Skilled Workers, Immigration Options and Optimal Investment in Human Capital

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Outline

Motivation

- Investment in human capital as a highly irreversible decision
- Uncertainties about the future pay-off of investment
- Human capital investment and real options: education as a multi stage growth option
- Skilled labor immigration as a global problem affecting both developed and developing countries

Immigration

- Forced vs selective immigration
- Immigration possible only with human/financial capital (the US, Canada, Australia, Germany's IT programme) or without this requirement (most of Europe)
- Benefits/costs to both sender and host countries

Our Work

- Impact of immigration option on investing in human capital
- Innovation: two types of human capital, *local* and *global*
- Tradeoff between universal and local human capital
  - Full transferability of universal human capital
  - Only a portion $\alpha \in [0, 1]$ of local human capital can be put to productive use in the destination country
- Expected results: Immigration option affects the rate of investment in global human capital positively and the rate of investment in local one adversely
- Total effect?
- A partial not general equilibriun model, wage differential is exogenous

Literature Review

- Real Options and Investment in Human Capital: Mainly discrete time
  - Human capital and exit option: Katz and Rapoport(2005)
  - Higher return on human capital due to the existence of option to wait: Jacobs(2007)
  - Education and option to shutdown: Hogan and Walker(2007)
- Immigration and Investment in Human Capital: Vidal(1998)

Literature Review

- Immigration and Real Options
  - Option value of waiting: Burda(1995)
  - Immigration quotas and option value: Moretto and Vergalli(2008)
  - Uncertainty and option to wait before immigration: Locher(2002)
- Immigrants Human Capital
  - Complementarity of language: Chiswick and Miller(2003), Berman et al (2002)
  - No positive economic return from homeland education: Hartog and Zorlu(2007)

Variations in Modelling

- Finite vs. infinite time horizon
  - Likely to affect the optimal investment policy
  - May not be optimal to accumulate global human capital if "close" to the termination time
  - May not be optimal to migrate if close to the end of career
  - Acknowledge and start with the infinite horizon case
- Probability of immigration, e.g. Quotas
  - Aim is to capture the immigration policy of destination: friendly or hostile
  - Either add an exogenous probability, p of being able to immigrate or assume that there is some underlying process by which the destination becomes friendlier (eg. a Poisson process)
  - Acknowledge and ignore for the moment
- Continue or stop accumulating global human capital after immigration

The Agent and Her Decision Problem

- A risk-neutral skilled person with an option to work abroad
  - Interim or original country
- Two types of skills accumulation
  - Stock of *universal* human capital, g(t)

$$dg(t) = u(t)dt \tag{1}$$

  - Stock of *local* human capital, k(t)

$$dk(t) = q(t)dt \tag{2}$$

- Investment in human capital is costly:

$$c(u, q) = \frac{c_1}{2}u^2 + \frac{c_2}{2}q^2 \tag{3}$$

The Agent and Her Decision Problem

- Normalize the wage in the host country to 1
- Exercise of option leads to a wage gain (destination/host):

$$dw(t) = \mu w(t)dt + \sigma w(t)dz(t) \tag{4}$$

- There is a lump-sum (opportunity) cost of moving of I due, for instance, to losing one's social network, sentiments and memories, permanent residence, any current pension plans etc.

The Payoff of the Agent

- In the host country, before immigration, the agent's payoff is:

$$\Pi^h(g,k,w) = [g(t) + k(t) - c(u,q)] \tag{5}$$

- After immigration, the payoff is given by:

$$\Pi^d(g,k,w) = w[g(t) + \alpha\bar{k}] - c(u) \tag{6}$$

- Note: after immigration, only investment in global human capital continues, that is:

$$\bar{k} = k_T$$

Statement of the Problem

- Before immigration:

$$\left.\begin{array}{l} max_{u,q,\tau} Z(g,k,w) = E_0 \left\{ \int_0^\tau \Pi^h e^{-rt} dt + e^{-r\tau}[V(g,w) - I] \right\} \\ s.t.(1),(2),(4) \end{array}\right\} \quad (7$$

- After immigration:

$$\left.\begin{array}{l} max_u V(g,w) = E \left\{ \int_0^\infty \Pi^d e^{-rt} dt \right\} \\ s.t.(1),(4) \end{array}\right\} \qquad (8)$$

A First Attempt at Solution

Move backwards

- Suppose the option to immigrate has been taken. The Bellman equation is:

$$V = [w(g + \alpha\bar{k}) - \frac{c_3}{2}u^2]dt + (1 - rdt)E[V(g^{'}, w + dw)] \quad (9)$$

- Using Itô and optimizing over u yields:

$$u^* = \frac{V_g}{c_3} \quad (10)$$

- Analogous arguments establish that before the immigration decision, the agent accumulates according to:

$$\left.\begin{aligned} u^* &= \frac{W_g}{c_1} \\ q^* &= \frac{W_k}{c_2} \end{aligned}\right\} \quad (11)$$

Characterization of the Value Functions

- Plugging the optimal policies into the Bellman equations we get
  - After immigration

$$\frac{1}{2}\sigma^2 w^2 V_{ww} + \mu w V_w + \frac{1}{2c_3} V_g^2 - rV + w(g + \alpha \bar{k}) = 0 \quad (12)$$

  - Before immigration

$$\frac{1}{2}\sigma^2 w^2 Z_{ww} + \mu w Z_w + \frac{1}{2c_1} Z_g^2 + \frac{1}{2c_2} Z_k^2 - rZ + (g + k) = 0 \; (13)$$

An Attempt to Make the Model More Tractable

- Rewrite the motion of deterministic states

$$\left.\begin{array}{l} dg(t) = ug(t)dt \\ dk(t) = qk(t)dt \end{array}\right\} \tag{14}$$

- Also change the payoff functions. Assume, respectively, before and after immigration:

$$\left.\begin{array}{l} \Pi^h = p(g,k) - 0.5c_1u^2 - 0.5c_2q^2 \\ \Pi^d = \underbrace{wg}_{y} - 0.5u^2 \end{array}\right\} \tag{15}$$

with

$$dy(t) = (\mu + u)y(t)dt + \sigma y(t)dz(t) \tag{16}$$

Recast of the Problem

Move backwards: suppose the option has been exercised

- The problem is:

$$\left.\begin{array}{l} max_u E\left\{\int_0^\infty (y - 0.5u^2)e^{-rt}dt\right\} \\ s.t.(16) \end{array}\right\} \quad (17)$$

- Optimization yields:

$$u^* = yV_y \quad (18)$$

- The HJB now satisfies:

$$0.5\sigma^2 y^2 V_{yy} + \mu y V_y + \frac{y^2 V_y^2}{2} - rV + y = 0 \quad (19)$$

$\Rightarrow$ Second-order nonlinear ODE!

Before Immigration

- Seperate value function into "assets-in-place" and the option:

$$Z(g,k,w) = f(g,k) + h(y) \tag{20}$$

- Assets-in-place have the following structure:

$$f(g,k) = p(g,k) - 0.5c_1u^2 - 0.5c_2q^2 \tag{21}$$

- Optimization yields:

$$\left.\begin{aligned} u^* &= gf_g \\ q^* &= kf_k \end{aligned}\right\} \tag{22}$$

The HJB and the Solution

- The solution depends on the form of p(g,k). Some alternatives:
  1. Multiplicative
  $$p(g,k) = gk \tag{23}$$
  2. Cobb-Douglas
  $$p(g,k) = g^{\theta} k^{\gamma}, \theta, \gamma < 1, \theta + \gamma \leqslant 1 \tag{24}$$
  3. Additive
  $$p(g,k) = g + k \tag{25}$$
  Then, conjecture $f(g,k) = f_1(g) + f_2(k)$
  $$\left.\begin{array}{l} 0.5g^2(\frac{df_1}{dg})^2 - rf_1 + g = 0 \\ 0.5k^2(\frac{df_s}{dk})^2 - rf_2 + k = 0 \end{array}\right\} \tag{26}$$

What about the Option Component?

Analogous to an investment option *à la* Dixit&Pindyck

$$\left.\begin{array}{l} 0.5\sigma^2 y^2 h_y y + (\mu + u^*) y h_y - rh = 0 \\ s.t. \\ h(0) = 0 \\ h(y^*) = V(y^*) \\ h_y(y^*) = V_y(y^*) \end{array}\right\} \quad (27)$$

BUT: $u^* = gf_g \Rightarrow$ Not so trivial to solve!

Policy Implications

- An individual immigrant is modeled. What drives her decision?
- Goal: How could countries attract more skilled labor? What are the tools to accomplish that?
  - Providing tax relief?
  - Subsidies (e.g. reducing cost) for integration to the country: ease local human capital investment
  - Pension plans
  - Ease of immigration/bureaucracy
  - Force immigrants to gain local human capital prior to immigration
  - Make labor market requirements (specially in highly skilled sectors) more international
- Effect of transferability of local human capital: France vs Denmark

Conclusion

- Option based model for skilled workers' human capital investment decision
- Comments on:
  - Is the problem intersting enough?
  - Any idea for analytical solutions?
  - Further insights and policy analysis from the model?