Using Omega Measure for Performance Assessment of a Real Options Portfolio

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Abstract

The portfolio composition of assets is a classic theme in finance literature. Investors want to get highest return, minimizing as possible the involved risk.

In a portfolio composed by real assets, such as investment projects, it must be decided which of them carry on. So, it is always a challenge to measure the involved risk. Furthermore, in a portfolio of real assets it's possible to introduce and model for each individual project future investment decisions, like the right time to invest, making an expansion, reduction of operations, abandonment, etc. Bearing in mind the possibility of exercising these options, the model becomes more realistic.

For this type of portfolios, in order to do an adequate assessment of their risk-return performance, it is proposed to use the universal measure called Omega, developed by Keating and Shadwick (2002), which reflects all the properties statistics of the distribution of gains and losses, incorporating all moments of the distribution of returns, and not only the mean and variance as is done in the classical approach of portfolio composition in Markowitz (1952). It's demonstrated that using Omega measure for performance assessment and portfolio composition, is more advantageous than using the classic Mean-Variance approach.

The main objective of our research is to apply the Omega measure in portfolios containing Real Options in their Investment Projects.

Keywords: Portfolio, Real Options, Risk, Return, Omega Measure.

1. Introduction

In financial literature, it is well known the fact that investors always want to get highest returns on their investments, minimizing as possible the involved risk. Markowitz (1952) designed the foundations of the portfolio composition theory of investments. According to his theory, investors can determine all optimal portfolios, relating to risk and return, and to form a efficient frontier. The efficient frontier can be described as the best possible set of portfolios, that is, all portfolios have the minimum level of risk for a given level of return. Investors would focus on the selection of a portfolio on the efficient frontier and they would ignore others considered inferior.

Although the classical theory of Markowitz (1952) is considered easy for implementation and efficient in portfolio composition of assets, complications appear when it is taken into account uncertainty in the value of variables. In this case, a deterministic approach does not result longer valid, and it's necessary to do a probabilistic modeling of variables and employ simulation methods.

When the portfolio is formed by real assets, the problem is more complex, because of there isn't a history of returns and that doesn't let the calculation of an expected value or correlations among other assets. For example, a portfolio composed of investment projects, and we have to decide which investments perform in order to get at least a desired return by managers.

Moreover, in the portfolio of real assets is possible to introduce and model future investment decisions for each individual project, like the right time to invest, making an expansion, reduction of operations, abandonment, etc.. Bearing in mind the possibility of exercising these options (called real options for having its application in real assets), the modeling becomes more realistic, thus improving the attractiveness of projects.

Along with the composition of the portfolio, it is necessary to assess the risk. In literature there is a great amount of risk measures; the most popular risk measures are standard deviation and Value at Risk (VaR). There is also a measure called Expected Shortfall (ES) which is more informative than VaR, because it evaluates the expected mean loss in a confidence level. Accordingly, VaR responds to the question: "What is the minimum loss incurred by the portfolio in α% worst scenarios?". In turn, ES responds the question: "What is the mean loss incurred by the portfolio in α% worst scenarios?".

Recently, several authors have proposed measures of risk-return (known also as performance measures) more consistent with the expected distribution of gains observed in practice, that is, not normal distributions. Among them the measure Omega (Ω), introduced by Keating and Shadwick (2002), seeks to include every moment from the return distribution when assessing the risk of an asset.

This paper explores the characteristics of Omega measure, which is a relatively new and there are few studies in portfolio optimization using Omega. Similarly, it will be showed its application in optimization for composition a real asset portfolio, particularly investment projects with options, assessing their levels of risk and return, setting targets to be achieved.

The proposed methodology is intended to be easy to implement to any type of industry. Optimization techniques and Monte Carlo simulation are main tools in its application.

2. Theoretical Framework

2.1. The Portfolio Composition Model of Markowitz

Mathematically the risk can be treated as a random variable, and the first two moments of the probability distribution (mean and variance) are indicators that define the degree of risk exposure. It is relatively simple to study the risk of an asset under this approach. But in a portfolio with many assets, the complexity of measuring the risk is great due to the fact the probability distribution of the portfolio return may differ significantly from the probability distribution of individual assets.

For example, consider a universe of 'n' stocks. If r_j is the return from stock j (random variable) and x_j is the amount, in cash, to invest in stock j. Expected return from this portfolio is given by:

$$r(x_1,...,x_n) = E\left[\sum_{j=1}^{n} r_j x_j\right] = \sum_{j=1}^{n} E[r_j]x_j \tag{1}$$

where E [.] represents the expected value of the random variable. Furthermore, the standard deviation of return is:

$$\sigma(x_1,...,x_n) = \sqrt{E\left[\left\{\sum_{j=1}^{n} r_j x_j - E\left[\sum_{j=1}^{n} r_j x_j\right]\right\}^2\right]} \tag{2}$$

Markowitz used the variance of returns, as a risk measure. It's desired to get a portfolio with minimum risk, that is, with minimum variance subject to constraints of capital and minimum return. Thus the model can be written as the following optimization program:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j \\ \text{sujeito a} \quad & \sum_{j=1}^{n} R_j x_j \geq \rho M_0 \\ & \sum_{j=1}^{n} x_j = M_0 \\ & 0 \leq x_j \leq u_j, \qquad j = 1,...,n \end{aligned} \tag{3}$$

where M_0 is the capital available initially, $R_j = E[r_j]$, $\sigma_{ij} = E[(r_i - R_i)(r_j - R_j)]$ is the covariance between assets i and j, r_i or r_j represent individual returns from asset distribution i or j, ϱ is a parameter that represents the minimum return rate required by an investor, and u_j is the maximum amount of money that could be invested in j.

2.2. Real Options

Companies and financial institutions, over many years, have used traditional methods for project evaluation in order to review their investment decisions. Net Present Value (NPV) and Internal Rate of Return (IRR) are classic methods of evaluation. They argue that projects with NPV positive or IRR higher than the discount rate, these would be, in principle, investments to be made.

But a few years ago, those methods are being severely questioned. The main reason is that they do not deal properly with three important characteristics in investment decisions: 1) Irreversibility: that is, the fact that investment is a sunk cost, so investor is unable to recover it totally in case of regret, 2) Uncertainty about future profits from the investment, 3) Possibility of postponement of investment, which can bring benefits for new information about the environment. So, given these features, company could have flexibility to change the original terms of the project to save possible losses or generate additional earnings depending on the future scenario.

Real options theory can evaluate in a way more realistic flexibility in investment. It has an analogy with financial options: a call on an asset (present value of future earnings on investment) gives right but not obligation to buy it in the future, at a exercise price (initial cost of investment), in a maturity time (maximum time that project can be postponed). Option exercise (do investment) is irreversible, but firm has opportunity to preserve the value of its option (postpone investment) until market conditions become more favorable.

Real options increase the company value, due to flexibility that projects would have to adapt to future conditions, in response to market changes. Thus, it applies the following relationship:

Project Value = Project Value without option + Option Value (Calculated by NPV)

There are different types of Real Options, between them:

A) Simple Options

- Abandonment Option: If future conditions become unfavorable to the project, you can leave the business and sell the assets to a pre-established salvage value.
- Deferral Option: It is the option that determines the optimal time for invest, in order to generate highest profits.
- Contraction Option: This option gives the right to reduce a portion of investment capacity. Future value of assets is reduced, but it is received a money entry in the contraction year.
- Expansion Option: If project resulted be better than expected, this option gives right to expand the original project capacity. Thus, the underlying asset value increases, but we need to do prior an additional investment.

B) Compound Options

Compound options are a combination of simple options that can be performed simultaneously or in sequential order. For example, a firm that invests in Research and Development may need some initial time to obtain results of preliminary tests before doing the investment; then it may decide to expand, contract or abandon the project, according to information obtained by waiting. In most cases, there is always more than one option to exercise.

C) Switching Options

Switching options provide to the holder possibility of change between different types of resources, assets or technology. Also allow initiate and terminate operations, or enter and exit from a particular activity. This high flexibility over the project adds value to it, in case that value of some alternative becomes more profitable in the future.

D) Learning Options

Options described previously, it is considered that as time passes, uncertainties regarding asset value, and possibility of exercising the option or not, will be revealing. However, in many situations uncertainty does not resolve by itself. Efforts and investments are necessary to obtain more information about project conditions and reduce uncertainty. For example, in petroleum industry, when it decides invest more in geological research and discover the exact magnitude of reserves, or testing the market before scale sales, for a few cases.

2.3. Risk Measures

Below, a package of measures for traditional risk obtained from a series of gains or returns laugh, i = (1.2 ,..., m), and 'm' the total number of observations of return on active:

a) Standard Deviation:

$$DP = \sqrt{\frac{\sum_{i=1}^{m}(r_i - \bar{r})^2}{m}} \quad (4)$$

b) Semivariance:

$$SV = \sqrt{\frac{\sum_{i=1}^{m}\min\left[0;(r_i - \bar{r})\right]^2}{m}} \quad (5)$$

c) Downside Risk:

$$DR = \sqrt{\frac{\sum_{i=1}^{m}\min\left[0;(r_i - r_{min})\right]^2}{m}} \quad (6)$$

where r_{min} is the minimum required gain

d) Mean Absolute deviation:

$$DA = \frac{\sum_{i=1}^{m}\left|r_i - \bar{r}\right|}{m} \quad (7)$$

e) Mean Absolute Semideviation:

$$SDA = \frac{\sum_{i=1}^{m}\left|\min\left[0;(r_i - \bar{r})\right]\right|}{m} \quad (8)$$

f) Mean Absolute Downside Risk:

$$DRA = \frac{\sum_{i=1}^{m}\left|\min\left[0;(r_i - r_{min})\right]\right|}{m} \quad (9)$$

g) Value at Risk – VaR:

VaR methodology, developed by JP Morgan Bank (1996), started in order to quantify, systematically and simple, potential losses due to exposure to market risk, that is come from the volatility of market prices (exchange rate, interest rate, stock market, etc.). Since then, this methodology has been widely used in managing risk practice as total risk measure in project portfolios, being mentioned in various regulatory practices of international financial system.

VaR attempts to summarize in a single number the maximum expected loss in a certain period, whit a specific statistical confidence level. He evaluate the random variable representing gain (or loss).So, VaR (95%) indicates that there are 5 chances in 100 that injury is greater than that indicated by VaR, in a given period. It becomes a number of easy reading and understanding that depends on the term (N) and the degree (1-α)% of desired confidence. VaR = V can be read as: "We are (1 - α)% certain that we will not lose more than V monetary units in following N days". Statistically this statement is equivalent to:

$$\text{Prob}\,[r_j < \text{VaR}] = \alpha\% \quad (10)$$

VaR calculation is quite simple, since you know in detail the return distribution r_j, because VaR is, by definition, some quantile associated with a extreme percentile from portfolio distribution - usually α =1% or α=5%.

h) <u>Expected Shortfall</u>

Expected Shortfall (ES) is a measure which indicates the expected mean loss exceeding VaR, that is, it quantifies "how" great is the loss (risk), on average, that we are exposed in a specific portfolio, thus providing information about the tail of NPV distribution (this statistic is also known as VaR conditional, VaR in the tail). You can think ES as "how heavy" is the gain distribution tail in a portfolio.

So, while VaR responds the question "What is the minimum loss incurred by portfolio in α% worst scenarios?", ES responds the question "What is the mean loss incurred by portfolio in α% worst scenarios?".

Mathematically, you can set ES as a conditional expectation of portfolio losses higher than VaR.

$$ES = E\,[r_j / r_j < VaR] \qquad (11)$$

2.4. Portfolio Performance Assessment (Risk – Return)

The importance of using models for assessing performance of investments started with the principle of diversification proposed by Markowitz (1952) and his mean-variance, which argues that investors would prefer higher returns to the same risk level. Thus, metrics required for portfolio selection were based on the expected return and the standard deviation (risk) of returns. Later, a relation between risk and return was statistically formalized by Treynor (1965), Sharpe (1966) and Jensen (1968). They assume that returns are normally distributed and investors have a quadratic utility function.

Establishment of alternatives to calculate gain and risk is not a trivial task, and there are various measures to assess portfolio performance. Risk can be established as any of the measures listed above, that is, standard deviation, minimum expected loss or other measure that includes higher moments from the distribution. At this point, Figure 1 extracted from Keating and Shadwick (2002) is quite illustrative, as it indicates importance of higher moments for investment evaluation. Both distributions have the same mean (10) and standard deviation (152), but they differ in skewness, kurtosis and in all higher moments. However, according to some traditional performance indicators, like mean and standard deviation, both would be equivalent.

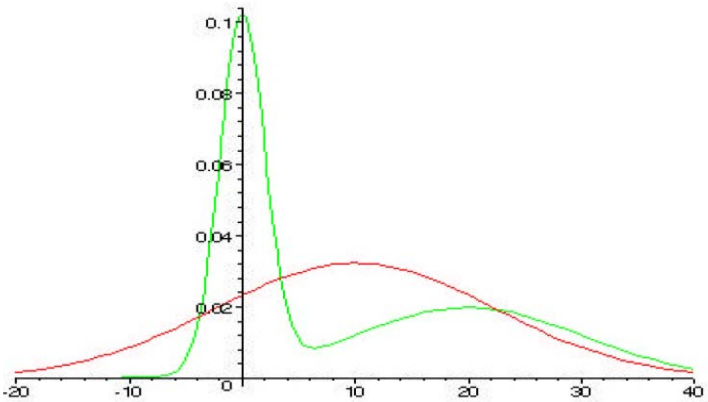

Figure 1 - Distributions with equal mean (10) and variance (152)

Best known ratios for evaluating portfolio performance are Sharpe (SR), Treynor (TR), Jensen (JR) and Sortino (SoR). Indicators developed by Jensen and Treynor evaluate performance taking into account the investment portfolio performance in relation to market performance. Sharpe ratio evaluates the investment performance taking into account only the portfolio behavior, while Sortino ratio uses Downside Risk concept to assess risks. Omega measure (Ω), proposed by Keating and Shadwick (2002), is considered more consistent because it can satisfactorily deal with gain distributions observed in practice (heavy tails and extreme values).

a) Sharpe Ratio:

Among statistics for performance evaluation, Sharpe Ratio (SR) is the best-known. Extremely acclaimed between academics and financial market practitioners, SR has been widely used in evaluation of investment funds. Formulated by William Sharpe (1966), SR is based on portfolio selection theory, pointing points on the capital market line, which represent optimum portfolios.

SR is defined in equation (17), where r_f is the risk free interest rate, and $E[R_p]$ e σ_p represent respectively expected return and volatility of portfolio.

$$SR=\frac{E[R_p]-r_f}{\sigma_p} \qquad (12)$$

Mean and variance theory of Markowitz determine optimum composition of portfolio on a risk-return space. It is easy to show that portfolios with highest SR are exactly optimum portfolios, considering normality in return distribution.

b) Sortino Ratio

According Duarte (2000), Sortino ratio (SoR) differs from Sharpe ratio, by tackling the risk concept called Downside Risk, which considers in calculation of variance just financial losses. Sortino perceived that standard deviation measures only the risk of failing to achieve a mean. However, the most important would be to capture the risk of not achieving the gain in relation to the goal.

Therefore, the Sortino ratio depends explicitly on the minimum acceptable return (MAR), for purposes of comparison between the analyzed portfolio or asset and that minimum.

Therefore, SoR is given by the following equation:

$$SoR=\frac{E[R_p]-r_{MAR}}{\sigma_{DR}} \qquad (13)$$

where,

$$\sigma_{DR}=\sqrt{\frac{\sum_{i=1}^{n}\left[\min(0;R_{p_i}-r_{MAR})^2\right]}{n}} \qquad (14)$$

3. A new performance measure: Omega Index (Ω)

Due to criticisms concerning to mean-variance approach proposed by Markowitz (1952), which is based on the assumption of normal distribution of gains, Keating and Shadwick (2002) present a universal performance measure called Omega, which reflects all statistic properties of gains distribution, incorporating all its moments, not only mean and variance.

Most performance indicators consider two main simplifications:

- Mean and variance completely describe the return distribution.
- Risk-return characteristics of a portfolio can be described only with mean of returns.

These simplifications are valid if it is assumed a normal distribution of returns, but it is generally accepted the empirical fact that returns on investments do not have a normal distribution. Thus, besides mean and variance, higher order moments would be required to better distribution description.

Omega measure (Ω) incorporates all moments of the distribution. It provides a complete description of the risk-return characteristics, so that results in a measure intuitively attractive and easily computable. Instead of estimating some individual moments, Omega measures total impact, which is certainly of interest of decision-makers.

In order to define Omega function (Ω), primarily, it's necessary to define exogenously the limit return (L). This divides probability distribution of returns in two areas: area of earnings, and area of losses. This limit varies by individual and by type of investment. Figure 2 illustrates the distribution of returns of an asset, which was established a limit L = 1,4.

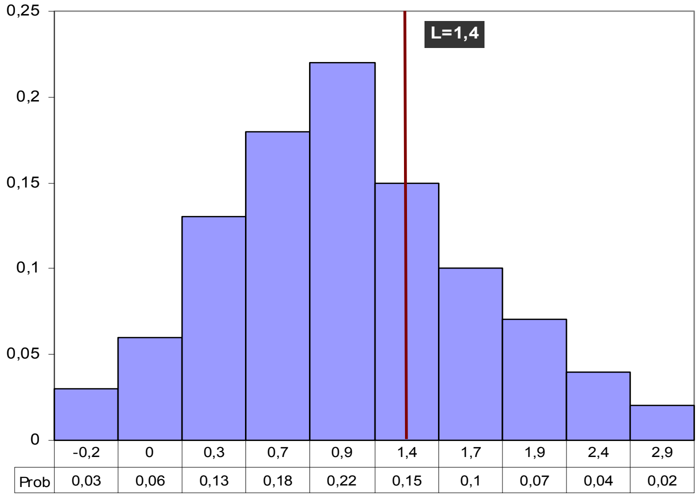

Figure 2 - Distribution of returns with limit L=1,4

Using the procedure described by Keating and Shadwick (2002), it is estimated Omega measure (Ω), through the cumulative distribution function, shown in Figure 3. Gains (g_i) and losses (l_i) may occur with some probability in gain areas ($r_i>L$) or loss areas ($r_i<L$).

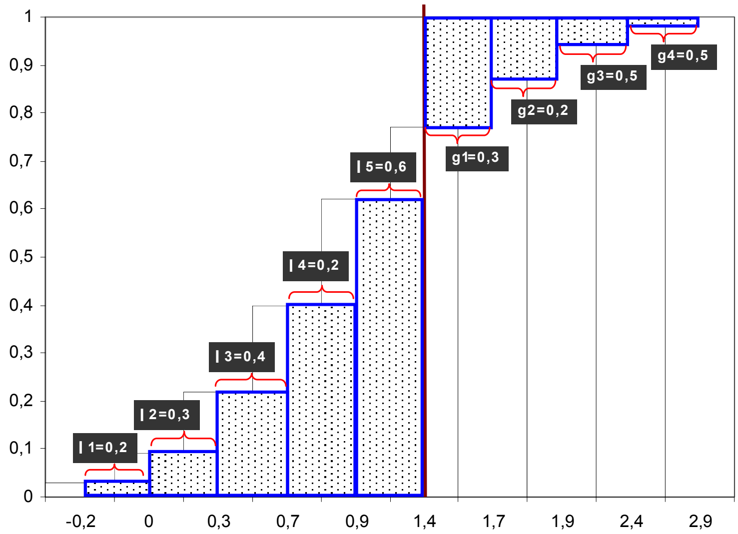

Figure 3 - By reducing intervals between returns to refine estimates of gains and losses relating to L =1,4

According to Figure 3, weighted total gain would be calculated:

r>=L	$g_i = r_{i+1}-r_i$	[1-F(r)]	g*[1-F(r)]
1,4	0,3	0,23	0,069
1,7	0,2	0,13	0,026
1,9	0,5	0,06	0,03
2,4	0,5	0,02	0,01
2,9			
Weighted Total Gain = $\sum g_i*F(r_i)$ =			**0,135**

and, weighted total loss would be:

r<L	$l_i = r_{i+1}-r_i$	F(r)	l*F(r)
-0,2	0,2	0,03	0,006
0	0,3	0,09	0,027
0,3	0,4	0,22	0,088
0,7	0,2	0,4	0,08
0,9	0,5	0,62	0,31
1,4			
Weighted Total Loss = $\sum l_i *F(r_i)$=			**0,511**

So, **Ω** = 0,135 / 0511 = **0,2642**

When probability distribution ceases to be discreet, it is said, a continuous density function, Figure 3 in the limit, when intervals become increasingly smaller, becomes Figure 4.

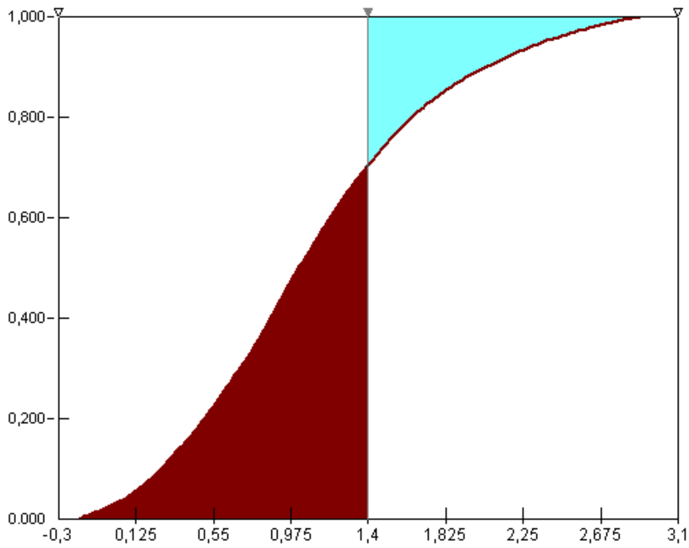

Figure 4 – Limit areas when profit and loss units tend to zero.

Considering a continuous density function, is defined:

- (a,b) = lower and upper limits, respectively, the range of returns distribution. Most of the time, a = -∞ e b=∞.
- $I_2(L)$ = weighted mean gains above a level L (upper area of Figure 4).
- $I_1(L)$ = weighted mean losses below a level L (lower area of Figure 4).

Performance measure Omega (**Ω**), in continue way, is defined by the following expression:

$$\Omega(L) = \frac{I_2}{I_1} = \frac{\int_L^b [1 - F(x)]dx}{\int_a^L F(x)} \quad (15)$$

Where:

F = Cumulative distribution function of gains
L = Minimum required level of gains
a = Minimum return
b = Maximum return

Function **Ω** (L) compares returns from different assets and rank them in relation to magnitude of their Omegas. A **Ω** (L) = 1, indicate that weighted gains equal to weighted losses. It's always desirable **Ω** (L)> 1.

Kazemi, Schneeweis and Gupta (2003) present Omega measure in a more intuitive way, demonstrating that equation (15) can be written as a division of two expected values. In equation (16), numerator is the expected value of gain excess (x-L) conditional to positive results (where 'x' is some return from the distribution), and denominator is the expected value of losses (L-x) conditional to negative results. Thus, these authors develop an analogy with the options theory and write the numerator like the maximum expected value between (x-L) and zero, similar to payoff of a call without discount. Denominator, therefore, would be the payoff of a put without discount.

$$\Omega(L) = \frac{\int_L^b [1 - F(x)]dx}{\int_a^L F(x)} = \frac{\int_L^b (x - L)f(x)dx}{\int_a^L (L - x)f(x)dx} = \frac{e^{-r_f}\, E[\max(x - L; 0)]}{e^{-r_f}\, E[\max(L - x; 0)]} = \frac{Call(L)}{Put(L)} \quad (16)$$

4. Optimization with Omega Measure

4.1. Optimization Program

In order to optimize the portfolio with Omega, Ick and Nowak (2006) follows the program:

$$\begin{aligned} \max_P \Omega(L) &= \frac{EC_P(L)}{ES_P(L)} \\ \text{sujeto a}: \sum_{j=1}^{n} w_j &= 1 \\ 0 \le w_j &\le 1 \end{aligned} \quad (16)$$

Where:

$ES_P(L) = E[\max(L - Rp_i; 0)]$ = Expected Shortfall Risk for portfolio P

$EC_P(L) = E[\max(Rp_i - L; 0)]$ = Excess Chance for portfolio P

$Rp_i = \sum_{j=1}^{n} w_j R_{ij}$ = Portfolio Return on period i

w_j = Part of portfolio invested in asset j

4.2. Application Example

It's optimized by Omega a portfolio composed of four assets, and then we compared with the optimization applying Markowitz model, with the purpose of better understanding the advantages between one and another methodology.

In Table 1, it's showed four assets of the portfolio, and main statistics. For each asset, it is being considered 500 observation periods, which it generates its return distribution.

Table 1 – Statistic Properties from Returns Historical Data of Four Assets

	Mean	Variance	Skewness	Kurtosis	Minimum Value	Maximum Value	JB Test
Asset X	0,15	0,25	-0,90	3,42	-1,69	0,84	71,63
Asset Y	0,20	1,44	1,96	8,38	-1,00	7,41	925,06
Asset Z	0,25	1,00	1,90	8,17	-0,80	6,39	856,89
Asset W	0,05	0,16	-1,42	5,51	-2,02	0,53	299,31

Four assets showed in Table 1 are very far from normal distribution, which is indicated by Jarque-Bera test. Observe that all values are much greater than 5.99 which is the value of chi-square with two degrees of freedom and probability of 95%, which is taken as a benchmark for testing normality hypothesis. Graphs of these distributions are shown in Figure 5.

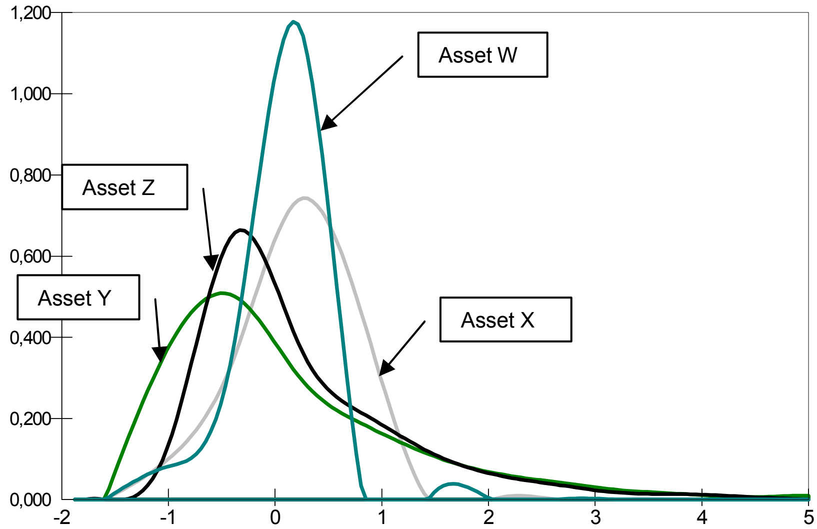

Figure 5 – Return Probability Distributions of Four Assets

Assets X and W have negative skewness and a slight excess of kurtosis, in contrast, assets Y and Z have positive skewness and a considerable excess of kurtosis and they have greater dispersion of their returns (high variance.)

Moreover, assets are correlated, which is shown in Table 2. Observe that assets X and Z, which have the same skewness sign, show a positive correlation between them and negatively with the rest of assets. A similar situation happens among assets Y and Z.

Table 2 – Correlation Coefficients Matrix

	X	Y	Z	W
X	1	-0,42	-0,46	0,44
Y	-0,42	1	0,46	-0,45
Z	-0,46	0,46	1	-0,48
W	0,44	-0,45	-0,48	1

Results from optimization of portfolio composition using Markowitz and using Omega measure with different levels of limit returns L, are shown in Table 3. Optimization through Mean-Variance methodology (Markowitz) does not take into account other distribution moments, and hence, composition percentages vary significantly in almost all assets, compared to optimization by Omega.

Table 3 –Portfolio Composition

	Methodology			
	Markowitz	Omega (L=0%)	Omega (L=3%)	Omega (L=15%)
w1 = %Asset X	27,12%	35,10%	44,54%	47,90%
w2 = % Asset Y	9,46%	10,10%	10,84%	2,74%
w3 = % Asset Z	15,34%	28,51%	28,95%	49,37%
w4 = % Asset W	48,08%	26,30%	15,66%	0,00%

In Markowitz, the goal is to achieve the minimum variance, so, asset W was chosen in greater proportion due to which presents the smallest variance in relation to others. When it's maximized the Omega measure, assets X and Z gain greater representation in the portfolio, because in these assets is more likely to obtain returns above the limit value L. Figure 6 shows the distribution shape of optimal portfolios, depending on the methodology used.

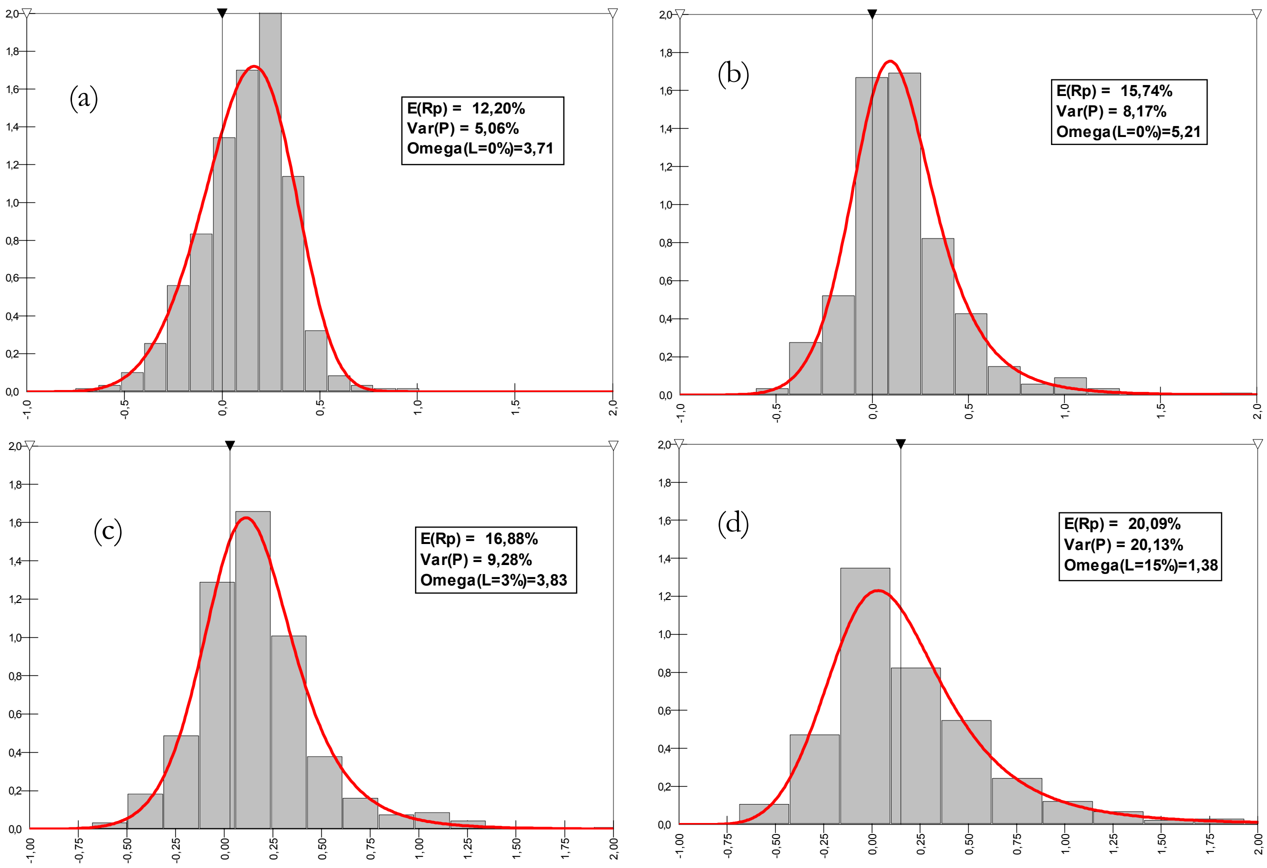

Figure 6 – Returns probability distribution of the optimized portfolio 'P'. (a) Mean-Variance (Markowitz) Optimization, (b) (c) and (d) Optimization by Omega measure with different levels of L.

Table 4 presents main statistics of the optimized portfolios using both methodologies (Markowitz and Omega).

Table 4 – Main Statistics of Optimized Portfolios

	Methodology			
	Markowitz	Omega (L=0%)	Omega (L=3%)	Omega (L=15%)
Mean	12,20%	15,74%	16,88%	20,09%
Variance	5,06%	8,16%	9,28%	20,13%
Skewness	-0,34	1,24	1,12	1,77
kurtosis	3,92	7,37	7,15	9,07
JB Test	27,06	524,85	462,03	1029,53
Minimum Value	-75,15%	-60,47%	-68,03%	-68,75%
Maximum Value	101,18%	198,08%	208,43%	324,49%
EC	16,70%	19,47%	18,80%	18,41%
ES	4,50%	3,74%	4,91%	13,32%
Omega	3,71*	5,21	3,83	1,38

* Omega is calculated with L=0

It's showed in Table 4, through Jarque Bera test, that all distributions of optimized portfolios aren't normality distributed. It also is appreciated that Markowitz methodology always finds the portfolio with the smallest variance, but its index Omega (L = 0) compared with the Omega index from the optimized portfolio with this measure (with the same limit return, L = 0) is considerably smaller (3,71 vs. 5,21). Ratio of weighted earnings vs. weighted losses is considered more significant when portfolio composition is optimized by Omega. Non-normal distributions are responsible by differences in results between a methodology and another, and Omega optimization is better for to deal with non-normal distributions.

Graphically, in Figure 7 are compared on the same scale (ES vs. EC) the efficient frontiers of Mean–Variance optimization and Omega optimization, with L = 0.

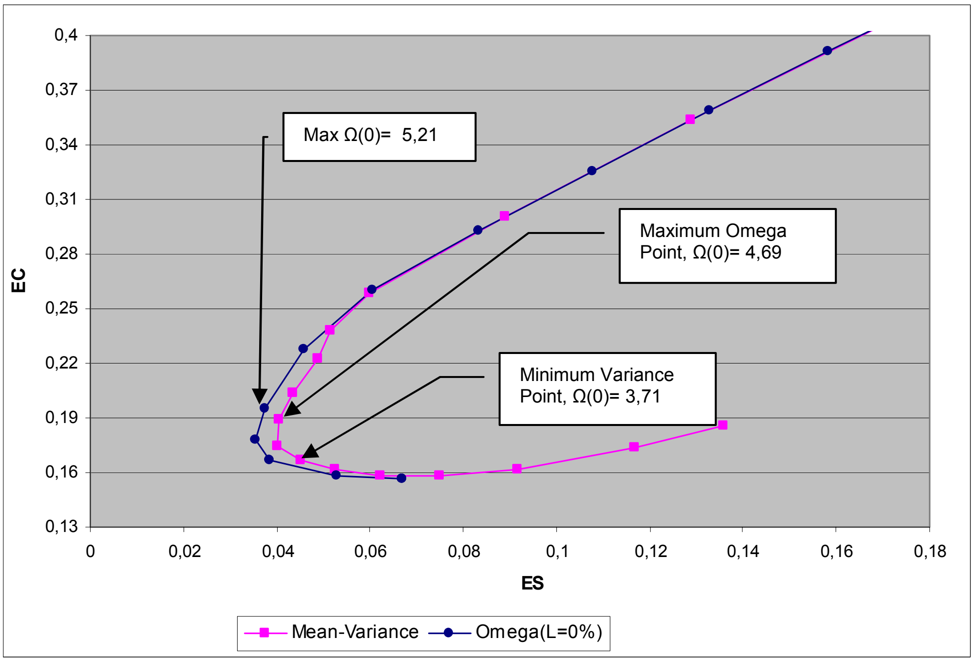

Figure 7 – Efficient frontiers in the scale ES vs. EC

It is observed in Figure 7 that the efficient frontier calculated by Omega measure is superior to the frontier calculated by Markowitz (Mean-Variance), especially in points with lower expected shortfall (ES). The maximum Omega index from Markowitz frontier is less than the maximum Omega index of the other frontier (omega optimization), in the same way, the Omega index of minimum variance is much lower. Thus, it is showed that for an expected shortfall (ES) level, through Omega measure optimization in found higher excess chance (EC) values.

By making the graph of both frontiers on the scale Variance – Mean, you get the graph shown in Figure 8.

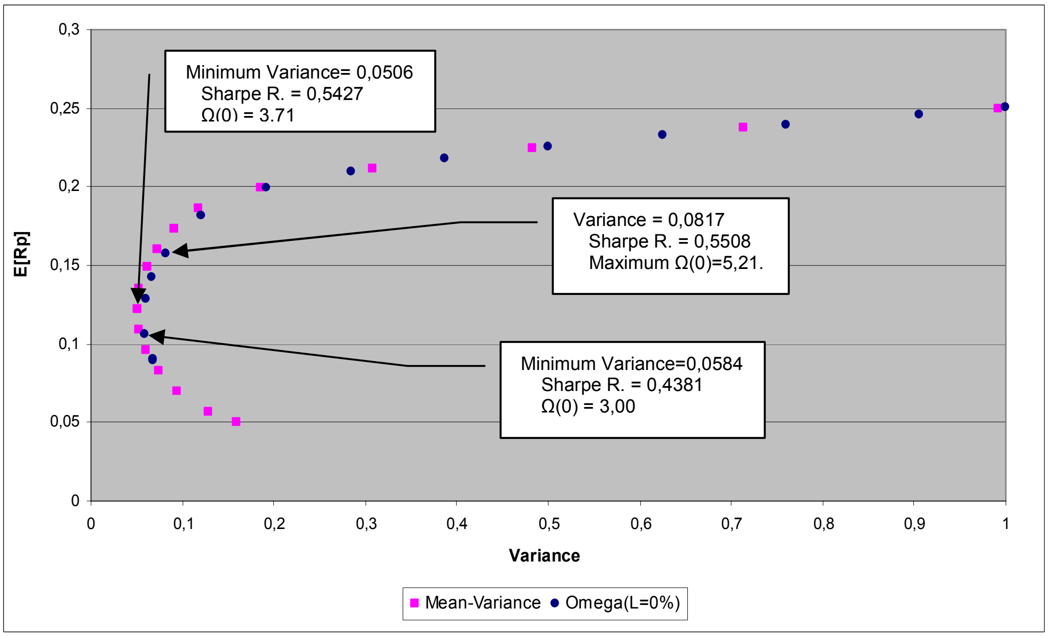

Figure 8 – Efficient frontiers in the scale Variance vs. Portfolio Mean

Given the assumption of normality in Markowitz methodology, its efficient frontier is slightly higher (Figure 8) than efficient frontier obtained by maximizing Omega (L = 0). If assets had a normal distribution, both frontiers would be equal, providing same results, but this difference appears because the frontier calculated by Omega, takes into account the shape of the portfolio returns distribution, which is not normal. In the frontier calculated by Markowitz, is assumed normal distribution for portfolio returns, so, it is always found the smallest variance. Nevertheless, both frontiers do not deviate much among them, and as the variance increases the curves will become equal.

Furthermore, for purposes of measuring performance using the Sharpe classic measure, the Markowitz frontier at the point of minimum variance, calculated a value for this index slightly lower than the Sharpe ratio obtained at the point of maximum Omega, showed on the efficient frontier calculated by this measure. But it is known that Sharpe ratio assume the normality of returns, therefore, that is not the most appropriate in other situations, as in our example.

4.3. Analysis of results and comparisons among both methodologies

Through the developed example in previous section, it is observed:

- When there is normality in the returns distribution of assets, and therefore normality in portfolio returns, optimization by Markowitz provides the same results as optimization through maximizing the Omega measure.
- When there is no normality, should be taken into account all the distribution moments

(skewness, kurtosis, heavy tails, extreme values, etc.), and in this way, to measure the real impact on expected gains and losses. All this can be incorporated through the Omega measure.

- The Figure 7 allows appreciate the difference between an efficient frontier optimized by Omega, and another using Markowitz. Using Omega is found a better frontier, which takes into account the actual format of the portfolio returns distribution and not just the first two distribution moments.
- In Table 4 is appreciated that if the limit return (L) increases, the Omega measure decreases, because as L moves to the right, the EC area will become smaller and ES area will become bigger. Thus, Omega index value tends to decrease. Doing L=0, it will provide the best Omega measure, which means that no losses are accepted.
- Efficient frontiers from Omega with three limit return levels are illustrated in Figure 9. Observe that the frontier with better performance is obtained when L = 0.

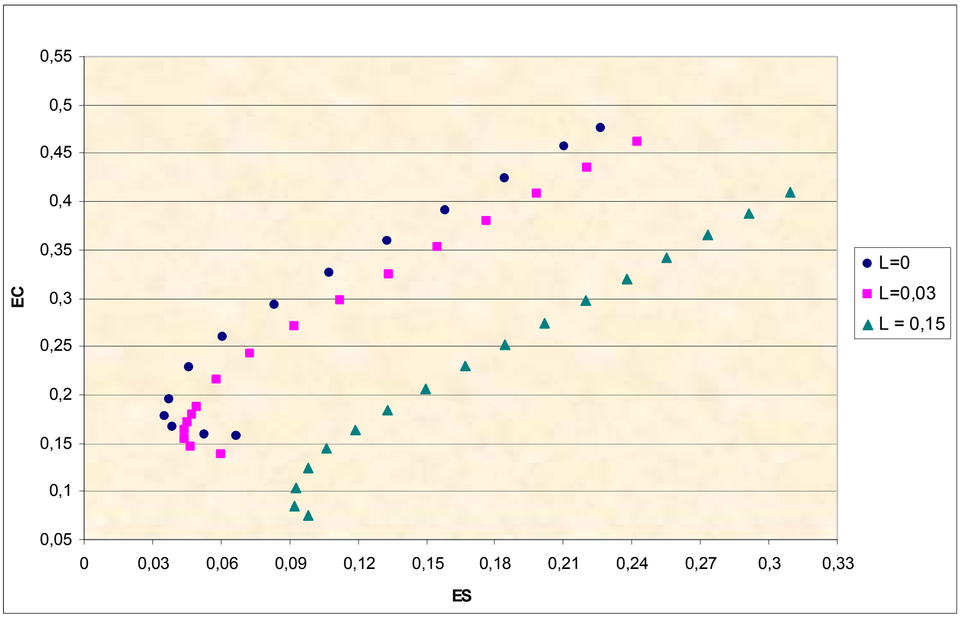

Figure 9 – Efficient frontiers for three limit return levels (L)

5. Optimization for a Real Options Portfolio

Next, it is described a methodology that allows optimizing portfolio composition for real assets, particularly projects with investment options. This optimization attends objectives for risk levels and desired returns, using the Omega measure (Ω). The proposed methodology consists of three steps:

Step 1:

Portfolio conformation of real assets becomes more complicated than in the case of tradable assets on the market. This is because there is no usually a history of similar projects in which one might assume a certain behavior of their return and/or volatility. And even, if there is a similar project, future context certainly will be different, doing unique each investment.

In order to estimate a returns distribution, Monte Carlo simulation technique is the most appropriate. It can model several future conditions for market variables that affect the project. Brandão et al. (2005) estimate the market value of a project, through Monte Carlo simulation, based on the MAD assumption (Market Asset Disclaimer). This assumption considers that the project value without options would be its market value, and hence, the project would be considered as a negotiable asset. It is proposed to use this approach to value projects from the portfolio, which it's previously required to define variables of the project that will be considered stochastic, for example, the product price and/or the production levels. Stochastic modeling with possible correlations between variables is highly recommended.

Step 2:

Second step is to insert options to the projects. This makes the value of them increase. The model considers possibility to exercise those options when market conditions are favorable. Modeling options by Monte Carlo simulation is very useful in that sense, doing parity with treatment given to American options. In those, exercise can be done within a time until the expiration.

Threshold curve concept will be very useful for identifying the right time for exercising the option. When project value reaches a value equal to some on the curve, it indicates that at that moment should be made the investment. If market conditions are unfavorable, it can be considered option to leave, or making a temporary cessation of operations.

Favorable or unfavorable conditions can also lead to decide expansion or reduction in operations, prior knowledge of the maximum production capacity and minimum capacity of operation.

Step 3:

Once modeled projects as if they were negotiable assets, now the task consist in forming a portfolio with several goals, as desirables risk and return levels. It is understood by risk the Expected Shortfall (ES), which is the denominator of Omega measure (**Ω**), and return is the Excess Chance (EC), which is the numerator of Omega measure (**Ω**). Both Omega components can be expressed in monetary units, or in return percentages.

As in Markowitz model, a goal could be to minimizing the risk for a given return level. Translating these concepts to Omega measure (prior stipulation of the limit return (L)), it is set up a level for EC, and on its efficient frontier, we find the lowest ES.

There are possibilities for manager choose a goal that he/she would like to achieve, and the methodology will allow him/her to know decisions about portfolio composition and options to be considered. It is very important a high flexibility, both in modeling of variables such as results to be obtained.

Finally, we desire to create basis for a project assessment, in which it's possible to combine in dynamic way, goals for risk, return and performance levels, taking advantage of new academic researches. This methodology can be easily adapted to any type of industry.

6. Conclusions and Final Considerations

Nowadays, companies face up to high degree of uncertainty regarding future performance of their investments. With market indicators changing constantly and vertiginous emergences of new enterprises, forecast based on the past is increasingly difficult to be justified, especially when they include market variables, on which we have no control.

Therefore, portfolio composition of investment projects must learn to deal with this dynamism, and be prepared to change its structure in a flexible way, depending on changes that may occur in the environment.

Monte Carlo simulation, aims to facilitate modeling of numerous scenarios for variables. For example, you can adopt stochastic modeling for sale price of products, production costs, production levels, etc. Similarly, inclusion of real options in projects is done easily by simulation. Over project life is modeled various scenarios and can be identified appropriate moments to exercise real options.

Doing an appropriate analysis for risk, returns and performance in portfolio of real assets, it is very crucial in making decisions. Flexibility in techniques and/or models is favorable in order to improve ability of the company for reaction.

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