A Real Option Valuation of Delivery Time Uncertainty

-The value of workforce effort

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Abstract

We attempt to value a workforce on generic consumer product line when there is uncertainty in the delivery time. The uncertainty is related to disruptions in the production line such as breakdowns etc. We focus on the delivery time as a performance measure of production activity related disturbances. In addition, we assume that the primary source of uncertainty originates from the disruptions (machine breakdowns etc.) that can be decomposed into two factors. The first factor being the delivery time uncertainty related directly to the impact from the breakdown and the secondary effect due to the recovery process. The second factor is examined and interpreted as the workforces' effort to recover from the disruptions (machine breakdowns etc.). Evaluating a workforce in this context may justify high labor costs associated with a skilled workforce when making outsourcing decisions.

Key Words: Real Option, Outsourcing, Production, Delivery Time, Mean Reversion
JEL:

1. Introduction

In recent years, generic consumer production lines have been relocated or have plans to be relocated to areas which are less costly. This may be due to the fact that many manufacturers operating in high wage countries cannot compete with those operating in a relatively cheaper environment overseas. In the past, various risks associated with factory relocations such as foreign exchange risk and operational hedges have been evaluated within the context real options (Triantis [2] among others). Models involving relocation usually assume the skill level of the workforce is equivalent to that of the current workforce. One factor that has not received much attention is the risk associated with changing to a new workforce as it may not be as skilled and experienced.

The value of the workforce operating the factory subject to relocation may be overlooked in the relocation decision making process due to the fact that the value of the workforce is often thought to be a minor factor in generic production facilities. However, the value of the workforce may potentially have a significant value that may override the cost related benefits from relocating; this of course assumes the operation does not remain in a steady state for prolong periods. The benefit gained by controlling operational fluctuations is the option value. Thus, the relocation decision should also take into account the value of the workforces' ability to bring the operational process back to the steady state. This implies that it may not be optimal to relocate the facility and, in turn, possibly justify the relatively higher wages paid to a high quality workforce.

The quality or versatility of the workforce will be referred to as the agility of the workforce. The value associated to agility has recently drawn much attention because of the increasing need to adapt quickly to demanding customers over a wide spectrum of goods. This trend does not only force the workforce to process a large variety of goods simultaneously[1] but also requires the quality of the process to improve rapidly (i.e. quick learning curve for new products) in order to reduce defects[2]. Understanding the value of the workforce should help in the relocation as

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1 The cross training of workforce provides the capability of overseeing the entire process that enhances the potential problems. This may also be considered important under the introduction of cell manufacturing.

2 The speed to adapt to the new manufacturing process becomes critical when the product life-cycles are short and rapidly changing because the defect rate may be reduced as much as 1/10 even in assembly lines.

well as the outsourcing decisions especially in situations where the operations may not remain in steady state for a long period.

Agile manufacturing is categorized into different areas such as system design, process planning, scheduling/control, material handling, facility design, information system, supply chain, business practice and human factors (see Sanchez, Nagi [5] for details). The key drivers of the agile manufacturing are *Organization, People and Technology*. In this paper, we focus on the *People* and try to value the workforces' ability to deal with disruptions in the manufacturing process. We assume delivery time[3] will serve as an appropriate index to measure workforce agility. In the event of a production disruption, the delivery time will fluctuate over time[4] and must be controlled back to the steady state.

In this paper, any change in delivery time (delivery time risk) due to production facility relocation is interpreted as the risk associated with changing the workforce. It should be clear that collaboration and training of the workforce are two key drivers controlling the production process and its quality. Workforce collaboration performance evaluation is examined in Oyen, Gel, Hopp [3], and cross training policy is discussed in Tekin [6]. Both cases support the importance of workforce training and quality as an important factor. (This supports the need for the valuation conducted in this paper). In the following, we will specify the underlying stochastic behavior of the production process so that the risk of switching the workforce can be quantified.

If there were very few disruptions in the production process under normal daily operations, one may conclude that the worker skill in such a generic production line may not be a major risk factor and thus worker skill does not possess significant value. On the other hand, if there exists many minor/major disruptions in the production process, that may or may not be clearly visible, the risk of carrying a workforce that is not capable of coping with such disruptions can potentially

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3 An alternative measure would to use the number of products produced. However, the number of products produced is a non-decreasing process that is hard to illustrate the dynamics of the process and to characterize the steady state. The delivery time process will be constant in the steady state process that is more intuitively appealing. Even though the total number of products produced may be the daily objective, the overtime work may be important if the work is outsourced.

4 The fluctuation may last for a few weeks if the disruption is caused by a product change. It may last for a few minutes in the case of minor machine troubles.

become a major risk factor depending on the operation characteristics. Because it is often hard to visualize the direct impact on the production process that a workforce has, we attempt to measure the variability indirectly by examining the entry and exit processes of the generic products produced.

Let us consider a relatively large-scale manufacturing environment that produces generic consumer goods. Due to the nature of the goods produced, the production facility primarily operates under very little variability in its daily production, i.e. there are no breakdowns etc. (i.e. steady state of production). However, once a machine breaks down (the production process halts for the duration to repair), or a major disruption[5] in the factory occurs that temporally stops or slows down production, there is often a resource reallocation in the workforce etc. to bring the post breakdown production process back to the steady state of production. This temporary resource reallocation often effects the recovery process of the post breakdown disruption in the production line. The recovery process may be longer or shorter depending on the resource allocation decisions made over time that and this in turn affects the production flow. One way to deal with this problem is to model the breakdowns explicitly and figure out what the optimal policy would be to operate under this situation. (Kamrad, Lele [1] assume that these failures occur according to a Poisson Process when integrating market risk and operational risk.) Another way is to model the production performance in an aggregate manner as done in this paper.

In order to define whether the production process is “back to normal”, let us define the steady state behavior of the production process. One way to measure this steady state behavior is to use the time it takes to deliver a particular product to the end of the production line as a finished good, and check whether it is a constant or a stationary process. The reason time is chosen as an index instead of the number of products is because the number of products that need to be produced is often preset on a daily basis and the number of work hours is adjusted to produce the preset number. This makes it easier to interpret the option on the delivery time. The option

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[5] The disruption is not necessarily a breakdown etc. that stops the production process and waits for recovery. The introduction of a new product initially increases defects tat need to be decreased over several weeks or so. The workforce needs to adjust to the new production environment quickly to decrease the defect rate of the products. The decreased defect rate would interpret as an increase in the output rate of products. This type of adaptability becomes critical when product lifecycles are very short.

is defined as the difference between the entry process and exit process of a particular "black box" production line. An advantage to the "black box" approach is that we do not necessarily need to explicitly model the behavior inside the "black box", and only need to observe the aggregate behavior of the production system that simplifies the evaluation process. Modeling the production process explicitly can potentially be very complex because the recovery process will be dependent on the dynamic decisions made by the workers responsible to bring the production process back to steady state. The drawback of this approach is that we loose the information on what we need to explicitly do to bring the process back to normal.

One way to illustrate this process is to consider a production line where we are interested in tracking how the production flow recovers after a disruption in the production line[6]. One may is to explicitly model the behavior of each item going down the line at each time point or one may choose to only observe time points of the departures/arrivals of products at a particular workstation. When there are many disruptions going on, it may become difficult to track the entire process of the products. In such a case, the choice to observe the aggregate behavior of the product flow may be a more appropriate method to value the production flow instead of tracking all of the products directly.

Although the usage of our model is not restricted to the post breakdown effect, let us focus on the case of breakdowns to simplify the recovery process characteristics. In order to evaluate the post breakdown effect through the delivery time of products, we introduce an option on the delivery time that is assumed to follow a mean reverting process. The mean reverting process introduced in our model will have two parameters that characterize the delivery time process. This first factor represents the dampening of the delivery time fluctuations, and the second factor corresponds to the tendency to converge to the long run mean delivery time. We assume the first factor is "given" or exogenously determined by the particular type of disruption occurring, where the second factor is interpreted as the work forces' effort to bring the delivery time back to a its steady state (long run) level. This provides us with one quantitative measure that enables us to measure the workforces' responsiveness in an aggregate manner. (Note that this is not the sole measure but only visualizes one facet of the entire workforce value.) Particularly, we

[6] This halt may be caused by anything from unanticipated machine breakdowns, scheduled product change/facility relocations to a major power outages etc.

interpret the mean reversion parameter as the effort level of the workers that try to bring the production process back to the steady state on an aggregate basis because this type of “skill” may be most critical when the production process frequently faces minor/major breakdowns[7].

Once a disruption occurs the delivery time will jump up to a higher level and gradually decrease over time[8] with some white noise fluctuations. The magnitude of this jump is determined on the length of delay caused by the breakdown etc. The white noise fluctuations correspond to the stochastic factor and the rate the delivery time decreases is governed by the mean reversion parameter. Thus, the value of the workforce will depend on the relative level of mean reversion for the delivery time that may change if the workforce differs. If the mean reversion is weak, the delivery time return to the steady state quickly and will result in a lot of overtime work. On the other hand, if the mean reversion is strong, the delivery time would be in steady state very quickly and would most likely not require overtime[9].

It should be noted that the detailed procedure on explicitly how to bring the process back to steady state would not be of our interest because we are trying to evaluate the aggregate effort of the workforce based on delivery time. An option set on the delivery time provides us with an aggregate measure where we can observe the overall effort level of the workforce based on “work time”. If necessary, we can work down the way to determine the specific procedures that need to be done in order to accomplish the recovery task. The aggregate approach is advantageous because analyzing the low level efforts may not be worth the time and effort when making decisions at the aggregate level. Thus, the mean reversion parameter represents the skill/motivation level of the workforce, i.e. the larger the mean reversion parameter, the better workforce.

The valuation of the workforce is performed by evaluating the price of a call option that corresponds to the value where the delivery time does not meet some threshold

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[7]Breakdowns may be minor such as machine breakdown, unscheduled maintenance that may stop or slow down the production process, or a more major disruption such as a catastrophic event that may halt the entire factory. In each case, this study focuses on the workforces’ role when recovering from such events.

[8] We assume that the workforce has a fundamental level of agility (capability) to bring the disrupted process back to the steady state that does not change over the time

[9] There is normally a cushion set up by the management that allows some delay in the production at a reasonable level.

level higher than the long run delivery time average (This makes the option out of the money.). For this call option, the underlying would be the delivery time that fluctuates over time, the strike of the option would be set equal to the delivery time threshold set to meet the final demand[10], the maturity of the option would be set to some time where the physical delivery takes place[11]. It should be clear that if the delivery time goes over the threshold, the exercise of the call option would insure whatever overtime required that accomplishes the production goal of the day.

When the value of the workforce is valued as a call option on the delivery time, the workforce value may become an absolute measure on how fast the workforce can adapt and correct the production process to disruptions. This provides us a way to measure the agility of the workforce. Recall that if the mean reversion is high, the workforce quality is high and if the mean reversion is low the workforce quality is low. When relocating the factory, we can simply compare the two workforce values by comparing the two call option prices. It is best to interpret the base case performance of the factory when there is no option value (workforce value) attached to the production. In this case, the performance may be potentially overvalued if the workforce does not adapt according to the long run mean performance. It should be noted that the risk associated with the delivery time is cancelled out by buying a call option on the delivery time. This means that subtracting the option value from the base case will provide the factory value by considering the value of the workforce. Because the absolute value of the option decreases as the mean reversion parameter increases, the workforce option with a smaller absolute value will provide a higher value (The amount subtracted from the base case would be smaller.).

Now, let us consider how the call option serves as a hedge to illustrate the relationship between the option value and the workforce value. The purchase of a call option will cancel out the delay amount in delivery time created by a disruption and as a consequence ‘hedges’ the factory against the risk of running overtime in the delivery process. If the factory possesses an agile workforce that can adapt quickly enough to avoid overtime, maintaining this agile workforce that has a higher wage

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10 For example, one may easily obtain such a threshold by using the average demand level and average inventory level. The threshold may not insure all deliveries, but will be set to meet them at a “reasonable” level.

11 This type of delivery may take place sequentially but will increase the number of options, so for illustrative purpose, we will focus on the single option case in the numerical calculation section.

may be justified if the higher payment can be recouped gradually by evading overtime work in the future. The higher pay to the workers may be interpreted as the option premium to maintain the workforce that can prevent the production process to run overtime. This type of option valuation serves as a measure for the factory to figure out the workforce agility value. Although we have given an example of in-house value of workforce when for the case of working overtime, the model is also useful in making outsourcing decisions and measuring the workforce value when product life cycles are short.

In the case of outsourcing decisions, considering an alternative hedge (an option issued by a third party) provides us with a framework to price the workforce value. It should be quite clear that it would not be optimal to outsource the process that has large variability when the factory possesses a high quality workforce. On the other hand, the factory should outsource and eliminate the variability of the overtime cost if the workforce quality is low. However, outsourcing and maintaining a high quality workforce does not come for free. By considering an alternative hedge that would carry the underlying asset (time) in addition to underwriting a put option with the same strike and maturity of the call option to a third party, allows us to create a synthetic call option on the delivery time[12]. In this case, one would be carrying overtime[13] credit to compensate if the delivery time runs over time because of a disruption in the production process. If a disruption actually causes the delivery time to be delayed, the overtime credit is consumed (the payoff equal to the call option above exercise price). If the delivery time is not delayed, the put option is exercised that cancels out any residual delivery time credit.

Because the option will need to be written between a third party (most likely interacting with the workers and factory owners) in this case, the prices of the options will be negotiated upon so that the put option and call option prices may fluctuate even when there is only one workforce involved in the evaluation process. The put option prices obtained through this interactive process represents more on how the workers feel their agility should be priced. On the other hand, the call option price would serve more as what the factory considers the workforce value

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[12] It should be clear that such a strategy should be equivalent to holding a call option, in theory, from the put-call-parity.

[13] The overtime credit may be carried in a form of contract or in physical inventory (overtime credit equivalent) to be delivered, that compensates for the overtime.

should be priced. This makes the pricing process interactive and thus represents a relative measurement tool to value workforce agility as opposed to the absolute measurement of using the call option alone.

2. Model

Before we introduce the option valuation framework, we would like to briefly note on how the costs are often calculated in a generic consumer good production environment[14]. It should be noted that volatility is not explicitly included in the pricing.

We examine the case for a typical generic consumer good production line. Because of the relatively large amount of production involved, the pricing of the final product is often based on a time average criteria such as the following.

Costs

A_i	per unit time per unit product cost of machine i. ($i = 1, \ldots, N$)
B	per unit product cost of raw material/intermediate products.
T_i	The total processing time of machine i.($i = 1, \ldots, N$)

Then the total cost is defined as

$$(\sum_{i=1}^{N} A_i * T_i + B).$$

The price of the product Z is set as by a multiple θ for the cost, i.e.

$$Z = \theta(\sum_{i=1}^{N} A_i * T_i + B).$$

where θ is a factor larger than one that accounts for the variability of the process, direct cost of breakdown, profit margin etc. Thus we may assume in this model that the only source of variability resides in the delivery time T_i.

Because the primary interest of this study is to explore the impact of the post breakdown effect, we may consider a special case where the breakdown time/cost is 0, and the final price already accounts for the non-conformable items in the long run and the profit is set to be zero for simplicity. In addition, we will consider the case where $T \equiv T_i$. Then we may define the additional factor that accounts for the variability of the delivery process

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14 The pricing process was formulated through interviews with Matsushita Electronics.

by the following mean reverting process.

$$\frac{dT}{T} = \mu_t dt \qquad (1)$$

$$d\mu_t = \kappa(\overline{\mu} - \mu_t)dt + \eta_t dW_2(t) \qquad (2)$$

where

T is the delivery time,

$\overline{\mu}$ is the long run average of the delivery time,

η_t is the volatility of the drift of the delivery time,

μ_t is the drift of the delivery time,

κ is the mean reversion parameter, and

$dW_2(t)$ is the increment of the Weiner Process.

In addition, we assume that the volatility of the delivery time η_t diminishes exponentially as time t passes according to a known parameter κ_2, i.e. follows the following differential equation.

$$d\eta_t = -\kappa_2 \eta_t dt \qquad (3)$$

In this formulation, the mean reversion parameter κ corresponds to "the impact of the workers" on the delivery time T. It should be noted that the delivery time T is not the clock time t and it is a stochastic process.

Note

Because we have been considering a manufacturing environment where the steady state has little variation, we obtained the process (1)-(3). However, this may be extended to the case where there is significant a variation even in the steady state of production. Please see the Appendix for details.

These equations (1), (2), (3) can be discretized and numerically evaluated (as preformed in [2]) to obtain the distribution of the delivery time process T. Because the delivery time T is targeted to be smaller than some threshold date T^* (where $T^* \geq \overline{\mu}$) in order to meet the demand of the products at a pre-specified level, a put option on this process with a strike T^* would be the quality of the "workers' effort" in the case of a machine breakdown. Unlike the exogenous factors like demand or supply, this factor is unique to the production facility and may be considered as the value of the workforce in a breakdown situation whose value will obviously zero when working in the steady state of production.

3. Numerical Results

In order to facilitate the calculations, we first derive the discrete time version of equations (1)-(3).

$$\frac{\Delta T_{i+1}}{T_i} = \mu_i \Delta t \qquad (4)$$

$$\Delta \mu_i = \kappa(\overline{\mu} - \mu_i)\Delta t + \eta_i \Delta Z_2(i) \qquad (5)$$

where

T_i is the delivery time at time i,

$\overline{\mu}$ is the long run average of the delivery time,

η_i is the volatility of the drift of the delivery time at time i,

μ_i is the drift of the delivery time at time i,

κ is the mean reversion parameter, and

$\Delta Z_2(i)$ is standard normal distribution at time i.

Δt is the time increment for each time step.

In addition, we assume that the volatility of the delivery time η_t diminishes exponentially as time i passes according to a known parameter κ_2, i.e. is governed by the following differential equation.

$$\Delta \eta_i = -\kappa_2 \eta_t \Delta t \qquad (6)$$

We run a simulation of 50 time steps to generate the distribution of the delivery times by changing the mean reversion parameter κ[15] from 0 to 1.5 (0.1 increment) and the volatility decay factor κ_2 from 0 to 1 (0.1 increment)[16]. Each distribution was

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15 The problem may be in estimation of the mean reversion parameter. One way to obtain it is through an interview with factory managers. If they can provide the speed they can recover from disruptions on average (historical basis), this may be one way to obtain the mean reversion parameter.

16 Small volatility reduction factors correspond to product change adaptation and large mean reversions will correspond to short disruptions.

generated with a sample of 10,000. The magnitude of the initial disruption is also altered by 10% to 100% (10% increment) of the steady state delivery time in the numerical study[17]. After the distribution is generated, we calculate the call option value of the distribution with a strike price that is 10% of the initial delivery time and risk free rate of 10%. For simplicity, we assume that the drift of the delivery time reverts to a long run average 0.

Table 1-1 to 1-10 and Figures 1-5 illustrate the changes of the option prices when the parameters are changed. Table 2-1 to 2-5 show the option prices change as time changes.

It should be noted that the steady state production does not necessarily have to converge to a 0 variability case, but may follow a geometric Brownian motion characterized by another stochastic factor as shown in the Appendix. Similar to the one factor case, this two factor mean reverting process may be easily generated and the call option prices can be easily calculated.

A general observation of Figure 1 and Figure 2 shows that the option values decrease (workforce value increases) as the mean reversion parameter increases. Because the increase of the mean reversion parameter interprets as the work force value increase, this coincides with our conjecture that the work force value increases as the mean reversion factor increases[18].

Figure 3 illustrates the change in option value against the mean reversion parameter for the call option case. A clear trend that all of the call option values flatten out after a certain amount of time can be observed, this is because the mean reversion effect stabilizes the delivery time process over time. More specifically, this output may be interpreted that the workforce "gets the process in control" by the time the option value flattens out. Another observation consistent with Figure 1 and Figure 2 is that the option value itself decreases as the mean reversion parameter increases.

4. Implications

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[17] Short disruption may occur around 10 % process in Japanese factories, which may take longer in other regions.

[18] The workforce agility value is the negative of the option value because the criteria is NOT to deliver over the delivery time threshold.

The evaluation of the delivery time under a stressed workforce may interpret as the value of the workforce to cope with breakdowns. This will differ from the regular Black-Scholes type uncertainty in the delivery time because there is a constant “force” that pushes the workforce to produce at the “steady state” mode. (We model this force as the mean reversion parameter.) This type of valuation may be of particular interest in the event of outsourcing a particular production procedure because it becomes critical to take into consideration the value of the workforce quality in addition to the other uncertainties attached to the production procedure. Overlooking this value in the workforce quality of a production process may lead to an overvaluation of the outsourcing decision if the option value of the workforce is not taken into account for the outsourcing party.

The need to take into account the workforce value is critical when the facility is relocated. Thus, the high wage expenses at facilities with high skills may be justified under customer demand with high variety and short product cycles.

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Appendix[19]

If we assume that the underlying production process itself has variability, the delivery time process may be defined as

$$\frac{dT}{T} = \mu_t dt + \sigma_t dW_1(t) \qquad (*)$$

where the drift process follows,

$$d\mu_t = \kappa(\overline{\mu} - \mu_t)dt + \eta_t dW_2(t) \qquad (**)$$

and the volatility terms follow

$$d\sigma_t = \kappa_1(\overline{\sigma} - \sigma_t)dt$$
$$d\eta_t = -\kappa_2 \eta_t dt$$
$$dW_1(t)dW_2(t) = \rho_t dt$$

It should be clear that by setting $\sigma_t \equiv 0$ one obtains (1), (2) and (3).

The following is the preliminary numerical results for the two factor model.

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[19] We follow the notation similar to Schwartz, Moon [2], thus the definition of the variables are omitted.

Figure 1 The Call/Put Options prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 01. to 1.5 at time step 40.

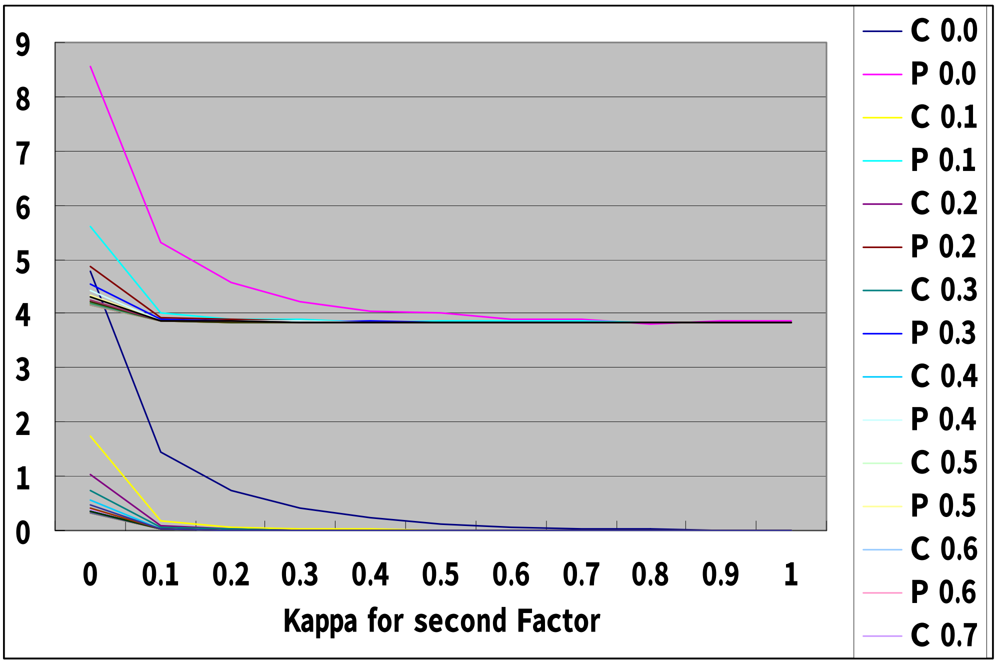

Figure 2 The Call/Put Options prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 01. to 1.5 at time step 50.

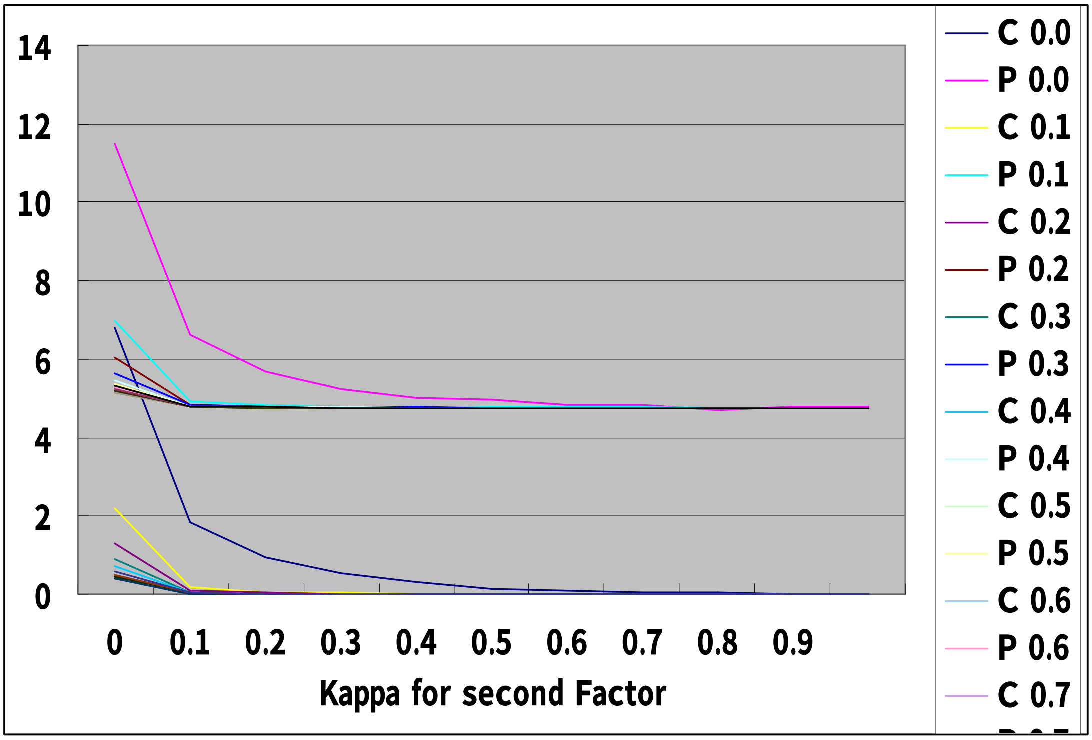

Figure 3 The Call Options prices v.s. Time steps (Kappa2=0.1) for Mean reversion parameter from 0.1 to 1.5 at time step 50.

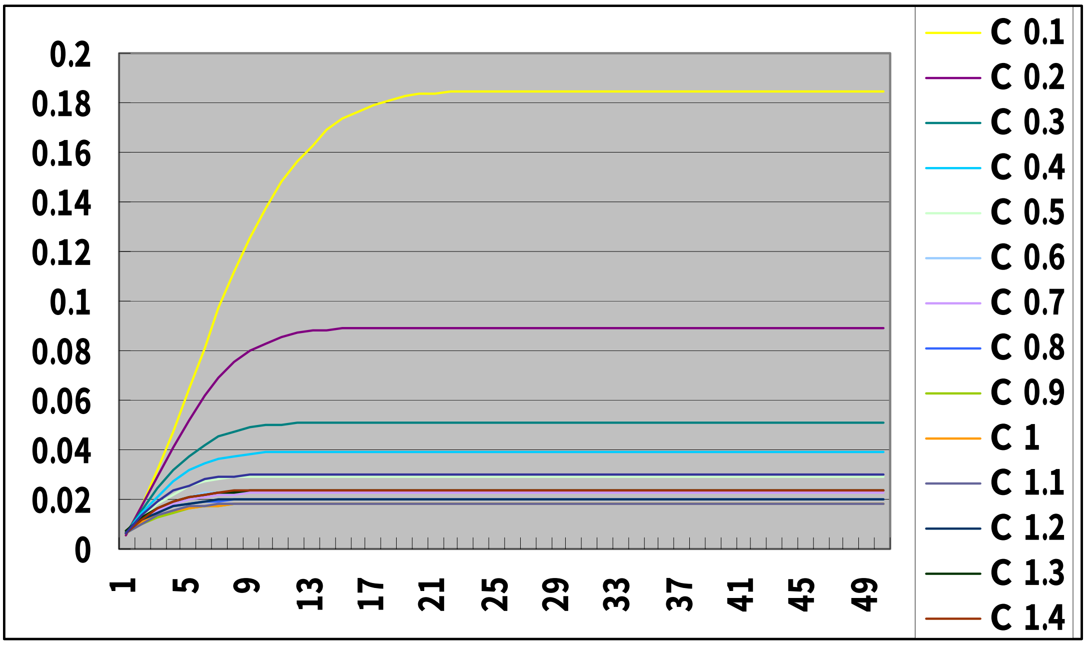

Figure 4 Volatility Surface v.s. time v.s. Mean reversion parameter with Volatility Reduction 0.1.

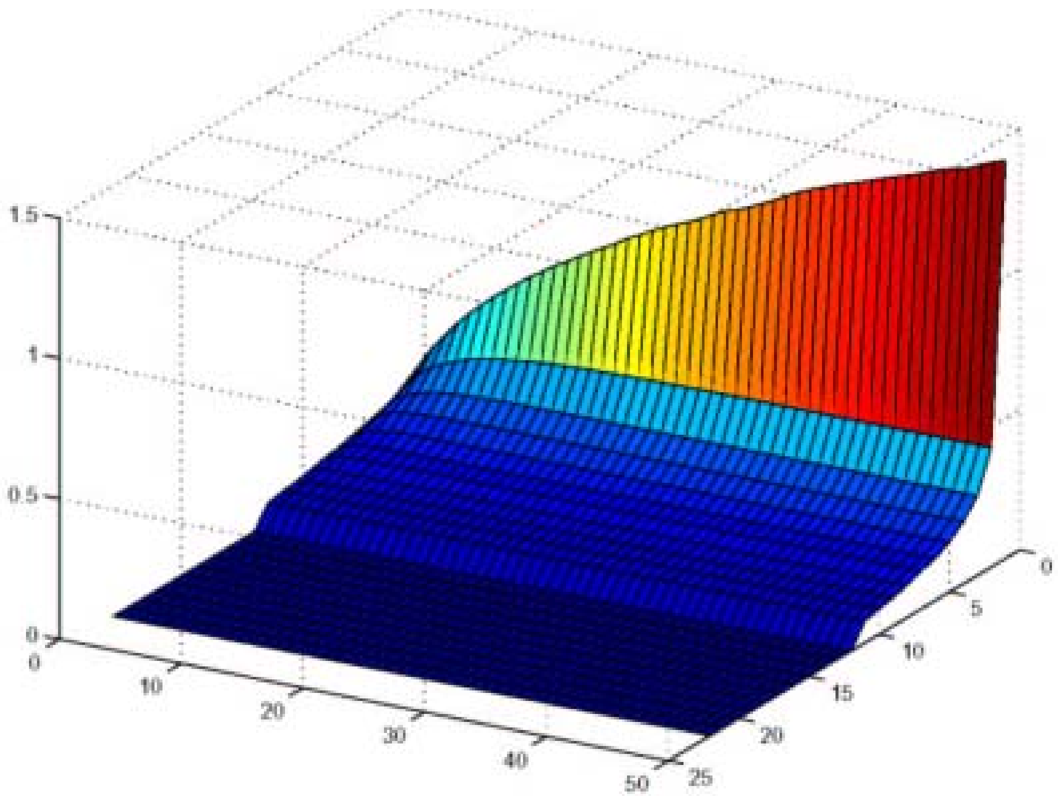

Figure 5 Volatility Surface v.s. time v.s. Volatility reduction parameter with mean reversion parameter 0.1.

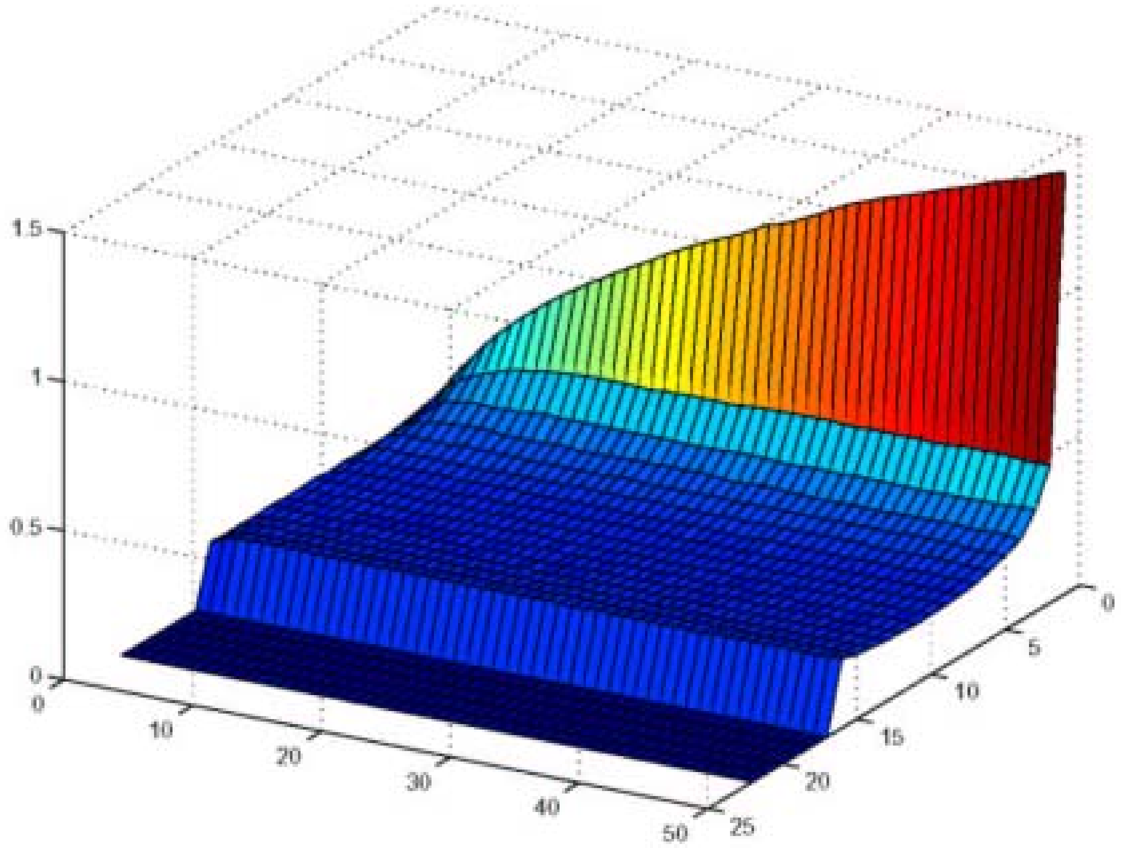

Table 1-1 The Call Option prices v.s. Time steps Kappa2=0.1 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.006	0.006	0.006	0.006	0.006	0.007	0.006	0.006	0.007	0.006	0.006	0.006	0.007	0.007	0.007	0.006
2	0.021	0.017	0.017	0.016	0.014	0.013	0.012	0.011	0.012	0.010	0.011	0.010	0.012	0.013	0.012	0.013
3	0.041	0.032	0.029	0.025	0.021	0.018	0.016	0.016	0.015	0.013	0.014	0.014	0.015	0.017	0.016	0.019
4	0.066	0.047	0.041	0.032	0.028	0.022	0.019	0.018	0.017	0.015	0.015	0.016	0.017	0.019	0.019	0.023
5	0.095	0.064	0.052	0.037	0.032	0.025	0.021	0.020	0.018	0.016	0.017	0.017	0.019	0.021	0.021	0.026
6	0.126	0.081	0.062	0.042	0.035	0.027	0.022	0.021	0.019	0.017	0.017	0.018	0.019	0.022	0.022	0.028
7	0.158	0.097	0.069	0.045	0.037	0.028	0.023	0.022	0.020	0.018	0.018	0.018	0.020	0.023	0.023	0.029
8	0.191	0.112	0.075	0.048	0.038	0.029	0.024	0.022	0.020	0.018	0.018	0.018	0.020	0.023	0.023	0.030
9	0.226	0.126	0.080	0.049	0.039	0.029	0.024	0.022	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
10	0.263	0.137	0.083	0.050	0.039	0.029	0.024	0.022	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
11	0.300	0.148	0.085	0.050	0.039	0.029	0.024	0.022	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
12	0.339	0.156	0.087	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
13	0.378	0.163	0.088	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
14	0.415	0.169	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
15	0.453	0.173	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
16	0.494	0.177	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
17	0.531	0.179	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
18	0.571	0.181	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
19	0.613	0.183	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
20	0.653	0.183	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
21	0.692	0.184	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
22	0.732	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
23	0.774	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
24	0.816	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
25	0.855	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
26	0.896	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
27	0.935	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
28	0.976	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
29	1.016	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
30	1.055	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
31	1.093	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
32	1.132	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
33	1.172	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
34	1.211	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
35	1.249	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
36	1.287	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
37	1.325	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
38	1.364	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
39	1.404	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
40	1.442	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
41	1.480	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
42	1.519	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
43	1.559	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
44	1.598	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
45	1.636	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
46	1.675	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
47	1.712	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
48	1.750	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
49	1.785	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
50	1.826	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030

Table 1-2 The Call Option prices v.s. Time steps Kappa2=0.2 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.004	0.004	0.004
2	0.014	0.012	0.011	0.010	0.009	0.008	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.006	0.008	0.008
3	0.027	0.020	0.018	0.014	0.012	0.011	0.008	0.008	0.008	0.007	0.007	0.008	0.008	0.007	0.009	0.010
4	0.043	0.029	0.023	0.016	0.014	0.012	0.009	0.008	0.008	0.007	0.007	0.008	0.008	0.008	0.010	0.011
5	0.060	0.037	0.027	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
6	0.078	0.044	0.029	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
7	0.098	0.049	0.030	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
8	0.117	0.053	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
9	0.137	0.056	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
10	0.157	0.058	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
11	0.177	0.060	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
12	0.197	0.061	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
13	0.216	0.061	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
14	0.236	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
15	0.256	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
16	0.275	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
17	0.295	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
18	0.314	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
19	0.334	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
20	0.353	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
21	0.373	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
22	0.393	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
23	0.412	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
24	0.431	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
25	0.451	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
26	0.470	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
27	0.489	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
28	0.508	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
29	0.527	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
30	0.546	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
31	0.566	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
32	0.586	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
33	0.606	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
34	0.625	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
35	0.643	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
36	0.663	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
37	0.681	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
38	0.700	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
39	0.719	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
40	0.737	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
41	0.757	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
42	0.776	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
43	0.794	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
44	0.813	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
45	0.832	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
46	0.852	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
47	0.871	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
48	0.889	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
49	0.908	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
50	0.927	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011

Table 1-3 The Call Option prices v.s. Time steps Kappa2=0.3 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.004	0.003
2	0.010	0.008	0.007	0.006	0.006	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.005	0.004
3	0.018	0.014	0.010	0.007	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
4	0.028	0.019	0.012	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
5	0.039	0.023	0.013	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
6	0.050	0.025	0.013	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
7	0.061	0.027	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
8	0.072	0.029	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
9	0.082	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
10	0.094	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
11	0.104	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
12	0.116	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
13	0.126	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
14	0.137	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
15	0.147	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
16	0.158	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
17	0.169	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
18	0.180	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
19	0.191	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
20	0.201	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
21	0.212	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
22	0.223	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
23	0.234	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
24	0.245	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
25	0.256	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
26	0.267	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
27	0.278	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
28	0.289	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
29	0.300	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
30	0.309	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
31	0.321	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
32	0.332	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
33	0.342	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
34	0.352	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
35	0.363	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
36	0.373	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
37	0.384	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
38	0.394	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
39	0.404	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
40	0.415	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
41	0.426	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
42	0.436	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
43	0.447	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
44	0.457	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
45	0.467	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
46	0.476	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
47	0.487	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
48	0.497	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
49	0.507	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
50	0.517	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005

Table 1-4 The Call Option prices v.s. Time steps Kappa2=0.4 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2	0.006	0.005	0.005	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
3	0.011	0.009	0.006	0.004	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
4	0.017	0.011	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
5	0.023	0.013	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
6	0.030	0.014	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
7	0.036	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
8	0.043	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
9	0.049	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
10	0.056	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
11	0.062	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
12	0.068	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
13	0.074	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
14	0.079	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
15	0.085	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
16	0.091	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
17	0.098	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
18	0.104	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
19	0.110	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
20	0.117	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
21	0.123	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
22	0.130	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
23	0.136	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
24	0.142	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
25	0.148	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
26	0.154	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
27	0.160	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
28	0.166	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
29	0.172	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
30	0.178	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
31	0.184	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
32	0.190	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
33	0.196	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
34	0.202	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
35	0.208	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
36	0.214	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
37	0.220	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
38	0.226	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
39	0.232	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
40	0.238	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
41	0.243	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
42	0.249	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
43	0.254	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
44	0.260	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
45	0.266	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
46	0.271	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
47	0.277	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
48	0.283	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
49	0.289	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
50	0.294	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003

Table 1-5 The Call Option prices v.s. Time steps Kappa2=0.5 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
2	0.004	0.003	0.003	0.002	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3	0.007	0.004	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4	0.010	0.005	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
5	0.013	0.006	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
6	0.016	0.006	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
7	0.019	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8	0.022	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
9	0.025	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
10	0.028	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
11	0.031	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
12	0.034	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
13	0.038	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
14	0.040	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
15	0.043	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
16	0.047	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
17	0.049	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
18	0.052	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
19	0.055	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
20	0.059	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
21	0.062	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
22	0.065	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
23	0.068	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
24	0.071	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
25	0.074	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
26	0.077	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
27	0.080	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
28	0.083	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
29	0.086	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
30	0.089	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
31	0.092	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
32	0.095	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
33	0.098	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
34	0.101	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
35	0.104	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
36	0.107	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
37	0.110	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
38	0.113	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
39	0.116	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
40	0.119	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
41	0.122	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
42	0.125	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
43	0.128	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
44	0.130	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
45	0.133	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
46	0.137	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
47	0.140	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
48	0.142	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
49	0.145	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
50	0.148	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Table 1-6 The Call Option prices v.s. Time steps Kappa2=0.6 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
2	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
3	0.004	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
4	0.006	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
5	0.008	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
6	0.010	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
7	0.012	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
8	0.014	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
9	0.016	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
10	0.018	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
11	0.020	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
12	0.022	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
13	0.024	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
14	0.026	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
15	0.028	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
16	0.029	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
17	0.031	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
18	0.033	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
19	0.035	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
20	0.037	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
21	0.039	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
22	0.040	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
23	0.042	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
24	0.044	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
25	0.046	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
26	0.048	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
27	0.050	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
28	0.051	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
29	0.053	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
30	0.055	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
31	0.057	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
32	0.059	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
33	0.060	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
34	0.062	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
35	0.064	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
36	0.066	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
37	0.068	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
38	0.069	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
39	0.071	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
40	0.073	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
41	0.074	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
42	0.076	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
43	0.078	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
44	0.080	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
45	0.082	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
46	0.083	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
47	0.085	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
48	0.087	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
49	0.089	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
50	0.090	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Table 1-7 The Call Option prices v.s. Time steps Kappa2=0.7 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.001	0.001	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
3	0.002	0.001	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
4	0.003	0.001	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
5	0.004	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
6	0.005	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
7	0.006	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
8	0.007	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
9	0.008	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
10	0.009	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
11	0.009	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
12	0.010	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
13	0.011	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
14	0.012	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
15	0.013	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
16	0.014	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
17	0.015	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
18	0.015	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
19	0.016	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
20	0.017	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
21	0.018	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
22	0.019	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
23	0.020	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
24	0.021	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
25	0.021	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
26	0.022	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
27	0.023	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
28	0.024	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
29	0.025	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
30	0.026	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
31	0.027	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
32	0.027	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
33	0.028	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
34	0.029	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
35	0.030	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
36	0.031	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
37	0.032	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
38	0.033	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
39	0.034	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
40	0.035	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
41	0.035	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
42	0.036	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
43	0.037	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
44	0.038	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
45	0.039	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
46	0.040	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
47	0.041	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
48	0.042	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
49	0.042	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
50	0.043	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000

Table 1-8 The Call Option prices v.s. Time steps Kappa2=0.8 and mean reversion parameter from 0.1 to 1.5 for 50 time steps

Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.010	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.010	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.011	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.011	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.012	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	0.012	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27	0.013	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28	0.013	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
29	0.014	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30	0.014	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
31	0.015	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
32	0.015	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
33	0.016	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
34	0.016	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
35	0.017	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
36	0.017	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
37	0.018	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
38	0.018	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
39	0.019	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
40	0.019	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
41	0.019	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
42	0.020	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
43	0.020	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
44	0.021	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
45	0.021	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
46	0.022	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
47	0.022	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
48	0.023	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
49	0.023	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.024	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2-1 The Call Option prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 0.1 to 1.5 at time step 10.

Kappa2	Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
0	10	0.476	0.319	0.220	0.171	0.141	0.115	0.105	0.094	0.087	0.082	0.084	0.083	0.087	0.092	0.103	0.116
0.1	10	0.263	0.137	0.083	0.050	0.039	0.029	0.024	0.022	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
0.2	10	0.157	0.058	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
0.3	10	0.094	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
0.4	10	0.056	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
0.5	10	0.028	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.6	10	0.018	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.7	10	0.009	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.8	10	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	10	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	10	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2-2 The Call Option prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 0.1 to 1.5 at time step 20.

Kappa2	Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
0	20	1.536	0.792	0.489	0.361	0.278	0.238	0.208	0.185	0.171	0.167	0.165	0.164	0.172	0.179	0.204	0.236
0.1	20	0.653	0.183	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
0.2	20	0.353	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
0.3	20	0.201	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
0.4	20	0.117	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
0.5	20	0.059	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.6	20	0.037	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.7	20	0.017	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.8	20	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	20	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	20	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2-3 The Call Option prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 0.1 to 1.5 at time step 30.

Kappa2	Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
0	30	2.990	1.253	0.759	0.547	0.422	0.354	0.308	0.277	0.257	0.251	0.245	0.246	0.257	0.270	0.304	0.352
0.1	30	1.055	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
0.2	30	0.546	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
0.3	30	0.309	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
0.4	30	0.178	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
0.5	30	0.089	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.6	30	0.055	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.7	30	0.026	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.8	30	0.014	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	30	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	30	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2-4 The Call Option prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 0.1 to 1.5 at time step 40.

Kappa2	Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
0	40	4.795	1.731	1.023	0.727	0.559	0.471	0.409	0.370	0.340	0.331	0.321	0.326	0.341	0.358	0.400	0.463
0.1	40	1.442	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
0.2	40	0.737	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
0.3	40	0.415	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
0.4	40	0.238	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
0.5	40	0.119	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.6	40	0.073	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.7	40	0.035	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.8	40	0.019	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	40	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	40	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2-5 The Call Option prices v.s. Volatility Reduction Factor Kappa2 for Mean reversion parameter from 0.1 to 1.5 at time step 50.

Kappa2	Time	C 0.0	C 0.1	C 0.2	C 0.3	C 0.4	C 0.5	C 0.6	C 0.7	C 0.8	C 0.9	C 1	C 1.1	C 1.2	C 1.3	C 1.4	C 1.5
0	50	6.800	2.207	1.281	0.902	0.694	0.584	0.508	0.456	0.420	0.409	0.400	0.405	0.421	0.444	0.497	0.574
0.1	50	1.826	0.185	0.089	0.051	0.039	0.029	0.024	0.023	0.020	0.018	0.018	0.018	0.020	0.023	0.024	0.030
0.2	50	0.927	0.062	0.031	0.018	0.015	0.012	0.009	0.008	0.008	0.007	0.008	0.008	0.008	0.008	0.010	0.011
0.3	50	0.517	0.030	0.014	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.006	0.005
0.4	50	0.294	0.015	0.007	0.005	0.004	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003
0.5	50	0.148	0.007	0.003	0.003	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.6	50	0.090	0.003	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
0.7	50	0.043	0.002	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
0.8	50	0.024	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	50	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	50	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Appendix[20]

If we assume that the underlying production process itself has variability, the delivery time process may be defined as

$$\frac{dT}{T} = \mu_t dt + \sigma_t dW_1(t) \qquad (*)$$

where the drift process follows,

$$d\mu_t = \kappa(\bar{\mu} - \mu_t)dt + \eta_t dW_2(t) \qquad (**)$$

and the volatility terms follow

$$d\sigma_t = \kappa_1(\bar{\sigma} - \sigma_t)dt$$
$$d\eta_t = -\kappa_2 \eta_t dt$$
$$dW_1(t)dW_2(t) = \rho_t dt$$

It should be clear that by setting $\sigma_t \equiv 0$ one obtains (1), (2) and (3).

The preliminary numerical results for the two factor model are given in Table A-1 to A-4, and the results are summarized in Figure A-1, A-2 for the case where the correlation is set to zero. It should be noted that the underlying process does not converge to a steady state, but rather to an "in-control" state where the volatility is not zero but small enough to be managed.

[20] We follow the notation similar to Schwartz, Moon [2], thus the definition of the variables are omitted.

Figure A-1 The Call Options prices v.s. Time steps (Kappa2=0.1) for Mean reversion parameter from 0.1 to 1.5 at time step 50. (Two Factor Model)

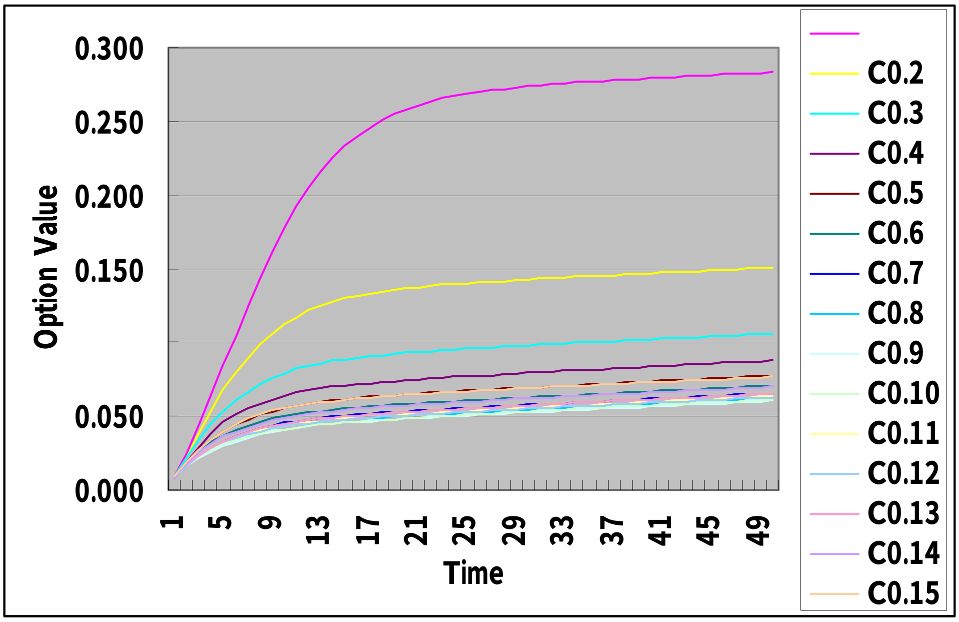

Figure A-2 The Volatility Change v.s. Time with Volatility Reduction Factor Kappa2=0.1 for Mean reversion parameter from 0.1 to 1.5.

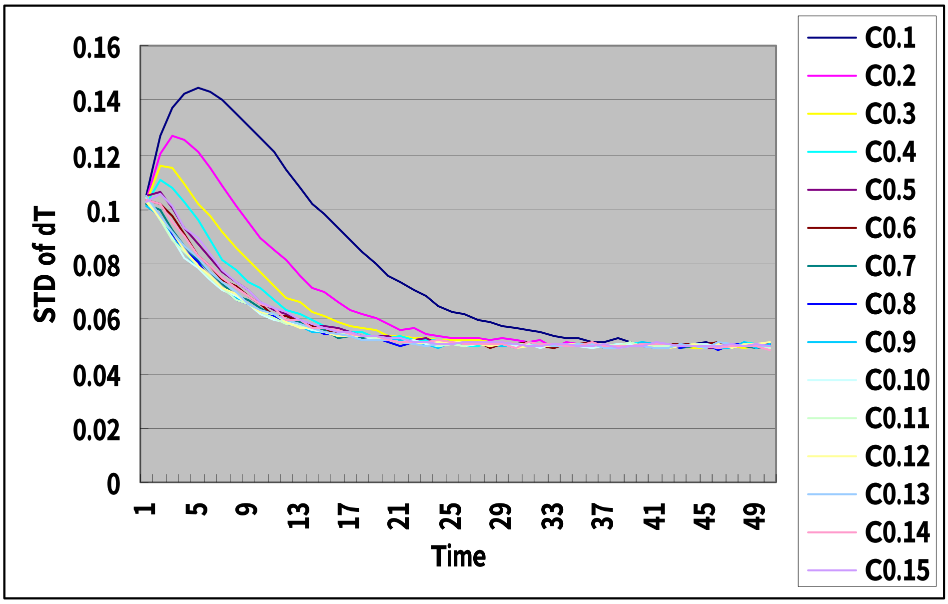

Table A-1 The Call Options prices v.s. Time steps (Kappa2=0.1) for Mean reversion parameter from 0.1 to 1.5 at time step 50. (Two Factor Model)

Time	C0.1	C0.2	C0.3	C0.4	C0.5	C0.6	C0.7	C0.8	C0.9	C0.10	C0.11	C0.12	C0.13	C0.14	C0.15
1	0.009	0.009	0.009	0.008	0.009	0.009	0.009	0.010	0.008	0.009	0.010	0.009	0.010	0.009	0.009
2	0.024	0.022	0.022	0.019	0.019	0.018	0.017	0.017	0.016	0.017	0.017	0.017	0.017	0.018	0.019
3	0.043	0.038	0.034	0.029	0.027	0.026	0.023	0.024	0.021	0.022	0.024	0.023	0.024	0.026	0.028
4	0.063	0.053	0.044	0.038	0.034	0.032	0.029	0.028	0.026	0.027	0.028	0.029	0.029	0.032	0.034
5	0.085	0.068	0.053	0.046	0.040	0.037	0.033	0.032	0.030	0.031	0.032	0.032	0.033	0.036	0.040
6	0.104	0.080	0.061	0.051	0.045	0.040	0.037	0.035	0.033	0.034	0.035	0.035	0.037	0.040	0.045
7	0.125	0.090	0.067	0.055	0.048	0.043	0.040	0.038	0.036	0.036	0.038	0.038	0.040	0.042	0.049
8	0.144	0.099	0.072	0.059	0.051	0.046	0.042	0.040	0.038	0.038	0.040	0.040	0.042	0.045	0.051
9	0.162	0.106	0.076	0.062	0.053	0.048	0.044	0.042	0.039	0.040	0.042	0.042	0.044	0.047	0.054
10	0.178	0.112	0.079	0.064	0.056	0.050	0.046	0.043	0.041	0.041	0.043	0.043	0.045	0.049	0.055
11	0.193	0.117	0.082	0.066	0.057	0.051	0.047	0.044	0.042	0.042	0.045	0.044	0.047	0.051	0.057
12	0.205	0.122	0.084	0.068	0.058	0.052	0.048	0.045	0.043	0.043	0.046	0.045	0.048	0.052	0.058
13	0.216	0.125	0.086	0.069	0.060	0.053	0.049	0.046	0.044	0.044	0.047	0.046	0.049	0.053	0.059
14	0.225	0.128	0.088	0.070	0.061	0.054	0.050	0.047	0.045	0.045	0.047	0.047	0.050	0.053	0.060
15	0.233	0.130	0.089	0.071	0.062	0.055	0.051	0.048	0.046	0.046	0.048	0.048	0.050	0.054	0.061
16	0.240	0.132	0.090	0.072	0.062	0.056	0.051	0.048	0.046	0.046	0.049	0.048	0.051	0.055	0.062
17	0.246	0.133	0.091	0.073	0.063	0.057	0.052	0.049	0.047	0.047	0.049	0.049	0.052	0.056	0.063
18	0.251	0.135	0.092	0.073	0.064	0.057	0.052	0.049	0.047	0.048	0.050	0.050	0.052	0.056	0.063
19	0.255	0.136	0.092	0.074	0.064	0.058	0.053	0.050	0.048	0.048	0.051	0.050	0.053	0.057	0.064
20	0.258	0.137	0.093	0.075	0.065	0.058	0.053	0.051	0.048	0.049	0.051	0.051	0.053	0.058	0.065
21	0.261	0.138	0.094	0.075	0.066	0.059	0.054	0.051	0.049	0.049	0.052	0.051	0.054	0.058	0.065
22	0.264	0.139	0.094	0.076	0.066	0.059	0.054	0.051	0.049	0.050	0.052	0.052	0.054	0.059	0.066
23	0.266	0.139	0.095	0.076	0.067	0.060	0.055	0.052	0.050	0.050	0.053	0.052	0.054	0.059	0.066
24	0.267	0.140	0.095	0.077	0.067	0.060	0.055	0.052	0.050	0.051	0.053	0.053	0.055	0.059	0.067
25	0.269	0.140	0.096	0.077	0.067	0.061	0.056	0.053	0.051	0.051	0.053	0.053	0.055	0.060	0.067
26	0.270	0.141	0.096	0.078	0.068	0.061	0.056	0.053	0.051	0.052	0.054	0.054	0.056	0.060	0.067
27	0.271	0.141	0.097	0.078	0.068	0.062	0.057	0.054	0.052	0.052	0.054	0.054	0.056	0.061	0.068
28	0.272	0.142	0.097	0.079	0.069	0.062	0.057	0.054	0.052	0.053	0.055	0.054	0.057	0.061	0.068
29	0.273	0.142	0.098	0.079	0.069	0.062	0.057	0.054	0.052	0.053	0.055	0.055	0.057	0.062	0.069
30	0.274	0.143	0.098	0.079	0.069	0.063	0.058	0.055	0.053	0.054	0.056	0.055	0.058	0.062	0.069
31	0.274	0.143	0.099	0.080	0.070	0.063	0.058	0.055	0.053	0.054	0.056	0.056	0.058	0.063	0.070
32	0.275	0.144	0.099	0.080	0.070	0.064	0.059	0.056	0.054	0.054	0.056	0.056	0.058	0.063	0.070
33	0.276	0.144	0.099	0.081	0.071	0.064	0.059	0.056	0.054	0.055	0.057	0.056	0.059	0.063	0.070
34	0.276	0.145	0.100	0.081	0.071	0.064	0.059	0.057	0.054	0.055	0.057	0.057	0.059	0.064	0.071
35	0.277	0.145	0.100	0.082	0.072	0.065	0.060	0.057	0.055	0.056	0.058	0.057	0.060	0.064	0.071
36	0.277	0.145	0.101	0.082	0.072	0.065	0.060	0.057	0.055	0.056	0.058	0.058	0.060	0.065	0.072
37	0.278	0.146	0.101	0.082	0.072	0.066	0.061	0.058	0.055	0.057	0.058	0.058	0.060	0.065	0.072
38	0.278	0.146	0.101	0.083	0.073	0.066	0.061	0.058	0.056	0.057	0.059	0.058	0.061	0.065	0.072
39	0.279	0.147	0.102	0.083	0.073	0.067	0.061	0.058	0.056	0.057	0.059	0.059	0.061	0.066	0.073
40	0.279	0.147	0.102	0.084	0.074	0.067	0.062	0.059	0.057	0.058	0.060	0.059	0.062	0.066	0.073
41	0.280	0.147	0.103	0.084	0.074	0.067	0.062	0.059	0.057	0.058	0.060	0.060	0.062	0.066	0.073
42	0.280	0.148	0.103	0.085	0.074	0.068	0.063	0.060	0.057	0.058	0.060	0.060	0.062	0.067	0.074
43	0.281	0.148	0.103	0.085	0.075	0.068	0.063	0.060	0.058	0.059	0.061	0.060	0.063	0.067	0.074
44	0.281	0.149	0.104	0.086	0.075	0.069	0.063	0.060	0.058	0.059	0.061	0.061	0.063	0.068	0.075
45	0.281	0.149	0.104	0.086	0.076	0.069	0.064	0.061	0.059	0.060	0.062	0.061	0.063	0.068	0.075
46	0.282	0.149	0.105	0.086	0.076	0.069	0.064	0.061	0.059	0.060	0.062	0.062	0.064	0.068	0.075
47	0.282	0.150	0.105	0.087	0.076	0.070	0.065	0.062	0.059	0.060	0.063	0.062	0.064	0.069	0.076
48	0.283	0.150	0.105	0.087	0.077	0.070	0.065	0.062	0.060	0.061	0.063	0.062	0.065	0.069	0.076
49	0.283	0.151	0.106	0.088	0.077	0.071	0.065	0.062	0.060	0.061	0.063	0.063	0.065	0.070	0.076
50	0.283	0.151	0.106	0.088	0.078	0.071	0.066	0.063	0.061	0.061	0.064	0.063	0.065	0.070	0.077

Table A-2 The Volatility Change v.s. Time with Volatility Reduction Factor Kappa2=0.1 for Mean reversion parameter from 0.1 to 1.5. (Two Factor Model)

Time	C0.1	C0.2	C0.3	C0.4	C0.5	C0.6	C0.7	C0.8	C0.9	C0.10	C0.11	C0.12	C0.13	C0.14	C0.15
1	0.105	0.103	0.103	0.102	0.105	0.104	0.102	0.104	0.102	0.104	0.104	0.103	0.105	0.103	0.103
2	0.127	0.121	0.116	0.111	0.106	0.102	0.100	0.097	0.098	0.096	0.096	0.097	0.098	0.102	0.105
3	0.137	0.127	0.116	0.108	0.101	0.097	0.092	0.091	0.089	0.090	0.089	0.092	0.091	0.096	0.101
4	0.142	0.126	0.109	0.103	0.093	0.091	0.087	0.085	0.084	0.082	0.084	0.085	0.087	0.090	0.093
5	0.145	0.121	0.102	0.096	0.088	0.083	0.081	0.081	0.078	0.079	0.079	0.079	0.082	0.083	0.090
6	0.143	0.115	0.098	0.089	0.082	0.079	0.077	0.074	0.076	0.074	0.075	0.076	0.077	0.079	0.084
7	0.140	0.108	0.092	0.081	0.077	0.074	0.073	0.071	0.071	0.070	0.071	0.071	0.073	0.075	0.078
8	0.135	0.101	0.086	0.078	0.074	0.072	0.068	0.067	0.068	0.068	0.067	0.069	0.069	0.071	0.073
9	0.131	0.096	0.082	0.074	0.071	0.068	0.067	0.065	0.065	0.066	0.065	0.065	0.065	0.068	0.071
10	0.126	0.090	0.077	0.071	0.066	0.065	0.064	0.062	0.063	0.061	0.062	0.063	0.063	0.066	0.066
11	0.121	0.085	0.072	0.067	0.063	0.061	0.061	0.061	0.061	0.060	0.060	0.061	0.062	0.064	0.064
12	0.114	0.081	0.068	0.063	0.062	0.061	0.059	0.058	0.059	0.058	0.058	0.059	0.060	0.060	0.062
13	0.108	0.076	0.066	0.062	0.060	0.059	0.059	0.058	0.057	0.057	0.057	0.057	0.058	0.060	0.059
14	0.102	0.071	0.063	0.059	0.057	0.057	0.056	0.057	0.055	0.056	0.056	0.056	0.057	0.057	0.058
15	0.099	0.070	0.061	0.058	0.057	0.056	0.056	0.054	0.055	0.055	0.055	0.055	0.055	0.056	0.057
16	0.094	0.066	0.059	0.057	0.056	0.055	0.053	0.054	0.054	0.054	0.054	0.053	0.055	0.056	0.055
17	0.089	0.063	0.058	0.055	0.055	0.053	0.054	0.054	0.053	0.054	0.054	0.053	0.054	0.054	0.055
18	0.084	0.062	0.057	0.055	0.053	0.054	0.053	0.052	0.052	0.052	0.053	0.054	0.052	0.054	0.053
19	0.080	0.060	0.056	0.053	0.053	0.054	0.052	0.053	0.052	0.053	0.053	0.053	0.052	0.053	0.054
20	0.076	0.058	0.053	0.053	0.054	0.052	0.053	0.051	0.052	0.052	0.052	0.052	0.052	0.052	0.053
21	0.073	0.056	0.054	0.053	0.052	0.052	0.051	0.050	0.052	0.051	0.052	0.051	0.051	0.052	0.053
22	0.071	0.056	0.053	0.052	0.052	0.051	0.051	0.051	0.051	0.052	0.051	0.051	0.051	0.051	0.051
23	0.068	0.054	0.053	0.052	0.053	0.051	0.051	0.051	0.051	0.051	0.052	0.051	0.050	0.051	0.051
24	0.064	0.054	0.052	0.052	0.051	0.051	0.051	0.051	0.049	0.050	0.050	0.051	0.051	0.052	0.051
25	0.062	0.053	0.052	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.050	0.052	0.050
26	0.062	0.053	0.052	0.050	0.051	0.051	0.050	0.051	0.050	0.050	0.050	0.051	0.051	0.051	0.051
27	0.060	0.053	0.052	0.050	0.051	0.051	0.050	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051
28	0.059	0.052	0.051	0.051	0.051	0.049	0.051	0.050	0.050	0.050	0.051	0.050	0.050	0.051	0.051
29	0.057	0.053	0.051	0.051	0.050	0.051	0.050	0.051	0.050	0.051	0.051	0.051	0.051	0.051	0.051
30	0.056	0.052	0.051	0.051	0.050	0.050	0.051	0.049	0.050	0.050	0.050	0.049	0.051	0.049	0.051
31	0.056	0.051	0.050	0.051	0.051	0.052	0.051	0.051	0.050	0.051	0.050	0.050	0.050	0.051	0.050
32	0.055	0.052	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.049	0.050	0.050	0.051	0.050	0.050
33	0.053	0.049	0.050	0.050	0.050	0.049	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.051	0.050
34	0.053	0.051	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050
35	0.053	0.051	0.050	0.050	0.051	0.050	0.051	0.050	0.050	0.050	0.050	0.050	0.051	0.050	0.050
36	0.052	0.050	0.051	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.051	0.051	0.050
37	0.051	0.050	0.049	0.050	0.050	0.050	0.049	0.050	0.050	0.050	0.050	0.050	0.049	0.050	0.050
38	0.053	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.051	0.050	0.049	0.049	0.050	0.049
39	0.052	0.050	0.050	0.051	0.051	0.050	0.050	0.050	0.050	0.051	0.051	0.050	0.050	0.050	0.050
40	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.050	0.050
41	0.051	0.050	0.050	0.050	0.050	0.051	0.050	0.049	0.051	0.050	0.050	0.049	0.050	0.051	0.051
42	0.050	0.050	0.050	0.051	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.050	0.049	0.050	0.051
43	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.049	0.050	0.050	0.050	0.050	0.050	0.050	0.050
44	0.051	0.050	0.049	0.051	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.050	0.050
45	0.051	0.051	0.050	0.050	0.049	0.050	0.051	0.051	0.050	0.051	0.050	0.049	0.049	0.050	0.050
46	0.050	0.050	0.050	0.050	0.049	0.051	0.049	0.049	0.049	0.050	0.050	0.051	0.051	0.049	0.050
47	0.049	0.050	0.050	0.049	0.051	0.050	0.050	0.050	0.049	0.050	0.050	0.049	0.050	0.050	0.051
48	0.051	0.050	0.049	0.051	0.051	0.050	0.050	0.051	0.051	0.050	0.050	0.050	0.050	0.050	0.050
49	0.050	0.050	0.050	0.051	0.050	0.051	0.049	0.050	0.049	0.050	0.050	0.051	0.050	0.050	0.050
50	0.050	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.049	0.050	0.052	0.050	0.049	0.050

Table A-3 The Call Option prices v.s. Volatility Reduction Factor Kappa2=0.1 for Mean reversion parameter from 0.1 to 1.5 at time step 10 and 20.

Kappa	Time	Put Value	Call Value	Kappa	Time	Put Value	Call Value
0	10	1.295706	0.298956	0	20	2.694119	0.737389
0.1	10	1.146245	0.178349	0.1	20	2.16911	0.258119
0.2	10	1.094747	0.112176	0.2	20	2.083254	0.136858
0.3	10	1.074613	0.079347	0.3	20	2.052743	0.09303
0.4	10	1.05445	0.064286	0.4	20	2.033407	0.074547
0.5	10	1.041039	0.055534	0.5	20	2.019294	0.065118
0.6	10	1.034404	0.049995	0.6	20	2.012895	0.058434
0.7	10	1.033965	0.045545	0.7	20	2.008865	0.053418
0.8	10	1.031922	0.042953	0.8	20	2.009655	0.050571
0.9	10	1.033102	0.040811	0.9	20	2.009879	0.048454
1	10	1.035684	0.041263	1	20	2.010803	0.048722
1.1	10	1.020671	0.043421	1.1	20	1.994431	0.051183
1.2	10	1.030248	0.043059	1.2	20	2.008076	0.05083
1.3	10	1.032503	0.045227	1.3	20	2.010573	0.053075
1.4	10	1.034418	0.049058	1.4	20	2.013512	0.057649
1.5	10	1.04916	0.055385	1.5	20	2.031018	0.064529

Table A-4 The Call Option prices v.s. Volatility Reduction Factor Kappa2=0.1 for Mean reversion parameter from 0.1 to 1.5 at time step 30 and 40.

Kappa	Time	Put Value	Call Value	Kappa	Time	Put Value	Call Value
0	30	4.075237	1.189249	0	40	5.42938	1.632281
0.1	30	3.131331	0.273702	0.1	40	4.063874	0.279368
0.2	30	3.046852	0.142902	0.2	40	3.982435	0.147027
0.3	30	3.006798	0.098258	0.3	40	3.942193	0.102213
0.4	30	2.988916	0.079456	0.4	40	3.921946	0.083887
0.5	30	2.980678	0.069453	0.5	40	3.910719	0.073625
0.6	30	2.968826	0.062849	0.6	40	3.903929	0.066926
0.7	30	2.961479	0.057729	0.7	40	3.895628	0.06178
0.8	30	2.96228	0.054874	0.8	40	3.898427	0.058821
0.9	30	2.964786	0.052764	0.9	40	3.904592	0.056609
1	30	2.96452	0.053518	1	40	3.900882	0.057645
1.1	30	2.946904	0.055518	1.1	40	3.87943	0.05973
1.2	30	2.964661	0.055183	1.2	40	3.895565	0.059277
1.3	30	2.960516	0.057553	1.3	40	3.901111	0.061501
1.4	30	2.969976	0.062144	1.4	40	3.911661	0.066012
1.5	30	2.985704	0.069115	1.5	40	3.918181	0.073025