The Ethanol-Gas Flex Fuel car: What is the option value of choosing your own Fuel?

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Abstract

Renewable energy sources are becoming more important as the world's supply of fossil fuels decrease and also due to environmental concerns. Since 2003, when the ethanol-gasoline *flex fuel* car became commercially available in Brazil, the growth of this market has been significant, to the point where currently 40% of the fuel consumption in Brazil is from renewable biofuels. This has been made possible due to the success of the flex fuel car, which can run on ethanol, gasoline, or any mix of these in the same fuel tank, and which is sold at a premium over the non flex models,

Flex fuel cars, on the other hand, provide the owner with the flexibility to choose fuels at each refueling stop. Given the uncertainty on future prices of ethanol and gas, this option adds value to the owner since he can always opt for the cheaper fuel whenever he fills up his car. We use the Real Options method to analyze the value of the flex fuel option assuming both a Geometric Brownian Motion and Mean Reverting diffusion processes and compare the results for both methods. We conclude that the flex option value is significant using either method and twice as high as flex premium charged by the car manufacturers, which help explain the success that this type of automobile has enjoyed in Brazil since 2003. Our results also indicate that consumers should be wiling to purchase flex fuel cars even if manufactures increase the flex premium.

Key Words: Finance; Real Options; *Flex fuel* car

1 Introduction

The search for renewable energy alternatives to fossil fuels has been growing rapidly in recent years. Of these, biomass based fuels are becoming increasingly popular, specially in Brazil, where a combination of favorable climate and large scale availability of arable land and water has turned the country into a major producer of agricultural commodities from which these fuels can be produced. The first large scale attempt to use renewable energy fuels in Brazil was a government sponsored program (PROALCOOL) to produce ethanol from sugar cane in the late 1970s which met with considerable success, such that by 1986, 73% of all vehicles produced in that year ran exclusively on ethanol fuel (E100). Despite its achievement, the program depended on high oil prices and government subsidies to farmers, and when these conditions ceased to exist, consumers began experiencing fuel shortages and public confidence in the program collapsed. As a result, by the early 1990s E100 car sales had all but ceased and the program was discontinued.

Although the PROALCOOL program eventually failed, the ethanol engine technology and the vast ethanol distribution network it created set the stage for the introduction of the flex fuel car almost three decades later. The development of a mixed fuel engine which could run on any proportion of ethanol and gas in the late 1990s, known as the flex fuel engine, helped dispel the concerns about ethanol shortages, and the rise of oil prices in recent years have made this alternative economically feasible again for the consumer. In only five years since its introduction to the market in 2003, the flex fuel car has completely dominated sales of new vehicles with a market share of 68% in 2007.

From the perspective of the car owner, the advantage of *flex fuel* technology over the gas engine is that the owner has the option to choose the cheapest fuel each he must fill the tank. When the flex fuel car was first introduced in the market, this option was provided free of charge by the manufacturers as a way of attracting customers to a new technology, still unknown to the public, and flex fuel cars sold for roughly the same price as gas cars. Beginning in 2006, however, manufacturers began to charge a premium for this flexibility, selling *flex fuel* cars at a price approximately 5% higher than the non flex models, which were then discontinued.

The analysis of the flexibility the *flex fuel* car is a real options problem which cannot be modeled by traditional valuation methods such as discounted cash flow methods. The investment is irreversible, since is partially lost in case the consumer decides to resell his car, since the resale price is generally lower than the dealer price. There is uncertainty about the future behavior of fuel prices, and finally, there is flexibility to choose the fuel which has a better cost-benefit relation each time the car needs to be filled up. These three conditions require that option pricing methods when valuing the advantages of a flex fuel car. The option to choose fuels is a classic switch option, where there is flexibility to switch between gasoline and ethanol. Gonçalves et al (2006) and Bastian-Pinto and Brandão (2007) analyzed the flexibility of a sugar cane plant as a switch option, where the production of the plant can be converted either into either sugar or ethanol depending on which alternative yields greater cash flows. Another interesting example of switch option analysis is presented in Kulatilaka (1993), where he values a dual-fuel industrial boiler, by modeling the switch option as a call on the cheapest fuel price. The authors have not found in the literature references to the analysis of the valuation of the *flex fuel* car option, probably due to the recentness of this innovation.

We analyze the value of the flexibility provided by a *flex fuel* automobile from the owner's point of view, in order to compare it with the premium charged by the automobile manufacturers for this technology. We model the problem using the real options method and solve by simulation, where the uncertainty in gasoline and ethanol prices are modeled both as correlated geometric Brownian motion and mean reverting stochastic processes. We consider the flexibility available to the owner of the flex car to choose the cheapest fuel alternative at each refueling time and model this as a series of switch options over the life of the asset.

This paper is organized as follows. In the next section we present the historical background on the evolution of flex fuel technology and the use of ethanol fuel in Brazil. In section three we discuss the stochastic processes for ethanol and gas and present a real options model of the flex fuel car. In section four we analyze the results obtained and in the following section we conclude.

2 Background

With a land area equivalent to that of the United States, transportation of goods and passengers in Brazil is highly concentrated on roads and highways, which makes it vulnerable to uncertainty in fuel prices. As a result, the oil crisis of 1973 had a strong impact on the country's trade deficit and within one year expenditures with fuel imports more than tripled (UNICA, 2004), since local oil production covered only 20% of the country's consumption at the time.

In response to this problem, in 1975 the Brazilian government created incentives for the production of sugar cane based ethanol known as the PROÁLCOOL program in order to reduce the country's dependence on imported oil. Initially the program was limited to the addition of anhydrous ethanol to gasoline (E22), but after the second oil crisis in 1979, production of ethanol fueled (E100) cars began and the availability of ethanol in pump stations became widespread. This was aided by the fact that as an additional incentive to reduce imported oil consumption, gas stations at the time were only allowed to sell gas on weekdays, while there were no restriction on ethanol sales. These incentives helped create a strong market for E100 vehicles which by 1986 accounted for 73% of all vehicles produced in the country.

By 1900, however, oil prices reached historical lows while on the other hand, high international market prices for sugar stimulated producers to direct their sugar cane processing capacity to the production of sugar for export. In this case, producers exercised their option to switch production output by choosing to produce sugar rather than supplying the internal market with ethanol fuel, since both are derived from the same biomass, sugarcane. Ethanol shortages at pump stations became increasingly common, which lead to the discredit of E100 car in the eyes of the consumer. As a result, ethanol car sales decreased sharply and mass production of the vehicles came to a halt, becoming only available under special order, although by the end of the 1990's a fleet of little over 4 million E100 vehicles was still in service.

Meanwhile, in the United States, the Alternative Automobile Fuel Act of 1988 stimulated the development of bi-fuel vehicles, which use a fuel known as E85, indicating a mixture of 85% ethanol and 15% gasoline. This limit was set so that engines would not have problems starting in extreme weather conditions, which are common in many regions of the USA. General Motors was the first company to introduce the bi-fuel technology to the North

American market in 1992, and after that, other manufacturers also began to make available similar products. In Brazil, research was underway to develop a mixed fuel engine which could run on any proportion of ethanol and gas, known as the flex fuel engine, and by the end of the decade this technology was already available to the major automobile manufacturers.

The difference between flex fuel and bi-fuel vehicles is that the bi-fuel engine works similarly to the conventional gasoline engine, accepting only a fixed proportion of ethanol and gasoline. On the other hand, flex fuel cars have a sensor located in the exhaust of combustion gases which detects which proportion, or mix of ethanol-gasoline is being used, and informs this to a computerized control center in the engine. With this information, the control center can adjust and optimize engine performance to the fuel mix in use by changing the ignition point, fuel injection time and the opening and closing of valves. The last fuel mix is memorized and no further adjustments are necessary till the next fueling alters the ethanol/gas proportion. In addition, a gas-only cold starting process is used if the fuel tank contains more than 80% ethanol and the temperature is bellow 20ºC (68ºF). The use of this technology allows any proportion of ethanol/gasoline to be used ranging from E22 (gasoline has 22% ethanol added in which is used for anti-knocking purposes) all the way to E100, and not only a fixed one as the E85 in the United States.

With the reduction in ethanol prices due to productivity gains and the rise of oil prices, mass production and sales of flex fuel vehicles began in 2003, with an initial production of 39,853 vehicles in that year. Production increased to 1,391,636 vehicles in 2006 and almost 2 million in 2007, representing 68% of the 2.9 million vehicles produced by the industry in that year (ANFAVEA, 2007). According to the Brazilian Agribusiness Association (ABAG), flex fuel car production is expected to stabilize at 75 % of all new vehicles. The success of the *flex fuel* cars in the Brazilian market is unrivaled in the world, and has been an example of how renewable energy sources can substituted fossil fuels while reducing the damage caused by gas emissions in the environment. Part of this success is due to the competitive advantages of Brazilian agriculture and the higher efficiency of the production of ethanol from sugar cane compared to other feedstock, such as corn or beet based ethanol. Table 1 shows the distribution of the Brazilian production of vehicles by type of fuel.

Year	Automobiles				
	Gas	Ethanol	Flex fuel	Diesel	Total
2000	1,471,050	10,106	0	115,726	1,596,882
2001	1,615,476	19,032	0	82,014	1,716,522
2002	1,576,418	56,594	0	67,134	1,700,146
2003	1,561,283	34,919	49,264	76,375	1,721,841

2004	1,682,167	51,012	332,507	115,445	2,181,131
2005	1,334,693	28,222	878,144	134,083	2,375,142
2006	977,134	775	1,391,636	241,489	2,611,034
2007	787,628	3	1,888,557	265,035	2,941,223

ANFAVEA - Anuário da Indústria Automobilística Brasileira

Table 1: Vehicle production by type of fuel - Brazil

3 Modeling and application

Although ethanol has a lower pump price than gasoline, this difference is not always advantageous to the consumer, since ethanol has a lower energy yield (in kilometers per liter) than gasoline. CEPEA-USO (Centro de Estudos Avançados em Economia Aplicada) recommends drivers not to use ethanol if its price is greater than 70% of the price of gasoline. In this article we assume that the flex fuel car owner always makes the optimum decision of choosing the most economic fuel each time he fills up his car, considering the 70% ethanol to gasoline yield rate. Therefore, ethanol will never be chosen whenever its price is greater than 70% of the price of gas.

In order to determine the value of the switch option that exists in the flex fuel car, we assume that the car is fueled once a month over a period of ten years, or 120 months. Although there are models with larger engines, most flex fuel cars in Brazil come with smaller 1.0 liter (61 cu in) engines that offer high mileage per gallon (MPG), so we conservatively assume that monthly consumption will he 100 liters (27.8 gallons) of gas equivalent per month. This way, if the owner adopts only ethanol in any particular month, he will purchase 100/0.70 liters. The risk free rate, based on the Brazilian long term government bonds rate (TJLP) from the National Development Bank (BNDES) is 6.85% per year in real terms, corresponding to a monthly rate of 0.55%.

The parameters required to model the stochastic behavior of the prices of ethanol and gasoline were derived from historical series of monthly prices from July of 2001 to October of 2006. These series were deflated by the IGP-M price index, which is the most widely used Brazilian inflation index. Figure 1 displays the plot of the series of deflated ethanol and gasoline prices, as well as ethanol prices adjusted by its yield ratio, that is, multiplied by: 1.4286 = 1/0.70.

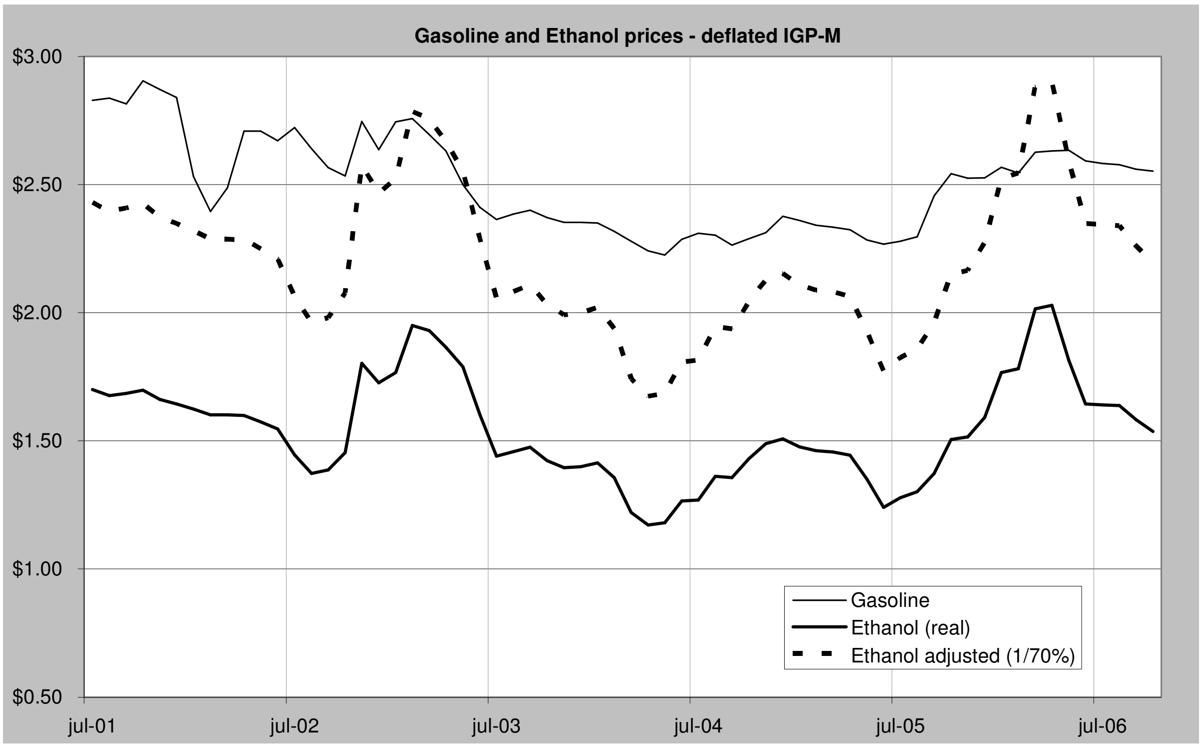

Figure 1: Deflated prices of gasoline, ethanol and ethanol adjusted (1/0.70)

By simple observation of the price series, it is clear that there were only two short periods within this time frame where the price of gasoline was more attractive than that of the adjusted ethanol, which helps explain the appeal and popularity of flex fuel cars as a lower costs alternative. But the high volatility of the later makes that the possibility of choosing the fuel relevant for a financial valuation with real options methodology.

We assume that at $t = 0$, the relative prices of gasoline and ethanol are such that the car owner is indifferent between these fuels. Thus it is assumed that the initial price for gasoline is R$ 2.50, and the price of ethanol, at the same instant, R$ 1.75, which corresponds exactly to 70% of that of gasoline. Therefore at $t = 0$, the owner would incur in the same expense, whether using gasoline or ethanol. It is assumed also that the consumption of ethanol will be: 1/0.7 = 1.4286 times greater than that of gasoline, so when computing the number of fuel tanks used with ethanol, it will be multiplied by 1.4286. The 2008 exchange rate is R$1.75 per US$ Dollar.

The initial cash flows expenditures for both fuels are:

- Expenditure with gasoline: number of fuel tanks used per month *x* tank capacity in liters *x* price of gasoline in R$/liter. Therefore the initial monthly expense with gasoline is R$ 250.00 (2.5 x 40 x R$ 2.50 = R$ 250.00).

- Expenditure with ethanol: number of fuel tanks used per month *x* tank capacity in liters *x* price of ethanol in R$/liter / ethanol efficiency factor. Therefore the initial monthly expense with ethanol is R$ 250.00 (2.5 / 0.7 x 40 x R$ 1.75 = R$ 250.00).

The fuelling decision is always a corner solution, since every time one fuel prevails economically over the other it will be optimal to fill 100% of the tank with that type of fuel. The real option valuation problem can then be solved with a Monte Carlo simulation, considering that the fuel switch option can be modeled as a bundle of European type options since the fueling decision at month *i* is totally independent from the fueling decision at any other month *j*, $\forall(i, j) \in [1{,}120], i \neq j$. Therefore we initially consider a cash flow expenditure for 120 months, assuming the fueling is done solely with gasoline, and similar one, assuming fueling only with ethanol.

The cash flows expenditures are determined as follows:

- Expense cash flow with gasoline in t:

  $CFG_t = G_t$ x 2.5 (tanks) x 40 (liter/tank) = G_t x 100 (1)

- Expense cash flow with ethanol in t:

  $CFE_t = E_t$ x 2.5 (tanks) / 0.7 (ethanol yield) x 40 (liter/tank) = E_t x 142,86 (2)

After determining these two cash flows, a third cash flow is defined by the exercising the option to choose the lowest cost alternative between these two each month:

Final Cash Flow_t = Minimum (CFG_t ; CFE_t) i

To model the switch option for simulation purposes, the optimized cash flows expenditures are then discounted at the risk free rate *r* in order to determine the present value of the total fuel expense during the life time of the flex fuel car (Equation (3)).

$$\text{PV of Total Cost} = \sum_{i=1}^{120} \frac{\text{minimum}\left(\text{CFG}_i \text{ ; } \text{CFE}_i\right)}{(1+r)^i} \quad (3)$$

3.1 Stochastic processes of fuel prices

In order to perform the Monte Carlo simulation we model the prices of gasoline G_t and ethanol E_t stochastically. For this, we first test the available price series for unit roots by modeling the price S_t as $\ln[S_t] = a + b \ln[S_t] + \varepsilon_t$, which can also be written as show in Equation (4):

$$\ln[S_t] - \ln[S_t] = a + (b - 1)\ln[S_t] + \varepsilon_t \quad (4)$$

where ε_t i.i.d ~ Normal $(0, \sigma^2/N)$.

Substituting S_t in equation (4) by the series G_t and E_t, and running simple linear regressions, we can test for unit roots, comparing the *t* statistics for the (*b*-1) coefficients obtained, with the critical values for Dickey-Fuller test (Wooldridge, 2000, p. 580). We are in fact testing if coefficient *b*, is statistically equal to 1, as the null hypothesis. If we fail to reject this, then we will have evidence that the prices series considered follow a random walk and can be modeled as a Geometric Brownian Motion (GBM) for which parameterization and discretization is well known. In the case of $0 < b < 1$, we will have indication of a stable auto regressive process for the series S_t, or, of mean reversion.

Running the regression described by equation (4) on both series G_t and E_t, respectively in Equations (5) and (6), we get the results in Table 2:

$$\log(G_t/G_{t-1}) = a_G + (b_G - 1)\log(G_{t-1}) + \varepsilon_{Gt} \quad (5)$$

$$\log(E_t/E_{t-1}) = a_E + (b_E - 1)\log(E_{t-1}) + \varepsilon_{Et} \quad (6)$$

	Gasoline	**Ethanol**
a	0.0913	0.0782
b-1	-0.1015	-0.1020
b	0.8985	0.8980
σ_ε	0.0291	0.0564
t statistic for (b-1)	-2.055	-1.863

Table 2 – Regression results

Both regressions on the series of prices for gasoline and ethanol are plotted in Figure 2.

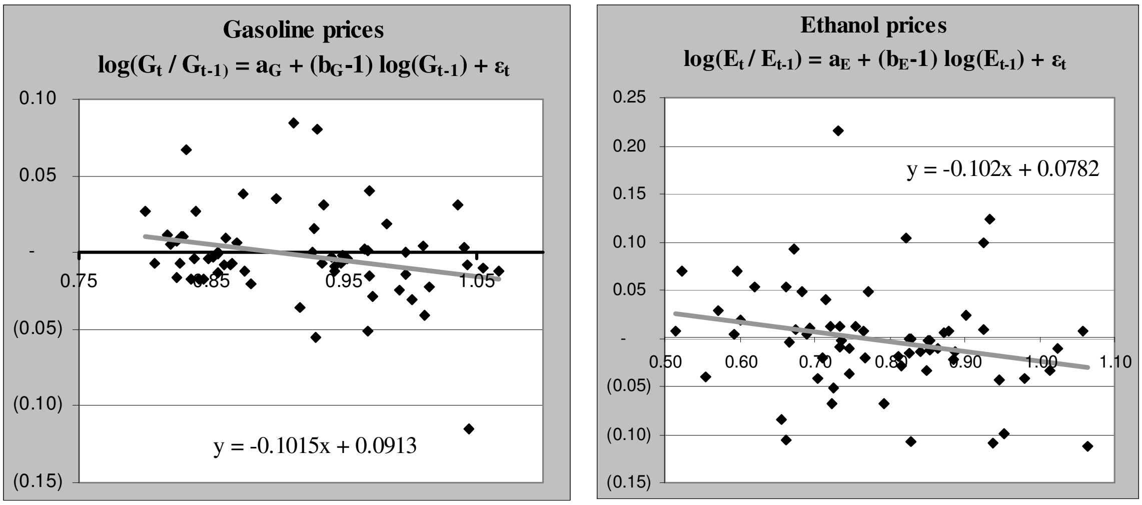

Figure 2: Regressions on deflated prices series

We can observe two important results. First the *t* statistics for both series of prices are above the critical value of 10% significance for the unit root test, which is -2.57, indicating failure to statistically reject the presence of a unit root. Therefore the series can be modeled by a geometric brownian motion (GBM). But we also note that both coefficients *b* are 10 % bellow the value of 1, indicating also the presence of mean reversion.

3.2 Modeling uncertainties using Geometric Brownian Motion (GBM)

In our first analysis therefore, we will assume that gasoline and ethanol prices follow a GBM, in the form:

Gasoline: $dG = \mu_G G dt + \sigma_G G dz_G$

Ethanol: $dE = \mu_E E dt + \sigma_E E dz_E$

Where:

dG and dE are the variations of prices during dt.

μ_G and μ_E , the respective drifts.

σ_e and σ_E , respective volatilities.

$dz_G = \varepsilon_G \sqrt{dt}$ and $dz_E = \varepsilon_E \sqrt{dt}$ are the Wiener processes for each of the uncertain variables, with: $\varepsilon \approx N(0,1)$.

and $dz_E.dz_G = \rho_{EG} dt$, where ρ_{EG} is the correlation between both series returns.

These parameters are calculated form the series as follows. For a price series S_t:

σ_S – Standard deviation of series: log(St)/log(St-1),

μ_S – Average of series: log(St)/log(St-1), plus: $\sigma^2_S/2$.

ρ_{EG} – Correlation of series: log[G(t)/G(t-1)], and: log[E(t)/E(t-1)]).

The values obtained are listed in Table 3:

	Month	Year
σ_E	5,75%	19,92%
σ_G	2,98%	10,33%
μ_e	0,005%	0,063%
μ_G	-0,119%	-1,425%
ρ_{EB}	51,68%	

Table 3: parameters for GBM modeling for Gasoline and Ethanol prices

It is worth noting that the volatility of ethanol is almost double of that of gasoline. This low volatility of gasoline prices can be attributed in part to the fact that the increase in value of the Brazilian Real in relation to the US Dollar during this period created a natural hedge against US Dollar denominated rise in oil prices. In addition, all gas in Brazil is refined by Petrobras and then sold to other distributors, and due to this dominant position and the fact that the Brazilian Government holds a majority stake in the company, gas prices tend to include a political component which leads to more stable prices. On the other hand Brazil's ethanol is almost entirely produced by private companies and its price is not only subject to seasonal factors but also highly correlated to that of sugar, since both are locally produced from sugar cane in the refining plants. The drifts for both series are almost zero, even slightly negative.

The stochastic processes for ethanol and gasoline are determined through Itô's integration (Dixit and Pindick, 1994) and are discretized in monthly time periods with the following equations:

$$G_t = G_{t-1}\exp\left[\left(\mu_G - \frac{\sigma_G{}^2}{2}\right)\Delta t + \sigma_G\varepsilon_G\sqrt{\Delta t}\right]$$

$$E_t = E_{t-1}\exp\left[\left(\mu_E - \frac{\sigma_E{}^2}{2}\right)\Delta t + \sigma_E\varepsilon_E\sqrt{\Delta t}\right]$$

where Δt is one month, or 1/12th of a year. The correlation $E[\varepsilon_G.\varepsilon_E] = \rho_{GE}$ was also considered in the simulation. In the simulation the real drift μ was replaced by the risk-free rate r in both equation, which makes these risk neutral simulation equations. This allows to discount the simulated expenses cash flow at the risk free rate *r*. Results obtained in the simulation after 10,000 interactions are listed in Table 4.

	Results (R$)
Present value of the total expenditure with gasoline only	29.863
Present value of the total expenditure with the cheapest fuel	25.575
Value of the real option	4.288
% value of the real option (relative to the expenditure w/o option)	14,36%

Table 4: Values for base case and of real option with prices modeled by GBM

3.3 Modeling uncertainties using Mean Reversion Model (MRM)

While modeling prices with a Geometric Brownian Motion (GBM) is mathematically simple, it may not be the most realistic form of modeling all diffusion processes. This is the case of projects where the cash flows are related to non financial commodity prices and interest rates, for instance, where these uncertainties may be better modeled by a long term Mean Reverting Model (MRM). MRM is also a Markov process, in which the direction and intensity of the drift are dependent on the current price level, and must revert to a long term market equilibrium level.

The logic underlying a MRM derives from micro-economics: when prices are depressed, or bellow their long term mean level, demand for these products tends to rise and production to fall. This is due to the fact that consumption of a commodity with low price grows while the low returns of the producers earn makes them postpone investments and close down less efficient units, reducing availability of the product. The opposite will happen when prices are high (or above their long term mean), and the frequency of this cycle will depend on the commodity considered.

Bastian-Pinto and Brandão (2007) value the switch option of a flexible processing plant, capable of producing either ethanol or sugar from the same input (sugarcane), by modeling the uncertain variables as GBMs as well as MRMs and demonstrate that the former may significantly overvalue the switch option. Therefore, as the regressions run before return coefficients $b < 1$, indicating the presence of mean reversion, we use a MRM modeling for both uncertain variables, ethanol and gasoline prices, in order to value the switch option with this different process and compare it to the GBM results obtained earlier.

The simplest form of MRM is the single factor Ornstein-Uhlenbeck process, also called arithmetic MRM, which is defined by Equation (7).

$$dY_t = \eta(\overline{Y} - Y_t)dt + \sigma dz_t \qquad (7)$$

where:

Y_t, is the logarithm of the price S_t: $Y_t = \log[S_t]$,

η, is the mean reversion speed,

σ, the process volatility, and

dz, a Weiner process, with a normal distribution.

From Itô's lema, we can determine that, being $\overline{S}$ the long term mean price, then:

$$\overline{Y} = \log\left[\overline{S}\right] - \sigma^2/2$$

The expected value, and variance of the Ornstein-Uhlenbeck process are given by (Dixit & Pindyck, 1994, p. 74-75):

$$E[Y_t] = \overline{Y} + (Y_0 - \overline{Y})e^{-\eta T}$$

$$Var[Y_t] = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta T}\right)$$

As with any arithmetic process such as the Ornstein-Uhlenbeck process, it is possible to have negative values of Y, which might make sense for returns rates, for instance, but not for prices, such as the prices of fuels that we need to model. So we will use the Ornstein-Uhlenbeck process to model the natural logarithm of prices, $\ln[S_t]$, since it is generally assumed that commodity prices are log-normally distributed. This is convenient since if $S = \exp(Y)$, then S cannot be negative.

Schwartz (1997) models commodity prices using an MRM, where the log of the price follows an Ornstein-Uhlenbeck process and proposes an analytical solution for the process solving Itô's integral. His solution includes the market price of risk, or normalized risk-premium $(\mu - r)/\eta$, where μ is the risk adjusted discount rate, and which is subtracted from the log of the long term average in order to have a risk neutral simulation equation. We are considering this risk premium as zero, since the flex fuel car owner is only subject the non-diversifiable market risk, regarding his fuel expenses. Therefore, for the mean reverting model of the discreet prices distribution of ethanol and gasoline in monthly time intervals, which we will use in a Monte Carlo simulation, we use equations (8) and (9), which are discretizations of the Schwartz (1997) model to model the prices of gasoline G_t, and ethanol, E_t at instant t.

$$G_t = \exp\left(\ln\left(G_{t-1}\right) \times e^{-\eta_G \Delta t} + \ln\left(\bar{G} - \frac{\sigma_G^2}{2\eta_G} \right) \times (1 - e^{-\eta_G \Delta t}) + \sigma_G \sqrt{\frac{1 - e^{-2\eta_G \Delta t}}{2\eta_G}} \varepsilon_G \right) \quad (8)$$

$$E_t = \exp\left(\ln\left(E_{t-1}\right) \times e^{-\eta_E \Delta t} + \ln\left(\bar{E} - \frac{\sigma_E{}^2}{2\eta_E} \right) \times (1 - e^{-\eta_E \Delta t}) + \sigma_E \sqrt{\frac{1 - e^{-2\eta_E \Delta t}}{2\eta_E}} \varepsilon_E \right) \quad (9)$$

where Δt is equivalent to one month, or 1/12 of a year, and the remaining parameters (indicated by subscripts (G) and (E) respectively for gas and ethanol) are:

η - mean reversion speed,

σ - processes volatility parameter, and

$\bar{G}$ and $\bar{E}$ respective long term means.

As with the GBM simulation, we will still consider the correlation ρ_{EG} between both uncertainties.

3.3.1 Parameters estimation for MRM

In order to determine the values for the parameters for the MRM diffusion processes we are modeling, we begin by the discretization of the Ornstein Uhlenbek process for a time interval Δt (Dixit & Pindyck, 1994, p. 76), which can be written as:

$$Y_t = \bar{Y} + \left(Y_{t-1} - \bar{Y}\right) e^{-\eta \Delta t} = \bar{Y}\left(1 - e^{-\eta \Delta t}\right) + Y_{t-1} e^{-\eta \Delta t}, \qquad \text{or:}$$

$$Y_t - Y_{t-1} = \bar{Y}\left(1 - e^{-\eta \Delta t}\right) + Y_{t-1}\left(e^{-\eta \Delta t} - 1\right)$$

Considering: $Y_t = \ln\left[S_t\right]$, and: $\bar{Y} = \ln\left[\bar{S}\right] - \sigma^2/2$ (obtained by applying Itô's lemma), then:

$$\ln\left(S_t / S_{t-1}\right) = \left[\ln(\bar{S}) - \sigma^2/2\eta\right]\left(1 - e^{-\eta \Delta t}\right) + \ln\left(S_{t-1}\right)\left(e^{-\eta \Delta t} - 1\right) \quad (10)$$

Rewriting this equation in the form:

$$\ln\left(S_t / S_{t-1}\right) = a + (b-1)\ln\left(S_{t-1}\right), \quad (11)$$

we have same equation (4) and we can now estimate the process parameters with the regression results already obtained and listed in Table 2.

Comparing (10) and (11): $b-1=e^{-\eta\Delta t}-1$, thus:

η = - ln(b)/ Δt (12)

Comparing (10) and (11) again: $a=\left[\ln\left(\bar{S}\right)-\sigma^2/2\eta\right]\left(1-e^{-\eta\Delta t}\right)$, as:

$1-b=1-e^{-\eta\Delta t}$, then:

$$\bar{S}=\exp\left[a/\left(1-b\right)+\sigma^2/2\eta\right] \quad (13).$$

Finally from the variance of errors obtained from the regression run, σ_ε^2, and from Dixit & Pindyck (1994, p. 74-75) we can determine the volatility parameter $\boldsymbol{\sigma}$, as follows:

From: $\sigma_\varepsilon^2=\dfrac{\sigma^2}{2\eta}\left(1-e^{-2\eta\Delta t}\right)$

as $b^2=e^{-2\eta\Delta t}$, then $\sigma_\varepsilon^2=\left(\dfrac{\sigma^2\Delta t}{2\ln\left(b\right)}\right)\left(1-b^2\right)$ or:

$$\sigma=\sigma_\varepsilon\sqrt{\frac{2\ln\left(b\right)}{\left(b^2-1\right)\Delta t}} \quad (14)$$

Note that with (12) and (14), then (13) can be written as:

$$\bar{S}=\exp\left[a/\left(1-b\right)+\sigma_\varepsilon^2/\left(1-b^2\right)\right] \quad (15)$$

which is only dependent on the regression outputs.

As we considered Δt = 1 month, or 1/12$^{\text{th}}$ of a year for the series of prices available, then we will have for our monthly series, Δt = 1/12 = 0.0833. From the results obtained in the regressions and listed in Table 2, the required parameters can be estimated with equations (12), (14) and (15). These are listed in Table 5:

	Gasoline	**Ethanol**
η	1.2848	1.2915
σ	10.61%	20.59%
Long term mean (R$/liter)	2.4585	2.1878
Correlation ρ_{GE}	0.5168	

Table 5–Parameters for MRM

Modeling the prices of gasoline and ethanol with equations (8) and (9) using the parameters of Table 5, we can obtain the expense cash flows to be used in the calculus of the switch option value with equations (1) and (2). And equation (3) will give the value of the option for each interaction of the simulation. Results obtained by the simulation after 10,000 interactions with a software @RISK® are listed in Table 6 below.

	Results (R$)
PV of total expense with gas only	21,883
PV of total expense with cheapest fuel	18,481
Value of the Flex Fuel Option	3,402
Flex Fuel Option as % of total expenditures	15.55%

Table 6: Values for the base case and the real option with prices modeled by MRM

4 Results and Sensitivity Analysis

Both the GBM and the MRM models show that the flex fuel option adds significant value to the owner of the vehicle by reducing fueling expenditures during the lifetime of the asset. The option value obtained with the GBM model is 24.4% higher than the value of the MRM model. (R$ 4,234.00 to R$ 3,402.00). The largest difference obtained in each model is in the results of the deterministic case value, when fueling only with gasoline. As the present value of this expenditure during the lifetime of the vehicle (assuming 10 years) is proportional to the projected fuel prices, this projection will be strongly affected by the stochastic model adopted. This is due to the fact that when using a GBM with a positive drift (r-$\sigma^2/2$), the expected value of fuel grows during the full period of projection. When using a mean reverting model, which seems more adequate for commodity prices such as Ethanol and Gas, the expected value of the projected price will revert to that mean and not grow indefinitely.

4.1 Sensitivity of option value to the correlation between uncertain variables

The high correlation factor between the return of the two price series ($\rho_{GE} = 0.5168$) has an important effect on the option value. Under the MRM modeling, when the correlation factor is zero, indicating that both prices are totally independent from each other, the option value increases from the base case value of R$ 3,402 to R$ 4,186, an increase of 23.0%. In the case of the GBM modeling, these values go from R$ 4.234 to R$ 5,537, or a 31% increase.

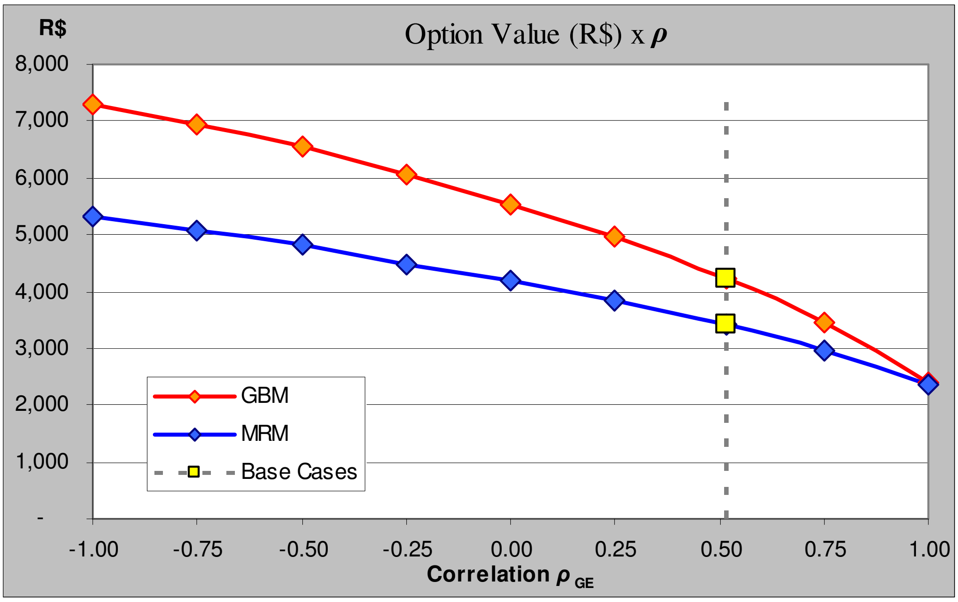

Figure 3: Sensibility of the option value to the correlation factor

It is also worth nothing that the option value is not zero even if both uncertainties are totally correlated (ρ_{GE} = 1) as can be seen in Figure 3. This is explained by the fact that the volatility factors of these variables are different, so even with fully correlated diffusion processes, the switch option can still be exercised and has a value of R$ 2,348 with the MRM modeling, very close to the R$ 2,391 obtained with the GBM modeling.

4.2 Sensibility of the option value the volatilities of gasoline and ethanol prices

Since the flex option value varies with the correlation factor, it is reasonable to expect that it is also affected by the volatilities of the fuels. The volatility of gasoline prices in Brazil has been relatively low, especially when compared to that of ethanol prices, which is subject to seasonality factor due to harvesting periods. This effect has been partially mitigated by changing the mix of anhydrous ethanol which is added to gasoline in Brazil. During harvest periods, when ethanol availability is at its maximum, this mix can be raised up to 25% without affecting the power yield of gasoline motors, which increases demand and reduces the drop in market prices for the producer. During the periods between harvests when supply is lower, this mix is brought back to 20% reducing pressure on demand and prices.

Figure 4 shows the effect of changes in the volatility of gas (σ_G) from 8% to 30.5% per year, while maintaining constant the volatility of ethanol at its base level σ_E = 20.59%. Likewise we varied the volatility of ethanol in the same range, maintaining gasoline volatility at its base case value of σ_G= 10.61%, assuming the MRM model. When the volatility of gas increases from 10.61% (base case) to 21.50% (close to ethanol's volatility) the option value increases to R$ 4,731 or 39.1% above the base case of R$ 3,402. Likewise with gasoline, a rise in the volatility of ethanol will cause an increase in the value of the flex fuel option. A ethanol volatility of 30.5% corresponds to an option value of R$ 3,798, a 11.6% increase above the base case. This growth in value of the flex switch option represents a hedge against eventual rises in the volatilities of gasoline and ethanol the consumer is always subject to.

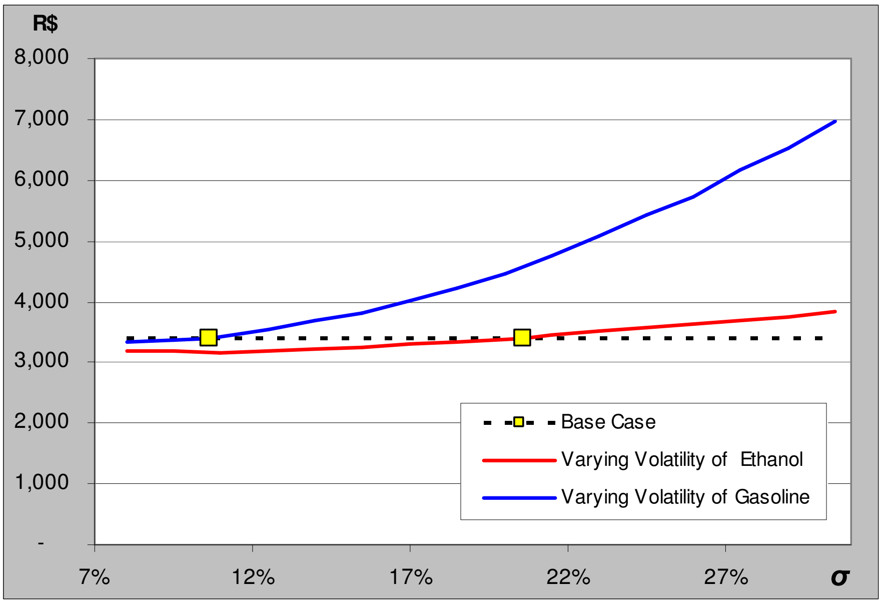

Figure 4: Sensibility to the volatility of gas and ethanol modeling with MRM

Likewise, Figure 5 displays the plotted results for changes in the volatilities of gasoline and ethanol under the GBM model.

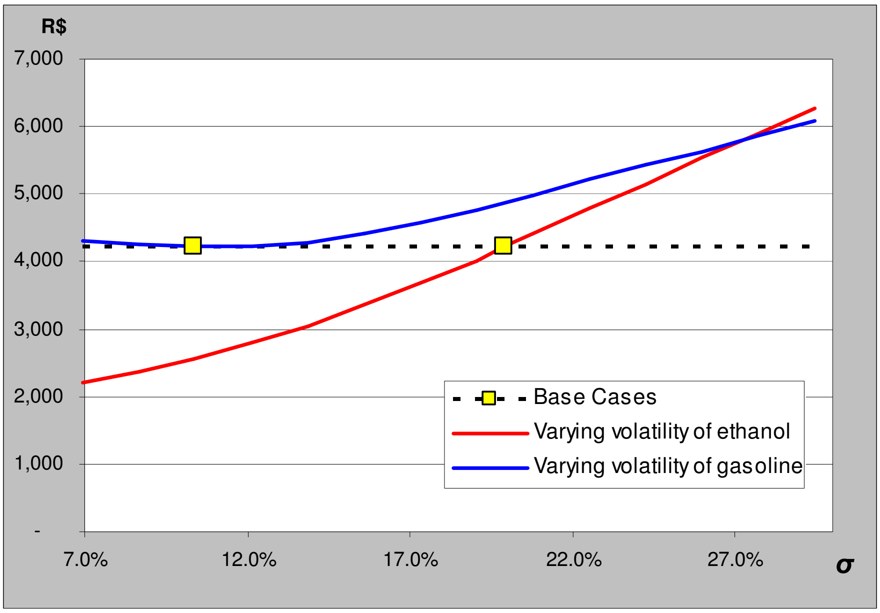

Figure 5 Sensibility to the volatility of gas and ethanol modeling with GBM

It is interesting to note that when modeling the fuel prices with GBM the option value is much more sensible to the volatility of ethanol price than when modeling with MRM. This is due to the characteristic of GBM's variance which grow indefinitely with *t*, contrary to the MRM where the variance is bounded.

5 Conclusion

The flex fuel car is a new technology developed in Brazil which allows consumers to choose any mixture of ethanol or gas each time the car must be refueled. Since its introduction to the market in 2003, the growth of this technology has been significant and currently represents 70% of the production of new vehicles in the country. In this article we analyze the value of the fuel switch option of a flex fuel car using the real options methodology, under both GBM and mean reverting diffusion processes for fuel price uncertainties.

Our results indicate that the flex option adds significant value to the car owner, and can generate savings in fuel costs of approximately 15% during the life of the vehicle, independent of the stochastic process used to model the option. The option value was R$ 4,288 under the GBM model and R$ 3.402 under the MRM model. Considering one of the cheapest cars available in the Brazilian market in 2007, a Renault Clio flex 1.0 with a cost of R$ 27,900.00 (approximately US$ 15,500), the value of the flex option represents 12.2% (MRM) to 15.4% (GBM) of the car value 10% the vehicle's value. For a more expensive model, a Honda Civic Flex 1.6 with a sticker price of R$ 65,300.00 (US$ 36,300) assuming double the fuel consumption, the flex option represents 10,4% (MRM) to 13,1% (GBM) of the car price. Considering that the price of a Honda Civic 1.6 gas in February 2007 was of R$ 61,740.00, we can see that the flex premium was less than 6%, which compares favorably with the flex option value of 12% in the average of both stochastic models. The options value of the flex fuel car may help explain the success achieved by this type of vehicle in Brazil, even if its price is higher than the non flex model. This also indicates that consumers should be willing to purchase flex fuel automobiles even if manufacturers increased the flex premium.

One aspect that was not considered in this analysis is that the power yield of the flex fuel engine when running solely on gasoline is lower than that of a car with a non flex gas powered engine. One of the problems is that gas and ethanol require different engine compression ratios which cannot be changed during operations. To overcome this issue, the flex engine adopts an intermediate compression ratio that is neither optimal for ethanol or gasoline. This results in a loss of efficiency when compared to engines designed exclusively for one of these two fuels. However this problem can be included in the model without great difficulty.

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