Investment, Exogenous Entry and Expandable Markets Under Uncertainty*

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Abstract

This paper proposes a real options model for a duopoly faced with an exogenous entry of a third competitor that can expand the market. Usually market positions appear as a stable *status quo* situation. Competition exists while the duopoloy places are available and both firms fight for them, but after the entry of the follower, no more competitive damages or benefits are considered. The proposed model tries to modify this picture by considering the hypothesis of a third entry in the market, which depends upon an investment that can produce a *market expansion*. The likelihood of entry, its impact on the first firms' market shares and the dimension of the expansion influences the behavior of the first two players.

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1 Introduction

1.1 Investment on oligopolistic markets

Real world investment decisions rarely occur in monopoly contexts. Also perfect competitive markets, with a large number of active firms, are not the typical structure for the majority of industries.

As argued by Bouis, Huisman and Kort (2009), the recent wave of mergers and acquisitions contributed for new oligopoly structures in several sectors. Also, the specific characteristics of some industries induce market structures with a few number of operating firms (typical examples are technology industries, telecommunication, infrastructures such as airports). The same oligopolistic market structure appears in some regulated markets, where the regulator, through licensing, determines the (small) number of firms allowed to operate. All this justifies the recent growing interest on real option games in oligopolistic structures, most of them dealing with duopoly markets (e.g.: Smets 1991, Grenadier 1995, Pawlina and Kort 2002, Paxson and Pinto 2003), and few concerning market structures with more than two active players (e.g.: Grenadier 2002).

In a recent contribution, Bouis, Huisman and Kort (2009) study the optimal decision to invest in markets where more than two competitors are allowed to enter. First they derive the value and the triggers for an oligopoly of three firms, and then extend the model to n firms. They make, however, a basic assumption about the market. They implicitly assume that market demand, as a whole, remains relatively stable and so, as firms enter the market, a reallocation of the market share occurs (e.g., 100% market share for the monopolistic firm, dropping to 50% when second firm enters, and again dropping to 33.33% with the entry of the third company). This assumption implies that none of the entries is considered to have some sort of impact on the market dimension itself.

However, frequently, a positive impact on market dimension occurs as a consequence of the introduction of some new product. A well known example is the positive impact of Apple's iPhone on the dimension of smartphones market as a whole.

This paper tries to relax this basic assumption, by considering that an entry of some competing firm can cause the expansion of the market. In this context, the market may not be exactly the same after the investment decision of a given player. Also, the possibility of asymmetric market shares after the expansion is considered, so the entry of a new competitor not only expands the market, but also may have some impact (positive or negative) on incumbent firms' market shares.

1.2 Expandable markets

Markets may expand due to advertising to capture totally new clients[3], a new usage for some existing product, or the introduction of a technological innovation.

Expandable markets consist of markets that can have a significant increment on its dimension, rather than presenting a smooth continuous growth. In this sense, a market expansion consists on some discrete increment of the market dimension, as a consequence of some external event (e.g.: the concession of a new license, the occurrence of a discovery, the construction of a new infrastructure). Also it can be promoted by a third party (not yet an active firm), as happens in advertising new products.

1.3 An overview of the competitive dynamics

We first provide an overview of the market and its competitive dynamics. Initially, the market is assumed to be a duopoly, where two[4] firms (which are assumed to be symmetric *ex ante*) compete for the two available places. As shown in the related literature, for lower levels of the state variable none of the two firm wishes to enter the market, since the follower's position is more valuable than that of the leader. At some level of the state variable, known as the *leader trigger*, one of the firms behaves optimally, preempting the other firm, by entering the market. When this happens, the other company remains idle until the optimal moment for the follower arrives (i.e. until the *follower trigger* is achieved).

While alone in the market, the leader receives the total net cash flows, but after the entry of the follower, both the leader and the follower share the market in equal parts. This corresponds to an *ex post* market symmetry.

Generally, in the related literature, market positions appear as a stable *status quo* situation. In fact, competition exists while the places are available and both firms fight for them, but after the entry of the follower, no more competitive damages or benefits are considered.

The proposed model tries to modify this picture by considering the hypothesis of a third entry in the market, which depends upon an investment that can produce a *market expansion*. Without this expansion, no space exists for any additional firm, but with that investment the market turns to be an oligopoly with three active firms. In practical terms, this is equivalent to assuming that the new entrant cannot take any existing market share from the installed firms, and so the only way is to obtain sales by increasing the market.

This entry depends upon some future random (exogenous) event (e.g.: the additional license, the discovery), about which the first two firms can only establish some expectations.

[3] In marketing context, expandable markets focus on adding new customers, rather than, existing customers Armstrong and Kotler (1996, p. 217).

[4] These can be considered *positioned firms* as in Armada, Kryzanowski and Pereira (2010).

Naturally, considering this option has a major impact on the optimal investment decision of the competing firms. The second follower decision depends on three important parameters: the *cost* and the *dimension* of the market expansion, as well as its *expected effect*. The two first firms' decisions, given the available information, are affected by *their expectations* about the *effect* of this possible expansion on their market share.

2 The Model

2.1 The second follower's option to expand

The model is developed following, as usual, a backwards procedure, starting with the last decision: the option to invest for the second follower.

Consider a market with two already active and operating firms, where no space for a third company exists, unless an additional investment is made, with the purpose of expanding the market, creating space for a new player, or, alternatively, unless a regulator opens the constrained market to a new player, by granting a new license. In this context, if a third firm intends to enter, it must invest in the market expansion, transforming the duopoly into a oligopoly of three players.

Let K_2 be the investment needed to expand the market. For the moment, it is assumed that the investment cost is constant over time, and also it is not proportional to the expansion dimension. The expansion consists of increasing by ϕ% the net cash flows available to all the firms acting in the market.

The whole market net cash flow (x) behaves according to a geometric Brownian motion, as follows:

$$dx = \alpha x dt + \sigma x dZ \tag{1}$$

where $x > 0$, $\alpha \in [0, r)$ and σ are, respectively, the drift parameter and the instantaneous volatility, r is the risk-free interest rate, and dZ is an increment of the Wiener process.

The firm is assumed to be risk neutral, or x in equation 1 is the certainty-equivalent cash flow.

By investing K_2 the firm increases the level of the (whole) market net cash flows from x to $x(1 + \phi)$.

After the entry of the second follower, the market share for the three companies are defined as follows: $\tilde{S}_l$ is the market share for the leader (the first company entering the market), $\tilde{S}_{f1}$ is the market share for the first follower, and finally, $\tilde{S}_{f2}$ represents the market share for the second follower. Obviously, $\tilde{S}_l + \tilde{S}_{f1} + \tilde{S}_{f2} = 1$. Consequently, the net cash flow for each player, after the market expansion, is: $x(1 + \phi)\tilde{S}_l$ for the leader, $x(1+\phi)\tilde{S}_{f1}$ for the first follower, and $x(1+\phi)\tilde{S}_{f2}$ for the second follower. Since we assume *ex post* symmetry for the first two players, we have $\tilde{S}_l = \tilde{S}_{f1}$.

The market expansion, and the entry of a new competing firm, may have some effect on the market share of the first two firms. Let this effect be defined by θ. If $\theta = 0$ there is no impact (neither positive or negative) from the market expansion for the firms already in place; this means that all the new (expanded) market is captured by the second follower. A $\theta > 0$ reflects a positive effect for the first two players from the expansion; in this situation the new firm only captures a part of the additional expanded market. Finally, a $\theta < 0$ implies a negative impact for the two installed firms: in this situation the second follower conquers a market share higher than the equivalent to the new market. The extreme values for θ are -1 and 1, meaning, respectively, the firms in place lose all their market share to the new follower, and the firms in place gain all the expanded market, leaving no space for the second follower. Accordingly, the market share for the firms is a function of θ: $\tilde{S}_l(\theta)$, $\tilde{S}_{f1}(\theta)$, and $\tilde{S}_{f2}(\theta)$.

Table 1 shows the market shares, before and after the market expansion, for the competing firms as a function of θ. Figure 1 shows the market shares for different levels of θ.

Player	**Before Expansion**	**After Expansion**	
	(Symmetric Duopoly)	$\theta \in [-1, 0]$	$\theta \in (0, 1]$
Leader/1^{st} Follower	$\frac{1}{2}$	$\frac{1+\theta}{2(1+\phi)}$	$\frac{1+\theta\phi}{2(1+\phi)}$
2^{nd} Follower	—	$\frac{\phi-\theta}{1+\phi}$	$\frac{\phi(1-\theta)}{1+\phi}$

Table 1: The market shares for the companies, before and after the market expansion.

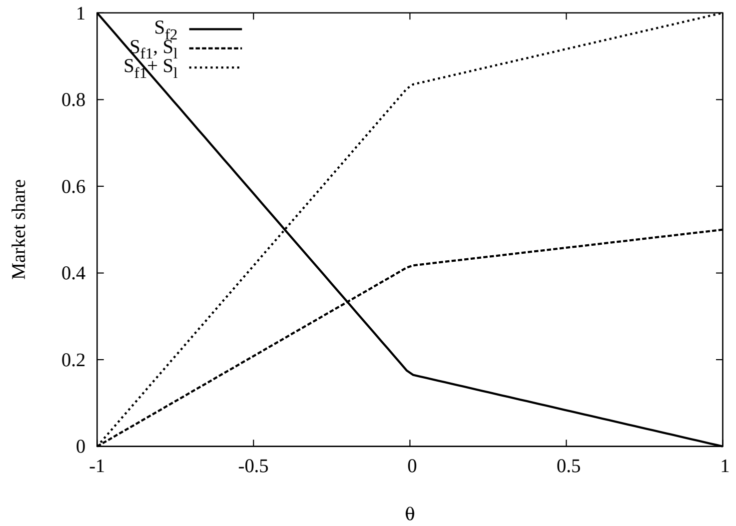


Figure 1: Market share as a function of θ ($\phi = 0.2$).

The value function for the second follower, $F_2(x)$, must satisfy the following ordinary differential equation (ODE), during the continuation period (the period for which it is not yet optimal to invest):

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F_2(x)}{\partial x^2} + \alpha x \frac{\partial F_2(x)}{\partial x} - rF_2(x) = 0 \tag{2}$$

subject to the following boundary conditions:

$$\lim_{x \to 0} F_2(x) = 0 \tag{3}$$

$$\lim_{x \to x^*_{f2}} F_2(x) = \frac{x^*_{f2} \tilde{S}_{f2}(\theta)}{r - \alpha} - K_2 \tag{4}$$

$$\lim_{x \to x^*_{f2}} \frac{\partial F_2(x)}{\partial x} = \frac{\tilde{S}_{f2}(\theta)}{r - \alpha} \tag{5}$$

where $\tilde{S}_{f2}(\theta)$ is as previously defined:

$$\tilde{S}_{f2}(\theta) = \begin{cases} \dfrac{\phi - \theta}{1 + \phi} & \text{if } \theta \in [-1, 0] \\[2ex] \dfrac{\phi(1 - \theta)}{1 + \phi} & \text{if } \theta \in (0, 1] \end{cases} \tag{6}$$

and x^*_{f2} corresponds to the optimal investment trigger for the second follower. Equations 4 and 5 are the well known value-matching and smooth-pasting conditions.

After considering the boundaries, the solution for $F_2(x)$ takes the form:

$$F_2(x) = \begin{cases} \dfrac{K_2}{\beta - 1} \left(\dfrac{x}{x^*_{f2}} \right)^{\beta} & \text{for } x < x^*_{f2} \\[2ex] \dfrac{x(1 + \phi)\tilde{S}_{f2}(\theta)}{r - \alpha} - K_2 & \text{for } x \geq x^*_{f2} \end{cases} \tag{7}$$

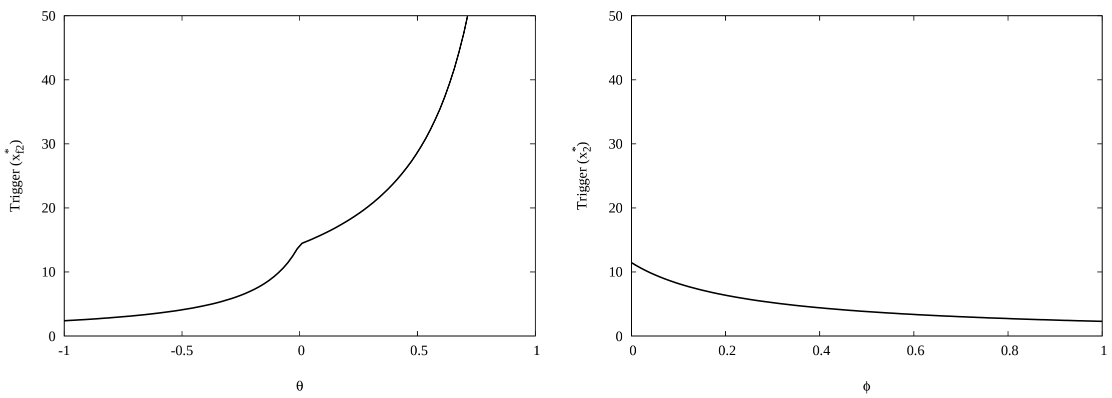
where β is the positive root of the fundamental quadratic $0.5\sigma^2\beta(\beta - 1) + \alpha\beta - r = 0$:

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{8}$$

and the trigger value is:

$$x^*_{f2} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{(1 + \phi)\tilde{S}_{f2}(\theta)} K_2 \tag{9}$$

The value and trigger functions exhibit the usual sensitivities to x, r, α and σ. If the market expansion is greater, the second follower invests sooner, $\frac{\partial x^*_{f2}}{\partial \phi} < 0$, and a higher market share (a lower market share for the players already in the market) also prompts the investment sooner, $\frac{\partial x^*_{f2}}{\partial \theta} > 0$.



(a) Impact of θ (b) Impact of ϕ

Figure 2: Optimal investment trigger for the second follower, x^*_{f2}, as a function of θ and ϕ. $\sigma = 0.25$, $r = 0.05$, $\alpha = 0.02$, $K_2 = 30$, $\theta = -0.25$, $\phi = 0.2$.

2.2 The optimal investment decision for the first follower

In order to compute the value and the optimal timing of first follower's investment opportunity, first the general value function for the active project needs to be determined.

2.2.1 The value of the active project

Since the value of the active project for the first follower ($V_1(x)$) is a contingent asset, i.e. its value depends on the state variable x, it is the solution to the following differential equation:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_1(x)}{\partial x^2} + \alpha x \frac{\partial V_1(x)}{\partial x} - rV_1(x) + xS_{f1} + \lambda \left[\frac{\left[x(1+\hat{\phi})\right] \tilde{S}_{f1}(\hat{\theta})}{r-\alpha} - V_1(x) \right] = 0 \quad (10)$$

This equation incorporates the instantaneous cash-flow (xS_{f1}, where S_{f1} is the firm's market share before the second follower's entry) that the firm receives after investing and during the *duopoly* period. However, there is some possibility of an additional entry, which depends on the decision of a third firm to invest on the market and/or other exogenous event. This possibility is modeled as an exogenous event[5], and the firms in place must

[5]Assuming that the two active firms do not have full information about the intentions of any second follower.

work with their expectations about the probability of occurrence, the dimension of the market expansion and its impact on their market share.

The expected impact on the first follower's project value caused by the entry of a second follower, is captured by the last term of the left-hand side of equation 10, where λ corresponds to the instantaneous frequency of entry of an additional competitor in the market, $\hat{\phi}$ is the firm's expectation about the dimension of the market expansion, and $\tilde{S}_{f1}(\hat{\theta})$ is its expected market share, after the expansion occurrence.

The solution for this non-homogeneous differential equation is:

$$V_1(x) = c_1 x^{\eta_1} + c_2 x^{\eta_2} + \frac{x S_{f1}}{r - \alpha + \lambda} + \frac{\lambda}{r - \alpha + \lambda} \frac{x(1+\hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r - \alpha} \tag{11}$$

Requiring that $V_1(0) = 0$, and in absence of speculative bubbles (see Dixit and Pindyck (1994) for the supporting arguments) $c_1 = c_2 = 0$, and the value of the first follower's active project equals:[6]

$$V_1(x) = \frac{x S_{f1}}{r - \alpha + \lambda} + \frac{\lambda}{r - \alpha + \lambda} \frac{x(1+\hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r - \alpha} \tag{12}$$

2.2.2 The value of the investment opportunity

Once $V_1(x)$ is determined, the value of the option to invest for the first follower can be obtained, $F_1(x)$, using the standard steps. Accordingly, we know $F_1(x)$ must satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F_1(x)}{\partial x^2} + \alpha x \frac{\partial F_1(x)}{\partial x} - r F_1(x) = 0 \tag{13}$$

subject to the following boundary conditions:

$$\lim_{x \to 0} F_1(x) = 0 \tag{14}$$

$$\lim_{x \to x^*_{f1}} F_1(x) = \frac{x^*_{f1} S_{f1}}{r - \alpha + \lambda} + \frac{\lambda}{r - \alpha + \lambda} \frac{x^*_{f1}(1+\hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r - \alpha} - K \tag{15}$$

$$\lim_{x \to x^*_{f1}} \frac{\partial F_1(x)}{\partial x} = \frac{S_{f1}}{r - \alpha + \lambda} + \frac{\lambda}{r - \alpha + \lambda} \frac{(1+\hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r - \alpha} \tag{16}$$

where x^*_{f1} corresponds to the optimal trigger for the first follower and K to the investment cost. Solving the equations using the boundaries, the trigger value is obtained:

[6] Note that for $\lambda = 0$, meaning no probability for the entry of an additional firm, the value of the active project is simply $V_1(x) = \frac{x S_{f1}}{r-\alpha}$, which corresponds to present value of the firm's future cash flows, sharing the market with the leader. For $\lambda = \infty$, meaning the sure entry of an additional firm, the value of the active project is simply $V_1(x) = \frac{x \tilde{S}_{f1}}{r-\alpha}$, which corresponds to present value of the firm's future cash flows in the new market structure with three players.

$$x^*_{f1} = \frac{\beta}{\beta - 1}\frac{K}{\gamma_1 + \psi\gamma_2} \tag{17}$$

where $\gamma_1 = \frac{S_{f1}}{r-\alpha+\lambda}$, $\gamma_2 = \frac{(1+\hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r-\alpha}$, and $\psi = \frac{\lambda}{r-\alpha+\lambda}$.

For the first follower, the value function is:

$$F_1(x) = \begin{cases} \dfrac{K}{\beta - 1}\left(\dfrac{x}{x^*_{f1}}\right)^{\beta} & \text{for } x < x^*_{f1} \\ (\gamma_1 + \psi\gamma_2)x - K & \text{for } x \geq x^*_{f1} \end{cases} \tag{18}$$

The effect of a higher probability of exogenous entry (higher λ) depends on the effect the entry has on the market share of the first follower. If entry reduces its cash flows ($\hat{\theta} < 0$), a higher λ delays investment of the first follower ($\frac{\partial x^*_{f1}}{\partial \lambda} < 0$), while the opposite occurs for $\hat{\theta} > 0$: if the first follower benefits from the entry, it will invest sooner the higher the probability of entry ($\frac{\partial x^*_{f1}}{\partial \lambda} > 0$). If the second follower expands the market and gets all the cash flows from the expansion ($\theta = 0$), the trigger for the first follower is independent of λ ($\frac{\partial x^*_{f1}}{\partial \lambda} = 0$).

The first follower's entry occurs sooner for higher values of $\hat{\theta}$ ($\frac{\partial x^*_{f1}}{\partial \hat{\theta}} < 0$): the more beneficial the entry for the first follower, the sooner it will invest. The dimension of the expansion $\hat{\phi}$ only influences the first follower's trigger for $\hat{\theta} > 0$: a higher expansion induces investment sooner given that the first follower captures part of the expanded market ($\frac{\partial x^*_{f1}}{\partial \hat{\phi}} < 0$).

Finally, the usual effect of uncertainty in the real option model holds: a higher uncertainty delays investment ($\frac{\partial x^*_{f1}}{\partial \sigma} < 0$), regardless of the values of $\hat{\theta}$, $\hat{\phi}$, and λ.

2.3 The optimal investment decision for the leader

Considering that the leader is already active in the market, its value function $L(x)$ must satisfy the following ODE, prior to the entry of the first follower:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 L(x)}{\partial x^2} + \alpha x \frac{\partial L(x)}{\partial x} - rL(x) + x = 0 \tag{19}$$

subject to the following boundary conditions:

$$\lim_{x \to 0} L(x) = 0 \tag{20}$$

$$\lim_{x \to x^*_{f1}} L(x) = \frac{x^*_{f1} S_l}{r - \alpha + \lambda} + \frac{\lambda}{r - \alpha + \lambda}\frac{x^*_{f1}(1 + \hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r - \alpha} \tag{21}$$

The second boundary (Eq. 21) ensures that when the optimal trigger for the first follower approaches, the value of the active project for the leader tends to the value for the first follower. This occurs because, at that moment, the monopoly period ends, and leader shares the market with the first follower. Given the *ex ante* symmetry assumption, the leader expectations about the occurrence of a market expansion are the same as those of the first follower.

The solution for the non-homogeneous ODE is:

$$L(x) = c_3 x^{\beta} + \frac{x}{r-\alpha} \tag{22}$$

where β is as presented in Eq. (8), and

$$c_3 = \frac{x^*_{f1}\left[(r-\alpha)(\gamma_3+\psi\gamma_2)-1\right]}{x^{*\beta}_{f1}(r-\alpha)} \tag{23}$$

where $\gamma_3 = \frac{S_l}{r-\alpha+\lambda}$; γ_2 and ψ are as previously defined.

Since $x^*_{f1} = \frac{\beta}{\beta-1}\frac{K}{\gamma_1+\psi\gamma_2}$, c_3 can be rearranged to[7]:

$$c_3 = \frac{\beta}{\beta-1}\left[1 - \frac{1}{(r-\alpha)(\gamma_3+\psi\gamma_2)}\right] K \frac{1}{x^{*\beta}_{f1}} \tag{24}$$

Accordingly, $L(x)$ is given by:

$$L(x) = \begin{cases} \dfrac{x}{r-\alpha} + \dfrac{\beta}{\beta-1}\left[1 - \dfrac{1}{(r-\alpha)(\gamma_3+\psi\gamma_2)}\right] K \left(\dfrac{x}{x^*_{f1}}\right)^{\beta} & \text{for } x < x^*_{f1} \\[2ex] \dfrac{xS_l}{r-\alpha+\lambda} + \dfrac{\lambda}{r-\alpha+\lambda}\dfrac{x(1+\hat{\phi})\tilde{S}_{f1}(\hat{\theta})}{r-\alpha} & \text{for } x \geq x^*_{f1} \end{cases} \tag{25}$$

For $\lambda = 0$ the model reduces to a duopoly. Figure 3 compares the value functions and the investment trigger values for the leader and the first follower, for $\lambda = 0$ and $\lambda = 0.1$, assuming that the entry of the third firm reduces the cash flows of the first two firms ($\hat{\theta} < 0$). For a given cash flow the possibility of an exogenous entry reduces the value of the firms. A positive probability of entry, delays the investment of the first follower and, consequently, reduces the leader's threshold, increasing the monopoly period (the wedge between x^*_l and x^*_{f1}), compared to the duopoly market. The opposite occurs if the first two players benefit from the third firm entry. Figure 4 shows how λ and $\hat{\theta}$ interact. For a negative impact of the exogenous entry, a higher entry likelihood reduces the leader's investment threshold (Figure 4(a)), and for a positive impact, increases the leader's investment threshold (Figure 4(b)).

[7] Note that $\gamma_3 = \gamma_1$, since $S_l = S_{f1}$ after the entry of the first follower.

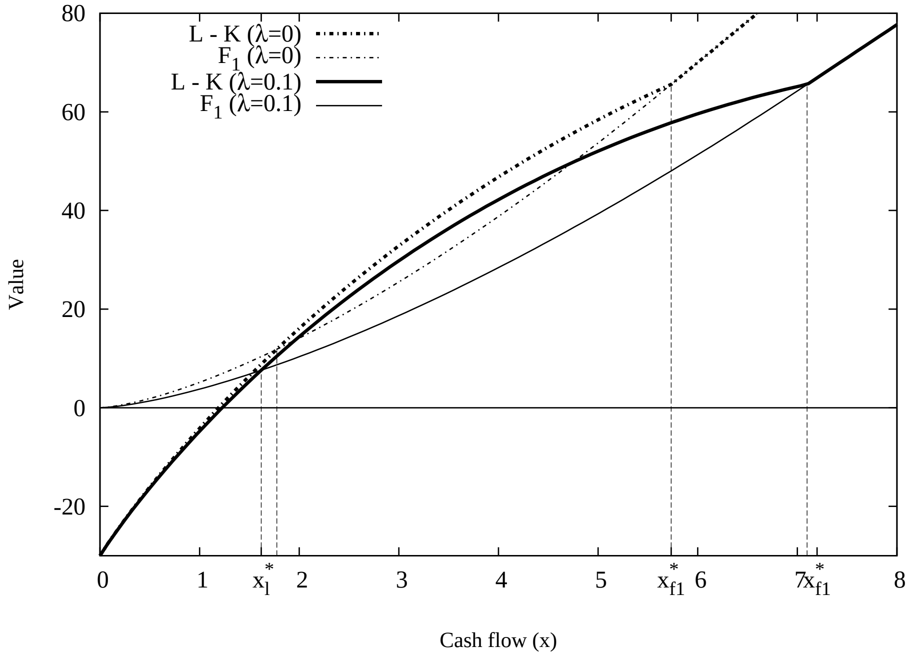
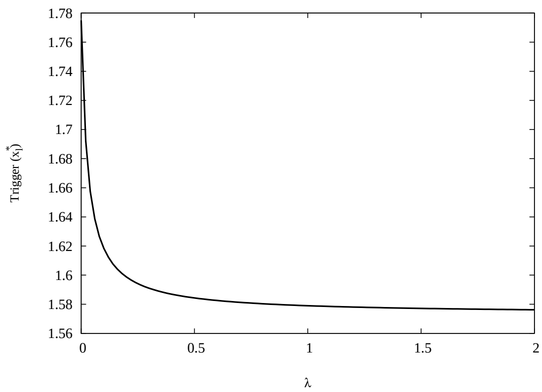


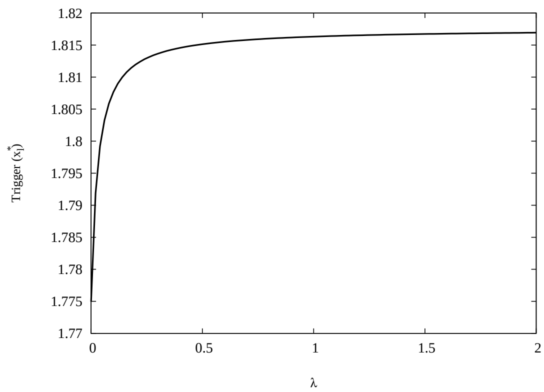
Figure 3: Leader's and first follower's value functions as a function of x and λ. $\sigma = 0.25$, $r = 0.05$, $\alpha = 0.02$, $K = 30$, $\hat{\theta} = -0.25$, $\hat{\phi} = 0.2$.

Figure 5 shows that if the entry of a third firm has a negative impact on the first two firms' cash flows, it increases the threshold entry for the first follower and decreases the leader's threshold, increasing the monopoly period, when compared to a less negative effect of the entry. A higher $\hat{\theta}$, i.e. a more (less) positive (negative) effect of the third exogenous entry, always increases the leader's threshold and reduces the first follower threshold (see previous section), irrespective of the entry probability, λ (Figure 4(c)), or the dimension of the market expansion, $\hat{\phi}$ (Figure 6(b)). The effect of $\hat{\theta}$ on the firms' values is not monotonous for a negative effect, while for a positive effect a higher $\hat{\theta}$ increases the value of leader (Figure 6(a)). For a deep negative effect of the third firm's entry, the leader may have its value increased (Figures 5 and 6(a)). The increase in the wedge between the triggers of the leader and first follower, i.e. the increase in the monopoly period, allows the monopolist (leader) to capture higher cash flows for a longer period, increasing its value.

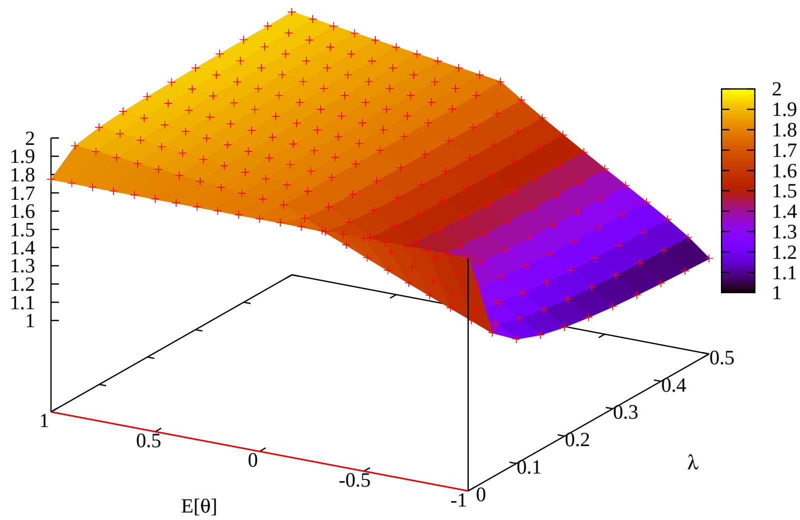
Table 2 summarizes the effects of $\hat{\theta}$, $\hat{\phi}$, and λ on the leader's and first follower's thresholds.



(a) $\hat{\theta} = -0.25$



(b) $\hat{\theta} = 0.25$



(c)

Figure 4: Optimal investment trigger for the leader, x_l^*, as a function of λ and $\hat{\theta}$. $\sigma = 0.25$, $r = 0.05$, $\alpha = 0.02$, $K = 30$, $\hat{\phi} = 0.2$.

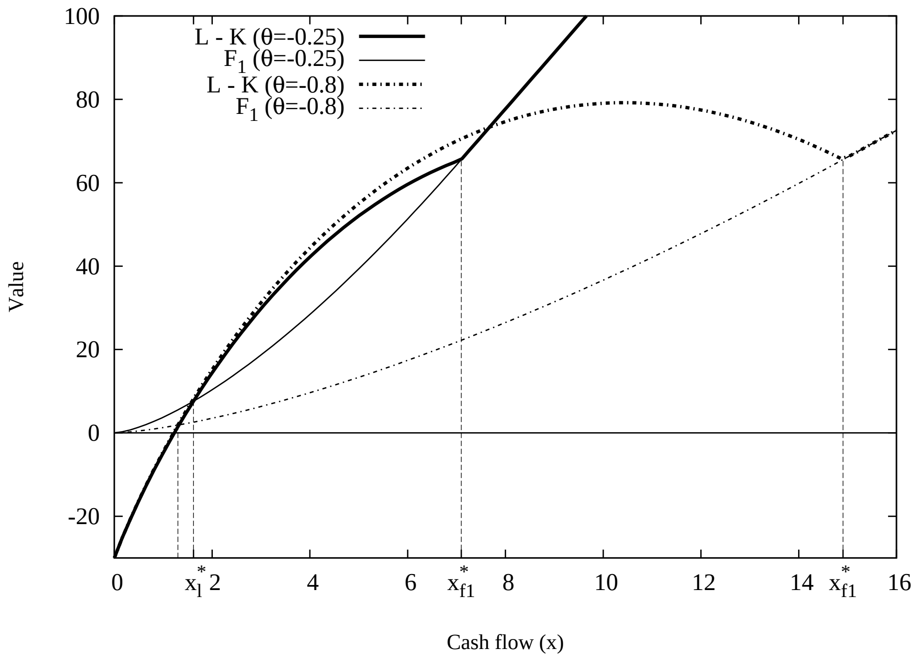
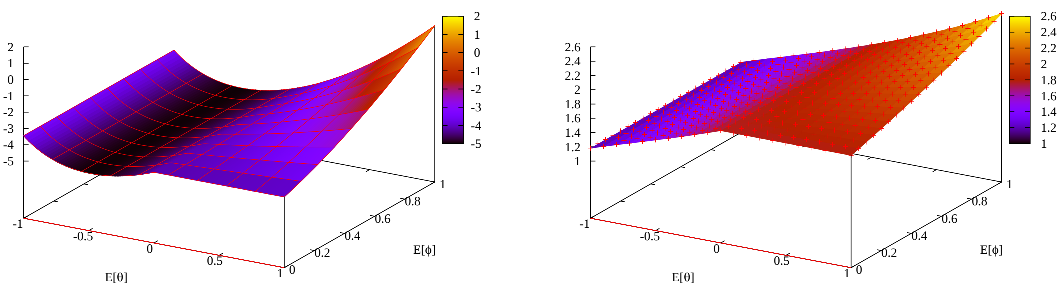


Figure 5: Leader and first follower value functions as a function of x. $\sigma = 0.25$, $r = 0.05$, $\alpha = 0.02$, $K = 30$, $\hat{\phi} = 0.2$, $\lambda = 0.1$.



(a) Value

(b) Trigger

Figure 6: Optimal investment trigger and value function for the leader, x_l^*, as a function of $\hat{\phi}$ and $\hat{\theta}$. $\sigma = 0.25$, $r = 0.05$, $\alpha = 0.02$, $K = 30$, $\lambda = 0.1$.

	x_l^*	x_{f1}^*	$x_{f1}^* - x_l^*$
λ, $\theta < 0$	$-$	$+$	$+$
λ, $\theta = 0$	$=$	$=$	$=$
λ, $\theta > 0$	$+$	$-$	$-$
θ	$+$	$-$	$-$
ϕ, $\theta \leq 0$	$=$	$=$	$=$
ϕ, $\theta > 0$	$+$	$-$	$-$

Table 2: Effect of $\hat{\theta}$, $\hat{\phi}$, and λ on the leader's and first follower's thresholds.

3 Conclusion

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