Operating Options and Commodity Price Processes

(Draft)

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Abstract

This paper discusses the short-run dynamics of commodity prices. It deals with the interrelationships between price, inventory and price volatility as well as the effects of inventory and the producers' operating flexibility on the dynamics of price in the short-run. It also illustrates how to model and estimate the stochastic process of commodity prices. We conclude that, in the short-run, producers' operating flexibility reduces price volatility when the spot price is higher than the threshold price causing expansion in the scale of operations. However, we also conclude that operating flexibility can increase price volatility when the spot price is lower than the threshold price resulting in a contraction of operations. We demonstrate the failure of currently used parametric models in describing the stochastic process of commodity prices and suggest using non-parametric methods. We also recommend including the time trend in such a model.

Key Words: Price, inventory, price volatility, operating options

1. Introduction

The stochastic behavior of commodity prices plays an important role in models used to evaluate resource investments[1]. Usually we represent the underlying price as a continuous-time diffusion process, satisfying a time-homogeneous stochastic differential equation[2],

$$dP = \mu(P)dt + \sigma(P)dZ \tag{1.1}$$

where $\mu(P)$ is the drift function, $\sigma(P)$ is the diffusion function and Z is a standard one-dimensional Brownian motion. Here we assume that both the drift term and the diffusion term are functions only of the underlying price. Equation (1.1) can be estimated by either parametric or non-parametric methods. However, in the real options literature, most studies have been conducted using parametric methods.

In resource economics, the most popular models used to describe the stochastic process of commodity prices are the geometric Brownian motion model and mean reverting model. For example, Clarke and Reed (1989) derived an optimal harvesting rule for the single rotation problem when price is assumed to follow geometric Brownian motion. Insley (2001) estimated the optimal cutting time assuming the lumber price are mean reverting. Recent studies also introduced some alternative models[3], such as stochastic volatility process[4] and Schwartz and Smith (2000)'s two-factor model.

One advantage of parametric models is that it may be possible to obtain analytical solutions for optimal investment rules. However, not all parametric models have such

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[1] See Schwartz and Smith (2000).
[2] See Stanton (1997).
[3] Since most jumps have been smoothed out in monthly or quarterly data, we do not use jump models in this paper. The discussion on the time smoothing effect can be found in Ait-Sahalia (2003).
[4] See Deng (1999).

analytical solutions. In any case, our objective is not to find an analytical solution, but a solution that is reliable. Thus, a question arises: are the prices of resource commodities and other goods best modeled as geometric Brownian motion, mean – reverting processes, or some alternative processes? A good parametric model describing the stochastic process of commodity prices should reflect the dynamics of commodity prices. To answer the above question, a detailed analysis on the dynamics of commodity prices is necessary[5].

Pindyck (2001) studied the short-run dynamics of resource commodity spot markets and explained the interrelationships among prices, rates of production and inventory levels. However, Pindyck did not consider the effects of producers' operating flexibility on price volatility. To evaluate resource investments, we usually use monthly or quarterly data. In some industries, a month or a quarter is a period long enough for producers to begin the adjustment of their operating scales, i.e., producers can expand, contract, and temporarily shut down and restart their production. Thus, producers' operating flexibility should be considered in the short-run dynamic analysis.

The main purposes of this paper are to analyze the short-run dynamics of resource commodity prices for the evaluation of resource investment and to provide a theoretical foundation for modeling and estimating the stochastic process of resource commodity prices. The outline of this paper is as follows: Section 2 describes the interrelationship between price, inventory and price volatility. Section 3 discusses producers' operating flexibility. Section 4 illustrates the short-run dynamics of commodity prices. Section 5 discusses the performance of parametric models in describing the stochastic process of

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[5] This paper only concerns storable resource commodities such as lumber, woodpulp, and rubber.. Since electricity is non-storable, it is not included here. Readers interested in the stochastic models of electricity

commodity prices and implications on the modeling of price process. Section 6 presents conclusions.

2. Pindyck's Model of Commodity Price Dynamics

Pindyck (2001) studied the short-run dynamics of commodity spot markets[6]. He explained how prices, rates of production and inventory levels are interrelated and that they are determined via equilibrium in two interconnected markets: a cash market for spot purchases and sales of the commodity and a market for storage. He also showed how equilibrium in these markets affects and is affected by changes in the level of price volatility. In this section, we will review Pindyck's model of the short–run dynamics of commodity spot markets.

2.1. Inventory and Net Demand for Inventory

Pindyck (2001) explained the function of inventories: In a competitive commodity market, both producers and industrial consumers hold inventories in order to mitigate the impacts of stochastic fluctuations in production and consumption. Producers can use inventory changes to smooth production and reduce adjustment cost, such as costs of hiring and training new or temporary workers, leasing additional capital, etc. and also to reduce the risk of being unable to satisfy unexpected customer orders.

Pindyck (2001) illustrated how price volatility and price affect the demand for inventory. An increase in price volatility implies an increase in the demand for inventory. Other

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can see Deng (1999).

things equal, market participants will want to hold greater inventories when prices are more volatile in order to buffer the effects of fluctuations in production and consumption. Denote N^D as the demand for inventory, σ as the price volatility, and then $\partial N^D / \partial \sigma > 0$.

The demand for inventory also depends on the spot price of the commodity. Other things equal, one should be willing to pay more to store a good when its price increases than one would be willing to pay if its price decreased, i.e., $\partial N^D / \partial P > 0$. Thus we can write the demand of inventory as

$$N^D = N^D(P; \sigma, z_N) \qquad (2.1)$$

Where P is the spot price, z_N is the vector of variables that can affect the demand of inventory other than spot price and price volatility[7].

When inventory can change, production in any period need not equal consumption. As a result, the market-clearing price in the spot market is determined not only by current production and consumption, but also by changes in inventory holdings. Denoting "net demand" as the difference between production and consumption, Pindyck(2001) characterizes the cash market as a relationship between the spot price and "net demand". Writing the demand function and supply function for the cash market as:

$$D = D(P; z_D, \varepsilon_D) \qquad (2.2)$$

And $$S = S(P; z_S, \varepsilon_S) \qquad (2.3)$$

where P is the spot price, z_D is a vector of demand-shifting variables, z_S is a vector of supply-shifting variables and ε_D and ε_S are random shocks.

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[6] Pindyck (2001) also studied futures market, which is not in the scope of this paper.

[7] For example, z_N includes the current inventory level.

Letting N_t denote the inventory level at time t. The change in inventory at time *t* is given by the accounting identity:

$$dN_t = S(P_t; z_{St}, \varepsilon_{S2t}) - D(P_t; z_{Dt}, \varepsilon_{Dt}) \qquad (2.4)$$

where dN_t is net demand, i.e., the demand for production in excess of consumption. Thus this equation implies that the cash market is in equilibrium when net demand equals net supply. Rewriting equation 2.4 as and inverse net demand function:

$$P_t = P(dN_t; z_{Dt}, z_{St}, \varepsilon_{Dt}, \varepsilon_{St}) \qquad (2.5)$$

Market clearing in the cash market therefore implies a relationship between the spot price and the change in inventories.

Because $\partial S / \partial P > 0$ and $\partial D / \partial P < 0$, the inverse net demand function is upward sloping in *N*, i.e., a higher price corresponds to a larger *S* and smaller *D*, and thus a larger *N*.

2.2. Price Volatility

Pindyck (2001) pointed out that price volatility is inversely related to inventory level, i.e., an increase in inventory level can reduce price volatility. Suppose a shock causes the price level to rise. Such shock, either a temporary increase in demand or a temporary decrease in supply, will also cause a decrease in the inventory level[8]. So a price increase always accompanies with a decrease in inventory level. Thus, price volatility will be positive sloping on price.

On modeling the commodity price, Pindyck concluded that, over the long run, price behavior seems consistent with a model of slow mean reversion, i.e.,

[8] Producers have to judge whether the price increase is caused by a shock or by regular fluctuation. Thus, producers will not adjust their operation immediately, and production will increase later than price rises.

$$dP_t = B \cdot (P_t - \overline{P}) \cdot dt + \sigma \cdot P_t dZ_t \qquad (2.6)$$

where B is the reverting rate, $\overline{P}$ is the mean of price and σ is a constant.

3. Producers' Operating Options and Market Supply

Pindyck (2001) did not consider the effects of producers' operating flexibilities on price volatility. Management is not passive. In the marketplace, producers have operating flexibilities, that is to say, they do not have to operate continuously at their base scales. As new information arrives, uncertainty about market conditions is gradually discovered, management may have valuable flexibility to alter its initial operating strategy. For example, in natural resource industries, producers may have the option to alter operating scales, i.e., to expand, to contract, to shut down and restart operations at various stages of the firm's useful operating life[9]. Such changes of operating scales do not have to happen immediately. Firms can wait till new information arrives to justify such changes. If market conditions are more favorable than expected, producers can expand the scale of production or accelerate resource use. Conversely, if conditions are less favorable than expected, producers can reduce the scale of operations. In extreme cases, production may be shut down and restarted.

In the following section, we will first explain the effects of these operating options on the supply of the underlying commodities at firm level. Later, we will derive the market supply curve from the individual producer's supply curve.

[9] Trigeorgis (1996) explained these managerial flexibilities in detail.

3.1. Producers' Operating Options

To simplify the illustration, we assume that at any operating scale, the individual producer's production is less elastic. Initially, the market price is P*, and the base scale of the producer is Q*, as shown in Figure 3-1.

Option to expand

If the market conditions turn out more favorable than expected, i.e., the commodity price increases significantly (above Pe in Figure 3-1), then producers can expand the scale of production by incurring a follow-on investment, Ie. This managerial flexibility is similar to a call option to acquire an additional part of the base-scale project, paying Ie as the exercise price. If we denote V as the present value of the complete project's expected operating cash flow, x as the percent rate of increase of the production scale, then the investment opportunity with the option to expand can be viewed as the base-scale project plus a call option on future investment, i.e., $V + \max(xV - Ie, 0)$.

Option to Contract

If market conditions are weaker than originally expected, i.e., the commodity price drops below the average total cost of production (below Pc in Figure 3-1), then the producer can reduce the scale of operations to save part of the planned investment outlays. This flexibility to mitigate loss is analogous to a put option on part of the base-scale project, with exercise price equal to the potential cost savings, Ic. If we denote c as the percent rate of decrease of the production scale, then the investment opportunity with the option to contract can be viewed as the base-scale project plus a put option on future investment, i.e., $V + \max(Ic - cV, 0)$.

Option to shut down and restart operations

Producer does not have to operate in each and every period. In case the price drops such that cash revenues are not sufficient to cover variable operating costs (below Ps in Figure 3-1), it might be better not to operate temporarily - Producers can restart operations later once prices rise sufficiently. If we denote R as the annual cash revenues, Iv as the variable costs of operating, then operation in each year is similar to a call option to acquire that year's cash revenues by paying the variable costs of operating as exercise price, i.e., $\max(R - Iv, 0)$.

3.2. The Market Supply Curve

Figure 3-1 is the individual producer's supply curve. Usually, a natural resource industry consists of many competitive producers. Although these producers may have similar supply curves, their operating scales and production costs may not be identical, so different producers may have different threshold prices to expand, contract and temporarily shut down their production. Thus, the market supply curve will be smoother than the individual producer's supply curve.

The market supply curve is shown in Figure 3-2. Note that Figure 3-2 is based on the assumption that producers have all the operation options mentioned above. This assumption is reasonable for the forestry industry, but might not be completely correct for other natural resource industries, such as some mine industries in which the option to expand seems to be not available. Thus, the dynamics of commodity prices and price volatility might not be identical for all commodities.

4. Dynamics of Commodity Price

To evaluate resource investment, we usually use monthly or quarterly data. Usually a resource investment will last for several decades, even over a hundred years, thus we need to consider the evolution of commodity price over a long period, say, several decades. In such a long period, most often only monthly or quarterly data are available.

For producers, a month or a quarter is a period long enough to begin the adjustment of their operating scales. Thus, the producers' operating flexibility should be considered in the short-run dynamic analysis. In this reason, this paper will use 'short-run' to denote the period in which the producers face only one shock and have time to adjust their production.

In order to analyze the short-run dynamics of commodity prices in more detail, I will consider how inventory, price and price volatility change in response to a small or large exogenous shock[10]. Define a shock as positive if it raises price and negative if it decreases price. Both positive and negative shocks can originate either on the supply or the demand side of the market. For example, country M's high economic growth rate increases the demand for lumber. A forest fire in country N reduces the lumber supply. Both events will raise the lumber price, so they are positive shocks. On the contrary, if a

[10] Pindyck (2001) did not consider the issue of the magnitude of shocks. We differentiate between small and large shock. Small shocks do not change the price enough to cause producers to change operating scales, while large shocks do.

new material is invented to replace lumber in furniture, or a new biotechnology can be applied to significantly increase the growth rate of trees, then the lumber price will be pushed down. These events are called negative shocks in this paper.

To illustrate the dynamics of commodity prices, I will analyze the evolution of commodity price when the commodity market faces a positive shock and a negative shock, respectively, and then provide some empirical examples.

4.1. Positive Shock[11]

Small Positive Shock

Suppose there is a small positive shock on the market. According to Pindyck (2001), the dynamics of price and change of inventory can be expressed by the inverse net demand function, $P(dN)$, as shown in Figure 4-1. Initially, the net demand function is $P_1(dN)$, and net demand is zero. Denote the current spot price as $P*$. A small positive shock will immediately push up the spot price. The inverse net demand function $P_1(dN)$ will shift upward to $P_2(dN)$. Before the producers can respond to the shock, the inventories will decrease ($dN_1 < 0$), and this will limit the size of the price increase (from $P*$ to P_1 in Figure 4-1).

Point B is not a steady state, (because inventory must be positive, it cannot keep decreasing. The new equilibrium can only occur when the net demand curve crosses the

[11] The definition of "shock" in this paper is different form Pindyck (2001). Pindyck analyzed the overall effects of a temporary event on the dynamics of commodity price. We find that it is more convenient to use two separate shocks to describe an event: one shock at the occurrence of the event, and one shock when the effect of the event disappeared. It is useful when the effect of a "temporary" event might last several years.

vertical axis and dN = 0), so price will move along the $P_2(dN)$ curve upward to P**, the new equilibrium price.

Since the price does not rise over the threshold price to expand, Pe, the producer will not change the scale of the operation, but will adjust its production at the base – scale level. As the production increases, the inventory level increases. Since the inventories will be re-accumulated[12], production will have to exceed consumption (i.e., dN > 0). Thus in Figure 4-1, the spot price rises from P_1 to $P_2 > P^{**}$(from B to C in Figure 4-1). (The increase in the balanced spot price is due to the increase of marginal cost.) This accumulation of inventories will continue until the inventory level reaches $N^D(P^{**})$, the new balanced inventory level, i.e., $dN^D(P^{**}) = 0$. When there is no further accumulation of inventories, the spot price will fall to P** (from C to P** in Figure 4-1).

Figure 4-3 shows the short-run behavior of price volatility, which is indicated by mean-reversion model (equation (2.6)).. Initially, correspond to spot price P_0, the price volatility is E. If producers have no operating options, the diffusion curve will be the ABECD line in Figure 4-3. A small positive shock raises the price, and causes the price volatility go upward along the EC line.

Large Positive Shock

When the shock is large, i.e., the shock pushes the price to rise above the threshold to expand, the consequence of such a shock is not like that of small shock. First, the shock will immediately push up the spot price. The inverse net demand function $P(dN)$ will shift upwards to $P_2^{LP}(dN)$ in Figure 4-2. If we neglect producer's option to expand, we would think that the inverse net demand function stay at $P_2^{LP}(dN)$, and conclude that, similar to the small positive shock case, the market will return to equilibrium at price P_2. However, since at $dN = 0$, $P_2 > P_e$, the producer will choose to expand the scale of operation. Before spot price reaches Pe, the producer will not exercise the option to expand, and the inventories will decrease ($dN_1 < 0$). When the spot price reaches Pe, the producer will expand the scale of operation, and the inverse net demand function will shift downwards to $P_3^{LP}(dN)$. Thus, in Figure 3-2, instead of $P^* \rightarrow A \rightarrow B \rightarrow P_2$, the dynamics of commodity price will be $P^* \rightarrow C \rightarrow D \rightarrow P^{**}$.

What is the effect of operating options on the price volatility curve? When a positive shock pushes the spot price over Pe, producers will expand operating scales. Since production increases with operating options is greater than production increases without, the inventory level with operating options will be higher than the inventory level without. Thus, the expansion of operating scale will reduce the price volatility level, and bend the volatility curve downward from CD to CD_1 or CD_2 shown in Figure 4-3.

4.2. Negative Shock

Small Negative Shock

The consequence of a small negative shock is similar to that of a small positive shock, but in the opposite direction. The small negative shock will immediately push down the spot price. The inverse net demand function $P(dN)$ will shift downwards. (In Figure 4-4,

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[12] To mitigate the positive shock, inventory has been reduced to the required level.

this is shown as a shift from $P_1(dN)$ to $P_3(dN)$.) Before the producers can respond, the inventories will increase ($dN_1 > 0$), and this will limit the size of the price decrease (from P_0 to $P_{1"}$ in the figure).

Since the price does not drop below the threshold price to contract, Pc, the producer will not change the scale of operation, but will adjust its production at the base – scale level. As the production decreases, the inventory level decreases. Since the inventories will be cut down, production will have to be less than consumption (i.e., $dN < 0$). Thus, in Figure 4-4, the spot price drops from P_1 to $P_3 < P_{**}$. This de-accumulation of inventories will continue until the inventory level reaches $N^D(P**)$, which is less than N_0. At point N^D, there will be no further de-accumulation of inventories, and the spot price will rise to P^{**}.

As shown in Figure 4-3, a small negative shock raises the price, and causes the price volatility to go downward along the EB line.

Large Negative Shock

A large negative shock will immediately push up the spot price. The inverse net demand function $P(dN)$ will shift downwards to $P_2^{LN}(dN)$ as in Figure 4-5. If the producer does not have the option to contract, the inverse net demand function will stay at $P_2^{LN}(dN)$. Then similar to the small negative shock case, the market will return to equilibrium at price P2.

However, since at $dN = 0$, $P_2 < P_c$, the producer will choose to contract the scale of operation. Before the spot price reaches Pc, the producer will not exercise the option to contract, and the inventories will increase ($dN_1 > 0$). When the spot price reaches Pc, the producer will contract the scale of operation, and the inverse net demand function will shift upwards to $P_3^{LN}(dN)$. Thus, instead of $P^* \rightarrow J \rightarrow K \rightarrow P_2$, the dynamics of commodity price will be $P^* \rightarrow L \rightarrow M \rightarrow P^{***}$ in Figure 3-5.

If the negative shock is so large that the spot price may be less than some producers' threshold price to temporarily shut down at some point before the market reaches a new balance, then these producers will choose to suspend production for a while, and resume operations later when the spot price rises again. Since the effect of the operating option to temporarily shut down on the commodity market is similar to the effect of the operating option to contract, I will not talk about this process in detail.

A large negative shock pushes down the spot price for P0 to Pc, and increases the inventory level. Since production with operating options decreases more than without, the inventory level with operating options is lower than the inventory level without. Thus, the contraction of operating scale will raise the price volatility level and bend the diffusion curve upward from AB to A_1B or A_2B shown in Figure 4-3.

The locus of the bended price volatility depends on the property of the commodity market. Of course, producers' managerial flexibility will depend on the characteristics of the industry so it is not surprising to see that different commodities have differently shaped diffusion curves.

4.3. The Dynamics of Commodity Spot Price

Mean Reverting Property

Market uncertainty can be regarded as the result of a sequence of positive and negative shocks, no matter whether these shocks are caused from the shift in the supply function or demand function. A temporary exogenous effect can be regarded as consisting of two shocks with opposite directions. As the exogenous effect disappears, production, consumption and inventory, as well as commodity price and price volatility, will return to the initial level. A permanent exogenous effect will change these market variables permanently and these market variables will not return to the initial level. However, if there is no long term upward or downward trend, we can assume that the permanent exogenous effects have been symmetric and have neutralized each other. Thus, in the long run, the process of commodity spot prices expresses the mean reverting property.

Time Trend of Commodity Spot Price

For some commodities, however, and for some longer time periods, long term upward or downward trends have been observed. The existence of a time trend will change the reverting target, and consequently shift the price volatility function, thus, when we have data for a long period, it may be necessary to include a time trend when we model the process of a commodity spot price[13].

4.4. Empirical Examples

Figure 4-6 and 4-7 present time series data of real Canadian softwood lumber prices and real US lumber prices, respectively. The Canadian softwood lumber price data are a Monthly Price Index for Canadian softwood lumber from January 1956 to December

[13] The discussion of long-run trend is beyond the scope of this paper.

2003. The original series is a nominal price index, defined so P_{1997}=100. We deflated this index by the monthly Canadian Consumer Price Index[14]. The U.S. lumber price data, ranged from January 1947 to December 2003, were obtained by deflating monthly nominal price index of U.S. lumber by the monthly U.S. Consumer Price Index.

Figure 4-8 and Figure 4-9 compare estimations of drifts and diffusions of real prices of Canadian softwood lumber and U.S. lumber estimated by a non-parametric method and a mean reverting model[15]. For non-parametric estimation, we employed a local linear method and choose bandwidth $h = 6 * n^{-2/9} * std(P)$, where n is the number of observations. The drifts of both Canadian price and US price show mean-reverting property – the expected change of price has downward sloping on price. The Figures of price volatility indicate that both curves estimated by non-parametric method are roughly in line with the approximate S-shape shown in Figure 4-3 and break the 95% confidence intervals of estimations using mean-reverting model[16]. These Figures indicate that estimation by mean-reverting model might give us biased results on the diffusion estimation.

An Additional example, the comparison of estimations of diffusions of real prices of U.S. Crude Oil estimated by a non-parametric method and a mean reverting model, is presented in Figure 4-10, in which the curve estimated by non-parametric method also

[14] It seems that choosing different deflators does not have significant effect on the results. If we use industrial price index to be the deflator, we can get similar curves.

[15] Mean Reverting Model: $dP_t = \eta \cdot (\overline{P} - P_t) \cdot dt + \sigma \cdot P \cdot dZ_t$

[16] Some researches pointed out that non-parametric estimation of price volatility might cause spurious non-linearity. (See Fan and Zhang, 2001). To test the hypothesis that the non-linearity of the volatility is spurious, I estimate the price process and use Monte Carlo method to generate 500 simulations with same size. The quantiles of estimations seem to indicate that the S shape of volatility curve cannot be explained only by non-linearity. The results of the test can be seen in Appendix I.

breaks the 95% confidence intervals of mean-reverting model, and indicates that estimation by mean-reverting model might give us the wrong result.

5. Implications on Modeling the Commodity Price Process

A good parametric model describing the stochastic process of commodity prices should capture the characteristics of the evolution of commodity prices. In this section, we examine whether those currently used parametric models can reflect the dynamics of commodity prices.

5.1. Parametric Models

The most popular parametric models describing the stochastic process of commodity prices in the real options literature include the geometric Brownian motion (GBM) and mean reverting motion (MR),

$$\text{GBM} \quad dP_t = B \cdot P_t \cdot dt + \sigma \cdot P_t dZ_t \qquad (5.1)$$

$$\text{MR} \quad dP_t = B \cdot (P_t - \overline{P}) \cdot dt + \sigma \cdot P_t dZ_t \qquad (5.2)$$

Other parametric examples used in finance include models from Vasicek (VAS) (1977), Cox, Ingersoll and Ross (CIRSR) (1985), Cox, Ingersoll and Ross (CIRVR) (1980), and Chan, Karolyi, Longstaff and Sanders (CKLS) (1992).

$$\text{VAS} \quad dP_t = (A + B \cdot P_t) \cdot dt + \sigma \cdot dZ_t \qquad (5.3)$$

$$\text{CIR SR} \quad dP_t = (A + B \cdot P_t) \cdot dt + \sigma \cdot \sqrt{P_t} \cdot dZ_t \qquad (5.4)$$

$$\text{CIR VR} \quad dP_t = \sigma \cdot P_t^{3/2} dZ_t \qquad (5.5)$$

$$\text{CKLS} \quad dP_t = (A + B \cdot P_t) \cdot dt + \sigma \cdot P_t^{\gamma} dZ_t \qquad (5.6)$$

In model (5.3) – (5.6), A, B, σ, γ are constant. Comparing model (5.3) – (5.6), we can find that model (5.1) – (5.5) are generalized by model (5.6).

Can these models correctly reflect the dynamics of the price process? The diffusion term in all these parametric models assume a constant functional form, $\sigma(\bullet) = \sigma \cdot P_t^{\gamma}$. Such functional form indicates that in the whole domain, the curve is concave or convex. However, the S shape of the volatility curve requires that the change of concavity or convexity within the domain. Thus, this functional form cannot describe the S shape of the volatility curve. Even though the dynamics of price shows mean reverting property, since the producers have the operating options to expand or contract, the reverting rate at different price level might be different. Thus, these models might be oversimplified[17].

Some studies have considered alternative models. One is the stochastic volatility model. For ease of illustration, consider a simple stochastic volatility model, i.e., assuming that, for geometric Brownian motion

$$dP_t = B \cdot P_t \cdot dt + \sigma_t \cdot P_t dZ_t \qquad (5.7)$$

σ in the diffusion term follows Brownian motion

$$\sigma_t = \kappa \cdot dt + \upsilon \cdot \cdot dW_t \qquad (5.8)$$

where *B*, κ and υ are constants and W is a standard one- dimensional Brownian motion.

Another one is Schwartz and Smith (2000)'s two-factor model. Schwartz and Smith (2000)' developed a two-factor model of commodity prices that allows mean-reversion in short-term prices and uncertainty in the long-term equilibrium level to which prices revert. In their parametric model, the long-run equilibrium price is assumed to evolve according to geometric Brownian motion with drift reflecting expectations of the exhaustion of existing supply, improving technology for the production and discovery of the commodity, inflation, as well as political and regulatory effects. The short-term deviations, defined as the difference between spot and equilibrium prices, are expected to revert toward zero following an Ornstein-Uhlenbeck process. These deviations may reflect short-term changes in demand resulting from variations such as intermittent supply disruptions, and are tempered by the ability of market participants to adjust inventory levels in response to changing market conditions.

However, both stochastic volatility model and Schwartz and Smith's two-factor model indicate the positive relationship between price volatility and price. Thus, they cannot explain the S shape of the volatility curve. The proof is provided in Appendix I.

5.2. Impact of Misspecification on Investment Decisions

All parametric estimators have some model assumptions. Inaccurate assumptions can lead to model misspecification. Model misspecification can lead to biased estimators.

Misspecification might also cause bias in investment decisions. For example, suppose we want to determine the entry and exit threshold prices of an industry. Figure 5-1[18] shows the entry and exit threshold prices curves as functions of the price volatility of the underlying commodity. The entry threshold price has positive relationship with the price volatility, while the exit threshold price has a negative relationship with the price volatility. (The intuition here is that, the higher the volatility, the greater the option value to enter or to exit, and the later the producer will exercise the option. Thus, a higher

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[17] Empirical examples will be provided later in this section.
[18] See Dixit & Pindyck (1994), p.226.

volatility will be accompanied by a higher entry threshold price and a lower exit threshold price.)

Assume the real volatility curve is ABCD in Figure 5-2, which is derived from Section 3. However, we impose an assumption that the diffusion term has a functional form as $\sigma(\bullet) = \sigma \cdot P$, where σ is constant. The estimated diffusion curve is OE in Figure 5-2.

Suppose the entry threshold price is P_H, and the exit threshold price is P_L. In the neighborhood of P_H, the estimated price volatility is lower than the real value, so the estimated entry threshold price, P_H^*, will be lower than P_H; In the neighborhood of P_L, the estimated price volatility is higher than the real value, so the estimated exit threshold price, P_L^*, will also be lower than P_L. We can see that, in this special case, misspecification might cause investors to enter too early or to exit too late.

5.3. Estimation Using Non-parametric Method

In general, we do not know the exact functional form of the drift term and diffusion term. We test hypotheses on the functional form of the commodity price process. However, based on the analysis on the dynamics of commodity prices in section 2, it seems that the functional form might be very complicated, so it is logical to ask whether or not a specific functional form is necessary.

Consider the value of a contingent claim, $F(P,t)$, which is a function of an underlying commodity price, P, and time, t, only. To evaluate the contingent claim, we need to solve the partial differential equation satisfied by the value function:

$$\frac{1}{2} \cdot \frac{\partial^2 F}{\partial P^2} \cdot \sigma^2(P) + \frac{\partial F}{\partial P} \cdot \mu(P) + \frac{\partial F}{\partial t} - \rho \cdot F = 0 \qquad (5.9)$$

where $\sigma(P)$ is the diffusion function, $\mu(P)$ is the drift function, and ρ is constant. To solve the partial differential equation (5.9) numerically, we only need to know the conditional value of the drift function and diffusion function on price. However, we can use a non–parametric method to obtain the conditional value of the drift and diffusion. Re-stating equation (1.1), the time-homogeneous stochastic differential equation that represents the underlying prices as a continuous-time diffusion process:

$$dP = \mu(P)dt + \sigma(P)dZ \tag{5.10}$$

Where $\mu(P)$ is the drift function, $\sigma(P)$ is the diffusion function, and Z is a standard one-dimensional Brownian motion. Here we assume that both drift and diffusion terms are determined only by the underlying price. The drift $\mu(P)$ and the diffusion $\sigma(P)$ can be identified by:

$$\lim_{dt\to 0}\frac{1}{dt}E(dP_t \big| P = P) = \mu(P) \tag{5.11}$$

$$\lim_{dt\to 0}\frac{1}{dt}E[(dP_t)^2 \big| P_t = P] = \sigma^2(P) \tag{5.12}$$

Equation (5.11) and (5.12) can be estimated by Kernel methods[19]. Kernel estimators of the $j^{th}\ (j = 1,2)$ conditional moments are:

$$\frac{1}{dt}\hat{E}[(dP_t)^j \big| P_t = P] = \frac{\sum_{i=1}^{T} K(\frac{P_i - P}{h_j}) * \frac{(P_{i+1} - P_i)^j}{dt}}{\sum_{i=1}^{T} K(\frac{P_i - P}{h_j})} \tag{5.13}$$

where h_j is the optimal bandwidth in the j^{th} estimator. Thus, a functional form of drift or diffusion is not necessary.

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[19] See Fan and Zhang (2001).

One advantage of non-parametric estimators is that they relax model assumptions, so the possible modeling biases are reduced. Because nonparametric estimators require little prior information relating to the functional form of the conditional expectations, it is very convenient to employ these non-parametric estimators.

6. Conclusions

This paper discusses several aspects of the short-run dynamics of commodity prices and the interrelationships between price, inventory, and price volatility. The paper analyzes the effects of inventory and the producers' operating flexibility on the dynamics of price in the short-run. The paper also examines the performance of currently used parametric models of commodity price processes in the real options literature, and illustrates how to model and estimate the stochastic process of commodity prices. Some conclusions can be made from the above discussion:

- Our model of inventories, price and operating flexibility indicates that producers' operating flexibility reduces price volatility when the spot price is higher than the threshold price for expanding operating scales, but raises price volatility when spot price is lower than the threshold price for contracting operating scales. Data for several commodities support the conclusions of this model.
- Currently used parametric models to describing the stochastic process of commodity prices, including geometric Brownian motion or mean reverting processes fail to reflect the dynamics of commodity prices.

- To evaluate the underlying contingent claim, we need to obtain the conditional value of the drift term and the diffusion term of the commodity price process, which can be estimated by non-parametric methods. However, it is not necessary to find the specific functional form of the commodity price processes.

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Appendix I

Some researches pointed out that non-parametric estimation of price volatility might cause spurious non-linearity. (See Fan and Zhang, 2001). To test the hypothesis that whether the S shape of the volatility curve can be explained only by spurious non-linearity caused by non-parametric estimation, I estimate the price process using mean – reverting model, use Monte Carlo method to generate 500 simulations with same size, and then use non-parametric method to estimate the quantiles of the simulations.

The estimations of real prices of Canadian softwood lumber and US softwood lumber using non-parametric method and using mean-reverting model, as well as the quantiles of simulation estimations are shown in Figure A-1 and Figure A-2, respectively, from which we can that, in some area, the estimated curve using non-parametric method breaks the boundaries – the 5% and 95% quantiles of the simulation estimations. It seems to indicate that the S shape of volatility curve cannot be explained only by non-linearity.

Figure A-1 Simulation Estimations on Real Price of Canadian Softwood Lumber

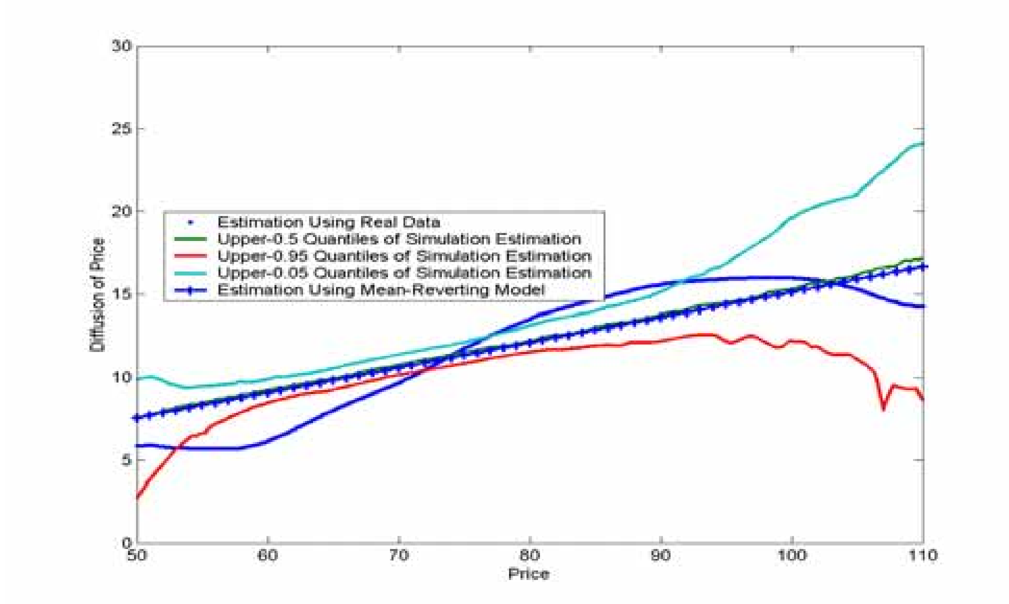
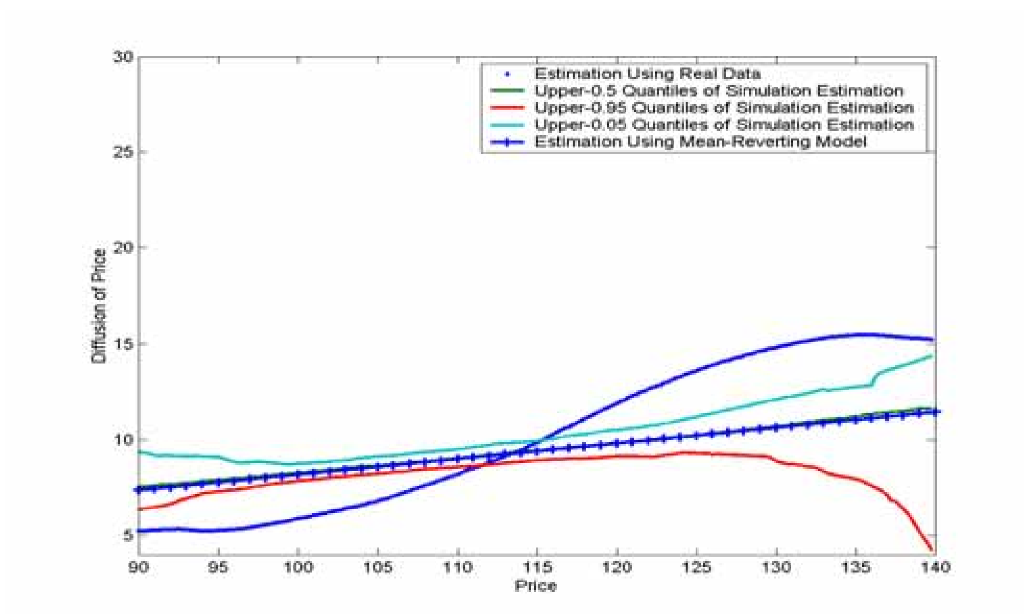


Figure A-1 Simulation Estimations on Real Price of US Softwood Lumber



Appendix II

The simple stochastic volatility model:

$$dP_t = B \cdot P_t \cdot dt + \sigma_t \cdot P_t dZ_t \tag{I.1}$$

σ in the diffusion term follows Brownian motion

$$\sigma_t = \kappa \cdot dt + \upsilon \cdot \cdot dW_t \tag{I.2}$$

where B, κ and υ are constant and W is a standard one- dimensional Brownian motion. The second order conditional moment of price increment in equation (I.1) can be written as

$$M2|(P_t = P_0) = E[(dP_t - \mu(P_t|P_t = P_0) \cdot dt)^2] = E[(\sigma_t \cdot P_t|P_t = P_0)^2] = \upsilon^2 \cdot P_0^2 \tag{I.3}$$

Equation (I.3) indicates positive relationship between price and the second order conditional moment, which implies positive relationship between price and price volatility.

Schwartz and Smith (2000) developed a two-factor model of commodity prices that allows mean-reversion in short-term prices and uncertainty in the long-term equilibrium level to which prices revert. Let S_t denote the spot price of a commodity at time t. Schwartz and Smith decompose the spot price into two stochastic factors as

$$\ln(S_t) = \chi_t + \xi_t \tag{I.4}$$

where χ_t is referred to as the short-term deviation in prices and ξ_t the long-term equilibrium price level.

In their parametric model, the short-term deviations, defined as the difference between spot and equilibrium prices, are expected to revert toward zero following an Ornstein-Uhlenbeck process:

$$d\chi_t = -\kappa \cdot \chi_t \cdot dt + \sigma_\chi \cdot dz_\chi \quad \text{(I.5)}$$

The long-term equilibrium price is assumed to evolve according to geometric Brownian motion:

$$d\xi_t = \mu_\xi \cdot dt + \sigma_\xi \cdot dz_\xi \quad \text{(I.6)}$$

In equation (I.5) and (I.6), dz_χ and dz_ξ are correlated increments of standard Brownian motion processes with $dz_\chi \cdot dz_\xi = \rho_{\chi\xi} \cdot dt$, and $\kappa, \sigma_\chi, \mu_\xi, \sigma_\xi$ are constants.

The expectation of logarithm of price increment in equation (I.4) can be written as:

$$\begin{aligned} &E(d\ln(S_t)) = E(d\chi_t + d\xi_t) \\ &= E(-\kappa \cdot \chi_t \cdot dt + \sigma_\chi \cdot dz_\chi + \mu_\xi \cdot dt + \sigma_\xi \cdot dz_\xi) \\ &= -\kappa \cdot \chi_t \cdot dt + \mu_\xi \cdot dt \end{aligned} \quad \text{(I.7)}$$

The variance of logarithm of price increment in equation (I.4) can be obtained by:

$$\begin{aligned} &Var(d\ln(S_t)) = E[(d\ln(S_t)) - E(d\ln(S_t)))^2] \\ &= E[((-\kappa \cdot \chi_t \cdot dt + \sigma_\chi \cdot dz_\chi + \mu_\xi \cdot dt + \sigma_\xi \cdot dz_\xi) - (-\kappa \cdot \chi_t \cdot dt + \mu_\xi \cdot dt))^2] \\ &= E[(\sigma_\chi \cdot dz_\chi + \sigma_\xi \cdot dz_\xi)^2] \\ &= \sigma_\chi^2 + \sigma_\xi^2 + 2 \cdot \rho_{\chi\xi} \cdot \sigma_\chi \cdot \sigma_\xi \end{aligned} \quad \text{(I.8)}$$

Equation (I.8) indicates that the variance of logarithm of price increment is a constant. Since $d\ln(S_t) = dS_t / S_t$, we have $dS_t = S_t \cdot d\ln(S_t)$. Thus, the second order conditional moment of price increment can be expressed by:

$$\begin{aligned} &M2 \big| (S_t = S_0) = S_0^2 \cdot Var(d\ln(S_t) \big| S_t = S_0) \\ &= S_0^2 \cdot (\sigma_\chi^2 + \sigma_\xi^2 + 2 \cdot \rho_{\chi\xi} \cdot \sigma_\chi \cdot \sigma_\xi) \end{aligned} \quad \text{(I.9)}$$

Equation (I.9) indicates that, according to Schwartz and Smith's model, over a long period, variance of price increment will be expected to increase as the price rises, which also implies a positive relationship between price volatility and price.

Figure 3-1: Individual Producer's Supply Curve

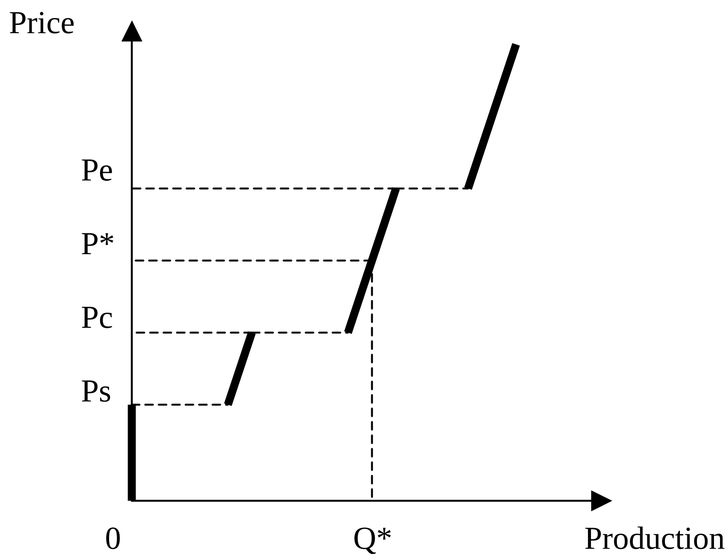


Figure 4-1: Effects of Small Increase in Net Demand

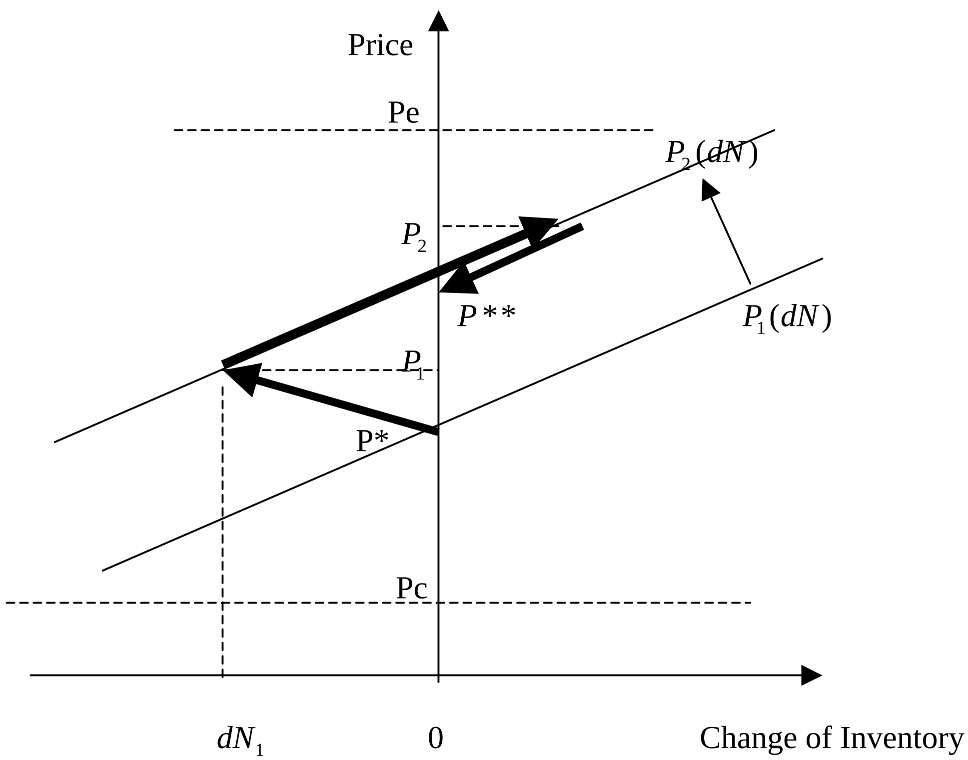


Figure 4-2: Effects of Large Increase in Net Demand

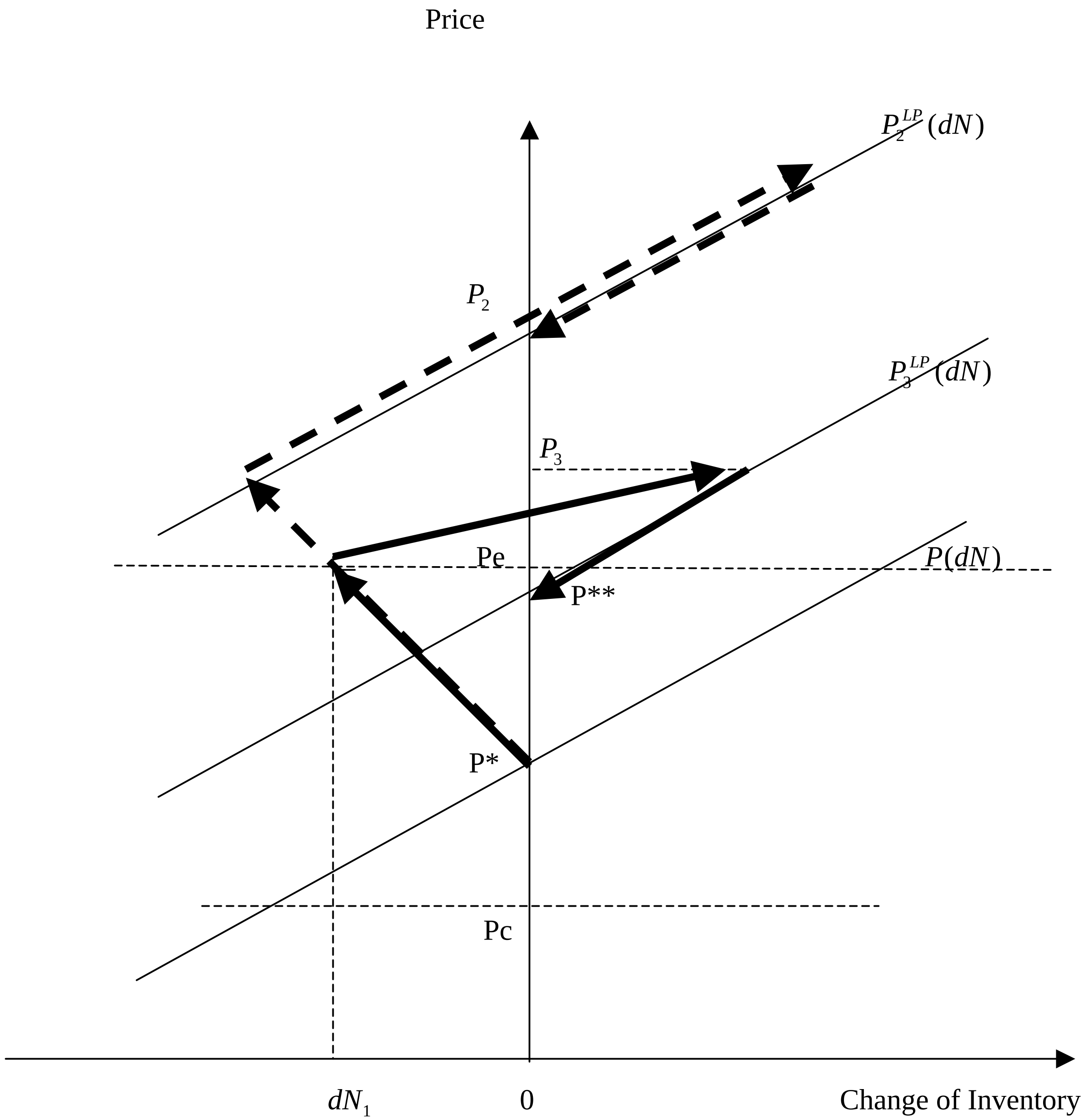


Figure 4-3: Effects of Operating Options on Price Volatility

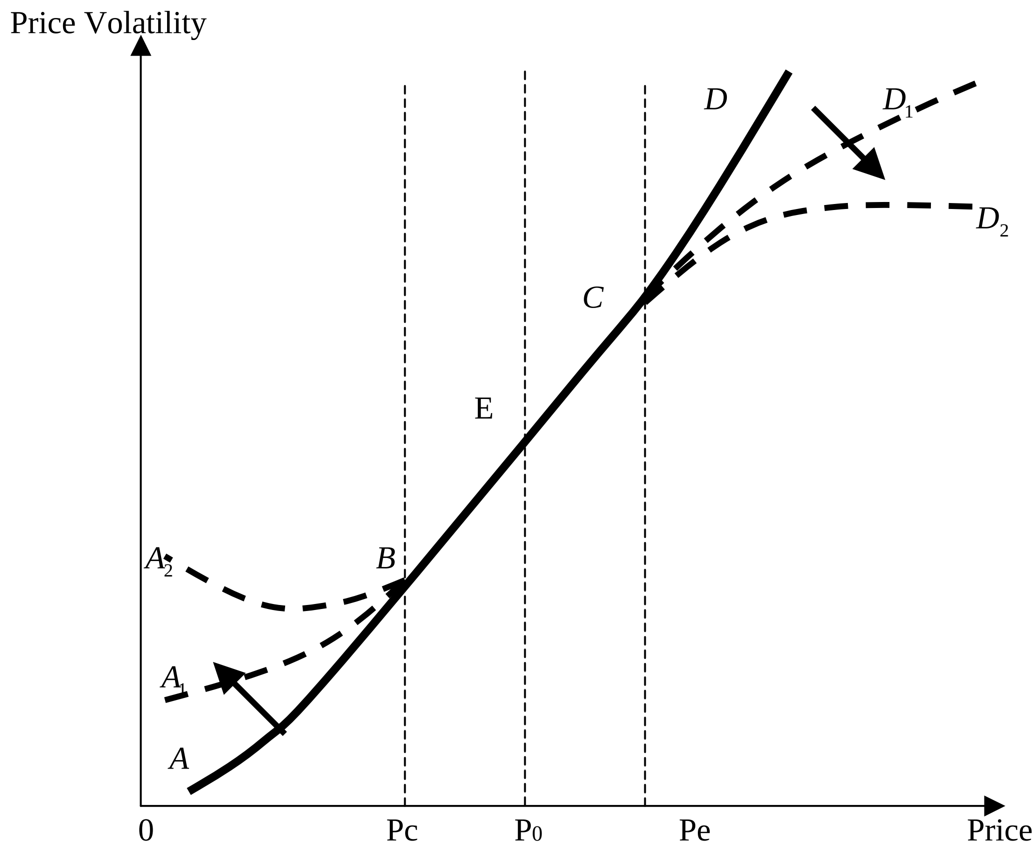


Figure 4-4: Effects of Small Decrease in Net Demand

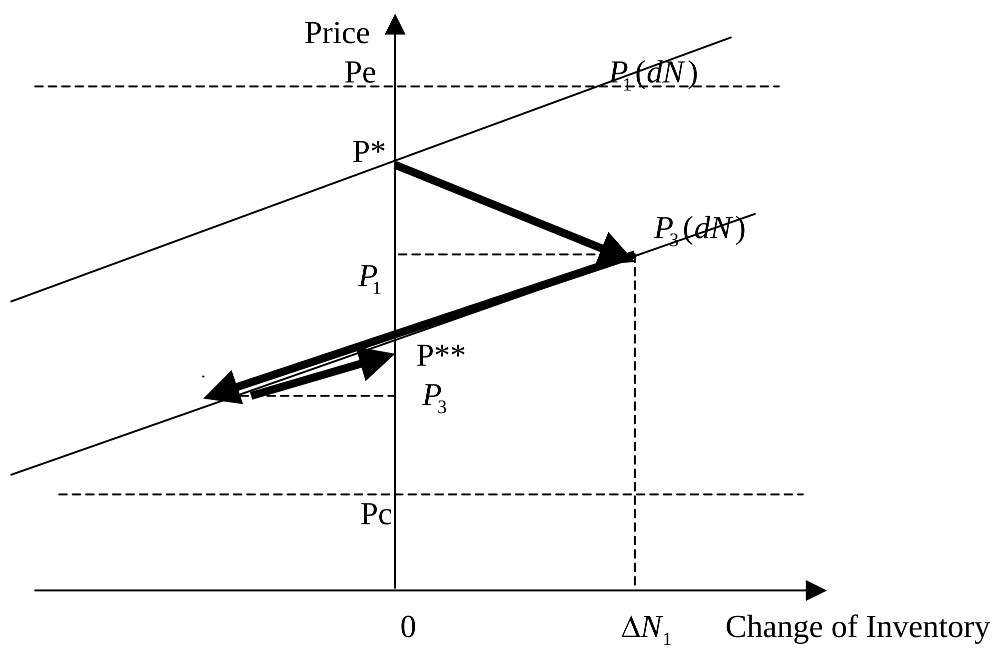


Figure 4-5: Effects of Large Decrease in Net Demand

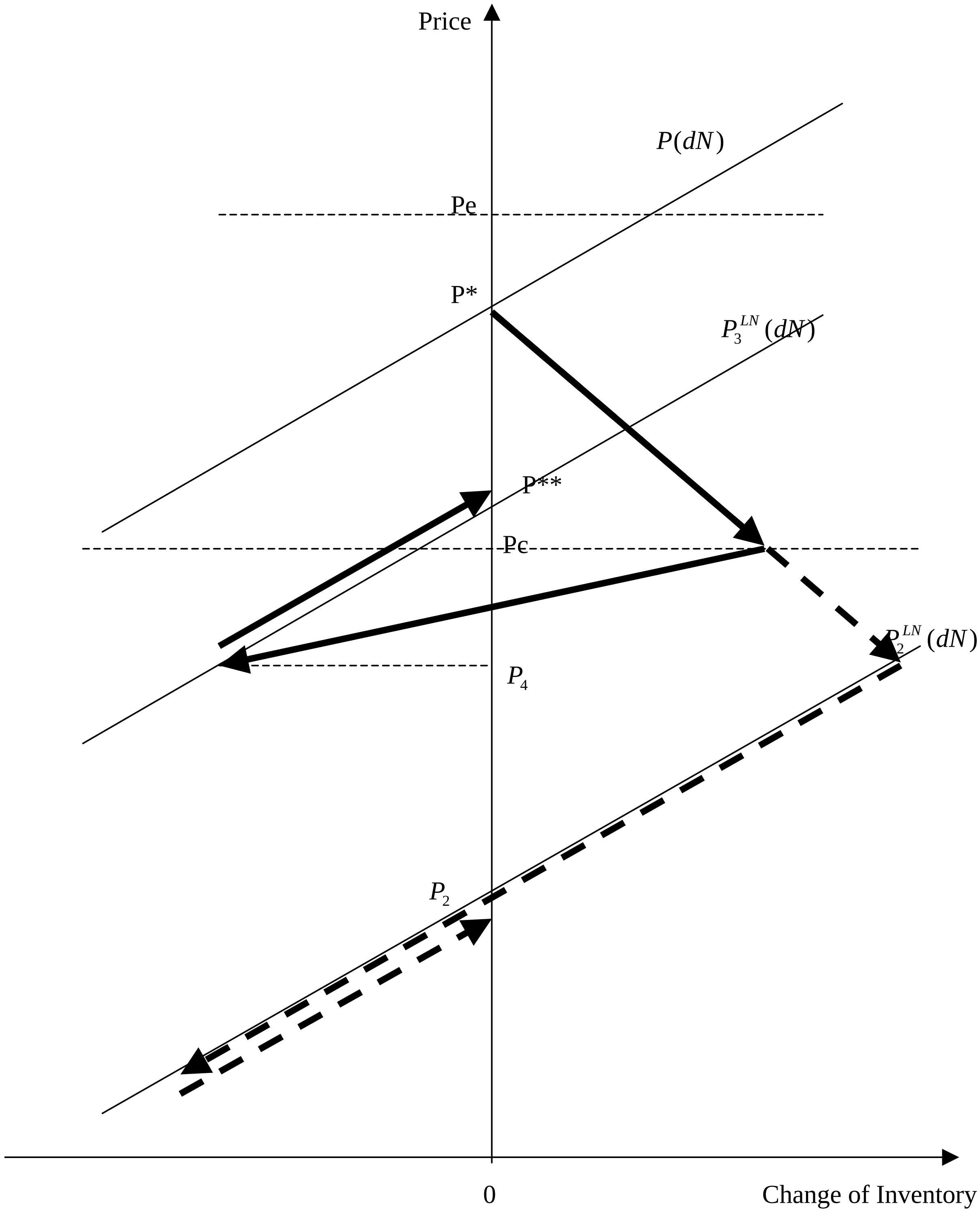


Figure 4-6: Monthly Time Series Data
- Real Price of Canadian Softwood Lumber[20]

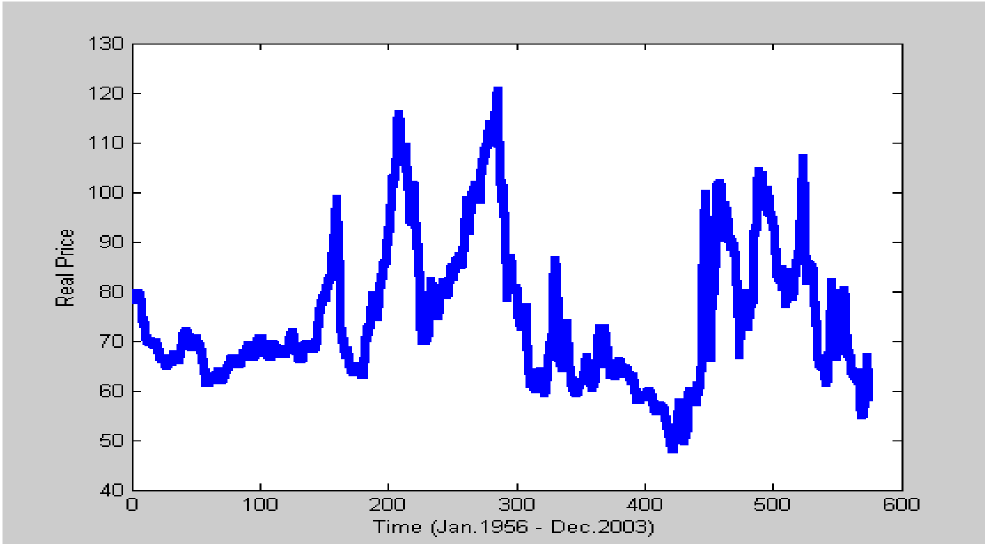
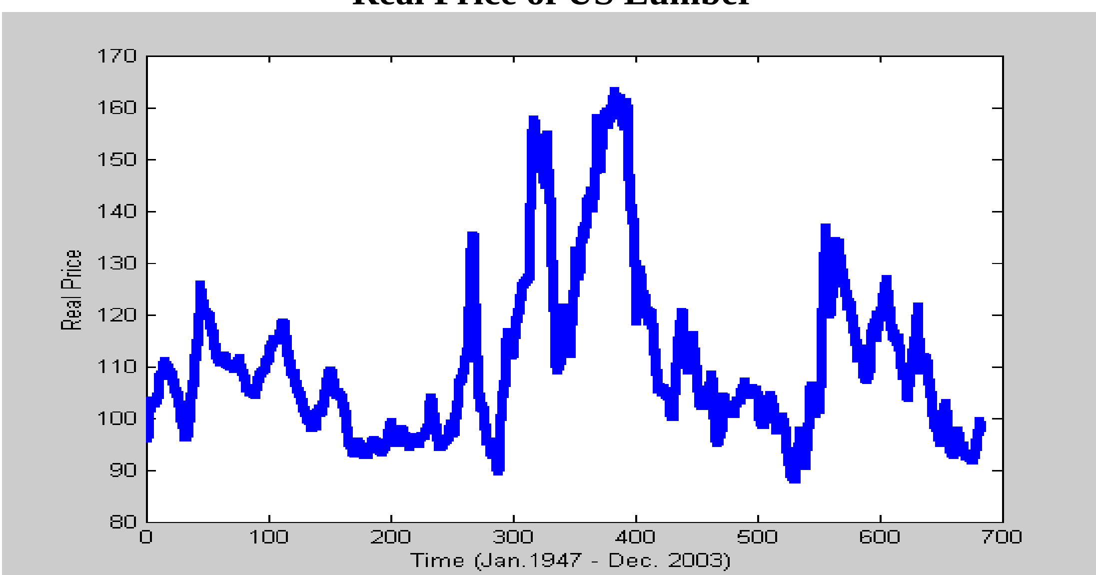


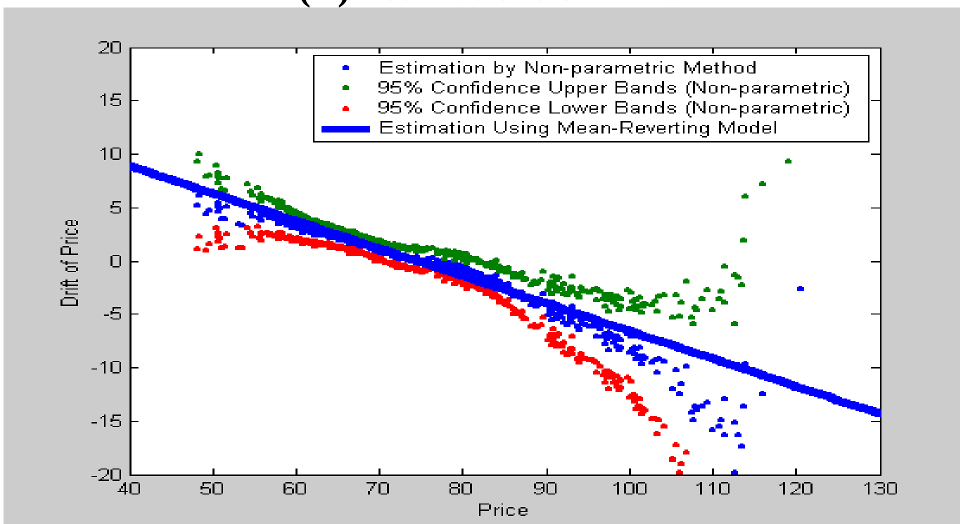
Figure 4-7: Monthly Time Series Data
- Real Price of US Lumber[21]



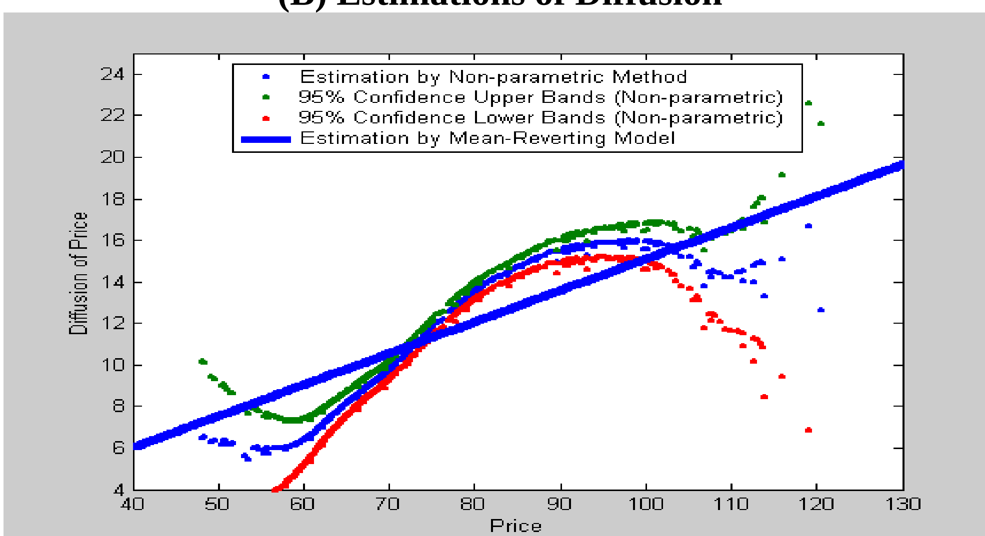
[20] Source: Statistics Canada
Monthly nominal Price Index for Canadian softwood lumber: Cansim II Series V1575009 (Jan. 1956 – Dec. 2003)
Monthly Canadian Consumer Price Index: Cansim II Series V735319
[21] Source: US Bureau of Labor Statistics
Monthly Nominal Price Index of US Lumber: Producer Price Index WPU081 (Jan 1947-Dec.2003)
Monthly US Consumer Price Index: All Items (1982-84 = 100)

Figure 4-8: Comparison of Estimations
- Real Price of Canadian Softwood Lumber[22]

(A) Estimations of Drift



(B) Estimations of Diffusion

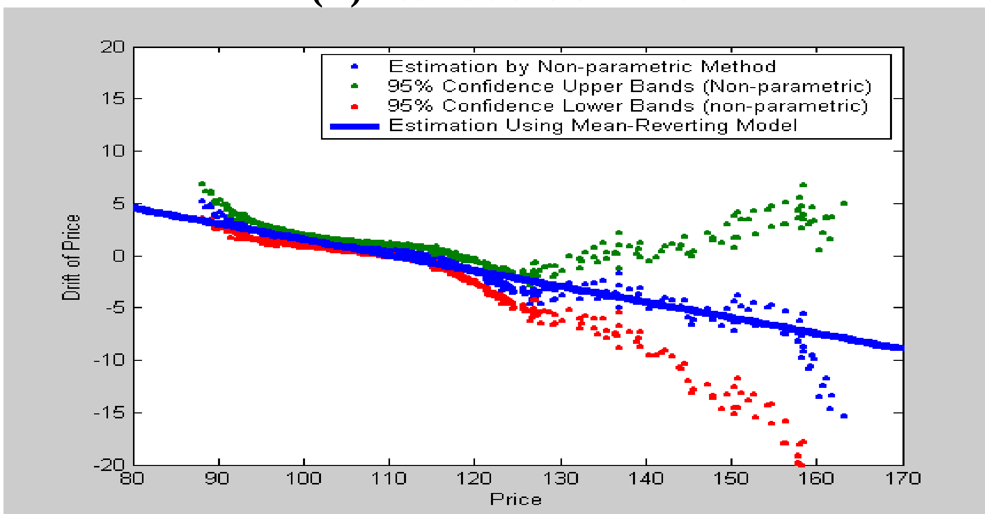


[22] Mean Reverting Model: $dP_t = \eta \cdot (\overline{P} - P_t) \cdot dt + \sigma \cdot P \cdot dZ_t$

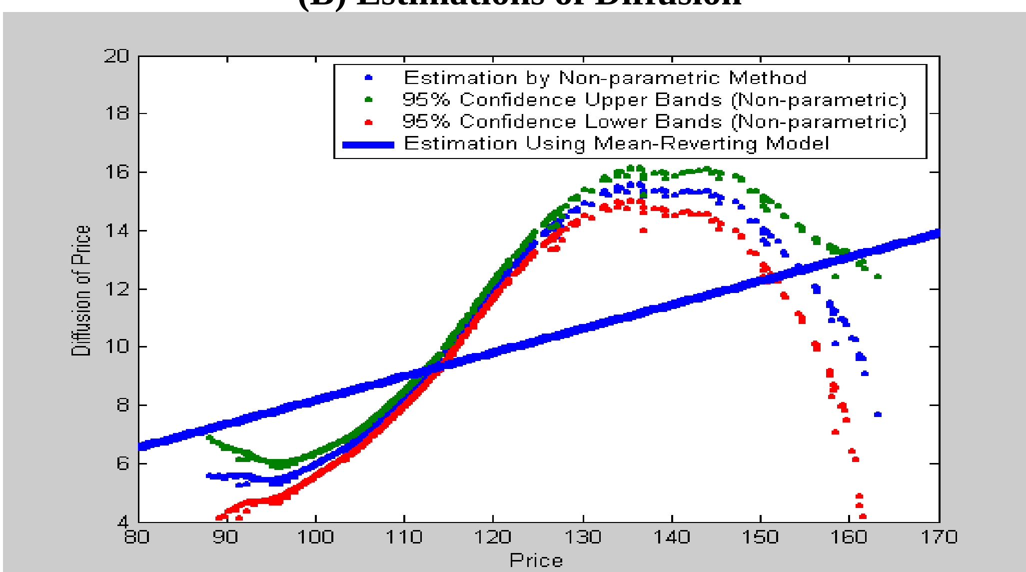
In Non-parametric estimation: bandwidth $h = 6 * n^{-2/9} * std(P)$, where n is the number of observations.

Figure 4-9: Comparison of Estimations – Real Price of US Lumber[23]

(A) Estimations of Drift



(B) Estimations of Diffusion



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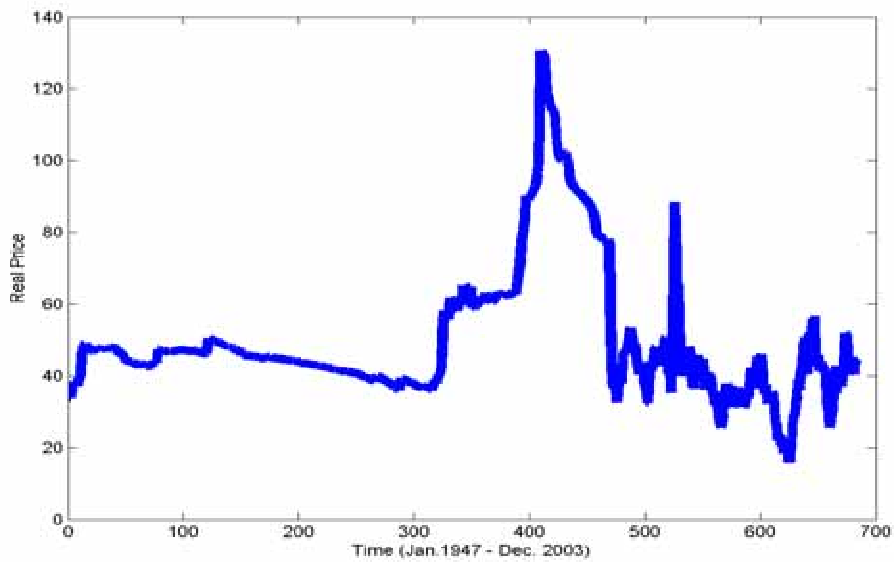
[23] Mean Reverting Model: $dP_t = \eta \cdot (\overline{P} - P_t) \cdot dt + \sigma \cdot P \cdot dZ_t$

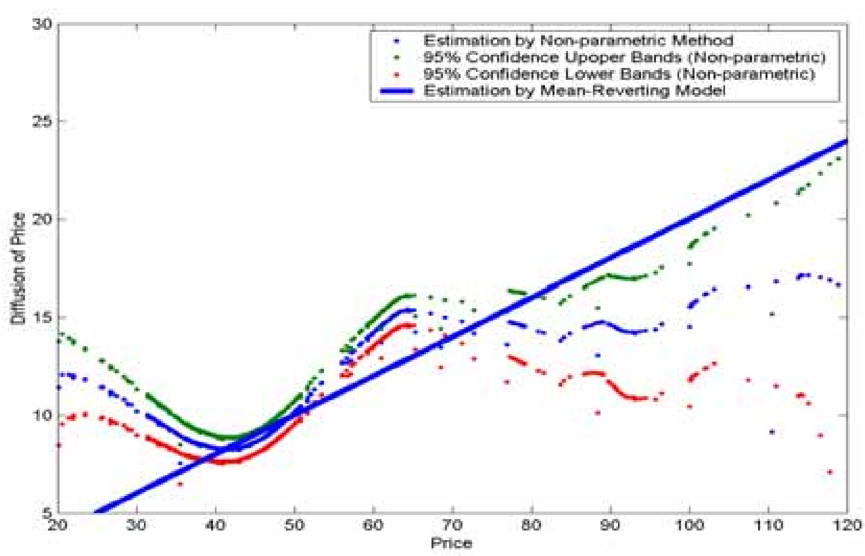
In Non-parametric estimation: bandwidth $h = 6 * n^{-2/9} * std(P)$, where n is the number of observations.

Figure 4-10: Comparison of Estimations – Real Price of US Crude Oil[24]

(A) Monthly Time Series Data



(B) Estimations of Diffusion



[24]Source: US Bureau of Labor Statistics
Monthly Nominal Price Index of US Lumber: Producer Price Index WPU0561 (Jan 1947-Dec.2003)
Monthly US Consumer Price Index: All Items (1982-84 = 100)

Mean Reverting Model: $dP_t = \eta \cdot (\overline{P} - P_t) \cdot dt + \sigma \cdot P \cdot dZ_t$

In Non-parametric estimation: bandwidth $h = 6 * n^{-2/9} * std(P)$, where n is the number of observations.

Figure 5-1: Entry and Exit Threshold Prices as Functions of Price Volatility[25]

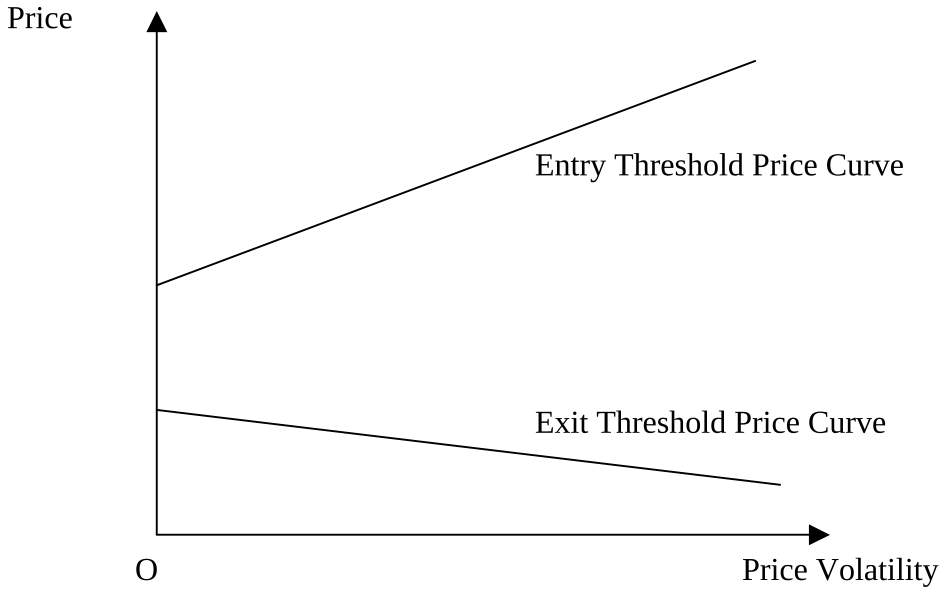
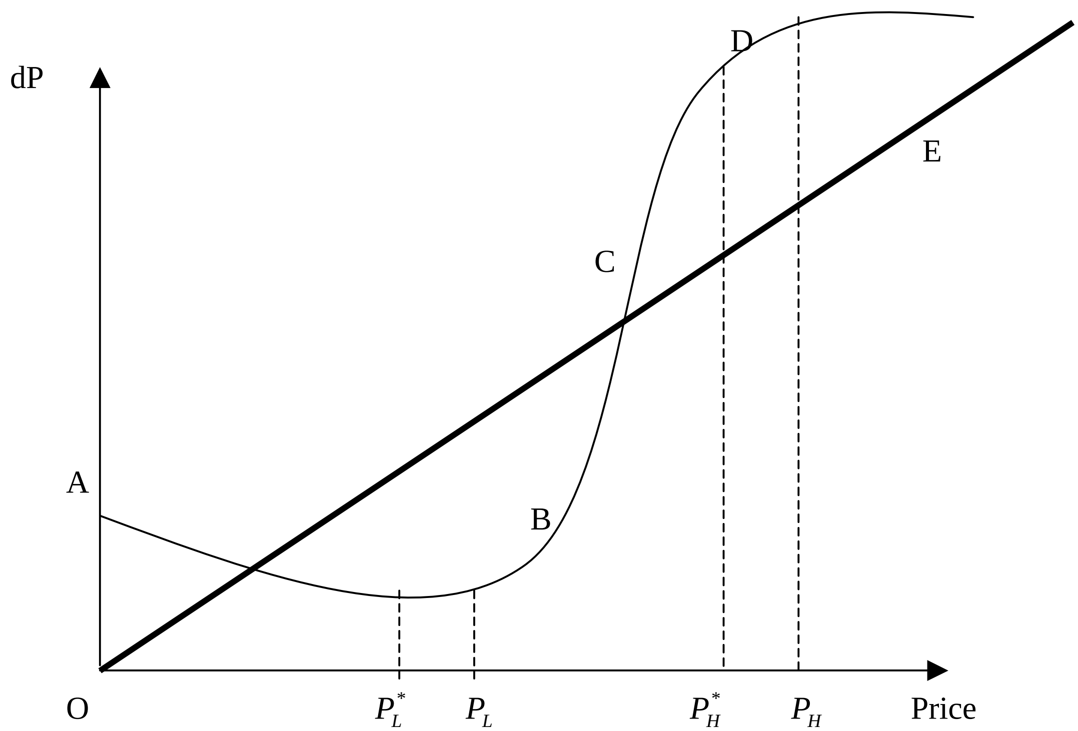


Figure 5-2: Impact of Misspecification on Option Evaluation



[25] Copy from Dixit & Pindyck (1994) p.226