Can Greater Uncertainty Hasten Investment?*

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Abstract

This paper examines irreversible investment in a project with uncertain returns, when there is an advantage to being the first to invest, and externalities to investing when others also do so. Pre-emption decreases and may even eliminate the option values created by irreversibility and uncertainty. Externalities introduce inefficiencies in investment decisions. Pre-emption and externalities combined can actually hasten, rather than delay investment, contrary to the usual outcome. These facts demonstrate the importance of extending 'real options' analysis to include strategic interactions.

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1. Introduction

The literature on irreversible investment under uncertainty teaches three major lessons. First, the net present value (NPV) rule for investment is generally incorrect, since it considers only a now-or-never decision and fails to appreciate that investment can be delayed. Secondly, an option value is created by the fact that the return is bounded below by the payoff from not investing; the effect of this option value is to delay investment, relative to the NPV rule. Finally, the greater the degree of uncertainty, the larger this delay: an increase in uncertainty increases the upside potential from investment, and so increases the value of the investment option.

Typically, literature on the 'real options' approach analyses investment decisions for a single agent in isolation. (Some exceptions to this are discussed below.) In many cases, however, investment takes place in a more competitive environment in which there are strategic interactions between investing agents. The purpose of this paper is to demonstrate that strategic interactions can have important consequences for irreversible, uncertain investments.

We analyse irreversible investment in a project with uncertain returns in a dynamic two-player model. Two types of strategic interactions are considered. The first is pre-emption. When there is some advantage to being the first to undertake an investment, there will be competition to be the first. In this situation, any benefit from delaying investment due to real option effects has to be balanced against the loss from being pre-empted. The second interaction arises when the value of an investment depends on the number of agents who have also invested. The interaction may affect value negatively: e.g., if it arises through a competitive effect; or it may have a positve effect, if there are complementarities between the agents' actions. In both cases, the timing of an agent's investment is influenced by the investment decisions of others.

It might be expected that the two effects—the real option in the investment opportunity and the threat of pre-emption—combine in a straightforward way. On their own, the former delays investment; the latter hastens it; with both, the outcome should be somewhere in between. In fact, we show that the story is more complicated than this. With pre-emption, greater uncertainty can hasten, rather than delay investment. To describe why, first note that two equilibrium patterns of investment are possible. Either the agents invest sequentially (i.e., the 'leader' invests early while the 'follower' invests late), or they invest simultaneously. We show that an increase in uncertainty can cause the leader in a sequential investment equilibrium to invest earlier. We also show that an increase in uncertainty can cause equilibrium to switch from sequential to simultaneous investment, or vice versa, in such a way that the first investment occurs sooner. We argue that these effects are present at plausible parameter values, and so can be empirically important. Overall, therefore, strategic interactions give rise to significant qualitative and quantitative effects that are omitted from the standard real options analysis of investment.

The strategic interactions in our model give rise to investment inefficiencies, an issue which does not arise in typical single-agent real options models. We identify three inefficiencies. A 'leader inefficiency' arises because each agent ignores the effect of its investment on the other. As a result, in equilibrium, sequential investment occurs inefficiently, compared to the co-operative solution. A 'follower inefficiency' arises because, when investment occurs sequentially, the follower does not consider the effect of its investment on the leader. As a result, the follower's equilibrium investment point is inefficient. Finally, a 'pre-emption inefficiency' arises because, when investment occurs sequentially, the leader does not consider the effect of its investment on the follower. As a result, the leader invests too early in equilibrium. In short: the leader leads inefficiently and invests too early in equilibrium; and the follower invests inefficiently when it has been pre-empted. We also show how the sizes of these inefficiencies are affected by the parameters of the model, such as the degree of uncertainty, the first-mover advantage, and externalities from investment.

There are many cases of technology investment in which uncertainty, externalities and pre-emption are important. Two examples are discussed here; clearly there are many others. While the focus of this paper is investment, a simple example concerns entry by firms into differentiated product markets; see Prescott and Visscher (1977), Lane (1980) and Neven (1987). Two entrepreneurs are considering opening shops on a street; they must decide when and where to open a shop. There are sunk costs (such as fitting) in opening a shop. If both entrepreneurs open a shop, they compete in prices against each other for custom—a negative externality. There may also be, however, positive externalities to being located on the same street; for example, because a common cost can be shared (such as a fixed cost of delivery of goods), or because aggregate demand is increased by lowering consumer search costs.

Finally, total demand (the mass of consumers) is growing over time but is uncertain. The outcome of this model is analysed in an appendix (available from the authors on request) to provide a micro-foundation for the reduced-form model of the next section.

A second example concerns two firms deciding whether to set up sites on the World Wide Web. There is some benefit to having a Web site; but the exact size of the benefit is uncertain.[1] Sunk costs are incurred in setting up a site: skilled labour is required to design and write the pages, a domain name must be purchased, marketing expenditures incurred etc..[2] An important reason to pre-empt is the ability of first-movers to buy their preferred domain names cheaply.[3] Generic Web addresses (such as `business.com` and `internet.com`), generally perceived to be the most valuable, are a limited resource. According to `www.names123.com`, an online domain name auction site, the top three domain names by price are `business.com` at US$7.5 million, `Aseenontv.com`, at US$5.1 million, and `altavista.com`, at US$3.3 million. At the time of writing, `21stcenturybusiness.com` is on offer at £90,000, and `hsbc.gb.net` at £250,000.[4] In the words of one industry newspaper, the "Internet equivalent of an uptown address just got a little bit pricier" (see CNET News.Com (1997)). A first-mover advantage may also arise because the firm that acts first to set up its Web site may face lower staff costs—site designers being relatively abundant—than later firms who have to hire when designers are more scarce. Finally, negative externalities arise through competition; positive externalities can also occur since a firm setting up a Web site benefits from the efforts of other firms, both directly (e.g., by being able to learn from the design of other sites) and indirectly (e.g., consumers already being accustomed to buying

[1] One study found that one-third of the small businesses that use the Internet increased their revenues by at least 10 per cent over the previous year. However, in the first nine months of 1999, consumer e-commerce in the U.S. initially fell and then plateaued; participation in online auctions has followed the same pattern. See InternetNews.Com (1999). Recent bankruptcies have emphasized the high degree of uncertainty facing internet-based businesses.

[2] Estimates of the cost of setting up the most basic web site range between US$225–1050, with an annual maintenance cost of between US$200–350; the most complex sites may cost several hundreds of thousands of dollars. See PC World Magazine (1999). Since its inception, marketing expenditure has been 25% of `amazon.com`'s revenues.

[3] Before 1994, Internic, the primary international authority for registration of domain names, did not charge; after this date, registration fees were instituted (in September 1999, US$70 per address for first 2 years, with a renewal fee thereafter). See Radin and Wagner (1996) for details.

[4] It might be argued that the most famous web addresses, such as `amazon.com` and `yahoo.com`, are non-generic. The point is, however, that generic web addresses are advantageous in attracting uninformed consumers who are unaware of specific brands.

online).

Two general strands of literature are related to this paper. Real options models have been used to explain delay and hysteresis arising in a wide range of contexts. McDonald and Siegel (1986) and Pindyck (1988) consider irreversible investment opportunities available to a single agent. Dixit (1989) and Dixit (1991) analyse product market entry and exit in monopolistic and perfectly competitive settings respectively. Jensen (1982) and Jensen (1983) examine adoption of a new technology by firm that is unable to estimate its value with certainty. The second strand of literature concerns timing games of entry or exit in a deterministic setting. There are several types of paper within this strand. Papers analysing pre-emption games include Fudenberg, Gilbert, Stiglitz, and Tirole (1983), Fudenberg and Tirole (1985), Katz and Shapiro (1987) and Lippman and Mamer (1993). Wars of attrition have been modelled by e.g., Fudenberg and Tirole (1986). Technology investment in the presence of network effects has been analysed by many papers, including Farrell and Saloner (1986) and Katz and Shapiro (1986).

Existing real option models typically assume a monopolistic or perfectly competitive framework, and do not allow for strategic interaction. Pre-emption models allow for incomplete information about the types of players, but not for common uncertainty about payoffs or externalities. Network papers have not (with the exception reviewed below) analysed explicitly the effect of 'option values'—created when there is exogenous uncertainty, investment is irreversible, and agents are able to choose the time of investment.

There are a number of papers more specifically related to this one. Choi (1994) examines a model in which there are positive network effects, uncertainty and the possibility of delay. Choi identifies two externalities (he calls them forward and backward externalities). In Choi's model, users are exogenously asymmetric: user 1 is able to choose which of two technologies (with random returns) to invest in either of two periods, while user 2 is able to invest only in the second period. This paper departs from Choi's in several respects. Most importantly, it does not impose exogenously an asymmetry between players, but instead allows the first mover to be determined endogenously. In our model, the leader invests at the point at which it is indifferent between leading and following; see section 3.[5] The fact that investment

[5]This is the rent equalization principle identified in Fudenberg and Tirole (1985).

by the leader is determined by indifference, rather than optimally (for the leader), makes an important difference to investment behaviour. We also allow for a more general payoff structure, including allowing for negative as well as positive externalities.

In Farrell and Saloner (1986), a model of technology investment with uncertainty about the timing of (rather than return from) investment, positive network effects, and irreversibility is analysed (see section II). Unlike Farrell and Saloner, we allow agents to invest at any time, not just at random opportunities. If this assumption were used in the Farrell and Saloner model, then many of the features would disappear (although the basic co-ordination problem due to network effects would remain). Here, delay is endogenously determined through the optimization decisions of the agents, rather than imposed exogenously.

Smets (1991) examines irreversible market entry in a duopoly facing stochastic demand. Simultaneous investment may arise only when the leadership role is exogenously pre-assigned. Consequently, he does not consider fully the pre-emption externality. Weeds (2002) presents a model in which two firms may invest in competing research projects with uncertain returns. She does not impose an asymmetry between the firms, but allows the leader to emerge endogenously. She does not include, however, more general externalities. Finally, Hoppe (2000) analyses a timing game of new technology investment in an uncertain environment. She considers second, rather than first, mover advantages and models uncertainty in a different way from this paper.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 analyses the non-co-operative equilibria of the model. Section 4 contains the analysis of the key question: does uncertainty delay or hasten investment? Section 5 determines the co-operative solution. Various inefficiencies in the model are analysed in section 6. Section 7 concludes. The appendix contains comparative static analysis and lengthier proofs.

2. The Model

This section develops a general model to capture the three effects that are the focus of this paper: (i) uncertainty, irreversibility and the possibility of delay in investment; (ii) investment externalities, where the return to investment depends on the number of investors;

and (iii) pre-emption, where early investors have an advantage. The section deals with a reduced-form model; a specific model (of entry into a differentiated product market) that conforms to the reduced-form structure is given in the appendix.

Two risk neutral agents, labelled $i \in \{1, 2\}$ each can invest in a project. There is a cost $K > 0$ to doing so, which is the same for both agents. Investment is irreversible (the cost K is entirely sunk), and can be delayed indefinitely. Time is continuous and labelled by $t \in [0, \infty)$. The timing of investment is the main concern of the analysis. Investment by the two agents may occur sequentially—that is, the two agents invest at distinctly different times—or simultaneously.

Consider first the outcome when the agents invest sequentially. Call the first investor the 'leader' and the second investor the 'follower'. The leader's instantaneous payoff at time t from investment, before the follower has invested, is

$$\pi_L^I = \theta_t, \tag{1}$$

where θ_t is the stand-alone benefit from investment—the instantaneous payoff received by an agent that is the sole investor. After the follower has invested, the leader's instantaneous payoff becomes

$$\pi_L^{II} = \gamma_L(1 + \alpha)\theta_t. \tag{2}$$

The follower's instantaneous payoff at time t from investment is

$$\pi_2^{II} = \gamma_F(1 + \alpha)\theta_t. \tag{3}$$

Now suppose that the agents invest simultaneously. The instantaneous payoff at time t from investment is the same for both agents:

$$\pi^{III} = \gamma_S(1 + \alpha)\theta_t. \tag{4}$$

The parameters $\gamma_L, \gamma_F, \gamma_S$ lie between 0 and 1; α is strictly greater than -1.

The instantaneous payoffs in equations (2)–(4) are parameterized to capture two separate effects. The parameters γ_L, γ_F and γ_S measure the payoffs to specific investors. Their relative sizes represent the extent of first-, second- or simultaneous-mover advantages (see below). The parameter α measures generally the extent of externalities between investors. Since the γ parameters are positive, a higher α corresponds to higher payoffs to both agents i.e., greater positive (or less negative) externalities. In the limit, as α tends to -1, payoffs when both agents have invested are zero (as would be the case if the the agents were Bertrand-competeting firms, for example). When $\alpha < 0$, externalities are negative; when $\alpha > 0$, they are positive. The particular way in which this parameter appears in the instantaneous payoff functions—as a multiplicative factor—is chosen for analytical convenience only. The important feature is that the α and γ parameters are complements (i.e, the marginal effect of an increase in α is positively related to the level of γ).

For most of the calculations, it is convenient to re-define variables as follows:

$$\gamma_L(1+\alpha) \equiv 1+\delta_L, \qquad \gamma_F(1+\alpha) \equiv 1+\delta_F, \qquad \gamma_S(1+\alpha) \equiv 1+\delta_S.$$

We will not investigate all possible configurations of the model parameters. Instead, we restrict attention to cases described in the following assumption:

Assumption 1:

$$-1 \leq \delta_F \leq 0, \tag{5}$$
$$\delta_F \leq \delta_S, \tag{6}$$
$$\delta_F \leq \delta_L \leq -\delta_F. \tag{7}$$

This assumption ensures several things.[6] First, there may be a first-mover advantage, since $\delta_L \geq \delta_F$. Secondly, the first-mover advantage cannot be too large: $\delta_L \leq -\delta_F$. Thirdly, there may be a second-mover disadvantage, in the sense that $\delta_S \geq \delta_F$. Fourthly, positive externalities cannot be too large, since $\delta_F \leq 0$ and $\delta_L \leq -\delta_F$. The role of particular aspects

[6] Assumption 1 can be expressed in terms of the parameters $\gamma_L, \gamma_F, \gamma_S$ and α: $0 \leq \gamma_F \leq \frac{1}{(1+\alpha)}$, $\gamma_F \leq \gamma_S$, and $\gamma_F \leq \gamma_L \leq \frac{2}{1+\alpha} - \gamma_F$.

of assumption 1 will be pointed out as the analysis progresses.

Even with assumption 1, our model encompasses many related papers. For example, in Fudenberg and Tirole (1985), when n firms have adopted the new technology, the payoff of a firm that has not adopted is $\pi_0(n)$, and of a firm that has adopted is $\pi_1(n)$. They assume that if $n' \geq n$, then $\pi_1(n') < \pi_1(n)$. A specific version of their payoffs can be represented in our model by supposing that $\pi_0(n) = 0 \; \forall n$, $\pi_1(1) = \theta$ and $\gamma_L = \gamma_F = \gamma_S = 1$ and $\alpha < 0$. Similarly, some of the payoff structures used in Katz and Shapiro (1987) can be replicated within our model. What they term the 'stand-alone incentive' is measured by δ_L in this model; their 'pre-emption incentive' is measured by $\delta_L - \delta_F$; the degree of imitation that is possible can be captured by δ_F. Lippman and Mamer (1993) analyse a model in which the first firm to innovate spoils the market for its rival; in this case, $\gamma_F = -1$. Notice also that by setting $\delta_S = (\delta_L + \delta_F)/2$, we can allow for the possibility that, in the event of simultaneous adoption, the roles of leader and follower are assigned randomly between the two agents.

θ_t is assumed to be exogenous and stochastic, evolving according to a geometric Brownian motion (GBM) with drift:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t \tag{8}$$

where $\mu \in [0, r)$ is the drift parameter, measuring the expected growth rate of θ, r is the continuous-time discount rate,[7] $\sigma > 0$ is the instantaneous standard deviation or volatility parameter, and dW is the increment of a standard Wiener process, $dW_t \sim N(0, dt)$. The parameters μ, σ and r are common knowledge and constant over time. The choice of continuous time and this representation of uncertainty is motivated by the analytical tractability of the value functions that result.

The strategies of the agents in the investment game are now defined. If agent i has not invested at any time $\tau < t$, its action set is $A^i_t = \{\text{invest, don't invest}\}$. If, on the other hand, agent i has invested at some $\tau < t$, then A^i_t is the null action 'don't move'. The agent therefore faces a control problem in which its only choice is when to choose the action 'invest'. After taking this action, the agent can make no further moves.

[7] The restriction that $\mu < r$ ensures that there is a positive opportunity cost to holding the 'option' to invest, and so that the option is not held indefinitely.

A strategy for agent i is a mapping from the history of the game H_t (the sample path of the stochastic variable θ and the actions of both agents up to time t) to the action set A_t^i. Agents are assumed to use stationary Markovian strategies: actions depend on only the current state and the strategy formulation itself does not vary with time. Since θ follows a Markov process, Markovian strategies incorporate all payoff-relevant factors in this game. Furthermore, if one player uses a Markovian strategy, then its rival has a best response that is Markovian as well. Hence, a Markovian equilibrium remains an equilibrium when history-dependent strategies are also permitted, although other non-Markovian equilibria may then also exist. (For further explanation see Maskin and Tirole (1988) and Fudenberg and Tirole (1991).)

The formulation of the agents' strategies is complicated by the use of a continuous-time model. Fudenberg and Tirole (1985) point out that there is a loss of information inherent in representing continuous-time equilibria as the limits of discrete time mixed strategy equilibria. To correct for this, they extend the strategy space to specify not only the cumulative probability that player i has invested, but also the 'intensity' with which each player invests at times 'just after' the probability has jumped to one.[8] Although this formulation uses mixed strategies, the equilibrium outcomes are equivalent to those in which agents employ pure strategies. (See section 3 of Fudenberg and Tirole (1985).) Consequently, the analysis will proceed as if each agent uses a pure Markovian strategy i.e., a stopping rule specifying a critical value or 'trigger point' for the exogenous variable θ at which the agent invests. Note, however, that this is for convenience only: underlying the analysis is an extended space with mixed strategies.

The possible states of each agent are denoted $n_i \in \{0, 1\}$ when the agent has not invested and has invested, respectively. The following assumptions are made:

ASSUMPTION 2: *If* $n_i(\tau) = 1$, *then* $n_i(t) = 1$ *for all* $t \geq \tau$, $i \in \{1, 2\}$.

[8] In Fudenberg and Tirole (1985), an agent's strategy is a *collection* of *simple strategies* satisfying an *intertemporal consistency condition.* A simple strategy for agent i in a game starting at a positive level θ of the state variable is a pair of real-valued functions $(G_i(\theta), \epsilon_i(\theta)) : (0, \infty) \times (0, \infty) \to [0, 1] \times [0, 1]$ satisfying certain conditions (see definition 1 in their paper) ensuring that G_i is a cumulative distribution function, and that when $\epsilon_i > 0$, $G_i = 1$ (so that if the intensity of atoms in the interval $[\theta, \theta + d\theta]$ is positive, the agent is sure to invest by θ). A collection of simple strategies for agent i, $(G_i^\theta(.), \epsilon_i^\theta(.))$, is the set of simple strategies that satisfy intertemporal consistency conditions.

Assumption 3: $\mathbb{E}_0\left[\int_0^\infty \exp(-rt)\theta_t dt\right] - K < 0$.

Assumption 2 formalizes the irreversibility of investment: if agent i has invested by date τ, it then remains active at all dates subsequent to τ. Assumption 3 states that the initial value of the project is sufficiently low that the expected return from investment is negative, thus ensuring that immediate investment is not worthwhile. (The operator $\mathbb{E}_0$ denotes expectations conditional on information available at time $t = 0$.)

3. Equilibrium

3.1. Sequential Investment

Start by assuming that the agents invest at different points. The possibility of simultaneous investment is considered below. As usual in dynamic games, the stopping time game is solved backwards; see e.g., Dixit (1989). Thus the first step is to consider the optimization problem of the follower who invests strictly later than the leader. Given that the leader has invested irreversibly, the follower's payoff on investing has two components: the flow payoff from the project, $(1+\delta_F)\theta_t$; and the cost of investment, $-K$. The follower's value function $F(\theta_t)$ at time t given a level θ_t of the state variable is therefore

$$F(\theta_t) = \max_{T_F} \mathbb{E}_t\left[\int_{T_F}^\infty \exp(-r(\tau - t))(1+\delta_F)\theta_\tau d\tau - K\exp(-r(T_F - t))\right] \tag{9}$$

where T_F is the random investment time for the follower, and the operator $\mathbb{E}_t$ denotes expectations conditional on information available at time t. The value function F has two components, holding over different ranges of θ: one relating to the value of investment before the follower has invested, the other to after investment. Let these value functions be denoted F_0 and F_1, respectively.

Prior to investment, the follower holds an option to invest but receives no flow payoff. In this 'continuation' region, in any short time interval dt starting at time t the follower experiences a capital gain or loss dF_0. The Bellman equation for the value of the investment opportunity is therefore

$$F_0 = \exp(-rdt)\mathbb{E}_t\left[F_0 + dF_0\right]. \tag{10}$$

Itô's lemma and the GBM equation (8) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 F_0''(\theta) + \mu\theta F_0'(\theta) - rF_0(\theta) = 0. \tag{11}$$

From equation (8), it can be seen that if θ ever goes to zero, then it stays there forever. Therefore the option to invest has no value when $\theta = 0$, and must satisfy the boundary condition $F_0 = 0$. Solution of the differential equation subject to this boundary condition gives $F_0 = b_F\theta^\beta$, where b_F is a positive constant and $\beta > 1$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$; i.e., $\beta = \frac{1}{2}\left(1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right)$.

Now consider the value of the agent in the 'stopping' region, in which the value of θ is such that it is optimal to invest at once. Since investment is irreversible, the value of the agent in the stopping region is given by the expected value alone with no option value terms. When the level at time t of the state variable is θ_t, this is

$$F_1(\theta_t) = \mathbb{E}_t\left[\int_t^\infty \exp(-r(\tau - t))(1 + \delta_F)\theta_\tau d\tau - K\right].$$

θ is expected to grow at rate μ, so that

$$F_1(\theta) = \frac{(1 + \delta_F)\theta}{r - \mu} - K. \tag{12}$$

The boundary between the continuation region and the stopping region is given by a trigger point θ_F of the stochastic process such that continued delay is optimal for $\theta < \theta_F$ and immediate investment is optimal for $\theta \geq \theta_F$. The optimal stopping time T_F is then defined as the first time that the stochastic process θ hits the interval $[\theta_F, \infty)$ from below. Putting together the two regions gives the follower's value function:

$$F(\theta) = \begin{cases} b_F\theta^\beta & \theta < \theta_F, \\ \frac{(1+\delta_F)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \tag{13}$$

given that the leader invests at $\theta_P < \theta_F$.

By arbitrage, the critical value θ_F must satisfy a value-matching condition; optimality requires a second condition, known as 'smooth-pasting', to be satisfied. (See Dixit and Pindyck (1994) for an explanation.) This condition requires the two components of the follower's value function to meet smoothly at θ_F with equal first derivatives, which together with the value matching condition implies that

$$\theta_F = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_F}\right)(r-\mu), \tag{14}$$

$$b_F = \frac{(1+\delta_F)\theta_F^{-(\beta-1)}}{\beta(r-\mu)}. \tag{15}$$

Equation (14) for the follower's trigger point can be interpreted as the effective flow cost of investment with an adjustment for uncertainty. The sunk investment cost is K, but this yields a flow payoff of $(1+\delta_F)\theta$; hence the effective sunk cost is $\frac{K}{1+\delta_F}$. With an effective interest rate of $r-\mu$ (i.e., the actual interest rate r minus the expected proportional growth in the flow payoff μ), this gives an instantaneous cost of $\left(\frac{K}{1+\delta_F}\right)(r-\mu)$. If a Marshallian rule were used for the investment decision, the trigger point would be simply this cost. But with uncertainty, irreversibility and the option to delay investment, the Marshallian trigger point must be adjusted upwards by the factor $\frac{\beta}{\beta-1} > 1$.

There are three components to the leader's value function holding over different ranges of θ. The first L_0 describes the value of investment before the leader (and so the follower) has invested; the second L_1 after the leader has invested, but before the follower has done so; and the third L_2, after the follower has invested. The first and third components are equivalent to those of the follower, determined previously. The second component is new, and so is derived first.

After the leader has invested, it has no further decision to take and its payoff is given by the expected value of its investment. This payoff is affected, however, by the action of the follower investing later at θ_F. Taking account of subsequent investment by the follower, the leader's post-investment payoff is given by

$$L_1(\theta_t) = \mathbb{E}_t\left[\int_t^{T_F} \exp\left(-r(\tau-t)\right)\theta_\tau d\tau + \int_{T_F}^{\infty} \exp\left(-r(\tau-t)\right)(1+\delta_L)\theta_\tau d\tau - K\right]. \tag{16}$$

The Bellman equation for the leader is

$$L_1 = \theta dt + \exp(-rdt)\mathbb{E}_t\left[L_1 + dL_1\right]. \tag{17}$$

Using Itô's lemma and equation (8) gives

$$\frac{1}{2}\sigma^2\theta^2 L_1''(\theta) + \mu\theta L_1'(\theta) - rL_1(\theta) + \theta = 0. \tag{18}$$

As before, investment has no value when $\theta = 0$, and so $L_1 = \frac{\theta}{r-\mu} + b_{L1}\theta^\beta$, where b_{L1} is a constant. The first part of the value function L_1 gives the expected value of investment before the follower invests, while the second is an option-like term reflecting the value (due to externalities) to the leader of future investment by the follower.

The other components of the leader's value function follow immediately from the calculations of the previous section:

$$L(\theta) = \begin{cases} b_{L0}\theta^\beta & \theta < \theta_P, \\ \frac{\theta}{r-\mu} + b_{L1}\theta^\beta - K & \theta \in [\theta_P, \theta_F), \\ \frac{(1+\delta_L)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \tag{19}$$

given the leader's trigger point θ_P and investment by the follower at the higher θ_F.

The value of the unknown constant b_{L1} is found by considering the impact of the follower's investment on the payoff to the leader. When θ_F is first reached, the follower invests and the leader's expected flow payoff is altered. Since value functions are forward-looking, L_1 anticipates the effect of the follower's action and must therefore meet L_2 at θ_F. Hence, a value-matching condition holds at this point (for further explanation see Harrison (1985)); however, there is no optimality on the part of the leader, and so no corresponding smooth-pasting condition. This implies that

$$b_{L1} = \frac{\delta_L \theta_F^{-(\beta-1)}}{r-\mu}. \tag{20}$$

The leader cannot choose its investment point optimally, as the follower can. Instead, the first agent to invest does so at the point at which it prefers to lead rather than follow, not the point at which the benefits from leading are largest. Clearly, it cannot be that the first agent invests when the value from following is greater than the value from leading—if this were the case, the agent would do better by waiting. Likewise, it cannot be that the first agent invests when the value from leading is strictly greater than the value from following , since in this case without pre-assigned roles, the other agent could pre-empt it and still gain. Hence the investment point is defined by indifference between leading and following. The trigger point θ_P in the pre-emption model is given by indifference: $L(\theta_P) = F(\theta_P)$. This is in contrast to the trigger point of the follower, which is determined by value matching and smooth pasting i.e., is chosen optimally.

The indifference relation $L(\theta_P) = F(\theta_P)$ gives a non-linear equation for θ_P:

$$\frac{\theta_P}{r-\mu} - K = \frac{K}{\beta - 1}\left(\frac{1+\delta_F - \beta\delta_L}{1+\delta_F}\right)\left(\frac{\theta_P}{\theta_F}\right)^{\beta}. \tag{21}$$

The next proposition establishes that there is a unique solution to this equation, and hence determines equilibrium in this case. (The remaining coefficient, b_{L0} is determined by value matching at θ_P i.e.,

$$b_{L0} = \theta_P^{-\beta}\left(\frac{\theta_P}{r-\mu} - K\right) + b_{L1}.) \tag{22}$$

PROPOSITION 1: *When equilibrium investment is sequential, the leader invests at θ_P and the follower at $\theta_F > \theta_P$. θ_P is the smallest value such that $L(\theta_P) = F(\theta_P)$ and $L(\theta) < F(\theta)$ for $\theta < \theta_P$, $L(\theta) > F(\theta)$ for $\theta > \theta_P$.*

PROOF: See the appendix.

One possibility for a solution to equation (21) is illustrated in figure 1 (in which it is assumed that $1 + \delta_F - \beta\delta_L > 0$). The left-hand side of equation (21) is the increasing, linear function; the right-hand side is the increasing, convex function. There are two intersection points of the two functions; the lower point is the relevant solution for the leader's trigger point θ_P. Section A.1 in the appendix analyses the comparative statics of the trigger point θ_P.

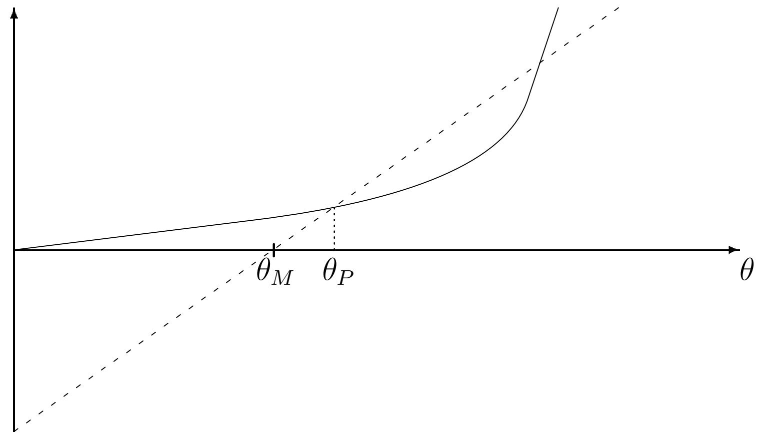

Figure 1: The solution for θ_P

3.2. Simultaneous Investment

Now consider the alternative case, in which investment is simultaneous at the trigger point θ_S. The previous analysis indicates that the value function of each agent is then

$$S(\theta) = \begin{cases} b_S\theta^\beta & \theta < \theta_S, \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_S. \end{cases} \tag{23}$$

(This value function can be derived from the appropriate Bellman equation, following the steps shown above.) There is a continuum of simultaneous solutions; it is straightforward to show that they can be Pareto ranked, with higher trigger points yielding higher value functions. In this case, it seems reasonable that the agents invest at the Pareto optimal point, given by both value matching and smooth pasting. So

Proposition 2: *The Pareto optimal trigger point for the simultaneous equilibrium is*

$$\theta_S = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_S}\right)(r-\mu).$$

The coefficient in the value function is

$$b_S = \frac{(1+\delta_S)\theta_S^{-(\beta-1)}}{\beta(r-\mu)}. \tag{24}$$

The next proposition describes when simultaneous investment is an equilibrium.

PROPOSITION 3: *Simultaneous investment occurs in equilibrium iff*

$$\lambda_E \equiv (1+\delta_S)^\beta - \left(1+\beta\delta_L(1+\delta_F)^{\beta-1}\right) \geq 0. \tag{25}$$

A sufficient condition is $\delta_S \geq 0 \geq \delta_L$.

PROOF: For equilibrium simultaneous investment, it must be that $S(\theta) \geq L(\theta)$ for $\theta \in [\theta_P, \theta_S]$. Due to the convexity of the value functions, this requires that $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_P]$, and so that $b_S \geq b_{L0}$. Substituting the expressions for these two coefficients gives the necessary and sufficient condition of equation (25). The sufficient condition follows directly from equation (25). ■

Whether simultaneous investment occurs in equilibrium is determined by whether the leader wishes to invest before the follower, or at the same time (i.e., by the comparison of $L(\theta)$ and $S(\theta)$). The proposition shows the reasonable condition that, in order for simultaneous investment to occur in equilibrium, it must be the case that δ_S is sufficiently large and/or δ_L and δ_F sufficiently small. (This is clearest in the sufficient condition.) Note that the simultaneous investment equilibrium, when it exists, Pareto dominates the sequential outcome; this is an immediate consequence of the condition for existence of the simultaneous investment equilibrium: $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_S]$. In section A.2 in the appendix, we consider how the payoff parameters affect whether the equilibrium without pre-emption involves sequential or simultaneous investment.

4. Does Uncertainty Delay or Hasten Investment?

In this section, we show that greater uncertainty can hasten, not delay investment. This runs counter to the usual real options effect, and shows that the combination of uncertainty and pre-emption does more than simply give an outcome that is an average of the effect of the two factors.

First note that the triggers θ_F and θ_S are increasing in σ, for the familiar real options reason. The intuition is that delay allows for the possibility that the random process (8) will go up; if it goes down, then the agent need not invest. The greater the variance of the process, the more valuable is the option created by this asymmetric situation, and so the more delay occurs for both agents. Notice that this result relies on the fact that all of these triggers are chosen optimally by the relevant agent(s).

There are two ways in which greater uncertainty can hasten investment. First, when equilibrium investment is sequential, the trigger point θ_P of the leader may decrease as σ increases. This possibility is examined in proposition 4. Secondly, a rise in σ can cause the pattern of equilibrium investment to switch, with investment in the new equilibrium pattern occurring earlier. This possibility is considered in proposition 5.

Proposition 4: *If* $\beta\delta_L > 1 + \delta_F$*, then* $\frac{\partial \theta_P}{\partial \sigma} < 0$.

Proof: See the appendix.

The result therefore raises the striking possibility that greater uncertainty lowers the leader's trigger point. The possibility arises from the lack of optimality in the choice of the pre-emption trigger point. An optimal trigger point is such that the marginal benefit from delaying investment for a period equals the marginal cost. The marginal benefit is the interest saved on the investment cost plus the expected gain from the possibility that the flow payoff increases. The marginal cost is the flow payoff foregone by not investing. In this marginal calculation, the agent does not consider the effect of its delay on the investment decision of the other agent, since in the models considered in this paper, each agent's trigger point (with the exception of θ_P) does not depend on the other's. Increased uncertainty raises the expected gain from delay, causing the (optimally chosen) trigger point to increase. This reasoning does not apply in the case of θ_P, however: it is not chosen according to a marginal equality, but an absolute equality between the value from leading and the value from following. The proposition shows that this difference in the determination of the trigger point can lead to θ_P decreasing as uncertainty increases.

In order for this unusual comparative static to hold, it must be that the leader's value function increases by more than the follower's when uncertainty rises, holding constant the leader's trigger point θ_P. (This statement follows directly from using the implicit function theorem on the non-linear equation (21) defining θ_P.) There are, therefore, two necessary and sufficient conditions for θ_P to be decreasing in σ:

1. The leader's value function is increasing in σ.
2. The increase in the leader's value function is larger than the increase in the follower's.

The leader's value function depends on uncertainty due to the option-like term that anticipates investment by the follower: $b_{L1}\theta^\beta$, where $b_{L1} \equiv \delta_L \theta_F^{-(\beta-1)}/(r-\mu)$ and $\theta \in (\theta_P, \theta_F)$. Hence this option-like term is positive only if $\delta_L > 0$; when this is the case, the option-like term increases in value with the degree of uncertainty (for the usual reasons), and so condition 1 holds. The follower's value function also depends on uncertainty, due to the option value of its investment: $b_F\theta^\beta$, where $b_F = (1+\delta_F)\theta_F^{-(\beta-1)}/\beta(r-\mu)$ and $\theta < \theta_F$. This option value increases with the degree of uncertainty.

If $\beta\delta_L - (1+\delta_F) > 0$, then the value of the leader's option-like term is greater than the option value of the follower. Both values are convex functions of θ; the leader's value is more convex than the follower's, since it lies above it. Therefore the same condition ensures that the value of the leader's option-like term, $b_{L1}\theta^\beta$, increases by more than the option value of the follower, $b_F\theta^\beta$, for any increase in σ and any value of $\theta \in (\theta_P, \theta_F)$. Hence if the sufficient condition in the proposition is satisfied, then the introduction of a small amount of uncertainty (corresponding to very high values of β) into the model increases all trigger points except the leader's, which decreases. More precisely, the condition ensures that $\frac{\partial \theta_P}{\partial \sigma}\Big|_{\sigma=0} < 0$. Notice that the sufficient condition requires that $\delta_L > 0$. This in turn requires two things: first, that γ_L, the first-mover advantage, be positive; secondly, given a

$\gamma_L > 0$, that the externality parameter α is sufficiently large (greater than $1/\gamma_L - 1$). The larger is γ_L—the stromger the first-mover advantage—the lower can be the extent of positive externalities.

Now consider the second possibility for greater uncertainty to hasten investment: as a result of a switch in the equilibrium pattern of investment as uncertainty increases. There are two cases to consider. First, equilibrium investment switches from simultaneous to sequential, and $\theta_S > \theta_P$. In this case, the investment point of the first investor decreases; but the follower adopts at a higher value of θ, since $\theta_F > \theta_S$. Secondly, equilibrium investment switches from sequential to simultaneous, and $\theta_S < \theta_P$. In this second case, the investment points of both agents clearly decrease. There are two steps to get to proposition 5. The first analyses whether the necessary and sufficient condition in proposition 3 for equilibrium to be simultaneous is easier or more difficult to satisfy as σ increases (i.e., whether λ_E is increasing or decreasing in σ). The second analyses whether θ_S is greater or less than θ_P.

LEMMA 1: *1. Joint sufficient conditions for λ_E to be a decreasing function of σ are: $\delta_S \geq 0$ and either (i) $\delta_L \geq 0$ and $\delta_F \leq e^{-1} - 1$ or (ii) $\delta_L \leq 0$ and $\delta_F \geq e^{-1} - 1$.*

2. Joint sufficient conditions for λ_E to be an increasing function of σ are: $\delta_S < 0$ and either (i) $\delta_L \geq 0$ and $\delta_F \geq e^{-1} - 1$ or (ii) $\delta_L \leq 0$ and $\delta_F \leq e^{-1} - 1$.

PROOF: See the appendix.

The dependence of λ_E on the degree of uncertainty is more complicated than other comparative statics of λ_E. Recall that two terms in θ appear in the two parts of the leader's value function before the follower's investment: L_0 contains a direct option value associated with the leader's own investment, while L_1 has an option-like term relating to the follower's investment.[9] Consider the effect of an increase in σ when $\delta_L < 0$. The leader's value increases due to the first, direct option term—this is the standard comparative static of an option value. But the leader's value decreases due to the second term: the magnitude of

[9] Refer to equation (19). Notice that both terms are important for $\theta \leq \theta_F$. This is explicit over the range $\theta \in [\theta_P, \theta_F)$, and implicit for $\theta < \theta_P$: for the latter, the two factors show up in the expression for b_{L0}—see equation (22).

the option-like value increases, but it is a negative value, since $\delta_L < 0$. Hence there are two conflicting effects when σ increases, and consequently the comparative static with respect to σ may be (and in fact is) non-monotonic.

In the cases identified in the lemma, however, the comparative statics are unambiguous. Consider part 1(i) of the lemma, in which $\delta_S \geq 0$ and $\delta_L \geq 0$. The value from simultaneous investment increases with σ, in line with the standard option value comparative static. The marginal effect on the simultaneous investment value function of an increase in σ is therefore positive; but it is decreasing in δ_S. This is because as δ_S increases, for any given level of σ, simultaneous investment occurs sooner (θ_S decreases). Hence an increase in δ_S acts in the opposite direction to an increase in σ, which increases θ_S. The direct option term in the leader's value function increases with σ; and the marginal effect of an increase in uncertainty is independent of δ_L and δ_F. The second term increases with uncertainty, since $\delta_L \geq 0$. In this case, the marginal effect of an increase in uncertainty is decreasing in δ_F: as δ_F increases, for any given level of σ, the follower invests sooner (θ_F decreases). Hence an increase in δ_F acts in the opposite direction to an increase in σ, which increases θ_F. This argument establishes that the value of the leader increases with uncertainty by more than the value of a simultaneous investor if (i) δ_S is sufficiently large; (ii) δ_L is sufficiently large; and (iii) δ_F is sufficiently small. Similar considerations underlie the sufficient conditions in the other parts of the lemma.

The second step is to compare θ_S and θ_P.

Lemma 2: *θ_S is greater (less) than θ_P iff*

$$\frac{\delta_S}{1+\delta_S} < (>) \frac{\delta_L}{1+\delta_F}.$$

Proof: The lemma follows from substitution of θ_S into equation (21). ■

The lemma gives the intuitive condition that θ_S is greater than θ_P if and only if δ_S is sufficiently small (since $\delta_S/(1+\delta_S)$ is increasing in δ_S) and/or δ_L sufficiently large and δ_F sufficiently small.

Lemmas 1 and 2 can be combined to give sufficient conditions for the trigger point of the first investor to decrease as σ rises, as a result of a change in the equilibrium pattern of investment.

PROPOSITION 5: *1. Suppose that the conditions in part 1 of lemma 1 hold, and that* $\delta_S/(1+\delta_S) < \delta_L/(1+\delta_F)$*. Then there exists a* $\sigma'' > \sigma' > 0$ *such that* $\lambda_E(\sigma') > 0 > \lambda_E(\sigma'')$*; and* $\theta_S > \theta_P$*.*

2. Suppose that the conditions in part 2 of lemma 1 hold, and that $\delta_S/(1+\delta_S) > \delta_L/(1+\delta_F)$*. Then there exists a* $\sigma'' > \sigma' > 0$ *such that* $\lambda_E(\sigma') < 0 < \lambda_E(\sigma'')$*; and* $\theta_S < \theta_P$*.*

Both cases give sufficient conditions for an increase in uncertainty from σ' *to* σ'' *to cause the trigger point of the first investor to decrease.*

(Proposition 5 follows directly from the two preceding lemmas, and so is stated without proof.) The proposition gives, then, a second reason why a model of investment under uncertainty with strategic interaction can be very different from the single-agent case. The reason now is that there are two types of equilibrium in the multi-agent case. An increase in uncertainty can cause a switch from one type to another in such a way as to decrease the trigger point of the first investor. Of course, this factor cannot arise in the single-agent case.

The final issue to consider is: how empirically relevant is this analysis? To focus the discussion, we concentrate on proposition 4. Recall that the proposition requires that the first-mover advantage δ_L must be large (certainly positive) and σ small. The first part of this condition may seem unusual—it requires that investment by a second agent increase the flow payoff to the first investor. If investment takes the form of entry into a product market, then this would require, for example, that the demand expansion effect of an additional firm outweighs increased competition. Note, however, that δ_L does not need to be very large at all. The sufficient condition is $\delta_L > (1+\delta_F)/\beta$. When σ is very low, β is very large; for example, setting $\mu = 0$ and $r = 0.05$, a standard deviation of the process (8) of 2% implies that β equals 16, approximately. The greatest lower bound on δ_L (when $\delta_F = 0$) in this case is 1/16 i.e., investment by the second agent increases the flow payoff of the first investor by around 6%. The ultimate test of the relevance of the proposition is how it matches data: the pattern of investment and the level of profits observed in a particular market. Nevertheless, these parametric conditions do not seem implausible.

Furthermore, the result and its empirical relevance is not specific to our model. The ratio of the leader's and follower's values anticipating the follower's investment is key for the result. In our model, the ratio is $\beta\delta_L/(1+\delta_F)$; when $\delta_L > 0$, this ratio is positive and tends to infinity as σ tends to zero (so that β tends to infinity). More generally, the result requires that, when the first-mover advantage and/or positive externalities are sufficiently large, the ratio increases above 1 as uncertainty decreases. The follower's option value at any level of the state variable below its trigger point decreases to zero as uncertainty is reduced. This fact is not specific to the particular form of process (see equation (8)) that we use, or the payoffs assumed. Hence the result requires that the follower's option value decrease more quickly than the value of the leader's option-like term. The leader does not hold an option i.e., its payoff is not determined by an optimal action by the leader. Instead, the leader's payoff is determined by value matching; as a consequence, the value of the leader's option-like term is less sensitive than the follower's option value to the level of uncertainty. Again, this fact is not specific to the modelling assumptions that we make. Therefore, provided that the value of the leader's option-like term is positive, a sufficient reduction in uncertainty ensures that the leader's value is greater than the follower's. Hence θ_P decreases with σ for small enough σ. This result is robust and extends beyond the assumptions used here.

5. Co-operative Solution

This section analyses the co-operative solution, in which the agents' investment trigger points are chosen to maximize the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies in the next section.

Consider first the co-operative solution when investment is sequential. Two trigger points, $\theta_1 < \theta_2$, are chosen to maximize the sum of the leader's and follower's value functions. Call the co-operative value function in this case C_{L+F}; using the same steps as before,

$$C_{L+F}(\theta) = \begin{cases} b_0\theta^\beta + b_1\theta^\beta & \theta < \theta_1, \\ \frac{\theta}{r-\mu} + b_2\theta^\beta - K + b_3\theta^\beta & \theta \in [\theta_1, \theta_2), \\ \frac{(2+\delta_L+\delta_F)\theta}{r-\mu} - 2K & \theta \geq \theta_2, \end{cases} \tag{26}$$

where $b_i,\ i = 0, 1, 2, 3$ are constants. The co-operative trigger points are determined by value matching and smooth pasting conditions at both points. Therefore

Proposition 6: *In the co-operative solution with sequential investment, the trigger points θ_1 and θ_2 of the first and second investments are, respectively,*

$$\begin{aligned}\theta_1 &= \left(\frac{\beta}{\beta-1}\right) K(r-\mu),\\ \theta_2 &= \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_L+\delta_F}\right)(r-\mu).\end{aligned}$$

Assumption 1 ensures that $\theta_2 > \theta_1$, since $\delta_L \leq -\delta_F$.

The comparative statics of θ_1 and θ_2 are standard and so are not discussed at length. The only difference in the comparative statics from previous ones is that θ_2 is decreasing in γ_L, while θ_F does not depend on γ_L. This fact is examined further in section 6.

Now consider the co-operative solution with simultaneous investment at the trigger point θ_3. The co-operative value function in this case is

$$C_S(\theta) = \begin{cases} b_4\theta^\beta & \theta < \theta_3, \\ \frac{2(1+\delta_S)\theta}{r-\mu} - 2K & \theta \geq \theta_3. \end{cases} \tag{27}$$

Again, value matching and smooth pasting determine θ_3:

Proposition 7: *The co-operative simultaneous investment trigger is*

$$\theta_3 = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_S}\right)(r-\mu) = \theta_S.$$

A similar analysis to that undertaken with the non-co-operative equilibria shows when co-operation involves simultaneous investment.

Proposition 8: *Simultaneous investment is the co-operative solution iff*

$$\lambda_C \equiv 2(1+\delta_S)^\beta - \left(1 + (1+\delta_L+\delta_F)^\beta\right) \geq 0. \tag{28}$$

A sufficient condition is $\delta_S \geq 0$.

Proof: The necessary and sufficient condition is that the value function for simultaneous investment $C_S(\theta) \geq C_{L+F}(\theta)$, for all $\theta \in [\theta_1, \theta_3]$. The strict convexity of the value functions means, however, that this requires that $C_S(\theta) \geq C_{L+F}(\theta)$ for all $\theta \in [0, \theta_1]$ i.e., $b_4 \geq b_0 + b_1$. From above,

$$\begin{aligned} b_0 + b_1 &= \left(\frac{1+(1+\delta_L+\delta_F)^\beta}{\beta-1}\right)\left(\left(\frac{\beta-1}{\beta}\right)\frac{1}{K(r-\mu)}\right)^\beta K, \\ b_4 &= \left(\frac{2}{\beta-1}\right)\left(\left(\frac{\beta-1}{\beta}\right)\frac{1+\delta_S}{K(r-\mu)}\right)^\beta K. \end{aligned}$$

It is immediate that $b_4 \geq b_0 + b_1$ iff condition (28) holds. The sufficient condition follows directly from equation (28), noting that assumption 1 implies that $\delta_L + \delta_F \leq 1$. ∎

Equation (28) is very similar to equation (25) and the intuition for propositions 3 and 8 is the same. The comparative statics of the co-operative solution are analysed in section A.3 in the appendix.

6. Inefficiencies

This section analyses the inefficiencies that arise in the non-co-operative equilibria. For the 'leader' inefficiency, identified in the next proposition, let $\lambda \equiv \lambda_C - \lambda_E$.

Proposition 9: *There are three investment inefficiencies:*

Follower*:* $\theta_F > (<)\ \theta_2$ *when* $\delta_L > (<)\ 0$*. In words, conditional on both equilibrium and the co-operative solution involving sequential investment, the non-co-operative follower invests too late (early) when* δ_L *is greater (less) than zero.*

Pre-emption*:* $\theta_P < \theta_1$*: conditional on both equilibrium and the co-operative solution involving sequential investment, the non-co-operative leader invests too early.*

Leader *(insufficient/excessive simultaneous investment in equilibrium): (i)* $\lambda = 0$ *for all values of* σ *iff* $\delta_L = \delta_F = 0$*; (ii) if either* $\delta_L > 0$ *or* $\beta > 2$*, then* $\lambda > 0$*; (iii) for any given* δ_F *and* $\beta < 2$*, there exists a critical value* $\underline{\delta_L} < 0$ *(which is a function of* δ_F *and* β*) such that* $\lambda > (<) 0$ *iff* $\delta_L < (>) \underline{\delta_L}$.

PROOF: See the appendix.

The follower inefficiency arises when investment is sequential. The follower does not consider the effect on the leader of its investment, and consequently invests either too soon (when $\delta_L < 0$) or too late (when $\delta_L > 0$). The pre-emption inefficiency arises when investment is sequential: the leader invests too early, relative to the co-operative solution. The leader inefficiency arises through inefficient simultaneous investment. In equilibrium, whether investment is sequential or simultaneous is determined by the leader's incentive to invest. The proposition gives the conditions under which the leader wishes to invest before the follower too often or too little, compared to the co-operative solution. For example, if the first-mover advantage and/or externalities are very strong, so that $\delta_L > 0$, or the degree of uncertainty σ is sufficiently low, equilibrium may involve sequential investment when co-operation would involve simultaneous investment. In short: the leader leads inefficiently and invests too early in equilibrium; and the follower invests inefficiently when it has been pre-empted.

The magnitude of the follower and pre-emption inefficiencies are measured by $\iota \equiv \theta_2 - \theta_F$ and $\kappa \equiv \theta_1 - \theta_P$. Proposition 6 shows that $\iota > (<) 0$ when $\delta_L < (>) 0$, and that $\kappa > 0$. In the rest of this section, we examine how the three inefficiencies depend on the model parameters. The first set of results, concerning ι and κ, follow directly from earlier results, and so are stated without proof.

LEMMA 3: ι *is (i) decreasing in* γ_L*; (ii) increasing (decreasing) in* γ_F *iff* $\delta_L > (<) 0$*; (iii) not a function of* γ_S*; (iv) decreasing in* α*; and (v) increasing (decreasing) in* σ *iff* $\delta_L < (>) 0$.

These comparative statics are, on the whole, straightforward. To simplify the discussion, suppose that $\delta_L < 0$: the follower imposes a negative externality on the leader, and $\theta_2 > \theta_F$.

Then an increase in γ_L diminishes the negative externality and hence decreases θ_2; θ_F is unchanged, and hence ι decreases. An increase in γ_F decreases both θ_2 and θ_F; but since the trigger points are convex in γ_F, the former decreases by more than the latter (when $\delta_L < 0$), because of the negative externality. Hence ι increases. A similar reasoning holds for the comparative statics with respect to α and σ.

LEMMA 4: *κ is (i) increasing in γ_L; (ii) decreasing in γ_F; (iii) not a function of δ_S; (iv) increasing in α; and (v) if $\partial\theta_P/\partial\sigma < 0$, increasing in σ.*

The unusual comparative static shown in proposition 4 gives rise to the strong result in part (v) of the lemma: if the relevant conditions hold, an increase in σ raises θ_1 but lowers θ_P. In this case, greater uncertainty exacerbates the pre-emptor inefficiency. The other parts of lemma 4 are straightforward; for example, it is intuitive that the pre-emptor inefficiency is increased by the degree of externalities (i.e., an increase in α).

LEMMA 5: *(i) λ is (a) increasing (decreasing) in γ_L iff $\delta_L < (>)0$; (b) if $\delta_L \leq 0$, increasing in γ_F; (c) increasing in γ_S.*

(ii) If $\lambda \geq 0$ for $\alpha = \alpha'$, then $\lambda \geq 0 \;\forall\; \alpha'' \geq \alpha'$.

In both equilibrium and the co-operative solution, simultaneous investment is favoured by an increase in the flow payoff from simultaneous investment (i.e., higher γ_S) and a decrease in the flow payoff to the leader (i.e., lower γ_L). A change in γ_S has a larger effect on the co-operative solution, and so λ increases in this parameter. The least obvious effect comes from an increase in γ_F. If the flow payoff to being the follower increases, then simultaneous investment is less favoured in the co-operative solution. The same is true in equilibrium only if δ_L is negative. In equilibrium, the payoff to being the leader relative to a simultaneous investor determines whether investment is simultaneous. If the follower's payoff increases, then it invests earlier (θ_F decreases). If δ_L is negative, earlier investment by the follower decreases the payoff to being the leader, and so encourages simultaneous investment. Therefore, when δ_L is weakly less than zero, an increase in γ_F favours simultaneous investment in equilibrium but not in the co-operative solution, and hence increases λ. (When δ_L is sufficiently positive, the effect of γ_F on λ is ambiguous.) Finally, positive externalities exacerbate the leader inefficiency: although a greater positive externality causes less sequential investment in equilibrium, it causes even less in the co-operative solution.

Lemma 6: *Joint sufficient conditions for λ to be a decreasing function of σ are: $\delta_S \geq 0$ and either (i) $\delta_L \geq 0$ and $\delta_F \leq e^{-1} - 1$ or (ii) $\delta_L \leq 0$ and $\delta_F \geq e^{-1} - 1$.*

(The lemma is a direct consequence of lemmas 1 and A.3; the latter can be found in the appendix.) As with λ_E and λ_C, a general comparative static for λ with respect to σ is not available analytically. The lemma gives sufficient conditions for the leader inefficiency to be exacerbated by an increase in the degree of uncertainty.

7. Conclusions

This paper has analysed irreversible investment in a project with uncertain returns, when there may be an advantage to being the first investor, and externalities to investing when others also invest. It therefore extends standard 'real options' analysis to a setting where there are general strategic interactions between investing agents. This framework captures a variety of strategic situations and encompasses a number of earlier contributions.

We believe that this is an important area of research. The real options literature has taught us that an option value is created by irreversibility and uncertainty; this option value typically leads to delayed investment, where the degree of delay increases with uncertainty. Strategic interactions, omitted from the standard real options analysis, can change and may even eliminate this option value. This has significant qualitative and quantitative effects on investment. In particular, we have shown that due to the interaction of pre-emption with externalities, greater uncertainty can actually hasten, rather than delay, investment, contrary to the usual presumption.

APPENDIX

A.1. The Comparative Statics of θ_P

An immediate corollary of proposition 1 shows how pre-emption affects investment. Let θ_L be the investment time of an agent who invests first and does so optimally i.e., not according to rent equalization. It is straightforward to show that

$$\theta_L = \left(\frac{\beta}{\beta - 1}\right) \frac{K(r-\mu)}{\delta_L}. \tag{A1}$$

Secondly, let $\theta_M \equiv K(r-\mu)$; this can be viewed as the non-strategic Marshallian trigger i.e., the investment point of an agent who ignores uncertainty and any subsequent investment by other agents.

Corollary A.1: $\theta_P < \theta_L$. *θ_P is greater (less) than θ_M iff $1 + \delta_F - \beta\delta_L$ is greater (less) than zero.*

So, pre-emption causes the leader to invest earlier than it does when the leader can invest optimally. And pre-emption can drive the trigger point θ_P below the non-strategic Marshallian trigger $\theta_M = K(r-\mu)$. The condition in the proposition can be viewed in a number of ways. First, a necessary condition is that $\delta_L \geq 0$ (since $1 + \delta_F \geq 0$, from assumption 1). This requires that the first-mover advantage, measured by γ_L, and/or the level of externalities α be sufficiently large. Secondly, if $\delta_L \geq 0$, then the condition will hold if β is sufficiently large ($\beta > (1+\delta_F)/\delta_L$) i.e., σ sufficiently small. This finding extends the result of Fudenberg and Tirole (1985), who show in a deterministic setting that pre-emption drives the first adoption time below the (equivalent of the) Marshallian level. Corollary A.1 shows how much uncertainty there can be, as a function of other model parameters, for this result still to hold. Thirdly, if $\delta_L \geq 0$, then $\theta_P < \theta_M$ if δ_F is sufficiently small: $\delta_F < \beta\delta_L - 1$. This last condition requires that the second-mover disadvantage be sufficiently large i.e., γ_F sufficiently negative.

The comparative statics of θ_P with respect to the payoff parameters are examined in the next proposition.

Proposition A.1: *θ_P is (i) decreasing in γ_L; (ii) increasing in γ_F; (iii) not a function of δ_S; and (iv) decreasing (increasing) in α iff $\delta_L > (<) \delta_F/\beta$.*

PROOF: The comparative statics of θ_P follow from differentiation of equation (21). Only the comparative static with respect to α is not straightforward. Re-write equation (21) as

$$-\psi\theta_P^{\beta} + \frac{\theta_P}{r-\mu} - K = 0, \tag{A2}$$

$$\psi \equiv \frac{K}{\beta - 1}\left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\theta_F^{-\beta}. \tag{A3}$$

Total differentiation gives

$$\frac{\partial\theta_P}{\partial\alpha} = \left(\frac{\theta_P^{\beta}}{\frac{1}{r-\mu} - \beta\psi\theta_P^{\beta-1}}\right)\frac{\partial\psi}{\partial\alpha}.$$

In equilibrium, it must be that the leader's value function crosses the follower's value function from below (as functions of the state variable θ). This implies that the denominator is positive for the equilibrium value of θ_P. Hence $\text{Sign}\frac{\partial\theta_P}{\partial\alpha} = \text{Sign}\frac{\partial\psi}{\partial\alpha}$. Differentiation gives

$$\frac{\partial\psi}{\partial\alpha} = -\left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\alpha}\right)\left(\frac{\beta\delta_L-\delta_F}{1+\delta_F}\right)\theta_F^{-\beta}.$$

The proposition follows. ■

θ_P is decreasing in γ_L and increasing in γ_F because this trigger point is determined by indifference. If the gain to being the leader increases or to being the follower decreases, then indifference requires that the leader invests earlier. The last part of the result is not immediately obvious: at its trigger point, the leader is indifferent between leading and following; but both the leader's and follower's returns increase as α increases; and θ_P is decreasing (increasing) in α iff the increase in the leader's return is the stronger (weaker) effect. To gain an intuition for the result, consider the case in which θ_P is decreasing in α. Re-write the leader's indifference condition as $L(\theta_P;\alpha) - F(\theta_P;\alpha) = 0$. Then

$$\frac{\partial\theta_P}{\partial\alpha} = -\left(\frac{\partial L}{\partial\alpha} - \frac{\partial F}{\partial\alpha}\right) / \left(\frac{\partial L}{\partial\theta_P} - \frac{\partial F}{\partial\theta_P}\right).$$

Since $L < (>) F$ for $\theta < (>) \theta_P$, it must be that $\frac{\partial L}{\partial\theta_P} > \frac{\partial F}{\partial\theta_P}$. Hence the sign of $\frac{\partial\theta_P}{\partial\alpha}$ is determined by whether $\frac{\partial L}{\partial\alpha}$ is greater or less than $\frac{\partial F}{\partial\alpha}$. (The partial derivatives here hold θ_P constant, but allow θ_F to vary.)

There are two effects as α increases. First, the agents' flow returns once both agents have invested increase. Secondly, the follower invests earlier (θ_F decreases); when $\delta_L > 0$, this benefits both agents, while when $\delta_L < 0$, it benefits the follower but not the leader. For the leader, both effects are important. For the follower, only the first effect is of first-order significance: since the follower chooses θ_F optimally, any variation in the trigger point due to a small change in α induces only a second-order variation in returns. When $\delta_L > 0$, this suggests qualitatively that the leader's return increases more when α rises. The comparison is more complicated when $\delta_L < 0$; note, however, that the second effect for the leader is limited by the parametric condition in the proposition, which requires that $\delta_L \geq \delta_F/\beta$. In this case, the lemma shows that the first effect dominates for the leader, and to such an extent that the leader's value function increases with α by more than does the follower's.

A.2. The Comparative Statics of Simultaneous Investment

In this section, we consider how the payoff parameters affect whether the equilibrium without pre-emption involves sequential or simultaneous investment.

Lemma A.1: *(i) λ_E is (a) decreasing in γ_L; (b) increasing (decreasing) in γ_F if $\delta_L < (>) 0$; (c) increasing in γ_S.*

(ii) If $\lambda_E \geq 0$ for $\alpha = \alpha'$, then $\lambda_E \geq 0 \ \forall \ \alpha'' \geq \alpha'$.

Proof: All parts of the lemma are proved by differentiation of the expression for λ_E. For example, part (ii) is shown by differentiating λ_E with respect to α:

$$\frac{\partial \lambda_E}{\partial \alpha} = \frac{\beta \lambda_E}{1+\alpha} + \frac{\beta(1-(1+\delta_F)^{\beta-1})}{1+\alpha} \geq \frac{\beta \lambda_E}{1+\alpha}.$$

Hence if $\lambda_E \geq 0$, then $\partial \lambda_E / \partial \alpha \geq 0$. This proves part (ii) of the lemma. Part (i) follows as easily, and so the proof is omitted. ■

Simultaneous investment is favoured by an increase in the flow payoff from simultaneous investment (i.e., higher γ_S) and a decrease in the flow payoff to the leader (i.e., lower γ_L). Less obvious are the effects of increases in γ_F and α. In equilibrium, the payoff to being the leader relative to a simultaneous investor determines whether investment is simultaneous. If the follower's payoff increases (i.e., γ_F increases), then the follower invests earlier (θ_F decreases). If δ_L is negative, earlier investment by the follower decreases the payoff to being the leader, and so encourages simultaneous investment. Therefore, when δ_L is weakly less than zero, an increase in γ_F favours simultaneous investment in equilibrium. The converse argument holds for $\delta_L > 0$. A unit increase in α causes the flow payoff of the leader (when the other agent has invested) to rise by γ_L, and to being a simultaneous investor to rise by γ_S. In part (ii) of the lemma, however, that only cases in which $\lambda_E \geq 0$ are considered—that is, the payoff to being a simultaneous investor relative to being the leader is large enough that equilibrium investment is simultaneous. In these cases, γ_S is greater than γ_L, and so an increase in α favours simultaneous investment.

A.3. The Comparative Statics of the Co-operative Solution

The next two lemmas examine the comparative statics of λ_C; the proofs are similar to those of lemmas (A.1) and (1) and so are omitted.

Lemma A.2: *1. λ_C is (i) decreasing in γ_L; (ii) decreasing in γ_F; and (iii) increasing in γ_S.*

2. If $\lambda_C \geq 0$ for $\alpha = \alpha'$, then $\lambda_C \geq 0 \; \forall \; \alpha'' \geq \alpha'$.

Lemma A.3: *If $\delta_S \geq 0$, then λ_C is a decreasing function of σ.*

A.4. Proof of Proposition 1

Define

$$\Delta(\theta) \equiv \frac{\theta}{r-\mu} - K - \left(\frac{\theta}{\theta_F}\right)^{\beta} \left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right) \frac{K}{\beta-1}$$

i.e., $L(\theta) - F(\theta)$, where $L(\theta)$ is conditional on the leader having invested, and $F(\theta)$ is conditional on the leader having invested but not the follower. The pre-emption trigger θ_P is determined by the equation $\Delta(\theta_P)$, if a solution exists. There are three possibilities: that there are (i) no, (ii) one or (iii) two solutions to the the equation. We use the following facts: (i) $\Delta(\theta)$ is a continuously differentiable function of θ; (ii) $\Delta(0) = -K < 0$; (iii) $\Delta(\theta_L) = \frac{K}{(\beta-1)(1+\delta_F)}\left(\left(\frac{\theta_L}{\theta_F}\right)^{\beta}\beta\delta_L + (1-\left(\frac{\theta_L}{\theta_F}\right)^{\beta})(1+\delta_F)\right)$; (iv) since, from assumption 1, $\delta_L \geq \delta_F$, $\Delta(\theta_L) \geq \frac{K}{(\beta-1)(1+\delta_F)}\left(\left(\frac{\theta_L}{\theta_F}\right)^{\beta}\beta\delta_F + (1-\left(\frac{\theta_L}{\theta_F}\right)^{\beta})(1+\delta_F)\right)$; (v) for all $\delta_F \in [-1,0]$ (see assumption 1) and $\beta \geq 1$, $\left(\frac{\theta_L}{\theta_F}\right)^{\beta}\beta\delta_F + (1-\left(\frac{\theta_L}{\theta_F}\right)^{\beta})(1+\delta_F) \geq 0$. Hence, by the intermediate value theorem, there exists a value $\theta_P < \theta_L$ such that $\Delta(\theta_P) = 0$, and

$\Delta(\theta)$ is less (greater) than 0 for θ immediately less (greater) than θ_P. (If there are multiple values, the lowest is the relevant solution, since it is the first point at which the value of being the leader crosses from below the value of being the follower.)

A.5. Proof of Proposition 4

The difference between the values of the leader's option-like term and the follower's option associated with the follower's investment is

$$\Delta(\theta, \beta) \equiv (b_{L1} - b_F)\theta^\beta = \left(\frac{\beta\delta_L - (1 + \delta_F)}{1 + \delta_F}\right) F(\theta)$$

where $F(\theta) \equiv b_F \theta^\beta > 0$ for $\theta \in (\theta_P, \theta_F)$. Hence

$$\frac{\partial\Delta(\theta, \beta)}{\partial\beta} = \frac{\delta_L F(\theta) + (\beta\delta_L - (1 + \delta_F))\frac{\partial F(\theta)}{\partial\beta}}{1 + \delta_F}.$$

But

$$\frac{\partial F(\theta)}{\partial\beta} = F(\theta)\left(-\frac{1}{\beta - 1} + \ln\left(\frac{\theta}{\theta_F}\right)\right).$$

Hence

$$\frac{\partial\Delta(\theta, \beta)}{\partial\beta} = \frac{F(\theta)}{1 + \delta_F}\left(\frac{-(1 + \delta_L + \delta_F)}{\beta - 1} + (\beta\delta_L - (1 + \delta_F))\ln\left(\frac{\theta}{\theta_F}\right)\right).$$

Suppose that $\beta\delta_L - (1 + \delta_F) > 0$. Then clearly $\Delta(\theta, \beta) > 0$. Also, since $\delta_L > 0$ in this case, $1 + \delta_L + \delta_F > 0$; with $\theta < \theta_F$, this means that both terms in the expression for $\partial\Delta(\theta, \beta)/\partial\beta$ are negative. β is a decreasing function of σ; hence $\partial\Delta(\theta, \beta)/\partial\sigma > 0$. Therefore if $\beta\delta_L - (1 + \delta_F) > 0$, the leader's value increases by more than the follower's for any increase in σ for $\theta \in (\theta_P, \theta_F)$. The proposition follows.

A.6. Proof of Lemma 1

Differentiate λ_E with respect to β:

$$\frac{\partial\lambda_E}{\partial\beta} = (1 + \delta_S)^\beta \ln(1 + \delta_S) - \delta_L(1 + \delta_F)^{\beta - 1}(1 + \ln(1 + \delta_F)). \tag{A4}$$

It is sufficient for λ_E to be an increasing function of β that all terms in equation (A4) be positive. Hence joint sufficient conditions are: (i) $\delta_S \geq 0$, so that $\ln(1+\delta_S) \geq 0$; (ii) $-\delta_L(1+\ln(1+\delta_F)) \geq 0$, which in turn requires that either (a) $\delta_L \geq 0$ and $1 + \ln(1 + \delta_F) \leq 0$ i.e., $\delta_F \leq e^{-1} - 1$, or (b) the converse. To complete the proof of the first part, note that β is decreasing in σ. The proof of the second part is very similar, and so is omitted.

A.7. Proof of Proposition 9

The first two parts of the proposition (relating to the follower and pre-emptor inefficiencies) follow from equations (14), (A1), and propositions 1 and 6. The proof of the third part of the proposition (relating to the leader inefficiency) requires a comparison of the necessary and sufficient conditions (25) and (28). Rewrite the conditions as

$$\begin{aligned} 2(1+\delta_S)^\beta - 1 &\geq 1 + 2\beta\delta_L(1+\delta_F)^{\beta-1}, \\ 2(1+\delta_S)^\beta - 1 &\geq (1+\delta_L+\delta_F)^\beta. \end{aligned}$$

The proposition gives the conditions under which $1 + 2\beta\delta_L(1+\delta_F)^{\beta-1}$ is greater (less) than $(1 + \delta_L + \delta_F)^\beta$. Let $\Delta \equiv (1+\delta_L+\delta_F)^\beta - 2\beta\delta_L(1+\delta_F)^{\beta-1}$; the issue is whether Δ greater or less than 1.

(i) Consider the equation $\Delta = 1$, involving three variables: δ_L, δ_F and β. Since Δ is continuously differentiable for $\delta_F \in (-1, 0]$, $\delta_F \leq \delta_L \leq -\delta_F$ and $\beta > 1$, $\Delta = 1$ defines implicitly δ_L (say) as a function of δ_F and β. Sufficiency: if $\delta_F = \delta_L = 0$, then $\Delta = 1 \forall \beta$. Necessity: in order for $\Delta = 1$ for any given values of δ_L and δ_F, both terms in the expression for Δ must be independent of β. This requires that $1 + \delta_L + \delta_F = 1$ and $\delta_L(1 + \delta_F) = 0$ i.e., $\delta_L = 0$ and $\delta_F = 0$.

(ii) Since $\delta_L + \delta_F \leq 0$ and $-1 \leq \delta_F$, by assumption (1), a sufficient condition for $\Delta < 1$ is $\delta_L > 0$. If $\delta_L < 0$, then Δ is a decreasing function of δ_L:

$$\frac{\partial \Delta}{\partial \delta_L} = \beta\left((1+\delta_L+\delta_F)^{(\beta-1)} - (1+\delta_F)^{(\beta-1)}\right) - \beta(1+\delta_F)^{(\beta-1)},$$

$1 + \delta_L + \delta_F < 1 + \delta_F$ when $\delta_L < 0$, and $1 + \delta_F \geq 0$, from assumption 1. Therefore Δ is maximized when $\delta_L = \delta_F$, its minimum value by assumption 1. At this value of δ_L, $\Delta = \overline{\Delta} \equiv (1+2\delta_F)^\beta - 2\beta\delta_F(1+\delta_F)^{(\beta-1)}$. Note that $\overline{\Delta}$ is defined only for $\delta_F \geq -1/2$. When $\delta_F = -1/2$, $\overline{\Delta} = \beta(1/2)^{(\beta-1)}$; this is greater (less) than 1 iff β is less (greater) than 2.

(iii) From part (ii), Δ is a decreasing function of δ_L; and $\Delta = (1 + \delta_F)^{\beta} \leq 1$ when $\delta_F = 0$. The statement in the proposition follows immediately.

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