Continuous Rainbow Options on Commodity Outputs:

What is the Value of Switching Facilities?

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Abstract

This paper develops a theoretical model to value a real option on the best of two output commodities with continuous switching and temporary suspension possibilities and applies the model to a flexible fertilizer plant. The real options approach to flexibility sometimes suggests counter-intuitive decisions insofar as multi-million investments can be justified even if not currently profitable. Despite the high correlation between the two alternative commodities Ammonia and Urea, the value of flexibility may exceed the required investment cost for the downstream Urea plant, making a case for investment in flexible assets. The switching boundaries between the two operating states are found to be influenced more by the operating costs than by the switching costs. While the Continuous Rainbow option determines the shape of the asset value surface particularly for moderate to high commodity prices, the option to temporarily suspend the operation is highly relevant when prices are low. Both strategic and policy implications for stakeholders of flexible assets are discussed drawing on our numerical application to the fertilizer industry.

1 Introduction

When is the right time for an operator of a flexible asset to switch between two possible commodity outputs in order to maximise the value, and when to switch back? Which factors should be monitored to take these decisions? How much would an investor be prepared to pay to buy such an operating asset? And what are the strategy implications for the operator, investor and possibly policy makers?

Production and processing facilities typically require significant investments in fixed assets. They can be operated profitably if the capacity utilisation is as high as possible. Many of these assets could be designed as flexible, creating the option to produce the best of two outputs. This comes however at a cost: the facilities are more complex and usually need additional fixed assets. The dilemma for the investor is that one part of the flexible facility, which requires an additional investment cost, might not be used, thereby not contributing to earning the required return. In order to overcome this reluctance to invest in a flexible asset, the additional option value through “operating flexibility” (according to Trigeorgis and Mason, 1987) has to be thoroughly understood and correctly modelled. Examples of flexible assets include the shipping industry (combination carriers), the chemical industry (e.g. flexible fertilizer plants), electricity generation (combined cycle: natural gas/ coal gasification) and real estate (multiple property uses).

The traditional approach to determine switching boundaries between two operating states is the discounting of future cash flows and using Marshallian triggers. This methodology does not fully encompass the option values which arise due to the uncertainty in future commodity prices. The value of waiting to gain more information on future price developments and consequently on the optimal switching triggers can be best viewed in a real options framework.

Starting with an asset with a single uncertain variable, Dixit and Pindyck (1994) determine the triggers for temporary suspension in the presence of operating costs as well as entry and exit strategies. Paxson (2005) develops this further to account for multiple states applied to properties. Conceptually, the switch between two volatile assets or commodities can be modelled as an exchange option. Margrabe (1978) and McDonald and Siegel (1986) model the European and perpetual exchange options, respectively. Hopp and Tsolakis (2004) use an American exchange option based on a two-state variable binomial tree to value the option on the best of two assets where the asset choice is a one-time decision. An analytical model for flexible production capacity is presented by He and Pindyck (1992) where switching costs and product-specific operating costs are ignored, thereby eliminating the components which would lead to a non-linearity of the value function in the underlying processes. Triantis and Hodder (1990) take the approach of modelling the profit in the respective operating state as stochastic and following arithmetic Brownian motion, but also assume cost-less switching.

Geltner, Riddiough and Stojanovic (1996) develop a framework for a perpetual option on the best of two underlying assets, applied to the case of two different uses for properties, and provide a comprehensive discussion of relevant assumptions for such a contingent-claims problem. Childs, Riddiough and Triantis (1996) extend the aforementioned model to allow for redevelopment or switching between different uses. Sodal (2005, 2006) develops a framework for an option to operate in the best of two shipping markets where switching between the two is always possible by incurring switching costs. He addresses this problem by creating a mean-reverting stochastic process representing the difference between the stochastic processes of the freight rates in the two markets. This is based on the assumption that these two markets are integrated, i.e. the applications are similar and substitution is possible.

Pinto, Brandao and Hahn (2007) focus on the Brazilian sugar industry and model the price of the two possible output commodities, sugar and ethanol, as mean-reverting. A bivariate lattice is used to replicate the discrete and correlated development of the two commodity prices. However, they allow for switching at no cost, production costs are assumed static and an option to suspend is not being considered.

We develop an option model for an asset with perpetual switching opportunities between two commodity outputs, both modelled by correlated geometric Brownian motion processes, taking into account switching costs, operating costs and the possibility of temporary suspension. This multitude of factors makes the model more realistic, while at the same time causing the value function to be no longer linear in the underlying stochastic variables which complicates the solution procedure. For the numerical application we study a flexible fertilizer plant. The basic product of this plant is Ammonia which can be sold to the market or be further processed to Urea. Thus, the operator has the option to produce and sell the best of two fertilizers, Ammonia and Urea. Switching from one product to the other comes at a cost, since ramping up or down the Urea plant means an inefficient use of resources and takes time. Despite the significant amount of capital tied up in the fixed assets, there is the possibility that the downstream Urea plant will at times not be used, as explained by Yara (2008): If the Urea price drops below the floor set by the Ammonia price, more Ammonia will be sold instead until the relative price levels are restored. Even the Ammonia plant might be temporarily shut down if the Ammonia price falls below the cash cost for its production which is dominated by the natural gas component.

The rest of this paper is organised in four sections. Section 2 sets the framework, develops the Continuous Rainbow Option model and provides the numerical solution method. Section 3 introduces the empirics of the fertilizer industry, including commodity price behaviour and parameter estimation, and applies the model to a flexible Ammonia/Urea plant. Section 4 discusses policy and strategy implications and section 5 concludes and discusses issues for further research.

2 Continuous Rainbow Option on Commodity Outputs

2.1 Assumptions

Numerous assumptions have to be taken in order to provide a framework within which the research problem can be resolved. The major ones shall be named here.

First, the prices of the two commodities, x and y, are assumed to follow geometric Brownian motion stochastic processes and are correlated.

$$dx = (\mu_X - \delta_X)x\,dt + \sigma_X\,x\,dz_X$$
$$dy = (\mu_Y - \delta_Y)y\,dt + \sigma_Y\,y\,dz_y$$

with the notations as follows:

μ	Required return on the commodity
δ	Convenience yield of the commodity
σ	Volatility of the commodity
ρ	Correlation between the two commodities: $dz_x\,dz_y$ / dt
dz	Wiener process (stochastic element)

Two operating states are possible where the cash flows to be earned in each state is the respective commodity price times the capacity per time unit less operating costs. In operating state '1', the cash flow per time unit is p_1 $(x – c_x)$ and correspondingly in operating state '2' p_2 $(y – c_y)$. The parameters p_1 and p_2 represent the theoretical production capacity per time unit (year) in the respective operating state, c_x and c_y are operating costs per unit produced. S_{12} and S_{21} are switching costs from state '1' to '2' and vice versa.

Definitions

Variable production cost (Cash cost)	c_X, c_Y
Production capacity	p_1, p_2
Switching cost	S_{12}, S_{21}
Risk-free interest rate	r

The price of the input material is assumed to be deterministic and constant. This is actually not an unrealistic scenario for our application scenario where the input is natural gas, depending on the country. The lifetime of the plant is assumed infinite, and the company is not restricted in the product mix choice by way of selling commitments or otherwise.

Furthermore, the typical assumptions of real options theory apply, with interest rates, yields, risk premium, volatilities and correlation constant over time. The financial markets are perfect with no transaction costs, and lending and borrowing rates are identical.

2.2 Model development

Our Continuous Rainbow option represents an asset with continuous opportunities to switch between two operating states, by incurring a switching cost. Childs et al. (1996) provide a solution to a similar problem by valuing the redevelopment option for property uses. They ignore ,however, operating costs and the option to temporarily suspend operations which can be quite valuable as will be demonstrated in the numerical application further on. The model then comprises two main option features for operating the asset:

1. Flexibility to chose the best of two products (Continuous Rainbow option) by incurring a switching cost; and
2. Option to temporarily suspend operation of the asset to avoid net losses.

The stochastic processes of the two commodities in the form of gBm are used as underlying variables. With V_1 being the value of the asset in operating state '1', the first PDE can be given as:

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_1}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_1}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_1}{\partial x \partial y} + (r - \delta_X)x \frac{\partial V_1}{\partial x} + (r - \delta_Y)y \frac{\partial V_1}{\partial y} - rV_1 + p_1 x - p_1 c_X = 0 \quad (1)$$

V_2 defines the asset value when operating in state '2':

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_2}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_2}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_2}{\partial x \partial y} + (r - \delta_X)x \frac{\partial V_2}{\partial x} + (r - \delta_Y)y \frac{\partial V_2}{\partial y} - rV_2 + p_2 y - p_2 c_Y = 0 \quad (2)$$

The trigger levels for switching from production state '1' to '2' and vice versa depend on both commodity prices, so we have $y_{12}(x)$ and $x_{21}(y)$ as the switching boundaries. Switching will occur if the value of being in the new operating state exceeds the value in the current state plus the switching cost, always taking into account the possibility to switch back. This gives us the following boundary and smooth pasting conditions:

$$V_1(y_{12}(x)) = V_2(y_{12}(x)) - S_{12} \quad (3)$$

$$V_2(x_{21}(y)) = V_1(x_{21}(y)) - S_{21} \quad (4)$$

$$\frac{\partial V_1(y_{12}(x))}{\partial x} = \frac{\partial V_2(y_{12}(x))}{\partial x} \text{ and } \frac{\partial V_1(y_{12}(x))}{\partial y} = \frac{\partial V_2(y_{12}(x))}{\partial y} \quad (5)$$

$$\frac{\partial V_1(x_{21}(y))}{\partial x} = \frac{\partial V_2(x_{21}(y))}{\partial x} \text{ and } \frac{\partial V_1(x_{21}(y))}{\partial y} = \frac{\partial V_2(x_{21}(y))}{\partial y} \quad (6)$$

Next, the fixed boundaries need to be defined. On the fixed boundaries the problem reduces from two-factor to single-factor. The pictures below visualise these boundaries for both of the operating states and the corresponding mathematical value functions are states thereafter.

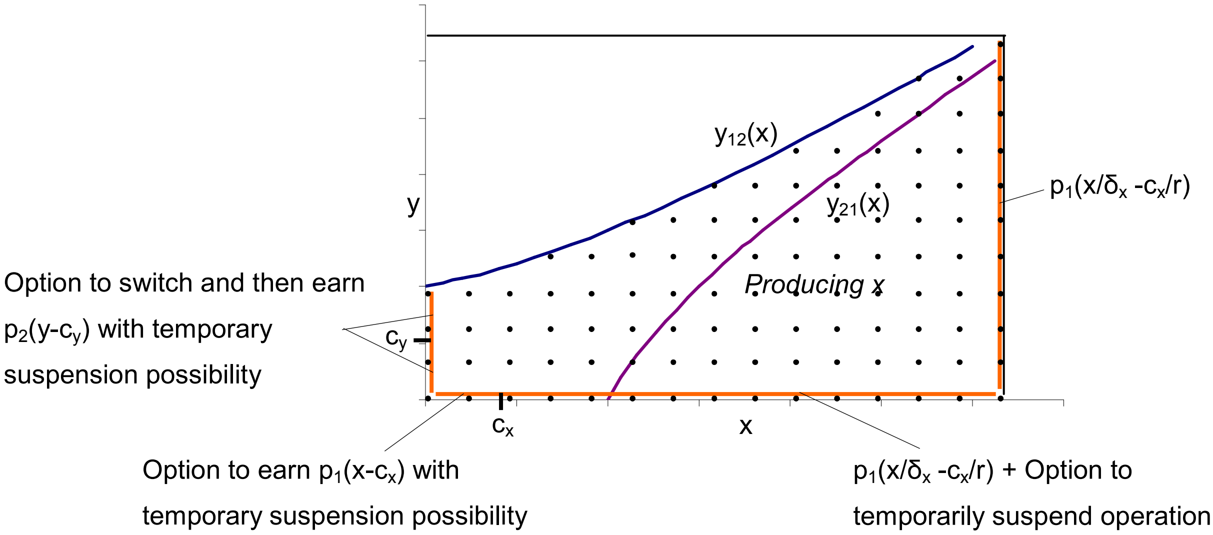


Figure 1 – Grid with Asset value V_1 on the fixed boundaries

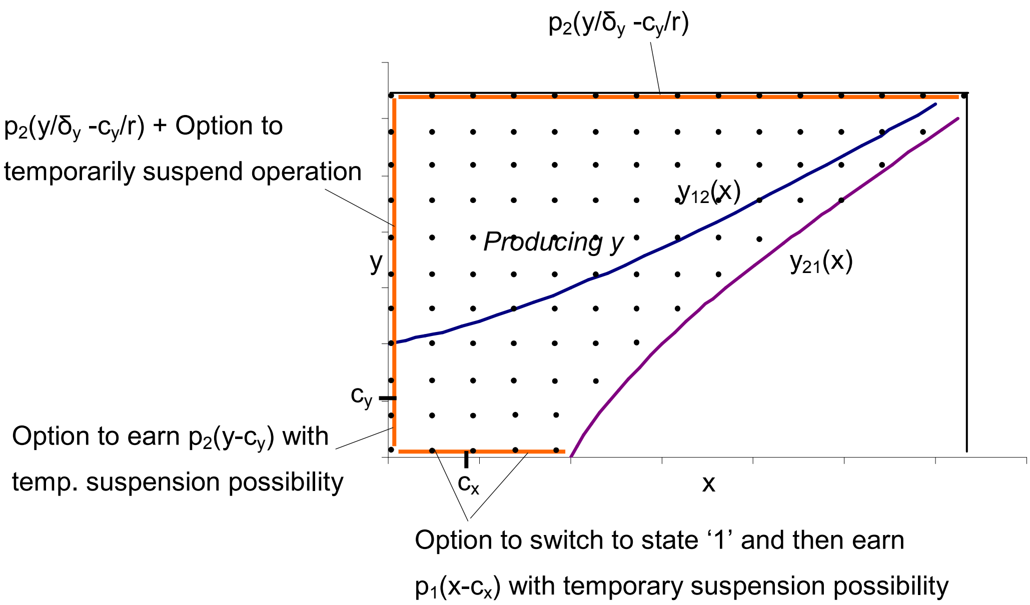


Figure 2 – Grid with Asset value V_2 on the fixed boundaries

Case: x=0 and y=0

$$V_1(x=0;y=0)=V_2(0;0)=0 \tag{7}$$

Case: y=0

Dixit & Pindyck (1994) provide models to value an asset based on a single underlying stochastic cash flow where operating costs are incurred and temporary suspension is possible.

$$V_1(x; y=0) = \begin{cases} A_1 (p_1 x)^{\beta_{11}} & \text{if } x < c_x \\ B_1 (p_1 x)^{\beta_{12}} + (p_1 x)/\delta_x - (p_1 c_x)/r & \text{if } x > c_x \end{cases} \tag{8}$$

where $A_1 = \frac{(p_1 c_x)^{1-\beta_{11}}}{\beta_{11} - \beta_{12}} \left(\frac{\beta_{12}}{r} - \frac{\beta_{12} - 1}{\delta_x} \right)$,

$$B_1 = \frac{(p_1 c_x)^{1-\beta_{12}}}{\beta_{11} - \beta_{12}} \left(\frac{\beta_{11}}{r} - \frac{\beta_{11} - 1}{\delta_x} \right),$$

$$\beta_{11/12} = \tfrac{1}{2} - (r - \delta_x)/\sigma_x^2 \pm \sqrt{\left((r - \delta_x)/\sigma_x^2 - \tfrac{1}{2}\right)^2 + 2r/\sigma_x^2}$$

The same literature also provides the value of an option to invest into an asset with operating costs and the possibility of temporary suspension. This option is relevant for us because it represents the behaviour of V_2 if y=0.

$$V_2(x; y=0) = D_1 (p_1 x)^{\beta_{11}} \tag{9}$$

with $D_1 = \frac{B_1 (p_1 x^*)^{\beta_{12}} + p_1 x^*/\delta_x - p_1 c_x/r - S_{21}}{(p_1 x^*)^{\beta_{11}}}$

where x* is the switching boundary x_{21}(y=0) and fulfils the equation:

$$(\beta_{11} - \beta_{12}) B_1 (p_1 x^*)^{\beta_{12}} + (\beta_{11} - 1)(p_1 x^*)/\delta_x - \beta_{11}\left((p_1 c_x)/r + S_{21}\right) = 0$$

Case: x=0

$$V_2(x=0; y) = \begin{cases} A_2 (p_2 y)^{\beta_{21}} & \text{if } y < c_y \\ B_2 (p_2 y)^{\beta_{22}} + (p_2 y)/\delta_y - (p_2 c_y)/r & \text{if } y > c_y \end{cases} \tag{10}$$

where $A_2 = \frac{(p_2 c_y)^{1-\beta_{21}}}{\beta_{21} - \beta_{22}} \left(\frac{\beta_{22}}{r} - \frac{\beta_{22} - 1}{\delta_y} \right)$,

$$B_2 = \frac{(p_2 c_y)^{1-\beta_{22}}}{\beta_{21} - \beta_{22}} \left(\frac{\beta_{21}}{r} - \frac{\beta_{21} - 1}{\delta_y} \right),$$

$$\beta_{21/22} = \tfrac{1}{2} - (r - \delta_y)/\sigma_y^2 \pm \sqrt{\left((r - \delta_y)/\sigma_y^2 - \tfrac{1}{2}\right)^2 + 2r/\sigma_y^2}$$

$$V_1(x=0; y) = D_2 (p_2 y)^{\beta_{21}} \tag{11}$$

with $D_2 = \frac{B_2 (p_2 y^*)^{\beta_{22}} + p_2 y^*/\delta_y - p_2 c_y/r - S_{12}}{(p_2 y^*)^{\beta_{21}}}$

where y* is the switching boundary y_{12}(x=0) and fulfils the equation:

$$(\beta_{21} - \beta_{22}) B_2 (p_2 y^*)^{\beta_{22}} + (\beta_{21} - 1)(p_2 y^*)/\delta_y - \beta_{21}\left((p_2 c_y)/r + S_{12}\right) = 0$$

Case: x → ∞

$$V_1(x \to \infty; y) = p_1(x - c_x)/\delta_x \tag{12}$$

Case: y → ∞

$$V_2(x; y \to \infty) = p_2(y - c_y)/\delta_y \tag{13}$$

2.3 Solution method

Two-factor problems which are linear homogeneous, meaning $V(a \cdot x; a \cdot y) = a \cdot V(x; y)$, can typically be solved analytically by substitution of variables. The presented Continuous Rainbow option is designed in a way to encompass a number of complexities, such as switching cost, operating cost and multiple switching, in order to make it more realistic. The consequence is that the problem is no longer homogenous of degree one. This makes an analytical solution practically unavailable. We therefore resort to a numerical solution by backward dynamic programming which still allows a feasible analysis of the value surfaces together with the switching boundaries.

Definition of the lattice

Terminal values are required in order to solve the system of equations. Any profits which lie very far in the future do not affect the present value a lot. We can therefore make the assumption that switching beyond a distant point in time, e.g. beyond 50 years of operation, is no longer allowed. This allows us to determine the value of the asset in 50 years time.

Childs et al. (1996) provide a framework for the lattice numerical solution. The lattice will be spanned by x and y-values as follows:

$$x_{i,j,k} = x_{min}\, e^{i\sigma_x\sqrt{3\Delta t}}\, e^{k\left(r-\delta_x-1/2\sigma_x^2\right)\Delta t}$$

$$y_{i,j,k} = y_{min}\, e^{i\rho\sigma_y\sqrt{3\Delta t}}\, e^{j\sigma_y\sqrt{3\Delta t\left(1-\rho^2\right)}}\, e^{k\left(r-\delta_y-1/2\sigma_y^2\right)\Delta t},$$

where (i,j,k) defines a point in the three-dimensional x-y-t grid by indicating the number of increments in the respective variable. We can see from the above functions that y is dependent both on time and on x while x is dependent only on t. This interdependency is necessary to map the correlation between the two variables.

The lattice also defines the marginal probabilities, i.e. how x and y change within an increment of time:

$$\begin{array}{cc|c} & & \Delta j \\ & & \begin{array}{ccc} \text{up} & \text{over} & \text{down} \end{array} \\ \hline \Delta i & \begin{array}{c} \text{up} \\ \text{over} \\ \text{down} \end{array} & \begin{bmatrix} 1/36 & 1/9 & 1/36 \\ 1/9 & 4/9 & 1/9 \\ 1/36 & 1/9 & 1/36 \end{bmatrix} \end{array}$$

Model Implementation

A value grid has to be determined for each of V_1 and V_2. First, all fixed boundaries are computed and then the terminal values which are simply the values assuming no more switching. For instance, assuming the operating state of the asset would then be '1', the terminal value amounts to $p_1(x_{i,j,k}/\delta_x - c_x/r)$.

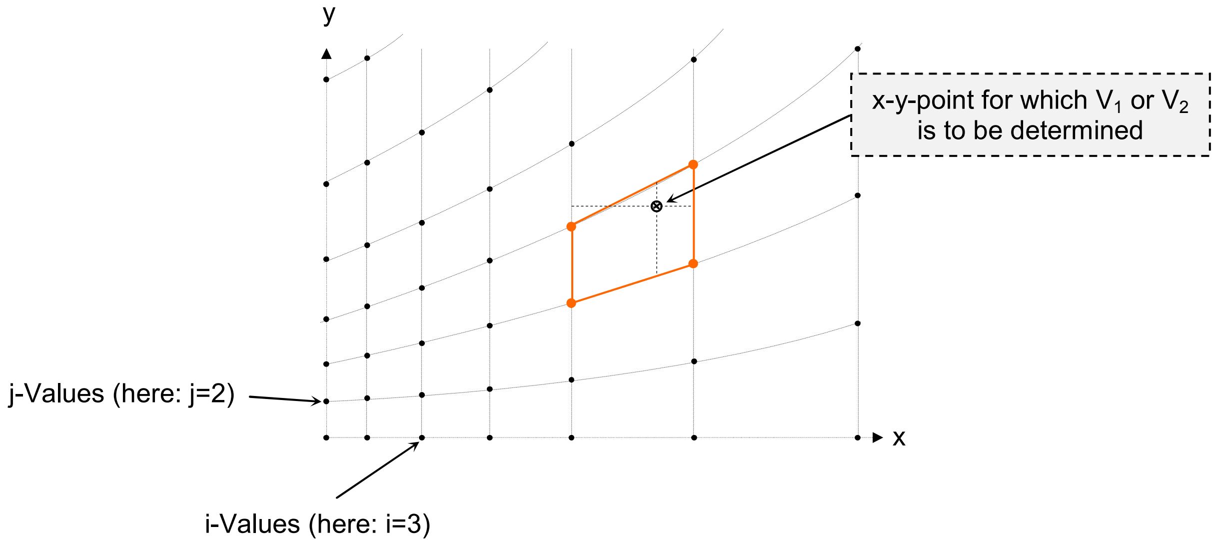
Starting from the terminal values backwards, V_1 and V_2 are determined at every point in time. $V1_{i,j,k}$ is the value at ($x=i\Delta x$, $y=j\Delta y$, $t=k\Delta t$), assuming the current operating state is '1'. It is equal to the discounted value of the higher of V_1 and V_2–S_{12} at the time t+1 according to the marginal probabilities.

$$V1_{i,j,k} = e^{-r\,\Delta t}\left[\tfrac{4}{9}\mathrm{Max}\left(V1_{i,j,k+1}, V2_{i,j,k+1} - S_{12}\right) + \tfrac{1}{9}\mathrm{Max}\left(V1_{i+1,j,k+1}, V2_{i+1,j,k+1} - S_{12}\right) + \ldots\right]$$

The asset value at the present time (t=0) can be represented by a surface spanned over the x-y-area.

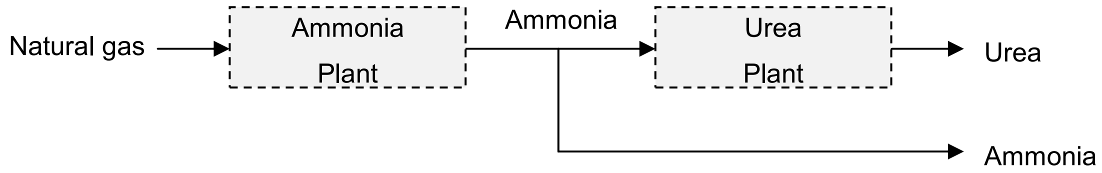
Determination of asset values at any x-y-point within the value grid

The value V_1 or V_2 can be determined at any point within the value grid by interpolating between known values. It has to be taken into account that the grid is not straight due to the dependence of y-values upon x.



3 Empirical application

The model developed in the previous section is applied to a flexible fertilizer plant. The following simplistic scheme depicts the required input, the basic transformation and the output of such a facility with production mix flexibility:



The investment cost for an Ammonia plant of 2,000 mt per day is estimated by industry experts to be around USD 550 m. The Urea plant can be seen to bring in the flexibility value; its investment for a capacity of 3,200 mt per year would be around USD 340 m.

The industry dynamics is such that in times of low demand for fertilizers the equilibrium price is supply-driven. The marginal producers with the highest cost base – typically based in regions of high gas prices (US, Western Europe) or inefficient facilities (e.g. Eastern Europe) – drop out until the prices have been stabilised. Estimates indicate that about 10% of the global Urea capacity was closed in January 2009 (Yara, 2009). In times of high demand on the other hand, prices are no longer determined by the cost base but by the marginal utility at full capacity of the industry.

3.1 Estimation of Commodity prices behaviour

As shown in the previous section the model is based on assuming commodity prices follow geometric Brownian motion. Alternative stochastic processes for commodities would include mean-reversion and Schwartz and Smith's (2000) "short-term variations and long-term dynamics" model. There is, however, no widespread consensus on the best stochastic model for commodities in general.

We assume the historic volatility of the commodity prices is a reasonable estimate of the future volatility. An analysis of the time series month-by-month reveals an annual volatility of 57% for Ammonia and 40% for Urea, based on monthly prices over ten years. It can also be seen from the graph that the price movements are slightly more marked for Ammonia. Furthermore, the graph suggests a high correlation between the two types of fertilizer. The numerical analysis confirms this with a correlation between Ammonia and Urea of 0.92.

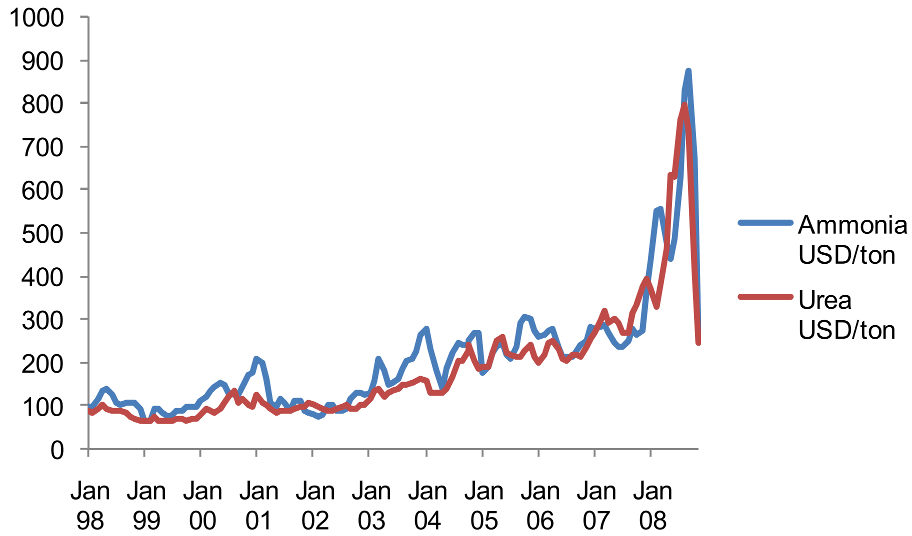


Figure 3 – Historic prices of Ammonia and Urea (Source: Yara)

There is limited evidence for estimating the convenience yields of the commodities, since futures or forward prices are not publicly available. Our assumption therefore is that the convenience yields range near average yields of other – more publicly traded – commodities. Since Urea is typically handled in the form of granules the storage and handling costs are much lower than for the gaseous or liquid Ammonia. This is reflected in a higher convenience yield of the former. The effect of the convenience yield on the option value is analysed in more detail in the sensitivities section.

The following set of commodity parameters is the basis for our empirical application.

Overview of Commodity Parameters

Current Ammonia price[1]	x	251	USD / mt
Current Urea price[1]	y	243	USD / mt
Ammonia convenience yield	δ_x	2	%
Urea convenience yield	δ_y	5	%
Ammonia volatility	σ_x	57	%
Urea volatility	σ_y	40	%
Correlation Ammonia/Urea	ρ	0.92	
Risk-free interest rate[2]	r	2.89	%

[1] Average of prices in November 2008

[2] Yield of a 10-year Treasury note, Source: Bloomberg, Feb 13th, 2009

3.2 Estimation of Parameters

Natural gas is the main raw material in the production of nitrogen-based fertilizers. At current market prices in the US, natural gas represents about 90% of the cash cost of both Ammonia and Urea. In some countries, fertilizer companies have fixed-price gas contracts with state-owned suppliers while companies in other places are exposed to the volatility of the (spot) natural gas market. For our case, we assume a fixed natural gas price of 7 USD / mmBtu. With 36 mmBtu of natural gas required to produce one metric tonne of Ammonia, the cash cost of Ammonia production can be determined.

Cash cost of Ammonia production[3]

Gas cost	252	USD / mt Ammonia
Other production cost	26	USD / mt Ammonia
Total cash cost	*278*	*USD / mt Ammonia*

Source: Yara Fertilizer Industry Handbook, Nov 2008

For the production of one metric tonne of Urea, 0.58 mt of Ammonia are required:

Cash cost of Urea production[4]

Ammonia cost	161	USD / mt Urea
Process gas cost	36	USD / mt Urea
Other production cost	22	USD / mt Urea
Total cash cost	*219*	*USD / mt Urea*

Source: Yara Fertilizer Industry Handbook, Nov 2008

Overview of Plant Parameters

Cash cost Ammonia production	c_X	278	USD / mt Ammonia
Cash cost Urea production	c_Y	219	USD / mt Urea
Capacity Ammonia plant	p_1	730,000	mt Ammonia per year
Capacity Urea plant	p_2	1,168,000	mt Urea per year
Switching cost[5] Ammonia → Urea	S_{12}	150	‘000 USD
Switching cost[5] Urea → Ammonia	S_{21}	150	‘000 USD

[3] All cost estimates are fob plant cash costs excluding load-out, depreciation, corporate overhead and debt service for a US proxy plant located in Louisiana (ca. 1,300 metric tons per day capacity)

[4] All cost estimates are fob plant cash costs excluding depreciation, corporate overhead and debt service for a US proxy plant located in Louisiana (~1,400 mt per day capacity).

[5] The switching cost has been estimated to correspond to the lost profit (assumed gross margin of 60 USD / mt Urea) over a twelve hours non-productive time plus 50% in addition for inefficient use of materials and energy during the switching process.

3.3 Empirical results

Applying the parameters to the numerical solution procedure provides the switching boundaries as shown in the picture below. It can be seen that the switching boundaries are spread apart for low commodity prices and close together for higher ones. This can be explained by the fact that for low prices the fixed switching cost outweighs the potential benefits from switching to the other state and earning a higher cash flow; The gross margin is too small to amortise the switching cost within a relatively short time period. Furthermore, when the prices are low enough that the cash cost of production exceeds the revenues, the asset is not operated at all. Hence, incurring a fixed switching cost without resuming operation would not be economical. The graph also makes evident that the switching boundaries are not centred around the 45% line which is due to different cash costs for the production of Ammonia and Urea.

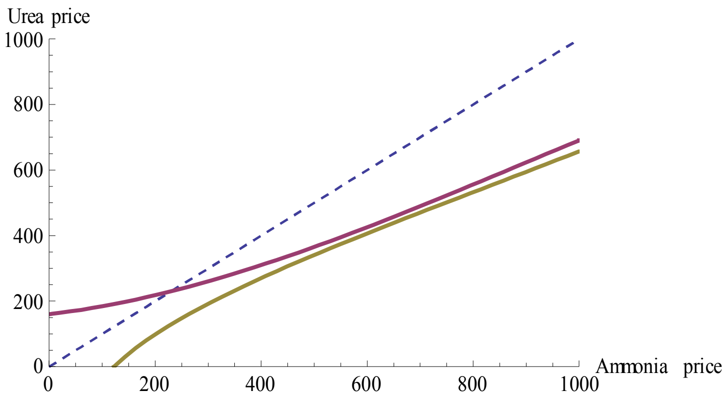


Figure 4 – Switching boundaries

Let us now investigate the value surface indicating the asset value for any combination of Ammonia/Urea prices, taking into account the operating state and switching boundaries. The light blue surface represents V_2 and the green one V_1.

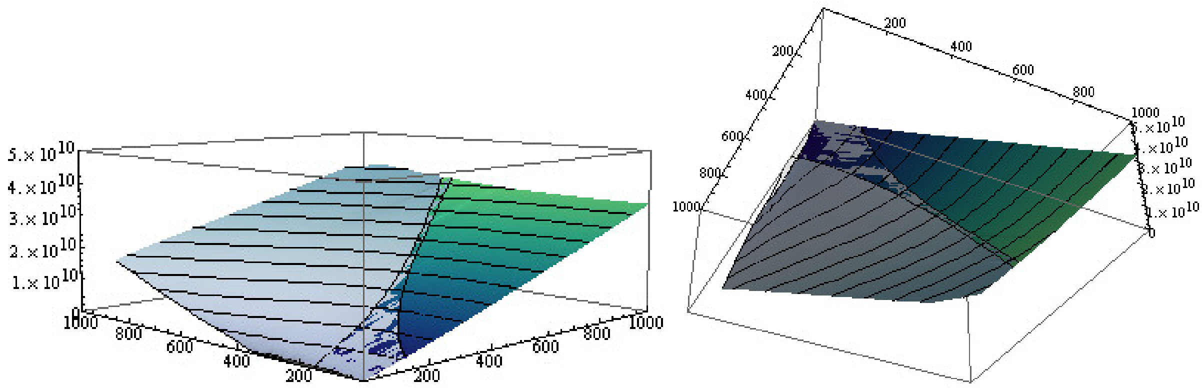


Figure 5 – Value surface V_1 and V_2 and switching boundaries (left: top view; right: view from below)

The value surface reflects the typical form one would expect from a two-factor output option model, increasing with both Ammonia and Urea prices. For low commodity prices the curvature is stronger

since an option to suspend temporarily is integrated. The value surfaces of V_1 and V_2 intersect so that their values differ at the switching boundaries by exactly the switching cost. The intersection would be better visible if the switching cost was higher compared to the scale of the value surface. If the Ammonia/Urea price combination is on the right side of the lower switching boundary, Ammonia is to be produced. If the price combination is on the left side of the upper switching boundary, Urea is to be produced. In between, the current operating state is to be continued (hysteresis effect).
For the current fertilizer prices (Ammonia: USD 251; Urea: USD 243), the plant should be in operating state '2' (Urea). The asset value is given by $V_2(251,243)$ as USD 8.13 bn. Taking into account that the investment cost for the flexible plant is around one billion USD, the magnitude of the operating asset value seems suspiciously high. We will come back to this in the Interpretation section.

The question of course is how valuable is the option to switch. Operating state '1' can be considered as the base case since the Ammonia plant is required anyway. The common reasoning is to assume that if a Urea plant is added to further process the Ammonia, it should always be used because value is added to the product. In the following picture, the monthly combinations of Ammonia/Urea from the year 2008 are depicted as vertical cylinders intersecting with the value surface. It can be observed that at least in two months only Ammonia should have been produced, meaning the downstream Urea plant would have been idle.

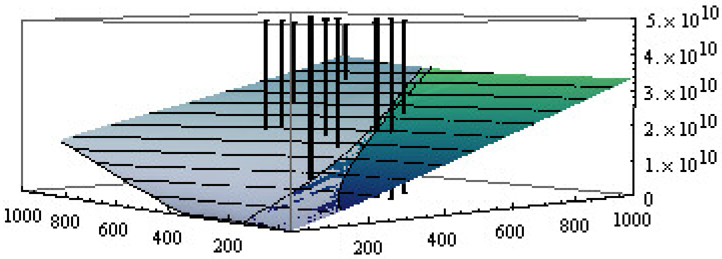


Figure 6 – Value surface and monthly Ammonia/Urea price combinations in 2008

However, we still do not know if the flexibility gained by the Urea plant is worth more than its investment cost. The picture below shows the known value surface for the Continuous Rainbow option, i.e. the flexible fertilizer plant, and an additional value surface representing the Ammonia plant as stand-alone. With increasing Urea prices the flexible asset value becomes worth much more than the Ammonia stand-alone. At current prices, the Ammonia stand-alone plant would be worth USD 7.50 bn. This gives us the value of the flexibility to switch as the difference between the two asset values (USD 8.13 bn and USD 7.50 bn) as USD 630 m. Recalling the investment cost for the Urea plant of USD 340 m, we can recommend to an investor to invest into the Urea plant to gain the flexibility between Ammonia and Urea.

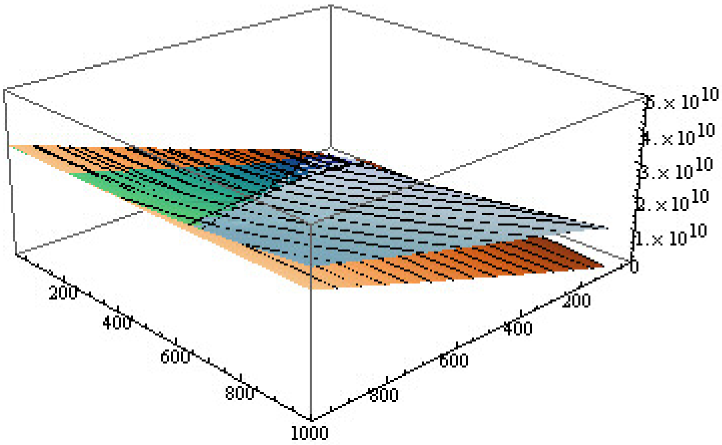


Figure 7 – Flexible asset value surface compared to single-product asset surface (brown)

It is interesting to note that at current prices a stand-alone Ammonia plant should be idle, whereas a Urea plant could be operated profitably.

Assuming the cash costs for production of Ammonia and Urea have not changed over the last three years, and according to our estimation, a comparison of the monthly prices with the cash costs show that the complete fertilizer plant should have been idle in the first two months of 2006 and from June to October 2006. This demonstrates that the operating cost and appropriate management reactions to low fertilizer prices have to be taken into consideration in order to capture the full value. As mentioned before, plant owners make use of the option to temporarily suspend the operation and thereby recognize it as a valuable asset management tool.

3.4 Sensitivities

Switching cost sensitivity

Varying the switching cost, the results confirm the intuition that the two switching boundaries are further apart for high switching cost. However the effect is weak which can be attributed to the almost negligible amount of the switching cost in comparison to the asset value and the gross margins to be earned.

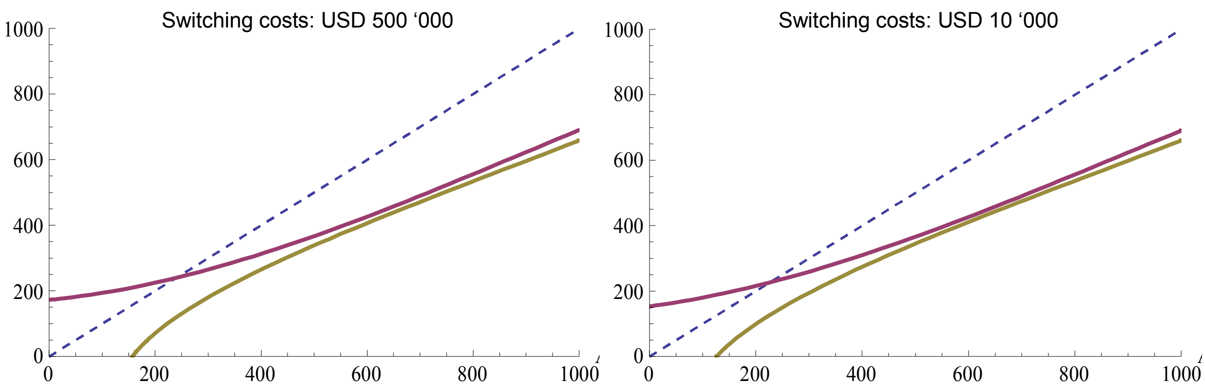


Figure 8 – Sensitivity of switching boundaries to switching cost

Convenience yield sensitivity

We now come back to the convenience yields of the commodities. In the risk-neutral environment which is constructed to solve real option problems, the convenience yield provides information on the expected growth rate of the commodity price ($\delta = r - \alpha_{risk\text{-}neutral}$). A higher convenience yield is equivalent to a lower growth rate since more of the required return is gained with the convenience (and less with price increase). This effect is reflected in the sensitivity analysis shown below where the asset values significantly increase with lower convenience yields. A decrease of the convenience yields from 5% (8%) to 2% (5%) more than doubles the value of the asset. A reduction to 1% visually lets the asset values go through the roof. At current prices, the asset value in these three cases varies from USD 2.5 bn to USD 32.0 bn.

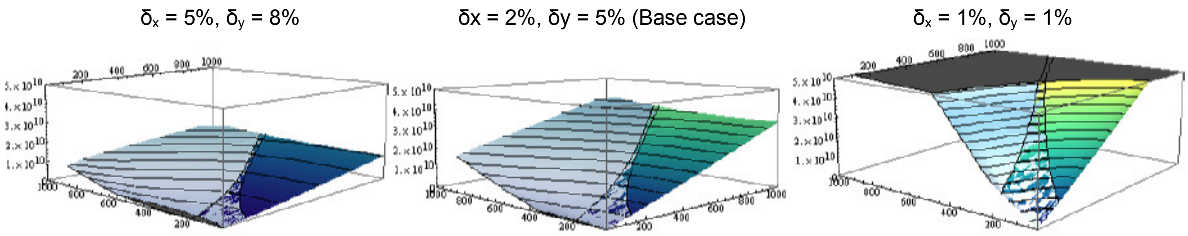


Figure 9 – Sensitivity of asset values to commodity convenience yields

Correlation sensitivity

A low or even negative correlation means that the underlying prices do not move in line. The chances are that while one price is high and the other one low, this might reverse. Therefore incurring the switching cost would be delayed in order to wait and see whether prices confirm the switching idea. The option value of waiting is much less if the two underlying prices move almost in line since a reversal of the relative prices is improbable. We have found a close correlation between Ammonia and Urea of 0.92 over the last decade. Simulating a correlation of 0.50 would lead to an asset value of USD 9.65 bn instead of USD 8.13 bn. In absolute terms the difference is huge, in relative terms an increase of slightly less than 20%. However, since the two fertilizers are to some degree substitutable and therefore in an integrated market, there is no reason to believe that the correlation will decrease significantly.

3.5 Interpretation

The asset values obtained from the model seem to be quite high in relation to the investment cost. According to the numbers there would be significant above-market returns to be earned by investing in fertilizer plants and any investor should be heading for this industry. Even at the currently moderate fertilizer prices the plant value would be around USD 8 bn. In the sensitivities section, we have seen the strong sensitivity to the convenience yields. In practice, the convenience yields are not constant because they depend on various market forces, including the level of demand, on inventories and on expected bottlenecks in supply. But this does not solve the issue that the asset values are so sensitive to the yields. It seems like the linkage of the convenience yields to the expected growth rates of the commodity prices cannot be fully justified.

Comparing the value of the flexible asset with a non-flexible plant has shown that they are in the same range of values, here about USD 7-8 bn. This means that the questionable high asset values are not due to the model on the best of two outputs but rather on the underlying commodity behaviour. The value of the flexibility option – exceeding the required investment cost – seems plausible. Investors actually do invest into Ammonia-plus-Urea plants.

4 Policy and Strategy implications

There are a number of stakeholders whose decisions and behaviour might potentially be influenced by the research results.

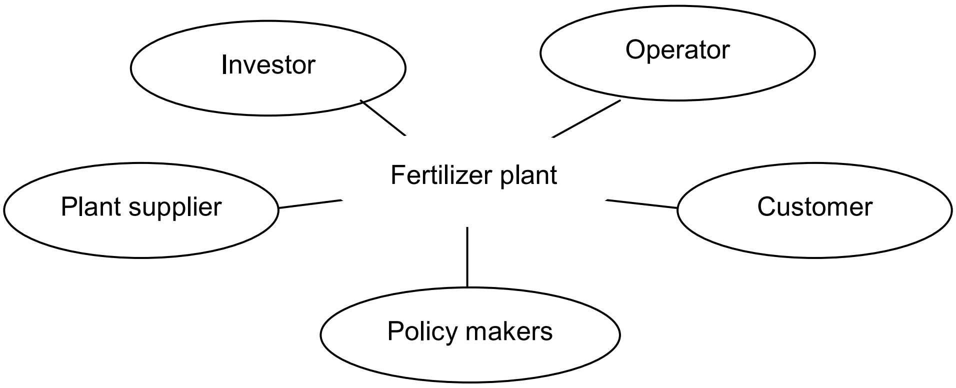


Figure 10 – Stakeholders of the project

Investors

The numerical results have shown that at current prices it is worth supplementing the Ammonia plant by a downstream Urea plant in order to have the flexibility to switch between the two products. For higher prices this will hold true even more. Only if the commodity prices go down so that a profitable operation is no more possible should the investor abstain from adding a Urea plant. It is important for the investor to keep in mind that Ammonia and Urea prices are highly correlated and will probably not diverge from each other. The major drivers to reap the benefits of the product choice are low switching costs and flexible supply contracts, meaning that the company will not be stuck in rigid contracts forcing it to supply a specific product to it's customers, even if it is better to produce the other product.

Operators

An obvious task of the operators is to minimise the operating costs as well as the switching costs in order to make the most of the available flexibility. Furthermore, the current supply commitments and inventory levels shall be monitored continuously in order to judge the practical level of flexibility. The operational management shall always be aware of the current market prices and regularly update the forward prices as indicators for the price development in the near future so that they are prepared for switching opportunities. Volatilities can be calculated in regular intervals. The convenience yields for fertilizers are not easily surveyed and their validity would be questionable since most of the variations can be attributed to short-term market conditions. Furthermore, with the investment already undertaken, the convenience yield hardly impacts on the operations.

Plant suppliers

It is usually difficult to market a product with a high up-front investment which is supposed to pay off during the asset lifetime through optimal operations. The above results prove a real opportunity for plant suppliers, because it supports the idea of more sophisticated (and expensive) assets. The strategy implication here is to aggressively market more flexible assets and to back it by demonstrating the financial benefits to the investor. Internally, the asset could be optimised for flexibility, i.e. the design focuses on minimising downtime and costs of switching.

Customers/Commodity traders

A commodity trader focused on arbitrage is not interested in long-term contracts and therefore is not in conflict with the increased flexibility request of the fertilizer supplier. Other traders might have long-term customer agreements which they need to back up by long-term supply agreements with the producers. Therefore they might insist on long-term supply commitments for a specific product or otherwise might turn to single-product producers.

Policy makers

The interest of the policy makers in this context can be considered to be the functioning of markets. Let us consider the example of a shift in fertilizer demand from Ammonia to Urea. The consumers and therefore the policy makers would be happy to see a quick response in supply in order to restore the market forces and to make the requested product available in sufficient quantities at acceptable prices. This will happen much faster if the assets are capable of multi-product operation. The political support might be put into place for instance by giving preference in permitting processes to extending current facilities to incorporate flexibility over new non-flexible investments.

5 Conclusion and Further Research

In this research paper we have developed a model to value and interpret a Continuous Rainbow Option on Commodity Outputs, representing an operating asset with the choice of the best of two outputs where switching between the two is always possible at a switching cost. Also included is the option to temporarily suspend the operation if revenues fall below operating costs. The numerical solution has been applied to the case of a flexible fertilizer plant.

We have found that despite the high correlation between the two alternative commodities Ammonia and Urea of 0.92, the value of flexibility exceeds the required additional investment cost. The historic price series indicate that this flexibility would have been used in 2008, which actually means a downstream value-adding facility of about USD 340 m would have been idle because it would have paid to produce the alternative product. The switching boundaries diverge for low commodity prices, due to the operating costs, and converge for high prices. The position of the switching boundaries is found to be rather insensitive to the magnitude of the switching cost. The interpretation is that the switching costs chosen – a range of values which seems to be reasonable – are still relatively low compared to the potential gross margins to be earned. The results also demonstrate that the possibility of temporary suspension shapes the asset value surface for low commodity prices, and this option is a practical, valuable management tool.

The results and interpretation also raised some further research questions. In particular, the overall asset value seems to be rather high and highly sensitive to the convenience yields. For fertilizer prices modelled as gBm, the concept of convenience yields and their influence on the expected growth rates of the commodity prices needs to be further investigated. Since prices following gBm are boundless, the question could be raised if gBm is the best stochastic process to model fertilizer prices. The results of this paper might be compared to the outcome based on different stochastic processes. One promising alternative approach for further research – which has recently been initiated by the authors – is to use a mean-reverting price differential between the two commodities to value the flexibility option.

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