

# Investment Applications in the Shipping Industry

Christian Hopp<sup>1</sup> , Stavros D. Tsolakis<sup>2</sup>

May 7, 2004

<sup>1</sup>University of Konstanz, Germany

<sup>2</sup>Center of Maritime Economics and Logistics, Erasmus University, Rotterdam

# Contents

<b>1</b>	<b>Abstract</b>	<b>2</b>
<b>2</b>	<b>Introduction</b>	<b>3</b>
<b>3</b>	<b>Real Options and Shipping-Paper Contribution</b>	<b>5</b>
<b>4</b>	<b>The Option to defer/wait</b>	<b>6</b>
<b>5</b>	<b>Choosing the best strategy</b>	<b>14</b>
<b>6</b>	<b>Option on the best of two assets</b>	<b>20</b>
<b>7</b>	<b>Conclusion</b>	<b>23</b>
<b>8</b>	<b>Appendix</b>	<b>24</b>

# 1 Abstract

This paper introduces Real Option Analysis and exotic options in particular as an alternative to the traditional capital budgeting technique for evaluating a series of shipping projects. The paper considers timing and deferment options, the option to choose the best operating strategy and the option to vary the firm's production methods. By evaluating investment opportunities using American Exchange Options, substantial differences are found compared to the NPV method in both the value of the investment opportunities and the timing of when the project is undertaken. Chooser options are employed to evaluate the various options open to a shipowner in order to optimise the strategic decision making process. Here, the model explicitly takes into consideration option interaction and shows how one can value a project when different mutually exclusive operating strategies are available. Finally, Exchange options are used to value the decision to invest in a new ship type, i.e. a new market yielding higher upside potential. Overall Real Options are useful tools for evaluating projects in an industry as volatile as shipping, where the agents need to value complex projects and make timely strategic decisions on a regular basis.

JEL Classification Numbers: G 13, G 31

## 2 Introduction

Shipping is a complex industry involving the management of units of varying carrying capacity and technological complexity. It is a risky business due to its high fixed and variable costs, and because both ship values and income are highly variable in time.

The remuneration value is affected by both economic (motion of market influence factors) and technical uncertainties (new technologies, obsolescence, new contract types and performance of new ships). In addition to market fluctuations, a vessel's value is depreciated by wear and tear and increasing maintenance requirements while the technological development makes it less competitive. Therefore, total returns can be reduced either by a fall in the ship's daily-rate or by its productive useful life decrease or a combination of both. As ships' day-rate is extremely variable, owning a ship is economically very interesting in periods of high demand since it yields very high returns and rather dull when the market is cooling off and freight rates fall. Moreover, the lead-time between a ship order and delivery is approximately two years, which means that upon delivery the market fundamentals may substantially differ from the ones when the ship was ordered thus creating opportunities for asset play or conditions for financial disaster.

Hence, a rational manager will stop the project (or reduce the investment) if the information is unfavourable (bad side), and continue the investment (or even speed it up) if it is a favourable one (good side). All these imply that ship managers are not passive: they must revise investment and operating decisions in response to market conditions, in order to maximize their company's wealth. They act to take advantage of "good times" (market's upside) and mitigate losses in "bad times" (market's downside). Therefore, due to economic uncertainty, active management adds value to investment opportunities, which is not captured by the traditional use of discounted cash flow (DCF) methods (Trigeorgis & Mason (1987). Such flexibility in timing of decisions about the firm's capabilities and opportunities give managers 'real options'. It is the way in which real options deal with uncertainty and flexibility that generates their value. Real options are not just about "getting a number", they also provide a useful framework for strategic decision making.

A real option is the right - but not the obligation - to acquire the gross present value of expected cash flows by making an irreversible investment on or before the date the opportunity ceases to be available. Although this sounds similar to NPV, real options only have value when an investment involves an irreversible cost in an uncertain environment. Thus, the beneficial asymmetry between the right and the obligation to invest under these conditions is what generates the option's value. According to Trigeorgis (1996) and Luehrman (1998) Real Options can be used in practice to value flexibility and the strategic character of investment de-

cisions under uncertainty. DCF and NPV analysis makes an implicit assumption concerning the expected scenarios of cash flows and assumes management's commitment to a certain strategy. In a real world setting consisting of uncertainty and competitive interaction, the realized cash flows will differ substantially from the expected values. As new information arrives and uncertainty concerning the cash flows can be resolved, the management can find that different projects offer varying degrees of flexibility to depart from the original strategy. As a result management has the option to defer, contract, expand, abandon or otherwise alter the project. A company thus holds a Real Option involving the right, but not the obligation to change the nature of the investment [Trigeorgis (1996)]. In the case of Real Options and financial options the holder of the option has the right to decide whether and when to make the investment. Management usually can wait and gather new information to reduce the uncertainty about a project in order to find the right timing for exercising the option.

The high freight rate volatility suggests that economic evaluations based on NPV (net present value) are inadequate. Thus, shipping companies support periods of negative cash flow in expectation of the situation reversal, as they know that the exit - and an eventual comeback - has a cost; and prevents (or makes difficult) realization of future profits in case of market recovery. However, it is usual that the nearer the end of a ship's useful life, the smaller the tendency to support such losses.

Therefore, an increase of uncertainty, increases the investment opportunity value (the opposite that tells the traditional DCF) in view of the asymmetric manager's action in response to uncertainty. This is the asymmetry on the value of the opportunity to invest in a project (or option to invest). However, increasing the value of the option to invest does not mean increasing the willingness to invest: an increase of economic uncertainty reduces the willingness to invest (or delays the investment decision), because the increment in the investment opportunity value is due to the waiting value.

In practice Real Options are embedded in projects with irreversible investments, asymmetric pay off structures, uncertainty and flexibility to act with respect to the uncertainty present. As the first two factors are present in almost every project that a company undertakes it is more important to focus on the last two factors, uncertainty and flexibility. Flexibility seems to be the most crucial factor in order to estimate the added value of Real Options, as it enables management to react to changes in the environment and opens up the possibility to directly influence the option value [Trigeorgis (1996)].

Real options focus on "dynamic complexity": the evolution of a few complex factors over time that determine the value of investment and cash flows. These are factors about which decisions can be taken at any time over a period. Triantis and Borison (2001) survey managers on their use of real options, identifying three

categories of real option usage:

- As an analytical tool
- As a language and framing device for investment problems
- As an organizational process

The article is organized as follows. In section 2, we give a brief overview about the industry specific literature concerning the use of real option techniques in pricing investment projects. In section 3, we consider the option value of waiting to invest and see how real options can help to estimate the true value of a project, taking into consideration the uncertainty about both, the investment costs as well the underlying project value. In section 4, we extend the analysis by using exotic options to model the effect of option interaction on the project value, thus explicitly dealing with option (non)-additivity. In section 5, we use the method developed in section 3 again to model the strategic decision to switch between different markets. Section 6 concludes.

### **3 Real Options and Shipping-Paper Contribution**

Shipping researchers were possibly the first to investigate and apply real options for project evaluation. Svendsen (1957), Zannetos (1966) and Miyashita (1977) analyse extensively the decision to mothball (lay-up) a ship or scrap it (abandonment option) based on the ship's remuneration, the supply-demand fundamentals and the overall economic condition. Subsequent research in the shipping industry has focused exclusively on the option to abandon. Dixit and Pindyck (1994) use a tanker vessel example to explain the manager's decision to mothball the ship in anticipation of improved market conditions or to scrap it if she sees no hope for recovery. Goncalves de Oliveira (1993) applies the Brennan and Schwarz (1985) model on valuing natural mineral resources in bulk shipping while Siodal (2001, 2003) bases his research on Dixit and Pindyck's methodology. However, despite having the option to abandon exhausted, no researcher has applied real options in valuing other ship management decisions.

This paper aims at filling this gap in literature by evaluating the shipmanagers decision-making process within a real options framework. The strategies under investigation are:

- The option to wait or defer an investment

- The option to choose the best operating strategy and
- The option to vary the mix of output or the firm's production methods

Furthermore, the paper contributes to the general literature on real option theory by employing a series of exotic options for valuing projects with valuation methods adjusted to the needs of valuing real projects rather than exchange traded options.

## 4 The Option to defer/wait

Real Options are embedded in projects with irreversible investments, asymmetric pay off structures, uncertainty and flexibility to act with respect to the uncertainty present. As mentioned by Dixit and Pindyck (1994) the key value driver for real options is the uncertainty incorporated in an investment plus the flexibility to mitigate these uncertainties. Therefore the presence of the uncertainty in different dimensions will add substantial value to the project by increasing the value of the real options Flexibility enables management to react to changes in the environment and opens up the possibility to directly influence the option value

Concerning the flexibility value of waiting to invest up to the point where the uncertainty about the market development is resolved, Dixit and Pindyck (1994) state that instead of looking at the value of direct investment, or of delayed opportunity, one should focus more on the value of the investment opportunity. In an uncertain world where the value of the underlying might fluctuate the opportunity to invest can be more valuable than investing directly into a project.

For our example we will focus on bulk shipping, which is a competitive market and as a result we investigate the option to wait when uncertainty about the project can be resolved (as opposed to Ingersoll and Ross (1992) or Berg (1999) who derive decision rules based on interest rates as a proxy). However we will use different approaches in order to show that the option of waiting to invest carries value for its holder and then try to loosen some of the assumptions underlying the different approaches in order to better approximate the true value of a project.

Timing is of essence in an industry as volatile as shipping since higher profits can be made from asset play. In addition to this decision the shipowner also needs to consider whether or not to invest in a new or a second-hand ship. In this section we focus on different models to cope with the uncertainty inherent in the shipping industry and try to find the optimal timing for investments.

The investment decision to buy a ship is irreversible, as the ship cannot be used for a different purpose. However the decision to defer the investment is indeed

reversible. Thus we can derive an investment decision based on whether the benefits from investing exceed the costs of building the ship. Although the exercise price is fixed and known in advance (at the moment of the purchase of the option) in a typical ("vanilla") call option, this is rarely the case in a real options context. While a company may be able to make a fairly accurate estimate of the cost of current investment, there is much less precision about investment costs in the future.

As a consequence, the real option to invest in the future corresponds to an exchange option and not to a simple call option, because of its uncertain exercise price. The investment corresponds to the exchange of a risky asset, investment cost, for another one, the gross project value. So, generally, when we value an investment opportunity, we are exposed to two sources of uncertainty, i.e. to two stochastic variables.

McDonald and Siegel (1986) examine the option to defer investments by looking at the optimal timing of an investment decision for an irreversible project. They suggest that we have to compare the value of investing today with the properly discounted value of investing in the future. Here it is possible to find a critical project value above which it would be optimal to undertake the investment and defer the investment if the project value is below this critical level. This is the case for an investment where the investment trigger value can be estimated by using the concept of a perpetual investment opportunity. With the assumption that the life of the investment opportunity is independent of time McDonald and Siegel show that the decision rule for investing depends on the ratio of the gross present value  $V_t$  and the investment cost  $F_t$  reaching a fixed boundary.  $V_t$  and  $F_t$  are assumed to follow a stochastic process and the investment is irreversible, thus it can only be used for this specific investment. As a result  $V_t$  as well as  $F_t$  follow a geometric Brownian motion of the form:

$$\frac{dV}{V} = \alpha_v dt + \sigma_v dz_v \quad (1)$$

$$\frac{dF}{F} = \alpha_f dt + \sigma_f dz_f \quad (2)$$

Where  $V$  is the gross present value of the expected future cash flows,  $\alpha$  is the instantaneous expected return of the project,  $\sigma$  is the instantaneous standard deviation of the project value and  $z$  is an increment of a standard Wiener Process. As a result the value of the investment opportunity resolves to:

$$X = (C^* - 1)F_0 \left(\frac{V_0}{C^*}\right)^\varepsilon \quad (3)$$



McDonald and Siegel conclude that there is substantial value incorporated in the option to wait and that it is optimal to wait with investing until the gross value of the underlying project is twice the investment costs. However the rule of investing when the present value is greater than zero does not hold in an uncertain environment where the uncertainty is resolved over time. The NPV rule would only yield the same result as the decision criterion based on real option analysis when the variance of the present value of the expected future cash flows and the investment costs is zero.

As pointed out in Trigeorgis (1996) the type of analysis that McDonald and Siegel followed seems to be unrealistic and closed form solutions, as the one mentioned above, do not exist when we add characteristics for the project such as opportunity values of investing, dividend pay-outs, as well as loosen the very strict assumption that the investment opportunity is supposed to be infinite, i.e. the option is perpetual. In practice, most investment opportunities do not continue forever, so they cannot be accurately valued using this model.

The merit in the McDonald and Siegel approach, however, is that it gives an intuitive feeling for the existence of an option value of waiting to invest for projects into uncertain markets. Nevertheless, we have to add more realistic assumptions into our model in order to estimate the value of an investment into the shipping industry more closely.

Margrabe (1978) values an American exchange option, where one exchanges a risky asset against another risky asset. However he assumes that both assets do not pay out any dividends during the life of the asset. Thus with no dividends  $V$  and  $F$  can be interpreted as the project value and the salvage value respectively. Both are assumed to follow a diffusion process of the form stated in (1) and (2). Margrabe also shows that when we think of  $F$  as a numeraire this solution can be reduced to a Black-Scholes one, as  $V$  becomes  $X = \frac{V}{F}$ , when expressed in units of  $F$ . This transformation then gives the Black-Scholes value of a call option on  $X = \frac{V}{F}$  with the riskfree rate being equal to zero, since the asset will be returned in exchange for asset including full capital appreciation:

$$c = XN(d_1) - 1e^{-rt}N(d_2) \quad (4)$$

In addition, Margrabe argues that in absence of any dividends the option will be worth more alive than exercised. However, we have to notice that there are certain drawbacks associated with the Margrabe model to value the option to exchange one risky asset for another.

Margrabe's model is not fully adequate because his exchange option can only be exercised at maturity. This characteristic is unrealistic because a company owning an option to invest can, in principle, exercise that option at any time until maturity. In other words, investment opportunities are, generally speaking, American

options. The Margrabe model can value American options only in the particular situation where the underlying asset does not distribute dividends. The reason is that, in the absence of dividends, an American option should never be exercised prior to maturity. In a real options context, "dividends" are the opportunity costs inherent in the decision to defer an investment [Majd and Pindyck (1987)]. As in a financial options context, deferment implies the loss of the project's cash flows. These lost cash flows must be seen as foregone "dividends", and must be taken into account.

We can obtain a solution for our investment-timing problem in the context of the volatile shipping industry, when we make small adjustments to the Margrabe model mentioned above. According to Rubinstein (1991), the use of a binomial approach clarifies the intuitive economic intuition behind the derivation of an exchange option formulated by Margrabe. He shows that with small adjustments the binomial model can be used and is able to handle American exchange options. Rubinstein takes the ratio of the two variables V and F and models this ratio as being univariate binomial. Thus he restates the pay-off as:

$$c = \max\left[0, \frac{F}{V} - 1\right] \quad (5)$$

Moreover he shows that one can value an American exchange option binomially by working backwards through the binomial tree for relative prices of V and F. As a result the binomial argument for the option to exchange one asset for another is equivalent to the binomial argument for standard calls except that:

- We use relative prices instead of the underlying asset prices
- The interest rate will be replaced by  $\delta_v$
- The payout rate will be replaced by  $\delta_f$
- The strike price will be replaced by 1
- The volatility will be replaced by  $\sigma^2 = \sigma_v^2 + \sigma_f^2 - 2\rho_{vf}\sigma_v\sigma_f$

After making the adjustments we will be able to value the option to exchange one risky asset for another by making substitutions in the standard Black-Scholes formula. In the following we will turn towards a practical application to show that substantial value can be incorporated with respect to the option value of waiting to invest. In addition, we will see that the NPV methodology is not able to adequately capture the "true" value of a project when uncertainty over future income and costs exists and is resolved over time.

## Model Implementation: Assumptions and Inputs

### *Gross Project Value (V)*

Corresponds to the present value of the project's appropriately discounted expected cash flows, given the information available at the evaluation date.  $V$  is the value that the firm receives by paying the exercise price (by making the investment). While the value of  $V$  at the evaluation date is known, its future values are unknown. We assume that  $V$  is a stochastic variable that follows the geometric Brownian motion process defined in (1).

### *Investment Cost (F)*

The exercise price of the investment option or the amount of capital that the company needs to invest "today" in the project. We do not know the value of  $F$  in the future, when the option to invest will be exercised. As for  $V$ , we assume that  $F$  follows the geometric Brownian motion process presented in (2).

### *Time-to-Maturity (T-t)*

Based on the average turn of a shipping cycle, we assume 4 years before each opportunity disappears. Therefore, we adopt a 4-year maturity for each project's deferment option. Since the options are American, the investment option can be exercised anytime until (or at) the maturity date.

### *Dividend-Yield of V ( $\delta_v$ )*

Let  $\mu$  be the (total) expected rate of return on  $V$  and  $\alpha$  be the expected percentage rate of changes of  $V$ . We assume that  $\delta = \mu - \alpha$  so that investment before the maturity date may be optimal, as in Dixit and Pindyck (1994).

As with call options,  $\delta$  corresponds to the dividend yield of the stock. The total return earned by the owner of the stock is then:  $\delta + \alpha = \mu$ . In the absence of dividends on the underlying stock, the optimal decision is to hold the option until maturity. Since the total return on the stock is reflected in the prices of both the underlying stock and the option, there is no opportunity cost to maintaining the option "alive". In the case of a positive  $\delta$ , there is an opportunity cost in holding the option instead of the stock. This opportunity cost corresponds to the dividends paid on the stock that are foregone by option holders.

The expected return from owning the completed project is also given by  $\mu$ . In this case the expected rate of return is irrelevant given the current asset values, as in Black-Scholes (1973). This market-determined equilibrium rate includes an appropriate risk premium. If  $\delta_v > 0$ , then the (capital) gains on  $V$  will be lower than  $\mu$ , so  $\delta_v$  is the opportunity cost of deferring the project. If  $\delta_v = 0$ , no opportunity cost exists. Thus, it is never optimal to invest earlier than at maturity. For high values of  $\delta_v$  (for high opportunity costs associated with holding the option),

the value of the option goes to zero. This transforms the project into a "now or never" type, and makes the traditional NPV a valid assessment method. In practice,  $\delta_v$  may represent several types of opportunity costs. One such opportunity cost is the cash flows foregone. Some authors (e.g. Trigeorgis, 1996) argue that  $\delta_v$  may also incorporate another type of opportunity cost. Specifically, project deferment may contribute to the early entrance of a competitor in a competitive environment, which, in turn, may have a negative impact on the value of the project. Herein, we assume that the only cost resulting from the deferment decision is the lost cash flows.

As noted above,  $\delta_v$  can be calculated as the difference between the total expected or required return on the project (i.e., the cost of capital or  $\mu$ ), and the expected growth rate of the project's value ( $\alpha$ ). We calculate  $\alpha$  using  $\alpha = \frac{V_n}{V_0} - 1$  where  $V_n$  is the expected value of the project in year n, and  $V_0$  is the project's current value if completed. Using the estimates of  $\mu$  and  $\alpha$  yields  $\delta_v$  estimates of for projects A, B and C, respectively.

#### *Dividend-Yield of D ( $\delta_f$ )*

According to the assumptions of the model, the "dividend yields" are assumed to be nonnegative constants. While this is true for  $\delta_v$ ,  $\delta_f$  is negative when carrying costs are associated with the project's capital cost. In this model, we need to assume that such costs do not exist because  $\delta_f$  cannot be negative. As pointed out by McDonald and Siegel (1986), the gain from deferral may increase with larger  $\delta_f$ . In our application, we assume that  $\delta_f = 0$  by assuming that there are no carrying costs associated with a project's capital costs nor benefits (from the capital cost's level) from deferring the project.

#### *Volatility of V and D ( $\sigma_v, \sigma_f$ )*

We assume that the volatility of the company's stock is an adequate proxy for the volatility of V (see, for example, Davis, 1998; Paxson, 1999; and Amram and Kulatilaka, 1999). It is also necessary to assume that the volatility of V is constant during the life of the option. The  $\sigma_v$  is calculated based on the natural logarithm of the monthly returns  $\ln \frac{n}{n-1}$  of the time charter rate data obtained from January 1979 to January 2003 from Braemar Seascope. The annual  $\sigma_v$  corresponds to the monthly  $\sigma_v$  multiplied by the square root of the number of months in a year (12). As to the volatility of F, and knowing that the volatility of the price of second-hand and new vessels were obtained from Clarksons following the same methodology as with  $\sigma_v$ .

#### *Correlation between the changes in V and F [ $\rho(v, f)$ ]*

We assume that the correlation between the changes in V and F can be approximated by the correlation between the monthly returns on the corresponding freight rates for every ship type and the monthly returns on the ship's values for the period described above.

A major characteristic of these investment opportunities is that they can be delayed or deferred for up to four years in order to resolve the uncertainties governing each project's value. However, if the company decides to postpone a project, it faces the uncertainties associated with future investment costs. Projects with these characteristics are similar to finite-lived American exchange options. Specifically, they have a finite maturity, they can be implemented anytime before or at the maturity date, and both the present value of the projects' cash flows and the investment costs behave stochastically.

### Model Implementation and Results

A shipping company is planning to invest in three projects. Table 1 provides input values for the valuation of each of the three investment projects.

Table 1: *Input Values for the valuation of each project (in \$ Mil.)*

	Project A	Project B	Project C
V	18.5	20.5	18.5
F	15.5	20	20.5
NPV	3.0	0.75	(2.0)
Time to expiry	4	4	4
$\sigma_v$	39%	36.5%	42%
$\sigma_f$	52%	30%	43%
$\delta_v$	8%	8%	8%
$\delta_f$	0%	0%	0%
Risk Free Rate	2%	2%	2%
$\rho$	0.82	0.90	0.85

Using the methodology in the previous section and the inputs in table 1, we obtain the results reported in table 2.

Table 2: *NPV and Option Value of waiting (in \$ Mil.)*

	Project A	Project B	Project C
NPV	3.0	0.5	(2.0)
Perpetual Call Option	5.74	5.09	4.92
European Call Option	3.78	3.15	2.92
American Exchange Option	2.74	0.54	0.75

Starting with project A the results indicate that the project should be undertaken immediately, as the project is deeply in the money and not much can be gained from deferring the project. Consequently, it is more valuable to exercise the option to invest now than to keep that option alive. The option to invest should be exercised immediately as the direct costs of investing are likely to appreciate

more than the underlying project value, therefore the firm will lose more when waiting to invest.

The results for project B indicate that despite the positive NPV, the project should not be undertaken right away due to the high value of the deferment option.

Finally, project C has a negative NPV, which initially indicates that the project shall not be undertaken. Nevertheless, this project has a high deferment option value that gives the company the flexibility to wait and see along with the right to invest in the project in future should the uncertainties be resolved in the project's favour.

The values obtained from the two methodologies and the resulting investment decisions are summarised in Table 3. We can see that the NPV method undervalues projects B and C significantly and its implementation leads to the wrong decision. Only in project A both methodologies yield the same result and propose the same investment-timing signal. Therefore, table 3 illustrates that the traditional NPV methodology is not adequate to value investment opportunities in an uncertain environment, especially when investing in a project can be deferred to a later date.

Table 3: *Summary of the Investment timing decisions of the projects*

	Value		Timing	
	AmEx Option	NPV	AmEx Option	NPV
Project A	2.47	3.0	Invest Now	Invest Now
Project B	0.54	0.5	Defer	Invest Now
Project C	0.75	(2.0)	Defer	Don't Invest

[Insert graph about here]

As we can see from the above chart there is support that NPV analysis sometimes understates the value of a project substantially. However, we can also infer that an option based analysis that forgets to take into consideration the fluctuations in the associated costs for undertaken a project might even overstate the "true" value of a project. That is, the option to wait will carry a lower value when the investment costs follow a stochastic process. Moreover, when the volatility of the investment costs is higher than the volatility of the underlying project value, the NPV might even be higher when compared with an option-based analysis, which contradicts with standard real option thinking. Once we extent our analysis to a more dynamic setting in which project cash flows and the associated costs of investing are stochastic we can derive a dynamic version of the "extended NPV" criterion, that is able to better capture the true value of a project, especially for industries that are characterized by a high degree of variability in investment costs.

## 5 Choosing the best strategy

Projects in real life are merely structured in a way that we can simply use "plain-vanilla" options to value them. In most cases the analysis is simplified by considering the project value as just a bundle of real options, thus making the assumption that the options are purely additive in nature and neglecting the option interaction that will come into play. As a result, the value of a project is likely to be overstated when option interaction is not taken into consideration. Let us consider the case of a shipowner who owns two modern Aframax tankers, each worth \$25 million, which are on a two-year charter to an Oil Major. Within the next two years the shipowner has to decide what to do with the company. There are several options available to him:

- The first one is at the end of year two to buy from the Oil Major, a third ship of similar specifications for \$24 million.
- The second option is to sell one of his tankers to the Oil Major again for \$24 million.
- Finally, the Oil Major has made him an offer to buy his company at the end of year two for \$ 40 million. The shipowner needs to notify the Oil Major of his decision six months before the expiration of the contract, that is one and a half years from now.

Clearly, valuing a combination of real options by performing them individually and then summing them yields different and incorrect results. We need to account for the interaction of option types within the same project. According to Mun (2002), the reason for the sum of individual options being different from the interaction of the same options is due to the mutual exclusiveness and independent nature of these options. That is, the firm can never, for example, both expand and abandon at the same time. Trigeorgis (1993) values projects with embedded interacting real options and shows that they exhibit non-value additivity and that the nature of option interaction depends on the type, separation, moneyness and order of options. Trigeorgis (1993) points out that when we deal with options of the same type which are exercisable under opposite circumstances (for example an option to contract and the option to expand) , the interactions are small and the options are approximately value additive. Moreover the (European) options would be purely additive if they both mature at the same time. However, this would only be the case if the decision whether to expand or contract is made at the expiration date. Whereas it is quite more likely that due to organizational needs the company will choose in advance whether to pursue an expansion or contraction strategy and therefore the options will not be purely additive. Based on the set up above we can see, that the shipowner has to evaluate three options that are open to him, expansion, contraction and abandonment, and take the most economic sound choice.

Figure 2 represents the typical set up for an investment project, where the time frame can be divided into a building phase (with incorporated deferment and abandonment option) and an operating phase with multiple operating strategies (options).

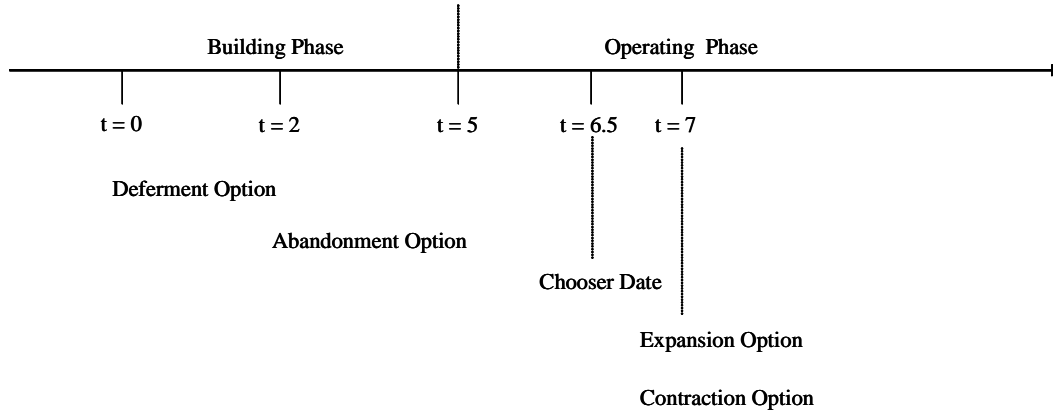


Figure 1: *Investment Project with embedded Real Options*

As we can see the option to wait and the abandonment option are clearly additive in nature. As the option to abandon expires before we enter the operating stage, we will focus on the expansion option and the contraction option and see how one can value the project when taking option interaction into account. Here, we have made the implicit assumption that the management only has a one-shot problem, as they are faced with an either-or decision. As a result the decision to expand or contract can be seen as irreversible in the short term (in the long term the company can place a new order for a ship but has to wait for quite some time due to the long lead times in ship construction). Consequently the company has to make a decision on the future strategy some time prior to the actual implementation phase and not ad-hoc in order to analyze potential consequences thoroughly and bring the necessary operational changes on track. In the end, the value resulting from the NPV expanded by the flexibility component inherent in the projects operating strategy, has to be compared to the alternative value of the (European) abandonment option.

To value the aforementioned operating strategy as an option, we can use the concept of a chooser option, or as-you-like-it option (Rubinstein 1991). Chooser options are somewhat similar to a standard straddle, will however be cheaper as they only include one leg of the straddle as one has to decide between a put or a call option. Specifically there exist two sorts of chooser options: A complex choser option and a simple chooser option. A complex chooser option gives its holder the right to select at a time  $T_0$  a call with a strike price of a and expiry at  $T_1$  or a put with a strike price of b and expiry at  $T_2$ . In the case of a simple chooser



option the expiry dates and the strike prices will be equal for both options, thus we would have  $T_1 = T_2$  and  $a = b$ . According to Buchen (2003) the payoff of such an option can then be stated as follows:

$$\text{Max}[C(S_t, X, T - t), P(S_t, X, T - t); t] \quad (6)$$

Rubinstein uses the following strategy to replicate the payoff of a chooser option:

(1) buying a call with underlying asset price  $S$ , striking price  $X$  and time-to-expiration (2) buying a put with underlying asset price  $S_d^{-(T-t)}$ , striking price  $X_r^{-(T-t)}$  and time-to-expiration  $t$

As a result the value of a standard chooser using the decomposition rule (shown in more detail in the appendix) is:

$$Sd^{-T}N(x) - Xr^T N(x - \sigma\sqrt{T}) - Sd^{-T}N(y) + Xr^T N(-y + \sigma\sqrt{T}) \quad (7)$$

Alternatively Buchen (2003) argues that one can also replicate the chooser option strategy by employing the methodology of binary options. He shows that dual expiry options, such as a complex chooser option, can be perfectly replicated with a particular set of first and second order binary options. His model returns results agreeing with published results for the case of log-normal asset prices and standard Black and Scholes assumptions.

One problem that arises with the set up mentioned above is that it is quite unlikely that strike prices and exercise dates of an expansion and a contraction will be equal. Therefore the use of a simple chooser option can only be justified when dealing with such a simplified model. In the following we will therefore extend the analysis and also deal with a more realistic model and explicitly make use of a complex chooser option that enables us to incorporate the more realistic scenario of differing characteristics, as for example strike prices.

Let us assume that the shipowner can sell one of his tankers to a third party for a higher price (\$30 million) than the one he can obtain from the Oil Major in two and a half years.

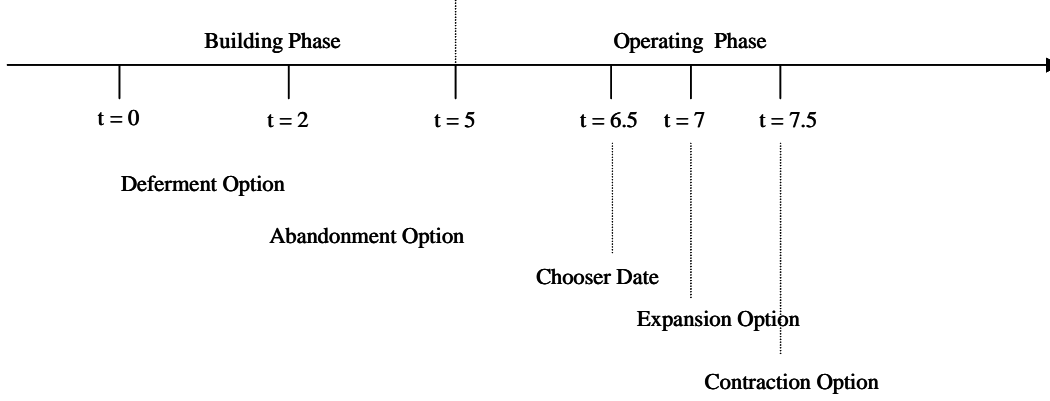


Figure 2: *Investment Project with embedded Real Options (Differing maturities and Strike Prices).*

In this case where the maturity and the strike price of the put and call options vary we can use a complex chooser option. A complex chooser option is similar to a standard chooser except that either the call/put striking prices, call/put time-to-expirations, or both are not identical. The payoff from a complex chooser can then be written as follows (Rubinstein (1991)):

$$\text{Max}[C(S_t, X_1, T_1 - t), P(S_t, X_2, T_2 - t); t] \quad (8)$$

implying the chosen call (put) has striking price  $X_1$  ( $X_2$ ) and time-to-expiration  $T_1 - t$  ( $T_2 - t$ ) on the choice date. As a result the valuation procedure will be more complicated and prevents the complex chooser option from being interpreted as a package of standard options. The "Black-Scholes" valuation formula for this option is:

$$\begin{aligned} & Sd^{-T_1}N_2(x, y_1, \rho_1) - X_1r^{-T_1}N_2(x - \sigma\sqrt{T_1}, y_1 - \sigma\sqrt{T_1}, \rho_1) \\ & - Sd^{-T_2}N_2(-x, -y_2, \rho_2) + X_2r^{-T_2}N_2(-x + \sigma\sqrt{t}, -y_1 + \sigma\sqrt{T_2}, \rho_2) \end{aligned} \quad (9)$$

The derivation of the formulae for both the simple and the complex chooser option can be found in the appendix.

Table 4 below provides an estimation of a chooser option for the above case. In order to calculate the option we calculate the volatility from monthly returns over a period of 24 years (1979-2003) to be 44%, while the risk free rate and the dividend yield is 2 and 10 per cent respectively. In order to make a sound decision we have to value the chooser option and then compare it to the alternative abandonment option. The chooser option incorporates the two mutually exclusive options (expand or contract).

Table 4: Results of the Option to choose among operating strategies

<b>Input Data</b>	
Current Price	50.00
Strike Price (Call)	24.00
Time to maturity (Call)	2.00
Risk Free Rate	2%
Dividend	10%
Volatility	44%
Time to Choice	1.50
<b>Additional Parameters for Complex Chooser</b>	
Time to maturity (Put)	2.50
Strike Price (Put)	30.00
<b>Output Data</b>	
Simple Chooser	20.85
Complex Chooser	21.80
<b>Single Options</b>	
BSM Value of Option to abandon	10.02
Value of Option to expand	19.67
Value of Option to contract	1.79 <sup>1</sup> / 4.83 <sup>2</sup>
Sum of Option to expand and contract: Scenario 1	21.46
Sum of Option to expand and contract: Scenario 2	24.50

Based on the results in the table above we can infer that when we simply add the separate values of the expansion and contraction option we will clearly overestimate the value of the project. As a result of not taking into consideration option interaction we will miscalculate the project value in the first scenario by some 3% and in the second scenario by some 12%. As we can see the more complex (in terms of multiple interacting operating options) the scenario gets, the less accurate will our estimation via "plain-vanilla" options be. Consequently we should take option interaction into account and make use of more advanced option pricing techniques.

<sup>1</sup>Scenario 1 with Put option having the same maturity and strike price as the Call option

<sup>2</sup>Scenario 2 with Put option having 2.5 years to maturity and a higher strike price

We can also infer that when dealing with two options written on the same underlying asset that both mature at the same time (and the decision to choose between the two options coincides with the maturity date), the joint probabilities of exercising both options at the same time is zero, as with no interaction the options will have their undistorted values and are additive in nature, and can therefore be added in order to derive an expanded NPV including the different option values. Which however does not happen in the case of additional flexibility by means of the right to decide on a certain irreversible capacity strategy, as can be seen when valuing the complex chooser option described.

In order to extend the analysis one could also turn towards the use of a compound option methodology to cope with multiple real options that are not mutually exclusive as for example the case of sequential expansion opportunities. In this case the two options are written on the same underlying project as before but the exercise of one option will directly affect the value of the other option by increasing (when exercised) the value of the project. Thus we could employ the methodology of Geske (1979) and value the compound nature of these interacting options. Valuing a compound option is different from valuing an ordinary option in part for mathematical rather than for conceptual reasons [McDonald (2003)]. The Black-Scholes formula assumes that the stock price is lognormally distributed. However, the option price cannot be lognormally distributed because there is a significant probability that it will be worthless. Therefore, while an option on an option is conceptually similar to an option on a stock or an asset, it is mathematically different. The difficulty in deriving a formula for the price of a compound option is to value the option based on the value of the stock/asset, which is lognormally distributed, rather than the price of the underlying option, which is not.

Our results for the valuation of differing operating strategies indicate that taking a project as a bundle of real options and thus adding all options together will clearly overstate the project value. Consequently, the effect of option interaction has to be taken into account in order to estimate the "true" value of a project. Our value is in line with the results of Trigeorgis (1991 and 1993), whereas our model is able to incorporate the characteristics of a valuation procedure when dealing with mutually exclusive options and additional flexibility available to management prior to the exercise of the options. Trigeorgis values projects with multiple interacting options using the concept of a log-transformed variation of a binomial option pricing technique, which is based on a backward iterative process, where at each time when a real option is encountered the opportunity value is revised. In comparison, the model described above uses a more intuitive technique, that in addition is able to incorporate a number of more realistic scenarios. For example, the case where one decides at the expiry date which option he wants to exercise (and when values are additive in nature) represents just a special case of the complex chooser option. Consequently, the use of a chooser option

methodology reflects not only the flexibility inherent in operating strategies, but also the flexibility of deciding on certain strategies and can therefore capture a more realistic estimate of the "true" project value.

## 6 Option on the best of two assets

As we have seen in the first part of the paper, projects can be regarded as an exchange option leading to more realistic results regarding the true project value under uncertainty for both the underlying project values and the associated investment costs. In the following we will extend the analysis by considering that management usually has to make decisions not only regarding the "go" or "no-go" decision for one specific project but could decide between more projects, or more specifically markets, and make the most economic choice. Especially with shipping as a derived good, that is dependent on the overall situation of the economy sectors, there are a variety of different opportunities that management can exploit.

Ship owners are always on the lookout for opportunities to invest into other ship types or in different ship sizes either for diversification or speculation or both. Consider for example a shipping company owning a five-year old handymax size bulk carrier that is exploring the possibility of investing instead in an Aframax tankers of the same vintage. You can think of the company as having an option to 'buy' a tanker vessel in exchange for a bulk carrier one. If freight rates and ship values were certain, this would be a simple call option on a tanker vessel with a fixed exercise price (the value of the ship). If the freight rates and ship values in the tanker market are sufficiently high it pays to exercise the option and switch to oil trades.

In practice, both dry bulk and tanker freight rates and ship values are likely to vary. This means that the exercise price of the company's call option changes as freight rates and vessel prices change. Uncertainty about this exercise price could reduce or enhance the value of the option, depending on the correlation between the prices of the two assets. If dry bulk and oil tanker freight rates moved together dollar for dollar, the option to switch trades would be valueless. The benefit of a rise in the value of the underlying asset (the handymax size bulk carrier) would be exactly offset by a rise in the option's exercise price (the Aframax tanker value). The best of all worlds would occur if the prices of the two rates were negatively correlated. In this case whenever tanker rates increased, bulk carrier rates would go down. In these (unlikely) circumstances the option to switch between two trades would be particularly valuable.

We can value such real options by using an exchange option. We saw a more extended variation of such an option in the timing option analysis. An exchange

option, also called an outperformance option, pays off only if the underlying asset outperforms another asset, called the benchmark. According to McDonald (2003), exercising any option entails exchanging one asset for another and that a standard call option is an exchange option in which the asset has to outperform cash in order for the option to pay off. In general, the exchange option provides the owner the right to exchange one asset for another, where both may be risky.

By setting the dividend yields and volatility appropriately, with an exchange call we have the option to give up K (the Aframax tanker) for acquiring S (Handymax size bulk carrier). For a put option we give up the underlying asset S for K.

As we have seen earlier American exchange options can be valued using a two-state variable binomial tree. This is because with the binomial model it is possible to check at every point in an option's life (i.e. at every step of the binomial tree, following the methodology of Cox, Ross and Rubinstein (1979)) for the possibility of early exercise. Where an early exercise point is found it is assumed that the option holder would elect to exercise, and the option price can be adjusted to equal the intrinsic value at that point. This then flows into the calculations higher up the tree.

Back to our example, we estimate both European and American call and put Exchange options, employing both the Black Scholes and the Binomial Method. According to the SSY Monthly Shipping Review July 2003 issue a five-year-old handymax bulk carrier of 45000 dwt is worth \$15 million. By the same token a five-year-old Aframax is worth \$33 million. Monthly data from 1979 to 2002 indicates volatility of 52% per annum for the handymax price and of 57% for the Aframax. The correlation between the two assets is 0.867. Based on industry data we assume a 15% dividend yield for the bulk carrier and zero yield for the tanker since the company does not own it. We assume that the company has to decide whether to leave dry bulk carriers for tankers within a year, either at the end of the period, European Exchange Option valued with Black-Scholes, or within the one year, American Exchange Option valued with a binomial model. The results are reported in Table 5. As we can see both the European and the American call options are valueless due to the high correlation of the price of the two assets. On the other hand however, we see that both the American and the European put options, the option to give up the bulk carrier business in exchange for the tanker have a value of approximately \$20.1 million. If you add up this figure to the \$15 million the company can obtain by selling the bulk carrier gives a total value that exceeds the tanker's price by \$2.1 million. Therefore, the price premium on this option suggests that the firm will be better off selling the bulk carrier during the year and investing in an Aframax tanker.

[Insert Graph 2 about here]

We can see in this case how real option theory can help the managers evaluate their decision to diversify or enter new markets in a way that traditional Dis-

Table 5: Choosing the Best of Two Assets with an Exchange Option

Inputs		Option Price	
Underlying (Handymax Bulk Carrier)		(Black-Scholes)	
Price	15	Call	Put
Volatility	52%	0	20.09
Dividend Yield	15%		
Strike (Aframax Tanker)		(Binomial)	
Price	33	Call	Put
Volatility	57%	0	20.089
Dividend Yield	0%		
Other			
Correlation	0.867		
Time to Expiration	1		
No. Binomial Steps	50		

counted Cash Flow techniques cannot. Real Option techniques incorporate the additional flexibility of revising and altering operating scale and strategies, and are able to attach a value to it. Thus, we can derive an expanded decision criterion that incorporates not only the static NPV but also the flexibility component measured by real options.

## 7 Conclusion

Since carriage of goods by sea is a derived demand, it is heavily dependent on the state of the world economy. In addition to that it is also prone to supply demand fluctuations within the industry as well as world politics. All these make the shipping industry highly volatile. As a result, ship managers have to be active in their decision making process in order to be able to adapt to the challenges that arise constantly.

Traditional Capital Budgeting Techniques are not suitable for valuing investments into an uncertain market. The reason is that they are not treating the risks involved as a source of value creation that might arise from managerial flexibility inherent in the project. This paper introduced Real Option Analysis and exotic options in particular as an alternative technique to cope with the value of flexibility incorporated in the process to capture the true value of a series of shipping projects. This way ship owners and managers can facilitate and optimise their financial decision making process.

The paper considered the following strategic options:

- The timing and deferment option
- The option to choose the best operating strategy and
- The option to vary the mix of output or the firm's production methods

Some adjustments, suggested by Rubinstein (1991), were made to the McDonald and Siegel (1986) model, in order to value the option to wait as an American exchange option with an uncertain underlying project value as well as uncertainty about the future strike price. By evaluating investment opportunities using the American Exchange Option methodology, substantial differences were found compared to the traditional NPV method in both the value of the investment opportunities and the timing of when the project is undertaken.

Furthermore, simple and complex chooser options were employed to evaluate the various options open to a shipowner in order to optimise his strategic decision making process.

Finally, European and American Exchange options were used to value the decision to invest in a new ship type or optimise the performance of an asset.

Overall, this paper found that Real Options are useful tools for evaluating projects in an industry as volatile as shipping, where the agents need to value complex projects and make timely strategic decisions on a regular basis.



## 8 Appendix

### Option Valuation Formulae

*Chooser Options* (adopted from Rubinstein 1991c)

Simple Chooser Option: Rubinstein uses the put-call parity relation, which holds for European options at all points during their lives, to restate the payoff of a chooser option as:

$$\text{Max}[C(S_t, X, T - t), C(S_t, X, T - t) - S_t d^{-(T-t)} + X r^{-(T-t)}, t]$$

which is equivalent to:

$$C(S, X, T - t) + \text{Max}[0, X r^{-(T-t)} - S_t d^{-(T-t)}, t]$$

with:

- $S_t$  - uncertain value after elapsed time  $t$  of the underlying asset
- $d$  - one plus the payout rate of the underlying asset
- $r$  - one plus the rate of interest

The following strategy therefore replicates the payoff of a chooser option:

- (1) buying a call with underlying asset price  $S$ , striking price  $X$  and time-to-expiration
- (2) buying a put with underlying asset price  $S_d^{-(T-t)}$ , striking price  $X r^{-(T-t)}$  and time-to-expiration  $t$

For example, in the case of Black-Scholes, the value of a standard chooser using the decomposition rule is:

$$S d^{-T} N(x) - X r^T N(x - \sigma \sqrt{T}) - S d^{-T} N(y) + X r^T N(-y + \sigma \sqrt{T})$$

with:

$$x \equiv \frac{\log \frac{S d^{-T}}{X r^T}}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T}, y \equiv \frac{\log \frac{S d^{-T}}{X r^T}}{\sigma \sqrt{T}} + \frac{1}{2} \sigma \sqrt{T}$$

and:

- $\sigma$  - Volatility of the underlying asset
- $N(a)$  - Area under the normal distribution from  $-\infty$  to  $a$ .

### *Complex Chooser Options*

A "complex" chooser option is similar to a standard chooser except that either the call/put striking prices, call/put time-to-expirations, or both are not identical. The payoff from a complex chooser can then be written as follows (Rubinstein (1991)):

$$\text{Max}[C(S_t, X_1, T_1 - t), P(S_t, X_2, T_2 - t), t]$$

implying the chosen call (put) has striking price  $X_1$  ( $X_2$ ) and time-to-expiration  $T_1 - t$  ( $T_2 - t$ ) on the choice date. As a result the valuation procedure will be more complicated and prevents the complex chooser option from being interpreted as a package of standard options. The "Black-Scholes" valuation formula for this option is:

$$\begin{aligned} & Sd^{-T_1}N_2(x, y_1, \rho_1) - X_1r^{-T_1}N_2(x - \sigma\sqrt{T_1}, y_1 - \sigma\sqrt{T_1}, \rho_1) \\ & - Sd^{-T_2}N_2(-x, -y_2, \rho_2) + X_2r^{-T_2}N_2(-x + \sigma\sqrt{t}, -y_2 + \sigma\sqrt{T_2}, \rho_2) \end{aligned} \quad (10)$$

with:

$$x \equiv \frac{\log \frac{Sd^{-t}}{Xr^t}}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}, y_1 \equiv \frac{\log \frac{Sd^{-T_1}}{Xr^{T_1}}}{\sigma\sqrt{T_1}} + \frac{1}{2}\sigma\sqrt{T}, y_2 \equiv \frac{\log \frac{Sd^{-T_2}}{Xr^{T_2}}}{\sigma\sqrt{T_2}} + \frac{1}{2}\sigma\sqrt{T_2}$$

and:

$$\rho_1 \equiv \sqrt{\frac{t}{T_1}}, \rho_2 \equiv \sqrt{\frac{t}{T_2}}$$

- $\rho$  - Correlation of the two random variables
- $N_2(a, b; \rho)$  - Area under the standard bivariate normal distribution covering the portion from  $-\infty$  to  $a$  and  $b$  to  $+\infty$ .

## References

- Amram M. and Kulatilaka, N. (1999) *Real Options: Managing Strategic Investment in an Uncertain World*, Harvard Business School Press, Boston, MA.
- Berk, J. (1999): "A Simple Approach for Deciding When to Invest," *The American Economic Review*, 89, Issue 5, pp. 1319-1326.
- Black, F. and Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities" *Journal of Political Economy*, 81, 637-659
- Braemar Seascope (2002). Various Reports.
- Buchen (2003), "The pricing of dual-expiry exotics", Working Paper, University of Sydney.
- Brennan, M. and Schwartz, E.S. (1985), "Evaluating Natural Resource Investments", *Journal of Business*, 58, 135-157
- Clarksons (2002). Various Reports
- Cox, J. C., Ross, S.A., and Rubinstein, M. (1979). "Option Pricing: A Simplified Approach". *Journal of Financial Economics*, 7(3), 229-263
- Davis, G. (1998). "Estimating Volatility and Dividend Yield when Valuing Real Options to Invest or Abandon," Working Paper, Division of Economics and Business, Colorado School of Mines.
- Dixit, A., and R. S. Pindyck (1994): *Investment under Uncertainty*, Princeton University Press.
- Geske, R., (1979). "The valuation of Compound Options". *Journal of Financial Economics*, 7(1), 63-81
- Goncalves de Oliveira, F. (1993). *Freight Futures and Chartering: A Contingent Claims Analysis Approach to the Optimal Operational and Investment Decisions in Bulk Shipping*. Chapter 10 in Gwilliam, K. (1993). *Current Issues in Maritime Economics*. Norwell and Kluwer Academics, Massachusetts and Dordrecht.
- Ingersoll, J. and S. Ross (1992): "Waiting to Invest: Investment and Uncertainty," *The Journal of Business*, 56(1), 1-29.
- Luehrman, T. A (1998): *Investment opportunities as real options: getting started on the numbers*. *Harvard Business Review*, July-August, 51-67.

- Majd, S and Pindyck R.S. (1987). "Time to Build, Option Value and Investment Decisions," *Journal of Financial Economics*, Vol. 18, No. 1, 7-27.
- Margrabe, W., (1978), "The Value of an Option to Exchange One Asset for Another." *Journal of Finance*, Vol. 33, No. 1, 177-186.
- McDonald, R. and D. Siegel (1985 ): "Investment and Valuation of Firms when there is an option to shut down", *International Economic Review*, Vol. 26, No. 2, 331-349.
- McDonald, R. and D. Siegel (1986): "The Value of Waiting to Invest", *The Quarterly Journal of Economics*, Vol. 101, No. 4, 707-728.
- McDonald, R (2003). *Derivatives Markets*. Addison - Wesley: Boston
- Mun, J. (2002). *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*. Wiley Finance. John Wiley & Sons, Hoboken, New Jersey
- Paxson, D. (1999). "R&D Strategy Using Real Option Values," Finance Research Seminar, Universidade do Minho, Braga, Portugal.
- Rubinstein, M. (1991). "One for Another", *Risk*, Vol. 4, No. 10, 30-32
- Rubinstein, M. (1991b). "Double Trouble", *Risk*, Vol. 5, No. 1, 73.
- Siodal, S. 2001. Entry and Exit Decisions Based on a Discount Factor Approach. Working Paper 2, Agder University College, Department of Economics and Business Administration.
- Siodal, S. 2003. Fuel Cells in Shipping: Higher Capital Costs and Reduced Flexibility. Real Options Conference, Washington DC, USA July 10-12
- Svendsen, A.S., 1958. *Sea Transport and Shipping Economics*. Bremen: Weltwirtschaftliches Archiv. Republication for the Institute for Shipping and Logistics, Bremen
- Triantis, A. and Borison, A., (2001); "Real Options: State of the Practice". *Journal of Applied Corporate Finance*, Vol. 14, No. 2, 8-24
- Trigeorgis, L. (1991): "A Log-Transformed Binomial Numerical Analysis Method for Valuing Complex Multi-Option Problems," *Journal of Financial and Quantitative Analysis*, Vol. 26, No. 3, pp. 309-326.
- Trigeorgis, L. (1993): "The Nature of Option Interaction and the Valuation of Investments with Multiple Real Options," *Journal of Financial and Quantitative Analysis*, Vol. 28, No. 1, pp. 1-20.

Trigeorgis, L (1996): Real Options: Managerial Flexibility and Strategy in Resource Allocation, MIT Press

Trigeorgis, L. and Mason S.P, (1987) "Valuing Managerial Flexibility," *Midland Corporate Finance Journal*, Vol. 5, No. 1, pp. 14-21.

Zannetos, Z. S. (1966). The Theory of oil Tankship Rates, MIT Press, Cambridge, MA.