Strategic Alliance and International Merger Evaluation Models Applying Real and Game Options

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Abstract

This paper reveals an international merger valuation model using stochastic real exchange rate which follows geometric Brownian motion and square root of mean reverting process as an index to investigate how the decision making of international merger when two domestic firms match two foreign firms under two single-channel strategy-alliances in duopolistic market. The proposed model applies game options to evaluate the before and after project values of international merger and analyzes the threshold of international merger which are dealt with the real exchange rate. Under the assumption that the specific profit function of firms is given, the thresholds of real exchange rate are evaluated and the numerical examples of sensitivity analysis are also made. The results of numerical analysis could provide the decision maker to refer either to continue strategy alliance or make international merger decision. The analysis result of this paper finds that with high volatility in real exchange rate, the firm is unlikely to taking an international merger action. The reactions of the leader and the follower are different in many concepts. The follower is more risk aversion than the leader and the leader usually has more advantages than the follower because the earlier wins the more market share. This paper concludes that the investment cost and the return affect the decision of merger. In addition, from the results of analysis using numerical examples, after mergering, a large firm scale gets more benefit than a small firm scale.

Keywords: Merger strategy, duopolistic market, game options, means reverting process,geometric Brownian motion.

1 Introduction

This study uses real exchange rate as an index to provide decision-maker of international merger guidance on competitive market impact. Real exchange rate follows GBM (geometric Brownian motion) and MRP (mean reverting process) to simulate the real world. In addition, this study applied game option to simulate the duopolistic market and the profit function parameters are given to deduce the impacts of parameters. This study wants to find the impact of the parameters for

providing guidance to decision-makers. The scale economic is expected to have influence on the international merger. Hence, we expect that the large scale could bring more benefit than small scale.

Exchange rate volatility affects decision of international merger. Kiymaz (2004) indicated that exchange rate volatility play a significant role in explaining wealth gain which affects firm's willing to take international merger. Exchange rate exposure is a firm's main managing technique in uncertainty under internationalization and global environment, and is also a very hard iron to the firm in influence (Dominguez, 2001a; 2001b). Jorion (1990) emphasized on exchange rate fluctuating the profit in firm's operation revenue based on the transaction exposure, economic exposure and transfer exposure, and found that the exchange rate change even more ten times greater than the inflation rate change and four times greater than the interest rate change to the influence degree of firm's revenue. The exchange rate moving volatility will have a direct or indirect impact on the international firms' operations and the non-internationalized firms can also be exposed to the exchange rate risk by the outside competition indirectly. The change of exchange rate will have apparently affected the performance to the firms (Bodnar and Gentry, 1993; Amihud, 1994; Griffin and Stulz, 2001).

In recent years, academic studies have argued that traditional valuation model, NPV (net present value), can not adequately capture the value of managerial flexibility to grow, delay, scale down or abandon projects. Therefore, ROA (real options approach) becomes very important as it allows explicit valuations of flexibility (Smith, 2003).

Game theory is a regular tool that came down of industrial organization and model in imperfect competition. However, standard game theory is ignored risk return of finance theory and managerial flexibility value under future uncertainty. Therefore, game theory and ROA could be supplementary methods (Dias and Teixeira, 2003). Very few studies deal with strategic interaction is chosen in uncertainty by researchers (Grenadier (2000)). One of the main reasons is that those applications of game theory which is continuous time model are not well developed and often very craftily continuously to add. Grenadier (2000) edited a lot of game option paper. Behind that, the real options combined with game theory of pioneering textbook is published with Huisman (2001), concentrated on choosing the model of important theory of option in continuous time. Hereafter, there are probable a lot of model combines the real option and game theory.

This study introduces the game theory to construct some models to simulate the competitive market. Moreover, many studies in the past used NPV method to assess the value of the firm and ignore the dynamics of uncertain environment. In contrary, real

options consider the flexibility of environment. Hence, it could be use to manage the flexibility effects. Furthermore, how the cost and profit affecting the decision are also discussed when real exchange rate which follows two different kinds of stochastic models: GBM (Vinod, 2004) and square root of MRP (Roger, 1999).

In summary, the purposed of this paper are listed as follows.

(1) The strategic behaviors and the relationship between the leader and the follower in duopolistic market are the other main focus to be analyzed in this study. Furthermore, this study will define and prove the gains of international merger and discuss the competitor's impact which affects the firm's strategic decision. The impact of firm's scale is applied to assess the decision of the international merger and how it affects the threshold of real exchange rate, are also addressed. The real exchange rate is used as an index while the other variables are changed, to know is the firm's tendency in order to switch its' production to the foreign firm. The thresholds of real exchange rate are applied to both monopoly and duopolistic market with game theory. The closed-form solutions of the thresholds of real exchange rate are calculated.

(2) Compare the difference of real exchange rate which follows GBM and square root of MRP and also compare these two different stochastic processes in the monopolistic or duopolistic markets. In addition, numerical and sensitivity analyses also be made.

This study combines above literature review and uses international merger as model of real exchange rate which follows stochastic process under two strategic alliance groups in the duopolistic market. The remainder of this paper is organized as follows. The real exchange rate of two proposed models is evaluated by following GBM and square root of MRP in Section 2. The sensitivity analysis explains with some numerical examples in Section 3. Finally, the conclusions and remarks are presented in Section 4.

2. Proposed Models

2.1 The Leader Determined by the Large Firm Scale

A specific relationship between foreign firm and domestic firm which take the strategic alliance with each other are regard as a linked strategic alliance group to take their international merger strategy. In this study, assumes that there are only two parallel strategy alliance groups in the duopolistic foreign market. The leader group with large firm scale takes the international merger decision first and shares the competition advantages before the follower taking the international merger decision. In the contrary, the follower group with small firm scale takes the international merger decision would only follow the leader group to create the value in waiting for adopting international merger. Both groups consider the international merger as a strategy to expand the same foreign duopolistic market and in this

subsection assumes the leader has larger market share than it of the follower because of large firm scale of the leader. Figure 1 is illustrated of these two groups here, it denotes that the leader is set as first group (Group I) and the follower is a set as the second group (Group II). In Figure 2, 1_D and 1_F represent the domestic and foreign firms of Group I, respectively. Similarly, 2_D and 2_F denote to the domestic and foreign firms of Group II. Furthermore, assume V_{m1} and V_{m2} present the values of Group I and Group II after merger and V_{g1} and V_{g2} present the values of Group I and Group II before merger. $V_{m1}^{'}$ and $V_{g2}^{'}$ implying the values of Group I and Group II affected by the merger have been taken.

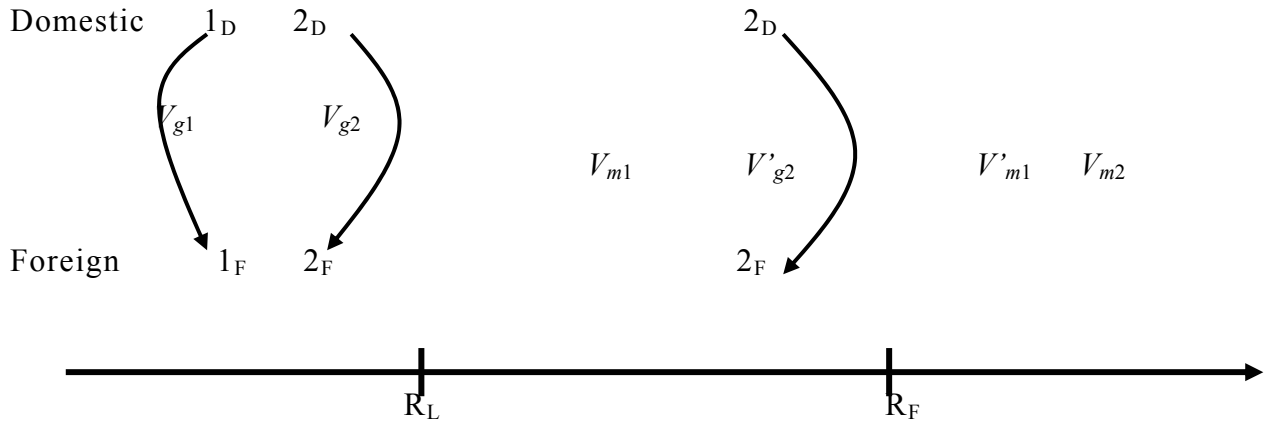

Figure 1 The Relationship of Two Groups

Two firms in both groups and all investors have complete information reveals with respect to all parameter in this proposed model. Since all firms are perfectly informed, the duopolistic foreign market will perfectly anticipate the merger and nobody can make arbitrage profit in advance. The firms' instantaneous profit functions before (j = 1 denotes domestic firm, 2 denotes foreign firm) and after ($j = m$), and i = 1 denotes Group I, i = 2 denotes Group II. Here, the profit function dealt with the international merger is given by

$$L_{ij}^{a_{ij}} - K_{ij}^{b_{ij}} - L_{ij}, \tag{1}$$

where L_{ij} denotes the variable production inputs (e.g. labor) whereas K_{ij} is fixed cost (e.g. invested capital). a_{ij} and b_{ij} imply the powers of per L_{ij} and per K_{ij}. The first item of profit function $L_{ij}^{a_{ij}} - K_{ij}^{b_{ij}}$, which is common to both firms, determines the output for operating revenue. The profit function displays increasing returns to scale with respect to both variable of inputs (i. e., $a_{ij} + b_{ij} > 1$), but decreasing returns to scale when the input is variable (i. e., $a_{ij} + b_{ij} < 1$).

Both groups are price takers in foreign duopolistic market. This study considers real exchange rate, R_t, follows GBM and square of root MRP.

These two groups face the profit flow $D(N_i, N_j)$, and possible values of $D_n(N_i, N_j)$ are dependant on what situations locates in market condition:

$D_n(0, 0)$ means that both groups have not been taken international merger yet;

$D_n(1, 0)$ means that the leader has taken international merger and the follower has not been yet followed;

$D_n(1, 1)$ means that both groups have been taken international merger as strategy already;

$D_n(0, 1)$ means that the follower takes international merger before the leader,

and where $n = L$ represents the leader and $n = F$ represents the follower.

The deterministic profit flow has the additional constraint of negative externality (the option exercise form one group reduces the value of the other group) given by the inequality:

$D_L(1, 0) > D_L(1, 1) > D_L(0, 0) > D_L(0, 1)$ and $D_F(0, 1) > D_F(1, 1) > D_F(0, 0) > D_F(1, 0)$

The other assumption is to match the first mover advantage which is given by the inequality:

$D_L(1, 0) - D_L(0, 0) > D_L(1, 1) - D_L(0, 1)$.

This study assumes that profit flows are determined by the variable cost (i.e. labor) and the fixed cost (i.e. capital). To fixed the profit flows, this study regard it isn't affected by the real exchange rate but is affected by the merger taken of these two groups. For simple explanation, the profit flows are defined as follows.

$D_L(1, 0)$ denotes the international merger cash flow of the leader, as leader has already merged and will have more market share. Hence, the value of $D_L(1, 0)$ should be larger than other situations. Therefore, a multiplier, c_1, of $D_L(1, 0)$ is bigger than one, and it should be added into the function of profit flow calculation. Therefore,

$$D_L(1, 0) = (c_1(L_{11} + L_{12}))^{a_{1m}} (c_1(K_{11} + K_{12}))^{b_{1m}} - c_1(L_{11} + L_{12}), \quad c_1 > 1. \qquad (2)$$

$D_L(1, 1)$ denotes the international merger cash flow of the leader when the follower followed the leader's strategy and the leader would loss more some market shares than the previous stage in duopolistic condition. Consequently, the international merger value of $D_L(1, 1)$ would be smaller than the previous stage in duopolistic condition but larger than the international merger value before the leader taking the international merger strategy. Hence, a multiplier, c_2, of $D_L(1, 1)$ is bigger than one but is smaller than c_1, and it should be added to calculate the profit flow of the leader. Therefore,

$$D_L(1, 1) = (c_2(L_{11} + L_{12}))^{a_{1m}} (c_2(K_{11} + K_{12}))^{b_{1m}} - c_2(L_{11} + L_{12}), \; c_1 > c_2 > 1. \quad (3)$$

$D_F(1, 1)$ denotes that the international merger cash flow of the follower follows the leader to merge and will win more market share than it of before merging in the initial stage. A multiplier, c_3, of $D_F(1, 1)$ is bigger than one, and it should be added to calculate the international merger cash flow of the follower. Therefore,

$$D_F(1, 1) = (c_3(L_{21} + L_{22}))^{b_{1m}} (c_3(K_{21} + K_{22}))^{b_{1m}} - c_3(L_{21} + L_{22}), \quad c_3 > 1. \qquad (4)$$

$D_F(1, 0)$ denotes the international merger cash flow of follower as leader has already merged and will win less market share than it of before merging in initial stage. As the results, the international merger cash flow of $D_F(1, 0)$ should be smaller as usual. A multiplier, c_4, of $D_F(1, 0)$ is smaller than one, and it should be added to calculate the international merger cash flow of follower. Therefore,

$$D_F(1, 0) = (c_4 L_{21})^{a_{21}} (c_4 L_{21}))^{b_{21}} + (c_4 K_{22})^{a_{22}} (c_4 K_{22})^{b_{22}} - c_4 (L_{21} + L_{22}), \quad c_4 < 1. \tag{5}$$

In this study, the leader is assumed to take a merger strategy before the follower taking the international merger strategy. In Figure 2, there are three periods of time zone in different international merger situation.

Since there are three separated time zones by two time points, this paper considers two time points of making international merger strategy which are the initial stage (no one taking international merger) time zone, the leader's action (only the leader taking international merger) time zone, and the follower's action (the leader and the follower exist in duopolistic market) time zone. In these two time points, three kinds of discount factor are considered, they are in the initial stage, the leader's action stage and the both groups action stage.

The discount factor at the time zone of follower's action is deduced. Suppose an expected discount factor in continuous time zone, with a risk-free discount factor r is:

$$f(R) = E[\exp(-rt_2)], \tag{6}$$

where the first hitting time at t_2 that denotes R is equal to or greater to the threshold R_F in the first hitting time, and represents when the option to merge by the follower will be optimally

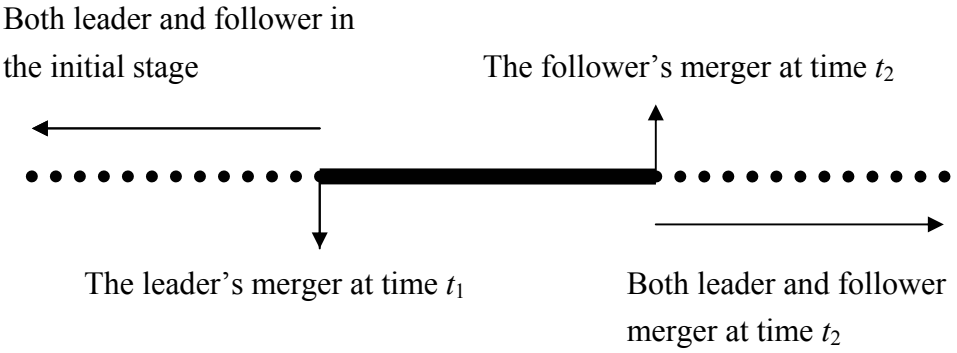

Figure 2 Three Periods of Time Zone of International Merger

exercised, the expect discount payoff from t_2 back to current time period is exactly the current value of the option to merge by the follower.

By choosing an very small interval dt that hitting the threshold R_F in the next short time interval dt and assumed the current $R < R_F$. The problem restarts from a new level ($R + dR$). Therefore, the following expression can be derived.

$$\begin{aligned} f(R) &= \exp(-rdt) E[f(R + dR) \mid R] \\ &= \exp(-rdt)\{f(R) + E[df(R)]\} \end{aligned}$$

Note that:

(a) R follows a GBM with drift μ and volatility σ; and

(b) using the ItÔ Lemma for expanding $df(R)$, and using the subscripts to denote derivatives:

$$df(R) = f_R(R)(\alpha R dt + \sigma R dz) + 0.5 f_{RR}(R)(\sigma^2 R dt)$$
$$= f_R(R)\alpha R dt + 0.5 f_{RR}(R)(\sigma^2 R dt) + f_R(R)\alpha R dz, \quad (7)$$

By noting that $E[dz] = 0$ and by substituting $df(R)$ into the previous equation, and letting $\exp(-rdt)=1 - rdt$ for a significantly small dt, the following expression cab be derived:

$$f(R) = (1 - rdt)\{f(R) + f_R(R)\alpha R dt + 0.5 f_{RR}(R)(\sigma^2 R dt)\}, \quad (8)$$

with a few algebra calculation, the following deferential equation can easy to obtain as

$$f_R(R)\alpha R dt + 0.5 f_{RR}(R)(\sigma^2 R dt) - rf(R) = 0, \quad (9)$$

and the general solution is :

$$f(R) = A_3 R^{\beta_1} + A_4 R^{\beta_2}, \quad (10)$$

where β_1 and β_2 are, respectively, the positive and the negative roots of the standard quadratic characteristic equation (Dixit and Pindyck, 1994) and A_3, A_4 are parameters.

Applying two boundary conditions, as R approximates to the threshold R_F, t_2 is probable to be small and the discount factor close to 1, $f(R_f) = 1$. When R is close to zero, t_2 is likely to be large and the discount factor is close to zero, therefore $f(0) = 0$.

After some manipulations, $A_3 = (1/R_F)^{B_1}$ and $A_4 = 0$. Hence, the solution for the expected discount factor of time zone of the follower's action is

$$f(R) = (\frac{R}{R_F})^{\beta_1}. \quad (11)$$

Figure 3 shows the discount factor of the time zone of follower's action

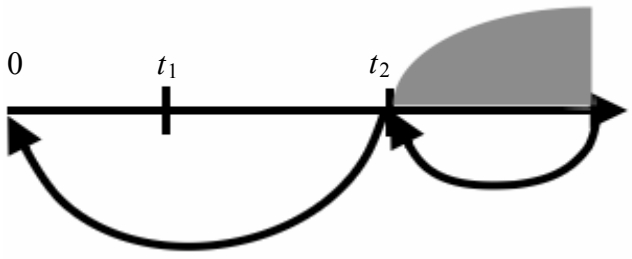

Figure 3 Discount Factor of Time Zone of Follower's Action

In the time zone of follower's action (see Figure 3), when the cash flow after t_2 discount back to t_2 and present value is regard a unit currency value, the discount factor is the present value ratio at the time t of the time zone of initial stage

In other word, the discount factor of the time zone of follower's action is the discount value per unit currency from infinity to t_2 and thus back to the initial stage.

Similarity, the discount factor in time zone of initial stage is

$$f(R) = (\frac{R}{R_L})^{\beta_1}. \tag{12}$$

Figure 4 shows the discount factor of time zone of initial stage

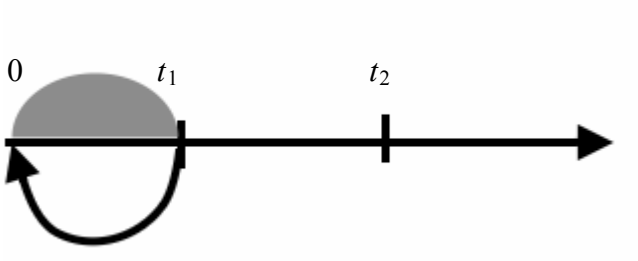

Figure 4 Discount Factor of Time Zone of Initial Stage

In the initial time zone stage, when the cash flows after 0 regards to one, the discount factor denotes one minus the present value ratio in the time *t* of the leader's action time zone.

In other word, the discount factor of time zone of initial stage is discounting value per unit currency minus discount rate from t_1 to 0 and thus back to the time zone of initial stage.

The discount factor in the time zone of leader's action converts to the time zone of initial stage is:

$$f(R) = (\frac{R}{R_L})^{\beta_1} - (\frac{R}{R_F})^{\beta_1}. \tag{13}$$

Figure 5 shows the discount factor of time zone of leader's action

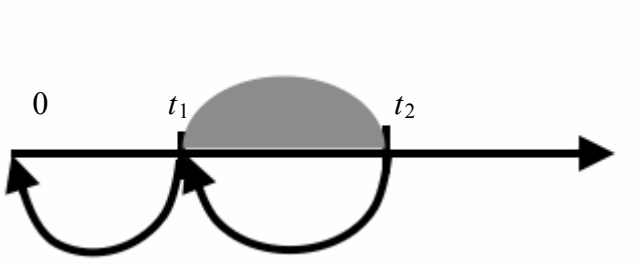

Figure 5 Discount Factor of Time Zone of Leader's Action

In the time zone of leader's action, the discount factor denotes that the present value of all cash flows after t_1 discounts to 0 minus present value of all cash flows after t_2 discounts to 0 as a unit currency value at time *t* of the initial stage action zone. In summary, the discount factor of the time zone of leader's action is discount value of per unit currency from infinity to t_1 minus discount value that from t_2 to infinity and then back to t_1 to the initial stage.

In this study, the follower's international merger value should be firstly determined. Let t_2 be the first time that the stochastic variable *R* hits a superior level $\overline{R}$ (where $\overline{R}$ = R_F). The follower's value of international merger has two components. One is the international merger profit flow before exercising the international merger option. The

other is the real international merger cash flow after the option exercise at R_F nets of initial investment cost, *invest*2. It summaries as an equation below

$$F(R) = E[\int_{t_1}^{t_2} e^{-rt} R(t) D_F(1, 0) dt] + OM(R_F)$$

$$= E[\int_{t_2}^{\alpha} e^{-rt} R(t) D_F(1, 1) dt] - invest2 . \quad (14)$$

where $OM(R_F) = A_1 R^{\beta}$.

The stochastic discount factor only depends on the R and r, and is given a simply equation below

$$E[\int_{t_2}^{\alpha} e^{-rt} dt] = (\frac{R}{R_F})^{\beta_1} \quad \text{and} \quad E[\int_{t1}^{t_2} e^{-rt} dt] = (\frac{R}{R_L})^{\beta_1} - (\frac{R}{R_F})^{\beta_1} \text{、}. \quad (15)$$

Similarly, let t_1 be the first time of R hits a superior level $\overline{R}$ (where $\overline{R}$ = R_L). The leader's value also has two components. One is the international merger profit flow before exercising the international merger option. The other is the real international merger cash flow after the option exercise at R_L nets of investment cost, *invest*1 of group with large scale, sees the integral below

$$L(R) = E[\int_{0}^{t_1} e^{-rt} R(t) D_F(0, 0) dt] + OM(R_L)$$

$$= E[\int_{t_1}^{t_2} e^{-rt} R(t) D_L(1, 0) dt] + E[\int_{t_2}^{\alpha} e^{-rt} R(t) D_L(1, 1) dt] - invest1 , \quad (16)$$

where $E[\int_{0}^{t_1} e^{-rt} dt] = 1 - (\frac{R}{R_L})^{\beta_1}$. $OM(R_L) = A_2 R^{\beta}$.

Notice that the investment cost *invest*2 of the group with small scale is less than the investment cost *invest*1 because the merger cost of leader whose scale is bigger than the followers must be more than the merger cost of follower.

By the value-matching and smooth-pasting rules, Eqs. (14) and (16), and below Eqs. (17) and (18), and if $R(t)$ follows GBM or square of root MRP, the value of R_L, R_F, A_1 and A_2 can be calculated.

$$dF(R) = d\{E[\int_{t_1}^{t_2} e^{-rt} R(t) D_F(1, 0) dt] + OM(R_F)\} / dR$$

$$= d\{E[\int_{t_2}^{\alpha} e^{-rt} R(t) D_F(1, 1) dt] - invest2\} / dR , \text{ and} \quad (17)$$

$$dL(R) = d\{E[\int_{0}^{t_1} e^{-rt} R(t) D_F(0, 0) dt] + OM(R_L)\} / dR$$

$$= d\{E[\int_{t_1}^{t_2} e^{-rt} R(t) D_L(1, 0) dt] + E[\int_{t_2}^{\alpha} e^{-rt} R(t) D_L(1, 1) dt] - invest1\} / dR . \quad (18)$$

Since the closed-form solutions can not be found due to the uncertainty of the parameters, a_{ij} and b_{ij}, this study uses sensitivity analysis to assess the change of parameters

in the paper.

2.2 The Leader Determined by the Small Firm Scale

This section is based on the small group viewpoint to evaluate groups' decisions of international merger and regards a group, with a smaller scale than the other group, as a leader. Usually, a group with a small scale may become the leader because the group with large scale lacks of mobile ability.

As similar as the pervious section of 2.1, the international merger profit flows of the leader and the follower are defined as follows.

$D_L(1, 0)$ denotes the international merger cash flow of leader, as leader has already merged and will have more market share Hence, the value of $D_L(1, 0)$ should be larger than other situations. Therefore, a multiplier, c_5, of $D_L(1, 0)$, here c_5 is bigger than one and it should be added into the function of profit flow calculation. As the group with large scale may have the economic scale, the multiplier c_5 should be smaller than c_1. Therefore,

$$D_L(1, 0) = (c_5(L_{22} + L_{21}))^{a_{2m}} (c_5(K_{22} + K_{21}))^{b_{2m}} - c_5(L_{22} + L_{21}), \quad c_1 > c_5 > 1 \quad (20)$$

$D_L(1, 1)$ denotes the international merger cash flow of the leader when the follower followed the leader's strategy and the leader would loss more some market shares than the time zone of leader's action. Consequently, the international merger value of $D_L(1, 1)$ would be smaller than the pervious stage in duopolistic condition but larger than the international merger value before the leader taking the international merger strategy. Hence, a multiplier, c_6, of $D_L(1, 1)$ is bigger than one but is smaller than c_5, and it should be added to calculate the function of profit flow of the leader. Therefore,

$$D_L(1, 1) = (c_6(L_{22} + L_{21}))^{a_{2m}} (c_6(K_{22} + K_{21}))^{b_{21m}} - c_6(L_{22} + L_{21}), \quad c_5 > c_6 > 1. \quad (21)$$

$D_F(1, 1)$ denotes that denotes that the international merger cash flow of the follower follows the leader to merge and will win the more market share than it of before merging in the initial stage. Because the value of $D_F(1, 1)$ would be bigger than of it in the initial stage, a multiplier, c_7, of $D_F(1, 1)$, is bigger than one but it should be added to calculate the function of profit cash flow of the follower. Therefore,

$$D_F(1, 1) = (c_7(L_{12} + L_{11}))^{a_{1m}} (c_7(K_{12} + K_{11}))^{b_{1m}} - c_7(L_{12} + L_{11}), \quad c_7 > 1. \quad (22)$$

$D_F(1, 0)$ denotes the international merger cash flow of follower as the leader has already merged and will win less market share than it of before merging in the initial stage. As the results, international merger cash flow of $D_F(1, 0)$ should be smaller as usual. A multiplier, c_7, of $D_F(1, 0)$ is smaller than one, and it should be added to calculate the function of profit flow of the follower. Since the group with larger scale may economic scale, it may have bigger c_4 than c_8. Therefore,

$$D_F(1,0) = (c_8 L_{12})^{a_{12}} (c_8 L_{12}))^{b_{12}} + (c_8 K_{11})^{a_{11}} (c_8 K_{11})^{b_{11}} - c_8 (L_{12} + L_{11}), \quad c_8 < c_4 < 1. \quad (23)$$

3. Numerical Examples and Sensitivity Analysis

This section conducts a simulation, the optimal threshold of real exchange rate is determined and project values for international merger using the proposed models in previous sections. For numerical analysis, this study manipulates a numerical example and the specific values of variables are list in Table 1.

"Mathematica 5.0" which is a software package for mathematical computation and is used in this study as a tool to calculate the numerical solutions and sensitivity analysis for these two proposed models of GBM or square of root MRP in the duopolistic market. The specific values of variables of these two proposed models are the same. The numerical results show the similar threshold values of real exchange rate of GBM and square root of MRP.

On Table 2, the values of R_F and R_L by GBM are 1.64521 and 0.769964, respectively, when large firm scale as the leader. The values of R_F and R_L by square root of MRP are 1.62077and 0.76079, respectively, when large firm scale as the leader. Similarly, the values of R_F and R_L by GBM are 1.18057 and 0.966377, respectively, when larger firm scale as the follower. The values of R_F and R_L by square root of MRP are 1.16889 and 0.958664, respectively, when large firm scale as the follower. The values above imply that the merger

Table 1 Values of Variables Assumptions for the Duopolistic Market

Variable	Meaning	Unit
L_{11}, L_{12}	Labor cost of domestic country and foreign country in the first group	7 million USD
L_{21}, L_{22}	Labor cost of domestic country and foreign country in the second group	5 million USD
K_{11}, K_{12}	Capital cost of domestic country and foreign country in the first group	7 million USD
K_{21}, K_{22}	Capital cost of domestic country and foreign country in the second group	5 million USD
a_1, a_{12}	The power of per labor cost in the first group, and in domestic or foreign country.	2[1]
b_{21}, b_{22}	The power of per labor cost in the second group, and in domestic or foreign country.	2
a_{m1}, a_{m2}	The power of per capital cost in the first group	2
b_{m1}, b_{m2}	The power of per capital cost in the second group	2

[1] Since the power is a kind of multiplier, it doesn't have the unit.

c_1, c_5	Multiplier affects the value of profit flow D (1, 0).	1.8, 1.7
c_2, c_6	Multiplier affects the value of profit flow D_L(1, 1).	1.5, 1.3
c_3, c_7	Multiplier affects the value of profit flow D_F(1, 1).	1.3, 1.5
c_4, c_8	Multiplier which affects the value of profit flow D_F(1, 0).	0.6, 0.5
σ	Risk attitude of the project bears	0.075
γ	The reversion power of square-root of MRP	0.01
R	The expected project return	0.4

Table 2 The Threshold Results in the Duopolistic Market

	Large Scale as a Leader		Large Scale as a Follower	
	R_F	R_L	R_F	R_L
GBM	1.64521	0.769964	1.1805	0.966377
Square root of MRP	1.62077	0.76079	1.16889	0.958664

is easier to execute in the long term than the short term since the real exchange rate is easier to achieve by square root of MRP than of it by GBM. In addition, the group with large firm scale is easier to merge than the group with small firm scale.

The threshold of real exchange rate is approach to 1 presents that the merge is easier to be occurred. In addition, higher exchange rate will appeal the domestic firm to take merger action than that of lower exchange rate

Figure 6 presents the sensitivity analysis of multiplier c_1 which affects the international merger cash flow D_L(1, 0). In Figure 6, "*g*" represents the GBM and "*m*" represents square root of MRP, R_L denotes the leader's threshold of real exchange rate and R_F denotes the follower's threshold of real exchange rate. As c_1 increased, the threshold of real exchange rate of the leader becomes farer from 1. This result implies that the increment of c_1 is hard to achieve merging, so the leader are hard to take international merger action as c_1 increases. However, under the numerical results shown, the leader is easier to take international merger action while the larger benefit than the lower benefit. In addition, the gap between the R_F and R_L becomes large states that the follower will be hard to take international merger action as c_1 is smaller. Thus, if the leader takes international merger action, the follower will be hard to follow the international merger strategy.

The similar results from Figure 6, the multiplier c_5 has the same numerical result. The difference between c_1 and c_5 is that the gap between R_F and R_L is much smaller in c_5. This numerical result can be explained that the group with large firm scale as the leader will have more advantages of economic scale than the small firm scale as the leader. Consequently, the leader with the larger firm scale can build more barriers to the follower.

Figure 7 also presents the sensitivity analysis of multiplier c_2 which affects the international merger cash flow D_L(1, 1). Based and the Eqs. (14), (16), (17) and (18), and as

presented in Figure 7, the follower does not affect by the increment of the leader's benefit. This result states that in a rational environment, if the benefit does not increase, the international merger action should not be taken. On the other hand, the multiplier c_2 of the leader increasing, it states that the leader has more benefit, eventually, but it is difficult to

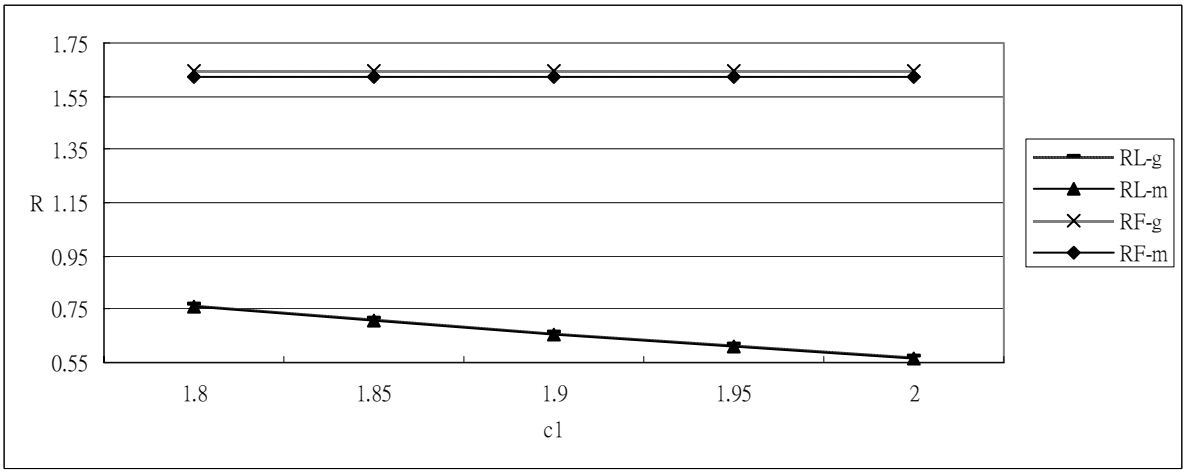

Figure 6 Sensitivity Analysis of Multiplier c_1

reach this situation. Therefore, $\overline{R}$ is difficult to achieve in the international merger strategy. The same result also shows in the multiplier c_5, but the gap between the R_F and R_L is smaller because the large firm scale has more power than the smaller one.

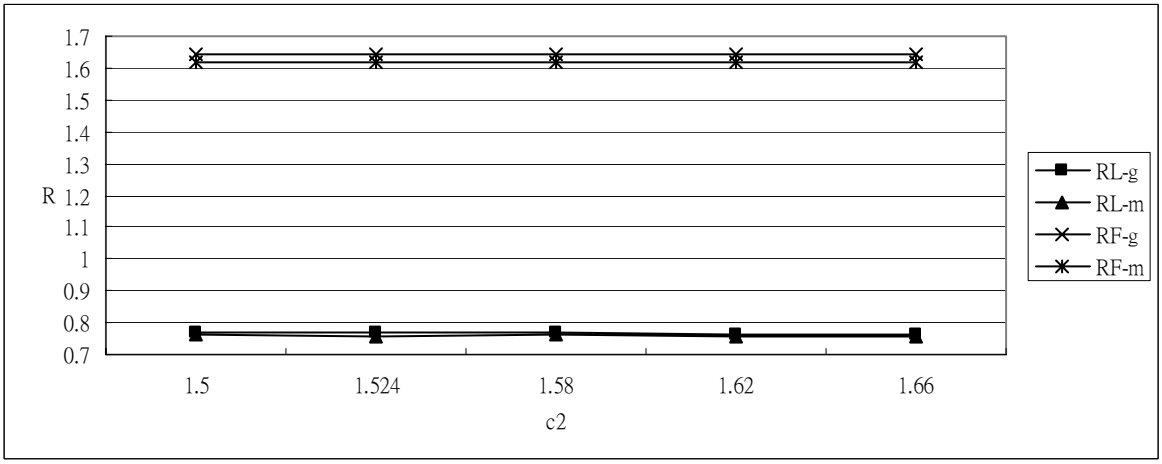

Figure 7 Sensitivity Analysis of Multiplier c_2

Figure 8 presents the sensitivity analysis of multiplier c_3 which affects the international merger cash flow $D_F(1, 1)$. As the follower's international merger action makes more benefit, the gap between the R_F and R_L becomes smaller and easier to achieve than before. Therefore, the increasing benefit of follower's action will encourage the follower to take international merger actions.

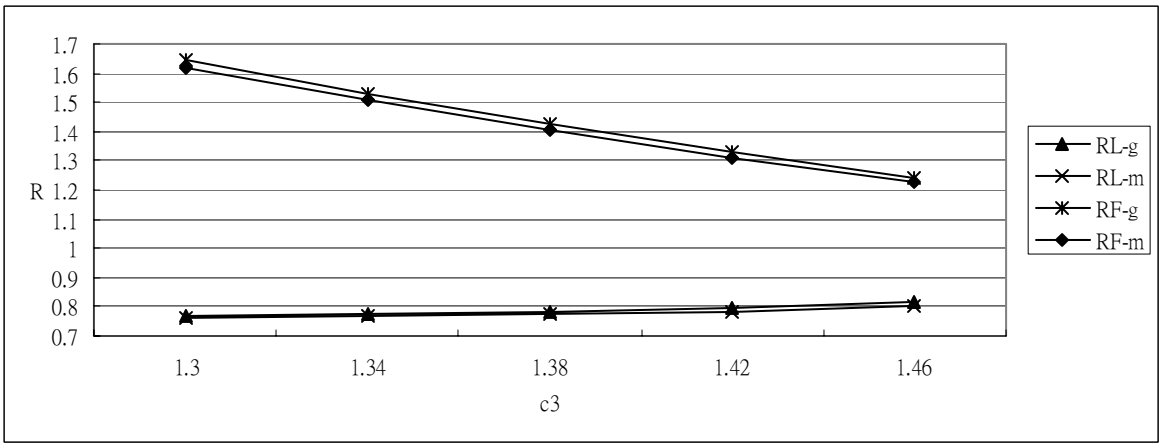

Figure 8 Sensitivity Analysis of Multiplier c_3

Figure 9 presents the sensitivity analysis of multiplier c_4 which affects the international merger cash flow $D_F(1, 0)$. As the retain market share of follower is big, the follower will be unwilling to take international merger action. Therefore, the leader is much easier to take action and $\overline{R}$ is approach to 1 as the leader achieves international merger strategy.

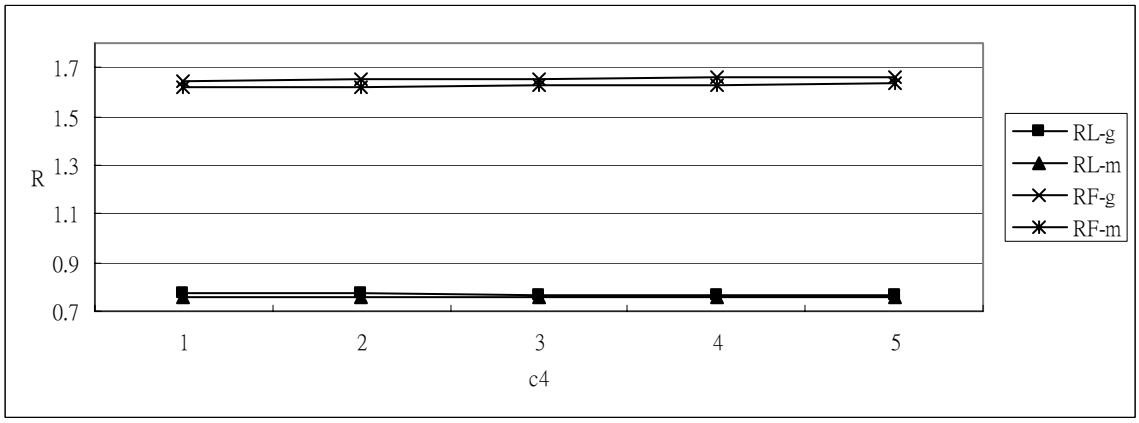

Figure 9 Sensitivity Analysis of Multiplier c_4

Multiplier c_7 also has the same results of R_F and R_L and only the difference in the gap between R_F and R_L which is smaller than the gap in Figure 9.

Figure 10 presents the sensitivity analysis of the scale of leader. From this figure, the follower is difficult to follow and the gap between R_F and R_L is large. The numerical result reflects that in the real world the larger the leader is, the harder the follower can be followed because of the economic scale and the resource fulfillment of the leader. However, the large firm lacks the mobile ability, thus, the large firm is more unwilling to take international merger action unless necessary.

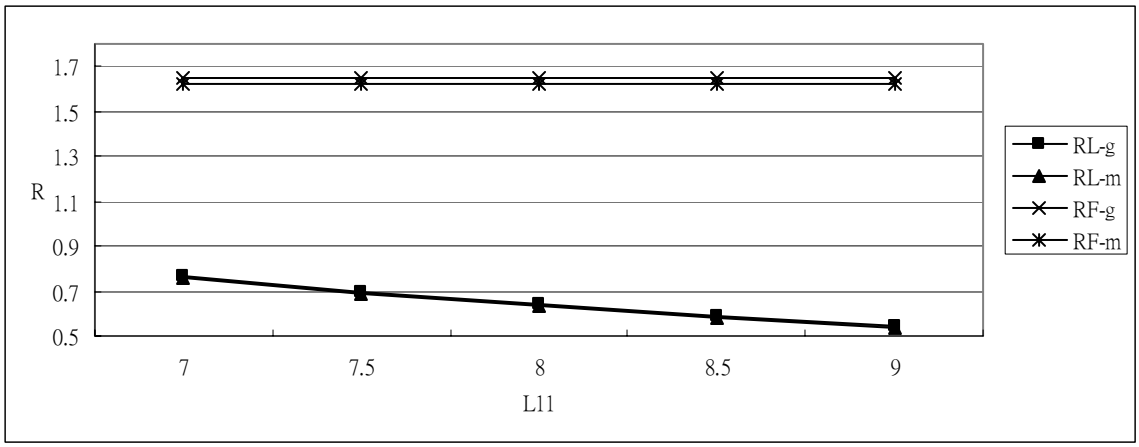

Figure 10 Sensitivity Analysis of Scale of the Leader

Figure 11 presents the sensitivity analysis of the scale of follower. From the Figure 11 the large scale of the follower makes the leader likely to take international merger action because the leader wants to get more advantages than the follower. In addition, the follower is more willing to take international merger action so that it can pursue more benefit. As the gap between R_F and R_L become narrowed, the follower's scale increasing shows that the follower is easier to pass the barriers built by the leader.

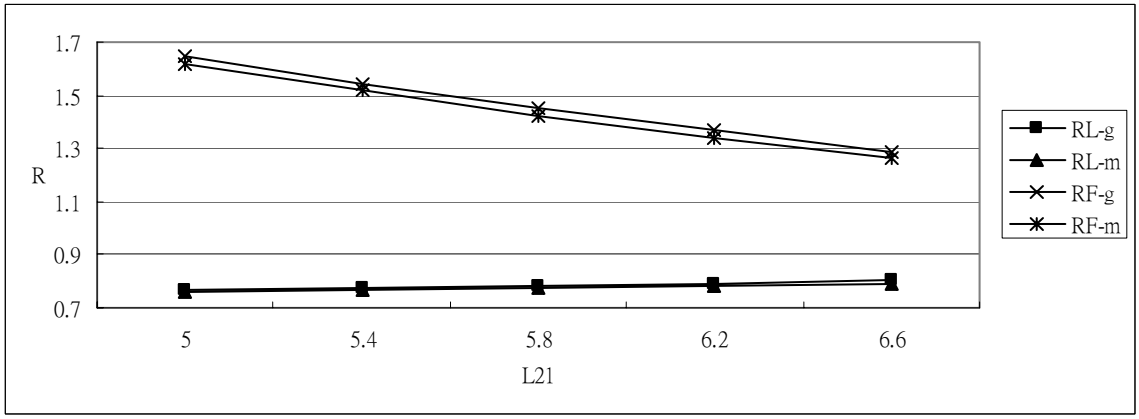

Figure 11 Sensitivity Analysis of the Scale of Follower

Figure 12 represents the sensitivity analysis of σ which denotes the risk bears by the group in the environment. In Figure 12, if there is more risk in the environment, the leader is more willing to change the current state to take international merger action because it may has some strength that the follower doesn't have. Moreover, the follower is more unable to take international merger action because it lacks of the competitiveness. Consequently, the follower is more likely to take international merger action when the risk is small.

Figure 13 represents the sensitivity analysis of the pull power γ of the mean reverting process. When taking the pull power γ into regard, it makes the international merger

action easier to occur and it can explain that the merger wave which happens all the time.

Figure 14 represents the sensitivity analysis of expected project return *r*. If the expected return *r* is high, the $\overline{R}$ will also be high. It implies that the more expected return *r* in first group, the more possible will the leader merged because it has the incentive to take international merger action. In the contrary, the high return is very difficult to achieve. Therefore, $\overline{R}$ of follower will become much more apart from 1, as the follower can not reach this state easily.

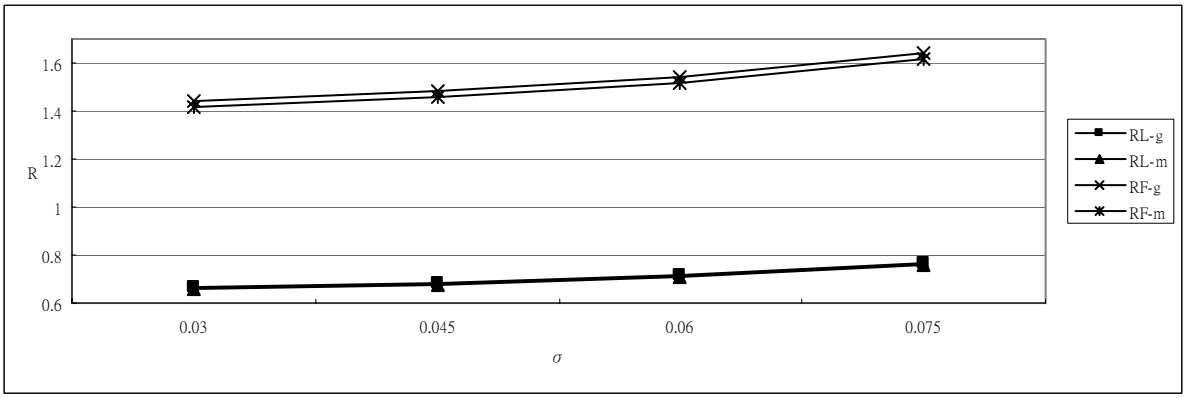

Figure 12 Sensitivity Analysis of σ

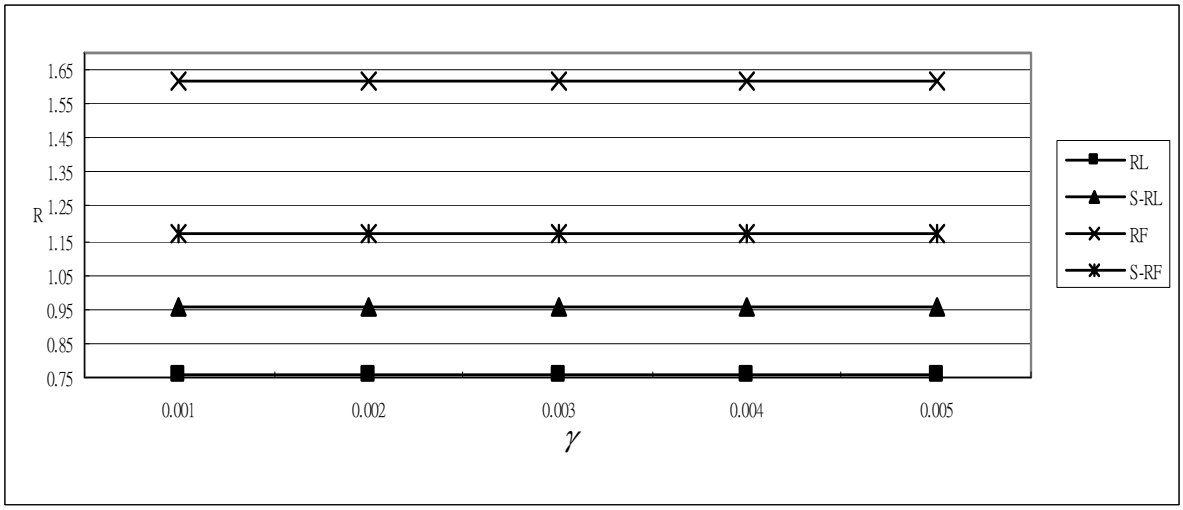

Figure 13 Sensitivity Analysis of Pull Power γ

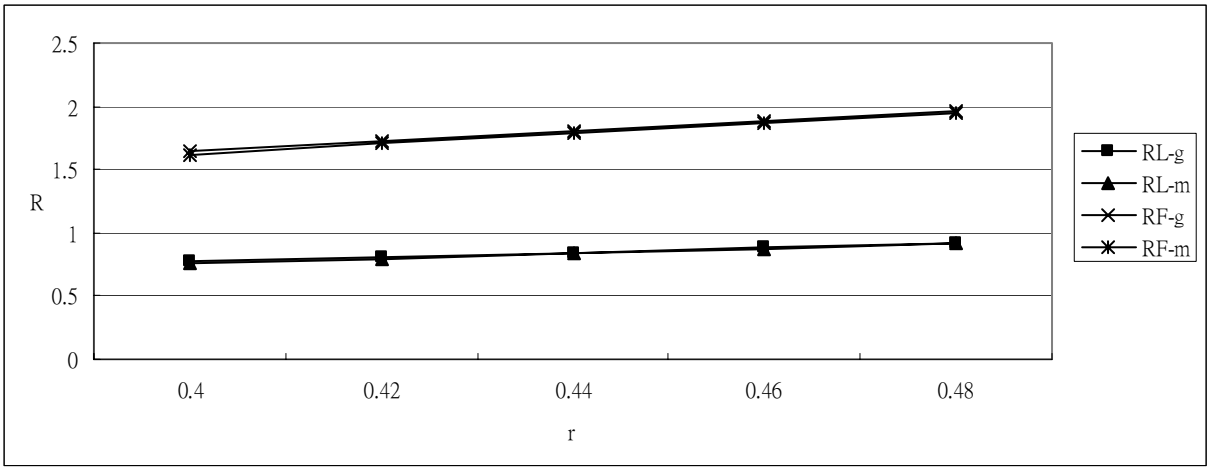

Figure 14 Sensitivity Analysis of Expected Return *r*

5. Conclusions

Since the failure rate of international merger is very high, this study desires to provide a guidance to evaluate the decision of international merger. This study has developed two proposed models with time zone of international merger in a dynamic economic environment in which the real exchange rate impacts the domestic firms to merge foreign firms follows stochastic process and in which the firms have completed information in the industry. As the results, the main conclusions and empirical implications of this paper are discussed as below.

1. This study considered the different scales of group as the leader and added the multipliers of the profit flow which originally from the action-taking gain or lose. Moreover, this study divided the time zone into three periods: (1) the time zone of initial stage, (2) the time zone of leader's action, (3) the time zone of follower's action and deduced the discount factors, and used to calculate the threshold of exchange rate. In order to assess decision of merger by the leader and the follower, this study organized four firms as two groups to calculate the threshold of real exchange rate.
2. This study considered the scale of group as the evaluators but Dias and Teixeira(2003) considered investment cost of firms as the evaluators. However, Dias and Teixeira only examine one period of time and unable to calculate the leader's value. Moreover, Dias and Teixeiras' profit flow is not affected by the additional multipliers and their stochastic demand shock only follows GBM.
3. This study considered the specific generalized profit function to estimate the international merger model which follows GBM or square root of MRP, and proposed the general solution from the high degree of power function by movement generating function in the first part of this paper.
4. This study used the real exchange rate which follows GBM or square root of MRP as

index to determine merger decision. Furthermore, the labor and capital are considered as input that affects the merger in both models.

5. This study found that a large firm scale has more advantage than the small firm scale. Hence, the large firm scale as the leader could build more barriers to the follower. In addition, the large firm scale as a follower could be easier to pass the barrier built by the leader. Consequently, being a leader would have more benefit than the follower.

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