Beyond Investment-Cash Flow Sensitivities: Using Indirect Inference to Estimate Costs of External Funds*

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Abstract

This paper estimates costs of external finance, applying indirect inference to a dynamic structural model where the corporation endogenously chooses investment, distributions, leverage and default. The corporation faces double taxation, costly state verification in debt markets, and linear-quadratic costs of external equity. Consistent with direct evidence on underwriter fee schedules, behavior is best explained by rising marginal costs of external equity, starting at 3.9%. Contrary to the notion that corporations are debt conservative, leverage is consistent with small (12.2%) bankruptcy costs. Investment-cash flow sensitivities are not a sufficient statistic for financing costs. The cash flow coefficient decreases in external equity costs and increases in bankruptcy costs. When the model is simulated using our parameter estimates, the cash flow coefficient across Fazzari, Hubbard, and Petersen's dividend classes is U-shaped. The difference between cash flow coefficients across dividend classes actually decreases as costs are increased.

The most studied question in empirical corporate finance over the past fifteen years is the source of positive investment-cash flow sensitivities.[1] In their influential paper, Fazzari, Hubbard, and Petersen (FHP) (1988) argue the significance of cash flow, particularly for low payout firms, demonstrates capital markets are imperfect. If this were the extent of their claim, it is doubtful that investment-cash flow sensitivities would have attracted so much attention. After all, few would argue that the necessary conditions identified by Modigliani and Miller (1958) for financial irrelevance are satisfied. However, there is considerable debate

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[1]See Hubbard (1998) for a survey.

regarding the *magnitude* of financial frictions. This brings up the second, more intriguing, part of FHP's claim, that investment-cash flow sensitivities increase monotonically in financing frictions. If this claim were indeed true, then corporate finance theorists and empiricists could look to a single number, the cash flow coefficient, in order to gauge the size of financial frictions.

The neoclassical theory of investment is our starting point for interpreting the findings of FHP. Hayashi (1982) shows that the shadow price of capital, or marginal q, is a sufficient statistic for investment in a setting with adjustment costs.[2] Facilitating empirical testing, Hayashi shows that an *observable* variable, average q, is equal to unobservable marginal q if: 1) capital is homogeneous; 2) the profit and adjustment cost functions are homogeneous degree one; and 3) financial markets are perfect. In a more general setting, Abel and Aberly (1994) show the second condition can be relaxed, with marginal q equal to $\rho *$average q if both functions are homogeneous degree ρ. In both models, average q is a sufficient statistic for investment under the maintained assumptions.

Some attribute the significance of cash flow to financial market imperfections. However, violations of the first two assumptions may also account for the significance of cash flow. Hayashi and Inoue (1991) show that a special "capital aggregator" restriction must be added if the firm uses multiple capital goods. In perfect capital markets settings, Gomes (2001), Alti (2003), and Abel and Eberly (2004) show that cash flow is significant when profits are concave in capital and the investment cost schedule is linear. Based upon indirect inference, Cooper and Ejarque (2001) conclude that market power is actually the main source of cash flow effects.

In his discussion of the paper by FHP (1988), Poterba (1988) argued that measurement error in average q potentially explains the significance of cash flow. Consistent with this view, Perfect and Wiles (1994) and Lewellen and Badrinath (1997) document substantial variation in average q values depending on the methods used to impute the replacement cost of capital and debt value. In addition, Erickson and Whited (2001) find that cash flow is insignificant when measurement error-consistent GMM estimators are exploited.

The sheer duration of the debate suggests that corporate finance economists will not reach consensus regarding the magnitude of financial frictions based on investment-cash flow sensitivities. In retrospect,

[2] Lucas and Prescott (1971), Mussa (1977), and Abel (1983) also link investment to marginal q.

it seems there was never any hope this line of research could deliver conclusive evidence regarding the magnitude of frictions. To see this, consider the imperfections identified by FHP (1988): corporate and personal taxation; bankruptcy and agency costs associated with debt; and costs of external equity, which may be fixed, proportional, or nonlinear according to various theories. Even if one could identify constrained firms and perfectly measure marginal q, it is clearly impossible to infer the magnitudes of the diverse costs based on a single regression coefficient.

This *underidentification* problem suggests the need for alternative approaches to the inference problem. To this end, the present paper offers a model-based procedure for estimating costs of external debt and equity. First, we formulate a dynamic model of corporate investment and financial policy under uncertainty, incorporating the financial frictions identified by FHP (1988). With the model in-hand, we employ the indirect inference technique in Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1996). This estimation procedure determines which vector of financial friction parameters best explains observed financial behavior, i.e. minimizes the distance between moments generated by the simulated model and a broad set of real-world data moments. By using an array of moments, indirect inference overcomes the underidentification problem.

Indirect inference also allows us to avoid three commonly cited pitfalls in the empirical investment literature. First, there is no need to assume the firm satisfies the homogeneity property discussed above. That is, the model permits the firm to have market power. Second, indirect inference does not require the zero-correlation restrictions that are necessary to identify OLS regressions. Finally, indirect inference permits a back-door method for dealing with $q-$measurement error. Rather than trying to perfectly measure q, one can simply add artificial noise to model-generated q values. In this way, the data generating process for the simulations mimics the real-world data generating process.

We find that corporate financial behavior is best explained by rising costs of external equity, starting at 3.9% for the first dollar raised, in conjunction with small (12.2%) bankruptcy costs. These estimates complement existing evidence on *direct* costs of external finance. Weiss (1990) estimates that legal and other professional fees amount to 2.8% of the book value of assets in default. On the equity side, Altinkilic and Hansen (2000) examine the shape of underwriter fee schedules. A potential shortcoming of such direct estimates is that they cannot measure indirect costs. For example, Weiss does not measure the indirect costs of bankruptcy, such as the loss of sales predicted by Titman (1984). Similarly, underwriter fees may not fully reflect the lemons premia predicted by Myers and Majluf (1984). We infer direct and indirect costs of external funds based on observed financing behavior.

Comparison of our parameter estimates with the direct estimates facilitates a rough test of the null hypothesis of maximizing behavior. For example, our low point estimate of bankruptcy costs is evidence in favor of the null that corporations are not "debt conservative." Similarly, our evidence indicates that corporations behave "as if" facing a low, convex cost of external equity. This is roughly consistent with the underwriting fee schedules estimated by Altinkilic and Hansen (2000), again supporting the null of maximizing behavior.

The model also allows us to identify which moments matter, i.e. which moments are informative about the magnitude of the various financing frictions. As intuition would suggest, the debt to asset ratio and the propensity to hold cash are informative about bankruptcy costs. The frequency, mean, variance, and skewness of equity issuance are informative costs of external equity. Clearly, investment-cash flow sensitivities are not a sufficient statistic for financing costs.

With the parameter estimates in-hand, we use simulations of the structural model to evaluate *the validity* of three null hypotheses central to the debate between FHP and Kaplan and Zingales (KZ) (1997, 2000). KZ argue, "there is no strong theoretical reason for investment-cash flow sensitivities to increase monotonically with the degree of financing constraints." This assessment is based on a static model with no distinction between stock and flow variables, no uncertainty, no debt/saving, and no retention decision. The restrictiveness of this setting is a potential concern. In addition, recent simulation-based papers by Gomes (2001), Alti (2003), Hennessy and Whited (2004), and Strebulaev (2004) suggest that static models are potentially misleading when interpreting regression coefficients in panel data.

We do not conduct standard hypothesis tests using real-world data. Rather, we use the structural model as a laboratory to evaluate whether cash flow coefficients behave in the way predicted by FHP. The first null hypothesis evaluated is *Monotonicity:* Firms facing higher costs of external funds will exhibit higher cash flow coefficients. Perhaps surprisingly, we find the cash flow coefficient is *decreasing* in fixed, proportional, and quadratic costs of external equity. The intuition is as follows. Even conditioning on average q, cash flow is a proxy for investment opportunities, due to concavity of the profit function. When faced with higher costs of external equity, the firm invests less aggressively when hit with a positive cash flow shock. Hence, the cash flow sensitivity of investment declines.

In contrast, the cash flow coefficient increases with bankruptcy costs. Interpreting the effect of bankruptcy costs is a bit more subtle. Higher bankruptcy costs cause the firm to choose less debt. This reduces the debt overhang problem. In addition, the propensity to hold cash increases dramatically when bankruptcy costs increase. A firm sitting on a pool of cash invests more aggressively when faced with positive shocks. Hence, the cash flow sensitivity of investment increases.

This set of results is closely related to the findings of Moyen (2004), who also simulates a dynamic stochastic model. There are two types of firms, constrained and unconstrained. Constrained firms cannot access external debt or equity. Unconstrained firms can issue debt and can access external equity at zero cost. Moyen finds that unconstrained firms exhibit higher cash flow coefficients. As in our model, cash flow is informative about marginal q. Relative to constrained firms, unconstrained firms invest more aggressively when hit with positive shocks, since they can utilize debt tax shields.

The second null hypothesis we evaluate is *Inversion:* If all firms face the same set of financing costs, cash flow coefficients will be inversely related to dividends. FHP argue that low payout firms are the "most constrained" and should invest more given an innovation to cash flow. To check this assertion, we mimic the sorting procedure in FHP (1988), splitting the firms into three classes, with "Class 1" having the lowest payouts. In our simulations, the Class 1 firms have the highest cash flow coefficient (0.134), followed by Class 3 firms (0.108), followed by Class 2 firms (0.039). Therefore, Inversion does not hold strictly. However, the fact that the Class 1 firms have the highest cash flow coefficient gives some support to FHP.

FHP (1988) state, "If the cost disadvantage is slight, then retention practices should reveal little about financing practices, q values, or investment behavior." This can be translated into a third null hypothesis, *Increasing Differences:* As financial frictions are increased, the difference between the cash flow coefficients across retention groups increase. This null is incorrect in our model. The difference between cash flow coefficients across dividend classes typically decreases as costs are increased.

We now discuss closely related papers. Moyen's (2004) model of financially unconstrained firms is closest to that presented here. Our model is a bit more general in that it features: 1) linear-quadratic costs of external equity; 2) progressive taxes on distributions; and 3) convex corporate taxes. The main difference between the papers is the empirical questions addressed. Our main objective is to use indirect inference to estimate structural parameters of the economy. In contrast, Moyen attempts to explain the seemingly contradictory evidence in the FHP-KZ debate.

Cooley and Quadrini (2001) analyze a firm that can issue defaultable debt and faces proportional costs of external equity. Their model of the debt market greatly influenced that presented in our paper. Our model is a bit more general, allowing for corporate and personal taxation and linear-quadratic costs of external equity. Cooley and Quadrini show that existing stylized facts regarding firm growth and exit can be explained by their model when one imposes a reasonable parameterization.

Cooper and Ejarque (2003) also employ indirect inference to estimate costs of external equity. There is no taxation, no debt, and costs of external equity are linear. Cooper and Ejarque do sketch the broad outlines of a model with corporate saving and riskless debt. However, no estimation is performed. They state, "The model is very difficult to estimate due to the additional state variable and the need for a fine state space." The present paper overcomes the dimensionality problem. Net worth is the only endogenous state variable. Finally, Cooper and Ejarque attempt to match investment moments. Our empirical focus is different in that we attempt to match financing moments.

Hennessy and Whited (2004) present a dynamic model with corporate and personal taxation, proportional costs of external equity, and credit rationing. An exogenous credit constraint ensures debt is riskless.[3] The primary objective of their paper is to show that a rational trade-off model can be reconciled with existing capital structure "anomalies." Leary and Roberts (2004) assume the firm's objective is to keep the leverage ratio within an exogenous band. A dynamic duration model is used to make inferences about the nature of restructuring costs. They conclude that a combination of fixed plus weakly convex costs of adjustment best explains observed hazard rates. Their results are informative about the nature of financial frictions, but leave open the question of magnitudes. Rauh (2004) uses mandatory pension contributions as a potentially

[3] We thank David Mauer for encouraging us to relax this assumption.

exogenous innovation to internal funds.[4] He finds a significant negative response of capital expenditures to required contributions. Rauh's evidence may serve as a reasonable basis for rejecting the null hypothesis of perfect capital markets, but does not address the nature and magnitude of financial market imperfections.

The remainder of the paper is organized as follows. Section 1 sets up the model. Section 2 derives the optimal financial and investment policies. Section 3 describes the numerical solution to the model and presents a baseline simulation. Section 4 describes the indirect inference procedure and presents the estimation results. Section 5 evaluates the validity of the null hypotheses central to the FHP-KZ debate. Section 6 concludes.

1. Economic Environment

A. Operating Profits

Time is discrete and the horizon infinite. There are two control variables, the capital stock (k) and the market value of one-period debt (b). Capital decays exponentially at rate δ. Negative values of b are properly interpreted as corporate saving. Variables with primes denote future values and minus signs denote lagged values. Subscripts denote partial derivatives.

An objective of the theoretical model is to specify the firm's problem in terms of primitives. We consider a firm with market power employing a constant returns to scale production technology in two inputs: capital and labor (l). Cooper and Ejarque (2001) find that, in the context of indirect inference estimation, the failure to account for market power causes one to incorrectly impute concavity in the profit function to convexity in the adjustment cost function. By analogy, failure to account for market power would cause us to confound concavity of the profit function with convex costs of external funds. The firm faces demand, productivity, and wage shocks. The timing assumption is that new capital becomes productive with a one-period lag. This means that k' is chosen before next period's shocks are observed. In contrast, the variable labor input is chosen optimally after next period's shocks are observed. Assumption 1 summarizes.

[4] Earlier papers by Blanchard, Lopez-de-Silanes, and Shleifer (1994) and Lamont (1997) also examine windfalls.

Assumption 1. The firm faces a stochastic constant elasticity demand schedule

$$q^d(p,\widehat{z}) \equiv \left[\frac{\widehat{z}}{p}\right]^{\eta}.$$

The production function has constant returns to scale

$$q^s(k,l,\widetilde{z}) = \widetilde{z}k^{\phi}l^{1-\phi}.$$

Labor inputs are variable, and the stochastic wage rate is ω. Capital requires a one-period time-to-build.

Under Assumption 1, the profit function admits a concave representation[5]

$$\begin{aligned} \text{Operating Profit} &= z\pi(k) \\ \pi(k) &\equiv k^{\alpha} \\ \alpha &\equiv \frac{\phi(\eta-1)}{1+\phi(\eta-1)} \\ z &\equiv \widehat{z}\left[\frac{1+\phi(\eta-1)}{\eta}\right]\left[\frac{\widehat{z}(1-\phi)(1-\eta^{-1})\widetilde{z}^{\frac{1}{1-\phi}}}{\omega}\right]^{\frac{(1-\phi)(\eta-1)}{1+\phi(\eta-1)}}. \end{aligned} \tag{1}$$

Assumption 2 imposes some structure on the shock z.

Assumption 2: The shock z takes values in the compact set $Z \equiv [\underline{z}, \overline{z}]$, $0 \leq \underline{z} < \overline{z} < \infty$, with its Borel subsets $\mathcal{Z}$. The Markovian transition function $Q : Z \times \mathcal{Z} \rightarrow [0,1]$ has no atoms, satisfies the Feller property, and is monotone (increasing).

B. Tax System

Fazzari, Hubbard and Petersen (1988) cite the tax system as being a potentially important factor affecting the financing hierarchy and cost of funds schedule. Our goal is to parsimoniously model the salient features of the U.S. corporate income tax.

Investors are risk neutral, and the risk-free asset earns a pre-tax rate of return equal to r. The tax rate on interest income at the individual level is τ_i, implying investors use $r(1-\tau_i)$ as their discount rate.

[5] Simply evaluate operating profits at the optimal labor input.

Corporate taxable income is equal to operating profits less economic depreciation less interest expense plus interest income. Consistent with the U.S. tax code, interest expense is computed as the product of the promised yield ($\widetilde{r}$) and the amount borrowed. As shown by Graham (1996a, 1996b), loss limitations create nonlinearities. Following Leland and Toft (1996), loss limitations are treated as a kink in the tax schedule. The tax rate when income is positive (τ_c^+) exceeds the tax rate when income is negative (τ_c^-). In the event that the firm does not default, the corporate tax bill is

$$T^c(k', b', z, z') \equiv [\tau_c^+ \chi + \tau_c^- (1 - \chi)] * [z'\pi(k') - \delta k' - \widetilde{r}(k', b', z)b'] \tag{2}$$

where χ is an indicator function for positive taxable income. In the event of default, interest deductions on the debt obligation are disallowed. This is consistent with the U.S. tax code, where recoveries in default are treated as principal first. An equilibrium bond pricing identity, derived below, is used to pin down $\widetilde{r}$. For now, it should be noted that the promised yield only hinges upon variables observable to the lender at the time of loan inception, and excludes the realized shock (z'). If the corporation saves, it earns r pre-tax, thus

$$b' < 0 \Rightarrow \widetilde{r}(k', b', z) = r \quad \forall (k', z). \tag{3}$$

The taxation of distributions is complicated by the fact that corporations pay out cash through dividends and share repurchases. Corporations should use share repurchases to disgorge cash if the marginal shareholder is a taxable individual due to the lower statutory rate historically accorded to capital gains, tax deferral advantages, and the tax free step-up in basis at death. Green and Hollifield (2003) present a model of optimal share repurchases. The first shareholders to sell into a tender offer are those with the lowest amount of locked-in capital gains. Under the optimal strategy, the effective tax rate on capital gains is only 60% of the statutory rate.

Complete substitution of repurchases for dividends is limited by the fact that the IRS prohibits replacing dividends with systematic repurchases. Given the historical reluctance of the IRS to challenge repurchase programs, the optimal plan would seem to entail a modest percentage of dividends. Another factor that may mitigate the substitution of repurchases for dividends is concern over SEC prosecution for stock price manipulation. SEC Rule 10b-18 provides safe harbor for firms adhering to certain restrictions on the timing and amount of shares repurchased. Cook et al. (2003) document that most corporations conform to the

SEC restrictions.

To capture these effects, we model the corporation as perceiving an increasing marginal tax rate on distributions. Intuitively, under an optimal distribution program, small distributions are implemented via share repurchases. Shareholders with high basis are the first to tender, implying that the capital gains tax triggered by the repurchase is low. As the firm increases the amount distributed, there are two effects. First, the basis of the marginal tendering shareholder is reduced. Second, the firm may be inclined to increase the percentage paid out as dividends due to the IRS and SEC regulations cited above. Both effects raise the marginal tax rate on distributions.

The marginal distribution tax rate is parameterized as follows

$$\tau_d(x) \equiv \overline{\tau}_d * [1 - e^{-\phi x}]. \tag{4}$$

In contrast, Hennessy and Whited (2004) assume distributions are taxed at a constant rate.[6] The total distribution tax liability at the shareholder level is

$$T^d(X) \equiv \int_0^X \tau_d(x)dx. \tag{5}$$

There is zero tax triggered on the first dollar distributed, while the limiting marginal tax rate reaches $\overline{\tau}_d$. Intuitively, such convexity creates an incentive for the corporation to smooth distributions. This insight is exploited in the indirect inference estimation of ϕ.

Assumptions regarding the tax system are summarized below.

Assumption 3: Corporate taxes are computed according to (2), where $0 < \tau_c^- < \tau_c^+ < 1$. At the individual level, interest income is taxed at rate $\tau_i \in (0, \tau_c^+)$. The marginal tax rate on distributions to shareholders is determined by (4), where $\overline{\tau}_d \in (0, 1)$.

C. Costs of External Equity and Debt

The main costs of external equity discussed by FHP (1988) are tax costs, adverse selection premia, and flotation costs. The tax cost associated with external equity is implicit in our parameterization of the tax system, which allows for double-taxation. Myers and Majluf (1984) show that informational asymmetries

[6] We thank Richard Roll for suggesting that we relax this assumption in light of tax rate heterogeneity.

can raise or lower the cost of external equity. The precise implications of this theory for the perceived cost of external equity are quite sensitive to the nature of the equilibrium one constructs and the type of firm being considered. For example, when the parameters of the problem are such that a pooling equilibrium can be supported, both types of firms issue equity, with low (high) quality firms receiving financing on better (worse) than fair terms. The more general conclusion the profession seems to have taken away from the model of Myers and Majluf is that equity issuance may send a negative signal to the market regarding insiders' assessment of firm quality. For example, the model presented in FHP (1988) treats the "lemons premium" as proportional. Finally, Atlinkilic and Hansen (2000) provide detailed evidence regarding underwriter fees, finding that average costs are U-shaped due to fixed costs and widening spreads for larger offerings.

The cost of external equity function is linear-quadratic, capturing the effect of flotation costs and lemons premia.

Assumption 4: The cost of external equity is equal to Λ, where

$$\begin{aligned} \Lambda(x) &\equiv \lambda_0 + \lambda_1 x + \lambda_2 x^2 \\ \lambda_i &\geq 0 \quad i = 0, 1, 2. \end{aligned}$$

Indirect inference is used to estimate the three unknown parameters of the cost of external equity function.

The borrowing technology consists of a standard one-period debt contract, analogous to that derived in the costly state verification models of Townsend (1978) and Gale and Hellwig (1985). The intermediary faces perfect competition. In order for him to verify net worth, he must incur a cost. If the promised debt payment is delivered, the intermediary does not verify and the original shareholders retain control. In the event of default, the intermediary verifies net worth. The informed intermediary then enters into renegotiations with the firm. The intermediary has full ex post bargaining power and extracts all bilateral surplus by demanding a payment that leaves the firm indifferent between continuing or not. This setup allows us to derive the firm's endogenous default rule, analogous to the smooth-pasting condition from continuous-time models with limited liability, since equity is worth zero in the event of bankruptcy.

The verification cost function is parameterized as follows.

Assumption 5: Verification costs are equal to $\xi(1-\delta)k'$.

Indirect inference is used to estimate the magnitude of ξ. It should be noted that this is not the only cost of debt incorporated in the model. Since there is some probability of default, equity recognizes that the lender may capture a portion of the return to capital accumulation. This is the debt overhang effect first analyzed by Myers (1977).

2. Model

A. Equity's Problem

The variable w denotes *realized net worth*

$$w(k', b', z, z') \equiv (1 - \delta)k' + z'\pi(k') - T^c(k', b', z, z') - (1 + \widetilde{r}(k', b', z))b'. \tag{6}$$

There is a single endogenous state variable $\widetilde{w}$ which denotes *revised net worth*. Revised net worth is equal to realized net worth if the firm does not default. In default, realized net worth is revised due to negotiations between the intermediary and firm. The precise nature of the adjustment is discussed in the next subsection, which treats debt market equilibrium.

To clarify the discussion below, it is useful to derive the firm's external funding requirement for a given desired capital stock (k'). Consider first a firm that did not default in the prior period. The direct cost of the investment is

$$k' - (1 - \delta)k. \tag{7}$$

Liquid internal funds are equal to

$$z\pi(k) - T^c(k, b, z^-, z) - (1 + \widetilde{r}(k, b, z^-))b. \tag{8}$$

The external funding requirement is equal to investment cost less liquid internal funds, which, in turn, is equal to the desired capital stock less revised net worth:

$$k' - (1 - \delta)k - [z\pi(k) - T^c(k, b, z^-, z) - (1 + \widetilde{r}(k, b, z^-))b] = k' - \widetilde{w}(k, b, z^-, z). \tag{9}$$

The external equity requirement is equal to

$$k' - \widetilde{w}(k, b, z^-, z) - b'. \tag{10}$$

Of course, when this amount is negative, the distribution to shareholders is positive. Next consider a firm that defaulted on the prior period's debt obligation. Once again, the external funding requirement is equal to the desired capital stock less revised net worth, while the external equity requirement is given by equation (10).

The construction of equilibrium proceeds in two steps. In this subsection equity's problem is formulated, while the next subsection analyzes the debt market. Consider first, the feasible policy correspondence

$$\Gamma : Z \to K \times B.$$

Without loss of generality, attention can be confined to compact K. The maximum allowable capital stock $\overline{k}$ is determined by

$$\overline{z}\pi'(\overline{k}) - \delta \equiv 0 \tag{11}$$
$$\Rightarrow \overline{k} = \left[\frac{\overline{z}\alpha}{\delta}\right]^{\frac{1}{1-\alpha}}.$$

Since $k > \overline{k}$ is not economically profitable, let

$$K \equiv [0, \overline{k}]. \tag{12}$$

Under the maintained assumption that $\tau_c^+ > \tau_i$, the optimal value of b is bounded below at some finite level, denoted $\underline{b} \in (-\infty, 0)$. To see this, note that for firms with positive taxable income, the after-tax return on corporate saving is below that available to the shareholder investing on his own account. As the firm's cash balance increases, the precautionary motive for retention becomes negligible and funds should be distributed. The upper bound on debt, i.e. the debt capacity of the firm, is denoted as $\overline{b}(k, z)$. Below, we show that debt capacity is finite.

The feasible policy correspondence can be expressed as

$$\Gamma(z) \equiv \{(k', b') : k' \in K \quad and \quad b' \in [\underline{b}, \overline{b}(k', z)]\}.$$

Let $C(\Theta)$ denote the space of all bounded and continuous functions on an arbitrary set Θ. The Bellman operator (T) corresponding to an abstract formulation of the equity's problem is

$$\begin{aligned}(Tf)(\widetilde{w}, z) \quad \equiv \quad & \max_{(k',b')\in\Gamma(z)} \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] \\ & + \left[\frac{1}{1 + r(1 - \tau_i)}\right] \int_Z f[\widetilde{w}(k', b', z, z'), z']Q(z, dz').\end{aligned} \tag{13}$$

subject to:

$$i.\quad \Gamma \text{ compact, convex, continuous and nondecreasing in } z$$

$$ii.\quad \widetilde{r} \in C(K \times B \times Z)$$

$$iii.\quad \widetilde{w}(k', b', z, z') \equiv \max\{\underline{w}(z'), w(k', b', z, z')\}$$

$$iv.\quad \underline{w} \in C(Z),\ \underline{w}(z') < 0 \quad \forall z' \in Z, \text{ and nonincreasing.}$$

The second constraint states that equity faces a continuous schedule determining the promised yield demanded by the intermediary. The third and fourth constraints state that revised net worth is bounded below by some schedule $\underline{w}$. The next subsection analyzes endogenous default and debt renegotiation. It will be shown that $\underline{w}$ necessarily satisfies condition (*iv*). The model is then closed by constructing a debt market equilibrium, pinning down a continuous $\widetilde{r}$ function.

The following Lemma will prove useful

LEMMA 1: *The operator* $T : C(\widetilde{W} \times Z) \to C(\widetilde{W} \times Z)$ is a contraction mapping with modulus $[1+r(1-\tau_i)]^{-1}$.
Proof. See Appendix.

Proposition 1 indicates that the value function exists, while Proposition 2 tells us that the value function can be determined by iterating on the Bellman equation, starting from an arbitrary conjecture regarding the solution.

PROPOSITION 1: *There is a unique continuous function* $V : \widetilde{W} \times Z \to \Re_+$ *satisfying*

$$V(\widetilde{w}, z) = \max_{k', b' \in \Gamma(z)} \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \left[\frac{1}{1 + r(1 - \tau_i)}\right] \int_Z V(\widetilde{w}(k', b', z, z'), z') Q(z, dz'). \tag{14}$$

Proof. Follows from Lemma 1 and the Contraction Mapping Theorem.

PROPOSITION 2: *For arbitrary* $v^0 \in C(\widetilde{W} \times Z)$, the sequence

$$v^{n+1} \equiv T(v^n)$$

converges to V, with

$$d_\infty(v^n, V) \leq \left[\frac{1}{1+r(1-\tau_i)}\right]^n * d_\infty(v^0, V).$$

Proof. Follows from Lemma 1 and the Contraction Mapping Theorem.

Propositions 3 and 4 establish some useful and intuitive properties of the value function.

PROPOSITION 3: *For each $z \in Z$, the equity value function $V(\cdot, z) : \widetilde{W} \rightarrow \Re_+$ is strictly increasing.*

Proof. See Appendix.

PROPOSITION 4: *For each $\widetilde{w} \in \widetilde{W}$, the equity value function $V(\widetilde{w}, \cdot) : Z \rightarrow \Re_+$ is nondecreasing.*

Proof. See Appendix.

B. Debt Market Equilibrium

In the event of default and renegotiation, original shareholders are pushed down to their reservation value of zero. Equity does not default if realized net worth is positive, since a positive continuation value can then be achieved even if the promised debt payment is delivered. There is some z'-contingent critical value of realized net worth, denoted $\underline{w}(z') < 0$, such that equity is just indifferent between defaulting and delivering the promised payment. The endogenous default schedule $\underline{w}(\cdot)$ is defined implicitly by the following equation

$$V(\underline{w}(z'), z') = 0 \qquad \forall \quad z' \in Z. \tag{15}$$

Proposition 5 establishes some useful and intuitive properties of the default schedule.

PROPOSITION 5: *The default schedule $\underline{w}$: $Z \rightarrow \Re$ is a negative valued, continuous, and nonincreasing function.*

Proof. If revised net worth is positive, so too is equity value, thus establishing negativity. Since V is strictly monotonic and continuous in its first argument, the inverse $\underline{w} = V^{-1}(0)$ is well defined and continuous. Weak monotonicity of $\underline{w}$ follows from Propositions 3 and 4.

Figure 1 depicts the default decision, plotting realized net worth and the default schedule as functions of the realized shock, z'. Since $w(k', b', z, \cdot)$ is strictly increasing, and $\underline{w}(\cdot)$ is nonincreasing, the two functions have at most one point of intersection, which is denoted $z_d(k', b', z)$. For shock values on the interval

$[z_d(k', b', z), \overline{z}]$ the firm delivers the promised payment. To see this, note that

$$z' > z_d(k', b', z) \Rightarrow w(k', b, z, z') > \underline{w}(z') \Rightarrow V(w(k', b', z, z'), z') > 0. \tag{16}$$

Alternatively, if $z' < z_d(k', b', z)$, equity prefers to default, since *revised* net worth exceeds *realized* net worth.

If debt is sufficiently low, equity does not default. To see this, note that the z'_d is implicitly defined as follows

$$w(k', b', z, z'_d) = \underline{w}(z'_d). \tag{17}$$

This condition may not be satisfied by any $z' \in Z$ if b' is sufficiently low. Returning to Figure 1, higher values of k' and z shift the $w(k', b', z, \cdot)$ schedule up, thus lowering the default threshold z'_d. Intuitively, high values of the capital stock imply that the realized shock must be very low in order to induce default. Similarly, high values of z are associated with lower bond yields $(\widetilde{r})$, which implies that worse shocks are required to induce default. On the other hand, high values of b' shift the $w(k', b', z, \cdot)$ schedule down, thus increasing the default threshold. Proposition 6 summarizes.

PROPOSITION 6: *The critical shock inducing default, $z_d : K \times B \times Z \to Z$, is a continuous function, decreasing in its first and third arguments, and increasing in its second.*

Proof. See equation (17). Continuity follows from w and $\underline{w}$ being continuous. Monotonicity in the various arguments follows from monotonicity of w.

In the event of renegotiation, the intermediary recovers a payment sufficient to drive net worth down to $\underline{w}(z')$. The intermediary's recovery in default, net of verification costs, is equal to

$$R(k', z') = (1 - \xi)(1 - \delta)k' + z'\pi(k') - [\tau_c^+ \chi + \tau_c^- (1 - \chi)] * [z'\pi(k') - \delta k'] - \underline{w}(z'). \tag{18}$$

The required bond yield is determined by a zero profit condition for the intermediary

$$b' = \left[\frac{1}{1 + r(1 - \tau_i)}\right] \left[[1 + (1 - \tau_i)\widetilde{r}(k', b', z)]b' \int_{z_d(k',b',z)}^{\overline{z}} Q(z, dz') + \int_{\underline{z}}^{z_d(k',b',z)} R(k', z')Q(z, dz')\right]. \tag{19}$$

Holding fixed the pair (k', z), for modestly risky debt $\widetilde{r}$ must be increasing in b'. However, there are limits to how much the firm can raise through debt, as it eventually reaches a debt capacity where further increases in $\widetilde{r}$ actually reduce b'. Attention is confined to pairs $(\widetilde{r}, b')$ where debt value is increasing in the promised yield, since other pairs are dominated on efficiency grounds. Along this region, equation (19) can be inverted. The required bond yield is

$$\widetilde{r}(k',b',z) = \left[\frac{1}{1-\tau_i}\right]\left[\frac{1+r(1-\tau_i) - \int_{\underline{z}}^{z_d(k',b',z)}[R(k',z')/b']Q(z,dz')}{\int_{z_d(k',b',z)}^{\overline{z}} Q(z,dz')} - 1\right]. \tag{20}$$

This analysis closes the model, since the bond market equilibrium is consistent with the maximization problem posited for the firm (13). Constraints *iii* and *iv* are implicit in the bond pricing equation. Equation (20) implies that the function $\widetilde{r}$ is continuous, thus satisfying *ii*. The fact that Γ is nondecreasing follows from maintained assumption that Q is monotone (increasing). This property of the transition function ensures that debt capacity is increasing in z. Other properties of Γ follow by construction.

C. Optimal Policies

To simplify the exposition, this subsection assumes V is concave and once differentiable.[7] In order to characterize the optimal financial policy, hold k' fixed and consider the choice of b'. Let b'_0 denote the amount of debt required to finance the investment program in its entirety, with

$$b'_0 \equiv k' - \widetilde{w}. \tag{21}$$

If $b'_0 < 0$, we will say that the firm is *unconstrained*, in the precise sense that it currently has sufficient internal resources to finance this period's desired capital stock.

Heuristically, we can view the CFO as performing financial optimization in two steps. First, he determines the optimal financing program ignoring fixed costs of external equity, i.e. treating $\lambda_0 = 0$. In the second step, he determines whether the intra-marginal benefits of equity issuance justify the fixed costs. For the first step in the optimization, the CFO evaluates the effect of a small positive perturbation in b' on the right-side of the Bellman equation

$$\begin{aligned}\frac{\partial V}{\partial b'} &= \Phi_i[1+\Lambda_1(k'-\widetilde{w}-b')] + \Phi_d[1-\tau_d(\widetilde{w}+b'-k')] \\ &\quad - \int_{z'_d}^{\overline{z}} \frac{\left[1+(1-\tau_c)\left(\widetilde{r}(k',b',z)+b'\frac{\partial \widetilde{r}}{\partial b'}\right)\right] * V_1(w',z')}{1+r(1-\tau_i)} Q(z,dz'),\end{aligned} \tag{22}$$

[7] This assumption is not utilized in the numerical analysis. In order to establish differentiability one must establish concavity. In the absence of fixed costs, Cooley and Quadrini (2001) establish concavity under restrictions on probability densities. The technical problem is that revised net worth is convex near default. A second issue is that fixed costs cause the "dividend" to be convex at zero.

The first line in (22) represents the marginal benefit to shareholders today from increasing debt. If $b' < b'_0$, then $\Phi_i = 1$ and debt replaces external equity. If $b' > b'_0$, then $\Phi_d = 1$ and marginal debt finances higher distributions. The MB schedule is strictly declining and exhibits a jump at b'_0 if the parameter λ_1 is positive. For the constrained firm, the first units of debt substitute for high levels of external equity, which are very costly due to the convexity of the Λ schedule. At the opposite extreme, high levels of debt are used to finance distributions to shareholders, who face an increasing τ_d schedule. Figure 2 graphs marginal benefit schedules under linear-quadratic costs of external equity. The figure depicts three firms, indexed by their beginning of period internal resources, with $\widetilde{w}_h > \widetilde{w}_m > \widetilde{w}_l$. For firm i, the marginal benefit schedule exhibits a jump at the point where the marginal increase in debt finances a distribution, as opposed to replacing external equity. In particular, $b'_0 = k' - \widetilde{w}_i$. Firm h is unconstrained, in the sense that it can finance the entire investment program with internal resources. Firm l is severely constrained, in that it must obtain a significant amount of external financing in order to fund the investment program.

The second line in (22) represents the marginal cost (MC) of debt service. The upward slope of the MC schedule is caused by four factors. First, higher amounts of debt increase the probability that interest expense will only be deductible at the lower rate τ_c^-. As in the detailed micro-simulations performed by Graham (2000), the expected marginal corporate tax rate in our model is flat at τ_c^+ up to a kink point, where it then becomes downward sloping. Second, Proposition 6 shows that increasing b' increases the probability of default, a standard effect. To compensate for higher default risk, the lender demands a higher promised yield ($\widetilde{r}$). Third, increases in $\widetilde{r}$ raise the cost of servicing intra-marginal units of debt. Finally, since V is concave, the shadow value of funds devoted to debt service increases with b'.

Returning to Figure 2, starting from the far left, the CFO evaluates whether the marginal benefit from increasing debt (decreasing savings) exceeds the marginal cost. Debt is increased so long as the MB schedule lies above the MC schedule. The optimal financing policy for firm h entails saving and making a positive distribution. Firm m finances the entire investment program with debt, neither issuing equity nor making a distribution. Firms will cluster in this zero distribution-zero equity issuance region whenever λ_1 is large. Firm l issues a large block of debt, using external equity to make up the rest of the financing gap, provided that the fixed costs of external equity are sufficiently low.

This analysis suggests the following insights which inform the choice of moments utilized in the indirect inference estimation. Under high fixed costs (λ_0), equity flotations are lumpy, as firms avoid issuing small blocks of new shares. If λ_1 is high, firms cluster around zero distributions. High values of λ_2 limit the variance and skewness of equity flotations. High bankruptcy costs (ξ) shift up the MC schedule, reducing optimal leverage. Finally, a high degree of curvature in the τ_d schedule limits the variance of distributions. In the next section, we perform numerical comparative statics on the model (under an exogenous parameterization) in order to illustrate the effect of various frictions on various model-generated moments.

At this point, we note that *our model supports the claim made by FHP (1988) that distribution policy is informative about the firm's "degree of constraint" as well as the current period's "cost of funds."*[8] To see this, return to Figure 2. The firm with high internal resources makes a positive distribution, with the cost of funds equal to $1-\tau_d$. That is, each dollar of real capital purchased today has an opportunity cost of $1-\tau_d$ from the perspective of shareholders. At the opposite extreme, the firm with low internal resources issues equity, with the marginal cost of funds equal to $1+\Lambda_1$. *Despite this consistency, the results presented in Section 5 cast doubts regarding the predictions made by FHP regarding the comparative static properties of the cash flow coefficient.*

The first-order condition for optimal financing simplifies further if the corporate tax schedule is linear. We begin by differentiating the bond pricing identity (19) with respect to b'. We can rearrange terms, obtaining

$$1+(1-\tau_i)\left[\widetilde{r}+b'\left(\frac{\partial \widetilde{r}}{\partial b'}\right)\right]=\frac{1+(1-\tau_i)r+(\partial z_d'/\partial b')[\xi(1-\delta)k'+(\tau_c-\tau_i)\widetilde{r}b']Q_2(z,z_d')}{\Pr(z'\geq z_d')}.$$

Substituting the term above into the first-order condition for optimal financing yields a simplified optimality condition

$$1+\Phi_i\Lambda_1(k'-\widetilde{w}-b')-\Phi_d\tau_d(\widetilde{w}+b'-k')=E\{V_1[w(k',b',z,z'),z']|z'\geq z_d'\}\times \tag{23}$$

$$\left[1+\frac{(\partial z_d'/\partial b')[\xi(1-\delta)k'+(\tau_c-\tau_i)\widetilde{r}b']Q_2(z,z_d')-\Pr(z'\geq z_d')(\tau_c-\tau_i)[\widetilde{r}+b'(\partial\widetilde{r}/\partial b')]}{1+r(1-\tau_i)}\right]$$

It is worth noting that the optimality condition (23) reduces to the traditional tradeoff theory when there are no distribution taxes or costs of external equity. In such cases, the shadow value of internal resources is

[8] In contrast, Moyen (2004) rejects the FHP sorting procedure.

always unity, and the first-order condition is

$$\left[\frac{\partial z_d'}{\partial b'}\right][\xi(1-\delta)k' + (\tau_c - \tau_i)\widetilde{r}b']Q_2(z, z_d') = \Pr(z' \geq z_d')(\tau_c - \tau_i)\left[\widetilde{r} + b'\left(\frac{\partial \widetilde{r}}{\partial b'}\right)\right]. \tag{24}$$

Intuitively, in the absence of distribution taxes and flotation costs, the optimal financing policy equates tax shield benefits with bankruptcy costs at the margin.[9]

In the presence of distribution taxes and costs of external equity, additional factors must be taken into account when deriving the optimal financing program. First, the firm must consider how the proceeds from the marginal dollar of debt will be utilized in the current period. Second, the firm must take into account the shadow value of internal resources next period. When the expected shadow value of resources next period is high, so too is the shadow cost of debt service, an effect which discourages leverage.

Under the working assumption of this subsection, that the value function is concave, the Envelope Theorem of Benveniste and Scheinkman (1979) tells us that the expected shadow value of funds can be linked to expectations regarding next period's "equity regime." In particular, for shock realizations such that the firm is issuing equity next period

$$V_1[w(k', b', z, z'), z'] = 1 + \Lambda_1'$$

which indicates that the shadow cost of debt service is high. For shock realizations such that the firm is making a positive distribution next period

$$V_1[w(k', b', z, z'), z'] = 1 - \tau_d'$$

which indicates that the shadow cost of debt service is low.

Consider next the effect of a perturbing the capital stock (k'). At an interior solution, the optimal investment policy satisfies

$$\frac{\partial V}{\partial k'} = -[\Phi_i(1+\Lambda_1(k'-\widetilde{w}-b'))+\Phi_d(1-\tau_d(\widetilde{w}+b'-k'))]\left[1 - \frac{\partial b'}{\partial k'}\right] + \int_{z_d'}^{\overline{z}} \frac{V_1(w', z')\left(\frac{\partial w'}{\partial k'} + \frac{\partial w'}{\partial b'}\frac{\partial b'}{\partial k'}\right)}{1 + r(1-\tau_i)} Q(z, dz') = 0. \tag{25}$$

Referring to equation (25), consider those firms for whom debt is the marginal source of funds. In Figure 2, debt is the marginal source of funds for those facing the MC_M schedule and those facing the MC_H schedule

[9] The analog of condition (24) in Moyen (2004), is equation 15. In her model, there is an additional term, since bankruptcy costs are proportional to the face value of debt. Our "verification costs" are proportional to the real capital stock.

with high fixed costs of equity issuance ($\lambda_0 > S$). For such firms, $\partial b'/\partial k' = 1$, so the optimality condition simplifies to

$$\int_{z'_d}^{\overline{z}} V_1(\widetilde{w}', z') \left[z'\pi_1(k') - [\widetilde{r}(k', b', z) + \delta] - b' \left(\frac{\partial \widetilde{r}}{\partial k'} + \frac{\partial \widetilde{r}}{\partial b'} \right) \right] Q(z, dz') = 0. \tag{26}$$

Stiglitz (1973) analyzes optimal financial policy and investment in a setting with no uncertainty and no default. He proves that for debt-financed investment: 1) the firm's first-order condition is unaffected by the corporate income tax; and 2) the marginal revenue product of capital is equated to $r + \delta$. Condition (26) shows that the first of Stiglitz' results carries over to a dynamic environment with uncertainty and default. Stiglitz' second result must be modified, given that the required bond yield is not constant. The optimal investment policy accounts for intra-marginal effects associated with changes in k' and b'.

Next consider firms with strictly positive equity issuance or strictly positive distributions. Rewriting (25) yields

$$\left[\Phi_i(1 + \Lambda_1(k' - \widetilde{w} - b')) + \Phi_d(1 - \tau_d(\widetilde{w} + b' - k')) + \int_{z'_d}^{\overline{z}} \frac{V_1(w', z') \frac{\partial w'}{\partial b'}}{1 + r(1 - \tau_i)} Q(z, dz') \right] \frac{\partial b'}{\partial k'} \tag{27}$$
$$-[\Phi_i(1 + \Lambda_1(k' - \widetilde{w} - b')) + \Phi_d(1 - \tau_d(\widetilde{w} + b' - k'))] + \int_{z'_d}^{\overline{z}} \frac{V_1(w', z') \frac{\partial w'}{\partial k'}}{1 + r(1 - \tau_i)} Q(z, dz') = 0.$$

For such firms, the first bracketed term in (27) is zero. Therefore, the optimal investment plan satisfies

$$\Phi_i(1+\Lambda_1(k' - \widetilde{w} - b')) + \Phi_d(1 - \tau_d(\widetilde{w} + b' - k')) = \int_{z'_d}^{\overline{z}} \frac{V_1(w', z') \left[1 + (1 - \tau_c)(z'\pi_1(k') - \delta - b' \frac{\partial \widetilde{r}}{\partial k'}) \right]}{1 + r(1 - \tau_i)} Q(z, dz'). \tag{28}$$

Intuitively, firms issuing equity are just indifferent between financing incremental investment with debt or equity. Hence, the marginal cost of funds is $1 + \Lambda_1$, which is equated with the expected discounted marginal benefit from installed capital. Similarly, firms making positive distributions are indifferent between financing incremental investment with debt or a reduction in distributions. Hence, the term $1 - \tau_d$ represents the marginal cost of investing. Clearly, the opportunity cost of investing is lower for firms making distributions than those issuing equity, thus encouraging capital accumulation.

3. Benchmark Simulation

This section presents a simulation of the model based on reasonable parameter values that are gleaned from previous studies. The intent is to provide the reader with an intuitive understanding of the mapping between the model and moments, before proceeding to our estimates of the underlying parameters. The analysis also serves as a robustness check of the comparative static properties of the model under alternative parameterizations.

A. Design

The shock z follows an $AR(1)$ process in logs:

$$\ln(z') = \rho \ln(z) + \varepsilon', \tag{29}$$

where $\varepsilon' \sim N\left(0, \sigma_e^2\right)$. We transform (29) into a discrete-state Markov chain using the method in Tauchen (1986), letting z have 5 points of support in $\left[-3\sigma_e \Big/ \sqrt{1-\rho^2}, 3\sigma_e \Big/ \sqrt{1-\rho^2}\right]$. We set $\alpha = 0.623$, which is mid-way between the point estimates of Cooper and Ejarque (2003) and Hennessy and Whited (2004). Also following Hennessy and Whited (2004), we set $\sigma_e = 0.118$ and $\rho = 0.740$.

The state space for (k, p, z) is discrete. The capital stock, k, lies in the set

$$\left[\overline{k}, \overline{k}(1-\delta)^{1/2}, \overline{k}(1-\delta), \ldots, \overline{k}(1-\delta)^{10}\right],$$

where $\overline{k}$ is defined by (12). The state space for b has the same number of points as the state space for k. We set the maximal value equal to k^{α}/r and the minimal value equal to the opposite of the maximal value. The maximal value represents a crude guess of the value of the firm. These state spaces for k and b appear to be sufficient for our purposes in that the optimal policy never occurs at an endpoint of either state space.

Next we define the tax environment. For $\bar{\tau}_d$, we use the estimate in Graham (2000) of 12%. We set the parameter ϕ in (4) equal to 0.02. We also set the tax rate on interest income, τ_i, equal to the Graham (2000) estimate of 29.6%. We set the maximal corporate tax rate $\tau_c^+ = 40\%$, which is close to the average combined state and federal tax rates. We set $\tau_c^- = 20\%$.

Next we parameterize the financial frictions. Following Gomes (2001), $\lambda_1 = 0.028$. Setting $\lambda_0 = 1.2$, gives us a ratio of fixed costs to equity issuance close to the 0.35% figure in Altinkilic and Hansen (2000). We set $\lambda_2 = 0.005$, to represent the increasing marginal costs of equity issuance found in Altinkilic and Hansen (2000). Following Moyen (2004), $\xi = 10$. The real risk-free interest rate is $r = 2.5\%$.

The model is solved via iteration on the Bellman equation, which produces the value function $V(\tilde{w}, z)$

and the policy function $(k', b') \equiv h(\widetilde{w}, z)$. The numerical solution proceeds in two steps. First, we guess $\tilde{r}(k', b', z) = r$, and solve for the value function given this guess. Second, we use the solution for the value function to identify default states and then recalculate $\tilde{r}(k', b', z)$ according to (20). We then iterate on this two-step procedure until the value function converges.

The model simulation proceeds by taking a random draw of the z shock and then computing $V(\tilde{w}, z)$ and $h(\tilde{w}, z)$. In the model simulation, the space for z is expanded to include 20 points, with interpolation used to find corresponding values of V, k, and b. The model is simulated for 1000 time periods, with the first fifty observations dropped in order to allow the firm to work its way out of a possibly sub-optimal starting point.

Knowledge of h and V also allows us to compute interesting quantities such as cash flow, Tobin's q, debt, and distributions. Specifically, we define our variables to mimic the sorts of variables used in the literature.

Investment/Book Real Assets	$(k' - (1-\delta)k)/\,k$
Cash Flow/Book Real Assets	$[zk^{\alpha} - T^c(k, b, z^-, z) - (1 + \widetilde{r}(k, b, z^-))b]/\,k$
Tobin's q	$[V(\tilde{w}, z) + b']/\,k + u$
Debt/Market Value Real Assets	$b'/\,(V(\widetilde{w}, z) + b')$
Equity Issuance/ Book Value Real Assets	$(k' - \widetilde{w} - b')/\,k$

Computation of average q using real-world data sets involves numerous judgment calls and imputations. Of course, this produces measurement error. In contrast, there is no measurement error when average q is computed from a structural model. Since it is impossible to remove measurement error from the real-world data, we put the model on equal footing by adding a pseudo-normal error term, denoted u, to model-generated q. We set $\sigma_u = 2.4$. The implied R^2 from the regression of $(V + b)/\,k + u$ on $(V + b)/\,k$ is approximately 0.4—a figure in line with the estimates in Erickson and Whited (2000).

B. Results

Table I presents the results from simulating the model. The first column provides moments from the simulated data. The rest of the table provides elasticities of these moments with respect to the underlying structural parameters, providing the reader with a sense of how various moments vary when financial frictions change.[10]

[10]See table notes for details of the elasticity computation.

First we discuss the more interesting moments. The simulated firm issues equity 4.65% of the time. Conditional upon issuing equity, the average flotation is equal to 6.34% of total assets. This average has a substantial variance and is highly skewed. Unlike equity issuance, distributions have a low variance. The average debt to asset ratio is 11.56%. The firm has negative leverage, i.e. holds cash, approximately 24% of the time. The coefficient on q is quite close to most of the estimates found in the literature. The coefficient on cash flow, however, is substantially smaller than most empirical estimates.

Next we turn to the elasticities. Not surprisingly, the frequency and size of equity issuance are quite sensitive to the three parameters that determine the cost of external equity. Leverage and cash holding are sensitive to bankruptcy costs (ξ) and to the parameters governing the driving process for z: ρ and σ_e. Intuitively, the more variable are the shocks, the less desirable is debt given costs of default, and the higher is the precautionary savings motive.

It is interesting to note that the cash flow coefficient is decreasing in all parameters of the external equity cost function and increasing in bankruptcy costs. This result underscores the idea that one number, cash flow sensitivity, cannot capture the magnitude of all external financial frictions.

4. Indirect Inference Estimation

A. Data

Our data are from the full coverage 2002 Standard and Poor's COMPUSTAT industrial files. We select a sample by first deleting firm-year observations with missing data. Next, we delete observations in which total assets, the gross capital stock, or sales are either zero or negative. To avoid rounding errors, we delete firms whose total assets are less than two million dollars and gross capital stocks are less than one million dollars. Further, we delete observations that fail to obey standard accounting identities. Finally, we omit all firms whose primary SIC classification is between 4900 and 4999 or between 6000 and 6999, since our model is inappropriate for regulated or financial firms. We end up with an unbalanced panel of firms from 1993 to 2001 with between 592 and 1128 observations per year. We truncate our sample period below at 1993, because our tax parameters are relevant only for this period.

B. Methodology

Because the model has no closed-form solution, we opt for an estimation technique based on simulation. Specifically, we estimate unknown parameters using indirect inference. This procedure chooses the parameters to minimize the distance between model-generated moments and the corresponding moments from actual data. Because the moments of the model-generated data depend on the structural parameters utilized, minimizing this distance will, under certain conditions discussed below, provide consistent estimates.

The goal is to estimate a vector of unknown structural parameters, say θ^*, by matching a set of *simulated moments*, denoted as m^*, with the corresponding *data moments*, denoted as M^*. The candidates for the moments to be matched include simple summary statistics, OLS regression coefficients, and coefficient estimates from non-linear reduced-form models.

Without loss of generality, the data moments can be represented as the solution to the maximization of a criterion function

$$\hat{M}_N = \arg\max_M J\left(Y_N, M\right),$$

where Y_N is a data matrix of length N. For example, the sample mean of a variable, x, can be thought of as the solution to minimizing the sum of squared errors of the regression of x on a constant. We first estimate $\hat{M}_N$. Then we construct S data sets based on simulations of the model under a given parameter vector, say θ. For each of these simulated data sets, we estimate m^* by maximizing an analogous criterion function

$$\hat{m}_n^s\left(\theta\right) = \arg\max_m j\left(y_n^s, m\right),$$

where y_n^s is a simulated data matrix of length n. Note that we express the simulated moments, $\hat{m}_n^s\left(\theta\right)$, as explicit functions of the structural parameters utilized in that particular round of simulations. The indirect inference estimator of θ^* solves

$$\begin{aligned}\widehat{\theta} &= \arg\min_\theta \left[\hat{M}_N - \frac{1}{S}\sum_{s=1}^{S}\hat{m}_n^s\left(\theta\right)\right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S}\sum_{s=1}^{S}\hat{m}_n^s\left(\theta\right)\right] \\ &\equiv \arg\min_\theta \hat{G}_N' \hat{W}_N \hat{G}_N,\end{aligned}$$

where $\hat{W}_N$ is an arbitrary positive definite matrix that converges in probability to a deterministic positive definite matrix W. The optimal weighting matrix is $\left[N \operatorname{var}\left(\hat{M}_N\right)\right]^{-1}$. Since our moment vector consists of both means and regression coefficients, we use the influence-function approach in Erickson and Whited (2000) to calculate this covariance matrix. Specifically, we stack the influence functions for each of our moments and then form the covariance matrix by taking the sample average of the inner product of this stack.

The indirect estimator is asymptotically normal for fixed S. Define $j^* \equiv p\lim_{n\to\infty}(j_n)$. Then

$$\sqrt{N}\left(\widehat{\theta}-\theta^*\right) \overset{d}{\longrightarrow} \mathcal{N}\left(0, \text{avar}(\widehat{\theta})\right),$$

where

$$\text{avar}(\widehat{\theta}) \equiv \left(1+\frac{1}{S}\right)\left[\frac{\partial j^*}{\partial\theta\partial m'}\left(\frac{\partial j^*}{\partial m}\frac{\partial j^{*\prime}}{\partial m}\right)^{-1}\frac{\partial j^*}{\partial m\partial\theta'}\right]^{-1}. \tag{30}$$

Further, the technique provides a test of the overidentifying restrictions of the model, with

$$\frac{NS}{1+S}\hat{G}_N'\hat{W}_N\hat{G}_N$$

converging in distribution to a χ^2 with degrees of freedom equal to the dimension of M minus the dimension of θ.

The success of this procedure relies on picking moments m that can identify the structural parameters θ. In other words, the model must be identified. Global identification of a simulated moments estimator obtains when the expected value of the difference between the simulated moments and the data moments equal zero if and only if the structural parameters equal their true values. A sufficient condition for identification is a one-to-one mapping between the structural parameters and a subset of the data moments of the same dimension. Although our model does not yield such a closed form mapping, we select our moments based on the underlying theory. In particular, we exploit moments that the underlying theoretical model indicates *should* be informative about the various frictions.

We use a minimization algorithm, simulated annealing, that avoids local minima. Finally, we perform a check of the numerical condition for local identification. Let $\hat{m}_n^s$ be a subvector of m with the same dimension as θ. Local identification demands that the Jacobian determinant, $\det\left(\partial\hat{m}_n^s(\theta)/\partial\theta\right)$, is non-zero. This condition can be interpreted loosely as saying that the moments (m), are informative about the structural parameters (θ). If this were not the case, not only would $\det\left(\partial\hat{m}_n^s(\theta)/\partial\theta\right)$ be near zero, but sample counterpart to the term $\partial j^*/\partial\theta\partial m'$ in (54) would be as well—a condition that would cause the parameter standard errors to blow up.

It is worth noting that indirect inference offers an important advantage over OLS and IV as a basis for parameter estimation. In particular, it does not suffer from simultaneity problems, since it does not require the zero-correlation restrictions that are necessary to identify OLS and IV regressions. Rather, as in a standard GMM estimation, it merely requires at least as many moments as underlying structural parameters.

To generate simulated data comparable to COMPUSTAT, we create $S = 6$ artificial panels, containing 10,000 *i.i.d.* firms.[11] We simulate each firm for 50 time periods and then keep the last nine, where we pick the number "nine" to correspond to the time span of our COMPUSTAT sample. Dropping the first part of the series allows us to observe the firm after it has worked its way out of a possibly suboptimal starting point.

One final issue is unobserved heterogeneity in our data from COMPUSTAT. Recall that our simulations produce *i.i.d.* firms. Therefore, in order to render our simulated data comparable to our actual data we can either add heterogeneity to the simulations, or take the heterogeneity out of the actual data. We opt for the latter approach, using fixed firm and year effects in the estimation of all of our data moments.

In order to estimate the eight unknown parameters $(\lambda_0, \lambda_1, \lambda_2, \xi, \phi, \sigma_e, \rho, \sigma_u)$ we must match at least eight model-generated moments with corresponding data moments. The parameters governing production, α and δ, are not estimated given that our focus is on financing. In addition, numerous other studies have already estimated these parameters. As discussed above, tax rate parameters are based upon estimates from Graham (2000).

We use twelve data moments in order to have an overidentified model. We start with the average, variance, and skewness of the ratio of equity issuance to assets. We also use the frequency of equity issuance; the fraction of firm years in which the firm neither issues equity nor distributes; the fraction of firm years in which the firm saves rather than borrows; the variance of the ratio of distributions to assets; and the average ratio of net debt to total assets, where net debt is defined as total long-term debt less cash.

[11] Michaelides and Ng (2000) find that good finite sample performance of an indirect inference estimator requires a simulated sample that is approximately ten times as large as the actual data sample.

The next two moments are the two slope coefficients from a regression of the ratio of investment to the capital stock on cash flow and Tobin's q. These three variables are calculated as in Erickson and Whited (2000). The final two moments capture important features of the driving process for z. We estimate a first-order panel autoregression of operating income on lagged operating income using the technique in Holtz-Eakin, Newey, and Rosen (1988). Operating income is defined as COMPUSTAT item #13 divided by item #6. The two moments that we match from this exercise are the autoregressive parameter and the shock variance.[12]

Assets are COMPUSTAT item #6, equity issuance is item #108 minus item #115, and net debt is item #9 plus item #34 minus item #1, and distributions are the sum of COMPUSTAT items 19 and 21 plus any negative equity issuance.

C. Parameter Estimates

The results from this estimation exercise are in Tables II and III. Table II compares the actual moments with those from the simulated model. We match most of the moments well. Indeed, one cannot reject the null hypothesis that the simulated moments equal the actual data moments. The χ^2 test of this null hypothesis reported in Table III (the test of the model overidentifying restrictions) does not produce a rejection at even the 10% level.

We have slight difficulty matching two of the twelve moments. This first is the frequency of equity issuance, with the simulated firm issuing equity fifty percent more often than the average real firm. Because high fixed costs of equity issuance should lower the frequency of issues, we suspect this result is due to the low point estimate for fixed costs of equity issuance (λ_0) reported in Table III.

The second moment that we have difficulty in matching is the investment-cash flow sensitivity: our model-generated sensitivity of investment to cash flow is just over half that of the corresponding figure seen in the data. We conjecture that this result is in part due to relatively low estimated measurement error variance for q reported in Table III. This estimate implies that 55% of the variation in "true q" can explained by

[12] As required by the Holtz-Eakin, Newey, and Rosen (1988) technique, we account for fixed effects via differencing our autoregression. For our other regressions, we simply remove firm-level means from the data. We opt for this method simply because it is the method most used in the empirical literature we are trying to understand.

"observed q." This figure is somewhat higher than the estimates in Erickson and Whited (2000); that is, our estimate of the measurement error variance is lower. Since cash flow and true q are positively correlated, the lower the measurement error variance, the lower the coefficient on cash flow.

Table III contains the point estimates of the structural parameters. As noted above, the estimate of λ_0 is quite small and insignificantly different from zero. This result stands in contrast to that in Altinkilic and Hansen (2000), who find significant fixed costs. Our estimates of λ_1 and λ_2, are, however, significantly different from zero and in line with their study, which finds that average variable costs of equity issuance are 4.4%. To calculate a comparable figure we take the ratio of total variable costs to equity issuance, finding an average value of 5.8%. The remaining wedge between these estimates could be easily be accounted for by adverse selection premia over and above those capitalized into underwriter fee schedules. Finally, the positive estimate of λ_2 mirrors the increasing marginal costs found in Altinkilic and Hansen.

The point estimate of bankruptcy costs is $\xi = 12.2\%$. Taken at face value, this point estimate casts doubt on the conventional wisdom that firms are debt conservative. Firms do not behave "as if" facing implausibly large bankruptcy costs. In fact, one cannot reject the null that bankruptcy costs are zero. However, this parameter estimate is noisy. Perhaps this is not surprising given that the structural model attempts to match the heterogeneous behavior of real-world firms who undoubtedly face different costs of bankruptcy and different risks to underlying cash flows.

5. The FHP-KZ Debate Revisited

This section uses simulations of the structural model to evaluate *the validity* of null hypotheses central to the debate between FHP and Kaplan and Zingales (1997). The structural model is used as a laboratory to evaluate whether cash flow coefficients behave in the way predicted by FHP. The three null hypotheses are as follows. *Monotonicity:* Firms facing higher costs of external funds will exhibit higher cash flow coefficients. *Inversion:* If all firms face the same set of financing costs, cash flow coefficients will be inversely related to dividends. *Increasing Differences:* As financing costs increase, the difference between the cash flow coefficients across dividend groups also increases.

Table IV is the analog of Table I, with the sole difference being that the moments and elasticities are based upon simulations of the model using the parameter estimates from Table III. This is a reasonable baseline, given that these parameters produce the "best fit." The tenth row of Table IV allows us to assess the Monotonicity conjecture. Apparently, the cash flow coefficient actually *decreases* in each cost of external equity. Therefore, Monotonicity is not a correct null hypothesis.

What explains this result? Close examination of Table IV suggests some answers. In our setting, cash flow is a proxy for investment opportunities, even when average q is included as a conditioning variable. If costs of external equity are low, the firm responds to a big positive shock by investing a large amount and is even willing to tap external equity, the last source of funds in the financing hierarchy. When faced with higher costs of external equity, the first four rows of Table IV show the firm becomes reluctant to tap external equity. That is, the firm invests less aggressively in response to a positive shock. Hence, the cash flow sensitivity of investment declines.

The cash flow coefficient actually increases in bankruptcy costs. Table IV suggests that changing any financing cost produces many subtle effects, as the firm re-optimizes. The most important response to higher bankruptcy costs is that the firm issues much less debt, which decreases the debt overhang problem. Second, the firm chooses to hold cash more often. The net effect is that the firm invests more aggressively in response to positive shocks.

FHP (1988) split their sample based on dividend policies. For each firm in their panel, there are 15 annual observations. Firms are then placed into three classes. Class 1 firms have a ratio of dividends to income less than 10% for at least 10 years. Class 2 firms have a ratio of dividends to income between 10% and 20% for at least ten years. Class 3 includes all other firms. FHP find that the cash flow coefficient is highest for Class 1 firms, declining monotonically across the other two classes.

In Table V, we mimic the FHP sorting procedure using the simulated model under the parameters in Table III. The results support their conjecture that low dividend firms will exhibit the highest cash flow sensitivity. However, strict Inversion does not hold, since the cash flow coefficient falls and then rises as one moves across Classes 2 and 3.

Finally, Figures 3-6 allow us to evaluate the Increasing Differences hypothesis. In each figure, we first estimate the cash flow coefficients generated when the model is simulated under the Table III parameters.

Then the appropriate parameter is increased, the model simulated, and a new cash flow coefficient is estimated for each class. As one can see, the Increasing Differences hypothesis is an incorrect null in our setting. Focusing first on Figures 3-5, we see that the cash flow coefficient declines across all distribution classes as the λ parameters are increased. This effect was discussed above. However, the rate of change across groups is apparently different. Finally, Figure 6 shows that cash flow coefficients for all three dividend classes increase in bankruptcy costs. There is no evidence that the differences in cash flow coefficients across classes widens.

6. Conclusions

This paper proposed an alternative to investment-cash flow sensitivities as a basis for estimating the magnitude of financial frictions faced by corporations. Starting with primitives, we first presented a dynamic structural model endogenizing all relevant choice variables of the firm: investment, distributions, leverage and default. This model is then taken to the data using indirect inference. We estimate which constellation of financial frictions best explains observed financing behavior, i.e. minimizes the distance between model-generated moments and real-world data moments. Consistent with direct evidence on underwriter fee schedules, behavior is best explained by rising marginal costs of external equity, starting at 3.9%. Contrary to the notion that corporations are debt conservative, debt issuance is consistent with small (12.2%) and statistically insignificant bankruptcy costs.

The cash flow coefficient is not a catch-all for financing costs, nor is it monotonic in the various frictions. It increases in bankruptcy costs, but decreases in external equity costs. When the model is simulated using our parameter estimates, the cash flow coefficient across Fazzari, Hubbard, and Petersen's dividend classes is actually U-shaped, achieving its highest value for the lowest dividend group. The difference between cash flow coefficients across dividend classes actually decreases as costs are increased.

Explaining the behavior of the cash flow coefficient is clearly difficult. However, this is to be expected. Firms optimize over time and at various margins, e.g. investment, distributions, and leverage. Changes in financing costs will bring about subtle changes as firms re-optimize, often rendering it humanly impossible to accurately predict the behavior of regression coefficients. Although demanding, simulation methods obviate the need for guess-work in such complex environments.

Appendix

Proof of Lemma 1

In the interest of brevity and keeping our notation consistent with that in Stokey and Lucas (1989), let

$$\begin{aligned} F(\widetilde{w}, k', b') &\equiv \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] \\ \beta &\equiv \frac{1}{1 + r(1 - \tau_i)}. \end{aligned}$$

We will show that, without loss of generality, the set of possible endogenous state variables can be treated as a compact set. For Lemma 1, $\widetilde{W} \times Z$ is treated as compact. Weierstrass' Theorem ensures that each $f \in C(\widetilde{W} \times Z)$ is bounded.

Partitioning the constraint correspondence as follows

$$\begin{aligned} \Gamma^+(z) &\equiv \{(k', b') \in \Gamma(z) : \widetilde{w} + b' - k' \geq 0\} \\ \Gamma^-(z) &\equiv \{(k', b') \in \Gamma(z) : \widetilde{w} + b' - k' \leq 0\}, \end{aligned}$$

we may express the Bellman operator (T) for this problem as follows, for arbitrary $f \in C(\widetilde{W} \times Z)$:

$$(Tf)(\widetilde{w}, z) \equiv \max \left\{ \begin{array}{cc} \max\limits_{(k',b') \in \Gamma^-(z)} & -[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz'), \\ \max\limits_{(k',b') \in \Gamma^+(z)} & \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz') \end{array} \right\}$$

where the constraints are as specified in (13).

We first claim that

$$T : C(\widetilde{W}, Z) \to C(\widetilde{W}, Z).$$

Fix $f \in C(\widetilde{W}, Z)$ and consider first the problem

$$\max_{(k',b') \in \Gamma^+(z)} \quad \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz').$$

Continuity of the function $\widetilde{r}$ implies continuity of $\widetilde{w}$. Lemma 9.5$'$ in Stokey and Lucas (SL) (1989), implies that the expectation above is bounded and continuous. From the Theorem of the Maximum, the value function

$$f^+(\widetilde{w}, z) \equiv \max_{(k',b') \in \Gamma^+(z)} \quad \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz')$$

is continuous, and hence bounded. By the same reasoning, the value function

$$f^{-}(\widetilde{w}, z) \equiv \max_{(k',b') \in \Gamma^{-}(z)} \quad -\left[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')\right] + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz')$$

is continuous, and hence bounded.

We can then write the Bellman operator as

$$(Tf)(\widetilde{w}, z) \equiv \max\left\{f^{+}(\widetilde{w}, z), f^{-}(\widetilde{w}, z)\right\},$$

which is also continuous, and hence bounded. This establishes the first claim.

We next show that T satisfies Blackwell's sufficient conditions for a contraction mapping, stated as Theorem 3.3 in SL. To establish monotonicity, consider arbitrary functions f_1 and f_2 in $C(\widetilde{W} \times Z)$, where $f_1 \leq f_2$ on $\widetilde{W} \times Z$. For $i = 1, 2$, we can define the same partitioned maximization problems as above, with

$$(Tf_i)(\widetilde{w}, z) \equiv \max\left\{f_i^{+}(\widetilde{w}, z), f_i^{-}(\widetilde{w}, z)\right\}.$$

Let (k'_*, b'_*) be the optimal policies corresponding to the value $f_1^{+}(\widetilde{w}, z)$. It follows that

$$\begin{aligned} f_1^{+}(\widetilde{w}, z) &= \widetilde{w} + b'_* - k'_* - T^d(\widetilde{w} + b'_* - k'_*) + \beta \int_Z f_1[\widetilde{w}(k'_*, b'_*, z, z'), z'] Q(z, dz') \\ &\leq \widetilde{w} + b'_* - k'_* - T^d(\widetilde{w} + b'_* - k'_*) + \beta \int_Z f_2[\widetilde{w}(k'_*, b'_*, z, z'), z'] Q(z, dz') \\ &\leq f_2^{+}(\widetilde{w}, z). \end{aligned}$$

The first inequality follows from the hypothesis $f_1 \leq f_2$ and the second follows from a standard dominance argument. By the same reasoning

$$\begin{aligned} f_1^{-}(\widetilde{w}, z) &\leq f_2^{-}(\widetilde{w}, z) \\ &\Rightarrow Tf_1(\widetilde{w}, z) \leq Tf_2(\widetilde{w}, z). \end{aligned}$$

Now fix scalar $a \geq 0$ and $f \in C(\widetilde{W} \times Z)$. We have

$$\begin{aligned} [T(f+a)](\widetilde{w}, z) &\equiv \max\left\{\begin{array}{ll} \max\limits_{(k',b') \in \Gamma^{-}(z)} & -\left[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')\right] + \beta \int_Z [f(\widetilde{w}(k', b', z, z'), z') + a] Q(z, dz'), \\ \max\limits_{(k',b') \in \Gamma^{+}(z)} & \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z [f(\widetilde{w}(k', b', z, z'), z') + a] Q(z, dz') \end{array}\right\} \\ &= \beta a + (Tf)(\widetilde{w}, z). \end{aligned}$$

This establishes discounting. Hence, T is a contraction mapping.■

Proof of Proposition 3

Let $C'(\widetilde{W} \times Z)$ and $C''(\widetilde{W} \times Z)$ be the space of all functions in $C(\widetilde{W} \times Z)$, that are, respectively, weakly and strictly increasing in their first argument. SL's Corollary 1 to the Contraction Mapping Theorem shows that

$$T[C'(\widetilde{W} \times Z)] \subseteq C''(\widetilde{W} \times Z) \Rightarrow V \in C''(\widetilde{W} \times Z).$$

Fix $f \in C'(\widetilde{W} \times Z)$ and $z \in Z$. Assume that the policy pairs (k_1', b_1') and (k_2', b_2') attain the supremum for the firm starting with revised net worth consider $\widetilde{w}_1$ and $\widetilde{w}_2$, respectively, where $\widetilde{w}_1 > \widetilde{w}_2$. Then

$$\begin{aligned}
(Tf)(\widetilde{w}_1, z) &= F(\widetilde{w}_1, k_1', b_1') + \beta \int_Z f[\widetilde{w}(k_1', b_1', z, z'), z']Q(z, dz') \\
&\geq F(\widetilde{w}_1, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z, z'), z']Q(z, dz') \\
&> F(\widetilde{w}_2, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z, z'), z']Q(z, dz') \\
&= (Tf)(\widetilde{w}_2, z).
\end{aligned}$$

The first inequality follows from that fact that (k_1', b_1') must weakly dominate (k_2', b_2') for the firm with revised net worth $\widetilde{w}_1$, since both firms have the same choice set $\Gamma(z)$. The second inequality follows from the fact that F is strictly increasing in its first argument. This establishes

$$T[C'(\widetilde{W} \times Z)] \subseteq C''(\widetilde{W} \times Z).■$$

Proof of Proposition 4

Let $C'(\widetilde{W} \times Z)$ be the space of all functions in $C(\widetilde{W} \times Z)$, that are nondecreasing in their second argument. SL's Corollary 1 to the Contraction Mapping Theorem shows that

$$T[C'(\widetilde{W} \times Z)] \subseteq C'(\widetilde{W} \times Z) \Rightarrow V \in C'(\widetilde{W} \times Z).$$

Fix $f \in C'(\widetilde{W} \times Z)$ and $\widetilde{w} \in \widetilde{W}$. Assume that the policy pairs (k_1', b_1') and (k_2', b_2') attain the supremum for the firm starting with the shocks z_1 and z_2, respectively, where $z_1 > z_2$. Then

$$\begin{aligned}(Tf)(\widetilde{w}, z_1) &= F(\widetilde{w}, k_1', b_1') + \beta \int_Z f[\widetilde{w}(k_1', b_1', z_1, z'), z']Q(z_1, dz') \\ &\geq F(\widetilde{w}, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z_1, z'), z']Q(z_1, dz') \\ &\geq F(\widetilde{w}, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z_2, z'), z']Q(z_2, dz') \\ &= Tf(\widetilde{w}, z_2).\end{aligned}$$

The first inequality follows from that fact that (k_1', b_1') must weakly dominate (k_2', b_2') for the firm facing the shock z_1, since $\Gamma(z_2) \subseteq \Gamma(z_1)$ by hypothesis. The second inequality follows from the fact that F is nondecreasing in z, $\widetilde{w}$ is nondecreasing in its third argument, and Q is monotone.$\blacksquare$

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Table I: Sensitivity of Model Moments to Parameters

	Baseline Moments	λ_0	λ_1	λ_2	ϕ	ξ	σ_e	ρ	σ_u
Average Equity Issuance/Assets	0.0634	-0.4010	-1.2576	-0.7251	-0.1288	0.4638	1.3129	-0.6129	0.0000
Variance Equity Issuance/Assets	0.8657	-0.3923	-1.3354	-0.8784	-0.1311	0.0420	0.9579	0.1834	0.0000
Skewness Equity Issuance/Assets	8.2353	-0.3329	-1.3677	-0.9467	-0.1264	0.1317	1.5959	1.3224	0.0000
Frequency of Equity Issuance	0.0465	-0.5667	-0.6302	-0.2677	-0.1081	0.0250	0.3468	-0.2505	0.0000
Frequency of Zero Dividend	0.1542	0.1145	0.0437	-0.0514	-0.0263	0.1564	-0.0575	-1.1716	0.0000
Frequency of Cash Holding	0.2361	-0.1256	0.3860	0.2196	0.7729	0.6360	0.9650	1.1520	0.0000
Variance Distributions/Assets	0.0143	-0.2983	-0.9237	-0.4320	-0.0707	0.0011	2.7675	-0.2092	0.0000
Average Debt-Assets Ratio (Net of Cash)	0.1156	1.0872	1.9597	-0.3136	-0.5485	-1.0192	-1.8896	-2.4520	0.0000
Investment q Sensitivity	0.0165	-0.5130	0.2057	0.0061	0.0145	0.0165	-0.4329	-0.5000	-1.5420
Investment Cash Flow Sensitivity	0.0543	-0.3690	-0.3212	-0.2415	0.0050	0.6708	-1.4352	0.2315	0.8564
Serial Correlation of Income/Assets	0.6433	0.0812	0.1360	0.1068	-0.0224	0.1085	0.1690	1.9624	0.0000
Standard Deviation of the shock to Incomes/Assets	0.1218	0.0686	0.0556	0.0471	-0.0042	0.0005	3.8226	0.1143	0.0000

This table presents elasticities of model moments with respect to the model parameters. The baseline parameters are $\lambda_0 = 1.2$, $\lambda_1 = 0.028$, $\lambda_2 = 0.005$, $\phi = 0.02$, $\xi = 0.1$, $\sigma_\varepsilon = 0.118$, $\rho = 0.740$, and $\sigma_u = 2.4$. Each elasticity is calculated by simulating the model twice: once with a value of the parameter of interest fifty percent below its baseline value, and once with a value fifty percent above its baseline value. Then the change in the moment is calculated as the difference between the results from the two simulations. This difference is then divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

Table II: Simulated Moments Estimation: Moment Estimates

	Actual Moments	Simulated Moments
Average Equity Issuance/Assets	0.042	0.056
Variance Equity Issuance/Assets	0.319	0.546
Skewness Equity Issuance/Assets	4.008	3.054
Frequency of Equity Issuance	0.099	0.156
Frequency of Zero Dividends	0.444	0.540
Frequency of Cash Holding	0.394	0.269
Variance Distributions/Dividends	0.001	0.001
Average Debt-Assets Ratio (Net of Cash)	0.075	0.078
Investment q Sensitivity	0.019	0.021
Investment Cash Flow Sensitivity	0.172	0.098
Serial Correlation of Income/Assets	0.583	0.620
Standard Deviation of the shock to Incomes/Assets	0.117	0.102

Calculations are based on a sample of nonfinancial firms from the annual 2002 COMPUSTAT industrial files. The sample period is 1993 to 2001. Estimation is done with the simulated moments estimator in Gourieroux, Monfort, and Renault (1993), which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from these data. The simulated panel of firms is generated from the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration. The simulated panel contains 10,000 firms over 50 time periods, where only the last nine time periods are kept for each firm. This table reports the simulated and estimated moments.

Table III: Simulated Moments Estimation: Structural Parameter Estimates

λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	σ_u^2	χ^2
0.369	0.039	0.0007	0.122	0.011	0.097	0.701	6.289	4.338
(0.273)	(0.018)	(0.0002)	(0.373)	(0.038)	(0.084)	(0.329)	(2.075)	(0.362)

Calculations are based on a sample of nonfinancial firms from the annual 2002 COMPUSTAT industrial files. The sample period is 1993 to 2001. Estimation is done with the simulated moments estimator in Gourieroux, Monfort, and Renault (1993), which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from these data. The simulated panel of firms is generated from the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration. The simulated panel contains 10,000 firms over 50 time periods, where only the last nine time periods are kept for each firm. This table reports the estimated structural parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a flatter tax schedule. ξ is the verication parameter, with total verification costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, and ρ is the serial correlation of $\ln(z)$. σ_u^2 is the variance of the measurement error in average Q. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

Table IV: Sensitivity of Model Moments to Parameters

	Baseline Moments	λ_0	λ_1	λ_2	ϕ	ξ	σ_e	ρ	σ_u
Average Equity Issuance/Assets	0.0557	-1.2647	-2.2407	-2.1076	0.0678	0.6401	1.0012	-0.7273	0.0000
Variance Equity Issuance/Assets	0.5455	-1.9209	-2.9099	-2.1886	0.2676	-0.0424	1.8444	0.1490	0.0000
Skewness Equity Issuance/Assets	3.0537	-2.2983	-1.5460	-3.5017	-0.1624	-0.0590	2.4726	0.8411	0.0000
Frequency of Equity Issuance	0.1557	-1.0526	-2.6140	-1.5848	0.1081	-0.0234	1.7661	-0.2383	0.0000
Frequency of Zero Dividends	0.5395	-0.0039	0.0650	0.0673	-0.0455	0.2007	-0.3203	-1.4709	0.0000
Frequency of Cash Holding	0.2690	-0.2178	0.5998	0.4210	0.4954	0.7452	1.6012	0.4018	0.0000
Variance Distributions/Assets	0.0012	-0.5565	-3.0370	-2.5483	0.1234	-0.0185	2.1674	-0.1946	0.0000
Average Debt-Assets Ratio (Net of Cash)	0.0784	-0.1501	0.0348	0.1770	-0.4190	-1.1087	-2.3411	-0.4708	0.0000
Investment q Sensitivity	0.0205	0.4487	-2.6677	-3.1464	0.0449	0.0213	1.7318	0.1933	-1.7654
Investment Cash Flow Sensitivity	0.0981	-0.8596	-2.9175	-2.1012	0.1527	0.7328	1.4838	0.1788	0.9148
Serial Correlation of Income/Assets	0.6201	0.0670	0.2375	0.1703	-0.0232	0.0021	-0.3369	0.9700	0.0000
Standard Deviation of the shock to Incomes/Assets	0.1015	-0.1175	0.0572	0.1650	-0.0385	-0.0004	2.0271	0.0859	0.0000

This table presents elasticities of model moments with respect to the model parameters. The baseline parameters are given in Table III. Each elasticity is calculated by simulating the model twice: once with a value of the parameter of interest fifty percent below its baseline value, and once with a value fifty percent above its baseline value. Then the change in the moment is calculated as the difference between the results from the two simulations. This difference is then divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

Table V: Summary Statistics and Cash Flow Sensitivity by Payout Category

	Class 1	Class 2	Class 3
Percent of Firms	0.092	0.154	0.754
Percent of Positive Distributions	0.643	0.883	0.860
Median Distributions/Income	0.094	0.280	0.352
Median Debt/Assets	0.066	0.035	0.064
Median q	1.086	1.017	1.132
Median Investment/Capital Stock	0.150	0.150	0.150
Investment q Sensitivity	0.017	0.013	0.022
Investment Cash Flow Sensitivity	0.134	0.039	0.108

The sample consists of 4000 simulated firms, each of which lasts for 15 periods. Class 1 firms have a ratio of dividends to income less than 10% for at least 10 out of 15 periods. Class 2 firms have a ratio of dividends to income between 10% and 20% for at least ten out of 15 periods. Class 3 includes all other firms.

Figure 1: Endogenous Default

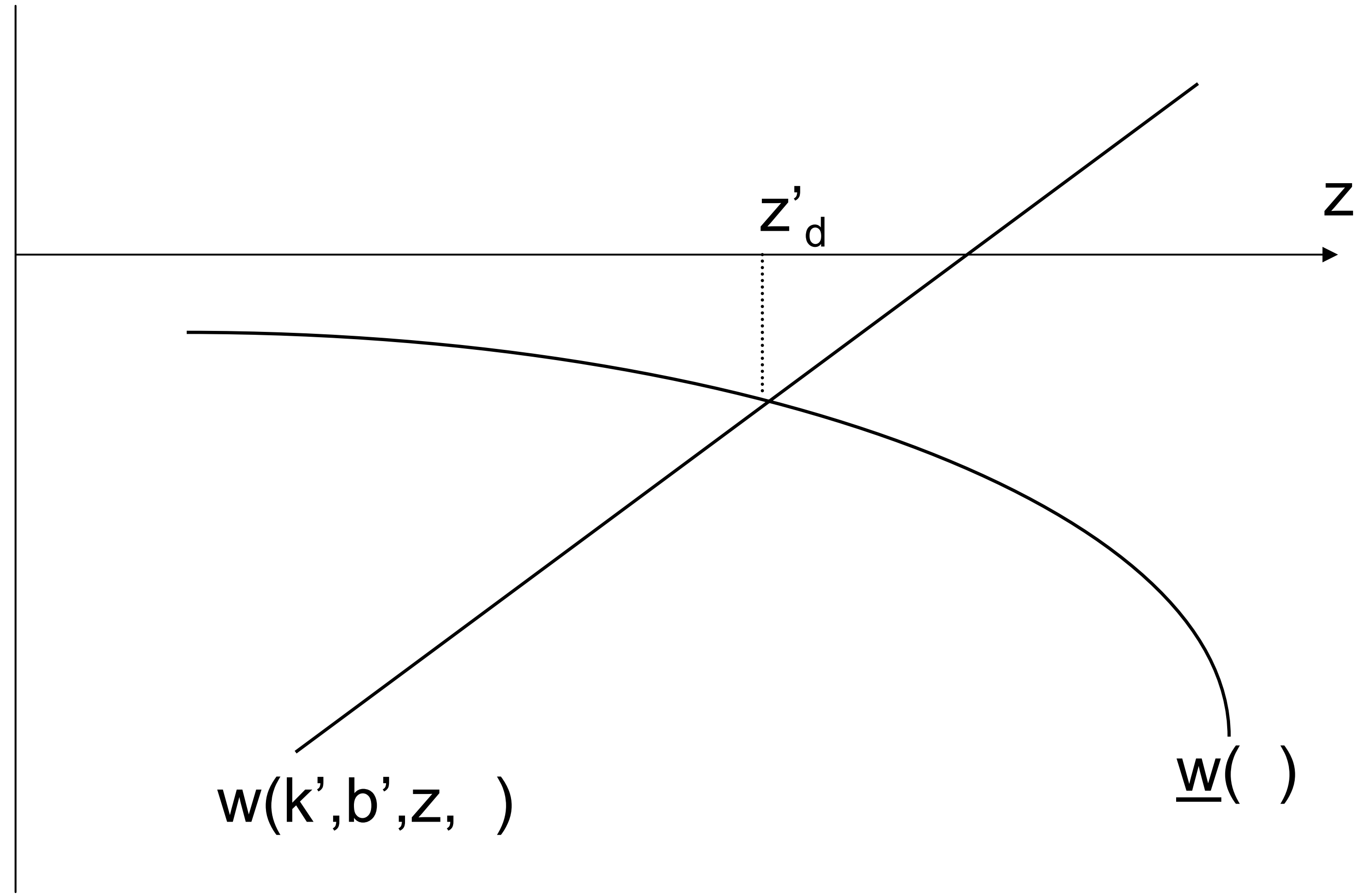
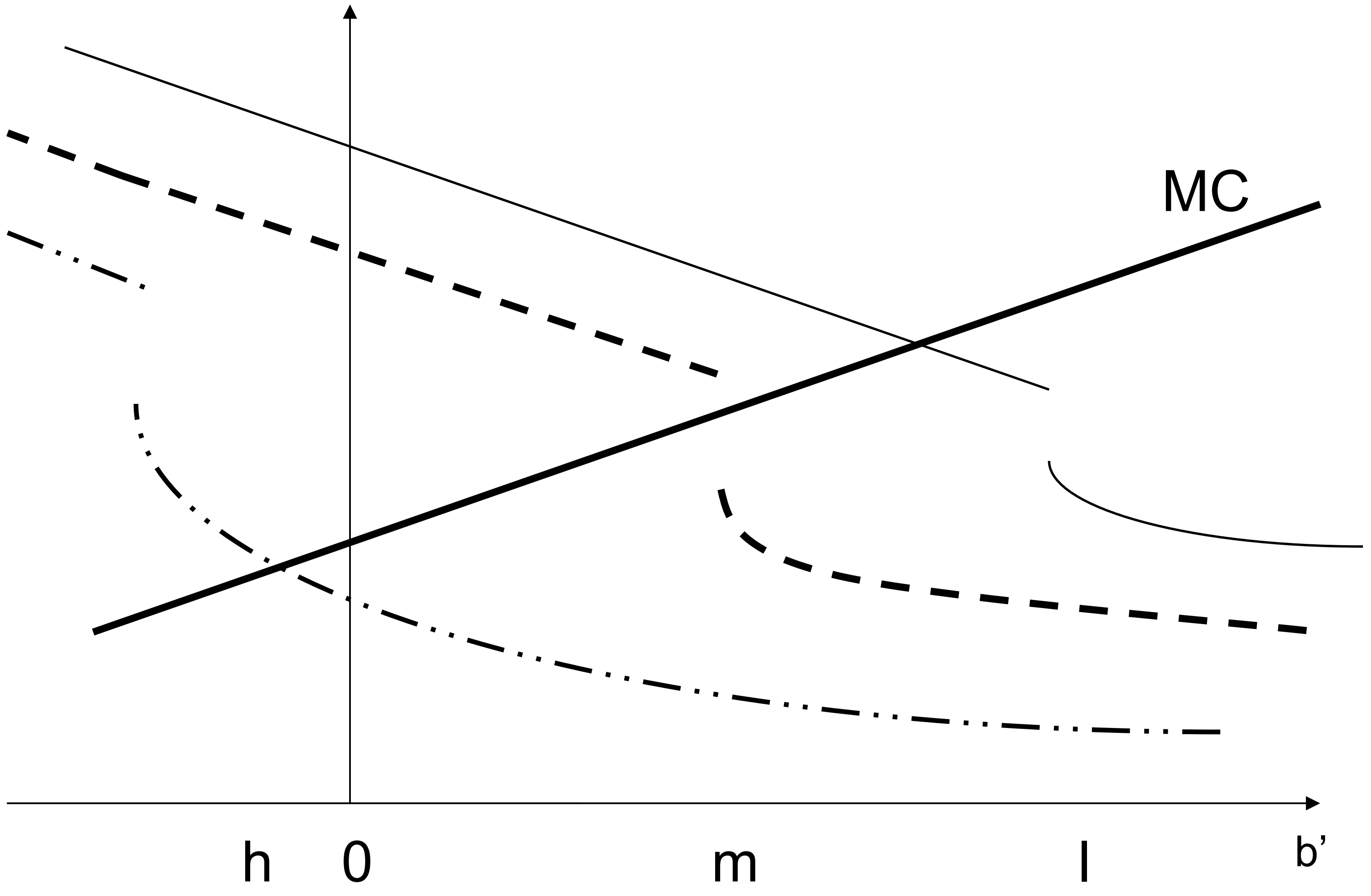


Figure 2: Optimal Financial Policy



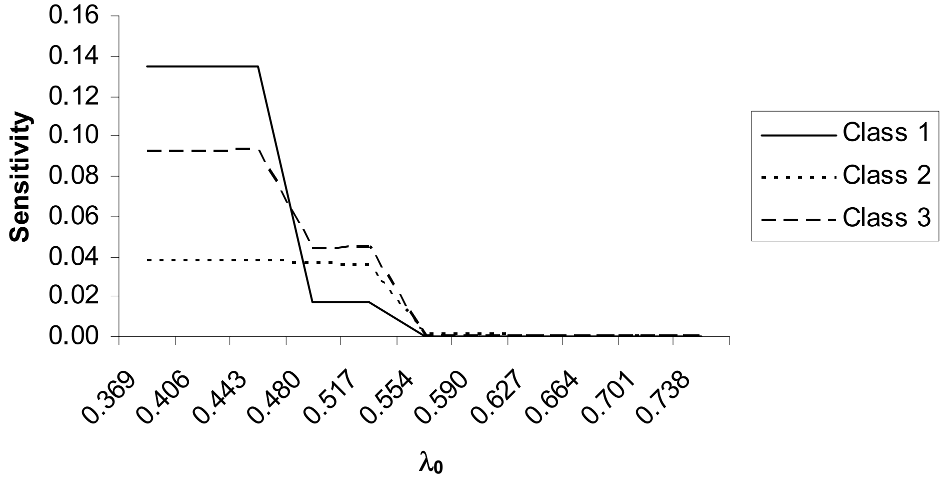


Figure 3: Investment Cash Flow Sensitivities as a Function of λ_0

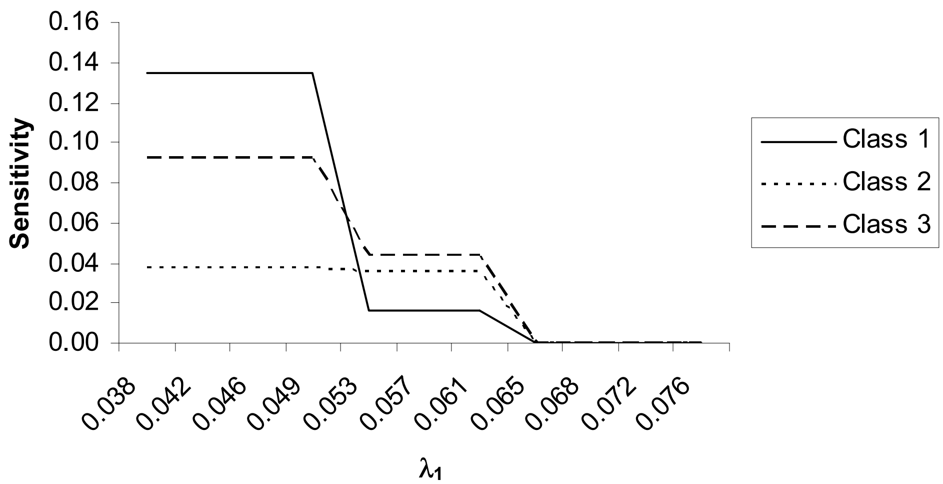


Figure 4: Investment Cash Flow Sensitivities as a Function of λ_1

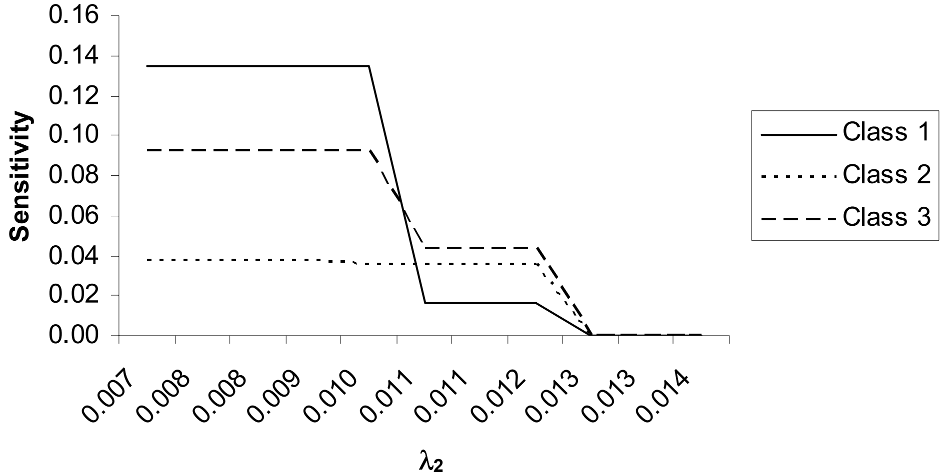


Figure 5: Investment Cash Flow Sensitivities as a Function of λ_2

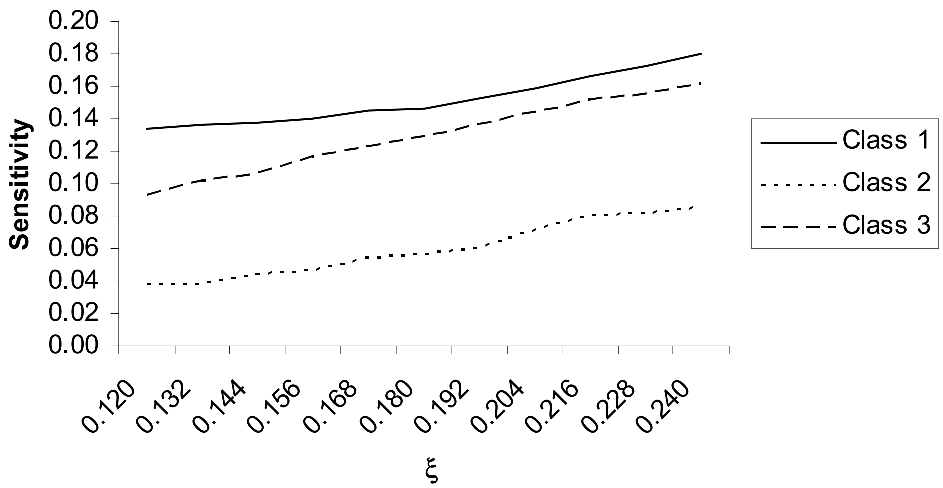


Figure 6: Investment Cash Flow Sensitivities as a Function of ξ