take on the values 1, 2 or 3. Assume that investors are risk neutral and can invest in a risk-free asset with a rate of return r. In equilibrium the value of the firm in booms and in recessions is the solution to:

$$
\begin{aligned}
r\overline{V}_{ij} &= \overline{\pi}_i + \overline{\lambda}\left[\underline{V}_{ij} - \overline{V}_{ij}\right] \\
r\underline{V}_{ij} &= \underline{\pi}_j + \underline{\lambda}\left[\overline{V}_{ij} - \underline{V}_{ij}\right] \qquad (16)
\end{aligned}
$$

Solving for $\overline{V}_{ij}$ and $\underline{V}_{ij}$ gives:

$$\overline{V}_{ij} = \frac{\overline{\pi}_i}{r}\left(1 - \overline{p}\right) + \frac{\underline{\pi}_j}{r}\overline{p} \qquad (17)$$

$$\underline{V}_{ij} = \frac{\underline{\pi}_j}{r}\left(1 - \underline{p}\right) + \frac{\overline{\pi}_i}{r}\underline{p} \qquad (18)$$

where

$$\overline{p} \equiv \frac{\overline{\lambda}}{r + \overline{\lambda} + \underline{\lambda}} \quad \text{and} \quad \underline{p} \equiv \frac{\underline{\lambda}}{r + \overline{\lambda} + \underline{\lambda}} \qquad (19)$$

$\overline{p}$ ($\underline{p}$) can be interpreted as the probability of the economy switching into recession (boom) given that it is currently in a boom (recession). As such the expressions for the firm values are very intuitive. For example $\overline{V}_{ij}$ is a weighted average of two perpetuities: $\frac{\overline{\pi}_i}{r}$ and $\frac{\underline{\pi}_j}{r}$. The former perpetuity represents the value of remaining in boom (and regime i) forever, whereas the latter represents the value of remaining in recession (and regime j) forever. The weights are given by the probability of staying in booms $(1-\overline{p})$ and the probability of switching to recession $(\overline{p})$.

The optimal production regime adopted in each state can be determined according to proposition 1. In booms, one determines the optimal switching points $\overline{p}_A^*$ and $\overline{p}_A^{**}$ by substituting p_A and p_B in proposition 1 by $\overline{p}_A$ and $\overline{p}_B$. For example, $\overline{p}_A^{**} = \overline{p}_B\left(1+\frac{c_A}{c_B}\right)$. The optimal production regime is determined analogously. For example, if $\widehat{p} < \overline{p}_B$ then:

regime 1 is optimal for $\overline{p}_A \in [\overline{p}_{A1min}, \overline{p}_A^*]$.

regime 2 is optimal for $\overline{p}_A \in]\max[\overline{p}_A^*, \overline{p}_{A2min}], \overline{p}_A^{**}]$.

regime 3 is optimal for $\overline{p}_A \in]\max[\overline{p}_A^{**}, \overline{p}_{A3min}], +\infty[$.

The optimal production regime in recessions can be determined analogously by replacing p_A and p_B in proposition 1 by $\underline{p}_A$ and $\underline{p}_B$. The dynamic optimization problem can therefore be solved by solving two static optimization problems (i.e. maximizing $\overline{\pi}_i$ and $\underline{\pi}_j$ in booms and recessions, respectively).

The simple nature of this solution and the reason for the optimality of 'myopic' behavior follows from the fact that there are no costs involved in changing the output level. In reality output capacity may be costly: it may be costly to increase the firm's production capacity and to have idle capacity. We discuss the case of costly capacity in next section and focus for the moment on the insights we can gain from the costless capacity model.

Proposition 1 shows that, all else equal, changes in p_A (and p_B) not only lead to changes in the optimal output level but potentially also in the production regime. A drop in p_A may cause the firm to switch from regime 3 to regime 2, or even regime 1. Furthermore, a drop in p_A (all else equal) creates excess capacity. This result is, of course, derived under the assumption that capacity is costless. We show later that when capacity is sufficiently costly, the firm may avoid excess capacity by adopting a much lower capacity level such that in booms and recessions the capacity is always fully utilized. Still, in what follows we find that when capacity is sufficiently cheap, there will be some idle capacity in recession. The problem of idle capacity is most obvious for price variations around p_A^*, where it could be the case that in recession more capacity is idle than in use. Another such critical point is the lowest viable price threshold p_{A1min} (or p_{A2min}), where the firm could stop producing altogether and all capacity is idle if output prices drop below this threshold.

Changes in p_B can have important effects too. All else equal if p_B drops below $\widehat{p}$ then the possibility of vertical integration could be eliminated for all levels of p_A. For example, if $\underline{p}_B < \widehat{p} < \overline{p}_B$, and $\overline{p}_A^* < \overline{p}_A < \overline{p}_A^{**}$, and output prices are viable then a switch from booms to recession implies a switch from vertical integration to outsourcing, irrespective of the output price level $\underline{p}_A$ in recession. This could, for example, explain the wave of outsourcing in the west under impulse of cheap export by countries like China and India.

3 Outsourcing and integration with costly capacity

In this section we examine the case where investment in capacity is costly and irreversible. We introduce therefore the following assumption:

Assumption 2 *Investment in capacity requires a constant and sunk cost k per unit of capacity.*

Without loss of generality, we assume that $\overline{p}_A \geq \underline{p}_A$, which means that market conditions with respect to selling product A are at least as good in booms as in recessions. Furthermore, we assume that the price p_B cannot drop "too" much, i.e. $\overline{p}_B - \underline{p}_B \leq \theta$ (where $\theta \geq 0$) so that is never optimal for firms to invest in extra capacity when the economy switches from a boom to a recession. (If this assumption is violated then one has to switch the labels from booms and recessions across states.)[8] Assume for simplicity that the game starts when the economy is in a boom so that the investment equilibrium is immediately reached.[9] When the firm enters the market in a boom it maximizes $\overline{NPV_{ij}} = \overline{V}_{ij} - kK$, where the value of the firm in booms, $\overline{V}_{ij}$, is as defined by equation (17). The firm has to determine the optimal capacity level K, the optimal output level in booms and recession $(\overline{q}, \underline{q})$, and the production regime adopted in booms (i) and recession (j). From previous section (see equation (15) we know that the following relation must be satisfied between capacity and output: $K = \overline{q}\,(1 + \overline{\varphi}_2) + \overline{\varphi}_3 \overline{Q}^{**}$ where $\overline{Q}^{**}$ is defined as the output level in booms at which the firm is indifferent between regime 2 and regime 3. Equivalently, we can define $\overline{K}^{**} = 2\overline{Q}^{**}$ as the capacity level at which the firm switches in booms from regime 2 to regime 3. As before $\overline{Q}^{**}$ needs to be determined as part of the solution to the problem.

The optimal investment strategy can take on two possible forms. A first possible outcome is the one where it is optimal for the firm to have some idle capacity in recession. A second possible outcome is that all capacity is fully used in booms as well as recessions. Loosely speaking the first strategy will be optimal if the cost per unit of capacity is small and if booms last sufficiently long. If, however, recessions last relatively long and capacity is quite costly then it is too expensive for the firm to have idle capacity during recession and the firm finds it optimal to adopt a capacity level that is fully utilized at all times.

[8]How high θ can be depends on the other parameters of the model. It is clear, however, that for $\theta = 0$ (and hence $\overline{p}_B \leq \underline{p}_B$), it can never be optimal to install extra capacity when switching from a boom to a recession.

[9]If the game starts in a recession then the firm may initially adopt a lower capacity level, and move to full capacity when the economy subsequently switches to a boom.

The solution method for each case is quite different, and we therefore cover each case separately. We start off with the case where some capacity is idle in recession, and subsequently consider the 'no-idle' capacity case.

3.1 Investment strategy with idle capacity in recession

If the firm has idle capacity in recession then capacity does not act as a constraint on the firm's output decision in recession and the firm can therefore choose its output in recession as if capacity is costless. This is similar to the case we analyzed in section 2, and the optimal output and production strategy is therefore as described in proposition 1. This strategy allows the firm to achieve in recession the optimal 'unconstrained' profit level that was characterized by proposition 1. In what follows we denote this profit level by $\underline{\pi}^o$. The optimality of $\underline{\pi}^o$ implies that $\frac{\partial \underline{\pi}^o}{\partial K} = \frac{\partial \underline{\pi}^o}{\partial \underline{q}} \frac{\partial \underline{q}}{\partial K} = 0$. In other words, at the optimal capacity level, the marginal profit in recession from an increase in capacity is zero because the capacity constraint is not binding.

Expressing K as a function of $\overline{q}$ allows us to formulate the optimization problem as a function of $\overline{q}$:

$$\underset{\overline{q},\overline{\varphi}_2,\overline{\varphi}_3}{Max} \ \frac{\overline{\pi}\left(\overline{q}\right)}{r}\left(1 - \overline{p}\right) + \frac{\underline{\pi}^o}{r}\overline{p} - k\left[\overline{q}\left(1+\overline{\varphi}_2\right) + \overline{\varphi}_3\overline{Q}^{**}\right] \tag{20}$$

The firm therefore optimizes with respect to the output level in booms ($\overline{q}$) as well as the production regime adopted (as reflected by the values for $\overline{\varphi}_2$ and $\overline{\varphi}_3$). The solution to this optimization problem is given in the following proposition.

Proposition 2 *If capacity is costly and if it is optimal for the firm to have some idle capacity in recession, then the output decision in recessions is as described in proposition 1 (but with p_A and p_B substituted by $\underline{p}_A$ and $\underline{p}_B$, respectively). In booms, investment in capacity is optimal only if the output price $\overline{p}_A$ exceeds a minimum* *threshold* $\overline{p}_{Aimin}$ *where* $\overline{p}_{Aimin}$ *is the solution to* $\overline{NPV}_{ij}(\overline{p}_{Aimin}) \equiv \overline{V}_{ij}(\overline{p}_{Aimin}) - kK = 0$.

If $\overline{p}_B \leq 2\sqrt{c_B f_B} + \frac{rk}{1-\overline{p}} \equiv \widehat{p}'_B$ *then pure outsourcing is optimal in booms for all price levels* $\overline{p}_A$ *at which production is viable. If* $\widehat{p}'_B < p_B$ *then in booms:*

regime 1 is optimal for $\overline{p}_A \in [\overline{p}_{A1min}, \overline{p}^*_A]$.

regime 2 is optimal for $\overline{p}_A \in]\max[\overline{p}^*_A, \overline{p}_{A2min}], \overline{p}^{**}_A]$.

regime 3 is optimal for $\overline{p}_A \in]\max[\overline{p}^{**}_A, \overline{p}_{A3min}], +\infty[$.

The price level in booms $\overline{p}^*_A$ $(\overline{p}^{**}_A)$ *at which it is optimal to switch from regime 1 to regime 2 (from regime 2 to regime 3) is given by:*

$$\begin{aligned} \overline{p}^*_A &= \overline{p}_B\left(1+\frac{c_A}{c_B}\right) - \frac{kr}{1-\overline{p}}\left(\frac{c_A}{c_B}-1\right) - \frac{\sqrt{\left(\left[\overline{p}_B - \frac{kr}{1-\overline{p}}\right]^2 - 4c_B f_B\right)(c_A+c_B)\,c_A}}{c_B} \\ \overline{p}^{**}_A &= \overline{p}_B\left(1+\frac{c_A}{c_B}\right) - \frac{kr}{1-\overline{p}}\left(\frac{c_A}{c_B}-1\right) \end{aligned} \tag{21}$$

The optimal output level in booms is given by $\overline{q}^o = \frac{\overline{p}_A - \overline{p}_B(1-\overline{\varphi}_2) - \frac{rk(1-\overline{\varphi}_2)}{1-\overline{p}}}{2(c_A+\overline{\varphi}_2 c_B)}$. $\overline{Q}^*$ *and* $\overline{Q}^{**}$ *(the optimal output level at* $\overline{p}^*_A$ *and* $\overline{p}^{**}_A$*, respectively) are the same as* Q^* *and* Q^{**} *in proposition 1, but with* p_B *everywhere replaced by* $\overline{p}_B - \frac{rk}{1-\overline{p}}$. *Furthermore,* $\overline{Q}^* = q^o_1(p^*_A) < q^o_2(p^*_A)$ *and* $\overline{Q}^{**} = q^o_2(p^{**}_A) = q^o_3(p^{**}_A)$.

For idle capacity to be optimal in recession the following condition has to be satisfied:

$$\overline{K} = \overline{q}^o(1+\overline{\varphi}_2) + \overline{\varphi}_3\overline{Q}^{**} > \underline{q}^o\left(1+\underline{\varphi}_2\right) + \underline{\varphi}_3\underline{Q}^{**} = \underline{K} \tag{22}$$

The proposition has a number of interesting implications. First, conditional on being in a particular production regime, a higher unit cost of capacity k reduces the optimal output $\overline{q}^o$ in booms. The formula for $\overline{q}^o$ shows that costly capacity has an effect similar to increasing the output price $\overline{p}_B$ by $\frac{rk}{1-\overline{p}}$. The quantity $\frac{rk}{1-\overline{p}}$ can be interpreted as the opportunity cost of investing in the marginal unit of capacity divided by the fraction of the time spent in booms. When $1-\overline{p}$ is small then the economy is in recession most of the time and the opportunity cost of the marginal unit of capacity is very high as this unit will be idle most of the time. If, however, the economy remains in boom forever ($\overline{p}=0$) then the opportunity cost of the marginal unit of capacity is merely rk.

Second, an increase in k reduces the interval of output prices $\overline{p}_A$ over which vertical integration occurs. Indeed, it is easy to show that $\frac{\partial[\overline{p}_A^{**}-\overline{p}_A^{*}]}{\partial k}<0$. In fact when $\overline{p}_B \leq \widehat{p}'_B \equiv 2\sqrt{c_B f_B}+\frac{rk}{1-\overline{p}}$ then vertical integration no longer occurs. The intuition is that vertical integration is a more capacity intensive production regime than outsourcing. As a result a higher unit of capacity, k, makes integration comparatively less attractive.

Third, when capacity is not too costly (as is assumed to be the case in proposition 2) then the behavior of the optimal output and capacity levels as a function of the output price $\overline{p}_A$ is similar as in the costless capacity case. In particular, we still have a discrete jump in both the output and capacity levels when at p_A^* we switch from regime 1 to regime 2 (i.e. $q_1^o(\overline{p}_A^*) < q_2^o(p_A^*)$), while output and capacity are continuous at p_A^{**} where we optimally switch from regime 2 to regime 3 (i.e. $q_2^o(\overline{p}_A^{**}) = q_3^o(p_A^{**})$). Obviously, the policy described in proposition 2 requires that capacity is not too costly so that it is optimal to have some idle capacity in recession. This "idle capacity" condition is expressed by inequality (22). The left hand of this inequality is the optimal capacity level adopted by the firm, whereas the right hand is the capacity level that the firm would like to have in recession if capacity were costless. If the latter capacity level is below K then the firm is not constrained in recession in the output level it adopts, which means that there is some idle capacity.[10]

[10]Strictly speaking there is also a knife edge case for which K exactly equals $\underline{q}^o\left(\underline{p}_A\right)\left(1+\underline{\varphi}_2\right)+\underline{\varphi_3 Q}^{**}$. In this limiting case the firm is not capacity constrained in recession, even though there is no idle capacity.

The above results shows that the economic cost of idle capacity is very much determined by two components: (1) the unit cost of capacity, k, and (2) the fraction of the time that the capacity is lying idle. It is intuitively clear that if either of these two components becomes prohibitively high, then the firm may no longer want to have idle capacity. We therefore now examine the case where it is not optimal for the firm to have idle capacity.

3.2 Investment strategy with no idle capacity in recessions

When the firm enters the market in a boom it maximizes $\overline{NPV}_{ij} = \overline{V}_{ij} - kK$, where $\overline{V}_{ij}$ is a weighted average of the profits in booms $\overline{\pi}$ and recession $\underline{\pi}$. With idle capacity in recession the marginal unit of capacity installed in booms has no effect on the profits that are achieved in recession. This reduces the optimization problem to a simple static optimization problem that maximizes the present value of all profits in booms net of the cost of investment in capacity.

Matters become more complicated when there is no idle capacity so the firm operates at full capacity at all times. In that case the marginal unit of capacity installed in booms not only affects the profits in booms, $\overline{\pi}$, but also the profits in recession, $\underline{\pi}$, because the capacity constraint in recession is being relaxed. Therefore, a different solution algorithm has to be adopted.

As a result, we directly optimize the net present present value $\overline{NPV}_{ij}$, that is, the difference between the weighted average of the present values of profits realized in booms, $\overline{\pi}/r$, and recessions, $\underline{\pi}/r$, and the investment cost kK. As we do not know the optimal regime choice *a priori*, we subsequently optimize over all possible combinations of production regimes. The optimization is done with respect to the amount of capacity installed, K, and – in certain regimes – to the amount of capacity earmarked for integrated production.

Such an algorithm is quite different from the solution method when the firm has idle capacity. Previously the firm could optimally and fully choose the output level as a function of the output price $\overline{p}_A$. It is true that with no idle capacity, the output level and the production regime are still functions of $\overline{p}_A$. Given that nine different combinations of regimes are generally possible now, the number of resulting cutoff values of $\overline{p}_A$ is twelve. Four of those cutoff values will effectively apply depending on which one of the six theoretically possible sequences of regimes prevails.[11] Therefore, we are generally unable to predict the prevailing production regimes in booms and recessions based on the knowledge of $\overline{p}_A$ (and of the other relevant parameters). Instead, we determine the production regimes across both states by directly solving the following maximization problem:

$$\max_{i,j} NPV_{ij}(K_{ij}^{opt}), \text{ where } i,j \in \{1,2,3\}, \tag{23}$$

and

$$K_{ij}^{opt} = \underset{K}{\operatorname{argmax}}\, NPV_{ij}(K), \tag{24}$$

subject to

$$\overline{Q}_{3j}^{**} \leq \frac{1}{2}K_{3j}, \tag{25}$$

$$\underline{Q}_{i3}^{**} \leq \frac{1}{2}K_{i3}. \tag{26}$$

The implementation of the above maximization program is relatively straightforward. For $i = j = 1$ (outsourcing in both states), maximization is unconstrained

[11] Cutoffs $\overline{p}_A^*$ and $\overline{p}_A^{**}$ can generally be calculated for the three regimes in recessions. Analogously, a pair of cutoff levels of $\overline{p}_A$ associated with the changes of the production regime in recessions can be calculated for the three different regimes in booms. This gives twelve cutoff levels of $\overline{p}_A$. Still, given the optimal sequence of the pairs of production regimes in booms and recessions, only four of the twelve cutoff levels are relevant. For example, the lowest cutoff level may denote the switch from outsourcing to integration in recessions, another would correspond to the switch from outsourcing to integration in booms, yet another would be associated with the switch from integration to the combined regime in recessions, and the final one – to the switch from integration to the combined regime in booms. In general, each of six different sequences of the pairs of production regimes can be optimal and each of them is associated with a different quadruple of the relevant cutoff levels of $\overline{p}_A$.

and performed solely with respect to K. On the opposite side of the spectrum, that is, for $i = j = 3$ (combined regime in both states), maximization is performed with respect to K, $\overline{Q}^{**}$ and $\underline{Q}^{**}$. Consequently, both constraints (25) and (26) have to be taken into account in this case. For $i = 3$ and $j \neq 3$ ($i \neq 3$ and $j = 3$) only constraint (25) ((26)) is relevant. How can we verify whether the optimal regime is that with integration occurring at least in one of the states? The solution is simple. Pure integration is never optimal if neither of conditions (25)–(26) binds in the optimum. Conversely, the combined regime cannot be optimal in booms (recessions) when regime j (i) prevails in recessions (booms) and condition (25) ((26)) is binding.

Consequently, finding the optimal production regimes across the two states requires solving the maximization problem (23) for the four following cases:

$$(i, j) \in \{(1,1), (1,3), (3,1), (3,3)\}.$$

Case (1,1) is straightforward. If condition (26) binds in the solution to case (1,3), the combined regime is dominated in recessions by pure integration.[12] Consequently, the constrained solution under regimes (1,3) simply corresponds to the solution under regimes (1,2). Analogously, condition (25) binding in the solution to case (3,1) is equivalent to the combined regime being dominated in booms by pure integration. As a result, the constrained solution under regimes (3,1) is identical to that under regimes (2,1). Finally, any binding conditions in the solution to case (3,3) are interpreted in the same way. To summarize, imposing constraints (25) and (26) on the solution to cases involving the combined regime in at least one of the states allows us naturally to embed the cases of pure integration. This property leads the number of regime combinations to fall from nine to four.

[12]In other words, $\underline{Q}_{i3}^{**} = \frac{1}{2}K_{i3}$ implies that the firm produces its entire required quantity of product B internally and uses no outsourcing at all. But such a "degenerate" combined production mode is simply equivalent to pure integration.

Numerical results of the model are presented in Table 1. The following parameter values have been adopted: $c_B = 1$, $f_A = 0.1$, $f_B = 0.1$, $\overline{p} = 0.5$, and $r = 0.1$. The values of c_A, k, $\overline{p}_A$, $\underline{p}_A$, $\overline{p}_B$, and $\underline{p}_B$ vary as described in the table.

[Please insert Table 1 about here.]

A number of conclusions can be drawn from Table 1. First, for relatively low costs of capacity and production of product A ($k = 0.02$ and $c_A = 1$; Panel A), an increasing variability of the output price p_A across states leads generally to a higher NPV and a greater capacity investment. The former result follows from the standard positive effect of a higher volatility of output price on profits when quantity is adjustable. The greater capacity investment is the result of higher capacity needs in booms, following from a higher level of $\overline{p}_A$. Furthermore, a higher dispersion of output prices across the two states results in some idle capacity being present in recessions. Finally, the higher dispersion is usually associated with a switch from integration to the combined regime (at least in some states of nature).[13]

The above conclusions are generally true also when the variable cost of product A is high ($c_A = 10$; Panel B). In this case, both the NPV of the project as well as the amount of capacity installed are lower. Moreover, a higher dispersion of output prices may lead to outsourcing being the optimal production regime. Contrarily to the above results, increasing the dispersion of output prices does not need to lead to a higher NPV or greater capacity installed if the cost of capacity is very high ($k = 3$; Panel C). (It does if it optimally leads to choosing less capitally intensive outsourcing in one of the states.)

[13] An increasing variability of the output price also leads generally to a greater dispersion in capacity utilized in booms and recession ($\overline{K} - \underline{K}$), and to a greater dispersion in output quantities across states ($\overline{Q} - \underline{Q}$).

Consistent with the intuition, a lower cost of input, p_B, results in a higher NPV. It also makes outsourcing (possibly combined with integration) a more frequently adopted production regime. Finally, increasing the spread on the cost of input across the states (from $\overline{p}_B = \underline{p}_B = 1.5$ to $\overline{p}_B = 2$ and $\underline{p}_B = 1$) has an ambiguous effect on the project's NPV: it reduces the NPV if outsourcing becomes more costly in the states in which it originally prevails and increases it if (cheaper) outsourcing is triggered in recessions.

As an illustration of the case of no idle capacity, consider the scenario in which the firm adopts the combined regime (regime 3) during booms and pure integration (regime 2) in recessions. By terminating all outsourcing and switching to pure integration the firm fills the overcapacity that has arisen as a result of the fall in demand. Given that the capacity is fully used in booms we can easily calculate the firm's output in recessions ($\underline{Q}$) as a function of the output in booms ($\overline{Q}$). For the optimal capacity level K, the output in booms ($\overline{Q}$) and recessions ($\underline{Q}$) must satisfy:

$$K = \overline{Q} + \overline{Q}^{**} = 2\underline{Q} \tag{27}$$

where $\overline{Q}^{**}$ is the quantity of B that is produced in house during booms when the firm adopts regime 3. Consequently, if the firm switches from regime 3 in booms to regime 2 in recessions, then the output in recessions is the average of the output sold in booms ($\overline{Q}$) and the amount of B produced in house ($\overline{Q}^{**}$): $\underline{Q} = \frac{\overline{Q}+\overline{Q}^{**}}{2} > \overline{Q}^{**}$. Therefore, in recessions the output level $\underline{Q}$ still exceeds $\overline{Q}^{**}$. However, instead of outsourcing the excess $\underline{Q} - \overline{Q}^{**}$ (as would normally be the case in booms), the firm produces B entirely in house. Similarly, when the economy reverts to a boom, the firm meets the increase in output ($\overline{Q} - \underline{Q}$) by reallocating capacity. By outsourcing the production of B in excess of $\overline{Q}^{**}$, the firm frees up an amount of capacity, $\underline{Q} - \overline{Q}^{**}$, which can be used to increase the output A by an equal amount $\overline{Q} - \underline{Q}$ (i.e. $\overline{Q} - \underline{Q} = \underline{Q} - \overline{Q}^{**}$). Outsourcing in booms in therefore a capacity efficient way of meeting excess demand. Investment in capacity is sunk and irreversible, whereas the decision on the amount to outsource is not (assuming the firm is not locked into a long term contract). As a result it is more efficient to meet the variable component of demand through outsourcing.

At this point we should note an important caveat. The above described switch from regime 3 in booms to regime 2 in recessions is quite different in nature compared to a switch from regime 2 to regime 1. The former is a switch to a more capacity-intensive regime, whereas the latter is a switch to a less capacity-intensive regime. A switch from regime 2 to regime 1 in response to a reduction in demand for product A would typically exacerbate the overcapacity problem. A switch from regime 2 to regime 1 with operation at full capacity in both states is, however, not impossible. As explained earlier, a drop in the output price p_B could imply that the firm prefers to outsource in recession. For example, if $\underline{p}_B < 2\sqrt{c_B f_B}$ then the firm prefers pure outsourcing in recession irrespective of the output price $\underline{p}_A$.

4 Conclusions

This paper examines a firm's choice between outsourcing and integration under output and input price uncertainty. With outsourcing the average unit cost of the input is constant, whereas with integration the average cost of the input is a U-shaped function of the quantity produced.

We find that up to three different production regimes can arise: pure outsourcing, pure integration, and in-house production complemented with outsourcing. Investment in capacity plays an important role in the choice of the optimal production mode. As capacity becomes more costly, internal production becomes more expensive compared to outsourcing, and therefore the range of output prices over which internal production is optimal shrinks. We show that the amount of capacity installed is determined by the opportunity cost of capital, the unit cost of capacity and the fraction of the time that the marginal unit of capacity is utilized. If capacity is cheap or if it is in use most of the time (for example, if recessions last relatively short), then it may still be optimal for firms to have idle capacity in recessions. If, however, the unit cost of capacity increases or the fraction of the time that capacity is lying idle is high, then the firm limits its investment in capacity in order to ensure that capacity is fully utilized in booms and recessions.

We find that firms can switch back and forth between production regimes in response to economic shocks, and this can be an efficient way to make the optimal use of capacity and minimize production costs. In response to a negative demand shock, firms that are operating above their ideal firm size can fill the resulting excess capacity by cutting back on some or all of their outsourcing. On the other hand, when demand is high in booms, firms can limit their capacity needs by using outsourcing as an overflow mechanism.

Our results have wider implications beyond the outsourcing literature and open up avenues for future research. For instance, there are implications for merger and takeover activity. Our results show that value can be created when a firm that outsources and is operating above its ideal size merges with a firm that is operating below its ideal size, particularly if the latter firm has excess capacity. The value creation arises from two potential sources. First, both firms can decrease their average cost of production by transferring production from the former to the latter firm. Second, the firms' capacity can be used more efficiently if the latter firm has excess capacity. This type of mergers reduces outsourcing activity and causes a shift towards large vertically integrated firms.

The paper may also have implications for research on barriers to entry and contestable markets. A large vertically integrated firm can use its option to outsource as an entry deterrence mechanism to protect its output market. A shift from pure integration to integration combined with outsourcing allows the firm to increase its output to fill preemptively any "gaps" that might arise in the market.

A Appendix

Proof of Proposition 1.

Optimizing the profit function with respect to q gives:

$$\frac{\partial \pi(q;\varphi_2,\varphi_3)}{\partial q} = 0 \Leftrightarrow q^o = \frac{p_A - p_B(1-\varphi_2)}{2(c_A+\varphi_2 c_B)} \tag{A.1}$$

The expression for q^o gives us the optimal output level conditional on a particular production regime, and conditional on a positive level of production being optimal in the first place. In the presence of fixed costs, the price level p_A has to be sufficiently high for the firm to produce at all. The lowest fixed costs are achieved through pure outsourcing. Regime 1 is therefore the natural candidate for optimal production when p_A is "very low". The optimal output level in regime 1 is:

$$q_1 = \frac{p_A - p_B}{2c_A} \tag{A.2}$$

Consequently, the maximum attainable profit level under pure outsourcing is:

$$\pi_1 = \frac{(p_A-p_B)^2}{2c_A} - \frac{(p_A-p_B)^2}{4c_A} - f_A = \frac{(p_A-p_B)^2}{4c_A} - f_A \tag{A.3}$$

The firm will only adopt a positive output level if and only if

$$\pi_1 \geq 0 \Leftrightarrow p_A \geq 2\sqrt{c_A f_A} + p_B \equiv p_{A1min} \tag{A.4}$$

The lowest viable output level is therefore $q_1 = \sqrt{\frac{f_A}{c_A}}$ and the fixed cost f_A works as a barrier to entry.

Consider next the case of pure integration. The optimal output level in regime 2 equals $q_2 = \frac{p_A}{2(c_A+c_B)}$. Therefore, the maximum attainable profits under pure integration are:

$$\pi_2 = \frac{p_A^2}{4(c_A+c_B)} - f_A - f_B \tag{A.5}$$

Consequently, pure integration is only viable if

$$\pi_2 \geq 0 \Leftrightarrow p_A \geq 2\sqrt{(c_A + c_B)(f_A + f_B)} \equiv p_{A2min} \tag{A.6}$$

To decide whether to opt for pure outsourcing or for pure integration, firms compare profits under either regime by calculating the difference $\pi_1 - \pi_2$. Substituting for the optimal output gives:

$$\pi_1 (q_1^o (p_A)) - \pi_2 (q_2^o (p_A)) = \frac{(p_A - p_B)^2}{4c_A} - \frac{p_A^2}{4(c_A + c_B)} + f_B \tag{A.7}$$

Since $c_B > 0$ it follows immediately that $\pi_1 - \pi_2$ is a convex quadratic function of p_A which reaches a minimum at $p_A = p_B \left(1 + \frac{c_A}{c_B}\right)$. The discriminant of this quadratic equation is negative (zero) for $p_B^2 < (=) 4c_B f_B$. Therefore, if $p_B^2 \leq 4c_B f_B$ then:$\pi_1 (q_1^o (p_A)) \geq \pi_2 (q_2^o (p_A))$ for all p_A and pure outsourcing always dominates pure integration. If $p_B^2 > 4c_B f_B$ then the function $\pi_1(p_A) - \pi_2(p_A)$ has two (real) roots. In what follows, we call p_A^* the negative root, which is given by:

$$p_A^* = p_B \left(1 + \frac{c_A}{c_B}\right) - \frac{\sqrt{(p_B^2 - 4c_B f_B)(c_A + c_B) c_A}}{c_B} \tag{A.8}$$

The above analysis implies that there exists an interval $[p_{A1min}, p_A^*[$ for which pure outsourcing is strictly better than pure integration. The switching point p_A^* satisfies the condition:$\pi_1 (q_1^o (p_A^*)) = \pi_2 (q_2^o (p_A^*))$.

Consider now the point p_A^* at which the firm is indifferent between pure outsourcing and pure integration. The optimal output level under pure outsourcing is then given by:

$$q_1^o (p_A^*) = \frac{p_B}{2c_B} - \frac{\sqrt{(p_B^2 - 4c_B f_B)(c_A + c_B) c_A}}{2c_A c_B} \equiv Q^* \tag{A.9}$$

On the other hand, the optimal output level under pure integration equals:

$$q_2^o (p_A^*) = \frac{p_B}{2c_B} - \frac{\sqrt{(p_B^2 - 4c_B f_B)(c_A + c_B) c_A}}{2c_B (c_A + c_B)} \tag{A.10}$$

It follows immediately that

$$Q^* = q_1^o(p_A^*) < q_2^o(p_A^*) \tag{A.11}$$

As p_A rises above p_A^* the optimal output $q_2^o(p_A)$ rises linearly in p_A. The marginal cost of producing product B rises, however, linearly in the output level and as a result it may become optimal for the firm to start outsourcing any production of B in excess of some critical level Q^{**}. Switching from regime 2 to regime 3 occurs at some price level p_A^{**}.

Our definition of the profit function in equation (5) implies that by construction the profit functions value match at Q^{**}, i.e. $\pi_2(Q^{**}) = \pi_3(Q^{**})$. The value matching condition is therefore satisfied for any value of Q^{**}, where Q^{**} is defined as the output level for which the firm switches from pure integration to integration combined with outsourcing (i.e. all production of B in excess of Q^{**} is bought). What we need to determine therefore is the optimal level for Q^{**}, or equivalently the corresponding price level p_A^{**} at which the firm switches optimally from regime 2 to regime 3. This optimal switching point is the solution to the following smooth-pasting condition:[14]

$$\frac{\partial \pi_2(q_2^o(p_A))}{\partial p_A}\Big|_{p_A=p_A^{**}} = \frac{\partial \pi_3(q_3^o(p_A))}{\partial p_A}\Big|_{p_A=p_A^{**}} \tag{A.12}$$

The condition states that the optimal profit function at p_A^{**} needs to be differentiable ("smooth") at p_A^{**}. The condition can be reformulated in the following way. The optimal profit level can be expressed as $\pi(q^o(p_A);\ p_A)$. The price level p_A has a direct effect on profits (through revenues), and an indirect effect (through the optimal output level). Consequently:

$$\begin{aligned}\frac{\partial \pi(q^o(p_A);\ p_A)}{\partial p_A} &= \frac{\partial \pi(q^o(p_A);\ p_A)}{\partial q^o}\frac{\partial q^o(p_A)}{\partial p_A} + \frac{\partial \pi(q^o(p_A);\ p_A)}{\partial p_A} \\ &= \frac{\partial \pi(q^o;\ p_A)}{\partial p_A} = q^o(p_A)\end{aligned} \tag{A.13}$$

[14]The second order condition for a maximum is given by $\frac{\partial^2 \pi_2(q_2^o(p_A))}{\partial {p_A}^2}\Big|_{p_A=p_A^{**}} < \frac{\partial^2 \pi_3(q_2^o(p_A))}{\partial {p_A}^2}\Big|_{p_A=p_A^{**}}$, or equivalently by $\frac{1}{2(c_A+c_B)} < \frac{1}{2c_A}$. This condition is always satisfied.

where we used the fact that the profit function is evaluated at the optimal output level, and therefore $\frac{\partial \pi(q^o;\, p_A)}{\partial q^o} = 0$. Our smooth-pasting condition can thus be written as:

$$\frac{\partial \pi_2 \left(q_2^o \left(p_A\right); p_A\right)}{\partial p_A} \Big|_{p_A = p_A^{**}} = q_2^o \left(p_A^{**}\right) = q_3^o \left(p_A^{**}\right) = \frac{\partial \pi_3 \left(q_3^o \left(p_A\right); p_A\right)}{\partial p_A} \Big|_{p_A = p_A^{**}} \tag{A.14}$$

Solving the equation $q_2^o \left(p_A^{**}\right) = q_3^o \left(p_A^{**}\right)$ for p_A^{**} gives:

$$p_A^{**} = p_B \left(1 + \frac{c_A}{c_B}\right) \tag{A.15}$$

The corresponding optimal output level is given by:

$$q_2^o \left(p_A^{**}\right) = q_3^o \left(p_A^{**}\right) = \frac{p_B}{2c_B} \equiv Q^{**} \tag{A.16}$$

While regime 3 strictly dominates regimes 1 and 2 over the interval $]p_A^{**}, \infty[$, there is no guarantee that production is viable in the first place. The maximum attainable profits in regime 3 are given by:

$$\begin{aligned} \pi_3(q^o(p_A)) &= \left(p_A - p_B\right) q^o - c_A {q^o}^2 - c_B Q^{**} + p_B Q^{**} - f_A - f_B \\ &= \frac{\left(p_A - p_B\right)^2}{4c_A} + \frac{p_B^2}{4c_B} - f_A - f_B \end{aligned} \tag{A.17}$$

Therefore, regime 3 is viable if and only if:

$$\pi_3 \geq 0 \iff p_A \geq p_B + 2\sqrt{c_A \left(f_A + f_B - \frac{p_B^2}{4c_B}\right)} \equiv p_{A3min}$$

■

Proof of Proposition 2.

The first order condition is given by $\frac{\partial \overline{\pi}(\overline{q})}{\partial \overline{q}} = \frac{kr(1+\overline{\varphi}_2)}{1-\overline{p}}$. Solving for $\overline{q}$ conditional on regime i being adopted gives:

$$\overline{q}_i^o \left(p_A\right) = \frac{\overline{p}_A - \overline{p}_B(1 - \overline{\varphi}_2) - \frac{rk(1-\overline{\varphi}_2)}{1-\overline{p}}}{2(c_A + \overline{\varphi}_2 c_B)} \tag{A.18}$$

where $\overline{\varphi}_2 = 1$ for $i = 2$ and $\overline{\varphi}_2 = 0$ otherwise.

Analogous as before we can use the value matching condition $\overline{NPV}_{1j}\left(\overline{p}_A^*\right) = \overline{NPV}_{2j}\left(\overline{p}_A^*\right)$ to determine the price level $\overline{p}_A^*$ at which the firm is indifferent between pure outsourcing and pure integration. Since the profit level $\underline{\pi}^o$ in recessions is unaffected by the output decision in booms, the term in $\underline{\pi}^o$ cancels out on either side of the equality and the value matching condition simplifies to:

$$\frac{\overline{\pi}_1\left(\overline{q}_1^o\left(\overline{p}_A^*\right)\right)}{r}\left(1-\overline{p}\right) - k\overline{q}_1^o\left(\overline{p}_A^*\right) = \frac{\overline{\pi}_2\left(\overline{q}_2^o\left(\overline{p}_A^*\right)\right)}{r}\left(1-\overline{p}\right) - 2k\overline{q}_2^o\left(\overline{p}_A^*\right) \tag{A.19}$$

Solving for $\overline{p}_A^*$ gives the expression in proposition 2. The optimal output function $\overline{q}^o\left(p_A\right)$ allows us to determine the optimal output and capacity at $\overline{p}_A^*$ under pure outsourcing as $\overline{Q}^* = \overline{q}^o\left(\overline{p}_A^*\right) = \overline{K}^*$.

The price level $\overline{p}_A^{**}$ at which the firm optimally switches between regime 2 and regime 3 is the solution to the smooth-pasting condition[15]

$$\frac{\partial\overline{NPV}_{2j}\left(\overline{q}_2^o\left(p_A\right)\right)}{\partial p_A}\Big|_{p_A=\overline{p}_A^{**}} = \frac{\partial\overline{NPV}_{3j}\left(\overline{q}_3^o\left(p_A\right)\right)}{\partial p_A}\Big|_{p_A=\overline{p}_A^{**}} \tag{A.20}$$

As shown before this condition is equivalent to $\overline{q}_2^o\left(\overline{p}_A^{**}\right) = \overline{q}_3^o\left(\overline{p}_A^{**}\right)$. Solving this equation for $\overline{p}_A^{**}$ gives the expression in proposition 2. The corresponding output level is given by $\overline{Q}^{**} = \overline{q}_2^o\left(\overline{p}_A^{**}\right) = \overline{q}_3^o\left(\overline{p}_A^{**}\right) \equiv \frac{\overline{K}^{**}}{2}$. The solution for $\overline{p}_A^*$ and $\overline{p}_A^{**}$ (see proposition 2) shows that $\overline{p}^* \leq \overline{p}^{**}$. As $\overline{p}_B$ declines towards $2\sqrt{c_B f_B} + \frac{rk}{1-\overline{p}} \equiv \widehat{\overline{p}}_B$, the threshold $\overline{p}^*$ converges towards $\overline{p}^{**}$ and the interval for which integration occurs, shrinks to zero. For $\overline{p}_B \leq \widehat{\overline{p}}_B$, pure outsourcing is optimal for all (viable) price levels of p_A. Note that (as in the costless capacity case) there is a jump in the optimal

[15] The second order condition is the same as in footnote 14, and is always satisfied.

output level at $\overline{p}_A^*$. Indeed

$$\begin{aligned}\overline{Q}^* &\equiv \frac{c_A\left(\overline{p}_B - \frac{rk}{1-\overline{p}}\right) - \sqrt{c_A\left(c_A + c_B\right)\left[\left(\overline{p}_B - \frac{rk}{1-\overline{p}}\right)^2 - 4c_B f_B\right]}}{2c_A c_B} = \overline{q}_1^o\left(\overline{p}_A^*\right) \\ &< \frac{\left(c_A + c_B\right)\left(\overline{p}_B - \frac{rk}{1-\overline{p}}\right) - \sqrt{c_A\left(c_A + c_B\right)\left[\left(\overline{p}_B - \frac{rk}{1-\overline{p}}\right)^2 - 4c_B f_B\right]}}{2c_B\left(c_A + c_B\right)} = \overline{q}_2^o\left(\overline{p}_A^*\right)\end{aligned}$$

As shown previously, the above optimization procedure does not guarantee that it is optimal for the firm to invest in the first place (i.e. $NPV_{ij}\left(p_A\right) \geq 0$). As before one can solve for the threshold $\overline{p}_{Aimin}$ at which the firm breaks even (i.e. $NPV_{ij}\left(\overline{p}_{Aimin}\right) = 0$). For example, the break-even threshold under pure outsourcing is the positive root to the following quadratic function:

$$NPV_{11}\left(p_A\right) = \left(\frac{M^2}{4c_A} - f_A\right)\frac{\left(1-\overline{p}\right)}{r} + \frac{\overline{\pi}^o}{r}\overline{p} - \frac{kM}{2c_A} = 0 \qquad \text{(A.21)}$$

where $M \equiv \overline{p}_A - \overline{p}_B - \frac{rk}{(1-\overline{p})}$. In general $\overline{p}_{Aimin}$ $(i = 1, 2, 3)$ is the value for $\overline{p}_A$ such that $NPV_{ij}\left(\overline{p}_A\right) \geq 0$ for all $\overline{p}_A \geq \overline{p}_{Aimin}$. While closed form solutions exist for each of those break even thresholds, they are lengthy and not so informative. We therefore omit them from the exposition.

Finally, we need to verify that the solution is consistent with our assumption that there is idle capacity in recession. For this condition to be satisfied it has to be the case that the capacity level K satisfies the following inequality:

$$K = \overline{q}^o\left(\overline{p}_A\right)\left(1 + \overline{\varphi}_2\right) + \overline{\varphi}_3\overline{Q}^{**} > \underline{q}^o\left(\underline{p}_A\right)\left(1 + \underline{\varphi}_2\right) + \underline{\varphi}_3\underline{Q}^{**} \qquad \text{(A.22)}$$

■

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Figure 1: Cost of Making versus Buying Product B

Cost of making and cost of buying q units of product B as a function of output quantity q. The convex curve is the cost of producing q units of B in house. The straight lines represent the cost $p_B q$ of outsourcing q units of B for different levels of p_B $\left(p_B = p'_B, p''_B, p'''_B\right)$. For $p'''_B < 2\sqrt{c_B f_B}$ it is cheaper for all output levels to outsource than to produce in house, whereas for $p'_B > 2\sqrt{c_B f_B}$ it is cheaper to produce in house than to outsource for $q \in \left]\widetilde{Q}\ ,\widehat{Q}\right[$. $Q^{**}(p_B)$ is the output level at which the firm optimally switches from pure integration to integration combined with outsourcing.

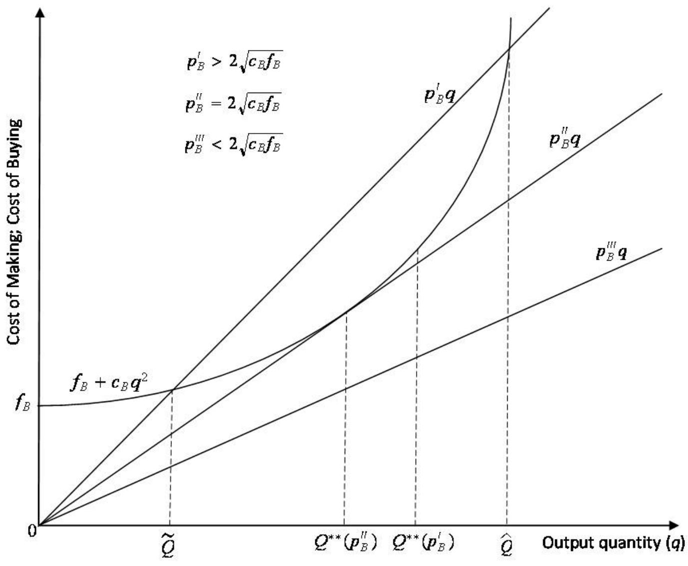

Figure 2: Optimal Output and Capacity Levels

Optimal output, q^o, and capacity, K, as a function of the output price p_A. p_{A1min} is the minimum output price level required to have production. For $p_{A1min} \leq p_A < p_A^*$ pure outsourcing dominates, whereas for $p_A^* \leq p_A \leq p_A^{**}$ pure integration always dominates. For $p_A^{**} < p_A$ the firm combines integration with outsourcing. Q^* and Q^{**} are the quantity levels at which the firm optimally switches from pure outsourcing to pure integration and from pure integration to integration combined with outsourcing, respectively.

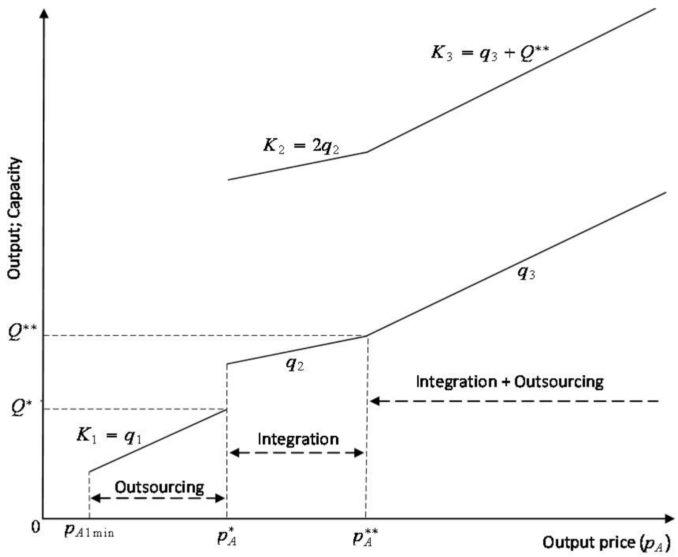

Table 1: Optimal Regime and Investment Strategy - Numerical Results

Optimal regime adopted in booms and recession (i, j), optimal investment in capacity in booms $\overline{K}$, capacity utilized in recession $\underline{K}$, NPV of investment strategy, and output quantity in booms and recession, $\overline{Q}$ and $\underline{Q}$, for different values of output prices, $\overline{p}_A$ and $\underline{p}_A$, input prices, $\overline{p}_B$ and $\underline{p}_B$, variable cost of the output, c_A, and unit cost of capacity, k. The other set of input parameters used is as follows: $c_B = 1$, $f_A = 0.1$, $f_B = 0.1$, $\overline{p} = 0.5$, $r = 0.1$.

Panel A: $k = 0.02$ and $c_A = 1$												
	$\overline{p}_B = \underline{p}_B = 2$			$\overline{p}_B = \underline{p}_B = 1,5$			$\overline{p}_B = \underline{p}_B = 1$			$\overline{p}_B = 2; \underline{p}_B = 1$		
$\overline{p}_A$	3.0	3.5	4.0	3.0	3.5	4.0	3.0	3.5	4.0	3.0	3.5	4.0
$\underline{p}_A$	3.0	2.5	2.0	3.0	2.5	2.0	3.0	2.5	2.0	3.0	2.5	2.0
NPV	9.22	9.53	10.46	9.22	9.68	11.09	10.47	11.09	12.96	9.85	9.68	10.46
$\overline{K}$	1.50	1.75	2.00	1.50	1.75	2.00	1.50	1.75	2.00	1.50	1.75	2.00
$\underline{K}$	1.50	1.25	1.00	1.50	1.25	1.00	1.50	1.25	1.00	1.50	1.25	1.00
$\overline{Q}$	0.75	0.87	1.00	0.75	1.00	1.25	0.75	1.25	1.50	0.75	0.87	1.00
$\underline{Q}$	0.75	0.63	0.50	0.75	0.63	0.50	0.75	0.75	0.50	1.00	0.75	0.50
Regime (i,j)	(2,2)	(2,2)	(2,2)	(2,2)	(3,2)	(3,2)	(2,2)	(3,3)	(3,2)	(2,3)	(2,3)	(2,2)
Idle Capacity?	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes

Panel B: $k = 0.02$ and $c_A = 10$												
	$\overline{p}_B = \underline{p}_B = 2$			$\overline{p}_B = \underline{p}_B = 1,5$			$\overline{p}_B = \underline{p}_B = 1$			$\overline{p}_B = 2; \underline{p}_B = 1$		
$\overline{p}_A$	3.0	3.5	4.0	3.0	3.5	4.0	3.0	3.5	4.0	3.0	3.5	4.0
$\underline{p}_A$	3.0	2.5	2.0	3.0	2.5	2.0	3.0	2.5	2.0	3.0	2.5	2.0
NPV	0.04	0.10	0.31	0.04	0.10	0.34	0.04	0.17	0.44	0.04	0.17	0.44
$\overline{K}$	0.27	0.32	0.36	0.27	0.32	0.36	0.27	0.32	0.36	0.27	0.32	0.36
$\underline{K}$	0.27	0.23	0.00	0.27	0.23	0.03	0.27	0.08	0.05	0.27	0.08	0.05
$\overline{Q}$	0.14	0.16	0.18	0.14	0.16	0.18	0.14	0.16	0.18	0.14	0.16	0.18
$\underline{Q}$	0.14	0.11	0.00	0.14	0.11	0.03	0.14	0.08	0.05	0.14	0.08	0.05
Regime (i,j)	(2,2)	(2,2)	(2,0)	(2,2)	(2,2)	(2,1)	(2,2)	(2,1)	(2,1)	(2,2)	(2,1)	(2,1)
Idle Capacity?	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes

Panel C: $k = 3$ and $c_A = 1$												
	$\overline{p}_B = \underline{p}_B = 2$			$\overline{p}_B = \underline{p}_B = 1,5$			$\overline{p}_B = \underline{p}_B = 1$			$\overline{p}_B = 2; \underline{p}_B = 1$		
$\overline{p}_A$	3.0	3.5	4.0	3.0	3.5	4.0	3.0	3.5	4.0	3.0	3.5	4.0
$\underline{p}_A$	3.0	2.5	2.0	3.0	2.5	2.0	3.0	2.5	2.0	3.0	2.5	2.0
NPV	5.20	5.20	3.40	5.20	5.36	4.34	6.45	6.93	7.45	5.83	4.62	4.65
$\overline{K}$	1.20	1.20	1.40	1.20	1.20	1.40	1.20	1.05	1.20	1.20	1.15	1.40
$\underline{K}$	1.20	1.20	0.00	1.20	1.20	0.25	1.20	1.05	0.50	1.20	0.75	0.50
$\overline{Q}$	0.60	0.60	0.70	0.60	0.73	0.95	0.85	1.05	1.20	0.60	0.58	0.70
$\underline{Q}$	0.60	0.60	0.00	0.60	0.60	0.25	0.85	0.65	0.50	0.85	0.75	0.50
Regime (i,j)	(2,2)	(2,2)	(2,0)	(2,2)	(3,2)	(3,1)	(3,3)	(1,3)	(1,1)	(2,3)	(2,1)	(2,1)
Idle Capacity?	No	No	Yes	No	No	Yes	No	No	Yes	No	Yes	Yes