Promoting Competition with Open Access under Uncertainty*

Keiichi Hori†

Faculty of Economics, Ritsumeikan University

Keizo Mizuno‡

School of Business Administration, Kwansei Gakuin University

February 2005

Abstract

This paper examines the effect of open access policy on competition in network industries under uncertainty. Comparing a competition under an open access policy with a facility-based competition, we confirm that allowing access to an essential facility makes the timing of a follower's entry earlier than that in a facility-based competition, irrespective of the level of access charge. Furthermore, a leader's (i.e., an incumbent's) incentive for network investment under open access policy can be larger than without open access, depending on the relative magnitude between the level of access charge and positive network externality generated by an additional network facility.

Keywords: Real options, Open access policy, Facility-based competition.

JEL classification: D92; G13; L43; L51

*Financial support from MEXT Grant No. 13630083 is gratefully acknowledged.

†Address: Faculty of Economics, Ritsumeikan University, 1-1-1 Noji-Higashi, Kusatsu, Shiga, 525-8577, Japan; Tel.: +81-77-561-4855; Fax.: +81-77-561-3955; E-mail: khori@ec.ritsumei.ac.jp

‡Address: School of Business Administration, Kwansei Gakuin University, 1-1-155 Uegahara, Nishinomiya, Hyogo, 662-8501, Japan; Tel: +81-798-54-6181; Fax: +81-798-51-0903; E-mail: kmizuno@kwansei.ac.jp

1 Introduction

Competition through *open access* has been prevalent in network industires such as airline, railroad, telecommunication, electricity, natural gas, especially since the trend of deregulation in early 1980s. In an open access environment, needless to say, an access charge is a crucial element that affects the degree of competition in those industries. For example, a lower access charge triggers high volume of entry of firms that do not have a network facility or an essential facility, whereas it induces an incumbent's incentive for exculsion/disclosure and deters a construction of a network facility.

In network industries characterized by high-capital intensiveness, both entry and network construction involves a large investment cost, which implies the explicit relatonship between the level of access charge and investment decision.[1] That is, a lower access charge induces entrants' incentives to invest, whereas it decreases an incumbent's incentive to build a network facility. Furthermore, the construction of additional network facility called *bypass* is usually allowed even in open access policy, since the bypass construction may introduce a positive externality such as the reduction of congestion.

These facts suggests that there are several interesting issues associated with the relationship between access pricing and investment incentives in open access environment. For example, does allowing open access induces a higher incentive to build network facilities or a higher degree of competition than without allowing it? How effective is an open access policy in promoting competition in network industries?

These questions are especially relevant in telecommunications, where policymakers are still wondering which type of entry is more effective in promoting competition

[1] Several issues on the relationship between access pricing and investment incentive are discussed in Valletti (2003). See also other articles in the same volume.

in order to secure lower prices and higher quality services, a *facility-based entry* or a *service-based entry* (i.e., a resale entry)? Although a facility-based entry of 'by-passers' such as cable TV operators involves dual investments for infrastructure, it may introduce a positve network effect through its construction. A service-based entry seems to easily enhance competitive environment without a new construction of network, whereas it may induce an incumbent's incentive for exclusion. One of the remarkable representations of an open access policy was the Telecommunications Act of 1996 that facilitated various ways in entering local exchange markets in telecommunications. The Act specifically provided entry routes into the local exchange by servicce-based providers with some requirements for access charge.

There are few literature on on the effect of open access policy or the comparison among several types of entry. An intersting research of Kaserman and Ulrich (2002) showed the effects of facility-based vs. resale entry on competition. According to their results in Table 3, resale entry seemingly has more drastic effect on competition than facility-based entry, in the sense that resale entry reduces incumbents' shares in long distance telecommunication market more than facility-based entry. Laffont and Tirole (2000) and Woroch (2002) provide useful discussion about the comparison between the two types of entry.

The purpose of this paper is to examine the effect of open access policy on the incentive for building network facilities or infrastructures and on the degree of competition in network industries. In particular, to address the questions mentioned above, we empoy a *real options* approach, since uncertainty and irreversibility of investment are influential factors when considering investment problems.

The real options is the application of option concepts to value real assets under uncertainty, and it has been an important growth area in investment theory. (See Dixit and Pindyck (1994) and Trigeorgis (1996) for its basic treatment of tools.[2]) It

[2] See also Smits and Trigeorgis (2004) for a real options approach to game-theoretic models.

is especially useful when a player has a sequential opportunity of investment timing. As is well-known, network industries comprise a production facility and a network facility. (For example, in the electricity industry, a plant for generating electricity is the production facility, whereas transmission and local distribution wires are network facilities.) In the industries, then, an entrant or follower has a sequential opportunity of investment timing; that of a construction of a *bypass* or another network facility. This is because an important characteristic of network industries is approval for a common use of network facilities. Since network (or essential) facilities are characterized by large sunk costs, their common use is recommended from a social point of view, as long as congestion problems do not occur. An entrant's decision to construct a bypass may be controversial with respect to improving welfare. In that case, the real options approach is suitable for examining the properties of an entrant's sequential investment decision (i.e., *from access to bypass*) because the application of a simple net present value (NPV) approach cannot provide adequate understanding of an entrant's incentives to construct a bypass when there is uncertainty and investment is irreversible. With an NPV approach, one would characterize the entrant's decision about whether (or when) to construct a bypass by comparing the net present value of profit under access with that under use of the bypass. However, such an approach would be inappropriate because it ignores the option value of delaying additional investment in the bypass. This is the main reason for adopting the real options approach to examine the incentives for investment in network industries. To sum up, this approach is useful for studying network industries and, in particular, for studying the effect of regulatory policies on the performance of these industries.

The effect of uncertainty on irreversible investment in public utility industries has been formally examined by Biglaiser and Riordan (2000). However, they neither analyzed a game between an incumbent and an entrant nor allowed an entrant to construct a bypass. A book edited by Alleman and Noam (1999) contains some existing debate on real options approach applied to telecommunications industries. We formally analyze the investment game in public utility or network industries by focusing on an entrant's decision to make an additional investment in bypass construction when there is stochastically growing demand.

In the model developed below, we assume that a bypass construction creates a positive network externality through which firms can benefit an increase in profit. Then, we firstly show that, in addition to the level of access charge, a choice of entry strategy by a follower (i.e., a second entrant) is affected by two effects, i.e., *the positive network externality effect* and *the transition-option effect*. In fact, the transition-option effect occurs only under uncertainty and irreversibility of investment. That is, once entering the market by access, a follower wants to implement the bypass project at a moment in the future, even though the total cost of bypass is larger than that of access. This is because, in addition to the positive network externality, the access payment can be avoided by exercising the option of the transition project (i.e., by builiding its own network facility), when the follower enters by access. In other words, a large investment cost of bypass can be cancelled by the existence of the option of the transition project only if the follower enters by access. Hence, the transition-option effect makes entry by access more preferable than entry by bypass.

Then, comparing a competition under an open access policy with a facility-based competition, we confirm that allowing access to an essential facility makes a follower's entry earlier than in a facility-based competition. Then, we show an incumbent's incentive for network investment under open access policy can be larger than without open access, depending on the relative magnitude between the level of access charge and a positive network externality generated by an additional network facility. In particular, when both the access charge and the positve network externality are zero, a leader, which later becomes an incumbent, in a competition under open access policy enter with the construction of a network earlier than in a facility-based competition. However, if the level of access charge is high and the positive network externality generated by a bypass construction is small, the leader in a competition under open access policy enters the market earlier than in the facility-based competition. Lastly, we discuss an effect of temporary access suspension on the follower's incentive to enter, which suggests that temporary access suspension makes the follower's entry earlier than without it.

Section 2 presents the framework of model. As a benchmark, we characterize a facility-based competition equilibrium in Section 3. Then, Section 4 characterizes an equilibrium in open access environment. Section 5 derives the effect of open access policy on the firms' incentives for entry. Section 6 discusses the effect of temporary access suspension. Concluding remarks are in Section 7.

2 The Model

There are two risk-neutral firms, $i = 1, 2$, which plan to enter into a network industry, such as the electricity, telecommunication, or natural gas industries, under imperfect competition. The network industry needs two types of facility to serve their customers: a production facility and a network facility. Each firm has the opportunity to invest in both types of facility, and the investment decisions in each type are assumed to be irreversible. The investment cost for the production facility is $I^e > 0$, whereas that for the network facility is $I^m > 0$. Both I^e and I^m are sunk costs.

Investments in the two types of facility may be undertaken simultaneously or sequentially. A firm builds the production facility at cost I^e, and at the same time or in the next stage, the network facility is built at an additional cost of I^m.

However, not all firms must invest in the network facility provided that at least one firm maintains the facility. That is, the firms without network facilities may utilize the existing network facility to distribute products. The firm that initially enters the market with both production and network facilities is called a *leader*, whereas the other firm, which may or may not have a network facility, is called a *follower*. We assume that the follower can access the existing network facility through a lump-sum access charge, v, which is given for each firm and determined by a policy maker.[3] In the main anaylsis of Section 4 below, we assume that once the follower enters the market by access to the incumbent's network, it has to continue to serve the market by paying v, even when the demand is accidentally low. This assumption may be justified by a universal service obligation to serve customers. However, one may imagine that the follower can stop serving customers in the period of an unexpected demand reduction by disconnecting the leader's network facility with an almost zero re-start production cost. We call it the case of "*temporary access suspension*". With uncertainty and irreversibility, the temporary access suspension introduces an additional option value, which complicates the analysis. This case will be discussed in Section 6.

When the follower uses the leader's existing network facility, the leader incurs a lump-sum access cost for the network facility, c, which is normalized to zero for analytical simplicity.[4] Production costs other than the access cost are also assumed to be zero.

Note that the follower, having access to other network facilities, may invest in its own network facility at some time in the future. This additional network facility is

[3] For analytical simplicity, we analyze only a lump-sum or fixed charge in this paper. See our companion paper (2003) for the analysis of the case of a usage access charge.

[4] If we set the assumption that $c > 0$, it changes the leader's profit flow in the access duopoly, which in turn changes the leader's value and the derived leader's trigger point in equilibrium. However, it does not change the qualitative results, which is derived by the comparison between with and without open access, on the effects of allowing open access on competition and investment incentive.

called a bypass facility, which is assumed to introduce a positive network externality, such as a reduction of blackout in electricity or a reduction of congestion.

We assume that the two firms compete in a market for a homogeneous good produced in the network industry. The profit flows of the firms are uncertain because the firms face an aggregate exogenous industry shock. The profit flow of a firm is represented by $\pi = Y\Pi(N)$, where Y is the aggregate exogenous shock, $N = 0, 1, 2$ is the number of active firms, and $\Pi(N)$ is interpreted as the non-stochastic part of the firm's profit flow at the industry equilibrium.

Y evolves exogenously and stochastically according to a geometric Brownian motion, with drift given by the following expression:

$$dY_t = \alpha Y_t dt + \sigma Y_t dW$$

where $\alpha \in [0, r)$ is the drift parameter measuring the expected growth rate of Y, r is the risk-free interest rate, $\sigma > 0$ is a volatility parameter, and dW is the increment of a standard Wiener process where $dW \sim N(0, dt)$. Note that when $\alpha > (1/2)\sigma^2$, the expected firm's profit flow is enhanced stochastically.

A firm's profit flow in the monopoly equilibrium is represented by $Y\Pi(1)$. When the two firms are active in the market, we distinguish two duopolistic market structures: a duopoly in which the follower has access to the leader's network facility; and a duopoly in which the follower maintains its own network facility. Let $Y\widehat{\Pi}(2)$ represent the profit flow of a firm in the former duopoly equilibrium, referred to as an "access duopoly", while $Y\Pi(2)$ represents the profit flow in the latter duopoly equilibrium, called a "bypass duopoly". The following relationship is assumed to hold for expositional convenience.

Assumption 1 $\Pi(1) > \Pi(2) > \widehat{\Pi}(2)$.

Although it is natural to assume that $\Pi(N)$ is a decreasing function of N, the assumption that $\Pi(2) > \widehat{\Pi}(2)$ needs some explanation. An idea behind this assumption is the existence of a positive network externality caused by additional supply of network facility. For example, we can imagine a decrease in the probability of blackout by a construction of another transmission wires in a local electricity market, or the capability of the provision of high calorie gas by a construction of additional gas pipeline in gas market. Let us illustrate this point by a numerical example. Suppose an inverse demand function is linear, $p = a - bQ$ when the market is monopoly or access duopoly, while $p = (a + \theta) - bQ$ (where $\theta > 0$) when the market is bypass duopoly. It is then easy to check that $\Pi(1) = a^2/4b$, $\widehat{\Pi}(2) = a^2/9b$, and $\Pi(2) = (a + \theta)^2/9b$, respectively. As long as $\theta < a/2$, the assumption that $\Pi(1) > \Pi(2)$ is satisfied.

Note that the follower has several strategies, depending on the relative size of network investment cost, I^m, to the net present value of access charge payment. First, the follower may want access to the network facility built by the leader because the investment cost is relatively cheaper than the net present value of access charge payment. Second, the follower may want to build its own network facility because the reverse relationship holds. A further possibility is that the follower initially accesses to the leader's network facility, but then it decides to build its own network facility.[5] We call these three alternatives "*access strategy*", "*bypass strategy*", and "*access-to-bypass strategy*", respectively. The follower may prefer, for example, the access strategy instead of the bypass strategy, or the access-to-bypass strategy, depending on the conditions regarding the level of investment costs, the equilibrium profit under product market competition, or the level of access charge. In Section 4, we examine the follower's choice of strategy before deriving the equilibrium of the game.

[5] Another possibility is that the follower first constructs its own network facility and then uses the leader's network with access charge payment. However, we can ignore this possibility because of the irreversibility of the network investment and the avoidability of the access charge payment.

3 Facility-Based Competition: A Benchmark

As a benchmark, we consider a situation where a follower is not allowed to access the leader's network facility. We call it the regime of "facility-based competition". Let us derive the equilibrium of this regime. As usual in dynamic-game contexts, the game can be solved backwards.

First, consider the follower's strategy. In this regime, the follower must invest not only in the production facility but also in the network facility to serve consumers, so that the total investment cost is $I^e + I^m$. Since the follower builds an additonal network (i.e., bypass), its profit flow is $Y\Pi(2)$ according to our setting. Following standard steps, we can find the trigger point above which the follower invests and enters the market[6]:

$$Y^B = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Pi(2)}(I^e + I^m), \tag{1}$$

where $\beta_1 = \frac{1}{2}\left\{1 - \frac{2\alpha}{\sigma^2} + \sqrt{\left(1-\frac{2\alpha}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right\} (> 1)$.

Before proceeding the analysis, two points deserve to be mentioned in order to clarify the standard properties of the investment timing derived under a real options approach. First, the trigger point Y^B is larger than the trigger point that would be derived under an NPV approach, i.e., $\widetilde{Y}^B = \frac{r-\alpha}{\Pi(2)}(I^e + I^m)$. Second, since β_1 is a decreasing function of a volatility parameter σ, an increase in uncertainty (i.e., an increase in σ) makes the firm deter entry even if it is risk neutral.

If $Y \geq Y^B$, the follower invests at once, so that it obtains the value $V(Y) = Y\Pi(2)/(r-\alpha) - (I^e + I^m)$. If $Y < Y^B$, it will wait until the trigger point is first reached. Hence, the value function of the follower is represented as follows.

$$V_F^B(Y) = \begin{cases} \left(\frac{Y}{Y^B}\right)^{\beta_1}\left[\frac{Y^B\Pi(2)}{r-\alpha} - (I^e + I^m)\right] & if \quad Y < Y^B \\ \frac{Y\Pi(2)}{r-\alpha} - (I^e + I^m) & if \quad Y^B \leq Y \end{cases} \tag{2}$$

[6] See Dixit and Pindyck (1994) for the technique of its derivation.

Next, we consider the leader's value in this regime. When the follower enters the market by building its own network facility, the value function of the leader is represented as follows:

$$V_L^B(Y) = \begin{cases} \frac{Y\Pi(1)}{r-\alpha}\left[1-\left(\frac{Y}{Y^B}\right)^{\beta_1-1}\right]+\left(\frac{Y}{Y^B}\right)^{\beta_1}\frac{Y^B\Pi(2)}{r-\alpha}-(I^e+I^m) & if \quad Y<Y^B \\ \frac{Y\Pi(2)}{r-\alpha}-(I^e+I^m) & if \quad Y^B \leq Y \end{cases} \tag{3}$$

The first term of $V_L^B(Y)$, when $Y < Y^B$, represents the expected monopoly profit until Y^B is reached, whereas the second term represents the expected value occurring from the probability that Y^B is reached.

According to the argument of Fudenberg and Tirole (1985), the form of the non-cooperative equilibrium depends on the relative magnitude of the leader's value and the value when both firms invest simultaneously. In our framework, it is apparent that two asymmetric leader-follower equilibria appear in the non-cooperative game, since there exists some $Y \in (0, Y^B)$ such that $V_L^B(Y)$ is larger than the value when both firms invest simultaneously. (The two equilibria differ only in the identities of the two firms.) We call the asymmetric equilibria the "*facility-based competition equilibria*".

[Insert Figure 1]

The facility-based competition equilibrium is depicted in Figure 1. When $Y \in [0, Y_L)$, where Y_L is the trigger point at which the leader enters the market, the two firms do not enter the market. When $Y \in [0, Y^B)$, the leader enjoys monopoly profits. When $Y \in [Y^B, +\infty)$, the follower serves its customers by constructing its own network facility.

The characterization of the equilibrium is the same as that discussed in Dixit and Pindyck (1994, Chapter 9). One important property of the equilibrium is the dominance of a *preemption* effect to a *real option* effect: Since firms compete for monopoly profits, a firm as a leader enters earlier than as a monopolist even under uncertainty and irreversiblity of investment. In fact, comparing the competition equilibrium with the monopoly equilibrium (i.e., no competition equilibrium), Nielsen (2002) showed that the trigger point of a leader in a competition regime is lower than that of a monopolist.[7]

4 Competition under Open Access Policy

In contrast to the previous regime, we now consider the case in which the follower is allowed to access the leader's network facility in order to serve customers. Let us derive the equilibrium under this open access policy.

4.1 The follower's choice of strategy

We first need to consider the follower's strategy choice, since in this environment the follower has three alternative strategies: the access strategy, the bypass strategy, and the access-to-bypass strategy. Then, we have to specify the condition under which the follower chooses one strategy instead of the other two strategies.

To do so, we must firstly derive the value of each project before obtaining the values of the three strategies.

When the access project is undertaken, its value is:

$$V^{A}(Y) = \frac{Y\widehat{\Pi}(2)}{r-\alpha} - \frac{v}{r}. \tag{4}$$

[7] Without uncertainty, the preemption effect was also reported by Fudenberg and Tirole (1985) and Katz and Shapiro (1987) in a framework of R&D racing game.

Here, v/r is the discounted value of access charge payment when assuming that once the follower enters the market by access to the incumbent's network, it has to continue to serve the market by paying v even if the demand is accidentally low.

On the other hand, when the bypass project is undertaken, the value of the project is:

$$V^B(Y) = \frac{Y\Pi(2)}{r-\alpha} \tag{5}$$

Then, using (4) and (5), we can define the value of *the transition project*, $\Delta V(Y)$, i.e., the difference in the values between the bypass project and the access project:

$$\Delta V(Y) \equiv V^B(Y) - V^A(Y) = \frac{Y\Delta\Pi(2)}{r-\alpha} + \frac{v}{r} \tag{6}$$

where $\Delta\Pi(2) \equiv \Pi(2) - \widehat{\Pi}(2)$ is called an *incremental profit flow from access to bypass*. Note that $\frac{v}{r}$ is involved in the transition project, $\Delta V(Y)$, since the follower can avoid paying the access charge after the construction of a bypass facility.

The access-to-bypass strategy is derived by a backward induction procedure. Suppose that the follower already implements the bypass project. Then, consider the problem of whether it should exercise the option of the transition project or not. We define $F^T(Y)$ as the option value of the transition project. From the standard procedure and $F^T(0) = 0$, we have $F^T(Y) = G_1 Y^{\beta_1}$.[8] The parameter G_1 and the trigger point Y^{B*} are the solutions that satisfy the following value-matching condition and the smooth-pasting condition:

$$F^T\left(Y^{B*}\right) = \Delta V\left(Y^{B*}\right) - I^m \tag{7}$$

$$F^{T\prime}\left(Y^{B*}\right) = \Delta V'\left(Y^{B*}\right) \tag{8}$$

Substituting (6) into (7) and (8), we derive a trigger point Y^{B*} at which the

[8] The reason of the requirement that $F^T(0) = 0$ stems from the observation that if Y goes to zero, it will stay at zero.

bypass project starts.

$$Y^{B*} = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\Delta\Pi(2)} \left(I^m - \frac{v}{r} \right). \tag{9}$$

Next, consider the follower's decision problem of whether it should really exercise the access project or not. Note that since there is an opportunity to implement the bypass project after an access to the leaders network, the *effective* value of the access project includes not only its own project value, but also the option value of the transition project. Then, defining the option value of the access project and using the value-matching and the smooth-pasting conditions and using a standard procedure, we derive the trigger point Y^{A*}.

$$Y^{A*} = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\widehat{\Pi}(2)} \left(I^e + \frac{v}{r} \right). \tag{10}$$

Let us chracterize the follower's choice of strategy. It is easy to realize that the follower does not adopt the access strategy, since $\Delta\Pi(2) > 0$ and the demand is stochastically growing.[9] Hence, we have the following lemma.

Proposition 1 *The follower adopts the access-to-bypass strategy (bypass strategy, respectively) if and only if*

$$I^e + \frac{v}{r} \leq (>) \frac{\widehat{\Pi}(2)}{\Pi(2)} (I^e + I^m) \tag{11}$$

Proof. Observe that, when $Y^{B*} < +\infty$ and $(0 <) Y^{A*} \leq Y^{B*}$, the follower in fact adopts the access-to-bypass strategy. The condition (11) is derived just by rewriting the condition that $Y^{A*} \leq Y^{B*}$. When $Y^{A*} > Y^{B*}$, only the bypass strategy remains

[9] From the assumptions that $\Delta\Pi(2; v) > 0$ and Y evolves according to a geometric Brownian motion such that it has the expected growth rate α, there necessarily exists a trigger point Y^{B*} at which the follower starts a bypass projct.

for the follower. In fact, this is confirmed, because

$$\begin{aligned}
Y^{B*} &\equiv \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Delta\Pi(2)}\left(I^m - \frac{v}{r}\right) < \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\widehat{\Pi}(2)}\left(I^e + \frac{v}{r}\right) \equiv Y^{A*} \\
&\Leftrightarrow \widehat{\Pi}(2)\left(I^m - \frac{v}{r}\right) < \left(\Pi(2) - \widehat{\Pi}(2)\right)\left(I^e + \frac{v}{r}\right) \\
&\Leftrightarrow \widehat{\Pi}(2)\left(I^e + I^m\right) < \Pi(2)\left(I^e + \frac{v}{r}\right) \\
&\Leftrightarrow Y^B \equiv \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Pi(2)}\left(I^e + I^m\right) < \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\widehat{\Pi}(2)}\left(I^e + \frac{v}{r}\right) \equiv Y^{A*}, \quad (12)
\end{aligned}$$

which means that the timing of a facility-based entry is earlier than that of an entry by access. ■

[Insert Figure 2 around here]

Figure 2 shows the follower's choice of strategy. In particular, there are three regions, depending on the values of v/r and $\Pi(2)$. When $(v/r) > -I^e + \left(\widehat{\Pi}(2)/\Pi(2)\right) \times (I^e + I^m)$ (i.e., Region I), the follower adopts the bypass strategy. When $(v/r) < -I^e + \left(\widehat{\Pi}(2)/\Pi(2)\right)(I^e + I^m)$ (Region II), the follower adopts the access-to-bypass strategy. When $(v/r) = -I^e + \left(\widehat{\Pi}(2)/\Pi(2)\right)(I^e + I^m)$ (Region III), both the two strategies are indifferent to the follower. To sum, roughly speaking, when the level of access charge is relatively low when compared with the positive externality generated by bypass construction, the follower adopts the access-to-bypass strategy, i.e., it prefers entry by access to a facility-based entry and then construct its own network facility when the demand grows sufficiently.

An interesting point in Lemma 1 deserves to be mentioned, which we illustrate by a numerical example. Suppose that $\widehat{\Pi}(2)/\Pi(2) = 0.8$ and $I^e + \frac{v}{r} = 10$. Then, if $I^e + I^m = 11$, $I^e + I^m = 11 > 10 = I^e + \frac{v}{r}$ and $I^e + \frac{v}{r} = 10 > 8.8 = \left(\widehat{\Pi}(2)/\Pi(2)\right)(I^e + I^m)$. Hence, the follower enters the market by a bypass construction, even though the total cost of bypass (i.e., $I^e + I^m$) is larger than that of access (i.e., $I^e + \frac{v}{r}$). This is because a positive network externality is generated by the additional network construction (*A positive network externality effect*). Next, consider the case in which $I^e + I^m = 30$. Then, $I^e + I^m = 30 > 10 = I^e + \frac{v}{r}$ and $I^e + \frac{v}{r} = 10 < 24 = \left(\widehat{\Pi}(2)/\Pi(2)\right)(I^e + I^m)$. In this case, the follower enters the market by access, which does not seem to be surprising. A surprising point is that once entering the market by access, the follower wants to implement the bypass project (i.e., to exercise the option of the transition project) at a moment in the future, even though the total cost of bypass (i.e., $I^e + I^m$) is larger than that of access (i.e., $I^e + \frac{v}{r}$). This is because, in addition to the positive network externality, the access payment can be avoided by exercising the option of the transition project. In other words, a large investment cost of bypass can be cancelled by the existence of the option of the transition project only if the follower enters by access (*A transition-option effect*). Hence, the transition-option effect makes entry by access more preferable than entry by bypass.

In sum, in addition to the level of access charge, the two effects (i.e., the positive network externality effect and the transition-option effect) are also the driving forces of the follower's choice of strategy in our model.

4.2 The equilibrium

There are two types of equilibria. The first type of the equilibrium is the one in which the follower adopts the bypass strategy. When the follower adopts the bypass strategy, the follower's value function is the same as in the facility-based competition equilibrium derived above. So is the leader's value function. Let us call it a "*bypass competition equilibrium*". In the bypass competition equilibrium, the trigger points of the leader and the follower are Y_L and Y^B respectively, both of which are the same as in the facility-based competition equilibrium.

The second type of equilibrium is the one in which the follower adopts an access-to-bypass strategy. We call it an "*access-to-bypass competition equilibrium*". Here, we report the value functions of the leader and the follower in the equilibrium, respectively. (The procedure of the derivation is the same as in the facility-based competition equilibrium.)

The follower's value function in the access-to-bypass competition equilibrium is dervied as follows.

$$V_F^{AB}(Y) = \begin{cases} \left(\frac{Y}{Y^{A*}}\right)^{\beta_1} \left\{ \frac{Y^{A*}\widehat{\Pi}(2)}{r-\alpha} - \frac{v}{r} - I^e \right. \\ \left. + \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1} \left[\frac{Y^{B*}\Delta\Pi(2)}{r-\alpha} + \frac{v}{r} - I^m \right] \right\} \\ \qquad\qquad if \ \ Y < Y^{A*} \\ \frac{Y\widehat{\Pi}(2)}{r-\alpha} - \frac{v}{r} - I^e \\ + \left(\frac{Y}{Y^{B*}}\right)^{\beta_1} \left[\frac{Y^{B*}\Delta\Pi(2)}{r-\alpha} + \frac{v}{r} - I^m \right] \\ \qquad\qquad if \ \ Y^{A*} \le Y < Y^{B*} \\ \frac{Y\Pi(2)}{r-\alpha} - (I^e + I^m) \qquad if \ \ Y^{B*} \le Y \end{cases} \tag{13}$$

The trigger points Y^{A*} and Y^{B*} are given by (10) and (9), respectively.

Next, we derive the leader's value when the follower takes the access-to-bypass strategy. In that case, the value function of the leader can be derived as follows.

$$V_L^{AB}(Y) = \begin{cases} \frac{Y\Pi(1)}{r-\alpha}\left[1 - \left(\frac{Y}{Y^{A*}}\right)^{\beta_1 - 1}\right] + \left(\frac{Y}{Y^{A*}}\right)^{\beta_1} \left\{ \frac{Y^{A*}\widehat{\Pi}(2)}{r-\alpha}\left[1 - \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1 - 1}\right] \right. \\ \left. + \frac{v}{r}\left[1 - \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1}\right] + \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1} \frac{Y^{B*}\Pi(2)}{r-\alpha} \right\} \\ \qquad - (I^e + I^m) \qquad if \ \ Y < Y^{A*} \\ \frac{Y\widehat{\Pi}(2)}{r-\alpha}\left[1 - \left(\frac{Y}{Y^{B*}}\right)^{\beta_1 - 1}\right] + \frac{v}{r}\left[1 - \left(\frac{Y}{Y^{B*}}\right)^{\beta_1}\right] + \left(\frac{Y}{Y^{B*}}\right)^{\beta_1} \frac{Y^{B*}\Pi(2)}{r-\alpha} \\ \qquad - (I^e + I^m) \qquad if \ \ Y^{A*} \le Y < Y^{B*} \\ \frac{Y\Pi(2)}{r-\alpha} - (I^e + I^m) \qquad if \ \ Y^{B*} \le Y \end{cases} \tag{14}$$

The access-to-bypass competition equilibrium is depicted in Figure 3. When $Y \in [0, Y_L^*)$, where Y_L^* is the trigger point at which the leader enters the market, the two firms do not enter the market. When $Y \in [0, Y^{A*})$, the leader enjoys monopoly profits. When $Y \in [Y^{A*}, Y^{B*})$, the follower can operate with access to the leader's network facility. Finally, when $Y \in [Y^{B*}, +\infty)$, the follower serves its customers by constructing its own network facility.

[Insert Figure 3]

5 The Effects of Open Access Policy

Now we are in a position to examine the effects of open access policy by comparing the equilibria of the two competition regimes.

First, consider the entry timing of the follower under open access policy. We can confirm that under open access policy, the follower enters the market earlier than without open access.

Proposition 2 $Y^{A*} < Y^B$*: Under open access policy, the follower enters the market no later than without open access, irrespecitve of the level of access charge.*

Proof. As mentioned before, two types of equilibrium can emerge under open access policy, depending on the level of access charge and the degree of positive network externality; the bypass competition equilibrium and the access-to-bypass competiton equilibrium. In the bypass competition equilibrium, the trigger point of the follower is exactly the same as in the equilibrium in the facility-based competition regime, i.e., Y^B. So we need to check the trigger points of the follower only in the access-to-bypass competiton equilibrium.

In the access-to-bypass competiton equilibrium, the relationship that $Y^{A*} < Y^{B*}$ holds. Then, we can confirm that $Y^{A*} < Y^{B*}$ implies $Y^{A*} < Y^B$, according to (12). ■

The results of Proposition 1 can be seen on Figure 2. In Region I where the follower adopts the bypass strategy under open access policy, the entry timing of follower under open access policy is exactly the same as under a facility-based competition equilibrium. On the other hand, in Region II where the follower adopts the access-to-bypass strategy, the entry timing of the follower by access is strictly earlier than that in the facility-based competition equilibrium.

Proposition 1 states a strong result: *as long as open access is allowed, the follower's entry cannot be later than that without open access.* When the level of access charge is low, the follower enters market earlier than without open access, which seems to be intuitively appealing. On the other hand, when the level of access charge is high, one may guess that the follower enters market later than without open access. However, this is incorrect. This is because the follower has an option of transition project in open access environment, and in fact it exercise its option, i.e., build is own network facility, whenever the level of access charge is high. Hence, the followers entry cannot be later than that without open access.

Furthermore, we can state the timing of a bypass construction by the follower. Although the follower has the option to build its own network facility when entring by access, the timing to exercise it is later than the entry timing of the follower in the facility-based competition. This means that the introduction of positive network externality generated by a bypass construction cannot be earlier than without open access.

Proposition 3 $Y^B < Y^{B*}$*: In the access-to-bypass competition equilibrium, the follower constructs its own network failicity later than in the facility-based competition equilibrium.*

Proof. See Appendix. ■

Next, we examine the entry timing of the leader under open access policy. In fact, it seems to be a hard task to derive the global property of the leader's entry timing by comparing the equilibria with and without open access policy, since the curvatures of the leader's and the follower's value functions depend not only on the policy variable v but also on the environmental variables such as the severeness of product competition (i.e., the level of the exogenously given duopoly profit), the degree of positive network externality, the volatility parameter σ. Hence, we try to develop the comparison of leader's entry timing between with and without open access by focusing on some key points in Figure 2.

At first, we examine the neighborhood of $(v, \Delta\Pi(2)) = (0, 0)$; the case in which both the access charge and the positve network externality are zero.

Proposition 4 *In the case where both the access charge and the positve network externality are zero, the leader in the facility-based competition equilibrium enter earlier than the one in the access-to-bypass competition equilibrium.*

Proof. See Appendix. ■

The intuition of Lemma 2 is as follows. When access charge is almost zero, the follower has a strong incentive to enter earliler by access than in the facility-based competition equilibirum. In addition, since the positive network externality is almost zero, the follower has a weak incentive to exercise the transition project. In sum, the follower has a long period to access the leader's network. Expecting this follower's behavior, the incentive to become a leader is weak, since the net present value of the access profit is sufficiently low. Therefore, the leader's entry timing in the access-to-bypass competition equilibrium is later than that in the facility-based competition equilibrium.

Next, let us restrict our attention to the neighborhood of Region III in Figure 2. Then, we can derive an important property of the leader's trigger point in competition under open access policy.

Proposition 5 *In the access-to-bypass competition equilibrium that is close to Region III, if the level of access charge is high (low, respectively) and the positive network externality generated by a bypass construction is small (large), the leader enters the market earlier (later) than in the facility-based competition equilibrium.*

Proof. See Appendix. ■

[Insert Figure 4]

In Figure 4, we draw a hyperplane of $v/r = \Pi(1)[I^e + I^m]\left[(\Pi(2))^{-1} - \widehat{\Pi}(2)/(\Pi(2))^2\right]$ in addition to the hyperplane of $v/r = -I^e + \left[\widehat{\Pi}(2)/\Pi(2)\right](I^e + I^m)$ in Figure 2. Suppose the level of access charge v is decreased marginally from Region III., leading to the access-to-bypass competition equilibirum. Then, above (below, respectively) the hyperplane of $v/r = \Pi(1)[I^e + I^m]\left[(\Pi(2))^{-1} - \widehat{\Pi}(2)/(\Pi(2))^2\right]$, we have $V_L^{AB}(Y_L) > (<) V_F^{AB}(Y_L)$, which means that the leader's entry timing in the access-to-bypass competition equilibrium is earlier (later) than that in the facility-based competition equilibrium.

From Proposition 5, we confirm that there exists a region in which the leader's entry timing in the access-to-bypass competition equilibrium is earlier than that in the facility-based competition equilibrium, as long as the positive network externality generated by a bypass construction is null and the level of access charge is high. The result is intuitively appealing: if the level of access charge is high and the positive network externality generated by a bypass construction is small, then the relative advantage of the leader to the follower increases, since it can not only gain a large access profit in an access duopoly regime, but also it can enjoy a monopoly profit in a longer period than in the facility-based competition equilibrium.

Lastly, we show that when $\Delta\Pi(2) \simeq 0$, as the access charge becomes higher, the leader's entry timing in the access-to-bypass competition equilibrium is earlier than that in the facility-based competition equilibrium.

Proposition 6 *When $\Delta\Pi(2) \simeq 0$, as the access charge becomes higher, the leader's entry timing in the access-to-bypass competition equilibrium becomes earlier than that in the facility-based competition equilibrium.*

Proof. See Appendix. ■

Since there is no monotonicity in the sign of $\frac{\partial \chi}{\partial v}\Big|_{\Delta\Pi(2)\to 0}$, it is difficult to obtain analytically the result on the comparison of the leader's entry timing.

As mentioned before, it is hard to obtain the global property of the leader's entry timing in all the ranges of the access-to-bypass competition equilibrium. In fact, we tried a numerical example in which the parameters are the followings; $\alpha = 0.02$, $\sigma = 0.1$, $r = 0.05$, $I^e = I^m = 20,000, \Pi(1) = 1,400$, $\widehat{\Pi}(2) = 1,000$, $\Pi(2) = 1,200$. The numerical example confirmed the characteristics derived above.

6 Discussion: Allowing Temporary Access Suspension

In the previous analysis of open access policy, it is assumed that the follower has to pay the access charge v even when it may face a negative profit (because of an unexpected demand reduction) after a payment of the access charge. One may imagine that the follower can stop serving customers in the period of an unexpected demand reduction by disconnecting the leader's network facility, whereas it can re-start production almost costlessly. We call it the case of "*temporary access suspension*".

The case of temporary access suspension might be controvertial in the network industries. This is, for example, because the goods provided by the network industries are under the restriction of a unversal service obligation. However, it seems to be necessary to examine the effects of temporary access suspension, since the introduction of competition in the network industries *itself* can allow the freedom to temporary suspension of activation of any players. Hence, in this section, we provide an analysis on temporary access suspension.

To derive the equilibrium under open access policy and temporary access suspension, we need to consider the follower's choice as in the previous analysis. For simplicity, we assume that after temporary access suspension, the follower can re-start production with no cost.

As examined before, we must firstly derive the value of each project before obtaining the values of the three strategies.

When the access project is undertaken, its value is:

$$V^{AA}(Y) = C_2 Y^{\beta_2} + \frac{Y\widehat{\Pi}(2)}{r-\alpha} - \frac{v}{r}. \tag{15}$$

where $C_2 \equiv \frac{\left(v/\widehat{\Pi}(2)\right)^{1-\beta_2}}{\beta_1-\beta_2}\left[(1-\beta_1)\frac{\widehat{\Pi}(2)}{r-\alpha}+\beta_1\frac{\widehat{\Pi}(2)}{r}\right]$. The first term of (15) represents the suspension value generated by costlessly temporary access suspension. This occurs because the follower can breakdown the access to the leader's network facility when it faces a negative profit. When the bypass project is undertaken, the value of the project is the same as (5):

$$V^{BB}(Y) = \frac{Y\Pi(2)}{r-\alpha} \quad \left(= V^{B}(Y)\right) \tag{16}$$

Then, using (15) and (16), we can define the value of *the transition project*,

$\Delta\widetilde{V}(Y)$, i.e., the difference in the values between the bypass project and the access project:

$$\Delta\widetilde{V}(Y) \equiv V^{BB}(Y) - V^{AA}(Y) = \frac{Y\Delta\Pi(2)}{r-\alpha} + \frac{v}{r} - C_2 Y^{\beta_2} \tag{17}$$

Note that the value of transition project with temporary access suspension is less than that without it by $C_2 Y^{\beta_2}$. Thus, we expect that the timing to exercise the option of the transition project with temporary access suspension is delayed, which will be shown below.

Defining the option value of the transition project and using the value-matching and the smooth-pasting conditions, we can derive the trigger point Y^{BB*}, which is characterized by the solution of the following equation.

$$C_2(\beta_1 - \beta_2)\left(Y^{BB*}\right)^{\beta_2} - (\beta_1 - 1)\frac{\Delta\Pi(2)}{r-\alpha}Y^{BB*} + \beta_1\left[I^m - \frac{v}{r}\right] = 0. \tag{18}$$

We next derive the trigger point Y^{AA*} at which the access project begins. Defining the option value of the access project and using the vale-matching and the smooth-pasting conditions, we have the trigger point Y^{AA*}, which is characterized by the solution of the following equation.

$$C_2(\beta_1 - \beta_2)\left(Y^{AA*}\right)^{\beta_2} + (\beta_1 - 1)\frac{\widehat{\Pi}(2)}{r-\alpha}Y^{AA*} - \beta_1\left[\frac{v}{r} + I^e\right] = 0. \tag{19}$$

The way to characterize the follower's strategy is the same as in the previous analysis. When $Y^{BB*} < +\infty$ and $(0 <) Y^{AA*} < Y^{BB*}$, we can say that the follower takes the access-to-bypass strategy. When $Y^{BB*} = +\infty$ and $Y^{AA*} (> 0)$ exists, the follower takes the access strategy. When Y^{BB*} is smaller than Y^{AA*}, the follower takes the bypass strategy. It is apparent that the follower does not adopt the access strategy under $\Delta\Pi(2) > 0$ and a geometric Brownian motion. Then, we can characterize the follower's choice of strategy.

Proposition 7 *The follower takes the access-to-bypass strategy (the bypass strategy) if and only if* $\Omega\left(Y^{B}\right) \geq (<)\, 0$*, where*

$$\Omega(Y) \equiv C_2(\beta_1-\beta_2)Y^{\beta_2} - (\beta_1-1)\frac{\Delta\Pi(2)}{r-\alpha}Y + \beta_1\left[I^m - \frac{v}{r}\right] \tag{20}$$

Proof. See Appendix. ■

Then, as in the analysis before, we have the bypass competition equilibrium and the access-to-bypass competition equilibrium, depending on the follower's choice of strategy.

Let us examine the effect of temporary access suspension on the trigger points of the players. First, we report the effect of temporary access suspension on the follower's trigger point in this paper.

Proposition 8 *(i)* $Y^{AA*} < Y^{A*}$ *and (ii)* $Y^{B*} < Y^{BB*}$

Proof. See Appendix. ■

The results of Proposition 8 are also intuitively appealing. In fact, the allowance of temporary access suspension has the same effect as the reduction of access charge. Hence, in the access-to-bypass competition equilibrium, the entry timing of the follower by access becomes earlier than without open access policy. However, the introduction of positive network externality generated by an additional network facility becomes later. If the follower adopts the bypass strategy (i.e., in the bypass competition equilibrium), the entry timing of the follower is exactly the same as in the facility-based competition equilibrium.[10]

[10] Since the curvatures of the leader's and the follower's value functions depend on several environmental variables such as the severeness of product competition (i.e., the level of the exogenously given duopoly profit), the degree of positive network externality, the volatility parameter σ, the leader's trigger pioint, which is characterized by the intersection between them, is hard to analyze. Hence, we postpone the formal analysis as a future work.

7 Concluding Remarks

The purpose of this paper was to examine the effect of open access policy on the incentive for building network facilities or infrastructures and on the degree of competition in network industries. In particular, to address this issue, we empoyed a *real options* approach, since uncertainty and irreversibility of investment are influential factors when considering investment problems.

In the model, we assumed that a bypass construction creates a positive network externality through which firms can benefit an increase in profit. We firstly showed that, in addition to the level of access charge, a choice of entry strategy by a follower (i.e., a second entrant) is affected by two effects, i.e., the positive network externality effect and the transition-option effect. The transition-option effect occurs only under uncertainty and irreversibility of investment, and it implies that, once entering the market by access, a follower wants to implement the bypass project at a moment in the future, even though the total cost of bypass is larger than that of access. This is because, in addition to the positive network externality, the access payment can be avoided by exercising the option of the transition project (i.e., by builiding its own network facility), when the follower enters by access. Hence, the transition-option effect makes entry by access more preferable than entry by bypass.

Then, comparing a competition under an open access policy with a facility-based competition, we confirmed that allowing access to an essential facility makes a follower's entry earlier than in a facility-based competition. Then, we showed an incumbent's incentive for network investment under open access policy can be larger than without open access, depending on the relative magnitude between the level of access charge and a positive network externality generated by an additional network facility. In particular, when both the access charge and the positve network externality are zero, a leader, which later becomes an incumbent, in a competition under open access policy enter with the construction of a network earlier than in a facility-based competition. However, if the level of access charge is high and the positive network externality generated by a bypass construction is small, the leader in a competition under open access policy enters the market earlier than in the facility-based competition. Lastly, we discussed an effect of temporary access suspension on the follower's incentive to enter, which suggests that temporary access suspension makes the follower's entry earlier than without it.

Appendix

Proof of Proposition 3

We show that $Y^B < Y^{B*}$ if and only if $Y^{A*} < Y^{B*}$. This is because

$$
\begin{aligned}
Y^{A*} &\equiv \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\widehat{\Pi}(2)}\left(I^e + \frac{v}{r}\right) < \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Delta\Pi(2)}\left(I^m - \frac{v}{r}\right) \equiv Y^{B*} \\
&\Leftrightarrow \left(\Pi(2) - \widehat{\Pi}(2)\right)\left(I^e + \frac{v}{r}\right) < \widehat{\Pi}(2)\left(I^m - \frac{v}{r}\right) \\
&\Leftrightarrow \Pi(2)\left(I^e + \frac{v}{r}\right) - \widehat{\Pi}(2) I^e < \widehat{\Pi}(2) I^m \\
&\Leftrightarrow \Pi(2)\left(I^e + \frac{v}{r}\right) + \Pi(2) I^m - \widehat{\Pi}(2) I^e < \Pi(2) I^m + \widehat{\Pi}(2) I^m \\
&\Leftrightarrow \Pi(2)(I^e + I^m) - \widehat{\Pi}(2)(I^e + I^m) < \Pi(2)\left(I^m - \frac{v}{r}\right) \\
&\Leftrightarrow Y^B \equiv \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Pi(2)}(I^e + I^m) < \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\Delta\Pi(2)}\left(I^m - \frac{v}{r}\right) \equiv Y^{B*}.
\end{aligned}
$$

Therefore, we have $Y^B < Y^{B*}$ in the access-to-bypass equilibrium. ■

Proof of Proposition 4

We define $P^{AB}(Y) \equiv V_L^{AB}(Y) - V_F^{AB}(Y)$ at $Y\left(< Y^{A*}\right)$. Substituting (13) and (14) into $P^{AB}(Y)$ and arranging, we have

$$\begin{aligned} P^{AB}(Y) &= \frac{Y\Pi(1)}{r-\alpha} - (I^e + I^m) \\ &\quad + \left(\frac{Y}{Y^{A*}}\right)^{\beta_1}\left\{I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right\} \\ &\quad + \left(\frac{Y}{Y^{B*}}\right)^{\beta_1}\left\{I^m - 2\frac{v}{r}\right\}. \end{aligned} \tag{21}$$

To prove the proposition, we will check the sign of $P^{AB}(Y_L)$ where Y_L is the trigger point of the leader under facility-based competition equilibrium.

Note that Y_L is characterized by $V_L^B(Y_L) = V_F^B(Y_L)$, so that we have

$$\begin{aligned} &\frac{Y_L\Pi(1)}{r-\alpha}\left[1 - \left(\frac{Y_L}{Y^B}\right)^{\beta_1 - 1}\right] + \left(\frac{Y_L}{Y^B}\right)^{\beta_1}\frac{Y^B\Pi(2)}{r-\alpha} - (I^e + I^m) \\ = &\left(\frac{Y_L}{Y^B}\right)^{\beta_1}\left[\frac{Y^B\Pi(2)}{r-\alpha} - (I^e + I^m)\right] \end{aligned}$$

or

$$\frac{Y_L\Pi(1)}{r-\alpha} - (I^e + I^m) = \left(\frac{Y_L}{Y^B}\right)^{\beta_1}\left[\frac{Y^B\Pi(1)}{r-\alpha} - (I^e + I^m)\right] \tag{22}$$

Substituting (22) into $P^{AB}(Y_L)$ gives

$$P^{AB}(Y_L) = (Y_L)^{\beta_1}\chi(\mathbf{x}),$$

where

$$\begin{aligned} \chi(\mathbf{x}) \equiv &\left(Y^B\right)^{-\beta_1}\left[\frac{Y^B\Pi(1)}{r-\alpha} - (I^e + I^m)\right] \\ &+ \left(Y^{A*}\right)^{-\beta_1}\left[I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right] + \left(Y^{B*}\right)^{-\beta_1}\left[I^m - 2\frac{v}{r}\right]. \end{aligned} \tag{23}$$

and $\mathbf{x} \equiv \left(v, \Pi(1), \Pi(2), \widehat{\Pi}(2), I^e, I^m\right)$.

When $v \to 0$ and $\Delta\Pi(2) \to 0$, $Y^B \to \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\widehat{\Pi}(2)}(I^e + I^m)$ and $Y^{B*} \to +\infty$. Hence, rearranging the terms, we have

$$\begin{aligned}
\chi(\mathbf{x})|_{v\to 0, \Delta\Pi(2)\to 0} &= \left(Y^B\right)^{-\beta_1}\left[\frac{Y^B\Pi(1)}{r-\alpha} - (I^e + I^m)\right] \\
&\quad + \left(Y^{A*}\right)^{-\beta_1}\left[I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right] \\
&= \left(\frac{\beta_1}{\beta_1 - 1}\frac{\Pi(1)}{\widehat{\Pi}(2)} - 1\right)\left(\frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{\widehat{\Pi}(2)}\right)^{-\beta_1} \\
\times \left[(I^e + I^m)^{1-\beta_1} - (I^e)^{1-\beta_1}\right] &< 0,
\end{aligned}$$

since $\Pi(1) > \widehat{\Pi}(2)$ and $\beta_1 > 1$. ■

Proof of Proposition 5

Note that at Region III $P^{AB}(Y_L) = 0$, since the follower's trigger point in the access-to-bypass competition equilibrium becomes identical to that in the bypass competition equilibrium (so to the facility-based competition equilibrium).

Let us evaluate $\frac{\partial\chi}{\partial v}$ at Region III in which $Y^B = Y^{A*} = Y^{B*}$. Then, we have the following result by rearranging the equation.

$$\begin{aligned}
&\left.\frac{\partial\chi}{\partial v}\right|_{Y^B = Y^{A*} = Y^{B*}} \\
&= \frac{(\beta_1)^2}{\beta_1 - 1}\frac{r-\alpha}{r}\frac{\left(Y^B\right)^{-\beta_1 - 1}}{\Delta\Pi(2)\widehat{\Pi}(2)\Pi(2)}\Phi(\mathbf{x}),
\end{aligned}$$

where $\Phi(\mathbf{x}) \equiv \Delta\Pi(2)\Pi(1)[I^e + I^m] + \Pi(2)\left[I^e\Delta\Pi(2) - I^m\widehat{\Pi}(2)\right]$. Here, we should also note that at Region III we have

$$\frac{v}{r} = -I^e + \frac{\widehat{\Pi}(2)}{\Pi(2)}(I^e + I^m)$$

or

$$\frac{v}{r} = \frac{-1}{\Pi(2)} \left[I^e \Delta\Pi(2) - I^m \widehat{\Pi}(2) \right]$$

Substituting it into the second term of $\Phi(\mathbf{x})$, we can rearrange it as

$$\Phi(\mathbf{x}) = \Delta\Pi(2)\,\Pi(1)\left[I^e + I^m\right] - (\Pi(2))^2 \frac{v}{r}$$

The sign of $\left.\frac{\partial P^{AB}(Y_L)}{\partial v}\right|_{Y^B = Y^{A*} = Y^{B*}}$ is identical to $\left.\frac{\partial \chi}{\partial v}\right|_{Y^B = Y^{A*} = Y^{B*}}$, which in turn is identincal to that of $\Phi(\mathbf{x})$. Figure 4 in the text shows the hyperplane of $\Phi(\mathbf{x}) = 0$ or $v/r = \Pi(1)\left[I^e + I^m\right]\left[(\Pi(2))^{-1} - \widehat{\Pi}(2)/(\Pi(2))^2\right]$. Above the hyperplane, we have $\left.\frac{\partial P^{AB}(Y_L)}{\partial v}\right|_{Y^B = Y^{A*} = Y^{B*}} < 0$, whereas we have $\left.\frac{\partial P^{AB}(Y_L)}{\partial v}\right|_{Y^B = Y^{A*} = Y^{B*}} > 0$ below it. Therefore, we have the claim in the proposition. ■

Proof of Proposition 6

Differentiating $\chi(\mathbf{x})$ with respect to v gives

$$\begin{aligned}
\frac{\partial \chi}{\partial v} &= -\beta_1 \left(Y^{A*}\right)^{-\beta_1 - 1} \left[I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha} \right] \frac{\partial Y^{A*}}{\partial v} \\
&\quad + \left(Y^{A*}\right)^{-\beta_1} \left[\frac{2}{r} - \frac{\Pi(1)}{r-\alpha} \frac{\partial Y^{A*}}{\partial v} \right] \\
&\quad - \beta_1 \left(Y^{B*}\right)^{-\beta_1 - 1} \left[I^m - 2\frac{v}{r} \right] \frac{\partial Y^{B*}}{\partial v} - \frac{2}{r}\left(Y^{B*}\right)^{-\beta_1} \\
&= \left(Y^{A*}\right)^{-\beta_1 - 1} \frac{\partial Y^{A*}}{\partial v} \left[-\beta_1 \left(I^e + 2\frac{v}{r} \right) + (\beta_1 - 1) \frac{Y^{A*}\Pi(1)}{r-\alpha} \right] \\
&\quad + \frac{2}{r} \left[\left(Y^{A*}\right)^{-\beta_1} - \left(Y^{B*}\right)^{-\beta_1} \right] - \beta_1 \left(Y^{B*}\right)^{-\beta_1 - 1} \left[I^m - 2\frac{v}{r} \right] \frac{\partial Y^{B*}}{\partial v} \qquad (24)
\end{aligned}$$

where $\frac{\partial Y^{A*}}{\partial v} = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\widehat{\Pi}(2)} \frac{1}{r}$ and $\frac{\partial Y^{B*}}{\partial v} = -\frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\Delta\Pi(2)} \frac{1}{r}$. Substituting (9) into Y^{B*}, the last term in (24) can be rewritten as

$$\begin{aligned}
& -\beta_1 \left(Y^{B*}\right)^{-\beta_1 - 1} \left[I^m - 2\frac{v}{r}\right] \frac{\partial Y^{B*}}{\partial v} \\
= & \left(\frac{\beta_1 (r-\alpha)}{\beta_1 - 1}\right)^{-\beta_1} (\Delta\Pi(2))^{\beta_1} \frac{\beta_1}{r(r-\alpha)} \left(I^m - \frac{v}{r}\right)^{-\beta_1 - 1} \left[I^m - 2\frac{v}{r}\right]
\end{aligned}$$

Hence, as $\Delta\Pi(2) \to 0$, this term goes to zero. Also, $\left(Y^{B*}\right)^{-\beta_1} \to 0$. Then we have

$$\begin{aligned}
& \left.\frac{\partial \chi}{\partial v}\right|_{\Delta\Pi(2)\to 0} \\
= & \left(Y^{A*}\right)^{-\beta_1 - 1} \frac{\partial Y^{A*}}{\partial v} \left[-\beta_1 \left(I^e + 2\frac{v}{r}\right) + (\beta_1 - 1) \frac{Y^{A*}\Pi(1)}{r-\alpha}\right] + \frac{2}{r}\left(Y^{A*}\right)^{-\beta_1} \quad (25)
\end{aligned}$$

Substituting $\frac{\partial Y^{A*}}{\partial v} = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\widehat{\Pi}(2)} \frac{1}{r}$ into (25) and rearranging it, we have

$$\begin{aligned}
\left.\frac{\partial \chi}{\partial v}\right|_{\Delta\Pi(2)\to 0} = & \left(Y^{A*}\right)^{-\beta_1 - 1} \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\widehat{\Pi}(2)} \frac{1}{r} \\
& \times \left\{\left[\beta_1 \left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 2\right) + 2\right] \frac{v}{r} + \left[\beta_1 \left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 1\right) + 2\right] I^e\right\} \quad (26)
\end{aligned}$$

Note that the second term in the bracket in (26) is positive, whereas the first term in it can be negative.

If $\left.\frac{\partial \chi}{\partial v}\right|_{\Delta\Pi(2)\to 0} > 0$ for any v/r $(\leq I^m)$, the statement of the proposition holds, since as $v/r \to I^m$ and $\Delta\Pi(2) \to 0$, we have $P^{AB}(Y_L) > 0$ becaue of the result of Porpposition 5.

Suppose $\left.\frac{\partial \chi}{\partial v}\right|_{\Delta\Pi(2)\to 0} < 0$.As $\Delta\Pi(2) \to 0$, $\chi(\mathbf{x})$ becomes

$$\begin{aligned}
\left.\chi(\mathbf{x})\right|_{\Delta\Pi(2)\to 0} = & \left(Y^B\right)^{-\beta_1} \left[\frac{Y^B \Pi(1)}{r-\alpha} - (I^e + I^m)\right] \\
& + \left(Y^{A*}\right)^{-\beta_1} \left[I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right]
\end{aligned}$$

The first term is definitely positive, because of $V_L^B(Y_L) > 0$ and (). The term in the brancket of the second term can be rewritten as

$$\begin{aligned} & I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha} \\ = \; & I^e + 2\frac{v}{r} - \frac{\beta_1}{\beta_1 - 1}\frac{\Pi(1)}{\widehat{\Pi}(2)}\left(I^e + \frac{v}{r}\right) \end{aligned}$$

Now, we can show that if $\frac{\partial \chi}{\partial v}\big|_{\Delta\Pi(2)\to 0} < 0$, then $\chi(\mathbf{x})|_{\Delta\Pi(2)\to 0} > 0$. In fact, when $\frac{\partial \chi}{\partial v}\big|_{\Delta\Pi(2)\to 0} < 0$, i.e.,

$$\left[\beta_1\left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 2\right) + 2\right]\frac{v}{r} + \left[\beta_1\left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 1\right) + 2\right] I^e < 0$$

which implies

$$\left[\beta_1\left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 2\right) + 2\right]\frac{v}{r} + \left[\beta_1\left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 1\right) + 1\right] I^e < 0$$

Then, we have

$$\begin{aligned} & (\beta_1 - 1)\left[I^e + 2\frac{v}{r} - \frac{\beta_1}{\beta_1 - 1}\frac{\Pi(1)}{\widehat{\Pi}(2)}\left(I^e + \frac{v}{r}\right)\right] \\ = \; & -\left\{\left[\beta_1\left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 2\right) + 2\right]\frac{v}{r} + \left[\beta_1\left(\frac{\Pi(1)}{\widehat{\Pi}(2)} - 1\right) + 1\right] I^e\right\} > 0, \end{aligned}$$

which means that $I^e + 2\frac{v}{r} - \frac{Y^{A*}\Pi(1)}{r-\alpha} > 0$, so that $\chi(.)|_{\Delta\Pi(2)\to 0} > 0$. This argument implies that if $\chi(.)|_{\Delta\Pi(2)\to 0} < 0$, $\frac{\partial \chi}{\partial v}\big|_{\Delta\Pi(2)\to 0} > 0$, which in turn means that even in this case, we can find the region in which $\chi(.)|_{\Delta\Pi(2)\to 0} > 0$.

In sum, when $\Delta\Pi(2) \simeq 0$, as v/r increases, there exists the lower bound of v/r above which $P^{AB}(Y_L) > 0$ holds. ■

Proof of Proposition 7

We define:

$$\Psi(Y) \equiv C_2(\beta_1 - \beta_2) Y^{\beta_2} + (\beta_1 - 1) \frac{\widehat{\Pi}(2)}{r - \alpha} Y - \beta_1 \left[\frac{v}{r} + I^e \right] \tag{27}$$

and

$$\Omega(Y) \equiv C_2(\beta_1 - \beta_2) Y^{\beta_2} - (\beta_1 - 1) \frac{\Delta\Pi(2)}{r - \alpha} Y + \beta_1 \left[I^m - \frac{v}{r} \right]. \tag{28}$$

Note that $\Psi\left(Y^{AA*}\right) = 0$ and $\Omega\left(Y^{BB*}\right) = 0$. It is easy to check the shape of $\Psi(Y)$ and $\Omega(Y)$. In fact, when $\Delta\Pi(2) > 0$, $\Psi'(Y) > 0$ (for $Y\widehat{\Pi}(2) > v + rI^e$), $\Psi"(Y) > 0$, $\Omega'(Y) < 0$, and $\Omega"(Y) > 0$. We can also confirm that $\Psi'\left(Y^{AA*}\right) > 0$ and $\Omega'\left(Y^{BB*}\right) < 0$.

Then, from $\Psi\left(Y^{AA*}\right) = 0$, $\Omega\left(Y^{BB*}\right) = 0$, $\Psi'\left(Y^{AA*}\right) > 0$ and $\Omega'\left(Y^{BB*}\right) < 0$, we find that the necessary and sufficient condition for $Y^{AA*} \leq Y^{BB*}$ is that at Y^+ where $\Psi(Y^+) = \Omega(Y^+)$ and $\Omega(Y^+) > 0$ (or $\Psi(Y^+) > 0$). Furthermore, from $\Psi(Y^+) = \Omega(Y^+)$, we have $Y^+ = \frac{\beta_1}{\beta_1 - 1} \frac{r-\alpha}{\Pi(2)} (I^e + I^m) = Y^B$. ■

Proof of Proposition 8

Substituting Y^{A*} into $\Psi(Y)$ (in the proof of Proposition 7), we have $\Psi\left(Y^{A*}\right) > 0$. From the shape of $\Psi(Y)$, we confirm the claim of (i). Similarly, substituting Y^{B*} into $\Omega(Y)$ gives $\Omega\left(Y^{B*}\right) > 0$, which confirms the claim of (ii). ■

References

[1] Alleman, J. and Noam, E. eds. (1999), *The New Investment Theory of Real Options and its Implications for Telecommunications Economics*, Boston, MA: Kluwer Academic Publishers.

[2] Dixit, A. K. and R. S. Pindyck, (1994). *Investment under Uncertainty*, Princeton, NJ: Princeton University Press.

[3] Fudenberg, G. and J. Tirole (1985). "Preemption and Rent Equalisation in the Adoption of New Technology", *Review of Economic Studies*, 52, 383-401.

[4] Hori, K. and K. Mizuno (2004). "Network Investment and Competition with Access-to-Bypass", mimeo.

[5] Kaserman, D. L. and M. Ulrich (2002). "The Competitive Effects of Resale versus Facility-Based Entry: Evidence from the Long-Distance Market", *Telecommunications Policy*, 26, 415-424.

[6] Katz, M. K. and C. Shapiro (1987). "R&D Rivalry with Licensing or Limitation", *American Economic Review*, 77(3), 402-429.

[7] Laffont, J-J. and J. Tirole (2000). *Competition in Telecommunications*, Cambridge, MA: MIT Press.

[8] Nielsen, M. J. (2002). "Competition and Irreversible Investments", *International Journal of Industrial Organization*, 20, 731-743.

[9] Smits H. T. J. and L. Trigeorgis (2004), *Strategic Investment*, Princeton, NJ: Princeton University Press.

[10] Trigeorgis, L. (2004), *Real Options*, Cambridge, MA: MIT Press.

[11] Valletti, T. M. (2003), "The Theory of Access Pricing and its Linkage with Investment Incentives", *Telecommunications Policy*, 27, 659-675.

[12] Woroch, G. A. (2002). "Local Network Competition", in *Handbook of Telecommunications Economics*, vol.1, (edited by Cave, M. E., S. K. Majumdar, and I. Vogelsang), Elsevier Science B.V.

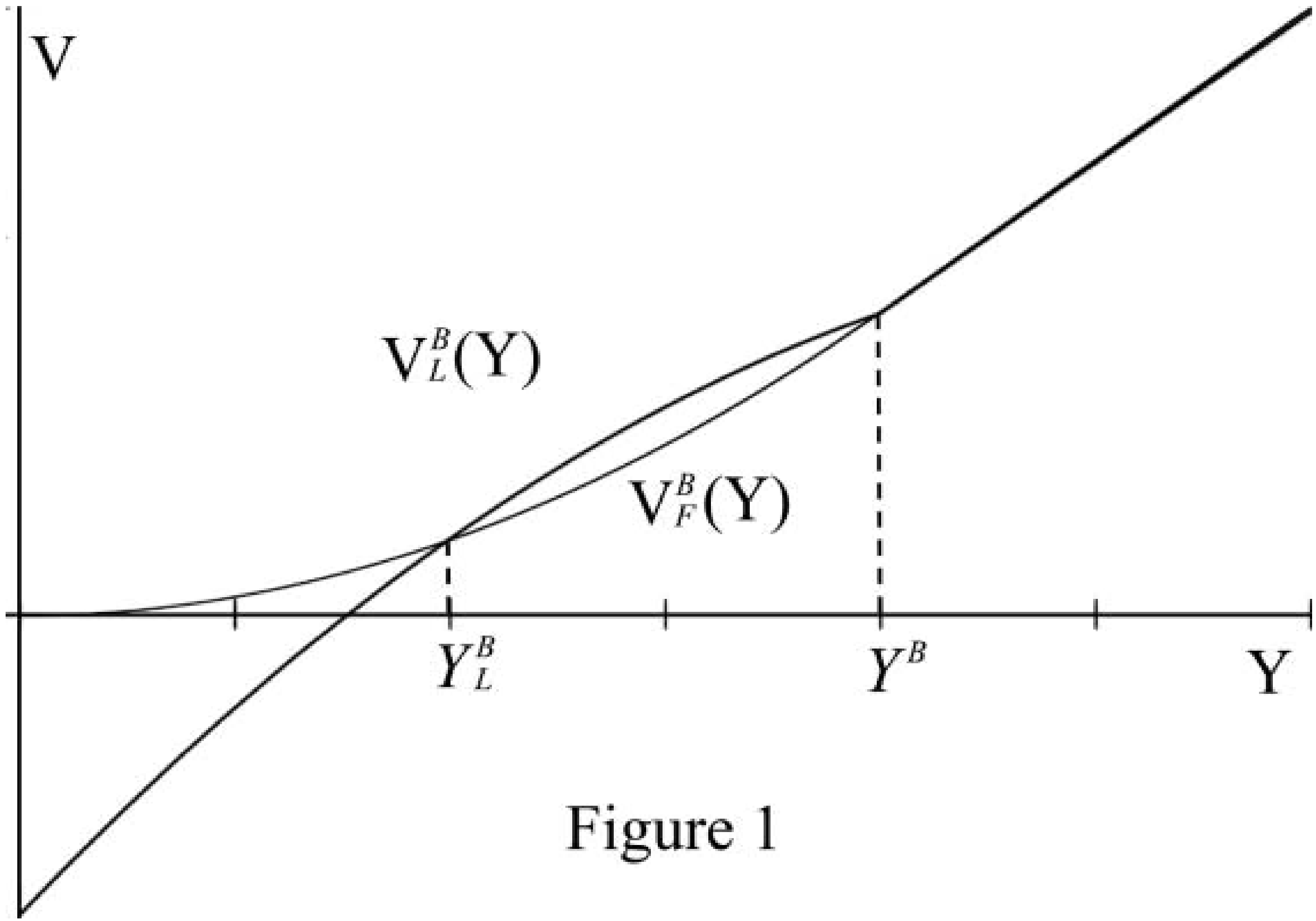

Figure 1

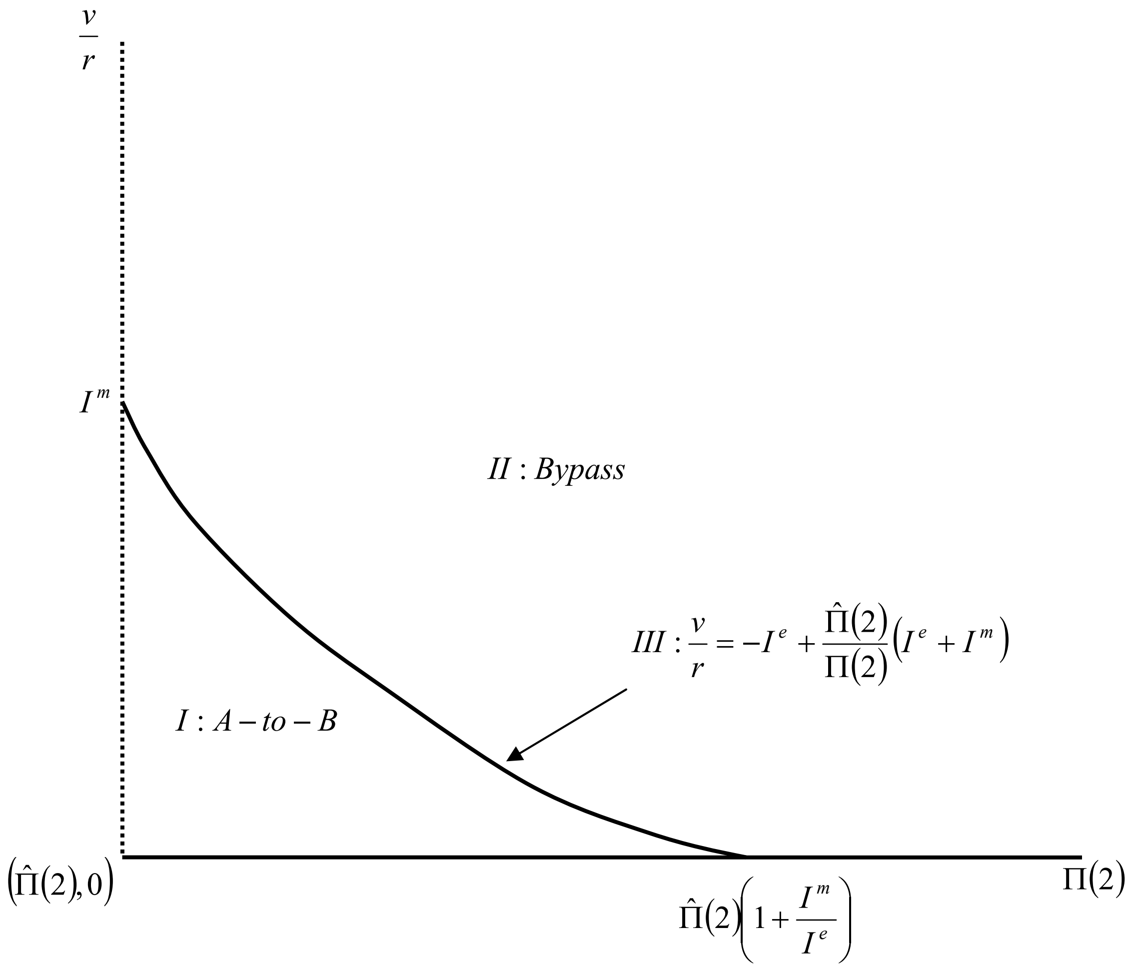

Figure 2: The Follower's Strategy

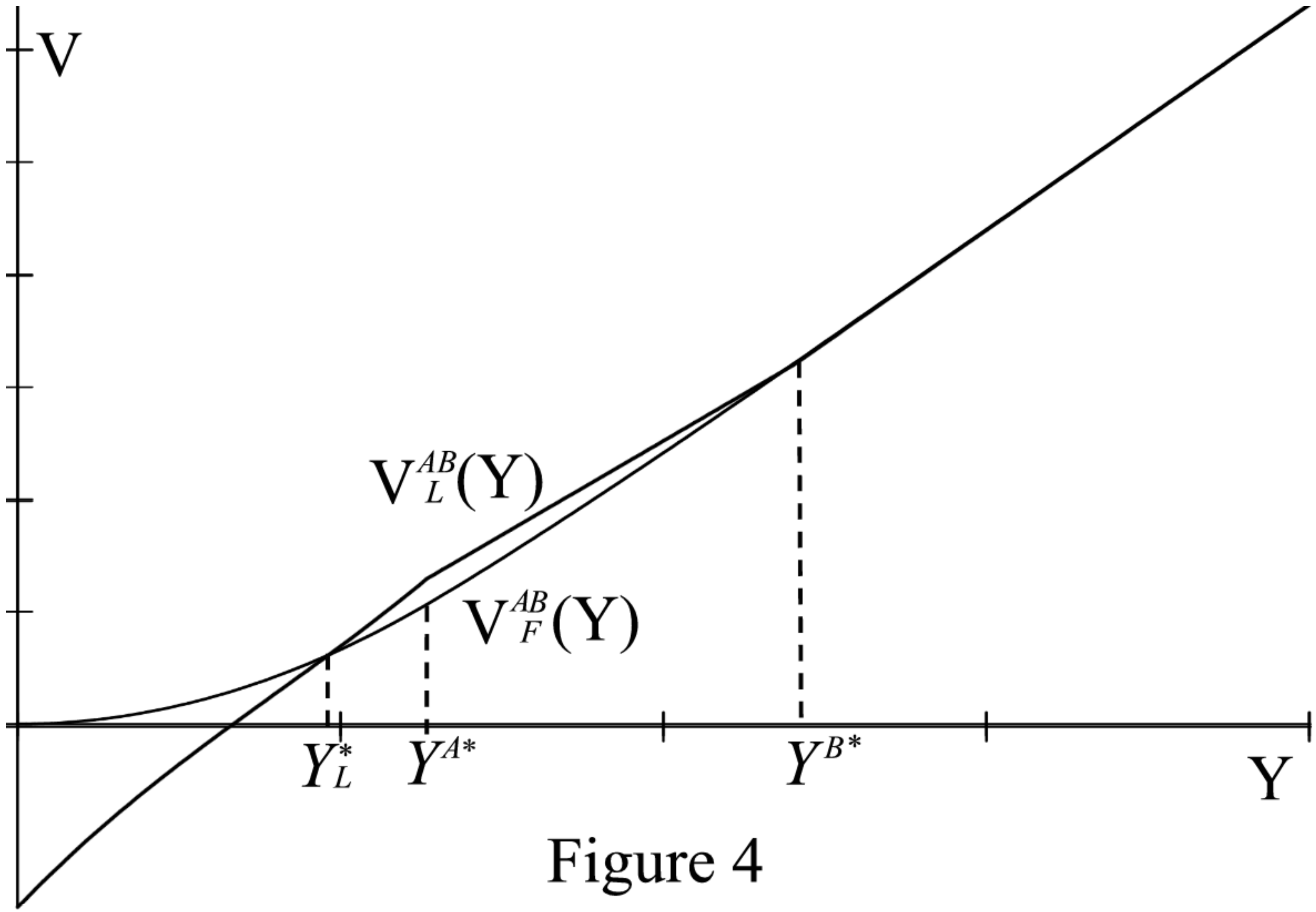

Figure 4

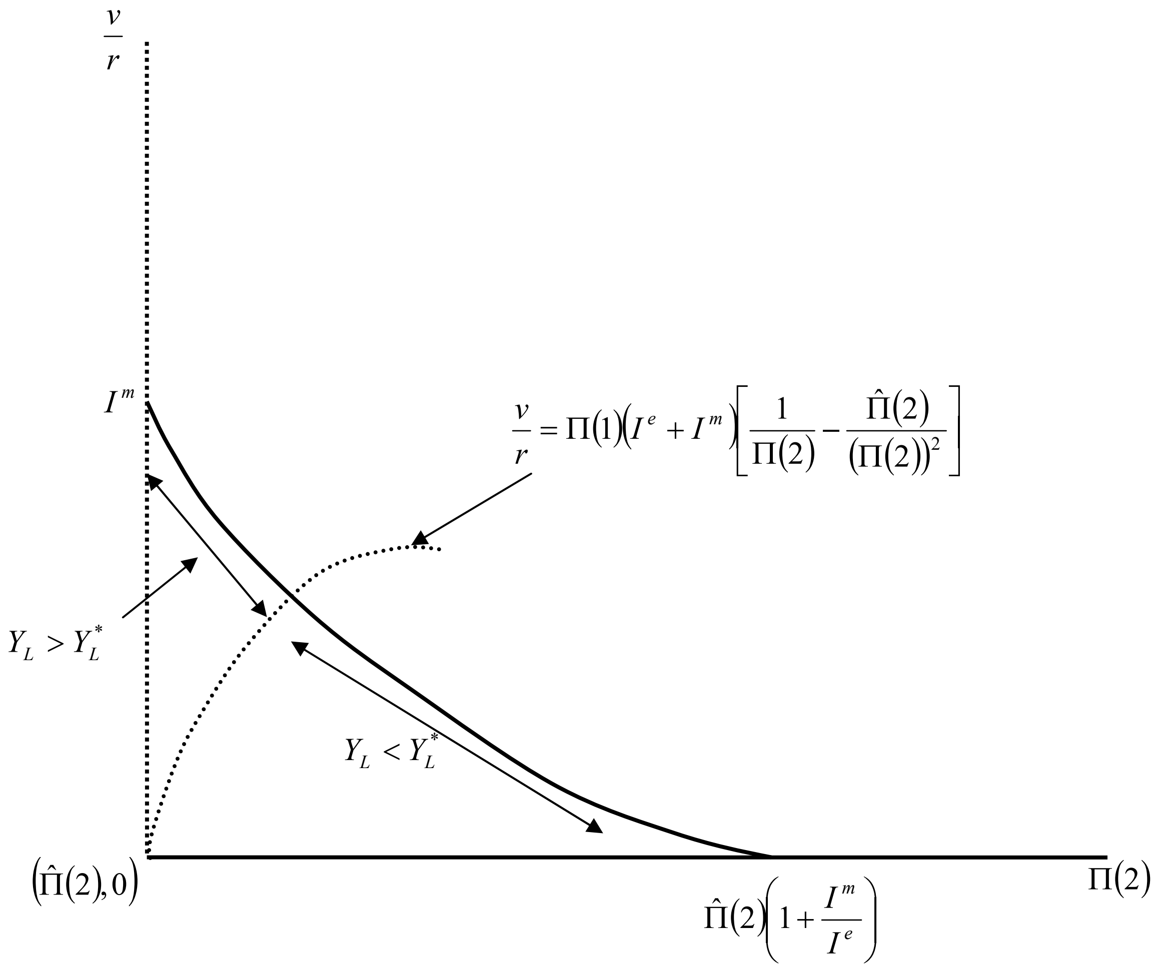

Figure 4: The Leader's Entry Timing