Uncertainty and Competition in the Adoption of Complementary Technologies

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Abstract

In this paper we study, for a duopoly market, the combined effect of uncertainty, competition and "technological complementarity" on firms' investment behaviour, in a game-choice setting where it is assumed that there is a first-mover advantage. Firms often use inputs whose qualities are complements such as computer and modem, equipment and structure, train and track, transmitter and receiver and, therefore, on such cases, investment decisions on upgrades or replacements must consider the degree of complementarity between investments. We derive analytical solutions for the investment thresholds of two firms which have the option to adopt one or two new technologies, whose functions are complements, in a context where the evolution of the saving costs that can be made through the adoption and the cost of the technologies are uncertain. We obtain analytical expressions for the value functions of the leader and the follower as well as for their optimal investment thresholds. Some of the illustrated results show nonlinear and complex investment criteria and sensitivities to expected cost savings. It might be optimal for both firms to adopt the two technologies at different times, first the technology whose price is decreasing at a lower rate and then the technology whose price is decreasing more rapidly, but the degree of complementarity between the two technologies determines the timing of such investments.

1. Introduction

Since the pioneering work of Smets (1993) the effect of uncertainty and competition on firms' investment behaviour has been extensively studied in the real options literature[3], but the influence of the degree of complementarity between the inputs of an investment on firms' investment decisions has been neglected. However, firms often use inputs whose qualities are complements, such as computer and modem, equipment and structure, train and track, and transmitter and receiver. In such cases, investment decisions on upgrades or replacements must consider the degree of complementarity between such assets.

Conventional wisdom says that when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously, i.e., when raising the quality of one input it should upgrade its complements at the same time[4]. However, this conclusion has been made for economic contexts where it is assumed that there is no uncertainty, competition or adjustment costs. We study the effect of the complementarity between two technologies on their optimal time of adoption, considering competition between firms and uncertainty about the revenues and the investment costs. For simplicity we neglect adjustment costs.

Our initial intuition is that when uncertainty is added to the investment problem, for instance uncertainty about the cost of the technologies, the conventional wisdom stated above may not hold. Due to technological progress the cost of a technology can decline rapidly and, therefore, when firms anticipate that the cost of technologies may not fall at the same rate, it may pay to adopt first the technology whose price is falling more slowly and wait to adopt the technologies whose price is falling more rapidly.

The concept of the complementarity between the elements of a (technological) "system" is studied here from the adopter's point of view. Our aim is to investigate to what extent the degree of complementarity between two technologies affects the adopter's investment behavior, in economic contexts where uncertainty and competition hold as well. However, the phenomenon of complementarity between the inputs of a "system" affects many other areas of the economy and, therefore, can be studied from other perspectives. For instance, it has been argued that the pace of

[3] Dixit and Pindyck (1994), chapter 9, Grenadier (1996), Lambrecht and Perraudin (1997), Huisman (2001), Weeds (2002) and Paxson and Pinto (2005) address such problems.
[4] See Milgrom and Roberts (1990, 1995) and Javanovic and Stolyarov (2000).

modernization of an industry is quite often influenced by the degree of technological complementarity between it and those whose activities are complements. This phenomenon was studied by Smith and Weil (2005) who investigated how changes in retailing and manufacturing industries, together, affected the diffusion of new information technologies in the U.S. apparel industry between 1988 and 1992. They show that the process occurs in a stepwise fashion[5], i.e., retailers typically adopted the new information technology systems first and the increased demand for rapid replenishment by retailers then stimulated suppliers to adopt new manufacturing practices and make greater investments in complementary information technologies, causing a "ratchet-up" process as the payoffs to adopting increased when more customers and suppliers, respectively, adopted. This case constitutes, according to the authors, an extraordinary example of the effect of the complementarity between new technologies on the pace of modernization of interlinked industries.

Another area where the concept of "complementarity" also plays an important role is research and development (R&D), in the sense that firms, when planning their R&D activities, do make strategic decisions regarding the degree of complementarity (sometimes called compatibility) between the new products they aim to launch in the future and the complement products that are already available in the market and those they conjecture will be launched by their rivals in the future[6]. We find in the market two distinct types of behavior (strategies): firms who do not have a dominant market position and want to growth tend to guide their R&D efforts in order to launch new products that are, as much as possible, complement those of their rivals who have dominant market positions. Firms who have a dominant market position tend to guide their R&D efforts in order to launch new products that are, as much as possible, not complements (compatible) of those from their opponents. An example of the later strategy is the nine-year battle between the European Union (EU) commission and Microsoft that culminated last October 2007 with a fine of € 497 million due to its supposed misconduct in developing software that does not allow open-source software developers access to inter-operability information for work-group servers used by businesses and other big organizations[7,8].

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[5] For a detailed description of how the new information technologies adoptions occurred in both industries see Smith and Weil (2005), pp. 494-495.

[6] Note that the diffusion of an innovation depends, to some extent, on the diffusion of complement innovations, which amplify its value.

[7] See Etro (2007), p. 221, and Financial Times, October 23, 2007, p. 1.

[8] Note that Microsoft has 95 per cent market share in desktop publishing and more than 70 per cent of work-group server operating systems.

Mamer and McCardle (1987) studied the effect on investment behaviour of technological complementarity and competition between firms. They derived a model where competition is modeled in a game theoretic setting and show that firms' optimal investment strategy, when uncertainty about the profitability of the innovation is unknown and competition is considered, is characterized by a monotone sequence of pairs of threshold values which delineate a cone-shaped continuation region. Milgrom and Roberts (1990), in an attempt to improve the understanding of the effect of the technological complementarity on the manufacturing modernization, derived an optimizing model of the firm that generates many of the observed patterns that mark modern manufacturing.

Milgrom and Roberts (1995b) use the theories of super-modular optimization and games as a framework for the analysis of systems marked by complementarity, and suggest that the ideas of complementarity and super-modularity in optimization and games can be quite useful to understand the relation between strategy and organizational structure. Colombo and Mosconi (1995) study the diffusion of flexible automation production and design/engineering technologies in the Italian metalworking industry, giving particular attention to the role of the technological complementarity and learning effects associated with the experience of previously available technologies.

The concept of "complementarity" was also used by Milgrom and Roberts (1995a), studying the Japanese economy between 1940 and 1995, in an attempt to interpret the characteristic features of Japanese economic organization in terms of the complementarity between some of the most important elements of its economic structure. Jovanovic and Stolyarov (2000) study the combined effect of complementary and non-convex cost of adjustment in the upgrade (or adoption) of new technologies, and show that if upgrading each input involves a fixed cost, the firm may upgrade them at different dates. Their results might be an explanation for why investment in structures is more spiked than equipment investment, and why plants have spare capacity.

We use the real options methodology to derive, for a duopoly market where it is assumed that there is a first-mover advantage and in a game-choice setting, analytical expressions for the value functions of the leader and the follower and their respective investment trigger values. In our investment problem we assume that the market is composed of two idle firms and that at the beginning of the investment game there are two new (complementary) technologies available, *tech*

1 and *tech 2*. Firms are allowed to invest twice[9], in *tech 1* and in *tech 2*; the costs firms can save due to the adoption(s) as well as the amount they pay for each technology at the time of the adoption are uncertain. We assume that the firms' cost savings are a proportion of their market revenues and that both market revenues and the cost of each technology follow independent, and possible correlated, geometric Brownian motion (gBm) processes.

The word "complementarity" between the two technologies means here the degree to which two technologies are better off when operating together rather than operating alone; γ in inequality $\gamma > \gamma_1 + \gamma_2$, is the parameter that represents the degree of complementarity between the two technologies, where, γ_1 and γ_2 are the proportion of the firm's revenues that are expected to be saved if *tech 1* and *tech 2*, respectively, are adopted alone (firms operate with just one technology), and γ is the proportion of the firm's revenues that are expected to be saved if both technologies are adopted together (firms operate with the two technologies).

The methodology used to set the investment game is similar to that developed by Smets (1993), and followed by Dixit and Pindyck (1994), chapter 9, and Huisman (2001).

The rest of this paper is organized as follows. In section 2, we outline the model assumptions and define the duopoly investment game. In section 3, we derive the firms' value functions and their respective investment trigger values. In section 4, we present the results. In section 5 we conclude and give some guidelines for possible extensions of this research.

2. The Investment Game

The investment game is characterized as follows: in a risk neutral world, there are two idle firms studying the possibility of entering in a market by adopting one or two new technologies, *tech 1* and/or *tech 2*, at the same time or at different times, for which they have to spend a sunk (and uncertain) cost $I_1^*(t)$ and/or $I_2^*(t)$, respectively. The two technologies are currently available and the firms' cost savings that are expected to be achieved through the adoption of the technologies are a proportion of their revenues, whose evolution is uncertain. The two firms are allowed to invest twice, in *tech 1* and *tech 2*, the life of each technology is assumed to be infinite and time is

[9] Note that, in the adoption of two new technologies, if there is uncertainty about the revenues from the adoption and the cost of the technologies, before adoption firms have the option to adopt either one or both technologies, at the same time or at different times.

continuous. In Figure 1 we represent the investment game using an extensive-form representation. For a detailed description of this type of game representation see Gibbons (1992).

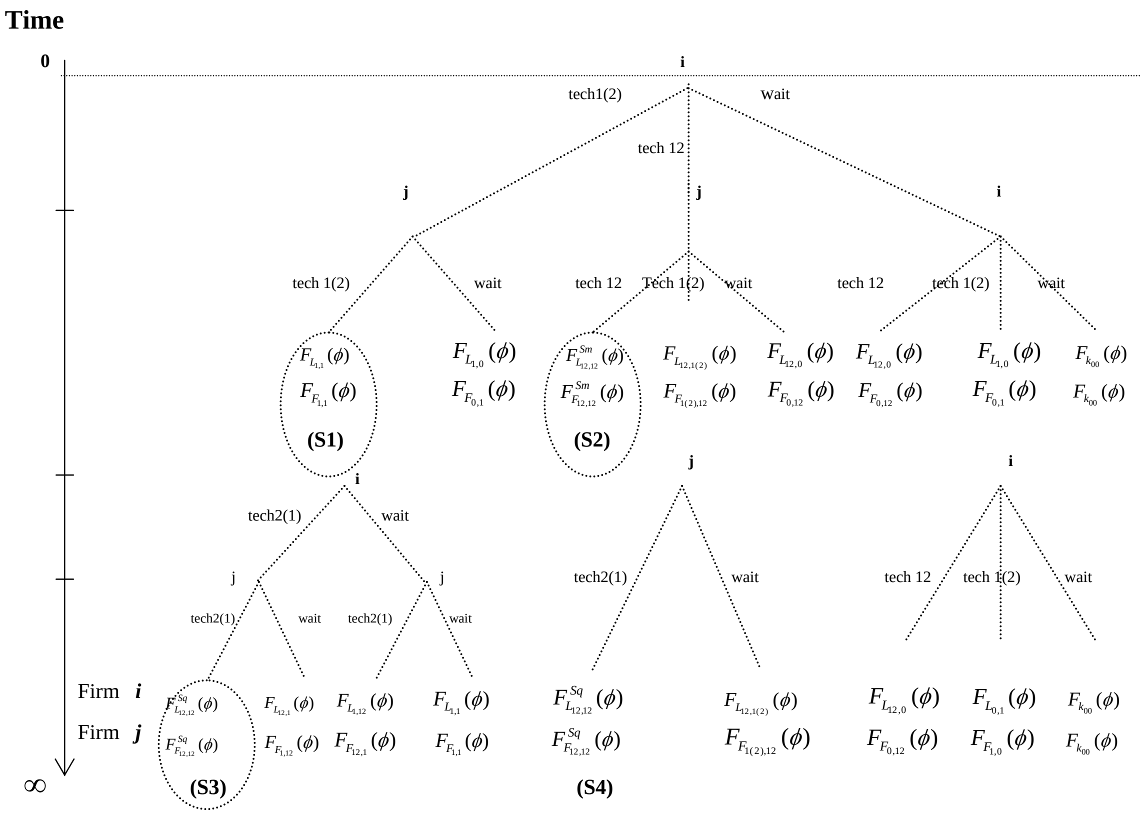

Figure 1 - Extensive-form representation of a Continuous Time Real Option Game (CTROG) with two firms and two complementary technologies.

Below we characterize four of the investment game scenarios described in Figure 1:

Scenario 1 **(S1):** firm *i* adopts first *tech 1(2)* and becomes the leader, firm *j* adopts later *tech 1(2)*, and becomes the follower. The payoffs for firm *i* and *j* are given, respectively, by $F_{L_{1,1}}(\phi)$ and $F_{F_{1,1}}(\phi)$. ***Scenario 2*** **(S2):** firm *i* adopts first *tech 1* and *tech 2* (*tech 12*) simultaneously, and firm *j* does the same later. Firm *i* becomes the leader and firm *j* the follower and their payoffs are, respectively, $F^{Sm}_{L_{12,12}}(\phi)$ and $F^{Sm}_{F_{12,12}}(\phi)$. ***Scenario 3*** **(S3):** in the first two rounds of the game, firms *i* and *j* adopt *tech 1* or *tech 2* (*tech 1(2)*). Firm *i* adopts first (first round) and becomes the leader, firm *j* adopts second (second round) and becomes the follower. Then, at the third and fourth rounds of the game, both firms adopt the remaining technology available *tech 2(1)*, again, one after the other, firm *i* first and firm *j* second[10], and the firms' payoffs are given by $F^{Sq}_{L_{12,12}}(\phi)$ and $F^{Sq}_{F_{12,12}}(\phi)$, respectively for firm *i* and *j*. ***Scenario 4* (S4):** in this scenario, in the first round, firm *i* adopts both technologies simultaneously (*tech 12*) first, and becomes the leader with a payoff given by $F^{Sq}_{L_{12,12}}(\phi)$. Firm *j* adopts then the two technologies sequentially, *tech 1(2)* first and then *tech 2(1)* second and gets $F^{Sq}_{F_{12,12}}(\phi)$ as payoff.

In the next section we derive analytical expressions for the firms' payoffs marked in Figure 1 with an ellipse (S1, S2 and S3). In Figures 2 and 3 below we represent in a time line the investment thresholds of the leader and the follower, for the case where the two technologies are adopted sequentially, and the case where both technologies are adopted simultaneously, respectively.

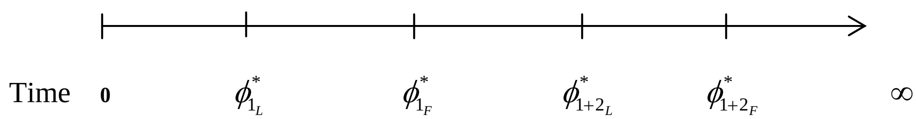

Figure 2 – Firms' Investment Thresholds when the two Technologies are adopted Sequentially.

Figure 2 represents the investment scenario where both firms adopt, one after the other, the two technologies sequentially; $\phi^*_{1_L}$ represents the leader's investment threshold to adopt *tech 1*, given that none of the technologies have been adopted; $\phi^*_{1_F}$ denotes the follower's investment threshold to adopt *tech 1*, when the leader is operating with *tech 1* and the follower is not yet in the market. $\phi^*_{1+2_L}$ is the leader's investment threshold to adopt *tech 2* given that *tech 1* is in place; and $\phi^*_{1+2_F}$ is the follower's investment threshold to adopt *tech 2* given that it has adopted *tech 1* and the leader is already operating with both *tech 1* and *tech 2*.

Note that in Figure 2 we assume that it is optimal for the follower to adopt *tech 1* before the leader has adopted *tech 2*. However, the initial conditions underlying our investment problem do not

[10] Note that here we have two possibilities: firm *i* adopts the remaining technology first and firm *j* second or the other way round. For simplicity, we assume that the first to adopt the first technology, *tech 1(2)*, is also the first to adopt the second technology, *tech 2(1)*.

restrict the occurrence of the scenario where the follower adopts *tech 1* after the leader having invested in both technologies[11].

Figure 3 represents the firms' investment threshold for the scenario where at the beginning of the investment game none of the technologies have been adopted and the two firms, one after the other, adopt the two technologies simultaneously; $\phi^*_{12_L}$ and $\phi^*_{12_F}$ represent, respectively, the leader's and the follower's investment thresholds for this investment scenario.

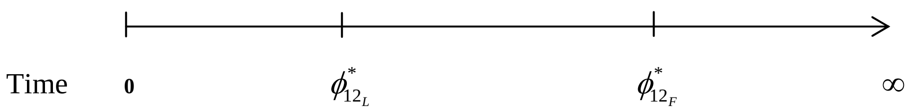

Figure 3 – Firms' Investment Thresholds when the two Technologies are adopted Simultaneously.

A summary of the firms' investment thresholds treated in the paper is given in Table 1.

Firms' Investment Trigger Values	**The Adoption of *Tech 1* or *Tech 2* alone***		**Sequential Adoption**** (***tech 1*/*tech 2***)**	**Simultaneous Adoption** (***tech 1 + tech 2***)**
Leader	$\phi^*_{1_L}$ **Equation (39)**	$\phi^*_{2_L}$ **Equation (39)**	$\phi^*_{1+2_L}$ **Equation (28)**	$\phi^*_{12_L}$ **Equation (43)**
Follower	$\phi^*_{1_F}$ **Equation (35)**	$\phi^*_{2_F}$ **Equation (35)**	$\phi^*_{1+2_F}$ **Equation (23)**	$\phi^*_{12_F}$ **Equation (41)**

*** The expressions for the firms' trigger value to adopt tech 1 and tech 2 are exactly the same, only the subscripts (1, 2) change.**
**** On the assumption that firms adopt first *tech 1*.**

Table 1 - Investment Thresholds for the Scenarios where Firms adopt the two Technologies, Sequentially and Simultaneously.

2.1 The Pre-emption Game

In games of timing the adoption of new technologies, the potential advantage from being the first to adopt may introduce an incentive for preempting the rival, speeding up the first adoption. The first contribution on new technology adoption games under rivalry is the Reinganum (1981) game-theoretic approach. In this model, the adoption of one firm has a negative effect on the profits of the

[11] For simplicity we will neglect this scenario and focus our analysis only on the investment scenario where the leader adopts the second technology only after the follower has adopted the first one (see Figure 2).

other firm and the increase in profits due to the adoption is greater for the leader than for follower. Fudenberg and Tirole (1985) studied the adoption of a new technology and illustrate the effects of preemption in games of time. We use the Fudenberg and Tirole (1985) concept of preemption to derive the value functions of the leader and the follower as well as their investment trigger values.

3. The Model

At time *t* there are two new (complementary) technologies available, *tech 1* and *tech 2*, and two idle firms, *i* and *j*, which are considering the adoption of the two technologies, one after the other or both simultaneously depending on which one of these choices is the best. In addition, we assume that there is a "first-mover cost savings advantage"[12], the firms' cost savings are a proportion of their revenues, whose evolution is uncertain, and the cost of each technology, *tech 1* and *tech 2*, is not known beforehand.

Given the economic context above, the firms' cost savings flow is given by the following expression:

$$X(t)\gamma_k \left[ds_{k_i k_j} \right] \tag{1}$$

where, $X(t)$ is the variable that represents the market revenues flow over time, γ_i is the proportion of firm i's revenues that is expected to be saved through the adoption of technology *k*, with $k = \{0,1,2,12\}$, where 0 means that the firm is not yet active and 1, 2 and 12 mean, respectively, that the firm operates with *tech 1* only, operates with *tech 2* only, or operates with *tech 1* and *tech 2*; $ds_{k_i k_j}$ is a deterministic factor that ensures a first-mover market/efficiency advantage, with $i, j = \{L, F\}$, where L means "leader" and F "follower", and represents the proportion of the market revenues that is held by each firm (*i*, *j*) for each investment scenario (see inequality 2).

The intuition used here to justify the first-mover "market/efficiency advantage" is similar to that used by Dixit and Pindyck (1994), following Smets (1993), for a first-mover "market advantage". In addition, we implicitly assume that firms are symmetric in their ability to learn how to operate

[12] i.e., the firm that adopts first tech 1(2) alone or *tech 1* and *tech 2* simultaneously gets a "cost savings advantage".

with the new technologies and that spillover information is not allowed, meaning that the firms' "first-mover cost savings advantage" holds forever. Consequently, inequality (2) holds:

$$ds_{12_i 0_j} > ds_{12_i 1_j} > ds_{12_i 12_j} > ds_{0_i 12_j} \tag{2}$$

The economic interpretation of inequality (2) is that for firm *i* (*j*), its best investment scenario is when it adopts both technologies (*tech 1* and *tech 2*) while its opponent *j* (*i*) is inactive ($ds_{12_i 0_j}$); its second best investment scenario is when it adopts both technologies first and its opponent adopts, later, only *tech 1* ($ds_{12_i 1_j}$); its third best investment scenario is when both firms adopt both technologies but it does so a little earlier ($ds_{12_i 12_j}$); and finally, its worst investment scenario is when it does not adopt any of the technologies and its opponent adopts both technologies ($ds_{0_i 21_j}$).

In our model, market revenues, $X(t)$, are described by the following equation:

$$dX = \mu_X X dt + \sigma_X X dz_X \tag{3}$$

where, μ_X is the trend rate of growth of market revenues, σ_X is the volatility of the market revenues and dz_X is the increment of a standard Wiener process.

We consider that *tech 1* alone provides a net cost savings, S_1, that is a fraction, γ_1, of the firm's market revenues, $X\left[ds_{k_i k_j}\right]$:

$$S_1 = \gamma_1 X\left[ds_{k_i k_j}\right] \tag{4}$$

Since the potential firms' cost savings are proportional to revenues and revenues follow a gBm process, so firms' cost savings also follows a gBm process. The equation for that process is:

$$dS_1 = \mu_{S_1} S_1 dt + \sigma_{S_1} S_1 dz_{S_1} \tag{5}$$

where, μ_1 is the trend rate of growth of the cost savings due to the adoption of *tech 1*, σ_1 is the volatility of the cost savings on the assumption that *tech 1* is adopted and dz_{S_1} is the increment of a standard Wiener process.

Similarly, the use of *tech 2* alone provides a potential cost savings equal to:

$$S_2 = \gamma_2 X \left[ds_{k_i k_j} \right] \tag{6}$$

with,

$$dS_2 = \mu_{S_2} S_2 dt + \sigma_{S_2} S_2 dz_{S_2} \tag{7}$$

where, μ_2 is the trend rate of growth of the cost savings due to the adoption of *tech 2*, σ_2 is the volatility of the cost savings on the assumption that *tech 2* is adopted and dz_{S_2} is the increment of a standard Wiener process.

The simultaneous use of both technologies yields cost savings equal to:

$$S = \gamma X \left[ds_{k_i k_j} \right] \tag{8}$$

with, *S* also following a gBm process, given by Equation (9):

$$dS = \mu_S S dt + \sigma_S S dz_S \tag{9}$$

where, μ_S is the trend rate of growth of the cost savings due to the adoption of both *tech 1* and *tech 2*, σ_S is the volatility of the cost savings on the assumption that firms adopt both technologies and dz_S is the increment of a standard Wiener process.

Given that we assume that the two technologies are complementary, so the following relation holds:

$$\gamma > \gamma_1 + \gamma_2 \tag{10}$$

Furthermore, we assume that the costs of adopting *tech 1* and *tech 2*, respectively, I_1 and I_2, follow gBm processes as well, given by:

$$dI_1 = \mu_{I_1} I_1 dt + \sigma_{I_1} I_1 dz_{I_1} \tag{11}$$

and

$$dI_2 = \mu_{I_2} I_2 dt + \sigma_{I_2} I_2 dz_{I_2} \tag{12}$$

where, μ_{I_1} and μ_{I_2} are the trend rates of growth of the cost of *tech 1* and *tech 2*, respectively; σ_{I_1} and σ_{I_2} are the volatility of the cost of *tech 1* and *tech 2*, respectively; and dz_{I_1} and dz_{I_2} are the increments of the standard Wiener processes, respectively, for the cases of *tech 1* and *tech 2*.

3.1 Technology 1 is in place

3.1.1 The Follower's Value Function

In this session we derive the follower's option value to adopt *tech 2* assuming that *tech 1* is in place, $f_{12}(X, I_2)$. Once we have $f_{12}(X, I_2)$, we will derive the expression for the total value $F_{12}(X, I_2) = V_1 + f_{12}(X, I_2)$, where V_1 is the follower's expected value from operating with *tech 1* forever, and given by expression (13):

$$V_1 = \frac{\gamma_1 X \left[ds_{k_i k_j} \right]}{r - \mu_X} \tag{13}$$

Setting the returns on the option equal to the expected capital gain on the option and using Ito's lemma, we obtain this partial differential equation (PDE) for the value function of an active follower (i.e., a follower which is operating with *tech 1*) in the region in which it waits to adopt *tech 2*:

$$\frac{1}{2}\sigma_X{}^2 X^2 \frac{\partial^2 F_{12}}{\partial X^2} + \frac{1}{2}\sigma_{I_2}^2 I_2{}^2 \frac{\partial^2 F_{12}}{\partial I_2{}^2} + X I_2 \sigma_X \sigma_{I_2} \rho_{XI_2} \frac{\partial^2 F_{12}}{\partial X \partial I_2} + \mu_X X \frac{\partial F_{12}}{\partial X} + \mu_{I_2} I_2 \frac{\partial F_{12}}{\partial I_2} + \gamma_1 X \left(ds_{1_L 1_F} \right) = r F_{12} \tag{14}$$

where, ρ_{XI_2} is the correlation coefficient between the market revenues, *X*, and the cost of *tech 2* , I_2 and *r* is the riskless interest rate.

Equation (14) must be subjected to two boundary conditions. The first is the *"value matching"* condition:

(i) There is a value of $F_{12}(X, I_2)$ at which the follower will invest and at that point in time the follower's value equals the present value of the cash flows minus the investment costs ($I^*_{2_F}$):

$$F_{12}(X, I_2) = \frac{(\gamma - \gamma_1) X^* \left[ds_{k_i k_j} \right]}{r - \mu_X - \mu_{I_2}} - I^*_{2_F} \tag{15}$$

where, $(\gamma - \gamma_1) X^* \left[ds_{k_i k_j} \right]$ represents the follower's cost savings at the time the follower adopts *tech 2*; $X^* \left[ds_{k_i k_j} \right]$ is the follower's revenues at the time of adoption; $(\gamma - \gamma_1)$ is the proportion of the follower's revenues that is expected to be saved due to the adoption of *tech 2* on the assumption that *tech 1* is already in place; $I^*_{2_F}$ is the cost of *tech 2* at the follower's adoption time.

The second boundary condition comes from the *"smooth pasting"* conditions, for the value of both the idle and the active follower:

(ii) The first derivative, with respect to both stochastic variables, $X(t)$ and $I_2(t)$, at the point where the value functions equal the present value of the cash flows, $(X / I_2)^*$. Therefore, it holds that:

$$\frac{\partial F_{12}(X, I_2)^*}{\partial X^*} = \frac{(\gamma - \gamma_1) \left[ds_{k_i k_j} \right]}{r - \mu_X - \mu_{I_2}} \tag{16}$$

$$\frac{\partial F_{12}(X, I_2)^*}{\partial I^*_{2_F}} = -1 \tag{17}$$

To obtain a closed-form solution for Equation (14) we use similarity methods. Therefore, we make the assumption that the option value is homogeneous of degree one, i.e., $F_{12}(X, I_2) = I_2 f_{12}(X / I_2)$, where f_{12} is now the variable to be determined. Changing the variable $\phi_2 = X / I_2$ and substituting the relations above in the PDE (14) yields[13]:

[13] For the derivation of Equation (18) see the Appendix.

$$\frac{1}{2}\sigma_{m_2}(\phi_2)^2\frac{\partial^2 f_{12}(\phi_2)}{\partial\phi_2{}^2}+\left(\mu_X-\mu_{I_2}\right)(\phi_2)\frac{\partial f_{12}(\phi_2)}{\partial\phi_2}+\gamma_1 X\left(ds_{1_L 1_F}\right)-(r-\mu_{I_2})f_{12}(\phi_2)=0 \tag{18}$$

where, $\sigma_{m_2}=\sigma_C^2+\sigma_{I_2}^2-2\rho_{CI_2}\sigma_C\sigma_{I_2}$.

Equation (18) is a homogeneous second-order linear ordinary differential equation (ODE) whose general solution has the form:

$$f_{12}(\phi_2)=A_{12}(\phi_2)^{\beta_1}+B_{12}(\phi_2)^{\beta_2} \tag{19}$$

where, $\beta_{1(2)}$ is the characteristic quadratic function of the homogeneous part of equation (18), given by:

$$\frac{1}{2}(\sigma_{m_2})^2\beta(\beta-1)+(\mu_X-\mu_{I_2})\beta-(r-\mu_{I_2})=0$$

Solving the equation above for β leads to:

$$\beta_{1(2)}=\frac{1}{2}+\frac{\mu_{I_2}-\mu_X}{\sigma_{m_2}}+(-)\sqrt{\left(\frac{1}{2}+\frac{(\mu_{I_2}-\mu_X)}{\sigma_{m_2}}\right)^2+\frac{2(r-\mu_{I_2})}{\sigma_{m_2}}} \tag{20}$$

Note that as the ratio of market revenues to cost of *tech 2*, ϕ_2, approaches 0, the value of the option to adopt *tech 2* becomes worthless; therefore, in Equation (19) $B_{12}=0$. Rewriting the boundary conditions we obtain:

$$f_{12}(\phi^*_{1+2_F})=\frac{(\gamma-\gamma_1)\left[ds_{k_i k_j}\right]\phi^*_{1+2_F}}{r-\mu_X-\mu_{I_2}}-1 \tag{21}$$

where, $\phi^*_{1+2_F}$ is the follower's investment threshold and,

$$\frac{\partial f_{12}(\phi^*_{1+2_F})}{\partial\phi^*_{1+2_F}}=\frac{(\gamma-\gamma_1)\left[ds_{k_i k_j}\right]}{r-\mu_X-\mu_{I_2}} \tag{22}$$

Solving together equations (19), (21) and (22) we get the following value for $\phi^*_{1+2_F}$ and the constant A_{12}:

$$\phi^*_{1+2_F} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_2})}{\left(ds_{12_F12_L} - ds_{1_F12_L}\right)(\gamma - \gamma_1)} \tag{23}$$

$$A_{12} = \frac{\left(\phi^*_{1+2_F}\right)^{-\beta_1}}{\beta_1 - 1} \frac{(\gamma - \gamma_1)\left(ds_{12_F12_L} - ds_{1_F12_L}\right)}{r - \mu_X - \mu_{I_2}} \tag{24}$$

where, $\varphi^*_{1+2_F}$ is the follower's threshold for adopting *tech 2* if *tech 1* has been adopted.

Finally, using equations (19), (23) and (24) we derive the follower's value function:

$$F^{Sq}_{F_{12,12}}(\phi_2) = \begin{cases} \dfrac{\gamma_1 X\left(ds_{1_F12_L}\right)}{r - \mu_X} - I^*_{1_F} + \dfrac{I_2}{\beta_1 - 1}\left(\dfrac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} & \phi_2 < \phi^*_{1+2_F} \\ \dfrac{\gamma X\left(ds_{12_F12_L}\right)}{r - \mu_X} - I^*_{2_F} & \phi_2 \geq \phi^*_{1+2_F} \end{cases} \tag{25}$$

Equation (25) tells us that, before $\phi^*_{1+2_F}$, the follower's value when it adopts the two technologies sequentially is given by the value of operating with *tech 1* forever, $\frac{\gamma_1\left(ds_{1_F12_L}\right)X}{r - \mu_X} - I^*_{1_F}$, plus its option to adopt *tech 2*, $\frac{I_2}{\beta_1 - 1}\left(\frac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1}$, as soon as $\phi^*_{1+2_F}$ is reached. The follower's value is equal to the net present value of the cost savings obtained by the follower if it operates with both technologies from $\phi^*_{1+2_F}$ until infinity, $\frac{\gamma X\left(ds_{12_F12_L}\right)}{r - \mu_X} - I^*_{2_F}$.

3.1.2 The Leader's Value Function

Assuming that both firms are operating with *tech 1* and that the follower will adopt *tech 2* at $\varphi^*_{1+2_F}$ (derived above), the leader's value function is described by the following expression[14]:

$$\varepsilon\left[\int_{T_{2_L}}^{T_{2_F}} \gamma X_\tau \left(ds_{12_L 1_F}\right) e^{-r\tau} d\tau - I^*_{2_L} + \int_{T_{2_F}}^{\infty} \gamma X_\tau \left(ds_{12_L 12_F}\right) e^{-r\tau} d\tau\right] \tag{26}$$

where, the first integral represents the leader's cost savings in the period where it operates with the two technologies and the follower operates with *tech 1*; the second integral represents the leader's cost savings for the period where both firms are operating with the two technologies, *tech 1* and *tech 2*; $I^*_{2_L}$ is the cost of *tech 2* at the leader's adoption time.

Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the following expression for the leader's value function:

$$F^{Sq}_{L_{12,12}}(\phi_2) = \begin{cases} \dfrac{\gamma X\left(ds_{12_L 1_F}\right)}{r-\mu_X} - I^*_{1_L} - I^*_{2_L} + \left(\dfrac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} \gamma\left(ds_{12_L 12_F} - ds_{12_L 1_F}\right) I_2 \dfrac{\beta_1}{\beta_1 - 1} & \phi_2 < \phi^*_{1+2_F} \\ \dfrac{\gamma X\left(ds_{12_L 12_F}\right)}{r-\mu_X} - I^*_{1_L} - I^*_{2_L} & \phi_2 \geq \phi^*_{1+2_F} \end{cases} \tag{27}$$

Expression $\dfrac{\gamma X\left(ds_{12_L 1_F}\right)}{r-\mu_X} - I^*_{1_L} - I^*_{2_L}$ corresponds to the leader's total payoff if it operates alone with the two technologies forever; $\left(\dfrac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} \gamma\left(ds_{12_L 12_F} - ds_{12_L 1_F}\right) I_2 \dfrac{\beta_1}{\beta_1 - 1}$ is negative, since $\left(ds_{12_L 12_F} - ds_{12_L 1_F}\right) < 0$ (see inequality (2)), and corresponds to the correction factor that incorporates the fact that in the future if $\phi^*_{1+2_F}$ is reached the follower will adopt *tech 2* and the leader's profits will be reduced.

We do not get a closed-form solution for the leader's trigger value. A numerical solution is required for the equation (28), where $\phi^*_{1+2_L}$ is the unknown. Equation (28) is derived by equalizing the value functions of the leader and the follower, for $\phi_2 < \phi^*_{1+2_F}$.

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[14] Note that it is assumed that *tech 1* is in place.

$$\frac{\gamma X\left(ds_{12_L1_F}\right)}{r-\mu_X}+\left(\frac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1}\gamma\left(ds_{12_L12_F}-ds_{12_L1_F}\right)I_2\frac{\beta_1}{\beta_1-1}-I^*_{1_L}-I^*_{2_L}-\frac{\gamma_1 X\left(ds_{1_F12_L}\right)}{r-\mu_X}+I^*_{1_F}-\frac{I_2}{\beta_1-1}\left(\frac{\phi_2}{\phi_{1+2_F}}\right)^{\beta_1}=0 \quad (28)$$

3.2 None of the Technologies have been adopted

Now that we have the value of the implicit option on *tech 2* if *tech 1* has been adopted, we can analyze the first-stage decision to adopt *tech 1*. We assume that *tech 1* is in place, here we derive the firms' value functions and investment trigger values for the scenario where neither of the technologies has been adopted.

3.2.1 The Follower's Value Function

Let $F(X,I_1,I_2)$ be the value of the option to adopt either one or both technologies. Setting the return on the option rF equal to the expected capital gain on the option and using Ito's lemma, we obtain this partial differential equation for the region in which the firm waits to invest:

$$\begin{aligned}0=\frac{1}{2}\Bigg(&\sigma_X^2X^2\frac{\partial^2F}{\partial X^2}+\sigma_{I_1}^2I_1^2\frac{\partial^2F}{\partial I_1^2}+\sigma_{I_2}^2I_2^2\frac{\partial^2F}{\partial I_2^2}+2\rho_{XI_1}\sigma_X\sigma_{I_1}XI_1\frac{\partial^2F}{\partial C\partial I_1}+2\rho_{XI_2}\sigma_X\sigma_{I_2}XI_2\frac{\partial^2F}{\partial X\partial I_2}+\ldots\\&\ldots+2\rho_{I_1I_2}\sigma_{I_1}\sigma_{I_2}I_1I_2\frac{\partial^2F}{\partial I_1\partial I_2}\Bigg)+\mu_XX\frac{\partial F}{\partial X}+\mu_{I_1}I_1\frac{\partial F}{\partial I_1}+\mu_{I_2}I_2\frac{\partial F}{\partial I_2}-rF\end{aligned} \quad (29)$$

where, ρ_{XI_1} and ρ_{XI_2} are the correlation coefficients between the market revenues and the cost of *tech 1*, and the market revenues and the cost of *tech 2*, respectively, and $\rho_{I_1I_2}$ is the correlation coefficient between the cost of *tech 1* and the cost of *tech 2*.

In the region where the firm is waiting to adopt, this value can be separated into the value of the option to acquire *tech 1* plus the value of the option to acquire *tech 2* as well. Assuming first-order homogeneity, i.e., $F(X,I_1,I_2)=I_1f_1(X/I_1)+I_2f_{12}(X/I_2)$, the relevant partial derivatives yield:

$$0=\left(\frac{1}{2}\sigma_{m_1}\phi_1\frac{\partial^2 f_1(\phi_1)}{(\partial\phi_1)^2}+(\mu_X-\mu_{I_1})\frac{\partial f_1(\phi_1)}{\partial(\phi_1)}-(r-\mu_{I_1})\phi_1 f_1\right)+\ldots$$

$$\ldots+\left(\frac{1}{2}\sigma_{m_2}\phi_2\frac{\partial^2 f_{12}(\phi_2)}{(\partial\phi_2)^2}+(\mu_X-\mu_{I_2})\frac{\partial f_{12}(\phi_2)}{\partial(\phi_2)}-(r-\mu_{I_2})\phi_2 f_{12}\right) \tag{30}$$

where, $\sigma_{m_1}=\sigma_X^2-2\rho_{XI_1}\sigma_X\sigma_{I_1}+\sigma_{I_1}^2$ and $\sigma_{m_2}=\sigma_X^2-2\rho_{XI_2}\sigma_X\sigma_{I_2}+\sigma_{I_2}^2$.

In the region where the current value of the ratio of revenues over the cost of *tech 2* is lower than the threshold to adopt the two technologies simultaneously, i.e., in the region where $\phi_2\le\phi_{12_F}^*$, the second bracketed expression is equal to zero, leaving this second-order linear differential equation[15] equal to:

$$0=\left(\frac{1}{2}\sigma_{m_1}(\phi_1)\frac{\partial^2 f_1(\phi_1)}{(\partial\phi_1)^2}+(\mu_X-\mu_{I_1})\frac{\partial f_1(\phi_1)}{\partial(\phi_1)}-(r-\mu_{I_1})\phi_1 f_1\right) \tag{31}$$

Therefore, the economically meaningful solution is:

$$f_1(\phi_1)=A_1(\phi_1)^{\beta_1}+B_1(\phi_1)^{\beta_2} \tag{32}$$

where,

$$\beta_1=\frac{1}{2}+\frac{\mu_{I_1}-\mu_X}{\sigma_{m_1}}+\sqrt{\left(\frac{1}{2}+\frac{(\mu_{I_1}-\mu_X)}{\sigma_{m_1}}\right)^2+\frac{2(r-\mu_{I_1})}{\sigma_{m_1}}} \tag{33}$$

As the ratio revenues over adoption cost, ϕ_1, approaches 0, the value of the option is worthless, so $B_1=0$. Using the "*value matching*" and the "*smooth pasting*" conditions at the threshold ratio, $\phi_{F_1}^*$, we obtain:

$$A_1=\frac{\phi_{1_F}^{*\,-\beta_1}}{\beta_1-1}\frac{\left(ds_{1_F1_L}-ds_{0_F1_L}\right)\gamma_1}{r-\mu_X-\mu_{I_1}} \tag{34}$$

[15] The assumption underlying our derivation is that since the optimal time to adopt both technologies simultaneously is given by $\phi_{12_F}^*$ and that, by assumption, is higher than the optimal time to adopt *tech 2* alone, $\phi_{2_F}^*$, so before $\phi_{12_F}^*$, the option to adopt *tech 2* given that *tech 1* is in place is worthless.

$$\phi_{1_F}^{*} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_1})}{\left(ds_{1_F 1_L} - ds_{0_F 1_L}\right)\gamma_1} \tag{35}$$

$$F_{F_{1,1}}(\phi_1) = \begin{cases} \dfrac{I_1}{\beta_1 - 1}\left(\dfrac{\phi_1}{\phi_{1_F}^{*}}\right)^{\beta_1} & \phi_1 < \phi_{1_F}^{*} \\ \dfrac{\gamma_1 X\left(ds_{1_F 1_L}\right)}{r - \mu_X} - I_{1_F}^{*} & \phi_1 \geq \phi_{1_F}^{*} \end{cases} \tag{36}$$

Since we did not differentiate the two technologies, the expressions for the case of *tech 2* are exactly the same as those derived above for the case of the adoption of *tech 1*. The only difference is the subscript used in the notation for the complementarity parameters and the deterministic factors that ensure the first-mover advantage, where the subscript “2” replaces “1”.

Notice that $\varphi_{1_F}^{*}$ is the follower’s threshold for adopting *tech 1* by itself and $\varphi_{1+2_F}^{*}$ is the follower’s threshold to adopt *tech 2* given that *tech 1* is in place. From these expressions we conclude that when the two technologies are complements, the degree of complementarity does not affect the decision to adopt either technologies alone, but does reduce the threshold for adopting the other if one technology is adopted.

3.2.2 The Leader’s Value Function

Focusing again on the adoption of *tech 1*, the leader’s expected value is given by:

$$\varepsilon\left[\int_{T_{1_L}}^{T_{1_F}} \gamma_1 X_\tau \left(ds_{1_L 0_F}\right) e^{-r\tau} d\tau - I_{1_F}^{*} + \int_{T_{1_F}}^{\infty} \gamma_1 X_\tau \left(ds_{1_L 1_F}\right) e^{-r\tau} d\tau\right] \tag{37}$$

The first integral represents the leader’s payoff when alone in the market; the second integral represents the leader’s payoff when operating with the follower, both with *tech 1*; $I_{1_F}^{*}$, is the cost of *tech 1* at the follower adoption time.

Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the following expression for the leader’s value function:

$$F_{L_{1,1}}(\phi_1) = \begin{cases} \dfrac{\gamma_1 X\left(ds_{1_L 0_F}\right)}{r-\mu_X} + \dfrac{\beta_1}{\beta-1} I_1 \left(\dfrac{\phi}{\phi_{1_F}^*}\right)^{\beta_1} \left(ds_{1_L 1_F} - ds_{1_L 0_F}\right) - I_{1_L}^* & \phi_1 < \phi_{1_F}^* \\ \dfrac{\gamma_1 X\left(ds_{1_L 1_F}\right)}{r-\mu_X} & \phi_1 \geq \phi_{1_F}^* \end{cases} \tag{38}$$

Again, we do not get a closed-form solution for the leader's trigger value. However, a numerical solution is required for the equation (39), where $\phi_{1_L}^*$ is the unknown. Equation (39) is obtained by equalizing the value functions of the leader and the follower, for $\phi_1 < \phi_{1_F}^*$.

$$\frac{\gamma_1 X\left(ds_{1_L 0_F}\right)}{r-\mu_X} + \frac{\beta_1}{\beta-1} I_1 \left(\frac{\phi}{\phi_{1_F}^*}\right)^{\beta_1} \left(ds_{1_L 1_F} - ds_{1_L 0_F}\right) - I_{1_L}^* - \frac{I}{\beta_1 - 1}\left(\frac{\phi_1}{\phi_{1_F}^*}\right)^{\beta_1} = 0 \tag{39}$$

The procedure used to get this equation was the same as that used for Equation (28).

3.3 Simultaneous Adoption

Following similar procedures we get the expressions for the firms' value functions and investment trigger values for the case where, for some technical/economic reasons, the two technologies have to be adopted simultaneously.

3.3.1 The Follower's Value Function

$$F_{F_{12,12}}(\phi_{12}) = \begin{cases} \dfrac{I_{12}}{\beta_1 - 1}\left(\dfrac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1} & \phi_{12} < \phi_{12_F}^* \\ \dfrac{\gamma X\left(ds_{1_F 1_L}\right)}{r-\mu_X} - I_{12_F}^* & \phi_{12} \geq \phi_{12_F}^* \end{cases} \tag{40}$$

Investment Trigger Value:

$$\phi_{12_F}^* = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_{12}})}{\left(ds_{12_F 12_L} - ds_{0_F 12_L}\right)\gamma} \tag{41}$$

3.3.2 The Leader's Value Function

$$F_{L_{12,12}}(\phi_{12}) = \begin{cases} \dfrac{\gamma X\left(ds_{12_L 0_F}\right)}{r-\mu_X} + \dfrac{\beta_1}{\beta-1} I_{12}\left(\dfrac{\phi}{\phi_{1_F}^*}\right)^{\beta_1}\left(ds_{12_L 12_F} - ds_{12_L 0_F}\right) - I_{12_L}^* & \phi_{12} < \phi_{12_F}^* \\ \dfrac{\gamma X\left(ds_{12_L 12_F}\right)}{r-\mu_X} - I_{12_L}^* & \phi_{12} \geq \phi_{12_F}^* \end{cases} \tag{42}$$

Investment Trigger Value:

$$\frac{\gamma X\left(ds_{12_L 0_F}\right)}{r-\mu_X} + \frac{\beta_1}{\beta-1} I_{12}\left(\frac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1}\left(ds_{12_L 12_F} - ds_{12_L 0_F}\right) - I_{12_L}^* - \frac{I_{12}}{\beta_1 - 1}\left(\frac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1} = 0 \tag{43}$$

4. Results and Sensitivity Analysis

In this section we analyse the sensitivity of our real options model to changes in some of its most important parameters. In the simulations below we use the following inputs: $X = 95.00$, $I_1 = 13.00$, $I_2 = 4.00$, $I_{1_L}^* = 12.00$, $I_{1_F}^* = 11.00$, $I_{2_L}^* = 3.50$, $I_{2_F}^* = 3.00$, $\sigma_X = \sigma_{I_1} = \sigma_{I_2} = 0.20$, $\mu_X = 0.05$, $\mu_{I_1} = -0.05$, $\mu_{I_2} = -0.10$, $r = 0.09$, $\rho_{XI_1} = \rho_{XI_2} = 0.50$, $\gamma_1 = 0.15$, $\gamma_2 = 0.05$, $\gamma = 0.25$, $ds_{12_L 1_F} = 0.61$, $ds_{1_F 12_L} = 0.39$, $ds_{12_L 12_F} = 0.55$, $ds_{12_F 12_L} = 0.45$, $ds_{0_F 12_L} = 0$.

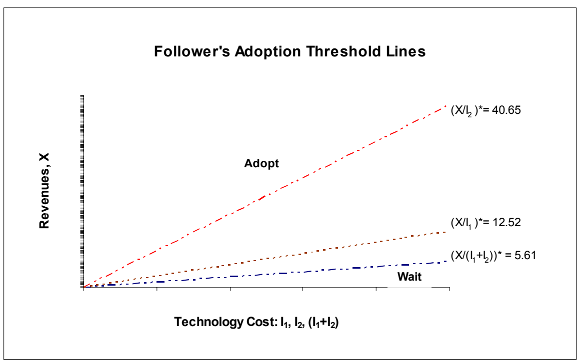

Figure 6

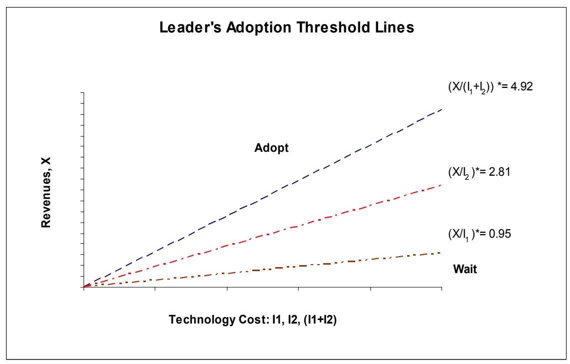

Figure 7

Current Values			Follower's Trigger				Leader's Trigger			
$(X/(I_{12}))$	(X/I_1)	(X/I_2)	$(X/(I_{1+2}))^*$	$(X/(I_{12}))^*$	$(X/I_1)^*$	$(X/I_2)^*$	$(X/(I_{1+2}))^*$	$(X/(I_{12}))^*$	$(X/I_1)^*$	$(X/I_2)^*$
5.59	**7.31**	**23.75**		5.61	12,52	40.65		4.92	0,95	2.81

Table 2

Table 2 shows that the leader should adopt tech 1 and tech 2, either each alone or both at the same time, since, for all scenarios, the current values of the variables (X / I_{12}), (X / I_1) and (X / I_2) are greater than its investment trigger values, respectively, $(X / I_{12})^*$, $(X / I_1)^*$ and $(X / I_2)^*$; the follower should defer the investment for all scenarios (the adoption of tech 1 alone, the adoption of tech 2 alone or the adoption of both tech 1 and tech 2 simultaneously), slightly in the case of the adoption of both technologies at the same time.

Figures 6 and 7 show, for the leader and the follower, the investment threshold lines for the scenarios where *tech 1* is adopted alone, *tech 2* is adopted alone and both technologies are adopted at the same time. Each investment threshold line separates two regions in the (X, I) space. The area above the straight lines represents an infinite number of ratios "operating costs over the cost of the technology(ies)" that, once reached, would lead firms to "*adopt*" the technology(ies); the area below the straight lines represents an infinite number of ratios "market revenues over the cost of the technology(ies)" that, once reached, would lead firms to "*wait*". Each straight line has a different slope, expressed by the ratios $(X / I_{12})^*$, $(X / I_1)^*$ and $(X / I_2)^*$, respectively, for the scenario where firms adopt both technologies at the same time, and the scenarios where firms adopt *tech 1* and *tech 2* alone. The higher the slope of these straight lines, the later the adoption of the respective technology(ies).

The results above were computed using different (negative) growth rates for the cost of the two technologies (-5% for t*ech 1* and -10% for *tech 2*). Our initial intuition is that, for a *ceteris paribus* analysis, firms would adopt first the technology whose price is decreasing more slowly (*tech 1*) and delay the adoption of the technology whose price is decreasing more rapidly (*tech 2*). For both firms, and for all investment scenarios, our results show that the slope of the investment threshold lines that regards *tech 2* are higher than the slope of the investment threshold lines that concerns *tech 1*, which confirm the intuition stated above.

Comparing the scenarios where each technology is adopted alone with that where both technologies are adopted at the same time, our results show that, for the follower, the slope of the investment threshold line that corresponds to the adoption of both technologies at the same time is lower than the slopes of the investment threshold lines that corresponds to the scenarios where each technology is adopted alone. For the leader our results show the opposite (adopt tech 1 first and tech 2 second). Since in our simulation we use different rates of decrease in the cost of the two technologies (-5%

for t*ech 1* and -10% for *tech 2*), so these results show also that, for the follower, a difference of 5% between the rates of the decrease in the cost of the two technologies, which constitutes an incentive for the adoption of the two technologies at different times, is not enough to offset the effect of technological complementarity between the two technologies (5% additional cost savings -given as a proportion of market revenues), which is an incentive to the adoption of both technologies at the same time.

When we compare, for each investment scenarios, the investment thresholds of the leader with those of the follower, the conclusion is that the slope of investment threshold lines of the leader are always lower than those of the investment threshold of the follower, meaning that the leader invests first in all scenarios.

In Table 3 and Figure 8 are the results of the sensitivity analysis of the effect of changes in the rate of decline of the cost of *tech 2* on the investment threshold lines of the leader and the follower. Since we do not differentiate the two technologies, these results apply also to the case of the adoption of *tech 1*.

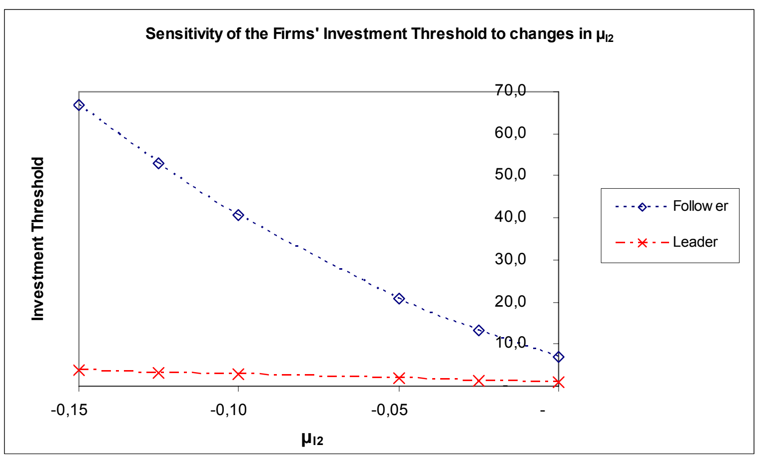

Figure 8

μ_{I2}	- 0.25	- 0.20	- 0.15	- 0.10	- 0.05	-
$\Phi^*_{2,F}$	66.69	52.90	40.65	20.78	13.12	6.96
$\Phi^*_{2,L}$	3.82	3.31	2.81	1.80	1.30	0.80

Table 3

The results show that when the rate of decrease of the cost of *tech 2* is 25%, the slopes of the investment threshold curves of the leader and the follower are 3.82 and 66.69, respectively. When the rate of decrease of the cost of *tech 2* is null, the slopes of the investment threshold curves of the leader and the follower are 0.80 and 6.96, respectively. The conclusion is that as the rate of decrease in the cost of *tech 2* increases, the slopes of the investment threshold curves of both firms increase substantially and, therefore, firms tend to adopt *tech 2* later. These results make economic sense because the higher the rate of decrease in the price of a technology the higher the incentive to defer the investment and (maybe) benefit from a potential decrease in price. This effect is more evident on the investment trigger value of the follower than in the investment trigger value of the leader.

Figure 9 shows the sensitivity of the firms' investment threshold ratios (slope of the investment threshold curve) of the leader and the follower to changes in the parameter γ_2. Note that γ_2 is a deterministic factor that represents the proportion of the net revenues (sales less operating costs, ignoring investment costs and taxation) that is saved through the adoption of tech 2. We perform this analysis for the case of the adoption of *tech 2*, but since we do not differentiate the two technologies, they apply also to the case of the adoption of *tech 1*.

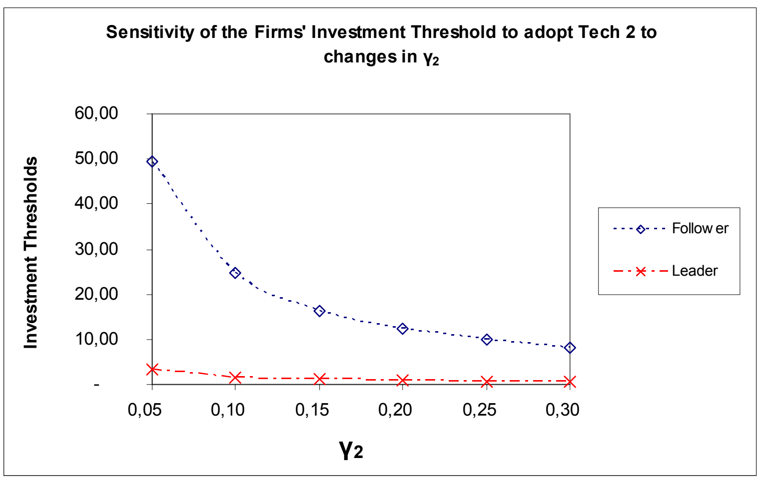

Figure 9

γ_2	0,05	0,10	0,15	0,20	0,25	0,30
$\Phi^*_{2,F}$	49.30	24.65	16.43	12.32	9.86	8.22
$\Phi^*_{2,L}$	3.31	1.65	1.10	3.00	0.66	0.55

Table 4

When the adoption of *tech 2* leads to a cost saving of 5% ($\gamma_2 = 0.05$) of the net market revenues, the slopes of the investment threshold curves of the leader and the follower are 3.31 and 49.30, respectively, and when the adoption of *tech 2* leads to a benefit (cost savings) of 30% of the net market revenues ($\gamma_2 = 0.30$), the slopes of the investment threshold curves of the leader and the follower are 0.55 and 8.22, respectively. In addition, the results also show that the investment thresholds of the leader and the follower are a non linear function of the parameter γ_2. When γ_2 increases, the slope of the investment threshold curve decreases significantly, leading to earlier investments for both firms, especially for the follower.

5. Conclusions and Further Research

We show that, in a *ceteris paribus* analysis, the higher the degree of complementarity between the two technologies, the earlier the adoption of both technologies at the same time and the more advantageous are such decisions compared with the adoption of each technology alone. Furthermore, we also show that when the degree of complementarity between the two technologies is low and the rate of decrease in the cost of the two technologies differs substantially, it might be optimal for both firms to adopt the two technologies at different times, first the technology whose price is decreasing at a lower rate and then the technology whose price is decreasing more rapidly.

We study the effect of the complementarity between two technologies on firms' investment decisions, considering uncertainty and competition. Our investment game setting was built under the assumption that there is a first-mover advantage (*pre-emption* game). However, an interesting extension for this research would be to derive a similar investment model, but for economic contexts where there is a second-mover advantage (*war of attrition* game).

In our framework we assume that there are two firms (the leader and the follower) and two technologies that can be adopted at the same time or at different times. Since it is quite common to find projects that have more than two inputs, whose functions are complements, an interesting extension would be to consider more than two complementary inputs.

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Appendix

Derivation of the Ordinary Differential Equation (18)

Equation (14) is written as:

$$\frac{1}{2}\frac{\partial^2 F_{12}}{\partial X^2}\sigma_X{}^2 X^2 + \frac{1}{2}\frac{\partial^2 F_{12}}{\partial I_2{}^2}\sigma_{I_2}^2 I_2{}^2 + \frac{\partial^2 F_{12}}{\partial X \partial I_2} X I_2 \sigma_X \sigma_{I_2} \rho_{XI_2} + \frac{\partial F_{12}}{\partial X}\mu_X X + \frac{\partial F_{12}}{\partial I_2}\mu_{I_2} I_2 - rF_{12} = 0$$

In order to reduce the homogeneity of degree two in the underlying variables to homogeneity of degree one, similarity methods can be used. Let $\phi_2 = \frac{X}{I_2}$, so:

$$F(X, I_2) = F(\phi_2)$$

$$\frac{\partial F(X, I_2)}{\partial I_2} = \frac{\partial F(\phi_2)}{\partial \phi_2} X$$

$$\frac{\partial F(X, I_2)}{\partial X} = \frac{\partial F(\phi_2)}{\partial \phi_2} I_2$$

$$\frac{\partial^2 F(X, I_2)}{\partial I^2} = \frac{\partial^2 F(\phi_2)}{(\partial I)^2} X^2$$

$$\frac{\partial^2 F(X, I_2)}{\partial X^2} = \frac{\partial^2 F(\phi_2)}{(\partial \phi_2)^2} (I_2)^2$$

$$\frac{\partial^2 F(X, I_2)}{\partial X \ \partial I_2} = \frac{\partial^2 F(\phi_2)}{(\partial \phi_2)^2} X I_2 + \frac{\partial F(\phi_2)}{\partial \phi_2}$$

Substituting back to Equation (14) we obtain Equation (18):

$$\frac{1}{2}\sigma_{m_2}(\phi_2)^2 \frac{\partial^2 f_{12}(\phi_2)}{\partial \phi_2{}^2} + \left(\mu_X - \mu_{I_2}\right)(\phi_2)\frac{\partial f_{12}(\phi_2)}{\partial \phi_2} + \gamma_1 X\left(ds_{1_L 1_F}\right) - (r - \mu_{I_2}) f_{12}(\phi_2) = 0$$