Extended Abstract

Compound Real Options with the Fuzzy Pay-off Method

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Abstract

Compound real options are combinations of real options, where an exercise of a real option opens another real option. Compound real options are commonly found in a number of industrial projects, but are especially relevant in, e.g., research and development (R&D) where the R&D projects give the real option to research further, or to start the implementation of the results. Valuation of compound options with the most commonly used option valuation methods is often very complex and the methods suffer from a number of problems when used for valuation of real options. This paper discusses the valuation of compound real options with the fuzzy pay-off method for real option valuation and shows that the method reduces complexity of the valuation of compound real options.

Keywords: *Fuzzy pay-off method, real option valuation, compound options, fuzzy logic*

1. Introduction

Real option valuation (ROV) is based on the observation that the possibilities financial options give their holder resemble the possibilities to invest in real investments and possibilities found within real investments, i.e., managerial flexibility - "an irreversible investment opportunity is much like a financial call option" (Pindyck, 1991). In other words, real option valuation is treating investment opportunities and the different types of managerial flexibility as options and valuing them with option valuation models. Real options are useful both, as a mental model for strategic and operational decision-making, and as a valuation and numerical analysis tool. This paper concentrates on the use of real options in numerical analysis, and particularly on the

derivation of the real option value for a given investment opportunity, or identified managerial flexibility.

Real options are commonly valued with the same methods that have been used to value financial options, i.e., with Black-Scholes option pricing formula (Black & Scholes, 1973), with the binomial option valuation method (Cox, Ross, & Rubinstein, 1979) , with Monte-Carlo based methods (Boyle, 1977), and with a number of later methods based on these. Most of the methods are complex and demand a good understanding of the underlying mathematics, issues that make their use difficult in practice. In addition these models are based on the assumption that they can quite accurately mimic the underlying markets as a process, an assumption that may hold for some quite efficiently traded financial securities, but may not hold for real investments that do not have existing markets or have markets that can by no means be said to exhibit even weak market efficiency.

Recently, a novel approach to real option valuation was presented in (Mathews & Datar, 2007a), (Mathews & Salmon, 2007b), and in (Datar & Mathews, 2004), where the real option value is calculated from a pay-off distribution, derived from a probability distribution of the NPV for a project that is generated with a (Monte-Carlo) simulation. The authors show that the results from the method converge to the results from the analytical Black-Scholes method. The method presented greatly simplifies the calculation of the real option value, making it more transparent and brings real option valuation as a method a big leap closer to practitioners. The most positive issue in this method is that it does not suffer from the problems associated with the assumptions connected to the market processes connected to the Black-Scholes and the binomial option valuation methods. The method utilizes cash-flow scenario based estimation of the future outcomes to derive the future pay-off distribution – this is highly compatible with the way cash-flow based profitability analysis is commonly done in companies.

Compound real options are possibilities that open new possibilities, i.e., a chain of contingent possibilities in which each possibility is opened by a selection to take the possibility before it. Compound real options can be found in a number of different domains, but in this paper we concentrate on multi-stage investment opportunities as compound real options.

HERE INTRODUCTION TO COMPOUND OPTIONS WITH REFERENCES

HERE WHAT FOLLOWS IN THE PAPER

2. Fuzzy Pay-off Method for Real Option Valuation

All of the above mentioned models and methods use probability theory in their treatment of uncertainty, there are however, other ways than probability to treat uncertainty, or imprecision in future estimates, namely fuzzy logic and fuzzy sets.

2.1 Shortly about fuzzy sets in financial modeling

In classical set theory an element either (fully) belongs to a set or does not belong to a set at all. This type of bi-value, or true/false, logic is commonly used in financial applications (and is a basic assumption of probability theory). Bi-value logic, however, presents a problem, because financial decisions are generally made under uncertainty. Uncertainty in the financial investment context means that it is in practice impossible, ex-ante to give absolutely correct precise estimates of, e.g., future cash-flows. There may be a number of reasons for this, see, e.g., (Knight, 1921), however, the at the end of the day we our estimations are less than fully accurate.

Fuzzy sets are sets that allow (have) gradation of belonging, such as "a future cash flow at year ten is about x euro". This means that fuzzy sets can be used to formalize inaccuracy that exists in human decision making and as a representation of vague, uncertain or imprecise knowledge, e.g., future cash-flow estimation, which human reasoning is especially adaptive to. "Fuzzy set-based methodologies blur the traditional line between qualitative and quantitative analysis, since the modeling may reflect more the type of information that is available rather than researchers' preferences" (Tarrazo, 1997) and indeed in economics "The use of fuzzy subsets theory leads to results that could not be obtained by classical methods." (Ponsard, 1988). The origins of fuzzy sets date back to an article by Lotfi Zadeh (Zadeh, 1965) where he developed an algebra for what he called fuzzy sets. This algebra was created to handle imprecise elements in our decision making processes, and is the formal body of theory that allows the treatment of practically all decisions in an uncertain environment. "Informally, a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not" (Bellman & Zadeh, 1970).

DEFINITION. Let $X = \{x\}$ denote a collection of objects (points) denoted generically by x. Then a fuzzy set A in X is a set of ordered pairs

Eq. 1. $$A = \{(x, \mu A\,(x))\}, \quad x \in X$$

where $\mu A\,(x)$ is termed the grade of membership of x in A, and $\mu_A : X \to M$ is a function from X to a space M called the membership space. When M contains only two points, 0 and 1, A is non fuzzy and its membership function becomes identical with the characteristic function of a crisp set. This means that crisp sets are a subset of fuzzy sets. *A* fuzzy number is a normal, convex fuzzy set whose referential set is the real numbers $X \in R$.

Fuzzy set theory uses fuzzy numbers to quantify subjective fuzzy observations or estimates. Such subjective observations or estimates can be, e.g., estimates of future cash flows from an investment. To estimate future cash flows and discount rates "One usually employs educated guesses, based on expected values or other statistical techniques" (Buckley, 1987), which is consistent with the use of fuzzy numbers. In practical applications the most used fuzzy numbers are trapezoidal and triangular fuzzy numbers. They are used, because they make many operations possible and are intuitively understandable and interpretable.

When we replace non-fuzzy numbers (crisp, single) numbers that are commonly used in financial models with fuzzy numbers we can construct models that include the inaccuracy of human perception, or ability to forecast, within the (fuzzy) numbers. This makes these models more in line with reality, as they do not simplify uncertain distribution-like observations to a single point

estimate that conveys the sensation of no-uncertainty. Replacing non-fuzzy numbers with fuzzy numbers means that the models that are built must also follow the rules of fuzzy arithmetic.

Fuzzy numbers (fuzzy logic) have been adopted to option valuation models in (binomial) pricing an option with a fuzzy payoff, e.g., in (Muzzioli & Torricelli, 2000), and in Black-Scholes valuation of financial options in, e.g., (Yoshida, 2003). There are also some option valuation models that present a combination of probability theory and fuzzy sets, e.g., (Zmeskal, 2001). Fuzzy numbers have also been applied to the valuation of real options in, e.g., (Carlsson & Fullér, 2003), (Collan, Carlsson, & Majlender, 2003), and (Carlsson & Majlender, 2005). More recently there are a number of papers that present the application of fuzzy RO models in the industry setting, e.g., (Chen, Zhang, Lin, & Yu, 2007; Tolga & Kahraman, 2008). There are also specific fuzzy models for the analysis of the value of optionality for very large industrial real investments, e.g., (Collan, 2004).

2.2. Fuzzy Pay-off Method for Real Option Valuation

In two recent articles (Mathews et al., 2007b) and (Mathews et al., 2007a) present a practical probability theory based method for the calculation of real option value (ROV) and show that the method and results from the method are mathematically equivalent to the Black-Sholes formula (Black et al., 1973). The method is based on simulation generated probability distributions for the NPV of future project outcomes. The method implies that: "the real-option value can be understood simply as the average net profit appropriately discounted to Year 0, the date of the initial R&D investment decision, contingent on terminating the project if a loss is forecast at the future launch decision date." The project outcome probability distributions are used to generate a payoff distribution, where the negative outcomes (subject to terminating the project) are truncated into one chunk that will cause a zero payoff, and where the probability weighted average value of the resulting payoff distribution is the real option value.

(Collan, Mézei, & Fullér, 2009 (Forthcoming)) present a novel method for the valuation and analysis of real options, the fuzzy pay-off method that uses fuzzy numbers in representing the expected future distribution of possible project costs and revenues, and hence also the profitability (NPV) outcomes. When using fuzzy numbers the fuzzy NPV itself is the payoff distribution from the project.

The method presented in (Mathews et al., 2007a) implies that the weighted average of the positive outcomes of the payoff distribution is the real option value; in the case with fuzzy numbers the weighted average is the fuzzy mean value of the positive NPV outcomes (which is nothing more than the possibility weighted average). Derivation of the fuzzy mean value is presented in (Carlsson & Fullér, 2001).

This means that calculating the real option value (ROV) from a fuzzy NPV (distribution) is straightforward, it is the fuzzy mean of the possibility distribution with values below zero counted as zero, i.e., the area weighted average of the fuzzy mean of the positive values of the distribution and zero (for negative values).

Definition 1.1. We calculate the real option value from the fuzzy NPV as follows

$$ROV = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} \times E(A_+)$$

Where A stands for the fuzzy NPV, $E(A_+)$ denotes the fuzzy mean value of the positive side of the NPV and $\int_{-\infty}^\infty A(x)dx$ computes the area below the whole fuzzy number A, and $\int_0^\infty A(x)dx$ computes the area below the positive part of A.

MORE WORK ON PRESENTATION CLARITY IN THIS SECTION

3. Compound Options with the Fuzzy Pay-off Method

HERE DESCRIPTION OF HOW COMPOUND OPTIONS CAN BE MODELLED WITH THE FUZZY PAY-OFF METHOD AND HOW THE MODELLING SIMPLIFIES THE CALCULATION AND ALSO WHAT PROBLEMS ARE NOT PRESENT WHEN USING THE FUZZY PAY-OFF METHOD

3.1. Case Example R&D

HERE PRESENTATION OF THE CASE

HERE NUMERICAL CASE WITH GRAPHICAL PRESENTATION AND DISCUSSION
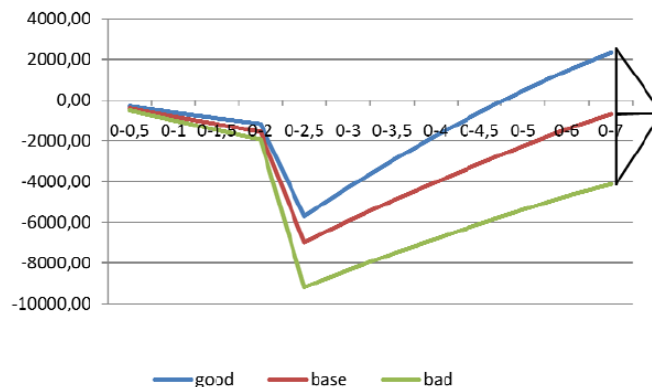
HERE DISCUSSION OF THE RESULTS FROM THE CASE & THE INTERPRETATION OF THE ANALYSIS

5. Discussion and Conclusions

6. References

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Appendix 1.

Investment / cost cash-flows (PV@"risk free" level)   Rf= 0,05

	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	6
CF Good	300,00	300,00	300,00	300,00	5000,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
CF Base	400,00	400,00	400,00	400,00	6000,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
CF Bad	500,00	500,00	500,00	500,00	8000,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PV Good	300,00	292,77	285,71	278,83	4535,15	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PV Base	400,00	390,36	380,95	371,77	5442,18	0,00	0,00	0,00	0,00	0,00	0,00	0,00
PV Good	500,00	487,95	476,19	464,71	7256,24	0,00	0,00	0,00	0,00	0,00	0,00	0,00

	0-0,5	0-1	0-1,5	0-2	0-2,5	0-3	0-3,5	0-4	0-4,5	0-5	0-6	0-7
ΣPV Good	300,00	592,77	878,48	1157,31	5692,46	5692,46	5692,46	5692,46	5692,46	5692,46	5692,46	5692,46
ΣPV Base	400,00	790,36	1171,31	1543,08	6985,26	6985,26	6985,26	6985,26	6985,26	6985,26	6985,26	6985,26
ΣPV Bad	500,00	987,95	1464,14	1928,85	9185,09	9185,09	9185,09	9185,09	9185,09	9185,09	9185,09	9185,09

Revenue cash-flows (PV @ "risk adjusted" level)   Ra1= 0,15   Ra2= 0,40

	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	6
CF Good	0,00	0,00	0,00	0,00	0,00	2000,00	2000,00	2000,00	2000,00	2000,00	2000,00	2000,00
CF Base	0,00	0,00	0,00	0,00	0,00	1500,00	1500,00	1500,00	1600,00	1600,00	1700,00	1700,00
CF Bad	0,00	0,00	0,00	0,00	0,00	1200,00	1200,00	1200,00	1300,00	1300,00	1350,00	1350,00
PV Good	0,00	0,00	0,00	0,00	0,00	1410,22	1315,03	1226,27	1143,51	1066,33	994,35	864,66
PV Base	0,00	0,00	0,00	0,00	0,00	1057,66	986,27	919,71	914,81	853,06	845,20	734,96
PV Good	0,00	0,00	0,00	0,00	0,00	846,13	789,02	735,76	743,28	693,11	671,19	583,64

	0-0,5	0-1	0-1,5	0-2	0-2,5	0-3	0-3,5	0-4	0-4,5	0-5	0-6	0-7
ΣPV Good	0,00	0,00	0,00	0,00	0,00	1410,22	2725,25	3951,52	5095,03	6161,35	7155,71	8020,36
ΣPV Base	0,00	0,00	0,00	0,00	0,00	1057,66	2043,94	2963,64	3878,45	4731,51	5576,71	6311,66
ΣPV Bad	0,00	0,00	0,00	0,00	0,00	846,13	1635,15	2370,91	3114,19	3807,30	4478,49	5062,13

	0-0,5	0-1	0-1,5	0-2	0-2,5	0-3	0-3,5	0-4	0-4,5	0-5	0-6	0-7
FNPV Good	-300,00	-592,77	-878,48	-1157,31	-5692,46	-4282,25	-2967,21	-1740,94	-597,43	468,89	1463,25	2327,90
FNPV Base	-400,00	-790,36	-1171,31	-1543,08	-6985,26	-5927,60	-4941,32	-4021,62	-3106,81	-2253,75	-1408,55	-673,60
FNPV Bad	-500,00	-987,95	-1464,14	-1928,85	-9185,09	-8338,96	-7549,94	-6814,18	-6070,90	-5377,79	-4706,60	-4122,96

Fuzzy NPV (FNPV) is the pay-distribution from the project. In this case it is calculated by deducting the bad case cost from the good case revenue, the base case cost from the base case revenue, and the good case cost from the bad case revenue - this way the resulting distribution includes the extremes (this is in line with the standard operations with fuzzy numbers).

Pay-off Good	2327,90
Pay-off Base	-673,60
Pay-off Bad	-4122,96

-4122,96	-673,60	2327,90
bad	base	good

108,59

Real Option Value
(by using the fuzzy pay-off method)

good — base — bad