# Real Extreme R&D Options[1]

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# REAL EXTREME R&D OPTIONS

## ABSTRACT

Clinical R&D is a highly uncertain venture where experiments achieve successful outcomes on an extraordinarily rare basis. Just one successful product could change the future of a company; the stage to discovery can often be an invaluable or disastrous experience. Developers should balance probability analysis against the potential profits that may result. With that objective, we propose extreme-value theory (EVT) as an extension to the standard perpetual American call option. We develop an R&D option model when discoveries follow extreme-value distributions. We examine the optimal trigger that justifies an investment, the effect of frequency in discoveries on real option values, the roles of tail-shape parameters of discovery distributions, and compare values to invest with models governed by other distributions. We find that effective premiums for options based on extreme-distributions should be lower (and triggers for optimal investment higher) than those governed by normally distributed underlying values, with otherwise similar parameters. The sensitivities of option values and triggers to changes in the shape parameter of the extreme distributions are simulated, and show interesting, perhaps counter-intuitive results.

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## 1. Introduction

Rare events in real options have been predominately described by stochastic processes such as Poisson and jump processes after Merton's (1976) work. In R&D literatures, rare events are frequently referred to as catastrophe or success. Schwartz and Moon (2000) assumed that asset value uncertainty and cost uncertainty are standard geometric Brownian diffusion processes, but they also acknowledge the possibility of a catastrophic failure represented by a Poisson process. Weeds (2002) considered discovery to occur following a Poisson distribution with a constant hazard rate in her competitive real option models. This hazard rate is independent of the duration of the research and number of rival firms which are rushing to find an identified compound to patent.

More recently, new studies used Extreme Value Theory (EVT) as an alternative to the Poisson process in modelling rare events. An example is Poon, Rockinger and Tawn (2004) analysis of rare events in the tails of return distributions. They modelled extreme values in different aspects of financial markets which can usually be controlled by a Poisson jump process in place of rare events. While extreme value theory is relatively new in application to real options, it has been used in value-at-risk (VAR) issues in insurance and risk management[3]. Together with extensive literature on non-Gaussian pricing models that involve high moments like Lévy processes, they commonly

[3] See Embrechts, Klüppelberg and Mikosch (2003) for treatment of extreme value methodology for random walk models, continuous-time stochastic processes and compound Poisson processes in both insurance and finance applications.

concentrate on upper tail ends of distributions which have a leptokurtic nature that decays more like a power law function, as compared to a normal bell curve[4].

Extreme value methodologies seem to be appropriate for valuing R&D projects due to the low frequency of rare discoveries and potential upside profits in a winner-takes-all outcome. The value of an R&D project at its initial stages of development can be rightly dependent on its prospect of a blockbuster product. One successful product or clinical compound could change the fortune of an R&D company. In the pharmaceutical industry, such events are rare and extreme. Myers and Howe (1997) pointed out that even as a drug enters into the market, there is a ten percent probability of an extreme popularity. So we model option pricing of R&D activities using extreme value theory (EVT).

Dahan and Mendelson (2001) were among the first to apply EVT to option pricing in innovation. They value abandonment options of concept tests when profits follow extreme patterns. They found that the optimal number of tests is related to cost of production for all three forms of extreme distribution. Rhys and Tippett (2003) also derived a closed-form solution for their value functions in the case when net present values follow a different class of probability distribution. They used a general class of Student distribution, which also has similar fat tail-end effects exhibited by extreme distributions. Brach and Paxson (2001) simulated skewed tail returns that look like extreme distributions by using the Merton (1976) jump process to model hot gene discovery.

[4] See Schoutens (2003) on introduction to option pricing using Lévy processes.

We develop a real option model based on extreme distributions, where our project values for a compound discovery follow a standard stochastic process. Then, we allow our expected value to follow some extreme density functions and derive our function values for the option to invest in a R&D project. We consider perpetual American call options for all of our models.

Our approach is different from Dahan and Mendelson (2001). Firstly, we are looking at the option to invest, hence the firm's opportunity to create success with its discoveries and its ensuing rewards. Secondly, our project value *x* follows a stochastic flow rate characterised by random walk behaviour.

The remaining sections are organised as follows: Section 2 lays out some background on extreme value theory. Section 3 includes description of extreme model in valuing perpetual call options. Section 4 shows results of sensitivity analyses of value functions. We also derive our partial derivatives of the value functions to changes in selected parameters and variables in this section. Section 5 concludes.

## 2. Extreme Results in R&D

In R&D business, a big 'transforming' success is hard to come by so project values frequently show extremity characteristics. We ask what is the project's option value and when the firm should enter to invest in the project, given the underlying project value is not normally distributed. In this section, we will introduce some extreme value theories.

The mathematical foundation of Extreme Value Theory (EVT) is a class of extreme value limit laws first derived by Fisher and Tippett (1928) and later by Gnedenko (1943). The summary of Fréchet, Weibull and Gumbel distributions can be summarised below:

***Table 1 – Summary of Three Distributions***

| | PDF | CDF | Mean | Variance |
|---|---|---|---|---|
| **Fréchet** | $\frac{\alpha}{\gamma}\left(\frac{x-x_0}{\gamma}\right)^{-\alpha-1} e^{-\left(\frac{x-x_0}{\gamma}\right)^{-\partial}}$ | $e^{-\left(\frac{x-x_0}{\gamma}\right)^{-\alpha}}$ | $E[x]=x_0+\gamma\Gamma\left(\frac{\alpha-1}{\alpha}\right)$ | $\gamma^2\Gamma\left(\frac{\alpha-2}{\alpha}\right)-\left[\gamma\Gamma\left(\frac{\alpha-1}{\alpha}\right)\right]^2$ |
| **Weibull** | $\frac{\alpha}{\gamma}\left(\frac{x_0-x}{\gamma}\right)^{\alpha-1} e^{-\left(\frac{x_0-x}{\gamma}\right)^{\alpha}}$ | $e^{-\left(\frac{x_0-x}{\gamma}\right)^{-\alpha}}$ | $E[x]=x_0-\gamma\Gamma\left(\frac{\alpha+1}{\alpha}\right)$ | $\gamma^2\Gamma\left(\frac{\alpha+2}{\alpha}\right)-\left[\gamma\Gamma\left(\frac{\alpha+1}{\alpha}\right)\right]^2$ |
| **Gumbel** | $\frac{1}{\gamma}e^{-\left(\frac{x-x_0}{\gamma}\right)} e^{-e^{\left(\frac{x-x_0}{\gamma}\right)}}$ | $e^{-e^{-\left(\frac{x-x_0}{\gamma}\right)}}$ | $E[x]=x_0+\gamma.\phi$ where ϕ is Euler's constant | $\gamma^2\frac{\Pi^2}{6}$ |

where α is the shape parameter, γ is the scale parameter, $X_0$ is the location parameter and $\Gamma[.]$ is incomplete gamma function. $x>0,\ 0<\alpha\le 2,\ \gamma>0,\ x_0\ge 0$.

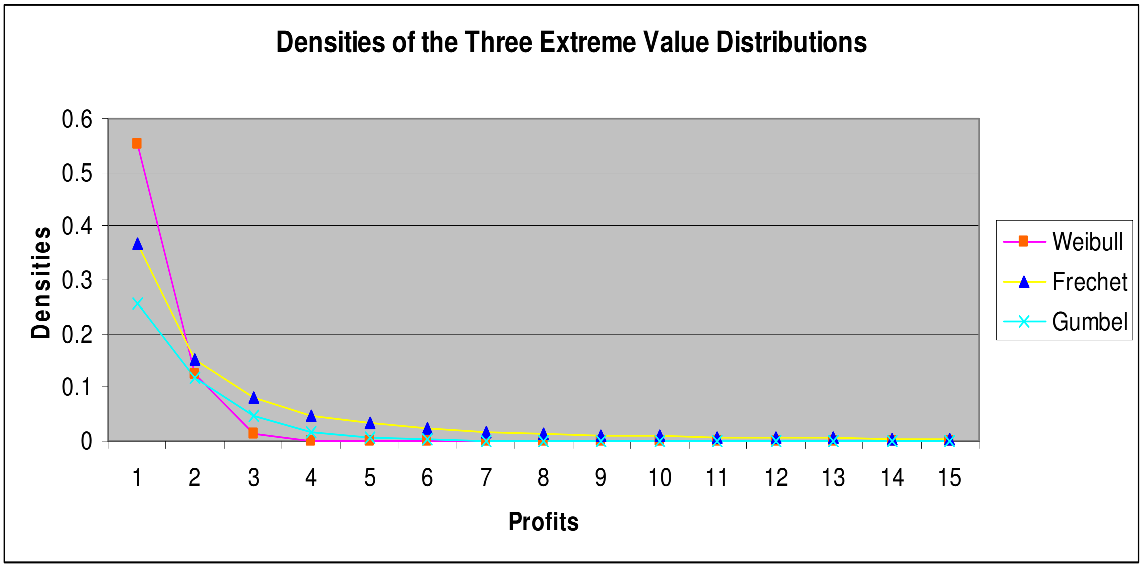
The Fréchet distribution generally has a longer and thinner tail on the right hand corner of the curve. It is also negatively skewed. It is not upper-bounded in its tail. This behaviour of the cash flows is particularly applicable to R&D activity. Consider a R&D compound with a great upside uncertainty as possible blockbuster drug. The tail-end pattern of Fréchet distribution can reflect the opportunistic return in the rare event of a discovery of an amazing drug. Biotechnology firms then can value their R&D projects given such extremist assumptions. This might give the management new insights into investment decisions as paths of events are assumed not to follow the normal distribution.

The Weibull distribution has an upper bound and hence a cap on the upside profit potential of the new drug. Hence the product does not have unlimited profits, which are prevalent with Fréchet distribution. We can use Weibull distribution if the management has a target profit to achieve and this target is regarded as a rare occurrence. Alternatively, it can also be used to check what value the project will be if they suffer an unlimited loss. This is because Weibull distribution has fat tails on its negative ends. However, this is not really our interest in this paper as we are concerned about maximising R&D returns and not minimising R&D losses.

In many industries, there seem to be no boundaries on gross profit potential of a product. However profit coming from outside the central range of the distribution seems extremely unlikely. Gumbel distribution might be appropriate where there can be unlimited gains or losses in projects.

Figure 1 shows an example of the three extreme-value distributions to profits, normalised to zero mean and unit variance.

**Figure 1**




***Densities for the Three Extreme Value Distributions***

## 3. The Model

Consider an R&D project to investigate and discover new and valuable genes. The project takes a certain amount of time to complete and for simplicity, we assume perpetuity here. Assume that the project has a constant total cost to completion, *K*. The number of 'prize' genes at the end of R&D project is denoted by *n*. When the project is successfully completed, the firm receives a payoff (e.g., a new compound to be sold to another external biotech) whose value, *V*, is determined as the present value of expected future net cash flows from the completed project. *F(V)* is the value of investment opportunity we derived and we referred to as the project value.

The estimated value of the asset received on completion of the project *V*, is assumed to follow the stochastic process:

$$dV = \mu V dt + \sigma V d\omega \tag{1}$$

where $\mu$ is the drift parameter, $\sigma$ is the standard deviation and $d\omega$ is the increment of a standard Wiener process. Schwartz and Moon (2000) highlighted the value of $\mu$, the drift component, which can be positive or negative in R&D programs due to various reasons. A negative drift could represent the opportunity cost of delaying the investment. Because we aim to value the upside of R&D returns, we shall assume $\mu > 0$ and $\sigma > 0$. The estimated asset value can be interpreted as the present value of the expected net cash flows to the project once the investment is completed, discounted at a risk-adjusted discount rate.

**3.1 Value of Investment Opportunity**

Let F(V) represent the value of the investment opportunity. This value function of opportunity to invest for the firm must satisfy the following Bellman equation:

$$rF(V)dt = E[dF(V)] \tag{2}$$

where r is the risk-free rate.

Using Ito's lemma, we obtain the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \mu V F'(V) - rF(V) = 0 \tag{3}$$

This is Euler's equation which has the general solution of:

$$F(V) = AV^{\beta_1} + BV^{\beta_2} \tag{4}$$

where A and B are constants, $\beta_1$ and $\beta_2$ are:

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{5}$$

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{6}$$

Our $\beta$ has to be positive and hence $\beta_2$ is eliminated and we will use $\beta_1$. We assume that if our variable, *V* reaches zero, our investment opportunity naturally falls to zero, and so we can obtain the following boundary condition:

$$F(0) = 0 \tag{7}$$

Since as the state variable goes to zero, the function has to decrease, B in equation (4) has to be equal to zero, hence our solution becomes:

$$F(V) = AV^{\beta_1} \tag{8}$$

This solution must also satisfy the boundary conditions of value-matching and smooth-pasting. Value-matching condition will ensures that our value function follows the discounted expected value after an investment point. Smooth-pasting condition as the name suggested will smooth out our value function. The two conditions are stated below:

$$F(V^*) = Z(V^*) - K \tag{9}$$

$$F^{'}(V^*) = Z^{'}(V^*) = 1 \tag{10}$$

**3.2 Present Values**

At this point, we depart from conventional approach of using Poisson distribution to estimate arrival rates of new drugs. We assume that in a pre-clinical environment, new genes or compounds are discovered following an extreme distribution. Hence, *n* number of discoveries in this case will follow an extreme distribution and it can be denoted by $F(n)$ where *F* is an extreme probability density function. The rate of discovery is not a focus here; we predict the number of discoveries adjusting to a probability throughout the life of the R&D process. The R&D process will have independent discovery outcomes, denoted by $N_i$, *I = 1, 2,…n,* that are observed at the conclusion of the test. The $n_i$ are i.i.d. random variables distributed as a random variable *N* with probability distribution *F(n)* i.e. $F(n) = \Pr\{N \le n\}$.[5] Discovery outcomes are directly related to expected value of

[5] *n* can be an integer in a discrete process or rational number in a continuous process, representing 'blockbuster status' or its equivalent. $N_i, i = 1,2 \ldots n$

project after completion. Our expected asset values will follow the probabilistic pattern of the discoveries and is given by,

$$Z(V) = V.f(N) \tag{11}$$

The NPV of the project would thus be

$$NPV = Z(V) - K \tag{12}$$

The optimal investment rule would be to proceed with the investment if and only if present value is greater than initial capital outlay, i.e. NPV is positive. Not surprisingly, (12) appears like one of the boundary conditions needed to satisfy differential equation (3).

**3.3 Option to Invest**

As *n* follows a univariate Fréchet distribution (a class of distribution under family of extreme distribution), the probability density function will be denoted by:

$$f(n) = \frac{\alpha}{\gamma}\left(\frac{n - n_0}{\gamma}\right)^{-\alpha-1} e^{-\left(\frac{n-n_0}{\gamma}\right)^{-\alpha}} \tag{11}$$

where $n_0$ is the location parameter of the distribution, $\gamma$ is its scale parameter which varies monotonically with variance, and $\alpha$ is the tail-shape parameter.

The boundary between the continuation region and the stopping region is given by a critical value of the stochastic process or trigger point, $V^*$. The optimal investment rule is found by solving for $V^*$. If the state variable is smaller than the trigger, the optimal decision for the investor is not to invest, i.e. to continue in the continuation region. If it exceeds the trigger, then the investor should invest in the project.

Applying equation (8) and (12) to our smooth pasting condition given by (9), we get

$$F(V^*) = V^* f(n) - K$$
$$A(V^*)^\beta = V^*\left[\frac{\alpha\gamma^\alpha}{(n-n_0)^{\alpha+1} e^{\left(\frac{\gamma}{n-n_0}\right)^\alpha}}\right] - K \tag{12}$$

where $V^*$ is the optimal asset value of the project where the firm should be investing.

The second condition, known as the value-matching condition, requires that the derivatives of the option value and present value match at the boundary:

$$F^{'}(V^*) = Z^{'}(V^*)$$
$$A\beta(V^*)^{\beta-1} = \frac{\alpha\gamma^\alpha}{(n-n_0)^{\alpha+1} e^{\left(\frac{\gamma}{n-n_0}\right)^\alpha}} \tag{13}$$

Solving equations (12) and (13) and rearranging terms, we obtain $A$ and $V^*$ as:

$$V^* = \frac{K\beta}{\left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1} e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right](\beta-1)} \tag{14}$$

and

$$A = \frac{\left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1} e^{\left(\frac{\gamma}{n=n_0}\right)^{\alpha}}}\right]^{\beta}}{\beta\left(\dfrac{K\beta}{\beta-1}\right)^{\beta-1}} \tag{15}$$

Putting equations (8), (12), (14) and (15) together we obtain the value function of the option as a function of asset value and number of discoveries as they following a Fréchet-distributed probability:

$$F(V)_{Frechet} = \begin{cases} \dfrac{\left[\dfrac{\alpha\gamma^{\alpha}}{(\mathrm{n} - \mathrm{n}_0)^{\alpha+1} e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right]^{\beta}}{\beta\left(\dfrac{K\beta}{\beta-1}\right)^{\beta-1}} \mathrm{V}^{\beta} & \text{if } V < V^* \\ V\left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1} e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right] - K & \text{if } V \geq V^* \end{cases} \tag{16}$$

This value function is the option to invest and is governed by three parameters that characterise the tail-end of Fréchet probability. Using a similar approach and computation, we apply two other classes of extreme distributions to value our possible project discoveries. $F(V)_{Weibull}$ and $F(V)_{Gumbel}$ denote project values when R&D breakthroughs follow Weibull and Gumbel distributions respectively. A summary is shown in Table 2.

### Table 2 – Summary of Results for the Three Extreme-Value Distributions

| | Fréchet | Weibull | Gumbel |
|---|---|---|---|
| Probability Density Function | $f(n)=\frac{\alpha}{\gamma}\left(\frac{n-n_0}{\gamma}\right)^{-\alpha-1} e^{-\left(\frac{n-n_0}{\gamma}\right)^{-\alpha}}$ | $f(n)=\frac{\alpha}{\gamma}\left(\frac{n_0-n}{\gamma}\right)^{\alpha-1} e^{-\left(\frac{n_0-n}{\gamma}\right)^{\alpha}}$ | $f(x)=\frac{1}{\gamma}e^{-\left(\frac{n-n_0}{\gamma}\right)}e^{-e^{-\left(\frac{n-n_0}{\gamma}\right)}}$ |
| $V^*$ | $\dfrac{K\beta}{\left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1}e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right](\beta-1)}$ | $\dfrac{K\beta}{\left[\dfrac{\alpha\left(\frac{n_0-n}{\gamma}\right)^{\alpha-1}}{\gamma e^{\left(\frac{n_0-n}{\gamma}\right)^{\alpha}}}\right](\beta-1)}$ | $\dfrac{K\beta}{\left[e^{\left(-\left(\frac{n-n_0}{\gamma}\right)-e^{-\left(\frac{n-n_0}{\gamma}\right)}\right)}\right](\beta-1)}$ |
| Option Value | $F(V)_{Frechet}\begin{cases}\dfrac{\left[\dfrac{\alpha\gamma^{\alpha}}{(\mathrm{n}-\mathrm{n}_0)^{\alpha+1}e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right]^{\beta}}{\beta\left(\frac{K\beta}{\beta-1}\right)^{\beta-1}}\mathrm{V}^{\beta} & \text{if } V<V^* \\ V\left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1}e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right]-K & \text{if } V\geq V\end{cases}$ | $F(V)_{Weibull}\begin{cases}\dfrac{\left[\dfrac{\alpha\left(\frac{n_0-n}{\gamma}\right)^{\alpha-1}}{\gamma e^{\left(\frac{n_0-n}{\gamma}\right)^{\alpha}}}\right]^{\beta}}{\beta\left(\frac{K\beta}{\beta-1}\right)^{\beta-1}}\mathrm{V}^{\beta} & \text{if } V<V^* \\ V\left[\dfrac{\alpha\left(\frac{n_0-n}{\gamma}\right)^{\alpha-1}}{\gamma e^{\left(\frac{n_0-n}{\gamma}\right)^{\alpha}}}\right]-K & \text{if } V\geq V^*\end{cases}$ | $F(V)_{Gumbel}\begin{cases}\dfrac{\left[e^{\left(-\left(\frac{n-n_0}{\gamma}\right)-e^{-\left(\frac{n-n_0}{\gamma}\right)}\right)}\right]^{\beta}}{\beta\left(\frac{K\beta}{\beta-1}\right)^{\beta-1}}\mathrm{V}^{\beta} & \text{if } V<V^* \\ V\left[e^{\left(-\left(\frac{x-x_0}{\gamma}\right)-e^{-\left(\frac{n-n_0}{\gamma}\right)}\right)}\right]-K & \text{if } V\geq V^*\end{cases}$ |

## 4. Results and Sensitivities

Figure 2 illustrates the comparison of project values as discoveries follow in three different forms of distributions. All parameters are the same except for the shape parameter, $\alpha$, which is endogenously different for each distribution because of their characteristic tail-ends.

**Figure 2 – Project Values As Function of Asset Values for Different Distributions**

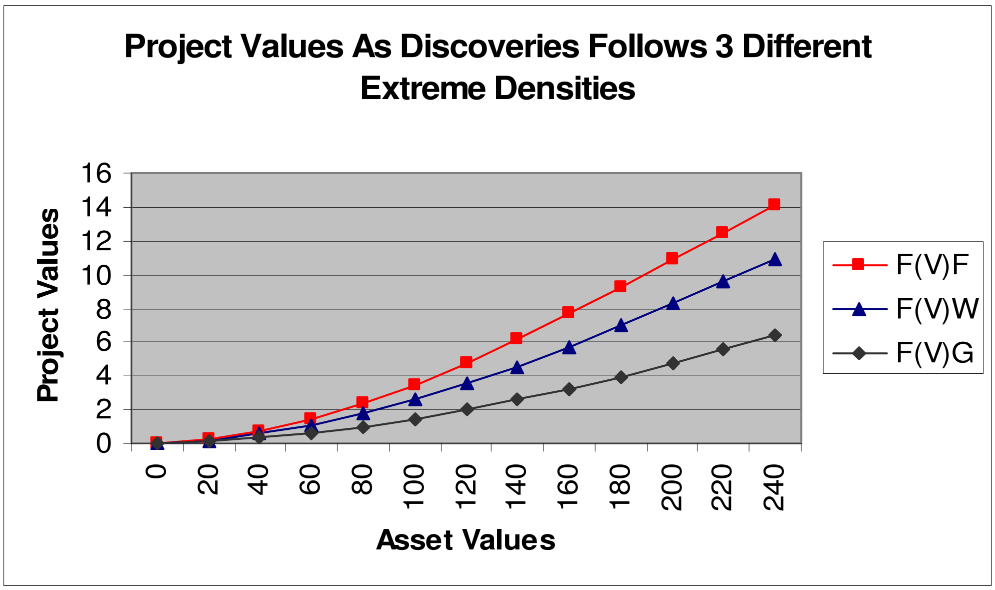



Fig 2 - The parameters are: $\sigma = 10\%$, r = 9%, $\mu$ = 5%, K = 5, $n_0$ = 0, $n$ = 3, $\gamma$ = 1, $\alpha_{Fréchet} = 1.5$, $\alpha_{Weibull}$ = -2.

The figures imply that our usual option to invest will increase as asset values increases. These option values are diminished by the extreme probabilistic tendencies to find new drugs at pre-clinical stages. Investment triggers are high and firms will have to wait for expected asset values to rise before it is optimal to invest. Hence these 'extreme' options are out-of-the-money for 'mega' returns in R&D. Project values dominated by a Fréchet-sort of pattern in discovery will generate the highest option values, given the parameters. This is evident from the fat and long tail-ends in Fréchet distribution. Project values from Weibull and Gumbel are also close reflection of their statistical distribution characteristics. Depending on the successes and failures of R&D unit in past researches, some firms might have distribution patterns that follow one of these three extreme scenarios.

In Figure 3 we plot and compare the sensitivity of value functions to volatility.

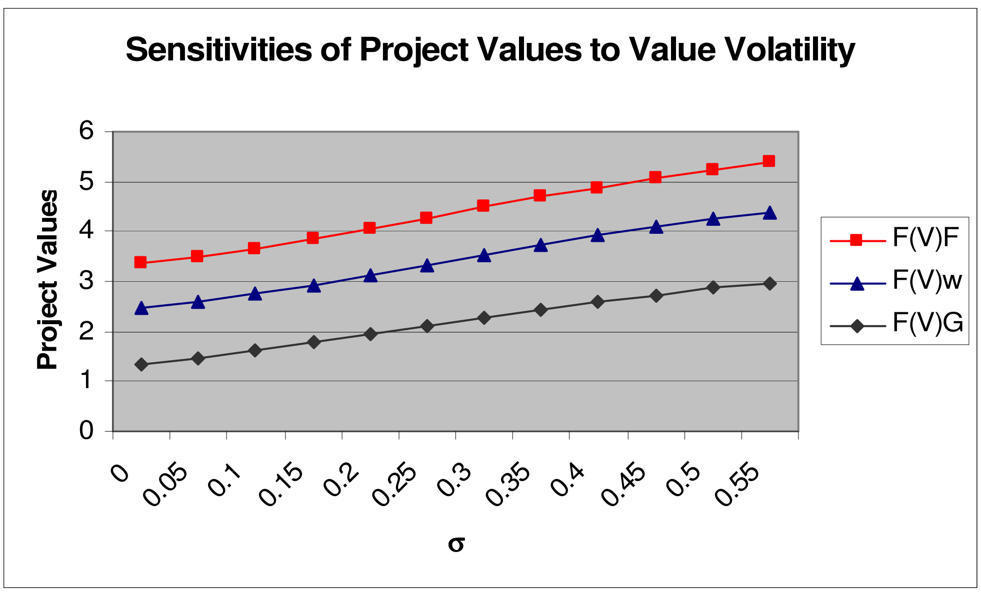



Fig 3 - The parameters are: V = 100, r = 9%, μ = 5%, K = 5, $n_0$ = 0, $n$ = 3, γ = 1, $\alpha_{Fréchet} = 1.5$, $\alpha_{Weibull}$ = -2.

As expected, our project values for all distributions increase with asset volatility. Fréchet has the highest sensitivity given the parameters. In the next Table, we will compare value volatility to our trigger values. We include values from a GBM model that does not follow any specific distribution for present values and one that uses Poisson distribution.

**Table 3 – Sensitivities of Trigger Functions to Value Volatilities**

| σ | V**F* | V**W* | V**G* | V**N* | V**P* |
|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.05 | 145.20 | 173.89 | 243.32 | 1.46 | 187.98 |
| 0.10 | 154.99 | 185.61 | 259.72 | 3.22 | 200.66 |
| 0.15 | 169.88 | 203.44 | 284.68 | 5.59 | 219.93 |
| 0.20 | 188.97 | 226.30 | 316.66 | 8.95 | 244.65 |
| 0.25 | 211.77 | 253.60 | 354.87 | 13.83 | 274.17 |
| 0.30 | 238.08 | 285.11 | 398.97 | 21.03 | 308.23 |
| 0.35 | 267.83 | 320.75 | 448.83 | 31.79 | 346.75 |
| 0.40 | 301.04 | 360.51 | 504.47 | 48.08 | 389.74 |
| 0.45 | 337.72 | 404.44 | 565.94 | 73.00 | 437.23 |
| 0.50 | 377.93 | 452.60 | 633.33 | 111.61 | 489.29 |
| 0.55 | 421.72 | 505.03 | 706.71 | 172.15 | 545.98 |
| 0.60 | 469.13 | 561.81 | 786.16 | 268.31 | 607.36 |

Table 3 - The parameters are: V = 100, r = 9%, μ = 5%, K = 5, $n_0$ = 0, $n$ = 3, γ = 1, $\alpha_{Fréchet} = 1.5$, $\alpha_{Weibull}$ = -2, $\lambda = 1$. $V_{*F}$, $V_{*W}$ and $V_{*G}$ symbolise trigger functions for Fréchet, Weibull and Gumbel. $V_{*N}$ refers to value triggers for call options with discoveries following a normal distribution. $V_{*P}$ is a trigger when discovery value is multiplied by a Poisson-distributed pattern in discovery and λ is size of Poisson jump.

In general, we can see that our trigger values increase with volatility. The 'extreme-led' triggers are larger than a process with Gaussian-expected R&D success, given these parameters. This implies that extreme distributions can cause firms to invest much later by rule of optimality. This is because firms that enjoy *n* number of discovery successes are rare and they should wait for expected asset values to rise higher before taking action to invest. Interestingly, Poisson-induced trigger values are above those of Fréchet and Weibull. It is possible that Poisson jumps have greater intensity on R&D outcomes. The term 'all or nothing' is more applicable to Poisson jump process. It also shows that

Poisson-distributed results are more reactive to changes in volatility than extreme distributions. Of the three extreme distributions, Fréchet has the lowest investment threshold and this could be due to the tail-end of Fréchet distribution decaying faster per unit time.

Figure 4 plots the graph shown in Table 3.

**Figure 4 – Sensitivities of Trigger Values to Volatility**

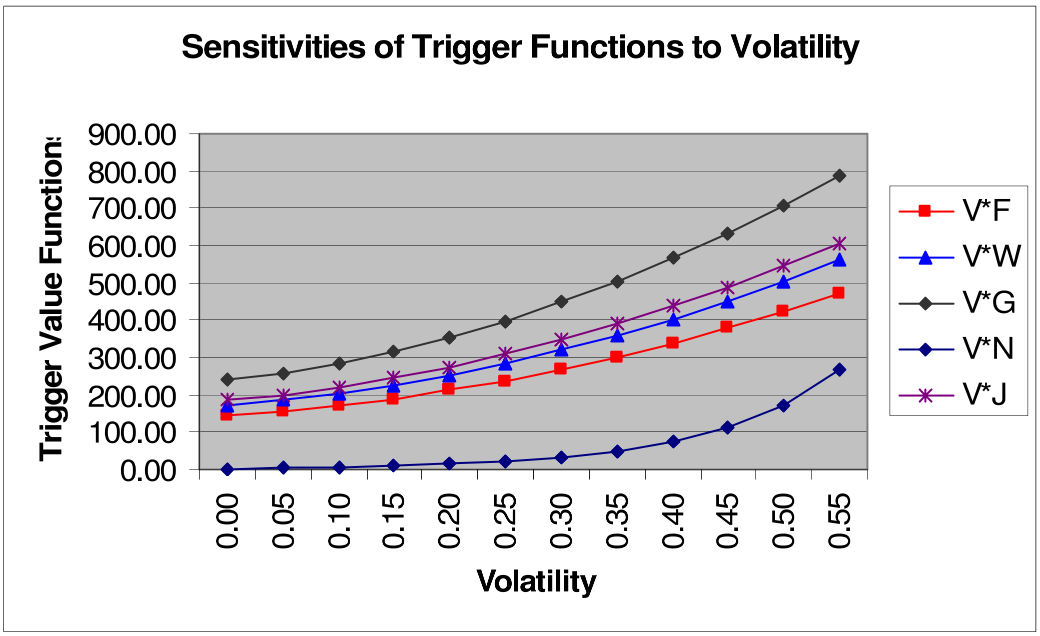



Figure 4 - The parameters are: V = 100, r = 9%, μ = 5%, K = 5, $n_0$ = 0, $n$ = 3, γ = 1, $\alpha_{Fréchet} = 1.5$, $\alpha_{Weibull}$ = -2, λ = 1

Figure 5 exemplifies how the number of discoveries can affect option values. At first sight, it appears interesting that the value of investment opportunity actually decreases with an increase in number of discoveries. This is not unusual since our discoveries are predicted by such extreme distributions and they imply that it is very rarely possible that one discovers a huge number of breakthroughs at one go. Because this is unrealistic, value of this investment opportunity will slide to zero as number of discoveries approach infinity.

**Figure 5 – Sensitivities of Project Values to Number of Discoveries**

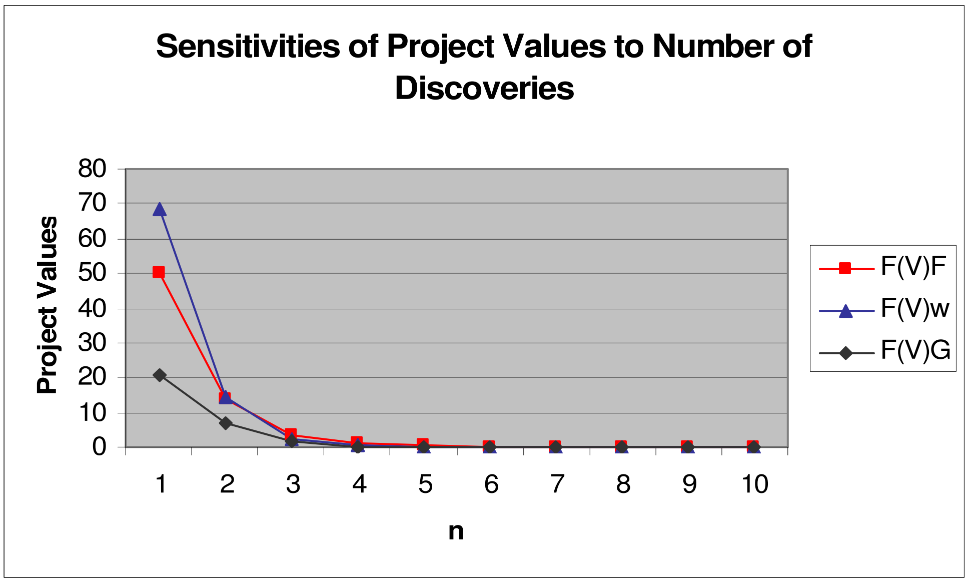



Figure 5 - The parameters are: V = 100, $\sigma$ = 10%, r = 9%, $\mu$ = 5%, k = 5, $n_0$ = 0, $\gamma$ = 1, $\alpha_{Fréchet} = 1.5$, $\alpha_{Weibull}$ = -2.

As the absolute value of the tail-shape parameter, $\alpha$ increases, the Fréchet and Weibull distribution converge to Gumbel. This is shown in Figure 6. This is consistent with extreme value theory put up Von Mises (1936)[6]. Figure 6 shows that both Fréchet and Weibull project values converge as $\alpha$ increases. This will also have a converging effect

[6] Von Mises (1936) stated that the three extreme distributions can be unified under a single continuous model $F(x) = e^{-\left(1+\frac{x}{\alpha}\right)^{-\alpha}}$ where the distribution is Fréchet is $\alpha > 0$, Weibull if $\alpha < 0$ and Gumbel as $\alpha \to \infty$.

on trigger values (see Figure 4). Also the project values appear to be highest between 1 < α < 2. Samorodnistsky and Taqqu (1994) noted these as stable α models. The slower α is, the slower the decay of the distribution at tail-end and the heavier the tails. Hence our option values are increasing between these α levels.

**Figure 6 – Sensitivities of Project Values to Shape Parameter, α**

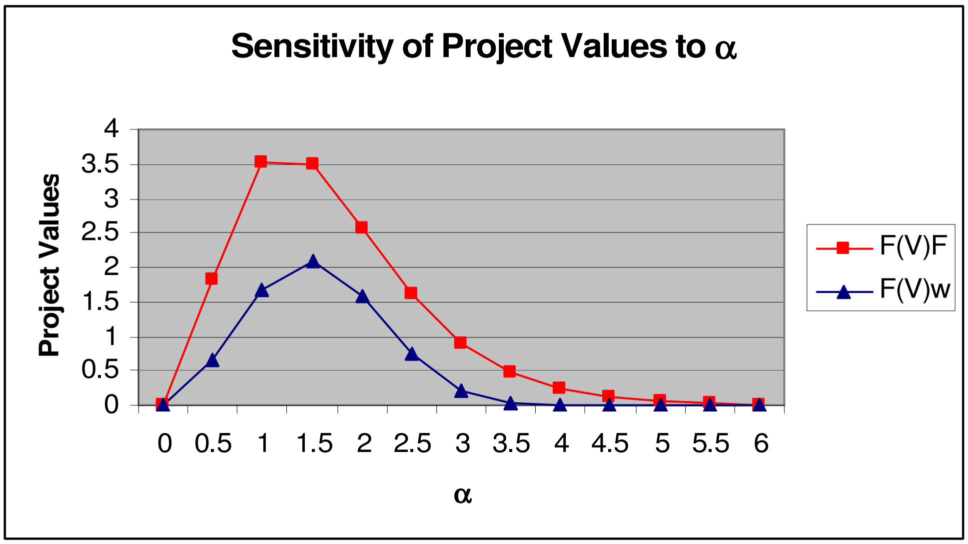



Figure 4 - The parameters are: V = 100, σ = 10%, r = 9%, μ = 5%, k = 5, $n_0$ = 0, $n$ = 3, γ = 1.

Thus the upper-tailed shape of the discovery distribution, as parameterised by α, plays a pivotal role in determining the optimal investment policy and the project values that result from that policy.

The sensitivity of the option value *F(V)*, to small changes in its determining variables can shed further light on the impact that extreme distributions can have on option values. We begin by computing the most common, option delta, which is the derivative of the option value *F(V)* with respect to the expected asset value of the project. The closed-form solutions of the deltas of the extreme distributions are summarised in the next table.

**Table 4 - Value of Delta Functions**

| | | | |
|---|---|---|---|
| Deltas | $F(V)_{Frechet}\begin{cases}\dfrac{\left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1}e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right]^{\beta}}{\left(\dfrac{K\beta}{\beta-1}\right)^{\beta-1}}V^{\beta-1} & \text{if } V<V^{*}\\ \left[\dfrac{\alpha\gamma^{\alpha}}{(n-n_0)^{\alpha+1}e^{\left(\frac{\gamma}{n-n_0}\right)^{\alpha}}}\right] & \text{if } V\geq V^{*}\end{cases}$ | $F(V)_{Weibull}\begin{cases}\dfrac{\left[\dfrac{\alpha\left(\dfrac{n_0-n}{\gamma}\right)^{\alpha-1}}{\gamma e^{\left(\frac{n_0-n}{\gamma}\right)^{\alpha}}}\right]^{\beta}}{\left(\dfrac{K\beta}{\beta-1}\right)^{\beta-1}}V^{\beta-1} & \text{if } V<V^{*}\\ \left[\dfrac{\alpha\left(\dfrac{n_0-n}{\gamma}\right)^{\alpha-1}}{\gamma e^{\left(\frac{n_0-n}{\gamma}\right)^{\alpha}}}\right] & \text{if } V\geq V^{*}\end{cases}$ | $F(V)_{Gumbel}\begin{cases}\dfrac{\left[e^{\left(-\left(\frac{n-n_0}{\gamma}\right)-e^{-\left(\frac{n-n_0}{\gamma}\right)}\right)}\right]^{\beta}}{\left(\dfrac{K\beta}{\beta-1}\right)^{\beta-1}}V^{\beta-1} & \text{if } V<V^{*}\\ \left[e^{\left(-\left(\frac{x-x_0}{\gamma}\right)-e^{-\left(\frac{n-n_0}{\gamma}\right)}\right)}\right] & \text{if } V\geq V^{*}\end{cases}$ |

**Figure 7 – Value of Delta for Options Following Extreme Distributions.**

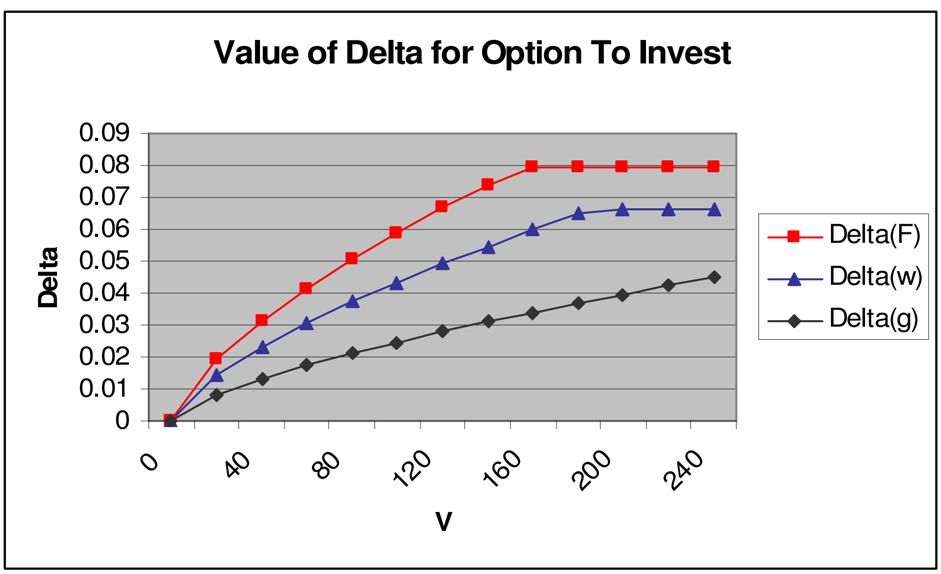



Figure 7 - The parameters are: σ = 10%, r = 9%, μ = 5%, k = 5, $n_0$ = 0, $n$ = 3, γ = 1, $\alpha_{Fréchet} = 1.5$, $\alpha_{Weibull}$ = -2.

Figure 7 contains the option delta for three different modes of discovery distributions. As expected, our delta values are positive and increasing with values. Furthermore it shows that the option delta is near to zero for deep-out-of-the-money options where it is not optimal to invest at all. The deltas of all three functions approach constants as trigger level are reached, when the options are in-the-money. Also note that the delta changes most quickly at near to zero for all cases. For out-of-the money options, Fréchet has the highest gamma (sensitivity of delta to changes in underlying value), while Gumbel has the lowest gamma. This is again due to the density functions of these respective distributions (see Fig 1). Gumbel has a lower peak at the centre of distribution and also declines slower at the tail-ends. On the other hand, Fréchet is the opposite with its high peak in the middle of distribution and steep decay towards the end-tails. Since Fréchet has the highest gamma, delta hedging of options governed by Fréchet will complex.

## 5. Conclusions

R&D activities are high-risk investment projects and their scale of returns is dependent on the success in producing break-through formulae, genetic compounds or blockbuster drugs. Occurrences of success are often rare and extreme. We have quantified the expected benefits of conducting high-risk R&D by taking an extreme and conservative perspective to the business. Our contributions include valuing the real option to invest in R&D, calculating the optimal point of entry into investment, determining the trend of discoveries with project values and identifying the important role played by the tail-shape parameter. Using the statistical theory of extreme values, we explore the option values generated under Fréchet-, Gumbel- and Weibull-distributed discoveries. Holding asset mean, variances and cost constant, we show that the upper tail-shape of the distributions of discoveries drives our probabilities of finding R&D success, the value of incentive to invest and the option to wait before investing.

Our results can be summarised as follows. Depending on rational assumptions of our parameters, the option value to invest has shown significant difference when using extreme distributed R&D data as compared to Gaussian distributed R&D data. This reinforced the worries and difficulties of firms heavily involved in R&D activities. Though extreme distribution and its close counterpart like Poisson distribution have shown evidence that they can create high expected values upon completion of these R&D projects, the large trigger values suggest that they should wait further before embarking on investment. Also interesting to note is that our number of discoveries follows an inverse relationship with project values. More discoveries do not suggest higher payoffs partly because in R&D discoveries, being a rare commodity, can sometimes mean only one researcher emerges as the winner. Fréchet seems to be most sensitive to changes in most parameters (except volatility) in our comparative study. Since its option value is the highest among the three distributions, it appears to be the best distribution to fit into an R&D search for top drugs. The manager's knowledge of the upper-tail shape of the discovery distribution should guide response on when to invest. Estimation of shape parameter of the distribution is a key factor. Though parameter estimation is not the focus of this paper, it is critical to simulate data on breakthrough emergence from the underlying distribution[7].

[7] See Reiss and Thomas (1997) on fitting observed maxima to the three-extreme-value distributions.

R&D has long been acknowledged to be a long, tedious and risky activity where returns may have fat-tailed proportions. Future research could include the uncertainty of cost in R&D to this paper. We can also introduce extreme value distribution into a stochastic process itself so as value parameters on basis of randomness rather than probabilities.

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