Real options approach to investment in base load coal fired plant

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keywords: real options, electricity generation, dark spread
JEL classification: C13, G31, Q4

May, 2010

Abstract

Using real options framework I analyze investment in base load coal fired power plant. Analysis is done using real options framework and assuming option to invest is a perpetual American option. I assume profitability of the power plant depends upon the value of dark spread. The paper has two objectives. First, to determine the most appropriate stochastic process to model evolution of dark spread prices. Second, to asses how does the choice of stochastic processes affect investment decision within the real options framework.

1 Introduction

Investment in electricity generation is characterized by large and sunk capital expenditures and great deal of uncertainty such as price, production cost, demand and construction cost uncertainty, just to name a few. To value such investments, utilities rely on discounted cash flow approach (DCF): a tool devised to value projects having predictable cash flows. By construction, DCF is not capable of evaluating investment in electricity generation characterized by large degree of uncertainty, managerial and operational flexibility. An appropriate way to evaluate new investments in electricity sector is to use real options (RO) approach. But, it was not until the late 1990s and early 2000s that we saw proliferation of RO literature applied to electricity sector. A probable reason why RO were applied to electricity sector so late lays in the fact that the sector was monopolized in most of the countries of the world. In the continental Europe it was not until early 1990s that countries embarked on the liberalization process, with Nordic countries being among the first. Rest of the countries followed, but for many of them liberalization did not occur in practice for many years. While operating in monopolized markets, utilities did not have an incentive to properly address the issue of new investment because all the risk and costs could be passed on to the final consumers.

In the early years of RO, authors used geometric Brownian motion (GBM) to describe the evolution of stochastic processes (e.g. McDonald and Siegel (1985), Brennan and Schwartz (1985), Dixit (1989)). Major advantage of using GBM is its analytical tractability: GBM allows authors to obtain closed form solution to an investment problem.

In most of the work on RO applied to electricity generation authors rarely go about estimating the most appropriate stochastic process for evolution of state variables: generally they assume GBM. Nevertheless, it is questionable whether GBM or its variant, arithmetic Brownian motion (ABM), can be used in real options analysis applied to investment in electricity generation. This is because electricity prices are mean reverting, while Brownian motion is an unbounded process.

In this paper I evaluate investment in a base load coal fired power plant using dark spread as a state variable. The paper has two objectives. First, to find a stochastic process which is capable of generating data similar to those observed in actual dark spread series. Quality of the process will be judged according to how closely do summary statistics of postulated processes come to observed dark spread prices. More precisely, I will run Monte Carlo (MC) simulations to determine distributional properties of simulated prices and compare them to observed prices. Furthermore, I will compute in sample root mean squared error (RMSE) as a second method to choose among different stochastic processes. Second objective of the paper is to estimate the effect of postulated processes on investment decision using RO. In other words, to asses how does the choice of different stochastic processes affect investment decision in a coal fired plant. Results of the paper are useful for practitioners who want to apply RO analysis to investment in coal fired power plant.

To my knowledge no one has yet evaluated investment in coal fired power plant using RO approach. Nevertheless, research has been done on investment within RO in other technologies. Relevant papers include Takizawa et al. (2001) who analyze investment in a nuclear power plant in Japan. Venetsanos et al. (2002) evaluate investment in wind farm in Greek electricity market. Gollier et al. (2005) investigates the impact of market liberalization on investment in large nuclear unit. Kjaerland (2007) and Bckman et al. (2008) evaluate investment in hydro power plant in Norway. Abadie and Chamorro (2008) investigate the choice of investment between Natural Gas Combined Cycle (NGCC) and Integrated Gasification Combined Cycle plant (IGCC).

The paper is organized as follows. Section 2 describes the investment problem and gives optimal investment threshold using NPV approach. In section 3 I approach the investment problem using RO based on different stochastic processes. Section 4 deals with the selection of the appropriate stochastic process. Last section concludes.

2 Investment details

2.1 Plant characteristics

I investigate optimal investment decision in a base load coal fired power plant undertaken by a merchant producer. Unlike state owned utilities that can have different goals, merchant producer has a single goal of profit maximization. I assume plant owner is not faced with emission allowance risk but only with risk stemming from uncertainty regarding coal and electricity prices: they are introduced into the model via dark spread prices. Dark spread is defined as a difference between price of electricity and price of coal required to generate one unit of electricity. One can think of dark spread representing the profit flow for the power plant.

Purpose of this paper is not to create a detailed investment study but rather to asses the impact of uncertainty on investment decision and to determine the difference in results obtained using traditional capital budgeting technique (NPV) and RO analysis. Thus, I use generic technical assumptions for the coal fired power plant. Plant characteristics are presented in Table 1.

Parameter	Value
Project life (T)	40 years
Capacity	500 MW
Capacity factor	85%
Annual production (q)	3,723,000 MWh
Investment cost per kW	1,500 €
Discount rate (r)	10%

Table 1: Investment parameters

Most of the technical assumptions are taken from International Energy Agency (IEA) publication on production costs of electricity (IEA (2005)). The report is based upon interviews / questionnaires with actual owners of power plants in the world. In the study it was assumed that economic life of the coal fired plant (amortization period) is 40 years, even though technically coal power plants can operate for much longer. The study also assumed capacity factor of 85% [1]. This capacity factor translates into annual production of 3,723,000 MWh of electricity.

Investment costs vary significantly and they are affected, among other, by demand for coal plants, cost of steel and concrete, and location (MIT (2007)). IEA (2005) cites investment cost of approximately 1,500 US$ /kW, but this number seems too low. More recent numbers on the cost of coal fired plants in the US, such as those revealed in an article *Power plant cost to top 1 billion USD* (2008), estimate cost per kW to exceed 3,000 US$, which seems to be a bit on a high side. Therefore, I assume investment cost of 1,500 €/kW. Assuming 1.45 US$/€ exchange rate this translates into 2,175 US$, which seems a reasonable number. In terms of operation and maintenance costs (O&M), I include only fuel costs. Reason is that other O&M costs are relatively small and given that

[1] Capacity factor equals the ratio of actual output during certain period of time divided by the output that would be produced if plant operated at maximum (rated) capacity during the same period.

they can be predicted with reasonable accuracy, they equally affect investment decision regardless of the approach used. Finally, for the discount rate I use 10%.

2.2 Price characteristics

Coal prices used in the analysis are obtained from McCloskey Co., and are published weekly. Prices are given in €/GJ. In order to calculate how much coal is needed to produce 1 MWh of electricity I use the following assumptions. It takes 3.6 GJ of coal to produce 1 MWh of electricity. I assume energy efficiency of the coal fired power plant is 36%, thus total requirement for production of 1 MWh of electricity is 10 GJ of coal. I use daily electricity prices obtained from the European Energy Exchange (EEX) in Leipzig, Germany. Given that coal prices are on weekly basis, electricity prices are converted to weekly basis also: I do this by generating an arithmetic average of daily electricity prices. Both coal and electricity price series go from January 1, 2002 to November 13, 2009. Figure 1 shows graphs of electricity and coal prices for the observed period, while plot of the dark spread prices is given in Figure 2. Summary statistics for electricity, coal and dark spread prices are shown in Table 2.

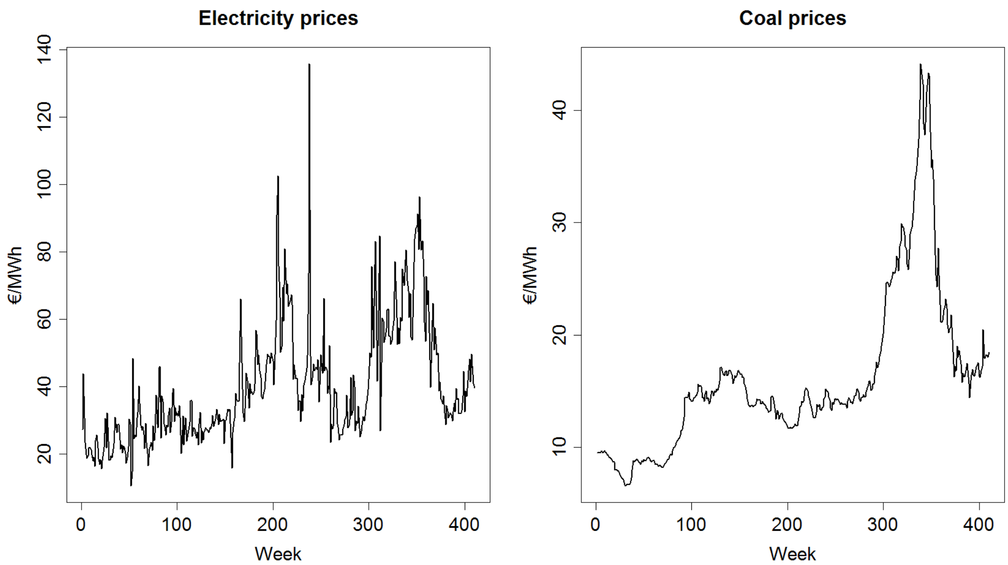

Figure 1: Weekly electricity and coal prices

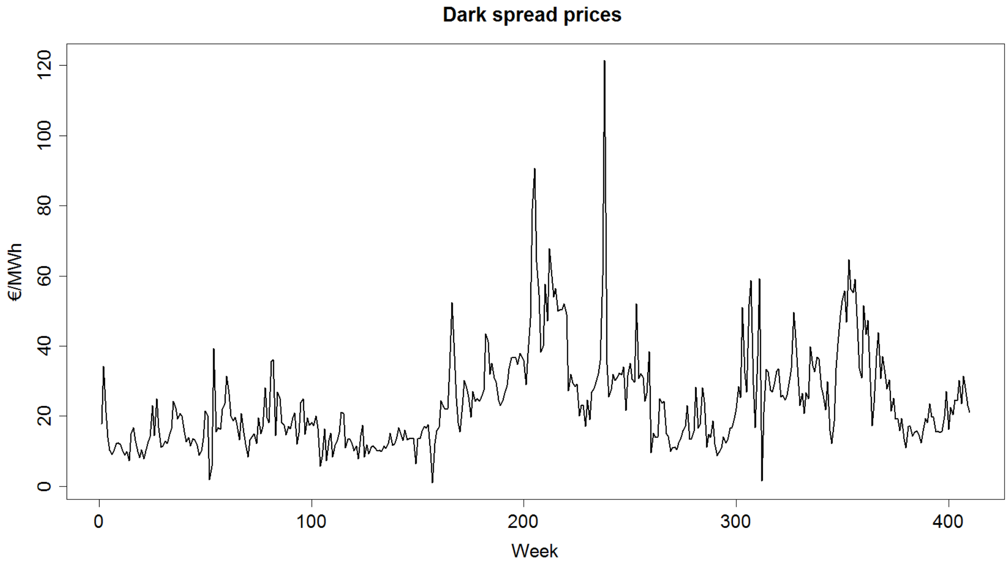

Figure 2: Weekly dark spread prices

	min.	median	mean	max.	se.	skew.	kurtosis
Electricity prices	10.70	35.52	40.07	135.69	17.51	1.34	5.39
Coal prices	6.60	14.40	16.05	44.10	7.27	1.72	6.23
Dark spread	1.09	20.27	24.01	121.39	14.07	1.93	9.86

Table 2: Summary statistics for electricity, coal and dark spread prices

2.3 NPV calculations

As a benchmark for investment analysis I calculate optimal investment threshold using NPV, an industry standard. Figure 3 shows what are the most popular capital budgeting techniques among US companies. It is well known that NPV (and other discounted cash flow methods such as IRR, payback period) underestimates critical values at which one should invest because it assumes investments are reversible and it does not take into account the opportunity cost of immediate action (option value) nor does it take into account stochastic behavior of state variables. In other words, NPV does not account for uncertainty but rather assumes that investment is made in a perfectly certain environment, which was the case in monopolized but not today's liberalized electricity markets.

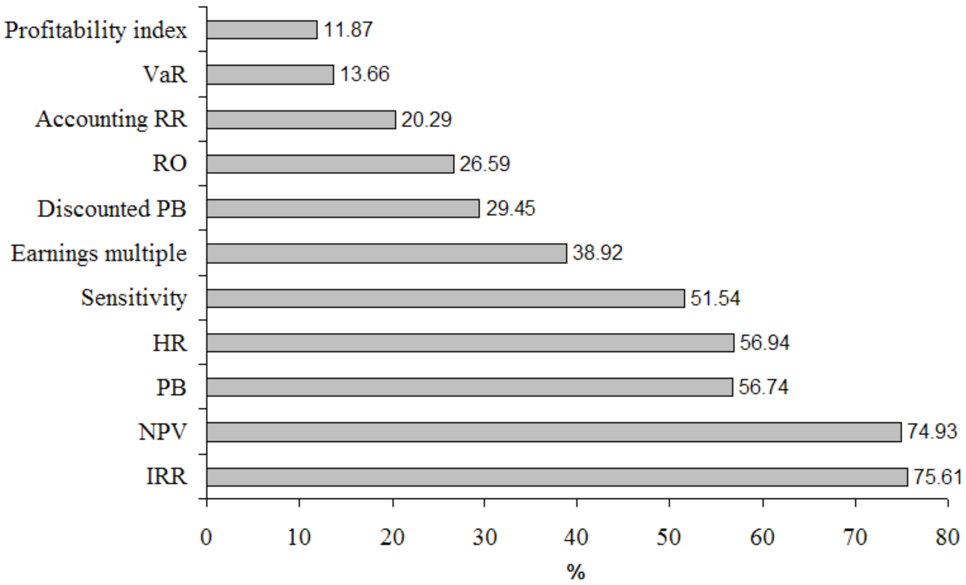

Figure 3: Popularity of different capital budgeting techniques (Graham and Harvey (2001))

Value of the project is obtained using the following expression:

$$NPV = \sum_{t=1}^{40} \frac{E(p_t \cdot q_t)}{(1+r)^t} - I \quad (1)$$

Where p_t stands for dark spread price at year t, q_t is annual production in MWh and I is investment cost. According to the NPV calculations, it is optimal to invest in the coal power plant if one believes price will be on average 20.6 €/MWh for each year during the life of the project. Figure 4 shows values of NPV as a function of the underlying dark spread price.

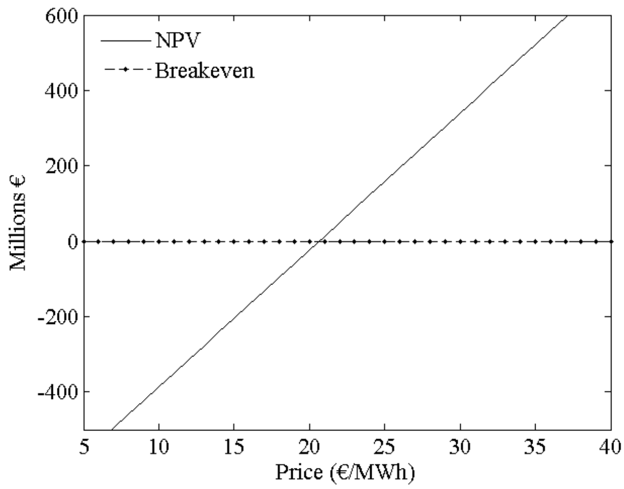

Figure 4: NPV - project value as a function of dark spread price

3 Real Options analysis

Previous section highlighted some drawbacks common to all discounted cash flow methods. Most of all, NPV assumed constant price throughout the life of the project: a realistic scenario in regulated but not in liberalized markets. Furthermore, it implicitly assumed investment is reversible: if the investment turns out bad it could be somehow undone. RO analysis can deal with these issues. In this section I look for an appropriate stochastic process that can be used to describe the evolution of dark spread prices. Following, for each stochastic process I will calculate investment threshold using RO.

In performing real options calculations one can take two approaches: contingent claims (CC) or dynamic programming (DP). CC approach rests on the assumption that one can use no arbitrage principle to derive optimal investment policy. Benefit of using CC approach is ability to use risk free interest rate as a discount rate. DP approach on the other hand relies on Bellman equation to determine the optimal investment timing using arbitrary discount rate. Because electricity can not be stored, one can not use CC approach [2]. Thus in this paper I will use DP to solve for the optimal investment threshold.

3.1 Arithmetic Brownian motion

One of the most simple stochastic processes which is commonly used in finance is Brownian motion. Despite the fact that Brownian motion does not appear to be a good candidate to model dark spread prices I evaluate it for the following reason[3]. Brownian motion is extremely simple process that has analytical solution. If by any chance Brownian motion could give results comparable to the ones I obtain by using more realistic processes, than in some cases it would be reasonable to sacrifice accuracy for the sake of simplicity. Namely, Brownian motion could be used as a first approximation to valuing more complex options.

Given that dark spread prices can become negative, I use arithmetic version of Brownian motion (ABM) to describe their evolution. ABM process for dark spread prices (p) is given by the following expression:

$$dp_t = adt + \sigma dz_t \tag{2}$$

Equation 2 states that change in the value of dark spread (dp) consists of two parts: a constant drift $a \in [-\infty, \infty]$ and stochastic component driven by constant volatility $\sigma \in [0, \infty]$ and i.i.d. random variable, i.e. $dz = \epsilon\sqrt{dt}$ where $\epsilon \sim N(0, 1)$ and $Cov(\epsilon_t, \epsilon_s) = 0$ for $t \neq s$. Expected value and variance of dark spread following ABM are given by the following two expressions:

$$E[p_t] = p_0 + at \tag{3a}$$
$$Var[p_t] = \sigma^2 t \tag{3b}$$

Equation 3a states that if dark spread follows ABM, its expected value will increase linearly at rate a as time passes. Also, Equation 3b states that variance of the ABM process increases with time as well.

[2]One potential way around this problem is to use futures contracts which can actually be stored. Unfortunately futures contracts on EEX have maturity of only a few years, and they are not very liquid, which rules out this option as well.

[3]Brownian motion is an unbounded process meaning variable can reach extremely high or low levels.

3.1.1 Parameter estimation

Given a vector of observed dark spread prices $\boldsymbol{p}$, I estimate parameters a and σ which I consider to be elements of parameter vector $\boldsymbol{\theta}$. For this I use maximum likelihood (ML) approach. Given that variable following ABM is normally distributed, log likelihood (lnL) function is given as follows:

$$lnL = (\boldsymbol{\theta}|\mathbf{p}) = -(n-1) \cdot ln\sigma - \sum_{i=2}^{n} \left[\frac{(p_i - p_{i-1} - a)^2}{2\sigma^2} \right] \tag{4}$$

Values of estimated parameters are given in Table 3. Value of a drift parameter is positive and rather economically insignificant: when converted to annual values drift equals 0.42 € per annum. On the other hand, volatility is very high and on annual basis it equals 71.4 €[4].

	Weekly values
Drift (a)	0.0081
Volatility (σ)	9.9

Table 3: Estimated weekly parameters for dark spread prices following ABM process

After estimating the parameters I determine how well does the process fit the real data. As a first measure of goodness of fit I run 10 000 simulations of ABM using estimated parameters. To simulate the ABM I use the expression in Equation 5. Given that I use weekly data to estimate the process and as I want to simulate weekly trajectories, for the value of Δt I put 1.

$$p_t = p_0 + a\Delta t + \sigma \cdot \sqrt{\Delta t} \cdot N(0,1) \tag{5}$$

In performing the simulations I need to select initial dark spread price from where the simulations will start. As the initial point for simulation I take the value of the first observed dark spread price, i.e. 17.84 €/MWh. Summary statistics for simulated dark spread prices together with statistics for observed dark spread prices are given in Table 4. Also, Figure 5 gives a histogram of simulated and observed prices.

Comparing the statistics of simulated to observed prices it appears ABM does a relatively good job in capturing the mean and median of observed data. In regard to all other statistics (minimum, maximum, standard error, kurtosis and skewness) simulation of ABM yields values which are substantially different from observed data.

	min.	median	mean	max.	se.	skew.	kurtosis
Simulations	-819.66	18.64	19.48	830.51	142.37	-0.01	4.03
Observed prices	1.09	20.27	24.01	121.39	14.07	1.93	9.86

Table 4: Summary statistics for observed and 10 000 simulated price trajectories for ABM process

[4]To obtain annual values of drift parameter I multiply weekly drift value by 52. To convert weekly volatility to annual volatility, I multiply it by $\sqrt{52}$ (Hull (2005)).

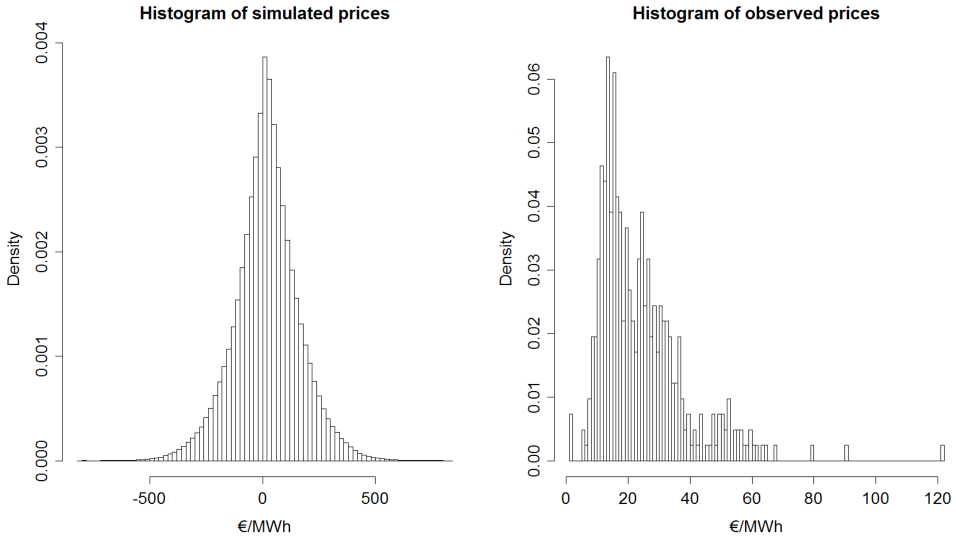

Figure 5: Histogram of observed and simulated prices for ABM process

Nevertheless, performance of ABM is even worse than it looks at first glance. Reason why ABM captures relatively well the mean and the median of observed prices is due to the selection of starting value for simulation. Starting value was 17.84 €/MWh which is relatively close to the mean of observed data (24.01 €/MWh). Because drift term in ABM is relatively small, one can expect that half of the time ABM will generate values grater than starting value, and other half of the time generated values will be lower than starting value. Thus, on average, mean of simulated prices should be around the value used to start the simulation (17.84 €/MWh), or a bit above it due to a positive drift. Therefore, if price at which the simulations are started is altered from 17.84 €/MWh to 0 €/MWh, values of descriptive statistics change and are given by Table 5. Now, it appears that ABM does not capture any of the distributional properties of observed prices.

	min.	median	mean	max.	se.	skew.	kurtosis
Simulations	-837.50	0.81	1.64	812.67	142.37	-0.01	4.03
Observed prices	1.09	20.27	24.01	121.39	14.07	1.93	9.86

Table 5: Summary statistics for observed and 10 000 simulated price trajectories for ABM process

Performing MC simulations shows ABM is not capable of matching distributional properties of dark spread prices. Furthermore, performance of ABM is greatly influenced by the starting value used for simulations. As an illustration, Figure 6 shows 4 random price realizations of ABM.

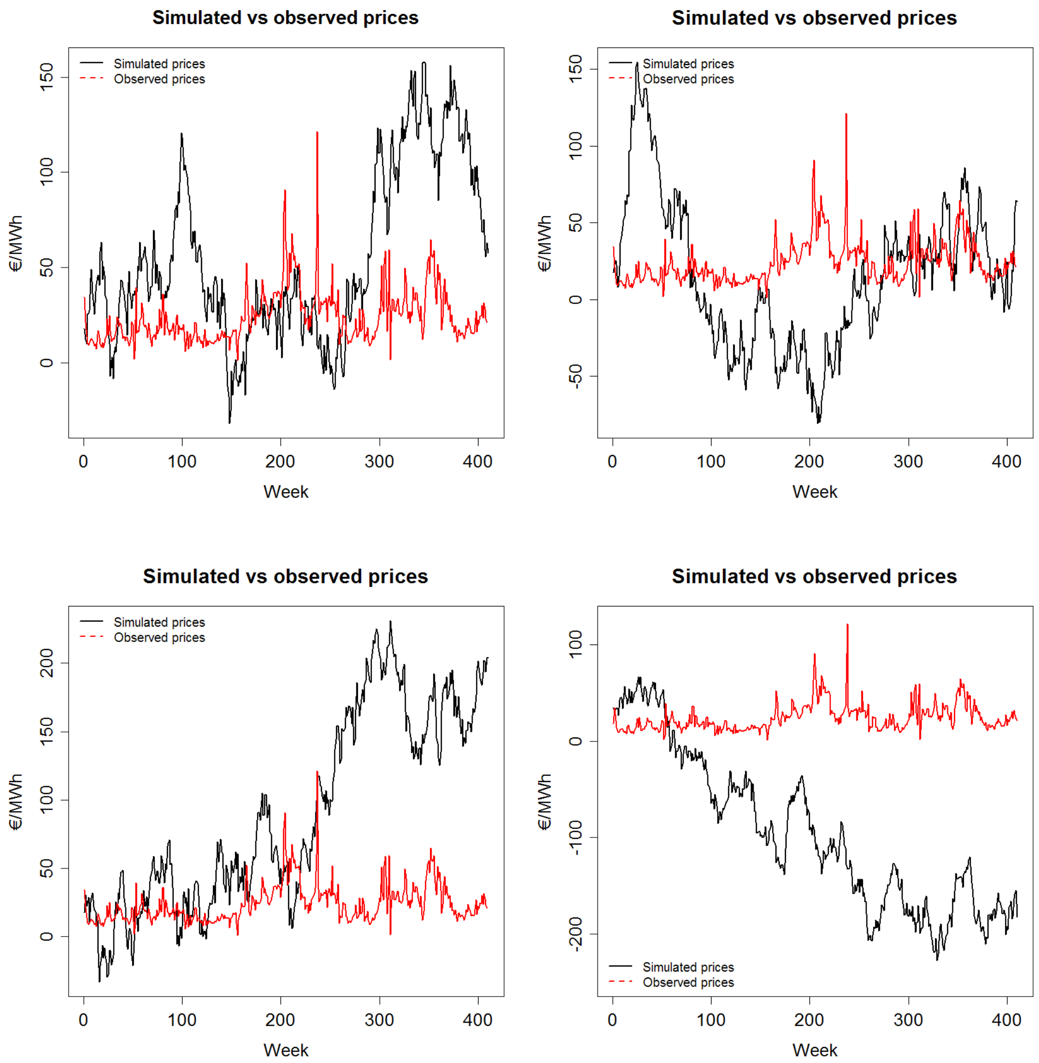

Figure 6: Simulated trajectories of arithmetic Brownian motion

As a further measure of goodness of fit I use in sample RMSE: I calculate RMSE for each of 10 000 sample paths. Table 6 reports mean value of RMSE together with the standard error of the estimate.

	mean	se.
RMSE	126.07	67.68

Table 6: RMSE for in sample forecasts for ABM process

3.1.2 Optimal investment threshold

Because I have assumed no options once the power plant is in operation, value of the project is a simple expected value of discounted future profits given by the following expression (where parameters have the same meaning as in Table 1):

$$V(p) = \int_0^T qE(p_t)e^{-rt}dt = q\int_0^T (at+p)e^{-rt}dt \tag{6a}$$

$$V(p) = \frac{q(a+pr-e^{-rT}(a+pr+arT))}{r^2} \tag{6b}$$

Following Dixit and Pindyck (1994), value of the option to invest (f) is given by:

$$E[df] = rfdt \tag{7a}$$

$$\frac{1}{2}f_{pp}\sigma^2 + af_p - rf = 0 \tag{7b}$$

Equation 7b is an ordinary differential equation which has the following general solution:

$$f(p) = Ae^{\beta_1 p} + Be^{\beta_2 p} \tag{8}$$

Where A and B are constants of integration and β is given by:

$$\beta_1 = -\frac{a}{\sigma^2} + \frac{\sqrt{a^2+2r\sigma^2}}{\sigma^2} > 0 \tag{9}$$

$$\beta_2 = -\frac{a}{\sigma^2} - \frac{\sqrt{a^2+2r\sigma^2}}{\sigma^2} < 0 \tag{10}$$

Because β_2 is negative, second part of Equation 8 ($Be^{\beta_2 p}$) implies that value of the option to invest decreases as value of dark spread increases. Intuition tells us this is incorrect and that value of the option to invest should increase with the increase in the value of underlying variable as we are more likely to make the investment: option to invest should be worth more the higher the value of dark spread. Thus, I can eliminate the second part of Equation 8 from general solution by setting $B = 0$. Therefore, solution to option value given in Equation 7b is given by Equation 11.

$$f(p) = Ae^{\beta_1 p} \tag{11}$$

To find optimal investment threshold and to determine the value of constant A, I use value matching and smooth pasting conditions (Dixit (1993), Dixit and Pindyck (1994)):

$$f(p) = V(p) - I \tag{12a}$$

$$f_p = V_p \tag{12b}$$

From smooth pasting condition I find the value of constant A:

$$A = \frac{qe^{-\beta_1 p}(1-e^{-rT})}{r\beta_1} \tag{13}$$

By inserting the expression for A and $V(p)$ into value matching condition in Equation 12a and solving for p, I obtain that price at which it becomes optimal to invest equals:

$$p = \frac{q(\beta_1 a - \beta_1 ae^{-rT} - \beta_1 ae^{-rT}Tr - r + re^{-rT}) - r^2\beta_1 I}{rq(e^{-rT}-1)\beta_1} \tag{14}$$

With the given parameters it becomes optimal to invest when dark spread price reaches 178.3 €/MWh. Critical price is extremely high and it has never been recorded in observed data. Thus, if one assumes ABM, it is very unlikely a coal power plant would be built. This is evidence showing that use of ABM in valuing investment in electricity generation cannot serve as a proxy to a more realistic mean reverting processes. Plot of the threshold price is given in Figure 7.

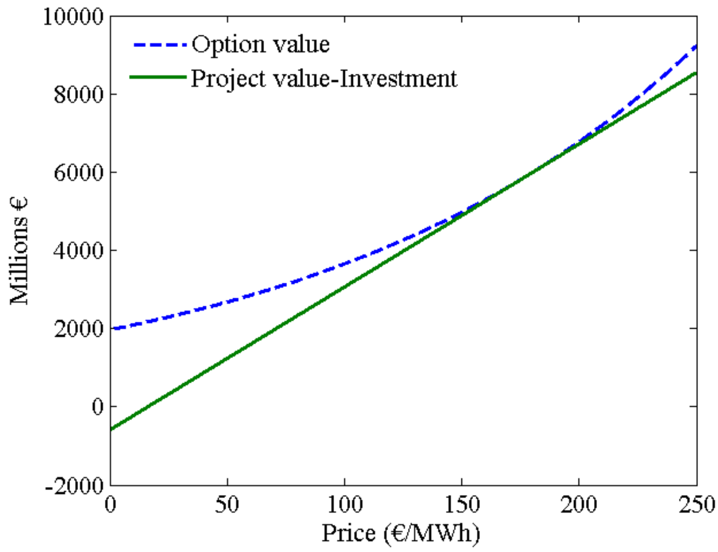

Figure 7: Optimal exercise price

Major culprit for such a high investment threshold is extremely large volatility of dark spread. Figure 8 shows dependence of investment threshold price on the standard error of the dark spread. As it can be seen from Figure 8, for lower values of standard error of dark spread, investment threshold is also lower.

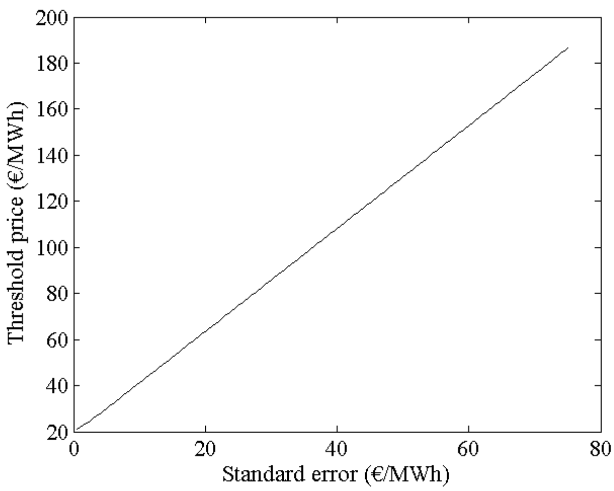

Figure 8: Threshold price as a function of standard error of dark spread

3.2 Ornstein Uhlenbeck process

In Section 3.1 it is shown that ABM cannot properly describe the behavior of dark spread prices. Economic logic suggests mean reverting processes seem best suited to model development of dark spread. Basic idea behind mean reverting process is that prices cannot stay away from some long run level for too long: soon after moving away, prices are pulled back to their long run level.

First mean reverting process I analyze is Ornstein Uhlenbeck (OU) process. The process is defined by the following stochastic differential equation:

$$dp_t = k[\mu - p_t]dt + \sigma dz_t \tag{15}$$

In Equation 15, μ represents average, long run price level: it can be thought of as a price level corresponding to average cost of production. k stands for speed of reversion i.e. how quickly prices are pulled back when they move away from their long run level. σ is volatility of price change, and p_t and dz_t represent price level and an increment of Wiener process.

3.2.1 Parameter estimation

To get the explicit solution to Equation 15 I define a new function $f(p,t) = pe^{kt}$ (Iacus (2008)). Applying Ito lemma to it I get: $f_t = pke^{kt}$, $f_p = e^{kt}$ and $f_{pp} = 0$. Furthermore:

$$df = f_t dt + f_p dp \tag{16a}$$
$$df = pke^{kt}dt + e^{kt}dp \tag{16b}$$

Inserting expression for dp from Equation 15 into Equation 16b and integrating I get the expression for dark spread following OU process:

$$p_t = p_0 e^{-kt} + \mu(1 - e^{-kt}) + \sigma \int_0^t e^{k(s-t)} dz_s \tag{17}$$

Thus, mean and variance of OU process are given by the following expressions:

$$E[p_t] = p_0 e^{-kt} + \mu(1 - e^{-kt}) \tag{18a}$$
$$V[p_t] = \frac{\sigma^2}{2k}\left(1 - e^{-2kt}\right) \tag{18b}$$

To estimate the values of parameters I use maximum likelihood approach where log likelihood (lnL) function is given by:

$$lnL(\boldsymbol{\theta}|\mathbf{p}) = -(n-1)\cdot ln\zeta - \sum_{i=2}^{n}\left[\frac{(p_t - (p_{t-1}e^{-k} + \mu(1-e^{-k})))^2}{2\zeta^2}\right] \tag{19}$$

Where I use ζ to denote standard error of dark spread prices, i.e.

$$\zeta = \sqrt{Var[p_t]} = \sigma\sqrt{\frac{1 - e^{-2kt}}{2k}}$$

The values of estimated parameters are given in Table 7.

	Weekly values
Mean reversion (k)	0.28
Long run price (μ)	24.05
Volatility (σ)	10.62
Standard error (ζ)	9.27

Table 7: Estimated weekly parameters for dark spread prices following OU process

To determine how well OU process fits the data I first simulate 10 000 trajectories using the following discretization:

$$p_t = p_0 e^{-k\Delta t} + \mu(1 - e^{-k\Delta t}) + \sigma\sqrt{\frac{1 - e^{-2k\Delta t}}{2k}} \cdot N(0,1) \tag{20}$$

Values of estimated parameters for simulated and observed dark spread prices are given in Table 8 while corresponding histograms are given in Figure 9. Table 8 shows that first and second moment of simulated prices are very close to values in observed prices. Due to the fact that OU process admits negative values which are not present in observed prices, simulated and observed prices differ in terms of skewness. Observed prices exhibit positive skew which is evident in histogram of prices shown in Figure 9. Furthermore, OU process is not able to generate such high kurtosis as the one present in observed data.

	min.	median	mean	max.	se.	skew.	kurtosis
Simulations	-44.40	23.95	23.99	102.42	14.05	0.01	3.01
Observed prices	1.09	20.27	24.01	121.39	14.07	1.93	9.86

Table 8: Summary statistics for observed and 10 000 simulated price trajectories for OU process

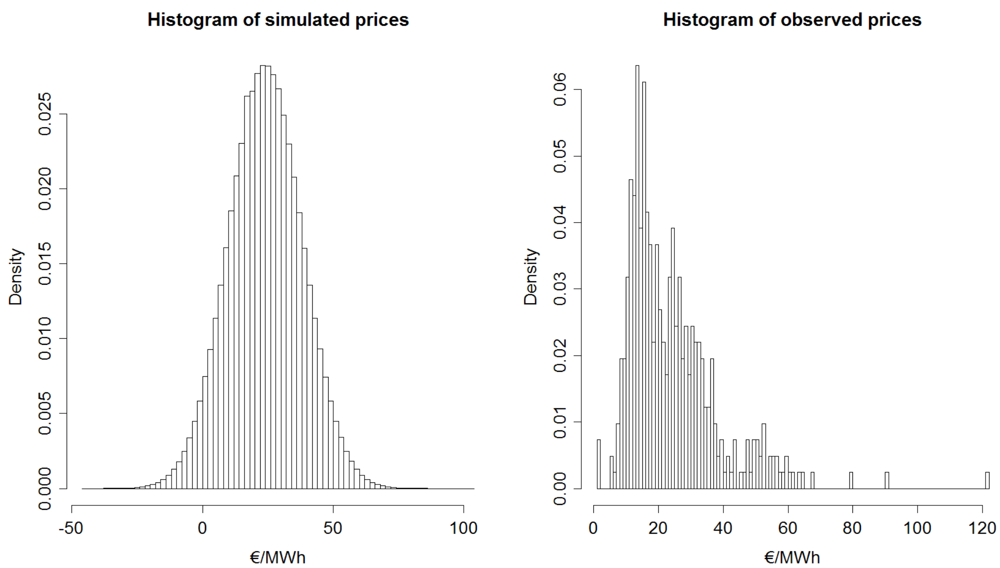

Figure 9: Histogram of observed and simulated prices for OU process

A sample trajectory of OU process is given in Figure 10. Taking into consideration summary statistics from Table 8 and comparing them with those for ABM, it is apparent that OU process is much better in describing evolution of dark spread prices.

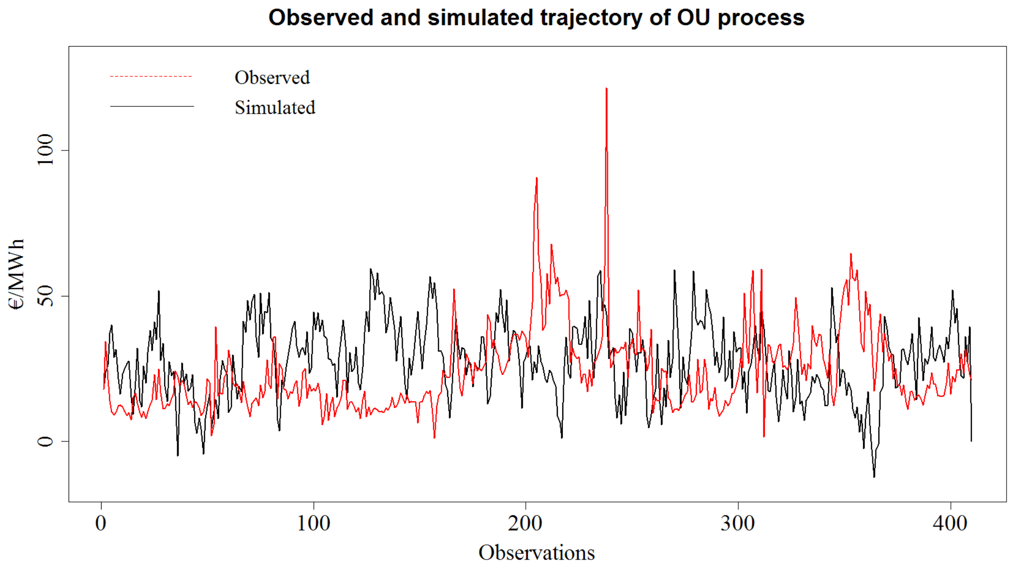

Figure 10: A sample path of dark spread following OU process

For a second test of goodness of fit I compute RMSE which is given in Table 9. Comparing it to the ABM, both mean and standard error are significantly improved.

	mean	se.
RMSE	19.82	1.22

Table 9: RMSE for in sample forecasts for OU process

3.2.2 Optimal investment threshold

Value of the project where dark spread follows OU process can be calculated analytically and is given by:

$$V(p) = \int_0^T qE(p_t)e^{-rt}dt \tag{21a}$$

$$V(p) = q\left[\frac{p_0(1-e^{-(k+r)T})}{k+r} + \frac{m(1-e^{-rT})}{r} + \frac{m(e^{-(k+r)T}-1)}{k+r}\right] \tag{21b}$$

Where I use a definition of $E(p_t)$ from Equation 18a. Value of the option to invest is given by:

$$\frac{1}{2}p_{pp}\sigma^2 + f_p k(m-p) - rf = 0 \tag{22}$$

Unfortunately, Equation 22 does not have an analytical solution, therefore, I have to resort to numerical procedures and implicit finite difference scheme (details are given in the Appendix).

For the parameters shown in Table 1 and estimated parameters for OU process, and using price increment of $dp = 0.2$ with $p_{min} = -70$ and $p_{max} = 200$ and time increment of $dt = 1/8760$ I obtain that optimal exercise price is 39 €/MWh. Plot of the exercise boundary as a function of time to maturity is given in Figure 11.

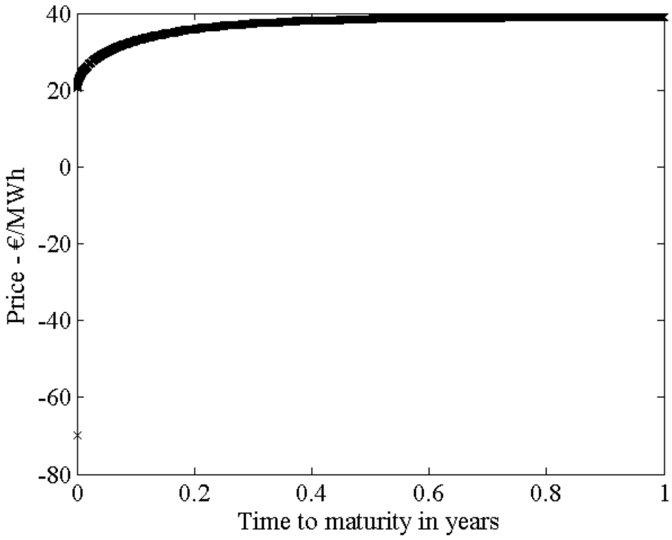

Figure 11: Exercise boundary for Orstein Uhlenbeck process as a function of option life

Threshold price obtained under OU is about twice as large as the one obtained under NPV analysis. This higher threshold is a consequence of stochastic behavior of dark spread price and irreversible nature of investment: factors that traditional capital budgeting analysis does not take into account. Nevertheless, threshold price is significantly lower than the one obtained when one assumes ABM for the evolution of dark spread prices. This is because project whose state variable follows OU process as opposed to ABM is less risky: price always reverts back to long run average.

3.3 Cox Ross Ingersoll model

Cox et al. (1985) introduced a model to describe the evolution of interest rates which can also be used to model dark spread prices (hence forth referred to as CIR process). Stochastic differential equation governing the process is given by the following expression:

$$dp_t = k(m - p_t)dt + \sigma\sqrt{p}dz_t \tag{23}$$

Main difference between CIR and OU process is in the volatility of the process which depends on the square root of price: this feature makes it very interesting for modeling dark spread prices as it implies non constant volatility. Another difference between the two processes is in the distribution of prices. Unlike OU process where prices are normally distributed, Cox et al. (1985) show that probability density function of the value of dark spread at time t, given the initial value at t_0 is given by the following expression:

$$f(p_t|p_{t0}) = ce^{-u-v}\left(\frac{v}{u}\right)^{q/2} I_q(2(uv)^{1/2}) \tag{24}$$

Where:

$$c = \frac{2k}{\sigma^2(1 - e^{-k(t-t0)})} \tag{25a}$$

$$u = cr_{t0}e^{-k(t-t0)} \tag{25b}$$

$$v = cr_t \tag{25c}$$

$$q = \frac{2k\theta}{\sigma^2} - 1 \tag{25d}$$

I_q modified Bessel function of the first kind of order q (25e)

Expected value and variance of variable p following CIR process are given by the following two equations.

$$E[p_t] = p_0 e^{-kt} + \mu(1 - e^{-kt}) \tag{26a}$$

$$Var(p_t) = p_{t-1}\frac{\sigma^2}{k}(e^{-kt} - e^{-2kt}) + \mu\frac{\sigma^2}{2k}(1 - e^{-kt})^2 \tag{26b}$$

Looking at the above equations, we see that expected value of CIR process is the same as in the case of OU process while standard error of the process is non constant and depends upon the square root of p. A useful property is that a variable following CIR process cannot become negative. More formally, if $2km/\sigma^2 > 1$ the process never reaches zeros; if $0 < 2km/\sigma^2 < 1$, zero serves as a reflecting barrier; and if $2km = 0$ zero is an absorbing barrier and it is reached surely in finite time (Overbeck and Ryden (1997)).

3.3.1 Parameter estimation

To estimate the parameters I use ML approach. Log likelihood function is given by:

$$lnL(\boldsymbol{\theta}|\mathbf{p}) = (n-1)lnc + \sum_{i=2}^{n}\left\{-u - v + 0.5qln\left(\frac{v}{u}\right) + ln\{I_q(2\sqrt{uv})\}\right\} \tag{27}$$

Using the above log likelihood function I estimate the following parameters:

	Weekly values
Mean reversion (k)	0.29
Long run price (μ)	24.05
Volatility (σ)	1.94

Table 10: Estimated weekly parameters for dark spread prices following CIR process

In order to test the goodness of fit of the model I simulate the price trajectories using estimated parameters. To simulate the trajectories it is not possible to use Euler discretization such as in the case of ABM or OU processes because of possibility of obtaining negative values for p. Formally, the following expression will not guarantee positive values of p, in which case value under the square root will be undefined:

$$p_{t+1} = p_t + k(m - p_t)dt + \sigma\sqrt{p_t}dz_{t+1} \tag{28}$$

Nevertheless, the transition density for p_t is known (Glasserman (2003)) and it can be used to simulate the price trajectories following CIR process. The transition density is given by the following expression:

$$p_t = \frac{\sigma^2(1-e^{-k(t-s)})}{4k}\chi_d^2\left(\frac{4ke^{-k(t-s)}}{\sigma^2(1-e^{-k(t-s)})}p_s\right) \quad t > s \tag{29}$$

Where

$$d = \frac{4mk}{\sigma^2} \tag{30}$$

Equation 29 says that given p_s, p_t is distributed as $\sigma^2(1-e^{-k(t-s)})/4k$ times a non central chi square random variable with d degrees of freedom and non centrality parameter λ given by:

$$\lambda = \frac{4ke^{-k(t-s)}}{\sigma^2(1-e^{-k(t-s)})}p_s \tag{31}$$

Using Equation 29 I run 10 000 simulations and obtain the summary statistics which I report in Table 11.

	min.	median	mean	max.	se.	skew.	kurtosis
Simulations	0.15	21.85	24.01	153.73	12.52	1.05	4.69
Observed prices	1.09	20.27	24.01	121.39	14.07	1.93	9.86

Table 11: Summary statistics for observed and simulated prices for CIR process

Histogram of observed versus simulated prices is given in Figure 12.

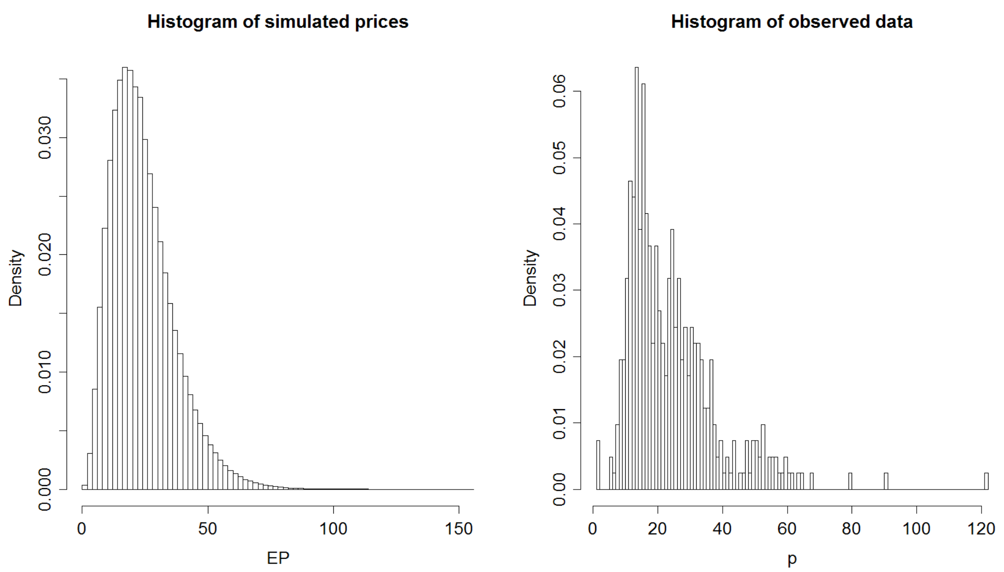

Figure 12: Histogram of observed and simulated prices for CIR process

Table 11 shows that simulated data comes very close to observed data in terms of mean and standard error, as was the case with OU process. Unlike OU process, CIR comes significantly closer to observed data in terms of skewness. A most likely reason for this improvement lays in the fact that CIR does not admit negative values, unlike OU. In terms of kurtosis, CIR also gives much better results compared with OU process, i.e. it is capable of generating larger price 'spikes'. Though, CIR is not able to come very close to the kurtosis in observed prices.

For an illustration, a plot of a simulated trajectory versus observed prices is given in Figure 13: one can see that a random CIR trajectory resembles the observed one rather well.

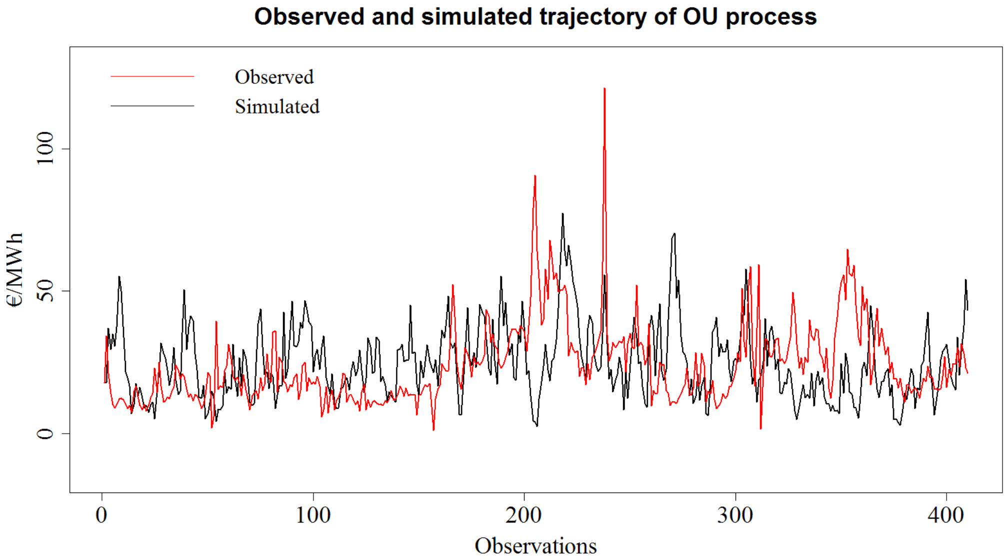

Figure 13: A sample path of simulated CIR process

Further, I test the goodness of fit using RMSE calculated from in sample forecast: values are reported in Table 12.

	mean	se.
RNSE	18.76	1.28

Table 12: RMSE for in sample forecasts for CIR process

3.3.2 Optimal investment threshold

Given that expected value of dark spread is the same as in OU case, project value ($V(p)$) is also the same as in the OU case, and is given by:

$$V(p) = q\left[\frac{p_0(1-e^{-(k+r)T})}{k+r} + \frac{m(1-e^{-rT})}{r} + \frac{m(e^{-(k+r)T}-1)}{k+r}\right] \tag{32}$$

To get the value of the option I need to solve the following differential equation:

$$\frac{1}{2}f_{pp}\sigma^2 p + f_p k(m-p) - rf = 0 \tag{33}$$

Unlike OU process, it is possible to obtain analytical solution to the investment problem. Following Ewald and Wang (2010), I first divide Equation 33 by k. Then I define:

$$f(p) = w(z) \tag{34a}$$
$$z = \frac{2kp}{\sigma^2} \tag{34b}$$
$$f_p = w_z \frac{2k}{\sigma^2} \tag{34c}$$
$$f_{pp} = w_{zz} \frac{4k^2}{\sigma^4} \tag{34d}$$

Using the above equations, Equation 33 becomes:

$$z w_{zz} + w_z(b - z) - aw = 0 \tag{35}$$

Where I define: $a = \frac{r}{k}$ and $b = \frac{2km}{\sigma^2}$. Solution to the Equation 35 is Kummer's M and U functions given by:

$$f(p) = A_1 KummerM\left(\frac{r}{k}, \frac{2km}{\sigma^2}, \frac{2kp}{\sigma^2}\right) + A_2 KummerU\left(\frac{r}{k}, \frac{2km}{\sigma^2}, \frac{2kp}{\sigma^2}\right) \tag{36}$$

Because $\lim_{p\to 0} f(p) << \infty$ I can eliminate KummerU function by assuming A_2 to be zero. Thus the solution is:

$$f(p) = A_1 KummerM\left(\frac{r}{k}, \frac{2km}{\sigma^2}, \frac{2kp}{\sigma^2}\right) \tag{37}$$

To determine threshold price at which the investment should be made I use value matching and smooth pasting conditions, namely:

$$f(p) = V(p) - I \tag{38a}$$
$$f_p = V_p \tag{38b}$$

Smooth pasting conditions gives value of the constant A_1 equal to:

$$A_1 = \frac{\left(1 - e^{(-k-r)T}\right) kmq}{r(k+r)\text{KummerM}\left(1 + \frac{r}{k}, 1 + \frac{2km}{\sigma^2}, \frac{2kp}{\sigma^2}\right)} \tag{39}$$

Using the expression for A_1 and value matching condition, I determine optimal investment threshold. Investment threshold is not given in analytical form but rather has to be found numerically. Value at which the investment should be undertake is 38.3 €/MWh which is lower than in the case of OU process. An explanation for lower threshold price when compared to OU process is that CIR process admits only positive values, which makes this process less risky than OU process. Plot of the option value and project value is given in Figure 14.

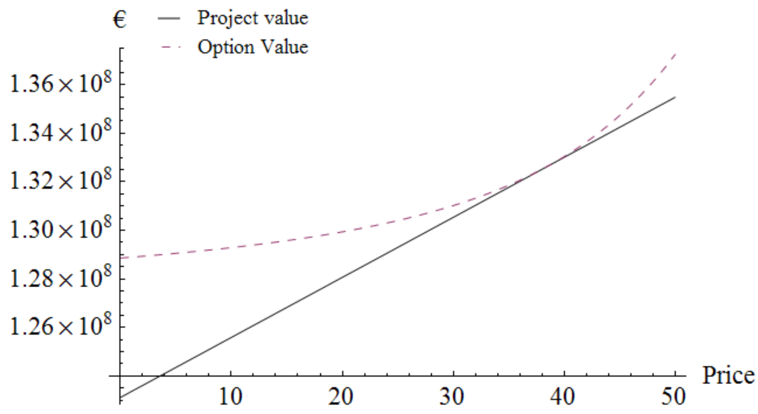

Figure 14: Optimal exercise price

3.4 Schwartz one factor model

Schwartz (1997) introduced a one factor mean reverting model for valuing commodities which is given by:

$$dp_t = k(a - lnp_t)pdt + \sigma pdz_t \tag{40}$$

The symbols have the same meaning as in case of CIR or OU process. This model also has non constant volatility which depends on price level. As a first step I transform the model into logarithm by defining $x = lnp$ and using Ito lemma I obtain the following expression (where $m = a - \frac{\sigma^2}{2k}$):

$$dx_t = k(m - x_t)dt + \sigma dz_t \tag{41}$$

To estimate the parameters I use log of price given in Equation 41, plot of which is given in Figure 15.

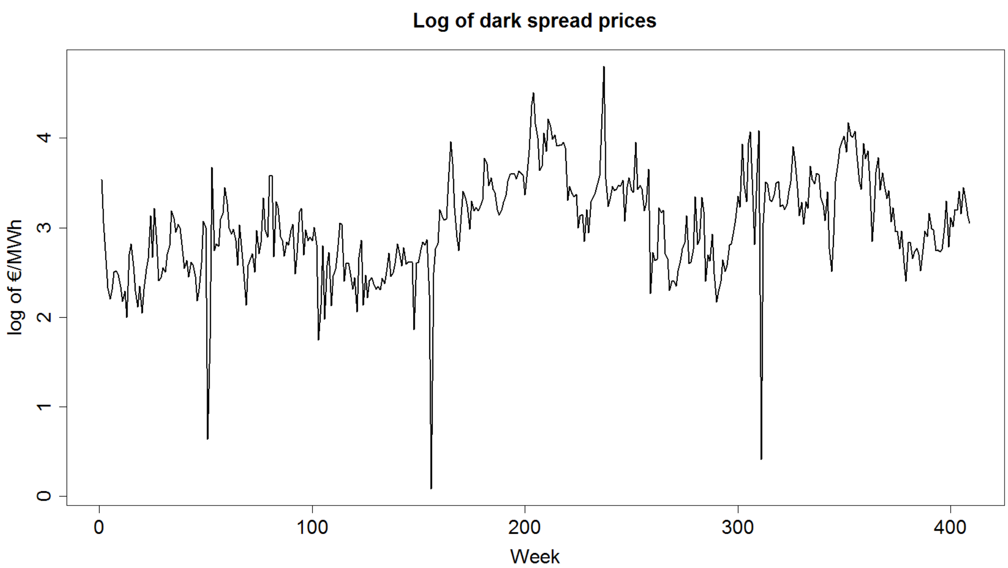

Figure 15: Logarithm of dark spread

Logarithm of dark spread given in Equation 41 is normally distributed. Expected value and variance of x_t are given by the following two expressions:

$$E[x_t] = \mu_x = x_0 e^{-kt} + m(1 - e^{-kt}) \tag{42a}$$

$$V[x_t] = \frac{\sigma^2}{2k}\left(1 - e^{-2kt}\right) \tag{42b}$$

3.4.1 Parameter estimation

To estimate the parameters of log of dark spread I use ML approach. Log likelihood function is given by the following expression:

$$lnL(\boldsymbol{\theta}|\mathbf{x}) = -(n-1)\cdot ln\zeta - \sum_{i=2}^{n}\left[\frac{(x_t - (x_{t-1}e^{-kt} + m(1-e^{-kt})))^2}{2\zeta^2}\right] \tag{43}$$

Where I use the following short hand notation:

$$\zeta = \sqrt{Var[x_t]} = \sigma\sqrt{\frac{1 - e^{-2kt}}{2k}} \tag{44}$$

The values of estimated parameters are given in Table 13.

	Weekly values
Mean reversion (k)	0.367
Long run price (a)	3.348
Standard error (ζ)	0.407
Volatility (σ)	0.484

Table 13: Estimated parameters for log of dark spread

To asses goodness of fit I use the expression from Equation 45 to simulate 10 000 trajectories of logarithm of prices following Schwartz model. Once the simulation has been performed, I take the exponential of simulated log prices to obtain level prices following Schwartz one factor model. Summary statistics are given in Table 14 and histograms of observed and simulated data are shown in Figure 16.

$$x_t = x_0 e^{-k\Delta t} + (a - \frac{\sigma^2}{2k})(1 - e^{-k\Delta t}) + \sigma\sqrt{\frac{1 - e^{-2k\Delta t}}{2k}}\cdot N(0,1) \tag{45}$$

What can be noticed is that Schwartz model comes close to the observed data in terms of first three moments of distribution based on 10 000 simulations. On the other hand, unlike CIR process, Schwartz process overestimates the kurtosis.

	min.	median	mean	max.	se.	skew.	kurtosis
Simulations	1.26	20.61	24.21	465.07	14.86	2.10	11.97
Observed prices	1.09	20.27	24.01	121.39	14.07	1.93	9.86

Table 14: Summary statistics for observed and simulated prices following Schwartz one factor model

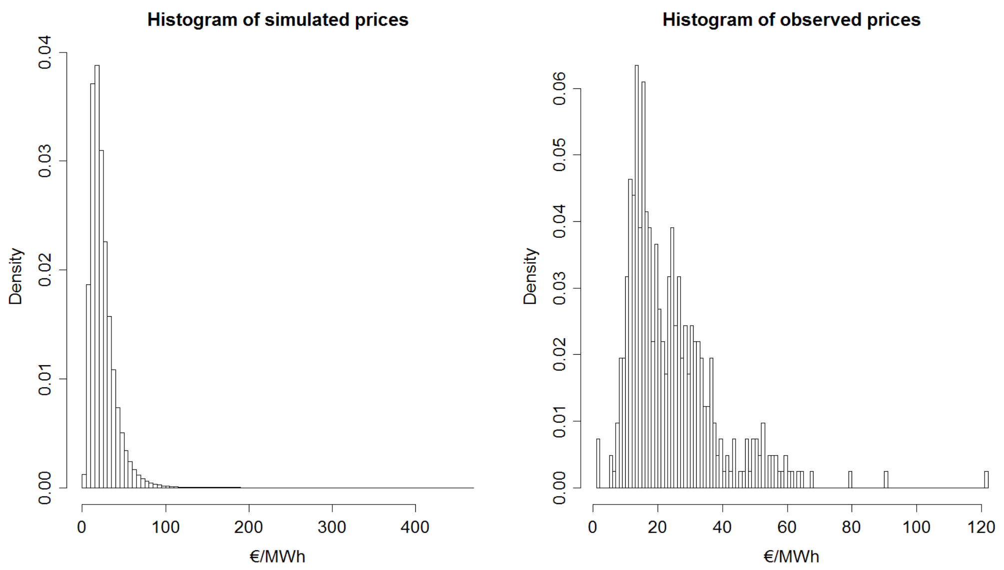

Figure 16: Histogram of simulated and observed data for Schwartz one factor model

As an example, Figure 17 shows a sample trajectory of Schwartz one factor model.

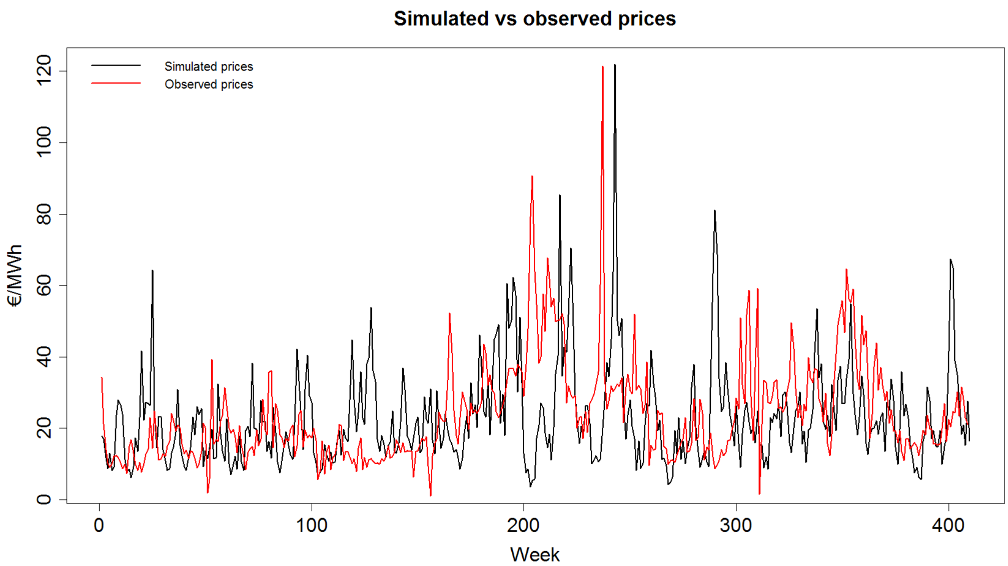

Figure 17: A sample path of simulated Schwartz one factor process

Next, to further asses goodness of fit I perform in sample forecasts and calculate RMSE. Value of RMSE is given in Table 15.

	mean	se.
RMSE	20.38	1.69

Table 15: RMSE for in sample forecasts for Schwartz model

3.4.2 Optimal investment threshold

Value of the project under Schwartz one factor process is given by:

$$V(p) = q \cdot \int_0^T E(p_t)e^{-rt}dt \tag{46}$$

To calculate the value of the project I need an expression for expected dark spread price. According to Schwartz (1997), level of dark spread is log normally distributed. Thus, if x is a logarithm of dark spread having mean and variance given by equations 42a and 42b, expected value of level of dark spread is given by (Aitchison and Brown (1957)):

$$E[p_t] = exp[\mu_x + \frac{1}{2}Var_x] \tag{47a}$$

$$E[p_t] = exp[x_0 e^{-kt} + m(1 - e^{-kt}) + \frac{\sigma_p^2}{4k}(1 - e^{-2kt})] \tag{47b}$$

Therefore, value of the project is given by:

$$V(p) = q \cdot \int_0^T exp\left[x_0 e^{-kt} + m(1 - e^{-kt}) - rt + \frac{\sigma_p^2}{4k}(1 - e^{-2kt})\right] \tag{48}$$

Nevertheless, integral in Equation 48 can only be calculated numerically. To determine the value of the option I need to solve the following equation:

$$\frac{1}{2}f_{pp}\sigma^2 p^2 + f_p k(a - lnp)p + f_t - rf = 0 \tag{49}$$

Just like in the case of OU process, the equation cannot be solved analytically, therefore, I resort to numerical procedures and use implicit finite difference scheme. Thus, using implicit finite difference method and the same approach as in OU case, I calculate that the optimal exercise price equals 41.4 €/MWh, which is higher value than what I got with CIR (38.3 €/MWh) or OU model (39 €/MWh). Plot of the exercise boundary is shown in Figure 18.

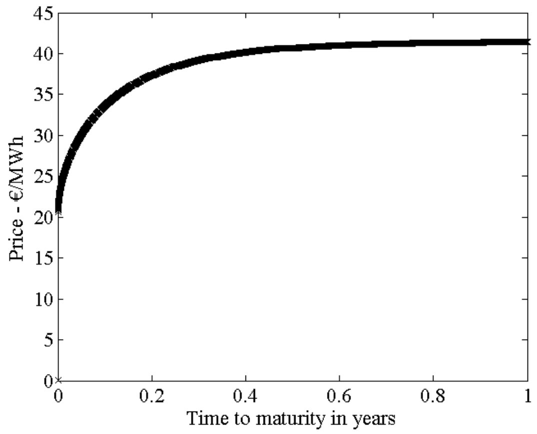

Figure 18: Exercise boundary for Schwartz one factor model as a function of time

4 Selection of appropriate stochastic process

In previous sections I estimated parameters of stochastic processes and determined optimal investment threshold for each stochastic process. The question I still have to answer is which process is the most appropriate for use in real options analysis. It should be the one that comes closest to the observe data. To gauge how close each process comes to observed data I use distributional statistics and RMSE.

In terms of distributional statistics it is apparent that mean reverting processes perform better than ABM process. The difference between each individual mean reverting process is not very significant, but Schwartz comes the closest to the observed data and is followed by CIR and OU process. This can be seen from Table 16 which shows absolute difference for each process and for each statistic from observed data[5].

	min.	median	mean	max.	se.	skew.	kurtosis
ABM	820.75	1.63	4.53	709.12	128.3	1.94	5.83
OU	45.49	3.68	0.02	18.97	0.02	1.92	6.85
CIR	0.94	1.58	0	32.34	1.55	0.88	5.17
Schwartz	0.17	0.34	0.2	343.68	0.79	0.17	2.11

Table 16: Absolute difference between simulated and observed data

Using distributional statistics, even though common in literature (e.g. Geman and Roncoroni (2006) and Seifert and Uhrig-Homburg (2007)), is not an exact measure. As a second measure of goodness of fit I use RMSE. Using this statistic it appears CIR process performs the best, while ABM performs the worst. On the other hand, performance of OU and Schwartz one factor process is rather comparable. The results are show in Table 17.

	mean	se.
ABM	126.07	67.68
OU	19.82	1.22
CIR	18.76	1.28
Schwartz	20.38	1.69

Table 17: RMSE for all stochastic processes

The question, though, is whether the differences in RMSE among stochastic processes are statistically significant. Diebold and Mariano (1995) develop a test for comparing prediction accuracy of two different models. Denote by y_{t+1} actual time series at time $t+1$ and by $\hat{y}_{A,t+1|t}$ and $\hat{y}_{B,t+1|t}$ two different forecasts of the actual series. Error associated with each forecast is given by:

$$e_{A,t+1} = \hat{y}_{A,t+1|t} - y_{t+1} \tag{50a}$$

$$e_{B,t+1} = \hat{y}_{B,t+1|t} - y_{t+1} \tag{50b}$$

[5] Absolute values are computed according to the following expression: $|\bar{y}_{p,i} - y_i|$: $\bar{y}_{p,i}$ represents value of a statistic i (minimum and maximum value, median, mean, standard error, skewness and kurtosis) for stochastic process p (ABM, OU, CIR and Schwartz) and y_i represents value of statistic i for observed prices.

The goal is to determine time $t+1$ loss associated with each forecast. To calculate the loss, I define a loss function g which is a direct function of forecast error, i.e. $g = g(e_{i,t+1})$ where $i = A, B$. I choose a quadratic loss function, $g = e^2_{i,t+1}$ Then, one can state a null hypothesis of equal forecast accuracy for both models against the alternative that their forecasts differ:

$$H_0: \; E[e^2_{A,t+1}] = E[e^2_{B,t+1}] \tag{51a}$$

$$H_1: \; E[e^2_{A,t+1}] \neq E[e^2_{B,t+1}] \tag{51b}$$

If one defines $d_t = e^2_{A,t+1} - e^2_{B,t+1}$, the null and the alternative hypothesis can be stated as:

$$H_0: \; E[d_t] = 0 \tag{52a}$$

$$H_1: \; E[d_t] \neq 0 \tag{52b}$$

Diebold Mariano (DM) test statistic is given by the following expression:

$$DM = \frac{\bar{d}}{Var(d)} \tag{53}$$

Where:

$$\bar{d} = T^{-1} \sum_{i=1}^{T} d_i \tag{54a}$$

$$Var(d) = T^{-1} \left[\gamma_0 + 2 \sum_{k=1}^{h-1} \gamma_k \right] \tag{54b}$$

And γ_k is k^{th} auto covariance that can be estimated by Equation 55.

$$\gamma_k = T^{-1} \sum_{t=k+1}^{T} (d_t - \bar{d})(d_{t-k} - \bar{d}) \tag{55}$$

To calculate DM statistic one can use Newey West robust standard error. As DM test statistic is a pairwise test, I report results for various combinations of stochastic processes. As each simulation of stochastic process is different from the previous one, I report results of DM test for all 10 000 simulations (calculation details for DM statistic are given in the Appendix).

4.1 ABM versus OU process

Both distributional properties and RMSE showed that ABM is the worst performing of all stochastic processes. Here, using DM statistic, I try to formalize this result by testing the null hypothesis of equal forecast accuracy between ABM and OU process against the alternative of ABM having worse forecast accuracy:

$$H_0: \; E[e^2_{ABM,t}] = E[e^2_{OU,t}] \tag{56a}$$

$$H_1: \; E[e^2_{ABM,t}] > E[e^2_{OU,t}] \tag{56b}$$

To perform hypothesis testing I use standard normal distribution, as it is suggested in Diebold and Mariano (1995). Therefore, for a one sided test where the alternative is that ABM gives worse forecast than OU process, critical value at 5% significance level is 1.65. Therefore, if the value of DM statistic is greater than critical value, I reject the null in favor of the alternative hypothesis.

In 75.1% of sample trajectories OU process has statistically lower forecast error, i.e. it performs better than ABM process. Therefore, I confirm that OU process is superior to ABM in generating data which resemble the observed prices. Figure 19 shows a histogram of computed DM statistics.

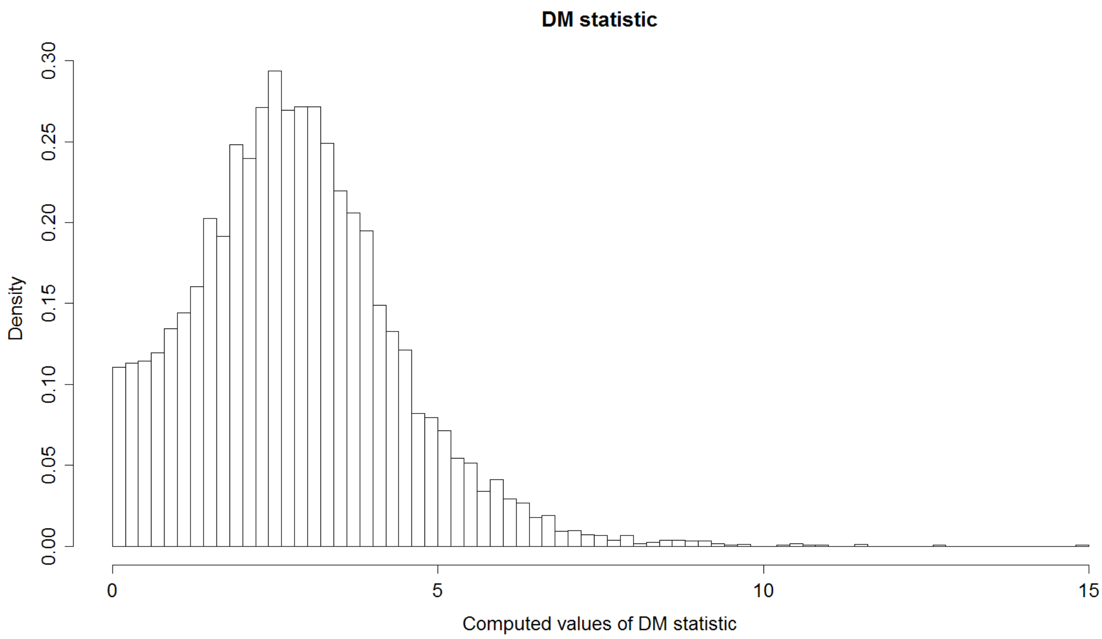

Figure 19: Histogram of computed values for DM test

4.2 ABM versus CIR process

I test the null hypothesis of equal forecast accuracy between ABM and CIR process against an alternative hypothesis that CIR has better forecast accuracy:

$$H_0 : \ E[e^2_{ABM,t}] = E[e^2_{CIR,t}] \tag{57a}$$
$$H_1 : \ E[e^2_{ABM,t}] > E[e^2_{CIR,t}] \tag{57b}$$

Results are very similar to the ones I get when comparing ABM and OU process. In this case I reject the null in 78.9% of sample trajectories, confirming that CIR process is statistically superior to ABM in terms of in sample forecast. Figure 20 gives a histogram of computed DM statistics.

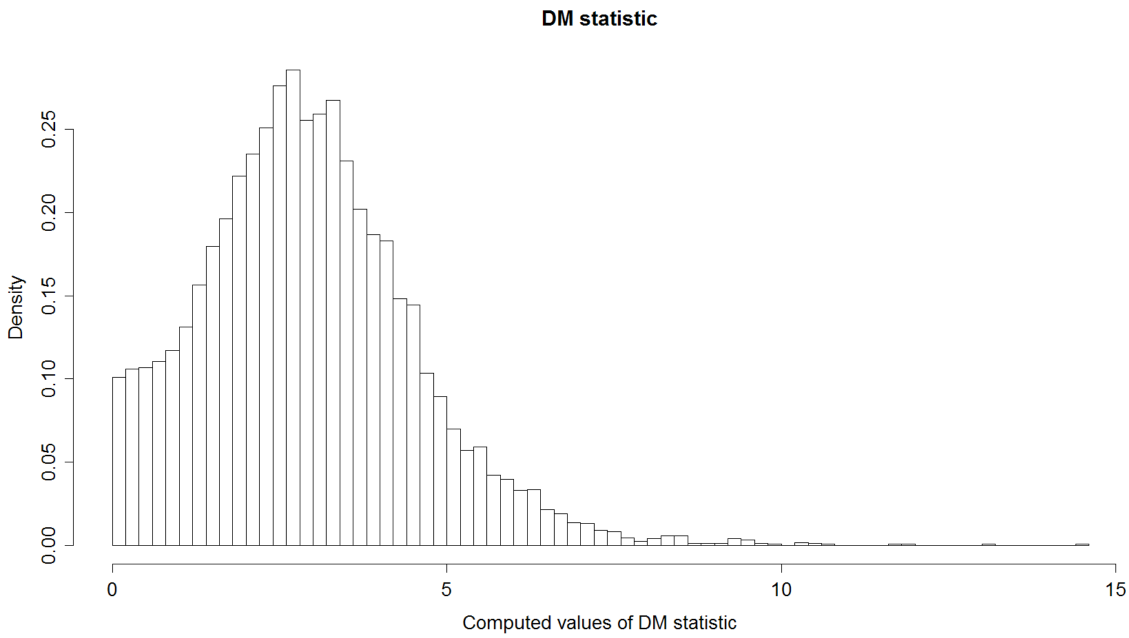

Figure 20: Histogram of computed values for DM test

4.3 ABM versus Schwartz process

Finally, I test ABM against Schwartz one factor model using the same null and alternative hypothesis as in the previous case:

$$H_0 : \ E[e^2_{ABM,t}] = E[e^2_{Sch,t}] \tag{58a}$$

$$H_1 : \ E[e^2_{ABM,t}] > E[e^2_{Sch,t}] \tag{58b}$$

Results are as expected and close to what I obtained when comparing ABM to OU and CIR process. For 72.6% of sample trajectories, Schwartz process results in statistically more accurate in sample forecasts. Histogram of computed values of DM statistic is given in Figure 21.

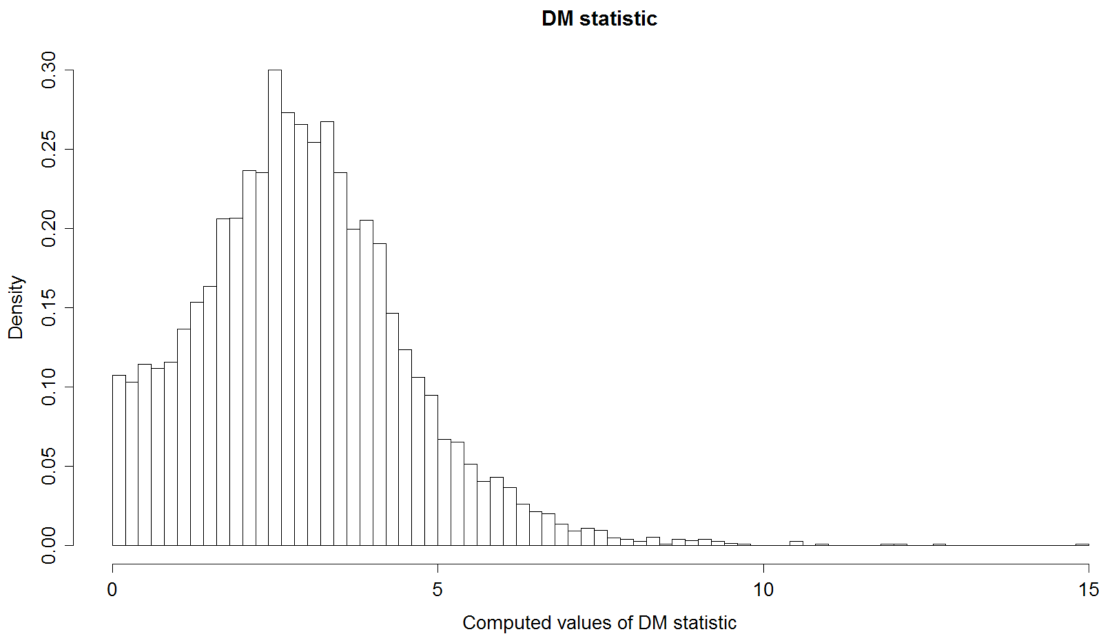

Figure 21: Histogram of computed values for DM test

4.4 OU versus CIR process

Next I test mean reverting processes against each other. First I test whether OU and CIR process have the same forecast accuracy against the two sided alternative that their forecasts differ. At 5% significance level, critical value is 1.95 and I reject the null if DM statistic in absolute value is greater than 1.95. For the given data, I reject the null in 13.9% of all sample paths.

$$H_0: \ E[e^2_{OU,t}] = E[e^2_{CIR,t}] \tag{59a}$$
$$H_1: \ E[e^2_{OU,t}] \neq E[e^2_{CIR,t}] \tag{59b}$$

I also test whether OU and CIR have the same forecast accuracy against the alternative that CIR has better forecast accuracy, i.e. lower forecast error. For a 5% significance level, one sided critical value is 1.65, and I reject the null if value of DM test exceeds 1.65. I reject the null in 20.3% of all sample paths.

$$H_0: \ E[e^2_{OU,t}] = E[e^2_{CIR,t}] \tag{60a}$$
$$H_1: \ E[e^2_{OU,t}] > E[e^2_{CIR,t}] \tag{60b}$$

I also try a different alternative hypothesis where forecast error of CIR model exceeds forecast error of OU model. Here again I use one sided alternative with 5% significance level: I reject the null if value of DM statistic is smaller than -1.65. In this case I reject the null in only 1.4% of all trajectories.

$$H_0: \ E[e^2_{OU,t}] = E[e^2_{CIR,t}] \tag{61a}$$
$$H_1: \ E[e^2_{OU,t}] < E[e^2_{CIR,t}] \tag{61b}$$

Therefore, CIR and OU process give forecasts of comparable accuracy. Nevertheless, it is possible that CIR process does perform better than OU process while the converse is not very likely. Histogram of computed values for DM statistic is given in Figure 22.

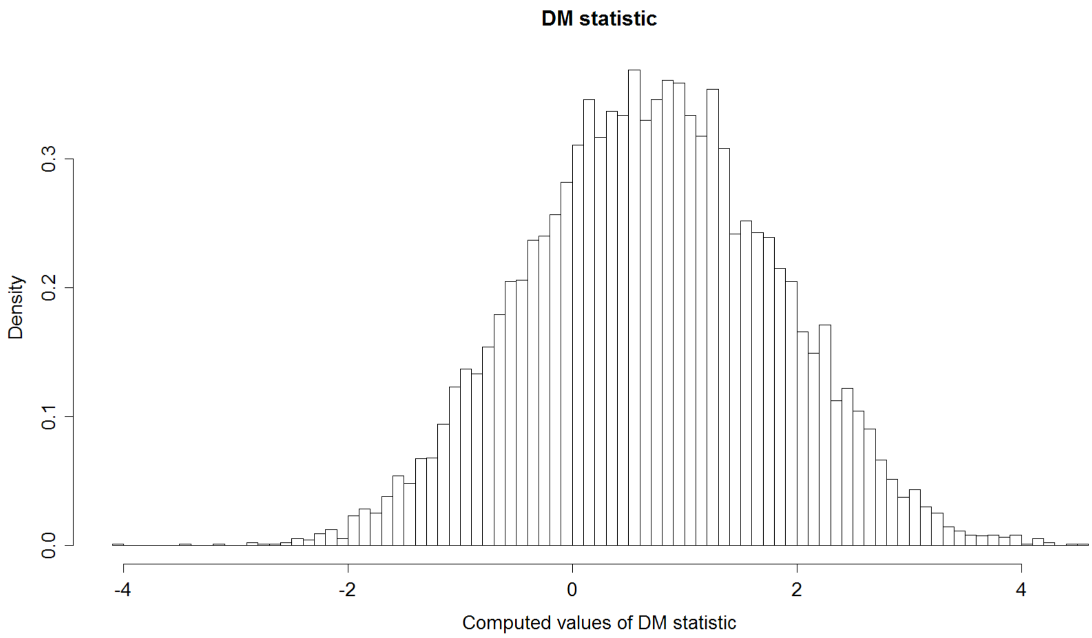

Figure 22: Histogram of computed values for DM test

4.5 OU versus Schwartz process

I test whether OU and Schwartz models generate forecasts of equal accuracy. At 5% significance level I reject the null in 6.5% of all trajectories.

$$H_0: \; E[e^2_{OU,t}] = E[e^2_{Sch,t}] \tag{62a}$$
$$H_1: \; E[e^2_{OU,t}] \neq E[e^2_{Sch,t}] \tag{62b}$$

I also evaluate the possibility that Schwartz model has lower forecast error. In this case I reject the null in only 4.7% of sample paths.

$$H_0: \; E[e^2_{OU,t}] = E[e^2_{Sch,t}] \tag{63a}$$
$$H_1: \; E[e^2_{OU,t}] > E[e^2_{Sch,t}] \tag{63b}$$

Furthermore, I evaluate the alternative possibility that OU performs better. Assuming this alternative, I reject the null in 8.6% of all sample paths.

$$H_0: \; E[e^2_{OU,t}] = E[e^2_{Sch,t}] \tag{64a}$$
$$H_1: \; E[e^2_{OU,t}] < E[e^2_{Sch,t}] \tag{64b}$$

Therefore, I conclude that OU and Schwartz model give forecasts that have statistically similar accuracy: it does not matter much which model is chosen. Histogram of computed values of DM statistic is given in Figure 23.

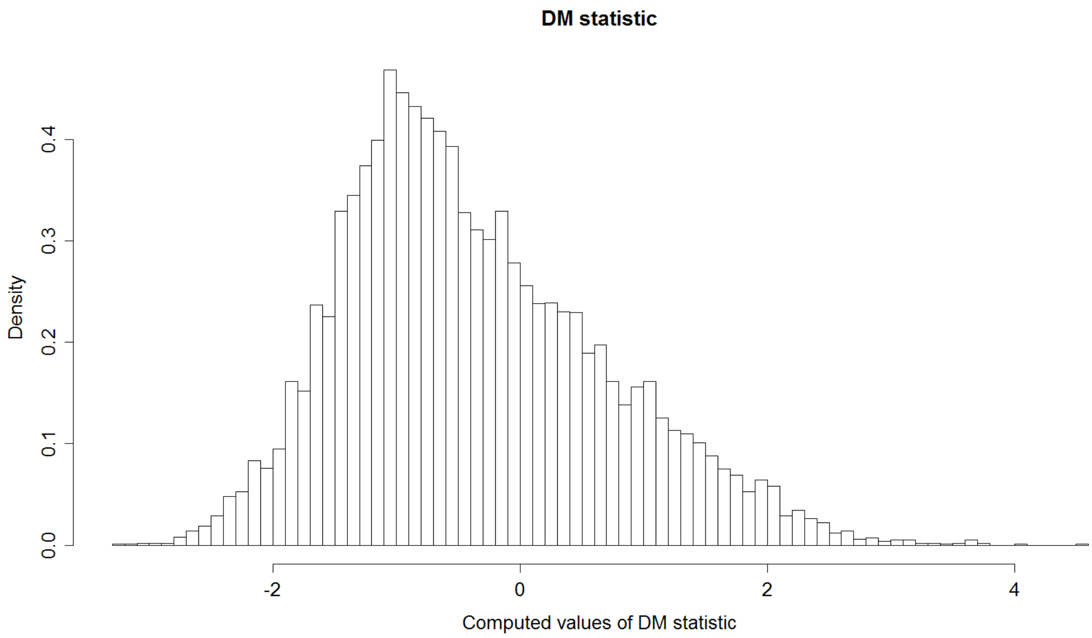

Figure 23: Histogram of computed values for DM test

4.6 CIR versus Schwartz process

First, I test an alternative that Schwartz and CIR model generate forecasts of different accuracy. I reject the null in favor of the alternative in 13% of sample paths.

$$H_0: \; E[e^2_{CIR,t}] = E[e^2_{Sch,t}] \tag{65a}$$
$$H_1: \; E[e^2_{CIR,t}] \neq E[e^2_{Sch,t}] \tag{65b}$$

Now I test whether CIR model performs better than Schwartz one factor model. I obtain this is the case in 20.5% of all trajectories.

$$H_0: \; E[e^2_{CIR,t}] = E[e^2_{Sch,t}] \tag{66a}$$
$$H_1: \; E[e^2_{CIR,t}] < E[e^2_{Sch,t}] \tag{66b}$$

Following I test whether CIR process performs worse than Schwartz process, i.e. whether CIR process has higher forecast error. This is the case in 1.4% of all trajectories.

$$H_0: \; E[e^2_{CIR,t}] = E[e^2_{Sch,t}] \tag{67a}$$
$$H_1: \; E[e^2_{CIR,t}] > E[e^2_{Sch,t}] \tag{67b}$$

I conclude that these processes result in either equal or forecasts where CIR process performs better. Therefore, CIR appears to be a better candidate to be used for modeling dark spread prices than Schwartz one factor model. Figure 24 gives histogram of computed DM statistics.

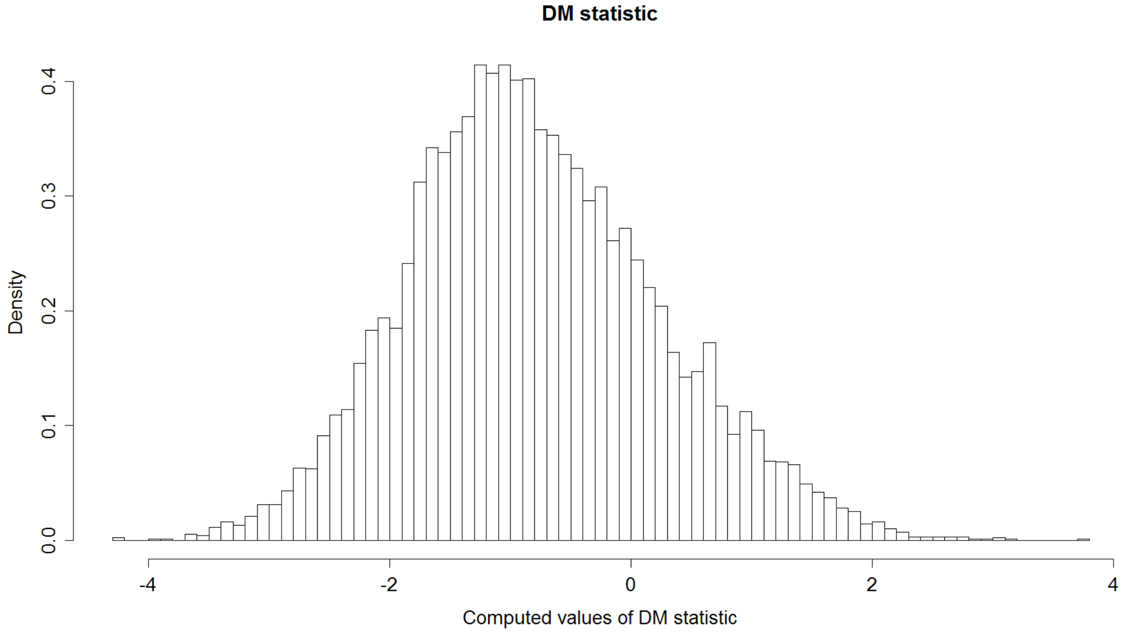

Figure 24: Histogram of computed values for DM test

5 Conclusion

First goal of the paper was to find an appropriate stochastic process to fit observed dark spread prices. I used four different processes: ABM, OU, CIR and Schwartz one factor model. After estimating the parameters I simulated 10 000 sample paths for each process and tried to determine how close are simulated trajectories to observed data. To gauge the difference I first used an informal, yet common procedure of computing distributional properties of simulated prices. According to distributional properties, ABM process was the worst in terms of replicating the observed data. Mean reverting processes performed rather well, with Schwartz one factor model performing best, followed by CIR and OU process.

Following I used RMSE, as a more formal measure to gauge the difference between selected models. Here again, ABM process performed the worst. On the other hand, performance of mean reverting processes changed. According to RMSE calculations, CIR process performed the best, while OU and Schwartz one factor model performed equally well. Finally I addressed the issue of whether RMSE difference between stochastic processes is statistically different. I used Diebold and Mariano (1995) test statistic and performed pairwise tests. As expected, ABM is statistically the worst performing process. Statistically, difference between mean reverting processes is not so dramatic. OU and Schwartz one factor model perform equally well, thus due to its simplicity, one should favor OU over Schwartz model. CIR process performed equally well as OU and Schwartz process or slightly better. At 5% significance level and in 20.3% of sample paths, I reject the null that CIR performs equally well as OU process in favor of the alternative that it performs better than OU process. Moreover, at the same significance level and in 20.5% of sample paths, I also reject the null that CIR performs equally well as Schwartz model in favor of the alternative that it performs better. Therefore, I conclude that all mean reverting processes equally well resemble observed data, while slight advantage should be given to CIR process.

Second goal of the paper was to asses to what degree does investment threshold depend upon the selected stochastic process. Of the four stochastic processes, ABM turns out to be the least useful - it is predicting that investment should be undertaken at extreme prices, which are very unlikely to ever occur. Other three processes are all mean reverting and prior to the analysis one would expect that OU process gives the highest investment threshold while CIR and Schwartz model give lower and perhaps rather comparable investment thresholds. Reason for this conjecture lays in the fact that OU process admits negative prices, therefore, one would require higher threshold price at which to invest.

Actual calculations do not completely confirm this conjecture. Investment threshold is the highest under Schwartz process and equals 41.4 €/MWh. Use of OU process results in second highest threshold of 39 €/MWh, while CIR process results in lowest investment threshold of 38.3 €/MWh. A possible explanation to why Schwartz process results in higher investment threshold lays in the fact that I work with logarithm of prices to estimate the parameters. Summary of threshold prices for all four processes is given in Table 18.

Process	Value (€/MWh)
NPV	20.6
ABM	178.3
OU	39
CIR	38.3
Schwartz	41.4

Table 18: Threshold prices for different stochastic processes

Given threshold values from Table 18 it seems that choice of mean reverting process does not play such a crucial role in making the investment decision. Reason why investment threshold does not vary significantly with mean reverting processes is due to the fact that they revert to long run price level soon after they move away from it. Table 19 shows values of half life for three mean reverting stochastic processes. As it can be seen, it takes approximately around two weeks for prices to revert half way back to the long run level from the starting price if there are no stochastic disturbances.

Process	half-life
OU	17 days
CIR	16.9 days
Schwartz	13.2 days

Table 19: Half life for mean reverting processes

Having regard for distribution properties and RMSE it would be most appropriate to use CIR process in valuing investment in base load coal fired power plant.

Appendices

A Implicit finite difference scheme

To calculate optimal investment threshold I first add time dimension (derivative f_t) to Equation 22 and then use implicit finite difference scheme (Wilmott (2006)). Derivatives in Equation 22 are approximated in the following way:

$$f_t = \frac{f_{i,j+1} - f_{i,j}}{dt} \qquad f_{pp} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{dp^2} \qquad f_p = \frac{f_{i+1,j} - f_{i-1,j}}{2dp} \tag{68}$$

Inserting derivative approximations from Equation 68 into Equation 22 I obtain the following expression for the value of the option:

$$\frac{1}{2}\frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{dp^2}\sigma^2 + \frac{f_{i+1,j} - f_{i-1,j}}{2dp}k(m - p_i) + \frac{f_{i,j+1} - f_{i,j}}{dt} - rf_{i,j} = 0 \tag{69}$$

After some manipulation Equation 69 can be written more compactly as:

$$f_{i,j+1} = A_i f_{i-1,j} + B_i f_{i,j} + C_i f_{i+1,j} \tag{70}$$

Where A, B, and C are defined as:

$$\begin{aligned} A_i &= \frac{dt}{2dp^2}\big((m - p_i)dpk - \sigma^2\big) \quad B = \frac{1}{dp^2}\big(dt\sigma^2 + dp^2(1 + dtr)\big) \\ C_i &= \frac{dt}{2dp^2}\big((p_i - m)dpk - \sigma^2\big) \end{aligned} \tag{71}$$

For the boundary conditions I assume *Dirichlet* boundary conditions, i.e. that for each time step j, value of the option remains constant at p_{min} and p_{max} and equals option value at expiration (T), i.e.:

$$f(p_{min}, j) = f(p_{min}, T) \qquad f(p_{max,j}) = f(p_{max}, T) \tag{72}$$

At expiration, the following terminal condition holds:

$$f(idp, T) = max[f(idp, T) - I, 0] \tag{73}$$

Where I denotes investment cost. Now I solve the following set of simultaneous equations for each time step:

$$\begin{aligned} f_{2,j+1} &= A_2 f_{1,j} + B_2 f_{2,j} + C_2 f_{3,j} \\ f_{3,j+1} &= A_3 f_{2,j} + B_3 f_{3,j} + C_3 f_{4,j} \\ &\dots \\ f_{n-1,j+1} &= A_{n-1} f_{n-2,j} + B_{n-1} f_{n-1,j} + C_{n-1} f_{5,j} \end{aligned} \tag{74}$$

Equations 74 can be written in matrix form as:

$$\begin{pmatrix} f_{2,j+1} \\ f_{3,j+1} \\ \cdots \\ f_{n-1,j+1} \end{pmatrix} = \begin{pmatrix} B_2 & C_2 & \dots & \dots \\ A_3 & B_3 & C_3 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & A_{n-1} & B_{n-1} \end{pmatrix} \cdot \begin{pmatrix} f_{2,j} \\ f_{3,j} \\ \dots \\ f_{n-1,j} \end{pmatrix} + \begin{pmatrix} f_{1,j} \cdot A_2 \\ \cdots \\ \cdots \\ f_{n,j} \cdot C_{n-1} \end{pmatrix} \tag{75}$$

Finally, starting at terminal condition $(j+1)$, and moving backwards in time, I solve for the option value at time j:

$$\begin{pmatrix} f_{2,j} \\ f_{3,j} \\ \cdots \\ f_{n-1,j} \end{pmatrix} = \begin{pmatrix} B_2 & C_2 & \dots & \dots \\ A_3 & B_3 & C_3 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & A_{n-1} & B_{n-1} \end{pmatrix}^{-1} \cdot \left[\begin{pmatrix} f_{2,j+1} \\ f_{3,j+1} \\ \cdots \\ f_{n-1,j+1} \end{pmatrix} - \begin{pmatrix} f_{1,j} \cdot A_2 \\ \cdots \\ \cdots \\ f_{n,j} \cdot C_{n-1} \end{pmatrix} \right] \tag{76}$$

To approximate perpetual American option I evaluate impact of option maturity on optimal exercise price. Basically I let *time* increase and observe what happens to optimal exercise price at the start of the option life. By making the option life sufficiently long I try to mimic perpetual option.

It turns out I need to make option life only be couple of years, as with increasing maturity optimal exercise price does not change. Optimal investment threshold is found at the point where:

$$f(p) = V(p) - I$$

Reason to how can option of such short time to maturity be used to approximate perpetual option lays in the fact that OU is mean reverting process implying that if the price deviates from the long run average, it will be pulled back. The stronger the mean reversion factor the shorter one has to make the option life to mimic perpetual option.

B Calculation of Diebold-Mariano statistic

Steps in calculation of Diebold Mariano statistic are as follows:

1. Calculate error associated with each model (A and B):

$$e_{A,t+1} = \hat{y}_{A,t+1|t} - y_{t+1} \tag{77a}$$
$$e_{B,t+1} = \hat{y}_{B,t+1|t} - y_{t+1} \tag{77b}$$

2. Define a variable d_t such that: $d_t = e^2_{A,t+1} - e^2_{B,t+1}$.

3. To determine the variance of d ($Var(d)$) I regress d_t on a constant and compute Newey West robust standard error of the constant. This standard error is then used in calculating the $Var(d)$.

4. For each simulated sample path, test the following hypothesis:

$$H_0 : \ E[d_t] = 0 \tag{78a}$$
$$H_1 : \ E[d_t] \neq 0 \quad \text{or } H_1 : \ E[d_t] > 0 \quad \text{or } H_1 : \ E[d_t] < 0 \tag{78b}$$

5. Determine the paths for which the DM statistic is above the critical value.
6. Report the percentage of sample paths for which I can reject the null in favor of the alternative hypothesis.

References

Abadie, Luis M. and Jos M. Chamorro, "Valuing flexibility: The case of an Integrated Gasification Combined Cycle power plant," *Energy Economics*, 2008, *30*, 1850–1881.

Aitchison, J. and J.A.C Brown, *The lognormal distribution*, Cambridge University Press, 1957.

Bckman, Thor, Stein-Erik Fleten, Erik Juliussen, Hvard J. Langhammer, and Ingemar Revdal, "Investment timing and optimal capacity choice for small hydropower projects," *European Journal of Operational Research*, 2008, *190* (1), 255 – 267.

Brennan, Michael J. and Eduardo S. Schwartz, "Evaluating Natural Resource Investments," *The Journal of Business*, 1985, *58* (2), 135–157.

Cox, John C., Jonathan E. Ingersoll Jr, and Stephen A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 1985, *53 (2)*, 385–407.

Diebold, Francis X. and Roberto S. Mariano, "Comparing Predictive Accuracy," *Journal of Business & Economic Statistics*, July 1995, *13* (3), 253–63.

Dixit, Avinash, *Art of Smooth Pasting* Fundamentals of Pure and Applied Economics, Routledge, 1993.

Dixit, Avinash K, "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, 1989, *97 (3)*, 620–638.

Dixit, Avinash K. and Robert S. Pindyck, *Investment under Uncertainty*, Princeton University Press, 1994.

Ewald, Christian-Oliver and Wen-Kai Wang, "Irreversible investment with Cox-Ingersoll-Ross type mean reversion," *Mathematical Social Sciences*, 2010, *In Press, Accepted Manuscript*, pages = " – ",.

Geman, Helyette and Andrea Roncoroni, "Understanding the Fine Structure of Electricity Prices," *Journal of Business*, 2006, *79 (3)*, 1225–1262.

Glasserman, Paul, *Monte Carlo Methods in Financial Engineering*, Vol. 53 of *Stochastic Modelling and Applied Probability*, Springer, 2003.

Gollier, Christian, David Proult, Franoise Thais, and Gilles Walgenwitz, "Choice of nuclear power investments under price uncertainty: Valuing modularity," *Energy Economics*, 2005, *27* (4), 667 – 685.

Graham, John R. and Campbell R. Harvey, "The theory and practice of corporate finance: evidence from the field," *Journal of Financial Economics*, 2001, *60* (2-3), 187 – 243.

Hull, John C., *Options, Futures and Other Derivatives*, Prentice Hall, 2005.

Iacus, Stefano M., *Simulation and Inference for Stochastic Differential Equations With R Examples*, Springer, 2008.

IEA, "Projected Costs of Generating Electricity: 2005 Update," Technical Report, OECD/IEA 2005.

Kjaerland, Frode, "A real option analysis of investments in hydropower–The case of Norway," *Energy Policy*, 2007, *35* (11), 5901 – 5908.

McDonald, Robert L. and Daniel R. Siegel, "Investment and the Valuation of Firms When There Is an Option to Shut Down," *International Economic Review*, 1985, *26 (2)*, 331–349.

MIT, "The Future of Coal Options for a Carbon Constrained World," Technical Report, MIT 2007.

Overbeck, Ludger and Tobias Ryden, "Estimation in the Cox-Ingersoll-Ross Model," *Econometric Theory,*, 1997, *13 (3)*, 430–461.
Power plant cost to top 1 billion USD

***Power plant cost to top 1 billion USD**, June 2008.*

Schwartz, Eduardo S.*, "The stochastic behavior of commodity prices: Implications for valuation and hedging," The Journal of Finance, 1997, 52(3), 923–973.*

Seifert, Jan and Marliese Uhrig-Homburg*, "Modelling jumps in electricity prices: theory and empirical evidence," Review of Derivatives Research, 2007, 10 (1), 59–85.*

Takizawa, Shinichiro, Ryota Omori, Atsuyuki Suzuki, and Kiyoshi Ono*, "Analysis of critical electricity price for the investment for constructing a nuclear power plant using real options approach," Journal of Nuclear Science and Technology, 2001, 38, No. 10, 907–909.*

Venetsanos, Konstantinos, Penelope Angelopoulou, and Theocharis Tsoutsos*, "Renewable energy sources project appraisal under uncertainty: the case of wind energy exploitation within a changing energy market environment," Energy Policy, 2002, 30, 293–307.*

Wilmott, Paul*, Paul Wilmott on Quantitative Finance, John Wiley & Sons, 2006.*