

THIRD GENERATION MOBILE GAMES – An application of real competition options

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The third generation technology will confer to mobile phones all the capacity and speed of a fixed line phone with the additional flexibility of mobility. Major investments in 3G installation facilities are planned in developed economies; the reported investment plans indicate “leader-follower” patterns.

Using three real competition options models, we determine the optimal timing of 3G investment of one Portuguese mobile company, Optimus, taken as the follower. In the first of those models both the number of units sold and the cash flow *per-unit* of the players follow separate but possibly correlated geometric Brownian motion. In the second model the investment cost and the operating cash flow are the state variables. The third model assumes that the investment cost and the operating cash flow stream of the total market follow separate geometric Brownian motion and that the market share of the follower occurs according to a Poisson process.

Consistent parameters are used to derive the leader and follower value functions for different models, which are compared to a traditional NPV valuation analysis. A positive NPV points to the acceptance of the investment and the immediate entry of all of the players in the market.

The results of all the models point to the delay of the entry of the follower, which might account for the observed behaviour of the actual players.

Keywords: Competitive real options, Empirical application of real options, two factor models, duopoly

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1. Introduction

One of the criticisms of academics and practitioners regarding real options is the lack of empirical applications of the theory². This paper is an application of three real competition option models. For practical purposes our main objective is to determine the optimal timing of investment of one mobile company, Optimus. The choice of the telecommunications market was due to the following reasons: it is a competitive sector and therefore a natural application for game theory models; it is a license race market; and it is subject to major technological changes, and hence the telecom companies should be analysed using real options. A major technological change is happening in developed economies, the introduction of the third generation of mobile phones. The 3G technology will confer to mobile phones all the capacity and speed that a fixed line phone has with the additional flexibility of mobility. Several companies and stockbrokers have valued telecommunication businesses, but there is no apparent evidence that real option models were used in those valuations. For these reasons we considered applying the valuation methodology to the introduction of third generation mobiles. The high penetration rate of mobile phones in Portugal determined our choice considering the country.

After this brief introduction the three real options models are presented in section 2. In section 3 Optimus is presented. In section 4 the parameter estimation is explained and the results are presented and analysed. Section 5 creates a new scenario where two companies merge, converting the market into a duopoly. Section 6 concludes.

² Quigg (1993), Wysocki (1998), Downing and Wallace (2001), Colwell et al. (2002), Eades and Marston (2002), Isimbabi (2002), Shackleton, Tsekrekos and Wojakowski (2002), Chen and Zhang (2003), Kallapur and Eldenburg (2003), Worner and Grupp (2003) are the few recent exceptions to this criticism.

2. The Models

Two of the models presented in this paper were developed in Paxson and Pinto (2003-a). The third model is an extension of Paxson and Pinto (2003-b) and consequently will be explained in more detail.

Two firms are contemplating the option to enter a new market. The firm that enters first, from now on defined as the leader, will acquire a first mover advantage defined as a higher share of the market. Following Fudenberg and Tirole (1985), the roles of the leader and the follower are defined endogenously. The two firms are ex-ante symmetrical but asymmetrical ex-post. The leader will always have a competitive advantage over the follower, so each firm will want to win the position of the leader, which creates a pre-emption effect.

2.1. Profit per Unit and Number of Units as Stochastic Variables

Considering that both the profit per unit and the number of units follow different but possibly correlated geometric Brownian motion processes, Paxson and Pinto (2003-a) obtained closed form solutions for the value function of the follower, $V(F)$, and the leader, $V(L)$, and also closed form solutions for the follower's trigger, X_F . Let V represent the profit per unit sold, M the number of units sold in a market by a follower and X the product of V and M , the follower's value function is given by:

$$V(F) = \begin{cases} \frac{K}{\beta_1 - 1} \left(\frac{X}{X_F} \right)^{\beta_1} & X < X_F \\ \frac{X}{2r - \omega - \mu} - K & X \geq X_F \end{cases} \quad (1)$$

Where:

$$\beta_1 = \frac{1}{z^2} \left(- \left(\sigma\alpha\rho + \mu + \omega - \frac{1}{2}z^2 \right) + \sqrt{2rz^2 + \left(\sigma\alpha\rho + \mu + \omega - \frac{1}{2}z^2 \right)^2} \right) \quad (2)$$

$$z^2 = \alpha^2 + \sigma^2 + 2\sigma\alpha\rho.$$

$$X_F = \frac{K(2r - \omega - \mu)\beta_1}{\beta_1 - 1} \quad (3)$$

σ and α are the volatilities of V and M respectively, ρ is the correlation coefficient; μ and ω are the expected growth rates of V and M respectively; r is the risk free rate and K is the investment cost.

Let now \bar{m} be a factor that when multiplied by M results in the maximum number of units that can be absorbed by the market at a certain moment in time and m a factor that when multiplied by the number of units results in the leader's number of units after the follower enters the market. The leader's value function is given by:

$$V(L) = \begin{cases} \left(\frac{X}{X_F}\right)^{\beta_1} \frac{K\beta_1}{\beta_1 - 1} (m - \bar{m}) + \frac{X\bar{m}}{2r - \mu - \omega} - K & X < X_F \\ \frac{Xm}{2r - \mu - \omega} - K & X \geq X_F \end{cases} \quad (4)$$

Figure 1 provides an illustration of the sensitivity of the value functions to profits per unit.

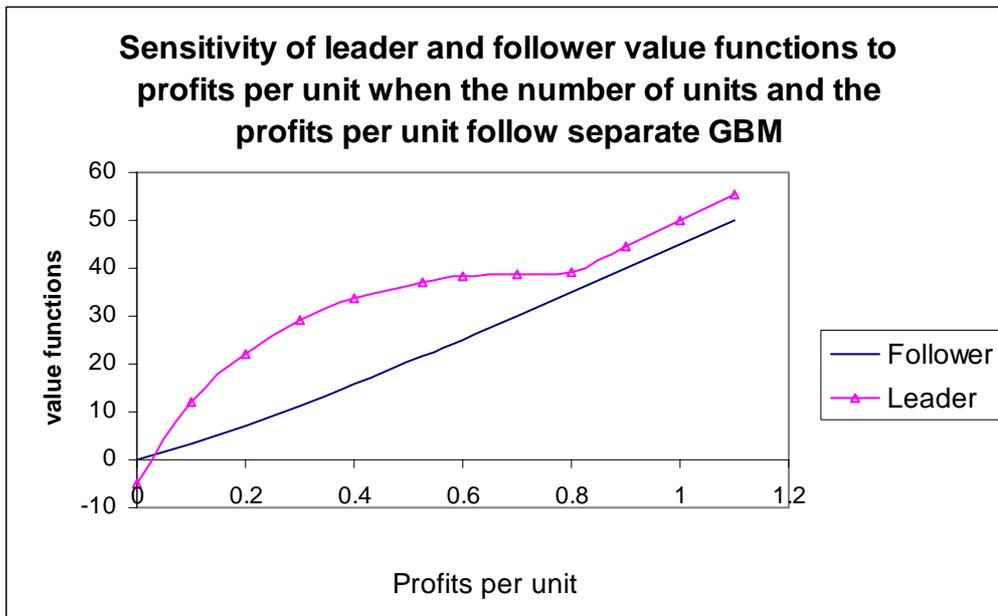


Figure 1 - The parameters are: $\sigma = 10\%$; $\mu = 2\%$; $\omega = 2\%$; $\alpha = 20\%$; $\rho = 0$; $M = 3$; $m = 1.1$; $\bar{m} = 10$; $K = 5$ and V varies from 0 to 1.15.

The leader's value function is almost always higher than the follower's value function, except when the unit profits are very low. Notice that at this point the follower has not yet entered the market, and if the leader were already in the market he would have been better off being a follower. Notice also that when the follower enters the market the two functions get closer. Additionally, if there were no competitive advantage, the two functions would be exactly the same from X_F onwards. Dixit and Pindyck (1994) describe this as a smooth-like-pasting condition of the present values.

The value function of the leader is more complicated than the one of the follower. It is concave until the moment the follower enters and at that precise moment in time it is discontinuous. This happens because although the total profits are increasing due to an increase in the profit per unit, they are also approaching the trigger function of the follower meaning that the negative effect of the entry of the follower increases. After the follower enters the two functions are very similar, with the leader having a higher value function because of a permanent advantage.

Also in Figure 1 we can see that there is a point where the two functions meet. Before that point a firm would be better off being a follower and after that a leader; this point represents the equalization principle of Fudenberg and Tirole (1985). If until that point a firm is better off being a follower and after it a leader, that point should be the trigger function of the leader, X_L .

2.2. Profit Flow and Investment as Stochastic Variables

Let now R stand for total profits and K for investment and let these two variables follow a geometric Brownian motion. The value function of the follower is now given by:

$$V(F) = \begin{cases} \frac{K}{\chi_1 - 1} \left(\frac{Y}{Y_F} \right)^{\chi_1} & Y < Y_F \\ \frac{R}{r - \varpi} - K & Y \geq Y_F \end{cases} \quad (5)$$

where:

$$\chi_1 = \frac{1}{2} - \frac{(\varpi - o)}{\gamma^2} + \sqrt{\left(\frac{\varpi - o}{\gamma^2} - \frac{1}{2}\right)^2 + \frac{2(r - o)}{\gamma^2}} \quad (6)$$

$$Y_F = \frac{(r - \varpi)\chi_1}{\chi_1 - 1} \quad (7)$$

and

$$\gamma^2 = \vartheta^2 + \zeta^2 - 2\vartheta\zeta\Lambda \quad (8)$$

ϖ and o are the expected gain of N and K respectively or, in other words the drift of the Brownian motion; ϑ and ζ are the volatilities and Λ the correlation coefficient. The value function of the leader is given by:

$$V(L) = \begin{cases} \frac{R\bar{m}}{r - \varpi} + \left(\frac{Y}{Y_F}\right)^{\chi_1} \frac{\chi_1 K}{\chi_1 - 1} (m - \bar{m}) - K & Y \leq Y_F \\ \frac{Rm}{r - \varpi} - K & Y > Y_F \end{cases} \quad (9)$$

2.3. Market Share and Poisson Distribution with Two Stochastic Factors

We will now consider that the investors are analysing the opportunity to enter a market where both the investment cost and the market profits (denoted by N) follow a geometric Brownian motion. We assume that after investing K the follower obtains a certain market share according to a Poisson distribution. Consequently, obtaining a certain share may take time, and in our setting time is random³. It may seem unlikely that the follower invests K and he does not obtain immediately a share of the market, but there are some examples that this may happen. In 1997 a mobile license was granted to the Portuguese company Optimus. In the beginning of 1998, the company launched a big advertising campaign, promising to its future clients a special mobile rate in the future. In order to access this rate, the potential clients had to sign a

³ Our approach combines the previous research of Grenadier (1996) and Weeds (2002). In Grenadier (1996) it takes time to obtain a certain revenue. In Weeds (2002), the time to make a discovery is random.

contract with the company. When, in the end of 1998, Optimus started operating it had already a considerable base of clients. Our setting is similar to this one, the follower invests K and he obtains the “desired” market share according to a Poisson distribution.

Since, obtaining a certain market share is a Poisson arrival, the time to the first event (here defined as the moment when the leader gets her share of the market (denoted by a) and consequently the follower gets $(1-a)$) is exponentially distributed with probability density function: $\Phi e^{-\Phi t}$. Where Φ denotes the rate of occurrence of a certain market share $(1-a)$ ⁴.

Let the profit flow of the market and the investment cost, K , be explained by two separate geometric Brownian motion of the kind:

$$dN = \nu N dt + \theta N dz_1 \quad (10)$$

and

$$dK = \omicron K dt + \zeta K dz_2 \quad (11)$$

where ν and \omicron are the expected growth of N and K respectively or, in other words the drift of the Brownian motion; θ and ζ are the volatilities and δ the correlation coefficient.

The partial differential of an idle follower is:

$$\frac{1}{2} \frac{\partial^2 V_0^F}{\partial N^2} \theta^2 N^2 + \frac{1}{2} \frac{\partial^2 V_0^F}{\partial K^2} \zeta^2 K^2 + \frac{\partial^2 V_0^F}{\partial N \partial K} NK \theta \zeta \delta + \nu N \frac{\partial V_0^F}{\partial N} + \omicron K \frac{\partial V_0^F}{\partial K} - r V_0^F = 0 \quad (12)^5$$

The value matching condition is given by:

$$V_0^F(N, K) = \frac{N(1-a)\Phi}{\phi} - K \quad (13)$$

⁴ For more details on the Poisson process see Karlin and Taylor (1975).

⁵ Where V_0^F denotes the value function of an idle follower.

where $\phi = r - \theta + \Phi$. The smooth pasting conditions:

$$\frac{\partial V_0^F(N, K)}{\partial N} = \frac{(1-a)\Phi}{\phi} \quad \text{and} \quad \frac{\partial V_0^F(N, K)}{\partial K} = -1 \quad (14)$$

Subjecting (12) to the above boundaries we obtain the value function of the follower:

$$V(F) = \begin{cases} \frac{K}{\lambda_1 - 1} \left(\frac{Y}{Y_F} \right)^{\lambda_1} & Y < Y_F \\ \frac{N(1-a)\Phi}{\phi} - K & Y \geq Y_F \end{cases} \quad (15)$$

where Y is as defined in the previous section and:

$$\lambda_1 = \frac{1}{2} - \frac{(v-o)}{c^2} + \sqrt{\left(\frac{v-o}{c^2} - \frac{1}{2} \right)^2 + \frac{2(r-o)}{c^2}} \quad (16)$$

where:

$$c^2 = \theta^2 + \zeta^2 - 2\theta\zeta\delta \quad (17)$$

The follower's trigger, Y_F , is given by:

$$Y_F = \frac{\phi\lambda_1}{(1-a)\Phi(\lambda_1 - 1)} \quad (18)$$

As common in these models, while the leader is alone in the markets he receives monopolistic profits. After the follower enters, the leader will share the market but has a competitive advantage. The leader has a larger share of the market, in other words it is assumed that $a > 50\%$. The value function of the leader, while alone in the market, can be explained by:

$$E \left[\int_0^{T_F} e^{-r\tau} N_\tau d\tau \right] + \int_{T_F}^{\infty} N_\tau a\Phi d\tau - K \quad (19)$$

The value function of the leader is now given by:

$$V(L) = \begin{cases} \frac{N}{r-\nu} + \frac{K\lambda_1}{\lambda_1-1} \frac{a\Phi(r-\nu) - \phi}{(1-a)\Phi(r-\nu)} \left(\frac{Y}{Y_F} \right)^{\lambda_1} - K & Y < Y_F \\ \frac{a\Phi N}{\phi} - K & Y \geq Y_F \end{cases} \quad (20)$$

3. Overview of the Portuguese mobile companies

There are three mobile companies with GSM licenses operating in Portugal: TMN, Vodafone and Optimus. In December 2000 four UMTS licenses were granted in Portugal. Three of those licenses were granted to the existing GSM mobile operators, another company, Oniway, purchased the fourth one. The end of 2002 was the date set by ANACOM (the Portuguese regulator) for the commercialisation of the UMTS services in Portugal. Similarly to what happened in other European countries the date of entry has been delayed. The new date is now the end of 2003, but until now none of the operators entered the market. In 2002 Oni Way dropped the license. TMN has recently announced its intention to enter the market in the beginning of 2004, Vodafone stated its intention to follow, but Optimus announced that it would further delay entry.

Optimus is one of the four major business areas of Sonae.com. The other areas are fixed communications (Novis), multimedia and internet (Publico and Clix), and software and systems integration (Enabler, WeDo, Biz Direct and Mainroad). Sonae.com owns about 46% of Optimus. The company was granted a GSM license in 1997, at the time the two other companies were already operating in the market. At the end of 1999, Optimus had already 17.5% of the market, showing a very competitive attitude towards the market. The fast initial growth stabilized in the last years with the company having a 20% market share at the end of 2002⁶.

⁶ With the penetration rate reaching almost 100% of the Portuguese population, the mobile market is entering a mature stage.

4. Parameters Estimation and Results

The three real competitive option models considered were developed for duopoly markets. Consequently the situation is not ideal, because either in the role of the leader or of the follower, two players have to be considered as one. Some market experts have noted that the Portuguese market is too small for three operators, suggesting that Optimus and Vodafone should merge. If this happened, and if we assume that the market share of the new company would result in the sum of the market share of the two companies, we would expect to have a fight for the leader's role and the perfect environment to apply the models. We analyse this possibility in section 5.

In this section we create a scenario where the company under analysis is alone and the other two merge. We decided to assume that the company under analysis would want to have the follower's role. Thus only the results concerning the follower are of importance; the other ones are merely illustrations of our models. Regarding the game played, none of the players know for sure the entry time of the other. Moreover, they all expect that the others want to enter the market first to achieve the first mover advantage. Therefore, the environment is pre-emptive.

The main objective of this paper is to infer on the timing of entry of Optimus. In other words assuming that the other two players will enter the market first, should the company start selling 3G mobile phones at the end of 2003 or later?

Some of the data necessary to estimate the models has proprietary characteristics, consequently it is difficult to obtain. Our parameters were estimated using data collected in stockbroker reports, the company annual reports, the Portuguese regulator reports and DataStream.

The estimations of number of subscribers⁷, ARPU and EBITDA margins were done using an equally weighted average of predictions obtained in stockbrokers' reports. We used annual data from 2004 until 2010. For 2011 the value of 2010 is considered as a perpetuity.

The investment cost was computed using the data of the most recent stockbroker report (Credit Suisse (February 2004)). According to that stockbroker

⁷ CSFB 2/04, BCP 7/03, Dresdner 5/02, CSFB 1/02, 11/01, Espirito Santo 5/01, Morgan Stanley 11/01, CSFB 7/00. Some early predictions extend only through 2005. Most stockbroker reports do not discriminate between 2G and 3G subscribers, or specific capex. It is assumed herein that the clients will shift from 2G to 3G at a rate of 15% annum.

report CAPEX will be 15% of sales from 2004⁸. Table 1 presents the predictions for EBITDA and CAPEX (values in million Euros).

Year	2004	2005	2006	2007	2008	2009	2010	2011
EBITDA	26.59	59.9	101.0	191.8	263.9	340.0	396.2	3961.7
CAPEX	31.9	67.1	104.7	149.7	200.4	253.6	292.3	2923.0

Table 1 – EBITDA and CAPEX annual predictions

Volatility is, as in most real option models, a very critical variable. We calculated three proxies for volatility: value, number of units and investment volatility. The proxy for value volatility was calculated as the standard deviation of the company shares. We used daily returns from June 2000 to June 2003 and obtained an annualised volatility of 43.5%. Using quarterly data from 1998 to 2003⁹ we calculated a proxy for the volatility of the number of units. The annual volatility of number of units is 14.4%. As a proxy for investment volatility we calculated the standard deviation of the returns of Ericsson and Nokia using daily returns from June 2000 to June 2003. Our intuition to choose volatility of these companies as a proxy for investment volatility was that since those companies are suppliers of equipment their business is associated with the investment cost. The investment cost volatility is 77.2%.

The correlation between EBITDA *per unit* and the number of subscribers is assumed to be zero. The correlation between EBITDA and investment cost is 21.1%. This correlation corresponds to the correlation of the daily returns of Sonae.com and the daily returns of Ericsson and Nokia.

The monopoly and first mover multiplier were defined using the predictions of the number of subscribers. The monopoly multiplier was calculated as an average of the yearly ratio of total market divided by Optimus' subscribers. The first mover advantage was calculated as the average of the yearly ratio of the number of subscribers of Vodafone plus TMN divided by Optimus' subscribers. For model 3 we considered that after entering the market Optimus will have a share of 20% and that

⁸ The investment cost prior to 2004 is considered sunk and therefore will not be considered in the analysis.

⁹ Source: Anacom.

the hazard rate is 100%. Notice that 20% is approximately the market share of Optimus in 2G and we are assuming that there is a high likelihood that the company will achieve the same market share. The risk free rate was assumed to be 5% and the drift rates 2%.

After computing the parameters we executed a traditional valuation analysis, i.e. Net Present Value. Using a 10% discount rate, as cost of capital, the NPV results in 1,423.23 million Euros pointing to the acceptance of the investment and the immediate entry of all of the players in the market. None of the three investors behaviour indicates that they believe in this result. Until March 2004 third generation mobiles were not commercialised in Portugal¹⁰. In Table 2 the main results are presented:

	State Variables	State Variable Value	Trigger Value	Follower's Value
Model 1	Units & EBITDA	161.27	1,233.4	2,207.5
Model 2	Investment & EBITDA	0.06	0.37	2,117.3
Model 3	Investment & EBITDA (share)	10.9	63.9	2,117.3

Table 2 – Main results of real option models.

All the models point to the delay of the entry of the follower. We decided not to present the leader's result because it was assumed that the company would have the follower's role. Using our assumptions the leader should already be in the market. Using model 2 we estimated the critical EBITDA and the critical investment cost. The critical EBITDA is 471 million Euros and the critical investment cost is 216.5. The critical investment cost represents 14% of the predicted investment cost. The commercialisation of 3G in Portugal is far from hitting the critical point. Also notice that according to the results, the composite state variable is a lot smaller than the trigger values. Consequently, unless a very significant change occurs in the short run, for instance a drop in CAPEX, an increase in the number of units resulting from one of the three mobile companies dropping their license or the merger of two of the three

¹⁰ Clix announced that it will launch in the second half of 2004 an internet service that will work under UMTS technology.

companies, it does not look like Optimus should commercialise UMTS in the near future.

5. Merger Scenario

Some analysts have been pointing out that the Portuguese market is too small to have three companies operating 3G. Considering this we decided to create a new scenario where Vodafone and Optimus merge. Under this scenario the commercialisation of 3G will be a duopoly and we would have the perfect environment for our models.

We will assume that ARPU will be the same for all companies and that the difference in earnings will be the result of different numbers of customers. The leader, that can be TMN or Vodafone+Optimus, will be able to obtain the number of customers resulting from adding the subscribers of Optimus and Vodafone. The follower would obtain the remaining market. In other words, the leader will have the share of the market of Vodafone+Optimus and the follower the one of TMN. Considering this, the multiplier defining monopoly like revenues in model 1 and 2 is 2.2. The multiplier defining the first mover advantage is 1.2. The number of subscribers of Optimus and Vodafone add to 54% of the market. Consequently, we assume that the leader will have 54% of the market after the follower enters. The hazard rate of obtaining 54% of the market is once again considered to be 100%.

The merger of the companies would result, according to some specialists, in network advantages¹¹. We will assume that the merger would result in a decrease in CAPEX of 35%, which results from outsourcing of towers and network sharing (Morgan Stanley 2001). Value volatility is now the average of the volatility of the shares of Vodafone and Optimus, resulting in 39%.

We start by calculating the present value of the EBITDA and of the investment cost of the follower, i.e. the company that will have the market share of TMN. The present value of EBITDA using a 10% discount rate is 4,761.58 million Euros. The present value of the investment cost is 1,773.7 million Euros. Consequently the net present value is positive pointing to the immediate entry of the companies in the market. Notice that since the follower's net present value is positive, all companies

¹¹ Network advantages in real competition options are analysed in Mason and Weeds (2000).

should already be in the market according to Marshallian rules. The main results of our models are presented in Table 3.

	State Variable	State Variable Value	Follower's Trigger Value	Leader's Trigger Value
Model 1	Units & EBITDA	285.7	1,526.5	196.6
Model 2	Investment & EBITDA	0.08	0.36	0.05
Model 3	Investment & EBITDA (share)	6.01	27	0.07

Table 3 – Main results of real option models – merger scenario.

According to all of our competitive option results, the leader should already be in the market, but the follower should wait. The primary difference between the leaders as the two dominant 2G operators, and as the merger scenario operations is roughly a market share of the follower of 20% versus 46%. The ratio of the follower's trigger to the state variable is 7.6, 6.2 and 5.8 compared to reduce ratios of 5.3, 4.5 and 4.5 indicating a shorter deferred time.

6. Conclusion

Consistent parameters were used to derive the results for the three real option models. A sensitivity analysis of some variables was performed, showing that our results are highly dependent on the assumptions and on an adequate and accurate estimation of some key parameters.

If instead of calculating the historical standard deviation of the shares of the company, we could have computed the implied volatility of share options as an alternative proxy for value volatility. Implied volatility reflects the expectations of the investors and is therefore a better market measure. Since there are no financial options on this company, implied volatility could not be computed. The assumption that volatility is constant is also subject to valid criticism. But restrictions on the amount of issues explored had to be imposed. This latter criticism can also be extended to the calculation of number of units and investment volatility.

All the assumptions of the models are obviously embedded in our analysis, for example, the duopoly setting, the processes describing the state variables, and the fairness of the game played. However, this paper is not meant to be a perfect and accurate description of the introduction of third generation mobiles in Portugal but an application of our models. Having in mind all the assumptions and limitations of our parameters, our models and our analysis, we think that the follower is taking the right decision not introducing 3G in the Portuguese mobile market in the beginning of 2004.

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