Underinvestment, Capital Structure and Strategic Debt Restructuring*

Grzegorz Pawlina[†]

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Abstract

In this paper the investment and liquidation policy of a levered firm is analyzed. The possibility of renegotiating the original debt contract is included. It is shown that the shareholders' option to restructure the outstanding debt exacerbates Myers' (1977) underinvestment problem. This result is due to a higher wealth transfer from the shareholders to the creditors occurring upon investment when the option to renegotiate is present. The problem can be eliminated only when all the bargaining power is given to the creditors. In such a case, the renegotiation commences at the shareholders' bankruptcy trigger and no additional wealth transfer occurs. Moreover, it is shown the liquidation policy under partial debt financing differs from the optimal policy when the firm is all-equity financed. Even after removing the effects of the tax shield by excluding taxes, it holds that the liquidation policy is affected by the second-best investment policy, thus it occurs inefficiently early. Finally, it is shown that the presence of a positive NPV investment opportunity increases the likelihood of a strategic default when the bargaining power of shareholders is high.

Keywords: irreversible investment, capital structure, Nash bargaining solution, endogenous bankruptcy, liquidation policy

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[†]Department of Econometrics & Operations Research and CentER, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, the Netherlands, email: g.pawlina@uvt.nl, phone: + 31 13 4663178, fax: + 31 13 4663280.
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1 Introduction

One of the consequences of debt financing is its influence on the firm's investment policy. As it is known from Myers (1977), the presence of risky debt in the company's books leads to underinvestment, i.e. a situation in which some positive NPV projects are foregone. Although the impact of the agency costs of debt on the firm's investment policy has been widely discussed in the literature in qualitative terms, relatively little has been done to analyze the magnitude of these costs. Moreover, the existing contributions yield differing predictions concerning the influence of the renegotiability of debt on the investment policy (cf. Mella-Barral and Perraudin, 1997, and Mauer and Ott, 1999). This paper uses the contingent claims approach to examine the firm's optimal investment and liquidation policy in the presence of debt financing and the equityholders' option to default and renegotiate the original debt contract.

The main objective of the paper is to investigate the impact of the renegotiation option, the distribution of bargaining power, and indirect bankruptcy costs on the optimal investment and liquidation policy of the firm. In particular, we are interested in the impact of those debt characteristics on the magnitude of underinvestment problem. Furthermore, the impact of a growth opportunity on the optimal bankruptcy and renegotiation timing is analyzed. In this way it can be investigated whether firms operating in sectors with significant growth opportunities are less likely to file for debt restructuring than their counterparts in more mature industries.

The motivation for this paper arises also from the ongoing debate on the differences in bankruptcy codes between the European Union (EU) and the United States, and the implications of the EU countries bankruptcy law for the firms' operating decisions .[1] Under Chapter 11 of the US bankruptcy law, financially distressed firms suspend their coupon payments and a reorganization plan, which includes writing new debt contracts, is implemented. The operations of a firm entering Chapter 11 reorganization usually remain unaffected by the negotiations process, which makes it relatively easy to remain in business if the financial restructuring is successful. In Europe, however, a distressed firm most likely goes under court administration, and its operations are suspended. As a result, the reputation of the firm deteriorates and there is a high chance that liquidation occurs.[2]

In our model debt renegotiation constitutes a good approximation of a private work-out. Under the work-out the initial debt contract is changed so that the equityholders, as the first-best users of assets, are better off running the company than declaring bankruptcy. Moreover, the creditors benefit from the fact that the modified debt contract reduces the

[1] See, e.g. *The Economist*, 23rd March 2002, 'Up from the ashes', and 7th September 2002, The firms that can't stop falling: Bankruptcy in America.

[2] McCahery et al. (2002) provides a collection of articles concerning the comparison of the countries' legal systems.

probability of bankruptcy. The case of bankruptcy better resembles the European system. A firm that defaults on its debt obligations goes bankrupt and its assets are foreclosed by the creditors. Such foreclosure leads in many cases to inefficiently early liquidation since the value of the assets to the creditors is lower than their value to the original owners. US Chapter 11 remains between these two cases as far as the time allowed for renegotiation is concerned, but it is more shareholder-friendly from the point of view of coupon suspension.

Our analysis also provides insight into the differences between the impact of a bank credit and diffusely held debt on the firm's operating policy. Bank credit is mostly associated with the possibility of debt renegotiation upon financial distress, whereas diffusely held debt makes renegotiation less likely (cf. Bolton and Scharfstein, 1996). The outcome of renegotiating the bank debt depends on the bargaining power of the equityholders vis-à-vis the bank and on both parties' outside options. Usually, the bargaining power of the bank is large, in particular when the firm is relatively small and uses a portfolio of its services. Consequently, the share of the renegotiation surplus received by the bank may be substantial (cf. Hackbarth et al., 2002). When corporate debt is held by dispersed bondholders, the bargaining power of the creditors is usually small and such is the surplus from renegotiation that accrues to the creditors (cf. Hege and Mella-Barral, 2002).

The model is based on the following assumptions. The firm has an investment opportunity to scale up its activities upon incurring an irreversible cost. The cash flow of the firm follows a random process and the firm has to pay an instantaneous coupon on its debt. Failure to pay the coupon triggers bankruptcy. Following Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000), we assume that the coupon payment can be renegotiated so that bankruptcy is avoided and the surplus is split among the equityholders and creditors.

A number of other models known from the literature can be nested in our framework. Setting the coupon level equal to zero leads to the basic model of Dixit and Pindyck (1996) with the firm scaling up its activities. Excluding the renegotiation possibility reduces our model to Mauer and Ott (1999). By setting the investment cost to infinity and liquidation value to zero, we arrive at Fan and Sundaresan (2000), whereas imposing prohibitively high investment cost in combination with take-it or leave-it offers and no taxes reduces our model to Mella-Barral and Perraudin (1997).

Consequently, this paper builds upon Mauer and Ott (1999), who analyze the interaction between the leverage and investment option when renegotiation is not allowed for, and both Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000), who focus on strategic debt service.[3] Bankruptcy and renegotiation concepts used in our paper coincide

[3]A far from complete list of references includes Vercammen (2000), analyzing how bankruptcy, triggered by the assets value falling below the face value of the debt, influences investment, Leland and Toft (1996), considering a finite maturity debt with a stationary structure, Anderson and Sundaresan (1996), Mella-Barral (1999), Acharya et al. (2002), and Hackbarth et al. (2002), analyzing debt renegotiation. Related work is presented by Mauer and Triantis (1994), Fischer et al. (1989), and Dangl and Zechner (2001), who focus on

with two polar cases analyzed by Morellec and Francois (2002), who model US Chapter 11 as costly reorganization with a limited duration. The extreme cases in which the renegotiation is not allowed for (duration equal to zero) and can last infinitely long, are analyzed by Leland (1994) and Mella-Barral and Perraudin (1997), respectively.

In the paper it is shown that the presence of the renegotiation option exacerbates the underinvestment problem. This is due to the fact that the wealth transfer to the debtholders, which occurs upon investment, is higher if the shareholders can default strategically on their original debt contract. In other words, the negative change in the value of the option to renegotiate the debt contract exceeds in absolute terms the negative change of the value of limited liability. The additional underinvestment does not occur if all the bargaining power is given to the creditors. Another implication of the renegotiability of the debt contract is that the problem of inefficient early liquidation can be reduced. This results from the fact that firm remains in the hands of the original shareholders, who can run it most efficiently.[4] However, it cannot be avoided fully, due to the impact of the suboptimal investment policy on the choice of liquidation trigger.

The firm's growth option influences its optimal debt restructuring policy. The presence of a positive NPV project, in combination with a high debtors' bargaining power, may result in an earlier timing of debt reorganization. However, the firm's liquidation policy determined, among others, by the magnitude of its tangible collateral, does not affect its optimal debt reorganization policy. This finding may be to some extent counterintuitive since the magnitude of collateral influences both the creditors' outside option and the value of the firm. It appears that these two effects cancel out when the debt renegotiation decision is made.

The paper is organized as follows. In Section 2 the basic model of the firm is described, whereas in Section 3 debt renegotiation is introduced. Comparative statics and some empirical implications are presented in Section 4. Section 5 concludes.

2 The Basic Model

As a starting point, we essentially use a version of the model of Dixit and Pindyck (1996, Ch. 6). Consider the following situation. A firm is producing a good that generates a random cash flow $x(t)$, where $x(t)$ is the time-t realization of a stochastic process. The firm has an option to make an irreversible investment, I, after which it will be entitled to a cash flow,

the optimal recapitalization policy.

[4]These results show the limitations of the two-period model of Myers (1977). In his case, the investment and the liquidation decisions are made simultaneously so that the possibility of renegotiation enhances investment and reduces liquidation. In the continuous-time framework of the present model, renegotiation reduces inefficient liquidation in bad states of nature but (anticipated by the shareholders in good states of nature) also impairs the investment activity.

$\theta x(t)$, where $\theta > 1$. Randomness of the cash flow is incorporated in our model by letting x follow the stochastic differential equation

$$dx(t) = \alpha x(t)\, dt + \sigma x(t)\, dw(t), \tag{1}$$

where α and σ are constants corresponding to the instantaneous growth rate and the volatility of the project's cash flow, respectively, and $w(t)$ denotes a standard Brownian motion.[5] Let r be the deterministic instantaneous riskless interest rate. It is assumed that all the agents are risk neutral and the drift rate of the cash flow, α, exhibits a shortfall δ below the riskless rate, i.e. $\alpha = r - \delta$. The uncertainty in the model is described by a complete filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in(0,\infty)}, \mathbb{P}\right)$, where Ω is the state space, $\mathcal{F}$ is the σ-algebra representing measurable events, and $\mathbb{P}$ is the (actual) probability measure. The filtration is the augmented filtration generated by the Brownian motion and satisfies the usual conditions.[6]

We begin the analysis with the simple case of an all-equity financed firm. In Subsection 2.1 the optimal liquidation and investment decisions of the unlevered firm are investigated. Subsequently, we introduce a mixed capital structure. The presence of debt results in a positive probability of bankruptcy and the shareholders' option to default. The optimal bankruptcy trigger and the impact of bankruptcy on the investment decision are analyzed in Subsection 2.2.

2.1 All-equity financing

The cash flow of the firm is subject to taxation and the corporate tax rate is τ. No other taxes are assumed. The firm may always decide to sell its assets and liquidate. Define an indicator $i \in \{0, 1\}$ to be equal to 0 if the investment has not yet been made, and 1 in the opposite case. Liquidation entails receiving a lump sum payment, γ_i, in return for the present value of the firm's expected future cash flow.

The standard no-arbitrage argument (cf. Dixit and Pindyck, 1996) implies that any claim, F, contingent on the process x and having an instantaneous payoff $Bx + C$, where $B, C \in \mathbb{R}$, satisfies the ordinary differential equation (ODE)

$$rF = (r - \delta)\, x \frac{\partial F}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F}{\partial x^2} + Bx + C. \tag{2}$$

For the value of the unlevered firm, V_i, parameters B and C are $\theta^i (1 - \tau)$ and zero, respectively. The general solution to (2) is of the form

$$F = \frac{B}{\delta} + \frac{C}{r} + M_1 x^{\beta_1} + M_2 x^{\beta_2}, \tag{3}$$

[5] We do not impose a constant positive marginal cost to avoid the need of tackling the issue of limited liability of the creditors in some states of nature.

[6] A filtration $\{\mathcal{F}_t\}$ satisfies the usual conditions if it is right continuous and $\mathcal{F}_0$ contains all the $\mathbb{P}$-null sets in $\mathcal{F}$ (see Karatzas and Shreve, 1991, p. 10).

where β_1 (β_2) is the positive (negative) root of the characteristic equation

$$\frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\left(r-\delta\right)\beta-r=0, \tag{4}$$

and M_1 and M_2 are constants determined from boundary conditions specific to the type of the contingent claim.

Let us first consider the value of the firm after the investment has been made. The only decision that is to be made by the shareholders at each instant is whether to continue running the firm or to liquidate it. The value of the firm after the investment, V_1, equals

$$V_1=\begin{cases}\gamma_1 & x<x_1^L,\\ \frac{x\theta(1-\tau)}{\delta}+\left(\gamma_1-\frac{x_1^L\theta(1-\tau)}{\delta}\right)\left(\frac{x}{x_1^L}\right)^{\beta_2} & x\geq x_1^L,\end{cases} \tag{5}$$

where x_1^L is the optimal liquidation threshold. The value of the firm prior to liquidation equals the present value of earnings in perpetuity and the value of the option to liquidate. Analogous to, e.g., Dixit and Pindyck (1996, Ch. 6), the solution to the liquidation problem equals

$$x_1^L=\frac{-\beta_2}{1-\beta_2}\frac{\gamma_1\delta}{\theta\left(1-\tau\right)}. \tag{6}$$

Before the investment, the strategy space of the firm consists of the three following elements

$$\{continue, liquidate, invest\}.$$

Liquidation occurs when earnings fall below a certain trigger, whereas investment takes place when earnings are sufficiently high. This results in a double-barrier problem where the optimal investment threshold and liquidation trigger before the investment have to be found simultaneously. The optimal investment and liquidation policies are found by solving ODE (2) for V_0 subject to

$$\begin{aligned} V_0\left(x^*\right) &= V_1\left(x^*\right)-I, && (7)\\ \left.\frac{\partial V_0}{\partial x}\right|_{x=x^*} &= \left.\frac{\partial V_1}{\partial x}\right|_{x=x^*}, && (8)\\ V_0\left(x_0^L\right) &= \gamma_0, && (9)\\ \left.\frac{\partial V_0}{\partial x}\right|_{x=x_0^L} &= 0, && (10)\end{aligned}$$

where x_0^L denotes the before-investment liquidation trigger and x^* is the optimal investment threshold.

2.2 Debt and equity financing

Now, let us assume that the firm is partially financed with debt. The debt contract is associated with a perpetual coupon stream b, which is tax deductible. The par value of debt is assumed to equal b/r. Because of the limited liability of equityholders in some states of nature it is optimal for them to default on debt obligations. A failure to pay the contracted coupon results in bankruptcy upon which creditors take over the firm. We impose the absolute priority rule (APR) so the equityholders receive nothing in the event of bankruptcy as long as the claim of debtholders is not fully satisfied.[7]

Since we are interested in the optimal debt restructuring policy, we assume an *endogenous bankruptcy* procedure. Such a procedure stipulates that equityholders declare bankruptcy so to maximize the value of equity. In such a case it is possible that for low cash flow realizations, the equityholders may actually inject cash to the firm. This modeling approach is consistent with, for instance, Leland (1994), Mella-Barral and Perraudin (1997), and Acharya and Carpenter (2002). It differs from the models of *exogenous bankruptcy,* which is triggered by the asset value falling below a prespecified level. For instance, in Merton (1974) bankruptcy occurs when the terminal value of assets is lower than the debt principal, whereas in Black and Cox (1976) it is triggered when the level of assets hits a deterministic barrier.[8] Yet another approach is taken by Kim, Ramaswamy and Sundaresan (1993), who assume that bankruptcy is triggered by illiquidity, i.e. when net profits fall negative.

The value of the firm operated by the creditors after bankruptcy is a function of the cash flow from output, denoted by $R_i(x)$. Following Fan and Sundaresan (2000), we abstain from analyzing the issue of dynamic recapitalization. As a consequence, the firm run by the creditors remains all-equity financed for ever, and the tax shield is irreversibly lost upon bankruptcy. Moreover, if bankruptcy occurs prior to the investment, the growth option expires unexercised. Finally, as in Mella-Barral and Perraudin (1997), it is assumed that the debtholders will run the firm less efficiently, so that the cash flow generated by the firm in the hands of the creditors equals $\rho\theta^i(1-\tau)x$, where $\rho \in (0,1)$.[9] The latter assumption reflects, among others, superior ability of existing management to run the firm and distraction of management upon bankruptcy, combined with impaired ability to contract and suboptimal investment in firm-specific human capital (see Hackbarth et al., 2002).

Since the value of the firm, V_i, its equity, E_i, debt, D_i, and creditors' reservation value, R_i, are securities contingent on the earnings process, x, they all satisfy ODE (2). The values of constants B and C defining their instantaneous payoffs are depicted in Table 2.1.

[7] Evidence presented by Franks and Torous (1989) indicates significant departures from the absolute priority rule in many bankruptcy settlements. Our assumption has been introduced for simplicity. Waiving this assumption would result in bankruptcy occuring for higher realizations of cash flow than with APR.

[8] See Bielecki and Rutkowski (2002) for a detailed reference list concerning related safety covenants.

[9] Hege and Mella-Barral (2000) develop a model in which the firm in the hands of new owners has exactly the same set options concerning new debt issues and subsequent reorganizations as under the management of incumbents. The assumption about proportional reduction of cash flow upon bankruptcy remains unchanged.

	V_i	E_i	D_i	R_i
B	$\theta^i(1-\tau)$	$\theta^i(1-\tau)$	$-$	$\rho\theta^i(1-\tau)$
C	$b\tau$	$-b(1-\tau)$	b	$-$

Table 2.1. Instantaneous payoffs associated with the value of the firm, V_i, equity, E_i, debt, D_i, and the creditors' outside option, R_i.

First, we determine the value of the firm run by the creditors, R_i. It is obtained by solving (2) with value-matching and smooth-pasting conditions reflecting the fact that the only option available to the firm run by the creditors is to liquidate. It holds that R_i is equal to

$$R_i = \begin{cases} \gamma_i & x < x_1^{LR}, \\ \frac{\rho x\theta^i(1-\tau)}{\delta} + \left(\gamma_i - \frac{\rho x_i^{LR}\theta^i(1-\tau)}{\delta}\right)\left(\frac{x}{x_i^{LR}}\right)^{\beta_2} & x \geq x_1^{LR}, \end{cases} \tag{11}$$

where

$$x_i^{LR} = \frac{-\beta_2}{1-\beta_2}\frac{\gamma_i\delta}{\rho\theta^i(1-\tau)} \tag{12}$$

is the optimal liquidation trigger of the creditors running the firm.

We determine the value of the firm and the optimal investment threshold by first considering the case in which the firm has already invested. We solve (2) for the firm's equity, E_1, and debt, D_1 with value-matching conditions at the bankruptcy trigger that correspond to the absolute priority rule. The value of the firm's equity, E_1, and debt, D_1, after the investment is made, can be described as follows

$$E_1 = \begin{cases} 0 & x < x_1^B, \\ (1-\tau)\left[\left(\frac{x\theta}{\delta} - \frac{b}{r}\right) - \left(\frac{x_1^B\theta}{\delta} - \frac{b}{r}\right)\left(\frac{x}{x_1^B}\right)^{\beta_2}\right] & x \geq x_1^B, \end{cases}, \tag{13}$$

and

$$D_1 = \begin{cases} R_1(x) & x < x_1^B, \\ \frac{b}{r} + \left(R_1\left(x_1^B\right) - \frac{b}{r}\right)\left(\frac{x}{x_1^B}\right)^{\beta_2} & x \geq x_1^B. \end{cases}, \tag{14}$$

The optimal equityholders' bankruptcy trigger is determined using the smooth-pasting condition for the equity value upon bankruptcy and equals

$$x_1^B = \frac{-\beta_2}{1-\beta_2}\frac{b\delta}{r\theta}. \tag{15}$$

The value of the firm equals

$$V_1 = E_1 + D_1 = \tag{16}$$

$$= \begin{cases} R_1(x) & x < x_1^B, \\ \frac{x\theta(1-\tau)}{\delta} + \frac{b\tau}{r} + \left(R_1\left(x_1^B\right) - \frac{x_1^B\theta(1-\tau)}{\delta} - \frac{b\tau}{r}\right)\left(\frac{x}{x_1^B}\right)^{\beta_2} & x \geq x_1^B, \end{cases}.$$

Figure 2.1 depicts the value of the firm and the claims written on it after the investment is made. The value of the firm approaches the present value of earnings increased by the tax shield for a high earnings level. For lower realizations of the earnings process, the concavity of the firm's value increases, which reflects the value of the equityholders' option to default. At the bankruptcy trigger, x_1^B, the firm's value function exhibits a kink which reflects the fact that bankruptcy is neither optimal nor reversible as seen from the perspective of the firm value maximization.[10] The value of equity approaches the present value of earnings minus the after-tax coupon payment. For lower realizations of earnings, its convexity increases due to the limited liability effect. At the equityholders' optimal bankruptcy trigger, the value of equity smooth-pastes to zero. Finally, the value of debt tends to its riskless valuation for high realizations of the earnings process, and equals the firm's value at the bankruptcy trigger.

Equipped with the value of the firm after the investment has been made, we are ready to determine the optimal exercise policy of the investment option. We calculate both the firm value-maximizing and the equity value-maximizing investment thresholds. Here, we use the framework of Mauer and Ott (1999) and correct two of their boundary conditions[11]. We start by observing that the value of the firm as well as its equity and debt before the investment, V_0, E_0, and, D_0, respectively, satisfy ODE (2). Therefore the corresponding values can be written as

$$V_0 = \frac{x(1-\tau)}{r-\alpha} + \frac{b\tau}{r} + K_0 x^{\beta_1} + B_0 x^{\beta_2}, \tag{17}$$

$$E_0 = \frac{x(1-\tau)}{r-\alpha} - \frac{b(1-\tau)}{r} + A_{01} x^{\beta_1} + A_{02} x^{\beta_2}, \tag{18}$$

$$D_0 = V_0 - E_0. \tag{19}$$

The component $K_0 x^{\beta_1}$ is the value of the growth option and $B_0 x^{\beta_2}$ reflects the value lost due to the potential future bankruptcy. $A_{01} x^{\beta_1}$ is the fraction of the value of the investment option that accrues to the equityholders and $A_{01} x^{\beta_2}$ is the equityholders' option to default. The constants K_0, B_0, A_{01}, A_{02}, the optimal bankruptcy trigger x_0^B and the firm value-

[10] If bankruptcy was optimal then the value function would be differentiable at x_1^B as a result of the smooth-pasting condition. Reversibility would imply the continuity of the first derivative of the value function at x_1^B due to the no-arbitrage condition (for details see Dumas, 1991).

[11] First, we replace the investment bankruptcy trigger in condition (9.20a) on p. 159 of Mauer and Ott (1999) that ignores the impact of the investment opportunity, by the one determined optimally. Second, we add a smooth-pasting condition necessary for calculating the optimal trigger in the presence of the investment opportunity.

maximizing investment threshold, x^*, are uniquely determined by the system of equations

$$\begin{aligned}
V_0(x^*) &= V_1(x^*) - I, && (20)\\
E_0(x^*) &= E_1(x^*) - I, && (21)\\
\left.\frac{\partial V_0}{\partial x}\right|_{x=x^*} &= \left.\frac{\partial V_1}{\partial x}\right|_{x=x^*}, && (22)\\
E_0\left(x_0^B\right) &= 0, && (23)\\
\left.\frac{\partial E_0}{\partial x}\right|_{x=x_0^B} &= 0, && (24)\\
V_0\left(x_0^B\right) &= R_0\left(x_0^B\right). && (25)
\end{aligned}$$

Equations (20) and (21) are the value-matching conditions ensuring the continuity of the value of the firm as well as of its equity and debt (by $D_0 = V_0 - E_0$) at the optimal investment threshold. (22) is the smooth-pasting condition associated with the firm value-maximizing property of the investment threshold. (23) and (24) are the value matching and smooth-pasting conditions for the equityholders at the bankruptcy trigger. (24) ensures that the bankruptcy trigger is chosen such that the value of equity is maximized. (25) is the value-matching condition for the firm at the bankruptcy trigger. Its RHS implies that the investment option expires upon bankruptcy.

It holds that x_1^B is lower than x_0^B. This is due to the fact that the cash flow is higher after the investment has been undertaken and the present value of the incremental cash flow from investment is worth more than the option to acquire it.

The debtholders benefit from undertaking the investment project in two ways. First, the probability of bankruptcy decreases so that the present value of the expected coupon stream is higher. Second, the outside option of the debtholders becomes more valuable. After bankruptcy is declared, the debtholders will run a firm that generates a higher cash flow than prior to the investment.

Since in most cases it is impossible to implement an investment schedule that maximizes the value of the firm, we compare the first-best solution with the second-best that maximizes the value of the equity.[12] The investment decision associated with maximizing the value of the equity requires replacing (22) by

$$\left.\frac{\partial E_0}{\partial x}\right|_{x=x^*} = \left.\frac{\partial E_1}{\partial x}\right|_{x=x^*} \qquad (26)$$

Constants K_0, B_0, A_{01}, A_{02}, and triggers x_0^B and x^* are completely described by the system of equations (20) - (25) with (22) replaced by (26). The optimal investment threshold is in

[12] In general, it is not in the interest of shareholders to align perfectly the incentives of the managers with their own in the presence of debt (cf. Brander and Poitevin, 1992, and John and John, 1993). The optimal compensation scheme should be constructed in such a way that the combined agency costs of equity and debt are minimized. However, in this paper's framework with a single owner-manager the first-best solution is not attainable.

this case higher since the wealth transfer to debtholders occurring upon investment causes that the equityholders invest later than the first-best solution would indicate. Furthermore, the optimal bankruptcy trigger is lower under the second-best investment rule than under the first-best policy. The reason for such a relationship is that under the second-best investment rule the value of the investment opportunity for the equityholders is higher than under the first-best policy. Therefore, at any x the continuation value is higher under the second-best that under first-best. As a consequence, the continuation value under the second-best smooth-pastes to the stopping value (equal to zero) at a lower x than under the first-best.[13]

3 Debt renegotiation

The divergence between the optimal liquidation trigger of the firm and the equityholders' endogenous bankruptcy trigger implies that there is a scope for debt renegotiation. The scope for renegotiation stems from the fact that upon bankruptcy the three following components of the firm's value are irreversibly lost. First, the investment opportunity ceases to exist when the creditors take over the company. Second, upon bankruptcy the firm forgoes the present value of the tax shield. Finally, creditors run the firm less efficiently so the instantaneous earnings of the firm are reduced by fraction $(1-\rho)$ of the current cash flow.

In this section we analyze the impact of debt renegotiation on the investment policy and the value of the firm. We assume that the renegotiation process has a form of Nash bargaining in which the bargaining power is split between the two types of the firm's stakeholders (cf. Perraudin and Psillaki, 1999, and Fan and Sundaresan, 2000). The distribution of the bargaining power is given exogenously and is described by parameter $\eta \in [0,1]$, where a high η is associated with high bargaining power of the shareholders. The take-it or leave it offers made either by the shareholders or by the creditors (as in Mella-Barral and Perraudin, 1997) are limiting cases of the Nash bargaining solution. Consequently, they correspond to the cases where $\eta = 1$ and $\eta = 0$, respectively. The former situation can be related to large corporations that are likely to be aggressive in negotiations, whereas the latter corresponds to small and young firms that use a portfolio of the bank's services.

The remainder of this section consists of two parts. In Subsection 3.1, we calculate the value of the firm as a function of the equityholders' renegotiation trigger and determine the optimal sharing rule. In Subsection 3.2 we simultaneously derive the values of debt and equity, and determine the optimal equityholders' renegotiation and investment policies and the firm's optimal liquidation rule.

[13] Mauer and Ott (1999) fail to incorporate this relationship in their model.

3.1 Nash Bargaining Solution

Debt renegotiation is assumed to have a form of a strategic debt service, i.e. it is associated with a lower than contractual coupon payment. The new coupon payment schedule has to satisfy both the shareholders' and debtholders' participation constraints associated with the renegotiation process. We follow Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) in assuming that the coupon is a function of the current cash flow. Such an approach allows for avoiding path-dependency leading to analytical intractability.[14] Repeated renegotiation is possible and occurs in equilibrium with positive probability.

It is also assumed that bargaining power is distributed among the shareholders and the creditors which results in the surplus from renegotiation being distributed with a certain proportion among the two groups. Moreover, we impose the assumption made by Fan and Sundaresan (2000) that during the renegotiation process the tax shield is temporarily suspended. As soon as the cash flow from operation recovers and debtholders are receiving coupon b again, the tax shield is restored.[15,16] Finally, it is assumed that γ_i, $i \in \{0, 1\}$, satisfies

$$\gamma_i < \frac{b}{r}\rho\,(1-\tau)\,. \tag{27}$$

Condition (27) implies that the liquidation value is small enough so that it will not be optimal for the creditors to liquidate the firm immediately after the original debt contract is infringed.[17]

First, we determine the value of the firm, V_i^{NB}, as a function of the optimal renegotiation trigger. Since the present value of the tax shield depends on the moment of commencing the debt renegotiation, the value of the firm as a whole depends on the renegotiation trigger. V_i^{NB} can be expressed as the sum of the present value of cash flow, tax shield, TS_i, growth option (for $i = 0$), $K_0 x^{\beta_1}$, and liquidation option, $L_i x^{\beta_2}$:

$$V_i^{NB} = \frac{\theta^i x\,(1-\tau)}{\delta} + TS_i + (1-i)\,K_0 x^{\beta_1} + L_i x^{\beta_2}. \tag{28}$$

In Appendix A we show that for a given choice of the renegotiation trigger, x_i^{NB}, the tax

[14] Hege and Mella-Barral (2000) assume that a once reduced coupon cannot be increased.

[15] According to Fan and Sundaresan (2000), p. 1072, the fact of temporary tax shield suspension in the renegotiation region "may be interpreted as debtholders agree to forgive some debt and the Internal Revenue Service (IRS) suspends tax benefits until contractual payments are resumed." An alternative approach is proposed by Hege and Mella-Barral (2000), and Hackbarth et al. (2002), who assume that the magnitude of the tax shield corresponds to the prevailing coupon payment.

[16] Fan and Sundaresan (2000) claim (footnote 12, p. 1073) that the optimal renegotiation trigger is lower when the tax benefits accrue during the strategic debt service. In fact, the optimal renegotiation trigger is higher when the tax shield is not suspended since the value of starting renegotiation is higher in such a situation. Therefore, our main results would be even stronger if we did not impose suspension of the tax shield. See also footnote 19.

[17] Since $\rho\,(1-\tau) < 1$, condition (27) also implies that the debt is risky.

shield, TS_i, equals

$$TS_i = \begin{cases} \frac{b\tau}{r}\frac{-\beta_2}{\beta_1-\beta_2}\left(\frac{x}{x_i^{NB}}\right)^{\beta_1} & x \leq x_i^{NB}, \\ \frac{b\tau}{r}\left(1-\frac{\beta_1}{\beta_1-\beta_2}\left(\frac{x}{x_i^{NB}}\right)^{\beta_2}\right) & x > x_i^{NB}. \end{cases} \tag{29}$$

The expressions on the right-hand side have an immediate interpretation. They are the products of the present value of the perpetual tax shield, $\frac{b\tau}{r}$, of the stochastic discount factor associated with hitting the renegotiation boundary, $\left(x/x_i^{NB}\right)^{\beta_2}$, and of the fraction $\frac{\beta_1}{\beta_1-\beta_2}$ (smaller than one) that reflects the fact that the tax shield operates only in the renegotiation region.

The constants K_0 and L_0 will be determined later, i.e. at the time of solving the firm's investment problem. The constant L_1 is given by

$$L_1 = \left(\gamma_1 - \frac{\theta\left(1-\tau\right)x_1^{LN}}{\delta} - \frac{-\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}\right)\left(x_1^{LN}\right)^{-\beta_2}, \tag{30}$$

where x_1^{LN} is the after-investment liquidation trigger. The latter is implicitly given by

$$\frac{1-\beta_2}{-\beta_2}\frac{x_1^{LN}\theta\left(1-\tau\right)}{\delta} + \frac{b\tau}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1} = \gamma_1 \tag{31}$$

(for derivation of (30) and (31) see Appendix A). It can be directly seen that in the absence of taxes, (31) reduces to (6) with $\tau = 0$. Upon comparing (31) with (6) it can be concluded that x_1^{LN} is lower than x_1^L as long as x_1^{NB} is finite. Consequently, in the presence of taxes the liquidation option is exercised later when the firm is partially financed with debt and renegotiation is possible.

Having determined the value of the firm, we are ready to calculate the solution to the bargaining game. Let φ_i^* be the outcome of the Nash bargaining process being equal to the fraction of the firm received by the shareholders. Given that the value of the firm is described by (28), the shareholders receive $\varphi_1^* V_i^{NB}$ and the debtholders get $\left(1-\varphi_i^*\right)V_i^{NB}$. The outside options (the off-renegotiation payoffs) of equityholders and debtholders are zero and R_i, respectively. Consequently, the solution to the bargaining game can be written as follows:[18]

$$\begin{aligned} \varphi_i^* &= \arg\max_{\varphi}\left[\left(\varphi V_i^{NB}\right)^{\eta}\left(\left(1-\varphi\right)V_i^{NB}-R_i\right)^{1-\eta}\right] \\ &= \eta\frac{V_i^{NB}-R_i}{V_i^{NB}}. \end{aligned} \tag{32}$$

[18] In the formulation of bargaining problem we follow Perraudin and Psillaki (1999), and Fan and Sundaresan (2000), (where for $\eta = 0.5$ the game is the one of Rubinstein, 1982, with $\Delta t \to 0$) who impose this multiplicative form of the objective function. The drawback of an alternative, additive formulation is that it yields bang-bang solutions.

From (32) it can be concluded that the fraction of the firm received by the equityholders in the renegotiation process critically depends on the creditors' outside option, R_i. If the creditors' outside option equals zero (i.e. if $\gamma_i = \rho = 0$), shareholders receive the fraction of the firm equal to their bargaining power coefficient. In the opposite case, i.e. when creditors outside option equals the value of the firm ($\rho = 1$, $\tau = 0$, and $i = 1$), shareholders receive nothing in the renegotiation process. Moreover, the optimal sharing rule again depends on the amount of the current cash flow, x.

3.2 Equity Valuation and Optimal Renegotiation Policy

Having calculated the value of the firm and the optimal sharing rule given the shareholders' renegotiation trigger, we now derive the optimal renegotiation policy. We begin by deriving the formulae for the securities values. Subsequently, we simultaneously determine the optimal renegotiation and investment policy by maximizing the value of the equity and the optimal liquidation policy by maximizing the value of the firm.

Given the value of the firm as a function of the underlying cash flow, we are ready to determine the after-investment value of equity, E_1^{NB}, and to find the optimal renegotiation trigger. The value of equity is determined by solving ODE (2) with an appropriate value-matching condition associated with the renegotiation trigger, x_1^{NB}. Consequently, E_1^{NB} equals

$$E_1^{NB} = \begin{cases} \eta\left(V_1^{NB}(x) - R_1(x)\right) \qquad \left(= \varphi^* V_1^{NB}(x)\right) & x \leq x_1^{NB}, \\ \frac{\theta x(1-\tau)}{\delta} - \frac{b(1-\tau)}{r} + \left(\frac{x}{x_1^{NB}}\right)^{\beta_2} \times & \\ \left(\eta\left(V_1^{NB}\left(x_1^{NB}\right) - R_1\left(x_1^{NB}\right)\right) - \frac{\theta x_1^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r}\right) & x > x_1^{NB}. \end{cases} \tag{33}$$

Applying the smooth-pasting condition allows for finding the optimal renegotiation trigger, x_1^{NB} (cf. Appendix A), which is equal to

$$x_1^{NB} = \frac{-\beta_2}{1-\beta_2} \frac{b(1-\tau+\eta\tau)\delta}{(1-\eta(1-\rho))\theta(1-\tau)r}, \tag{34}$$

It is straightforward to notice that the trigger x_1^{NB} increases with taxes. This is because the effect of taxes on the cash flow that accrues to the firm's shareholders dominates the effect of a temporarily suspended tax shield. Therefore, despite the fact that the tax shield is suspended under renegotiation, the shareholders prefer an earlier debt reorganization.[19]

[19] The impact of taxes on cash flow is not taken into account while analyzing the optimal bankruptcy trigger in Leland (1994) (see footnote 22 therein concerning the *ceteris paribus* assumption) and the renegotiation trigger in Fan and Sundaresan (2000) (see Assumption (6), p. 1061 therein). Consequently, the optimal renegotiation trigger in Fan and Sundaresan (2000) is reported to decrease in taxes since only the effect of the increasing tax shield is taken into account. Moreover, contrary to the result of Fan and Sundaresan (2000) obtained without the liquidation option, introducing taxes does not always imply that shareholders receive a higher fraction of the firm in the renegotiation process.

From (34) it can be seen that the renegotiation trigger is independent from taxes only if η is equal to zero. This is equivalent with the creditors holding the entire bargaining power. In such a case the optimal renegotiation trigger equals the optimal bankruptcy trigger and the latter has already been shown to be independent of taxes (cf. (15)). Moreover, the optimal renegotiation trigger does not depend on the liquidation value γ_1. This results from the fact that the change of the instantaneous payoff when the renegotiation commences is not influenced by the collateral.[20]

The after-investment value of the firm, V_1^{NB}, can be determined now by substituting (29) and (30) into (28). Having also calculated the value of equity, E_1^{NB}, and knowing the value of R_1 (see (11)), we are able to provide the value of its debt, D_1^{NB}. It holds that

$$D_1^{NB} = \begin{cases} (1-\eta)\, V_1^{NB} + \eta R_1 & x \leq x_1^{NB}, \\ \frac{b}{r} + \left((1-\eta)\, V_1^{NB} + \eta R_1 - \frac{b}{r}\right)\left(\frac{x}{x_1^{NB}}\right)^{\beta_2} & x > x_1^{NB}. \end{cases} \tag{35}$$

Figure 3.1 depicts the value of the firm and the claims written on it after the investment is made and there exists a possibility of renegotiation. The value of the firm remains within the band bounded from below by the present value of the cash flow and from above by the present value of the cash flow and of the perpetual tax shield. The value of the equity behaves as in the case without renegotiation with the only difference being that the option to default is replaced by a more highly valued option to renegotiate. The value of debt tends to its riskless valuation for high levels of cash flow as in the previous case, and it equals a fraction of the firm value, $(1-\varphi^*)\, V_1^{NB}$, for its low levels.

At the optimal equityholders' renegotiation trigger, the value of all the claims remain differentiable. For the equity it is the result of the smooth-pasting condition that guarantees optimality of the trigger. For the value of the firm and its debt it is a no-arbitrage condition. Since the renegotiation process is reversible, i.e. the equityholders will restore the original coupon flow, b, as soon as the earnings process again exceeds the critical threshold x_1^{NB}, the first-order derivative of the value of all the claims must be continuous. As a consequence, the value of the firm and of its debt does not exhibit kinks at the renegotiation trigger, x_1^{NB}.

In order to determine the optimal investment, renegotiation and liquidation triggers and the value of the corporate securities, we first observe that the value of equity before investment, E_0^{NB}, can be expressed as

$$E_0^{NB} = \frac{x\,(1-\tau)}{\delta} - \frac{b\,(1-\tau)}{r} + A_{01}x^{\beta_1} + A_{02}x^{\beta_2}. \tag{36}$$

$A_{01}x^{\beta_1}$ and $A_{02}x^{\beta_2}$ are the components of the value of equity associated with the investment and debt renegotiation option, respectively. Using equation (36) for E_0^{NB}, (33) for E_1^{NB}, (28)

[20] If γ_1 was high enough so that $R_1\left(x_1^{NB}\right) = \gamma_1$, then the renegotiation trigger would depend on γ_1. However, this is ruled out by assumption (27).

with $i=0$ and $i=1$ for V_0^{NB} and V_1^{NB}, respectively, and (11) for R_0, we obtain the optimal triggers and securities' values by solving the following system of equations

$$\begin{aligned}
V_0^{NB}(x^*) &= V_1^{NB}(x^*) - I, && (37)\\
E_0^{NB}(x^*) &= E_1^{NB}(x^*) - I, && (38)\\
\left.\frac{\partial V_0^{NB}}{\partial x}\right|_{x=x^*} &= \left.\frac{\partial V_1^{NB}}{\partial x}\right|_{x=x^*}, && (39)\\
E_0^{NB}\left(x_0^{NB}\right) &= \eta\left(V_0^{NB}\left(x_0^{NB}\right) - R_0\left(x_0^{NB}\right)\right), && (40)\\
\left.\frac{\partial E_0^{NB}}{\partial x}\right|_{x=x_0^{NB}} &= \eta\left.\frac{\partial\left(V_0^{NB}-R_0\right)}{\partial x}\right|_{x=x_0^{NB}}, && (41)\\
V_0^{NB}\left(x_0^{LN}\right) &= \gamma_0, && (42)\\
\left.\frac{\partial V_0^{NB}}{\partial x}\right|_{x=x_0^{LN}} &= 0. && (43)
\end{aligned}$$

Equations (37) and (38) are the value-matching conditions required for the value of the firm and equity to be continuous at the optimal investment threshold, x^*. The smooth-pasting condition (39) guarantees the optimality of the investment threshold, x^*. Conditions (40) and (41) are the value-matching and smooth-pasting conditions associated with the optimal renegotiation trigger chosen by the equityholders, respectively. The RHS of (40) is the share of the value of the firm received by the shareholders upon renegotiation. (42) is the value matching condition reflecting the value of the firm at the liquidation trigger. Finally, (43) is the smooth-pasting condition for the value of the firm at the closure point.

Now, we are ready to state the following proposition.

Proposition 1 *The optimal investment threshold, x^*, renegotiation trigger, x_0^{NB}, and liquidation trigger, x_0^{LN}, can be obtained by simultaneously solving the following equations*

$$\begin{aligned}
&\frac{(\theta-1)(1-\tau)}{\delta} - \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b\tau}{rx^*}\left(\left(\frac{x^*}{x_1^{NB}}\right)^{\beta_2} - \left(\frac{x^*}{x_0^{NB}}\right)^{\beta_2}\right)\\
&+\beta_2\left(L_1-L_0\right)\left(x^*\right)^{\beta_2-1} - \beta_1 K_0\left(x^*\right)^{\beta_1-1} = 0, && (44)\\
&\frac{1-\beta_2}{-\beta_2}\frac{x_0^{NB}(1-\tau)(1-\eta(1-\rho))}{\delta} - \frac{b}{r}(1-\tau+\eta\tau)\\
&-\frac{1-\beta_2}{-\beta_2}\left(\eta K_0\left(x_0^{NB}\right) - A_{01}\left(x_0^{NB}\right)\right) = 0, && (45)\\
&\frac{1-\tau}{\delta} + \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b\tau}{rx_0^{LN}}\left(\frac{x_0^{LN}}{x_0^{NB}}\right)^{\beta_1} + \beta_1 K_0\left(x_0^{LN}\right)^{\beta_1-1} = 0. && (46)
\end{aligned}$$

The constants K_0, L_0, A_{01}, and A_{02} are defined by equations (B.1) and (B.2) in Appendix B.

Proof. See Appendix B. ■

Unfortunately, an analytical solution to the above system of equations cannot be obtained. Therefore, we rely on numerical methods. Figure 3.2a depicts the values of the firm, its debt and its equity, in the presence of the investment and renegotiation options.

The boundary conditions for the equity value-maximizing investment policy are the same as for the firm value-maximizing policy, except that

$$\left.\frac{\partial E_0^{NB}}{\partial x}\right|_{x=x^*} = \left.\frac{\partial E_1^{NB}}{\partial x}\right|_{x=x^*} \tag{47}$$

replaces (39). This leads to the following proposition.

Proposition 2 *The shareholders' value-maximizing investment threshold is obtained by solving simultaneously equations (45), (46), and*

$$\frac{(\theta-1)(1-\tau)}{\delta} + \beta_2 (A_{12} - A_{02}) (x^*)^{\beta_2 - 1} - \beta_1 A_{01} (x^*)^{\beta_1 - 1} = 0. \tag{48}$$

Constants A_{01}, A_{02} and A_{12} are defined by equations (B.2) and (B.3) in Appendix B.

Proof. See Appendix B. ■

Figure 3.2b depicts the value of the firm, and its debt and equity in the presence of the investment and renegotiation options when the second-best investment policy is implemented. Now, it is the value of equity that is differentiable at x^* (cf. (47)).

Finally, we are able to present the optimal debt service prior to and after exercising the growth option. The coupon stream resulting from renegotiating the original debt contract, c_i^{NB}, can be expressed as follows

$$c_i^{NB} = \begin{cases} (1-\eta)\, x\theta^i (1-\tau) + \eta r \gamma_i & x \in \left(x_i^{LN}, x_i^{LR}\right], \\ (1-\eta(1-\rho))\, x\theta^i (1-\tau) & x \in \left(x_i^{LR}, x_i^{NB}\right], \\ b & x > x_i^{NB}. \end{cases} \tag{49}$$

The first regime of the strategic debt service corresponds to earnings remaining between the firm's optimal liquidation trigger, x_i^{LN}, and the level triggering liquidation if the firm was run by the creditors, x_i^{LR}. In this case the creditors receive a weighted average of cash flow from holding the collateral, $r\gamma_i$, and operating the firm as the first-best users, $x\theta^i (1-\tau)$. These streams are weighted with the shareholders' bargaining power coefficient, η. For the earnings level above x_i^{LR}, but still in the renegotiation region, the creditors receive a weighted average of the cash flow from operating the company as the second-best , $x\rho\theta^i (1-\tau)$, and as the first-best users, $x\theta^i (1-\tau)$. Outside the renegotiation region, the contractual coupon b is paid.

Note that for $\tau = 0$ and $\eta \in \{0, 1\}$ the coupon schedule corresponds to the outcome of the take-it or leave-it offers in Mella-Barral and Perraudin (1997), whereas setting γ_i to zero reduces the solution to the payment scheme of Fan and Sundaresan (2000).

On the basis of (49) it can be concluded that the presence of the growth opportunity does not change the coupon flow to the creditors within given regimes. This results from the following fact. In the bargaining process both groups of stakeholders receive the following portfolio: a fraction of the firm's value, V_i^{NB}, and the fraction of the creditors' outside option, R_i. Strategic debt service reflects cash flows to which these portfolios of securities are entitled. Since the investment opportunity that constitutes a part of the firm's value is not associated with any payment stream, the strategic debt service within a given regime is not influenced by its presence.

Although the growth option does not influence cash flows from the firm's securities, it affects, via its impact on optimal triggers, the regimes determining the structure of payoff under the strategic debt service. Let us observe that the following relationships hold

$$\frac{x_1^{LN}}{x_0^{LN}} > \frac{\gamma_1}{\theta\gamma_0} = \frac{x_1^{LR}}{x_0^{LR}} \lesseqqgtr \frac{x_1^{NB}}{x_0^{NB}}. \tag{50}$$

The first inequality is implied by the positive value of the growth option. Without the growth option, the liquidation trigger x_0^{LN} would be equal to $\theta\gamma_0 x_1^{LN}/\gamma_1$. However, the presence of growth option raises the opportunity cost of liquidating the firm. As a consequence, the firm is liquidated optimally at a cash flow level lower than $\theta\gamma_0 x_1^{LN}/\gamma_1$. The equality in the middle follows from the solution to the creditors' liquidation problem when the value of the firm run by the creditors is given by (11). The remaining relationship reflects an ambiguous sign of the impact of the growth opportunity on the renegotiation policy.

All the above relationships directly translate into the changes in the strategic debt service resulting from the presence of the growth option. First, the inequality on the left reflects the effect of the investment opportunity on the liquidation trigger. It implies that in the presence of the growth option, the debt will be strategically serviced for a longer period before the ultimate decision to abandon the firm. Furthermore, the boundary between the regimes delineated by the trigger x_1^{LR} is unaffected by the presence of the investment opportunity. After all, the creditors running the company after the bankruptcy do not hold the growth option anymore. Finally, the impact of the investment opportunity on the cash flow level that triggers the renegotiation is ambiguous. On the one hand, since the value of equity contains an additional component reflecting the value of the option to invest, the equityholders' value of the outside option increases, which makes renegotiation *ceteris paribus* less attractive. However, the value of the firm is also higher when the investment opportunity exists. Therefore, the value of renegotiation increases as well. Since these two effects work in the opposite directions, the presence of the investment opportunity can, in general, either raise or reduce the renegotiation trigger.

Proposition 3 *The optimal renegotiation threshold in the presence of the investment opportunity can either be lower or higher than the corresponding threshold in a situation where there is no such opportunity. The condition*

$$\eta K_0 > A_{01} \tag{51}$$

determines the range of η in which the presence of investment opportunity results in earlier renegotiation.

Proof. See Appendix B. ■

From Proposition 3 we conclude that it is possible to determine the critical level of the shareholders' bargaining power, η, that demarcates the two cases. It holds that under both the first-best and second-best solution, the optimal renegotiation trigger exceeds the one without the investment opportunity if and only if $K_0 - A_{01} > \frac{1-\eta}{\eta} A_{01}$. This condition describes the case where the present value of the wealth transfer to the creditors occurring upon investment exceeds the value of the option to invest that accrues to the shareholders by more than a factor $\frac{1-\eta}{\eta}$. This means that if the bargaining power of the shareholders is high enough, it is optimal for them to begin the renegotiation process earlier in the presence of an investment opportunity. By doing so, the shareholders forgo the component of the value of equity associated with the investment option $A_{01}x^{\beta_1}$, but they are more than compensated by receiving a fraction (dependent on η) of the firm's value including the firm's growth option $K_0 x^{\beta_1}$.

Introducing the option to renegotiate the debt may adversely affect the value of the debt itself. This happens in a situation where the renegotiation trigger is close, but the bankruptcy trigger (in the absence of renegotiation) lies much below the renegotiation trigger, i.e. when the shareholders' bargaining power, η, is sufficiently high and the efficiency of creditors as the would-be managers, ρ, is low. Naturally, for x close enough to the bankruptcy trigger, allowing for renegotiation increases the debt value since the creditors' renegotiation payoff is higher than the one received after the bankruptcy.

4 Numerical Results and Testable Implications

This section presents comparative statics concerning the firm's optimal investment, liquidation, and debt restructuring policies, the first passage time probabilities and securities' values. Moreover, it presents some testable implications of the model. The input parameters used for graphical illustrations are as follows: risk-free rate $r = 5\%$, drift rate of the earnings process $\alpha = 1.5\%$, volatility of earnings $\sigma = 20\%$, effective tax rate $\tau = 35\%$, instantaneous coupon $b = 0.66$, efficiency of the creditors as the second-best users of the firm's assets $\rho = 50\%$, bargaining power of the shareholders $\eta = 0.5$, liquidation value before investment $\gamma_0 = 1$, investment cost $I = 10$, earnings multiplier resulting from exercising the growth option $\theta = 2$, liquidation value after investment $\gamma_1 = 2$. In Subsection 4.1 we analyze the optimal policies, whereas in Subsection 4.2 we look at the first passage time probabilities. Subsection 4.3 discusses securities' valuation and Subsection 4.4 provides empirical implications.

4.1 Optimal Policies

The comparative statics for optimal investment, debt restructuring, and liquidation triggers are depicted in Table 4.1 below.

	σ	α, δ	r, δ	r, α	b	ρ	η	I, θ^{-1}	τ	γ_0	γ_1
x^*	+	−	+	(i)	(ii)	(iii)	(iv)	+	+	(v)	(v)
x_0^{NB}	−	−	+	−	+	−	+	+	+	−	+
x_0^{B}	−	−	+	(i)	+	(vi)	0	+	+	(vi)	(vi)
x_0^{LN}	−	−	+	+	+	−	+	+	+	+	−

Table 4.1. Comparative statics concerning the optimal investment, x^*, renegotiation, x_0^{NB}, bankruptcy, x_0^B, and liquidation, x_0^{LN}, thresholds. "+" ("−") denotes a positive (negative) derivative with respect to a given parameter. The numbers in brackets refer to the explanatory notes in the text.

The signs of first derivatives for both the first-best and second-best policy are included in Table 4.1. Below, we provide a discussion of those results that differ from the well-known results from the real options and corporate finance literature.

(i) From real options theory it is known that under all-equity financing the relationship between the optimal investment threshold and the risk-free *interest rate*, r, given constant return shortfall, δ, is increasing.[21] Such a relationship holds because the wedge between the Marshallian and optimal investment threshold increases in r, whereas the present value of the project does not change. Debt financing introduces another effect, which works in the opposite direction. Given that the coupon b is fixed, a higher r is associated with a lower debt value, and thus with a lower magnitude of the underinvestment problem. Consequently, a higher r can stimulate earlier investment since it is associated with a lower wealth transfer from shareholders to debtholders. The latter effect dominates if cash flow uncertainty is low. For low levels of uncertainty the optimal investment threshold is low, and this implies a relatively high leverage at the moment of undertaking the project. In such a case the impact of the change in r on the value of wealth transfer to debtholders is high and the wealth transfer effect dominates

[21] See Dixit and Pindyck (1996), Ch. 6.

the waiting option effect. As a result, for low cash flow uncertainty the relationship between interest rate and optimal investment threshold is U-shaped (cf. Figure 4.1).[22]

(*ii*) The impact of *leverage*, b, on the optimal investment threshold for the first-best and second-best solutions differs (see Figure 4.2). If the investment is made so as to maximize the value of the firm, the optimal investment threshold decreases with leverage. The latter relationship results from a higher increase in the present value of the tax shield upon completing the investment. The opposite is true in the situation where the investment threshold is chosen so to maximize the value of equity. In this case the optimal investment threshold increases with leverage. This can be explained by the wealth transfer from the equityholders to the debtholders, positively related to the level of leverage. The wealth transfer occurs since after undertaking the project the renegotiation trigger is lower than before the investment has been made.

(*iii*) The outside option of the debtholders, ρ, influences the optimal investment threshold either by delaying investment, if the threshold is chosen so that the value of the firm is maximized, or by accelerating it, if the shareholders choose the investment timing (cf. Figure 4.3). The reason for which the first-best investment threshold increases in ρ is that the optimal renegotiation trigger decreases in ρ. Consequently, since a lower renegotiation trigger is equivalent to a lower increase of the PV of the tax shield, the value of the project decreases in ρ and the investment is undertaken later. In the special case of $\tau = 0$, the tax shield argument is no longer present and the threshold is equal to the 100% equity one. Conversely, if the value of equity is maximized, a lower wealth transfer associated with high ρ (thus low x_0^{NB}) moves the investment threshold closer to the all-equity case. When the second-best solution is applied, the wealth transfer from debtors to creditors always occurs upon investment so that even in case of $\tau = 0$ the equity value-maximizing investment rule differs from the one given by the optimal all-equity threshold.

(*iv*) The shareholders' bargaining power, η, affects the optimal investment threshold in an opposite way than ρ (cf. Figure 4.4). If the timing of investment is chosen optimally so as to maximize the value of the firm, the optimal investment threshold decreases in η. This results from the fact that the value of the investment opportunity to the firm increases in η, since the present value of the additional tax shield (due to investment) increases. For an analogous reason as in (*iii*), the first-best investment threshold is insensitive to changes in η in the absence of taxes. However, if the timing of investment is chosen by the equityholders so that the value of equity is maximized, the optimal investment threshold increases in η. This is due to the fact that the renegotiation trigger

[22] When the first-best solution is applied, the wealth transfer to the debtholders does not directly influence the investment policy so that the optimal investment threshold always increases in r.

is positively related to η. Since the renegotiation trigger decreases upon investment, debtholders benefit most from investment when the initial trigger is high. A higher wealth transfer that accrues to the debtholders upon undertaking the project results in a later investment.

(v) The impact of the liquidation value of the firm on the optimal investment policy depends on the presence of the renegotiation option and on the fact whether the first-best solution can be implemented (cf. Figure 4.5). When the investment threshold is chosen as to maximize the value of the firm, the investment is always undertaken later (thus closer to the all-equity trigger) when the liquidation value γ_0 (γ_1) is higher (lower). This results from the fact that investment becomes less attractive if it is associated with a lower increase in the liquidation value. This effect is reversed if in the presence of the renegotiation option the choice of the investment trigger maximizes the equity value. Since a higher initial liquidation value negatively influences the probability of strategic default, the wealth transfer to the debtholders, which occurs at the moment of investment, is lower. This results in an earlier investment. The same argument can be applied to analyze the impact of γ_1. Finally, when renegotiation is not allowed for and the second-best solution is implemented, the investment trigger does not depend on the firm's liquidation value.

(vi) The bankruptcy trigger, x_0^B, is influenced neither by the firm's liquidation value nor by the efficiency of the creditors as the second-best users of the firm's assets as long as the investment threshold is chosen as to maximize the equity value. In a situation where the first-best solution can be implemented, the optimal bankruptcy threshold is positively related to the liquidation value γ_1 and negatively related to the creditor's efficiency and liquidation value γ_0. A positive change in a liquidation value and low creditor's efficiency make investment particularly attractive since it lowers the present value of the economic cost of bankruptcy. Consequently, investment occurs too early comparing with the case when the effect of the change of economic costs of bankruptcy is absent. This results in a lower value of the firm's claims as a going-concern and lower opportunity cost of bankruptcy.

4.2 First Passage Time Probabilities

Interactions between the options to scale up the operations and to reorganize debt can already be observed by analyzing the relevant optimal triggers. However, since equityholders face a double-barrier control problem, there is no one to one correspondence between the optimal triggers and the first passage time probabilities. Therefore, we extend the analysis and calculate the first passage time probabilities associated with the optimal renegotiation trigger and with the optimal investment threshold.

In order to evaluate the influence of a given option, or parameter, on the relevant decision trigger, we calculate the probabilities of reaching the trigger within a time interval of length T. For example, the probability of strategic debt restructuring is equivalent to the probability of the cash flow process hitting, either the renegotiation trigger, x_0^{NB}, or, first, the investment threshold x^* and then the renegotiation trigger, x_1^{NB}. Conversely, the probability of investment equals the probability of hitting the investment threshold, x^*, conditionally on not hitting the liquidation trigger, x_0^{LN}. The derivation of the relevant probabilities, based on solving a partial differential equation (PDE), is presented in Appendix C.

In Table 4.2 we present the comparative statics concerning the first passage time probabilities. The presented results have been obtained by numerical calculation of the relevant probabilities for an extensive range of input parameters.

	σ	α,δ	r,δ	r,α	b	ρ	η	I,θ^{-1}	τ	T	γ_0	γ_1
p^*	(vii)	$+$	$-$	$(viii)$	(ii)	(iv)	(iv)	$-$	$-$	$+$	(v)	(v)
p^{NB}	(vii)	$-$	$+$	$-^*$	$+$	$-$	$+$	$+$	$+$	$+$	$-^*$	$+^*$
p^B	(vii)	$-$	$+$	$-^*$	$+$	(vi)	0	$+$	$+$	$+$	(vi)	(vi)

Table 4.2. Comparative statics concerning the first passage time probabilities associated with investment, p^*, debt renegotiation, p^{NB}, and bankruptcy, p^B.* relationship can be reversed when $x^* - x$ is very small. The numbers in brackets refer to the explanatory notes in the text.

(vii) Non-monotonicity of the investment-uncertainty relationship has been already pointed out by Sarkar (2000) and analyzed further in Pawlina and Kort (2001). It crucially depends on the relationship between the horizon T and the time to reach the deterministic Jorgensonian threshold. From Pawlina and Kort (2001) it is obtained that if the horizon T is relatively short, the investment-uncertainty relationship is humped, while for high T it is negative. Another factor that influences the probability of investment in this double-barrier problem of the firm is the probability of bankruptcy (or of liquidation when renegotiation is possible) which is also sensitive to the changes in uncertainty. On the basis of Figure 4.6 one can conclude that higher uncertainty results in a lower probability of investment when cash flow is high. However, for lower levels of cash flow, uncertainty raises the probability of investment since bankruptcy becomes less likely. The latter holds since the bankruptcy threshold decreases with σ. The presence of renegotiation option affects the probability of investment twofold. First, it raises the optimal investment threshold. Second, it allows to preserve the investment opportunity for the levels of cash flow lower than the bankruptcy trigger. The smaller than one ratios of the probabilities with and without renegotiation illustrate that the effect of an increased investment threshold in the presence of renegotiation more than offsets the impact of losing the investment opportunity upon bankruptcy (see Figure 4.7).

(*viii*) An increase in the interest rate, r, when the return shortfall, δ, is kept constant, can change the probability of investment in both directions. If the investment threshold decreases in r (see (*i*)), then the probability of investment always increases in r. However, when the investment threshold is positively related to r (also see (*i*)), the sign of the investment-interest rate relationship is ambiguous. This results from the fact that such increase in the investment threshold is counterbalanced by the increase in the cash flow drift rate as well as by a decrease in the bankruptcy trigger (see (15)). The sign of the joint effect depends on the specific choice of model parameters.

What remains to be considered is the relationship between the presence of the growth option and the probability of strategic debt restructuring. Renegotiation is more likely when the debtors are given more bargaining power (cf. Figure 4.8). However, the magnitude of the influence of bargaining power on the renegotiation probability highly depends on whether the firm holds positive NPV growth opportunities. Such a comparison is illustrated in Figure 4.9. It appears that in the presence of a positive NPV project, the probability of debt renegotiation can be *higher* than without the investment option. Such a situation occurs when the actual renegotiation trigger x_0^{NB} exceeds the renegotiation trigger without the investment opportunity (equal to $x_1^{NB}\theta$), and the current cash flow is not excessively high.[23] This situation occurs when the shareholders' bargaining power, η, is large (cf. Proposition 3). This effect is magnified for moderate levels of uncertainty (high uncertainty relatively increases the shareholders' value of the investment option which makes renegotiation less likely).

4.3 Valuation of Securities

In this section the comparative statics concerning the valuation of the firm's securities are presented. Since the signs of the relevant relationships does not depend on the presence of the renegotiation option, the existence of such an option is assumed here. Table 4.3 depicts the direction of the impact of model parameters on the valuation of equity, debt and the entire firm.

[23] In the absence of the renegotiation option the bankruptcy triggers are the relevant ones. Since it holds that x_0^B is always lower than $x_1^B\theta$, the presence of the investment opportunity always reduces the default probability when there is no option to renegotiate.

	σ	α, δ	r, δ	r, α	b	ρ	η	I, θ^{-1}	τ	γ_0	γ_1
E_0^{NB}	$+$	$+$	$-$	$+$	$-+$	$-$	$+$	$-$	$-$	$-$	$-$
D_0^{NB}	(ix)	$+$	$-$	(xi)	$+-$	$+$	$-$	$-$	$-$	$+$	$+$
V_0^{NB}	(x)	$+$	$-$	(xii)	$+-$	$+$	$-$	$-$	$-$	$+$	$+$

Table 4.3. Comparative statics concerning the valuation of the firm, its debt and equity. "$+$" ("$-$") denotes a positive (negative) derivative with respect to a given parameter, and "$+-$" ("$-+$") indicates a humped (U-shaped) relationship. The numbers in brackets refer to the explanatory notes in the text.

Since changes in the valuation of the claims resulting from the changes in input parameters are mostly consistent with those reported in the dynamic capital structure literature (e.g. Leland, 1994), we mainly discuss the results that are directly influenced by the interactions between the option to invest and to restructure the debt.

(ix) The relationship between the *cash flow volatility*, σ, and the value of debt, D_0^{NB}, depends on the current level of the earnings process, x. When this level is high, the value of the debt decreases in volatility since higher volatility makes renegotiation, other things equal, more likely. However, for realizations of x sufficiently close to x_0^{NB}, two other effects result in a positive relationship between the value of the debt and uncertainty. First, for low x, the impact of x_0^{NB} decreasing in σ is stronger than the impact of a higher probability of hitting any *fixed* trigger lower than x.[24] Second, the renegotiation value of debt rises in σ. The latter relationship results from the fact that the value of the firm rises in σ, because of the included investment opportunity component.

(x) The relationship between the *cash flow volatility*, σ, and the value of the firm, V_0^{NB}, results from the impact of the volatility on the value of debt and equity. For a given σ and varying x, the value function is first convex (which mainly reflects the option value of the tax shield after the contractual debt service is restored), then it becomes concave (as a result of a short option on the tax shield once contractual service is restored), becomes once again convex (when the option component associated with the investment opportunity starts to dominate) and, eventually, becomes and remains concave (when it value-matches to $V_1^{NB} - I$, see (37)). Consequently, the effect of changes in σ is only unambiguous when the firm either is financially distressed (positive relationship) or close to the optimal exercise of its growth option (negative relationship).

(xi) The sign of the relationship between the value of debt and the risk-free *interest rate*, r, given constant return shortfall, δ, is in general ambiguous. The relationship is hump-shaped for low uncertainty combined with a high convenience yield, and decreasing

[24] Using a similar reasoning Leland (1994) explains the behavior of junk bonds.

otherwise. If the firm's debt was riskless, its value would always decrease in r irrespective of the drift rate, α, and return shortfall, δ. Here, the positive probability of renegotiation makes it risky. A very low interest rate, in combination with a positive return shortfall, is associated with a negative drift rate. If the uncertainty is small, the stochastic discount factor associated with renegotiation is high. Therefore, for low levels of uncertainty the value of the debt may benefit from an increasing interest rate if the latter is sufficiently low.

(xii) The relationship between the value of the firm and the risk-free *interest rate*, r, given constant return shortfall reflects the impact of r on the value of equity, E_0^{NB}, and debt, D_0^{NB}. Since the value of equity increases monotonically in r, the impact of the interest rate on the value of the firm depends on the relative slope of the debt value function comparing to equity. Since the former can both increase and decrease in r (see (xi)), the value of the firm is in general hump-shaped or increasing in r. For a very high r, the value of the firm levels off since the impact of changes in leverage becomes negligible (as $b/r \to 0$).

The comparative statics results from the last two columns of Table 4.3 coincide with the findings in the recent dynamic capital structure literature (cf. Flor, 2002, and references therein). It appears that ex post (i.e. when the capital structure is already fixed) the value of the firm's equity decreases in the asset resale value, γ_i. This results from the fact that the asset resale value increases the bargaining position of the creditors (who can always seize the assets upon the violation of the original debt contract by the equityholders), who are granted bigger concessions in the renegotiation process.

4.4 Empirical Implications

Testing empirical predictions of our model requires identifying proxy variables that can capture the effects of a different cost of renegotiation (in the model we consider only two polar cases: zero costs and costs offsetting entire benefits from renegotiation), equityholders' bargaining power η, and creditors' outside option, ρ. The costs of renegotiation (cf. Bolton and Scharfstein, 1996) are expected to be low when the firm is financed with a bank debt or, in general, when the number of its creditors is small. The distribution of bargaining power (cf. Hackbarth et al., 2002) crucially depends on the firm's size, age, and degree of diversification. Moreover, it is also influenced by the country's legal system (US Bankruptcy Code of 1978 is more shareholder-friendly than the codes in most continental European countries). Finally, creditors' efficiency as managers of the firm is expected to be higher when the brand recognition is low (cf. Mella-Barral, 1999) and in the sectors with low intensity of R&D.

In this section, we first analyze the sensitivity of investment to the firm's cash flow. Subsequently the stock price behavior and credit spreads are discussed. Finally, some social welfare results are presented.

Investment-cash flow sensitivity. The set-up of this paper's model stipulates that investment is triggered by a sufficiently high level of cash flow from operations. This implies that a higher magnitude of Myers' (1977) underinvestment makes the investment *ceteris paribus* less likely to be triggered by an incremental cash flow increase. As a consequence, the presence of the renegotiation option and high shareholders' bargaining power, which both result in higher underinvestment, is likely to decrease the sensitivity of investment to the firm's cash flow. Therefore, our model provides an alternative explanation of the empirical evidence that small and young firms exhibit relatively higher investment-cash flow sensitivity (cf. Lensink et al., 2001, Ch. 3, and references therein). Since small firms usually have a limited bargaining power in the debt renegotiation with banks, the magnitude of the additional underinvestment resulting from the renegotiation option will be in the most cases insignificant. This relatively lower magnitude of underinvestment implies that their investment-cash flow sensitivity is likely to remain high. The same argument can be used to claim that the capital investment of big and mature firms with dispersed bond market debt will be on average more sensitive to cash flow than investment of similar firms with a mixture of bank and bond market debt and with bank debt only (cf. Moyen, 2001).

Stock price behavior. Asymmetric returns are inherent to the equity of firms that hold a substantial portfolio of real options. As Bernardo and Chowdhry (2002) point out (cf. also Berk et al., 1997, and Pope and Stark, 1997), positive earnings surprises have a stronger effect on the prices of equity than negative ones. This is because the presence of a real option makes the payoff to equityholders convex in the stochastic variable that underlies the firm's cash flow. In the current model, the equity value function consists of two convex components, options to invest and to restructure the debt/declare bankruptcy, and one linear, present value of cash flow. Therefore, it is itself convex. As a consequence, the stock price returns exhibit right-skewness.

The presence of an investment and a renegotiation option has also implications for the responsiveness of the stock price to the earnings surprises. Upon introducing the renegotiation option alone, one can observe that the stock price becomes less responsive to the earnings surprises. This is associated with a decrease of the first derivative of the equity value function with respect to the process x. The reason for that is that the renegotiation option has a relatively higher value in the adverse states of nature (i.e. for low realizations of x). Consequently, any variation in x results in less drastic changes in E_0 in the presence of renegotiation option. The responsiveness of the stock price to the earnings surprises is magnified by introducing the growth option. This results from the fact that higher realizations of x

not only give rise to the present value of cash flow but also enhance the value of the growth option. As a consequence, the derivative $\frac{\partial E_0}{\partial x}$ increases and so does the responsiveness to the earnings surprises.

Credit Spreads. The riskiness of debt reflected by the credit spread is highly influenced by the presence of both an investment and a renegotiation option. On the basis of the formula for the credit spread (in bps), SPR, where

$$SPR = \left(\frac{b}{D_0} - r \right) * 100, \tag{52}$$

it can be concluded that for a given coupon and a riskless rate, the credit spread is inversely monotonic in the market value of debt. Consequently, the results of the analysis of Section 6.3 can be translated into implications for the credit spreads.

The first theoretical prediction is that the presence of growth options reduces *ceteris paribus* credit spreads. Anticipated future exercise of such options is associated with the prospect of lowering both the bankruptcy and renegotiation thresholds, which negatively affects the riskiness of the debt. In the absence of a renegotiation option, introducing the growth option the results not only in a decreasing the after-investment bankruptcy threshold but also in lowering the initial bankruptcy threshold. The latter holds since the opportunity cost of declaring bankruptcy is higher in the presence of the growth option. Consequently, in the absence of the renegotiation option, the impact of the investment opportunity on credit spreads is substantial.

When the renegotiation option is allowed for, a lower renegotiation threshold, which arises after completing the investment, reduces the riskiness of the debt even before the investment project is undertaken. However, there is a second effect that can increase the firm's credit risk. Contrary to the bankruptcy case, the impact of the growth option does not have to make the debt restructuring less likely. In the situation described in Proposition 3, the presence of the growth option increases the renegotiation trigger. This can lead to a higher riskiness of the debt, resulting in a higher credit spread. The magnitude of both opposing effects highly depends on the shareholders' bargaining power and the creditors' outside option. Higher shareholders' bargaining power results in a higher magnitude of the latter effect, whereas a higher creditors' outside option has an opposite effect. In general, for an extensive grid of the model parameters' values, the presence of the growth option reduces credit spreads even in the presence of strategic debt restructuring.

The impact of the market parameters such as interest rate, return shortfall and earnings volatility, as well as of the indirect bankruptcy costs is consistent with the literature on firm-value based models of credit risk (cf. Anderson and Sundaresan, 2000).

Social Value of the Firm. According to Hege and Mella-Barral (2000), the social value of the firm is not affected by the distribution of the bargaining power among the debtors and the creditors. The reason is that any loss of the tax shield, which is associated with premature renegotiation due to a higher bargaining power of the debtors, is just a transfer to the government. Contrary to that observation, in the current model the distribution of the bargaining power has an externality on the investment and the liquidation decision. Despite the fact that the changes in the present value of the tax shield do not directly influence the social value of the firm (they merely change the redistribution of wealth), they do affect the investment and liquidation policy. Consequently, in order to assess the impact of the distribution of bargaining power on the social value of the firm, one has to compare the first-best investment and liquidation thresholds calculated under all-equity financing assumption with the ones determined in the presence of a mixed capital structure.

In our set-up debt distorts the optimal investment and liquidation policies. As it can be seen from Figure 4.2, the optimal equityholders' investment threshold is higher than in the all-equity case. Moreover, the optimal investment threshold increases in the shareholders' bargaining power coefficient. Consequently, a high shareholders' bargaining power exacerbates the underinvestment problem, in this case the inefficiently late exercise of the option to expand (i.e. beyond the point at which the marginal cost of investing equalizes with the marginal revenue from expansion taking into account irreversibility and uncertainty).

Allowing for the possibility of renegotiating the original debt contract results in the liquidation trigger being a function of the shareholders' relative bargaining power. This is because the liquidation trigger is determined so as to maximize the value of the firm. The latter quantity is endogenous and depends on the renegotiation trigger that in turn is affected by the distribution of bargaining power. As it can be concluded on the basis of Table 4.1 the optimal liquidation threshold is an increasing function of η. The optimal liquidation threshold in the presence of debt financing and renegotiation lies between the all-equity liquidation threshold in the world without taxes and the all-equity threshold with when corporate tax is sufficiently high. Therefore, reducing the shareholders' relative bargaining power mitigates the negative externality of debt on the optimal liquidation decision.

We conclude that there are two negative welfare effects of a high bargaining power of the debtors. The first is associated with an excessively delayed investment, and the other with a too early liquidation.

5 Conclusions

The investment policy of the firm is affected by its capital structure. Introducing debt financing results in an inefficient delay in exercising the growth option. We show that eliminating costly bankruptcy by introducing the possibility of debt restructuring does not solve this problem. In fact, underinvestment is higher if the renegotiation option exists. This results from the fact that the wealth transfer from the equityholders to the debtholders, which occurs at the moment of exercising the investment option, is higher when the renegotiation option exists.

The departure from the all-equity financing affects the firm's liquidation policy. If renegotiation is not allowed for, the decision to liquidate the firm is made by the creditors who become the owners of the firm upon the bankruptcy. This results in an ex ante inefficient liquidation and this inefficiency constitutes part of the indirect bankruptcy costs. The introduction of a mixed capital structure combined with a renegotiation option influences the optimal liquidation policy twofold. First, the presence of the tax shield delays liquidation since *ceteris paribus* it enhances the value of the firm. Second, partial debt financing leads to the departure from the first-best investment policy, which results in the value of the firm being deteriorated and in the opportunity cost of its liquidation being lowered. For sufficiently high taxes the former effect dominates, thus liquidation occurs later than under all-equity financing but not as late as under the optimal liquidation all-equity financing in the world without taxes. Since there exists a positive relationship between the liquidation trigger and the shareholders' bargaining power, reducing this power brings the liquidation policy closer to the optimum.

Furthermore, we show that the debt restructuring policy is affected by the presence of the growth option. The growth option positively influences the renegotiation trigger if a high shareholders' bargaining power is combined with a substantial wealth transfer to the creditors occurring upon investment. In the opposite situation, this is when the creditors possess higher bargaining power and if they do not gain much upon investment, the renegotiation trigger falls.

Finally, we would like to indicate several extensions that may potentially constitute interesting research areas. A more realistic setting would include constructing a model with multiple investment opportunities (cf. Morellec, 2001). The model can also be extended to provide a pricing framework for a renegotiable debt with finite maturity where the coupon flow is a function of the underlying state variable (cf. Shackleton and Wojakowski, 2001). Moreover, the current analysis can be modified to incorporate the impact of product market interactions on the firm's investment behavior (the area pioneered by Fries et al., 1997, and Lambrecht, 2001). Another extension would include investigating the impact of Chapter 11 regulation on the intra-industry bankruptcy intensity. Current anecdotal evidence often indicates that artificially sustained capacity results in a lower sector profitability and, as a consequence, a higher chance of exit of other players.[25] The choice of the second-best solution in the current modeling set-up calls for an introduction of an executive compensation scheme that would allow for aligning the incentives of the self-interested managers with the value of the firm. Such alignment may prove to be ex post optimal from the equityholders' point

[25] Cf. *The Economist*, 7th September 2002, The firms that can't stop falling: Bankruptcy in America, and 14th December 2002, Testing the limits of Chapter 11.

of view. Finally, the divergence of the stakeholders' objectives may lead to an asset substitution problem, which will influence the equityholders' investment policy (cf. Leland, 1998, and Subramanian, 2002, in an agency, and Dangl and Lehar, 2002, in a banking regulation application).

A Derivation of Formulae

Derivation of (29). The value of the tax shield, TS_i, satisfies ODE (2) with the following instantaneous payoffs coefficients

$$(B,C) = \begin{cases} (0,0) & x < x_i^{NB}, \\ (0,b\tau) & x \geq x_i^{NB}. \end{cases}$$

Consequently TS_i can be written as

$$TS_i = \begin{cases} M_1 x^{\beta_1} + M_2 x^{\beta_2} & x < x_i^{NB}, \\ \frac{b\tau}{r} + M_3 x^{\beta_1} + M_4 x^{\beta_2}. & x \geq x_i^{NB}. \end{cases} \tag{A.1}$$

Since

$$\lim_{x\uparrow\infty} TS_i = \frac{b\tau}{r}, \text{ and} \tag{A.2}$$

$$\lim_{x\downarrow 0} TS_i = 0, \tag{A.3}$$

it holds that $M_2 = M_3 = 0$. The only remaining unknown constants are M_1 and M_4. They can be determined by applying the value-matching and smooth-pasting conditions at x_i^{NB}

$$\lim_{x\uparrow x_i^{NB}} TS_i = \lim_{x\downarrow x_i^{NB}} TS_i, \tag{A.4}$$

$$\left.\frac{\partial TS_i}{\partial x}\right|_{x\uparrow x_i^{NB}} = \left.\frac{\partial TS_i}{\partial x}\right|_{x\downarrow x_i^{NB}}, \tag{A.5}$$

which results in

$$M_1 = \frac{b\tau}{r}\frac{-\beta_2}{\beta_1-\beta_2}\left(x_i^{NB}\right)^{-\beta_1}, \text{ and} \tag{A.6}$$

$$M_4 = \frac{b\tau}{r}\frac{-\beta_1}{\beta_1-\beta_2}\left(x_i^{NB}\right)^{-\beta_2}. \tag{A.7}$$

■

Derivation of (30)-(31). The value of the firm at the optimal liquidation trigger satisfies the Bellman equation (2) with $B = \theta(1-\tau)$ and $C = 0$, subject to the following value-matching and smooth-pasting conditions

$$V_1^{NB}\left(x_1^{LN}\right) = \tag{A.8}$$

$$\frac{x_1^{LN}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1} + L_1\left(x_1^{LN}\right)^{\beta_2} = \gamma_1,$$

$$\left.\frac{\partial V_1^{NB}}{\partial x}\right|_{x=x_1^{LN}} = \tag{A.9}$$

$$\frac{\theta(1-\tau)}{\delta} + \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b\tau}{x_1^{LN} r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1} + \beta_2 L_1\left(x_1^{LN}\right)^{\beta_2-1} = 0.$$

The constant L_1 can be directly calculated from (A.8). Multiplying both sides of (A.8) by $\beta_2 x_1^{LN}$ and subtracting it from (A.9) yields the implicit formula for x_1^{LN}. ■

Derivation of (34). When the shareholders' optimal renegotiation trigger is approached from above, the value of equity satisfies the Bellman equation (2) with $B = \theta(1-\tau)$ and $C = -b(1-\tau)$, subject to the following value-matching and smooth-pasting conditions

$$\lim_{x \downarrow x_1^{NB}} E_1^{NB} = \frac{x_1^{NB}\theta(1-\tau)}{\delta} - \frac{b(1-\tau)}{r} + A_{12}\left(x_1^{NB}\right)^{\beta_2}, \tag{A.10}$$

$$\begin{aligned}
&\lim_{x \uparrow x_1^{NB}} E_1^{NB} = \eta\left(V_1^{NB} - R_1\right) \\
&= \eta\left[\frac{x_1^{NB}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r}\right. \\
&+ \left(\gamma_1 - \frac{x_1^{LN}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}\right)\left(\frac{x_1^{NB}}{x_1^{LN}}\right)^{\beta_2} \\
&\left. - \left(\gamma_1 - \frac{x_1^{LR}\rho\theta(1-\tau)}{\delta}\right)\left(\frac{x_1^{NB}}{x_1^{LR}}\right)^{\beta_2}\right],
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
&\lim_{x \uparrow x_1^{NB}} \frac{\partial E_1^{NB}}{\partial x} = \eta\left[\frac{\theta(1-\tau)}{\delta} + \frac{-\beta_1\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r x_1^{NB}}\right. \\
&+ \frac{\beta_2}{x_1^{NB}}\left(\gamma_1 - \frac{x_1^{LN}\theta(1-\tau)}{\delta} + \frac{-\beta_2}{\beta_1-\beta_2}\frac{b(1-\tau)}{r}\left(\frac{x_1^{LN}}{x_1^{NB}}\right)^{\beta_1}\right)\left(\frac{x_1^{NB}}{x_1^{LN}}\right)^{\beta_2} \\
&\left. - \frac{\beta_2}{x_1^{NB}}\left(\gamma_1 - \frac{x_1^{LR}\rho\theta(1-\tau)}{\delta}\right)\left(\frac{x_1^{NB}}{x_1^{LR}}\right)^{\beta_2}\right].
\end{aligned} \tag{A.12}$$

Calculating the derivative of (A.10), and applying value matching and smooth pasting at x_1^{NB} yields the formula for x_1^{NB}. ■

B Proofs of Propositions

Proof of Proposition 1. First, on the basis of (28), (33), (36) - (38), (40), and (42), we determine the constants K_0, L_0, A_{01}, and A_{02}:

$$\begin{aligned}
&\begin{bmatrix} K_0 \\ L_0 \end{bmatrix} = \\
&\frac{1}{(x^*)^{\beta_1}\left(x_0^{LN}\right)^{\beta_2} - (x^*)^{\beta_2}\left(x_0^{LN}\right)^{\beta_1}}\begin{bmatrix} \left(x_0^{LN}\right)^{\beta_2} & -(x^*)^{\beta_2} \\ -\left(x_0^{LN}\right)^{\beta_1} & (x^*)^{\beta_1} \end{bmatrix} \times \\
&\begin{bmatrix} \frac{(\theta-1)x^*(1-\tau)}{\delta} - \frac{\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\left(\frac{x^*}{x_1^{NB}}\right)^{\beta_2} - \left(\frac{x^*}{x_0^{NB}}\right)^{\beta_2}\right) - I + L_1(x^*)^{\beta_2} \\ \gamma_0 - \frac{x_0^{LN}(1-\tau)}{\delta} - \frac{-\beta_2}{\beta_1-\beta_2}\frac{b\tau}{r}\left(\frac{x_0^{LN}}{x_0^{NB}}\right)^{\beta_1} \end{bmatrix},
\end{aligned} \tag{B.1}$$

and

$$\begin{bmatrix} A_{01} \\ A_{02} \end{bmatrix} = \tag{B.2}$$

$$\frac{1}{(x^*)^{\beta_1}\left(x_0^{NB}\right)^{\beta_2} - (x^*)^{\beta_2}\left(x_0^{NB}\right)^{\beta_1}} \begin{bmatrix} \left(x_0^{NB}\right)^{\beta_2} & -(x^*)^{\beta_2} \\ -\left(x_0^{NB}\right)^{\beta_1} & (x^*)^{\beta_1} \end{bmatrix} \times$$

$$\begin{bmatrix} \frac{(\theta-1)x^*(1-\tau)}{\delta} + \left(\eta\left(V_1^{NB}\left(x_1^{NB}\right) - R_1\left(x_1^{NB}\right)\right) - \frac{\theta x_1^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r}\right)\left(\frac{x^*}{x_1^{NB}}\right)^{\beta_2} - I \\ \eta\left(V_0\left(x_0^{NB}\right) - R_0\left(x_0^{NB}\right)\right)\left(\frac{x^*}{x_0^{NB}}\right)^{\beta_2} - \frac{x_0^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r} \end{bmatrix}.$$

Moreover, on the basis of (48) we define

$$A_{12} \equiv \left(x_1^{NB}\right)^{-\beta_2}\left(\eta\left(V_1^{NB} - R_1\right) - \frac{\theta x_1^{NB}(1-\tau)}{\delta} + \frac{b(1-\tau)}{r}\right), \tag{B.3}$$

so that $A_{12}x^{\beta_2}$ is the equityholders' value of the option to renegotiate. The implicit formulae for the optimal investment threshold, x^*, optimal renegotiation trigger, x_0^{NB}, and liquidation trigger, x_0^{LN}, are obtained by rearranging equations (39), (41) and (43). ■

Proof of Proposition 2. Proposition 2 directly results from replacing equation (39) by (47) in the system of equations (37) - (43). ■

Proof of Proposition 3. The optimal renegotiation trigger can be calculated on the basis of equations (40) and (41). After multiplying (40) by β_2 and subtracting (40) from (41) we obtain that

$$\begin{aligned} & (1-\beta_2)\frac{x_0^{NB}(1-\tau)(1-\eta(1-\rho))}{\delta} + \beta_2\frac{b}{r}(1-\tau+\eta\tau) \\ = \; & (\beta_1-\beta_2)(\eta K_0 - A_{01})\left(x_0^{NB}\right)^{\beta_1}. \end{aligned} \tag{B.4}$$

This yields

$$\begin{aligned} x_0^{NB} = \; & \frac{-\beta_2}{1-\beta_2}\frac{b(1-\tau+\eta\tau)\delta}{(1-\eta(1-\rho))(1-\tau)r} \\ & + \frac{\beta_1-\beta_2}{1-\beta_2}\frac{\delta(\eta K_0 - A_{01})\left(x_0^{NB}\right)^{\beta_2}}{(1-\eta(1-\rho))(1-\tau)}. \end{aligned} \tag{B.5}$$

The first row in (B.5) equals the optimal renegotiation trigger in the absence of the investment opportunity (cf. (34)). Consequently, x_0^{NB} is higher than such a trigger if and only if $\eta K_0 - A_{01}$ is positive. ■

C Derivation of the First Passage Time Probabilities

In general, the probability that an event (i.e. bankruptcy, renegotiation or investment) will occur within the time interval of length T, denoted by $p(x,T)$, satisfies the following partial differential equation (PDE)

$$-\left(r-\delta\right)x\frac{\partial p}{\partial x}+\frac{1}{2}\sigma^{2}x^{2}\frac{\partial^{2}p}{\partial x^{2}}=-\frac{\partial p}{\partial T}, \tag{C.1}$$

subject to the following boundary conditions

$$p\left(\underline{x},T\right) = a, \tag{C.2}$$

$$p\left(\overline{x},T\right) = b, \tag{C.3}$$

$$p\left(x,0\right) = 0. \tag{C.4}$$

where the lower bound, $\underline{x}$, upper bound, $\overline{x}$, and parameters a and b are given in the following matrix.

	Probability	
$\underline{x},\overline{x};a,b$	of investment	of debt restructuring
Growth option present		
Renegotiation possible	$x_0^{LN},x^*;0,1$	$x_0^{NB},x^*;1,q\left(x^*,x_1^{NB}\right)$
Bankruptcy upon default	$x_0^{B},x^*;0,1$	$x_0^{B},x^*;1,q\left(x^*,x_1^{B}\right)$
No growth option		
Renegotiation possible	-	$x_0^{NB},\infty;1,0$

Function $q\left(x,y\right)$ denotes the the probability of reaching the lower trigger y before time T conditional on starting at x. It can be obtained by applying a change of variables to Corollary B.3.4 in Musiela and Rutkowski (1998), p. 470. Consequently, it holds that

$$\begin{aligned} q\left(x,y\right) &= 1+\left(\frac{x}{y}\right)^{-\frac{2\alpha}{\sigma^2}+1}\mathbf{\Phi}\left(\frac{-\ln\frac{x}{y}+\left(\alpha-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) \\ &\quad -\mathbf{\Phi}\left(\frac{\ln\frac{x}{y}+\left(\alpha-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right), \end{aligned} \tag{C.5}$$

where $\mathbf{\Phi}\left(\cdot\right)$ denotes the standard normal cumulative density function.

As an example, let us interpret the boundary conditions for the probability of debt renegotiation in the presence of the growth option. Condition (C.2) implies that the renegotiation is certain if the level of cash flow hits the boundary x_0^{NB}. Equation (C.3) means that upon reaching the investment threshold, x^*, the renegotiation trigger switches to x_1^{NB} and the probability of renegotiation is described by (C.5). Finally, when the length of the time interval tends to zero, the probability of renegotiation approaches zero as well.

Since an analytical solution to the PDE (C.1) with boundaries (C.2) - (C.4) has not been found, a numerical procedure has to be applied. To calculate the relevant probabilities, the explicit finite difference method is used (cf. Brennan and Schwartz, 1978).

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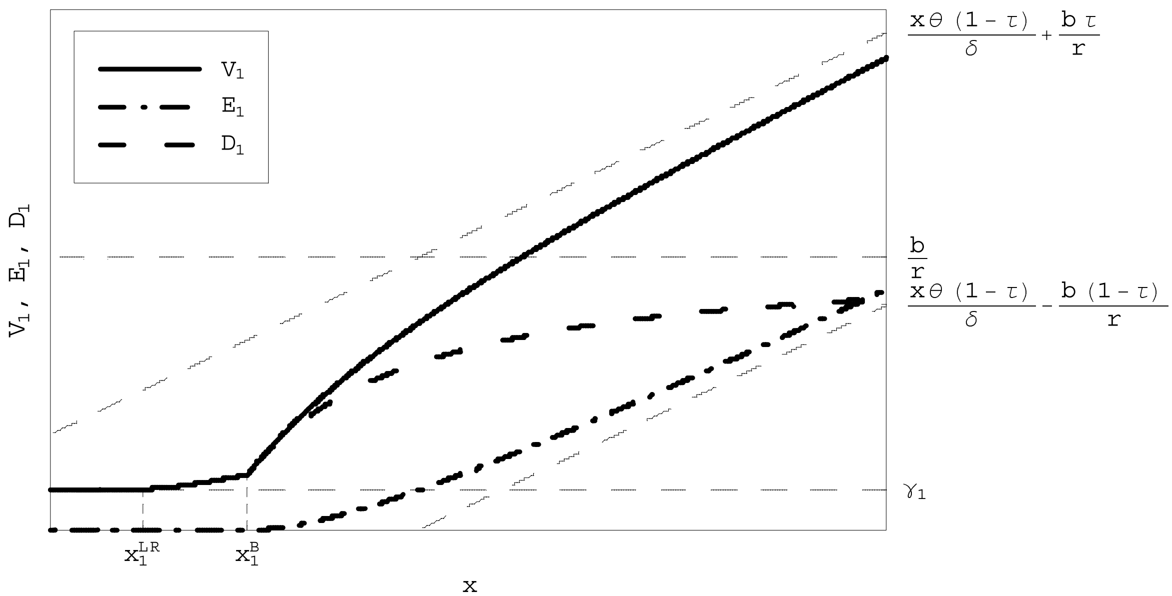

Figure 2.1. Valuation of the firm, V_1, its debt, D_1, and equity, E_1, with bankruptcy occurring upon default.

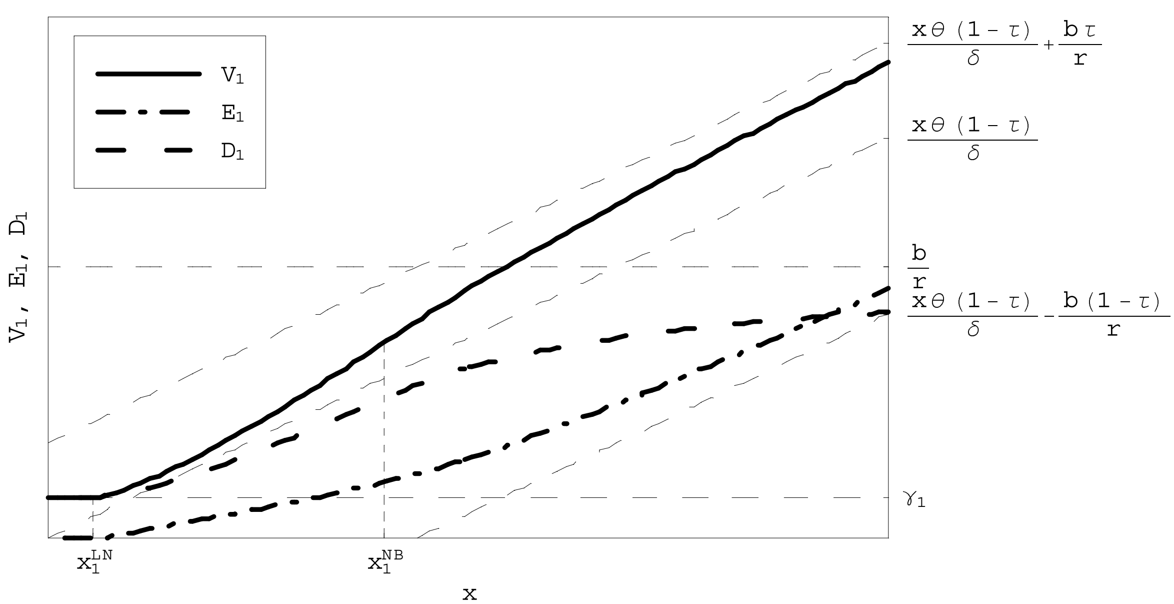

Figure 3.1. Valuation of the firm, V_1, its debt, D_1, and equity, E_1, with the shareholder's option to renegotiate the debt.

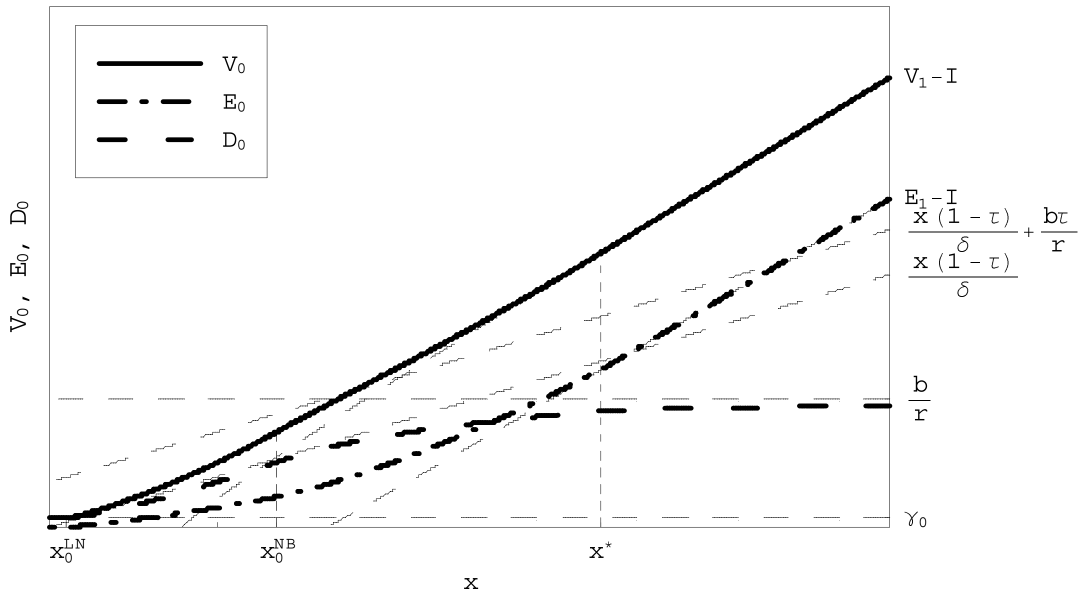

Figure 3.2a. Valuation of the firm, V_0, , its equity, E_0, and debt, D_0, with the shareholder's option to renegotiate the debt and the option to invest exercised at the firm value-maximizing level of earnings.

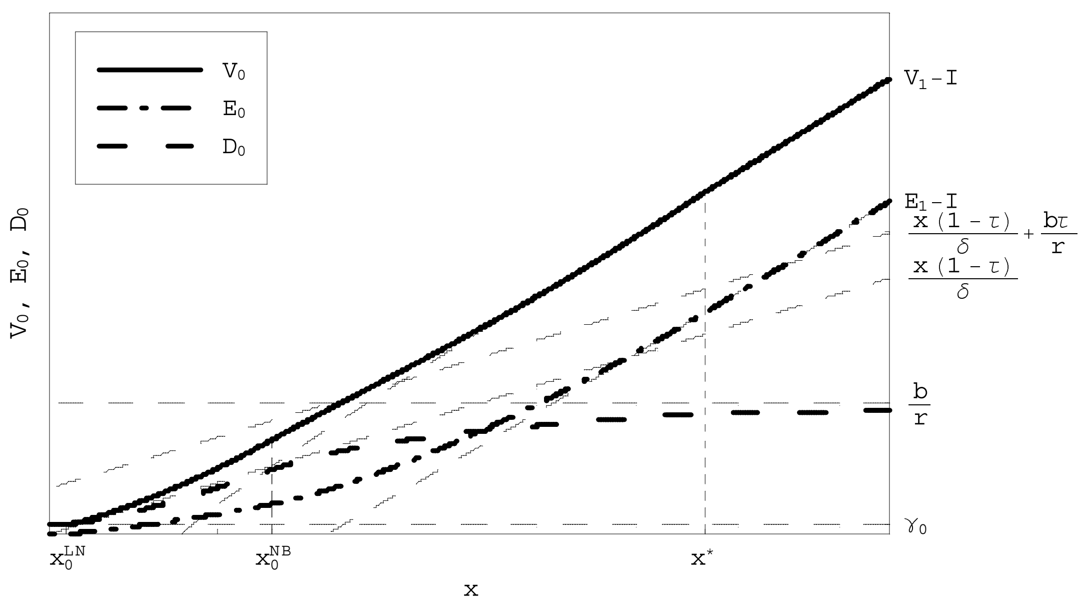

Figure 3.2b. Valuation of the firm, V_0, , its equity, E_0, and debt, D_0, with the shareholder's option to renegotiate the debt and the option to invest exercised at the equity value-maximizing level of earnings.

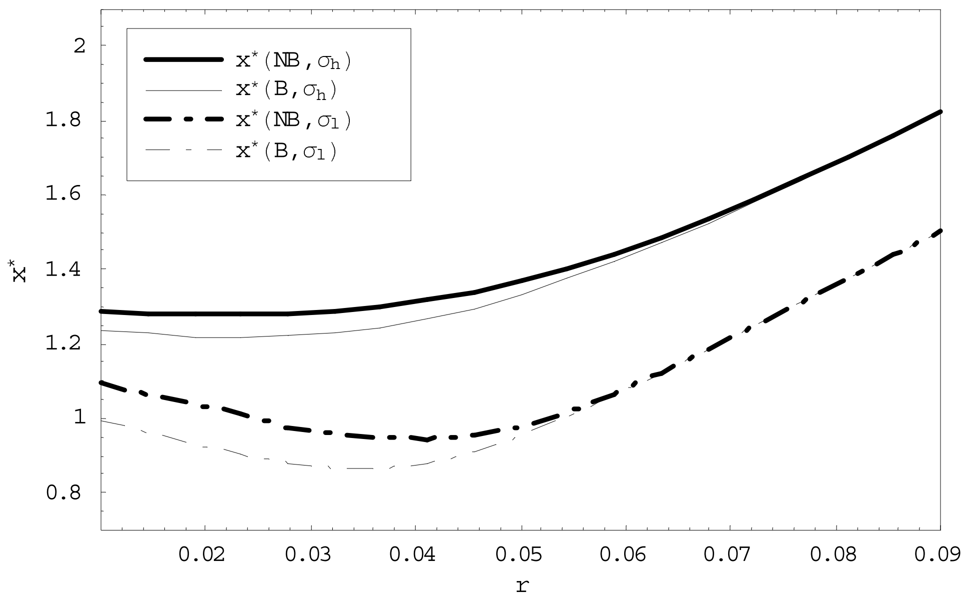

Figure 4.1. Equity value maximizing investment threshold in the presence of renegotiation option, $x^*(NB,\cdot)$, and without renegotiation, $x^*(B,\cdot)$, for $\sigma_l = 0.1$, $\sigma_h = 0.2$ and varying interest rate with the return shortfall rate, δ, kept constant at the 3.5% level.

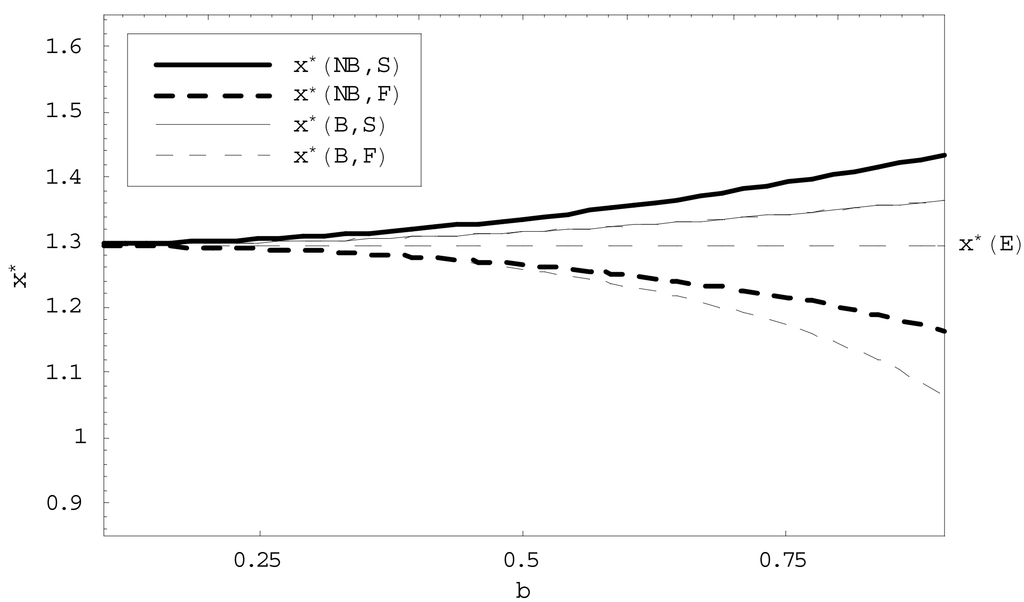

Figure 4.2. First-best, $x^*(NB,F)$, and second-best, $x^*(NB,S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B,F)$, and second-best, $x^*(NB,F)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for varying leverage (coupon rate), b.

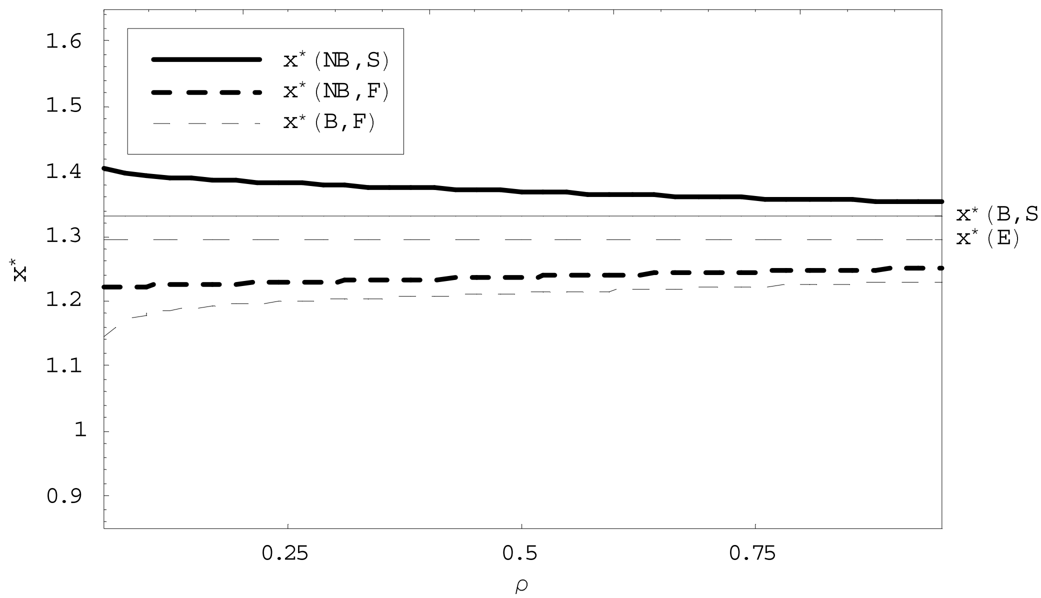

Figure 4.3. First-best, $x^*(NB,F)$, and second-best, $x^*(NB,S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B,F)$, and second-best, $x^*(B,S)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for varying magnitude of the creditors outside option, ρ.

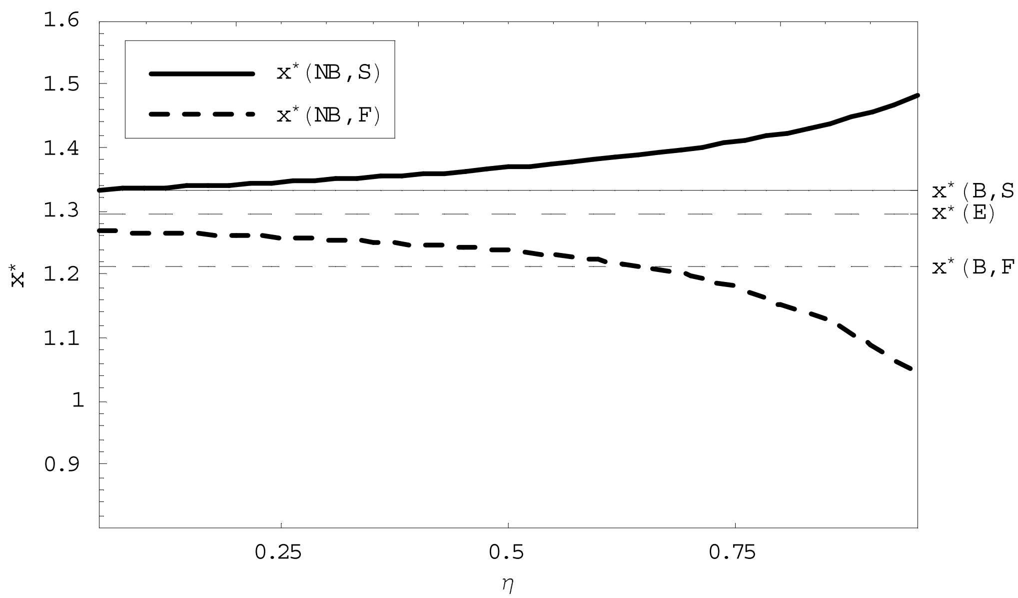

Figure 4.4. First-best, $x^*(NB,F)$, and second-best, $x^*(NB,S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B,F)$, and second-best, $x^*(NB,F)$, thresh-

olds without renegotiation, and with the all-equity threshold, $x^*(E)$, for varying distribution of bargaining power, η.

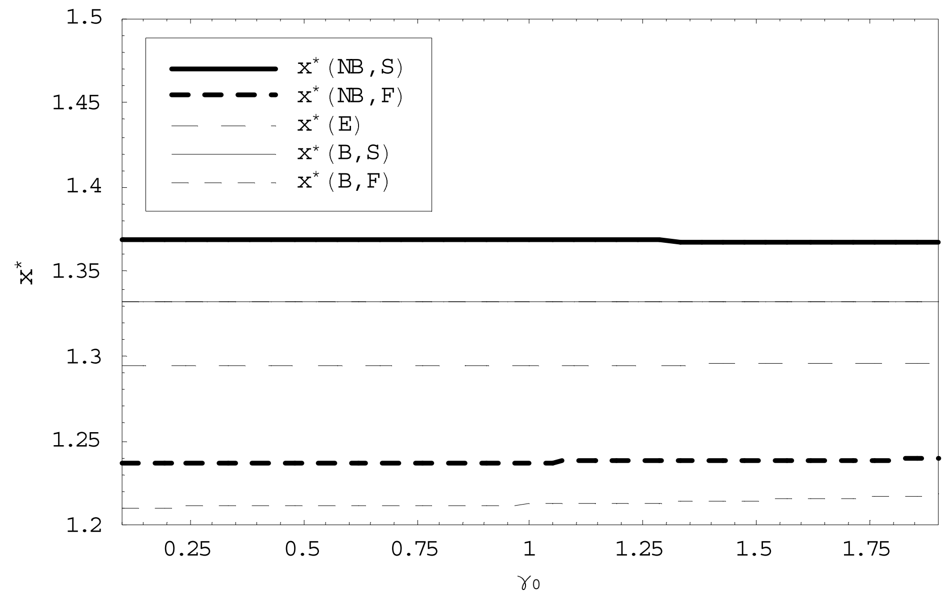

Figure 4.5. First-best, $x^*(NB, F)$, and second-best, $x^*(NB, S)$, investment thresholds in the presence of renegotiation option compared to first-best, $x^*(B, F)$, and second-best, $x^*(NB, F)$, thresholds without renegotiation, and with the all-equity threshold, $x^*(E)$, for different liquidation values, γ_0.

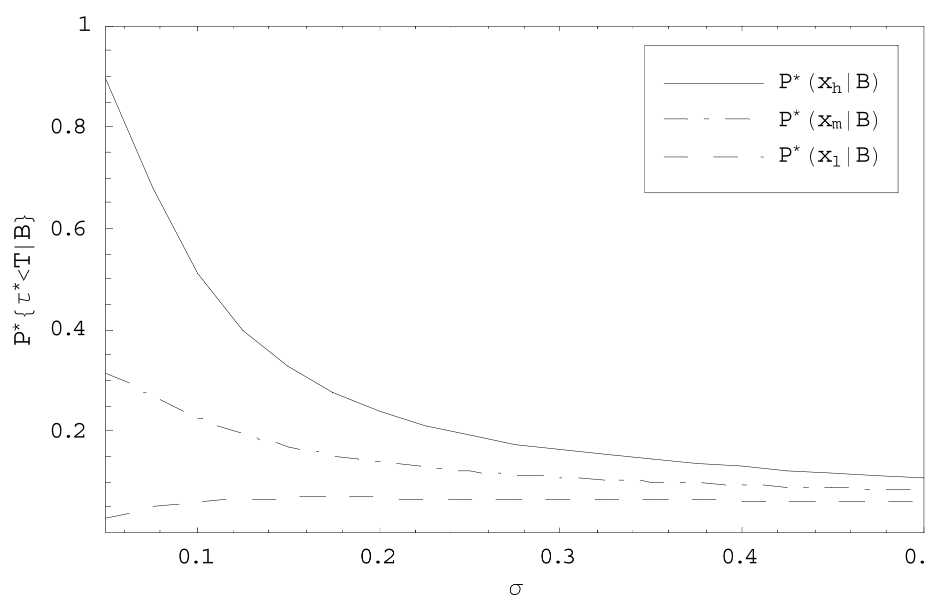

Figure 4.6. The probability of investment when the renegotiation is not possible for $x_l = 0.6$, $x_m = 0.7$, and $x_h = 0.8$, as a function of cash flow volatility, σ.

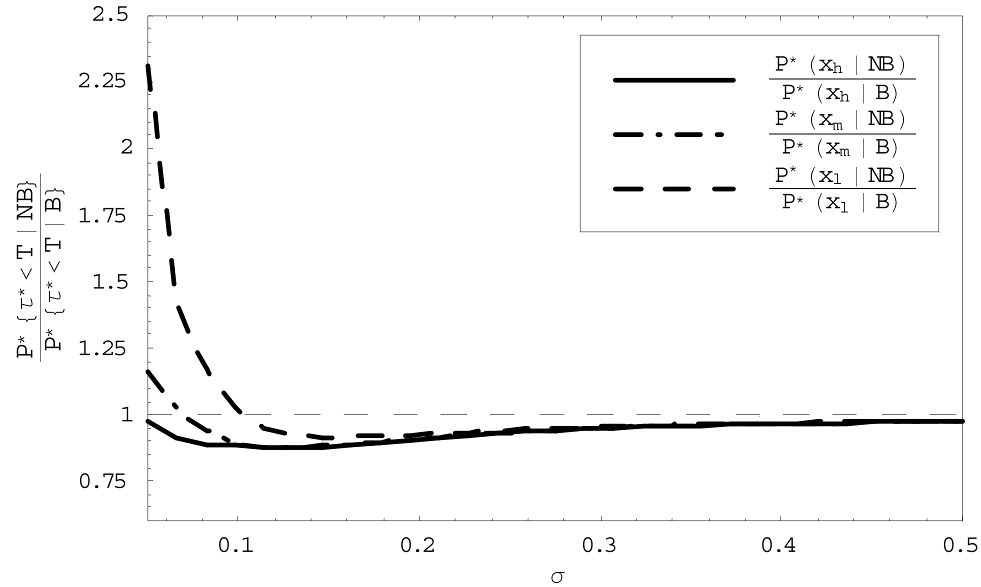

Figure 4.7. The ratio of probabilities of investment when the renegotiation is and is not possible for $x_l = 0.6$, $x_m = 0.7$, and $x_h = 0.8$, as a function of cash flow volatility, σ.

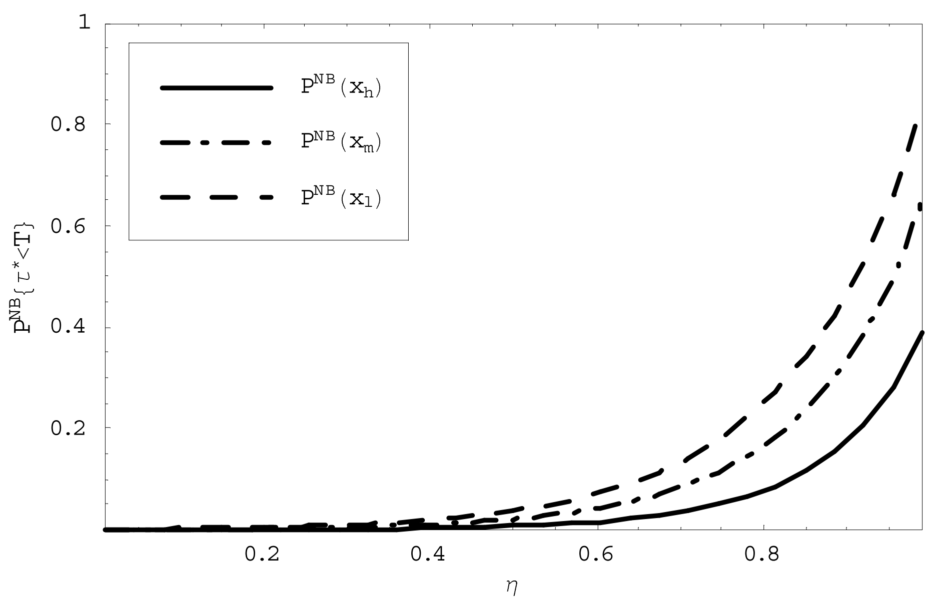

Figure 4.8. The probability of renegotiation for $x_l = 1.0$, $x_m = 1.1$, and $x_h = 1.25$, as a function of shareholders' bargaining power, η.

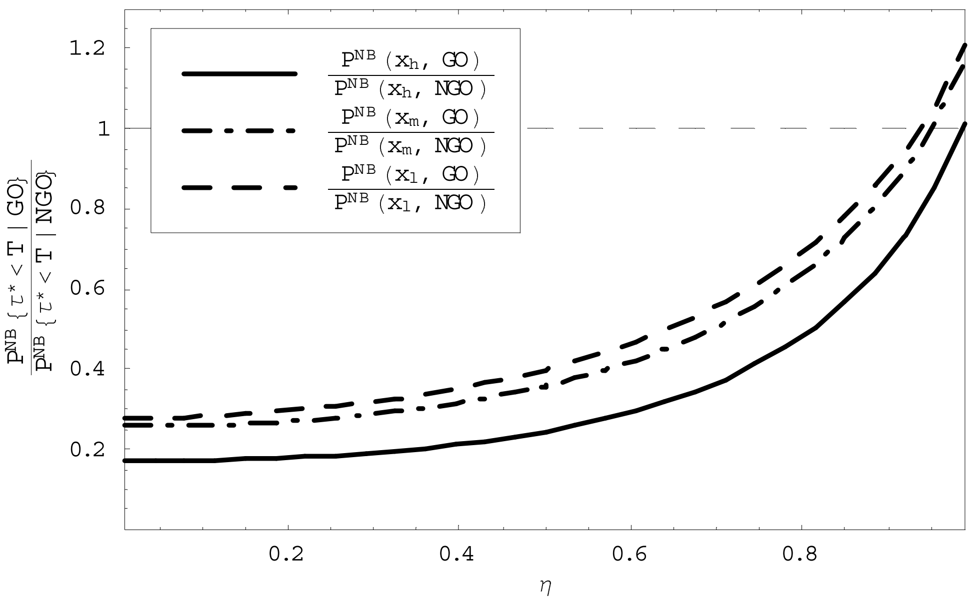

Figure 4.9. The ratio of probabilities debt renegotiation with, $p^{NB}(\cdot, GO)$, and without, $p^{NB}(\cdot, NGO)$, the growth option for $x_l = 1.0$, $x_m = 1.1$, and $x_h = 1.25$, as a function of shareholders' bargaining power, η.