REAL OPTION DECISION RULES FOR OIL FIELD DEVELOPMENT UNDER MARKET UNCERTAINTY USING GENETIC ALGORITHMS AND MONTE CARLO SIMULATION

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Abstract

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A decision to invest in the development of an oil reserve requires an in-depth analysis of several uncertainty factors. Such factors may involve either technical uncertainties related to the size and economic quality of the reserve, or market uncertainties. When a great number of investment alternatives are involved, the task of selecting the best alternative or a decision rule is very important and also quite complicated due to the considerable number of possibilities and parameters that must be taken into account.

This work proposes a model based on Genetic Algorithms and on Monte Carlo simulation which has been designed to find an optimal decision rule for oil field development alternatives, under market uncertainty, that may help decision-making with regard to: developing a field immediately or waiting until market conditions are more favorable.

This optimal decision rule is formed by three mutually exclusive alternatives which describe three exercise regions along time, up to the expiration of the concession of the field. The Monte Carlo simulation is employed within the genetic algorithm for the purpose of simulating the possible paths of oil prices up to the expiration date, and it is assumed that oil prices follow a Geometric Brownian Motion.

Keywords: Real Options, Genetic Algorithms, Monte Carlo Simulation

1. Introduction

A decision to invest in the development of an oil reserve requires an in-depth analysis of several uncertainty factors. Such factors may involve either technical uncertainties related to the size and economic quality of the reserve, or market uncertainties (e.g., oil price). Considering that the technical uncertainties are known, the analysis of market uncertainties will help decision-making with regard to investing in a field immediately or waiting until market conditions are more favorable. When a great number of investment alternatives are involved, the task of selecting the best alternative or a decision rule is very important and also quite complicated due to the considerable number of possibilities and parameters that must be taken into account.

This paper presents a Genetic Algorithm (GA) [1] [2] for obtaining the optimal investment decision rule for the development of an oil reserve under market uncertainty, particularly with regard to the price of oil. This optimal decision rule is formed by three mutually exclusive alternatives which describe three exercise regions along time, up to the expiration of the concession of the field. The Monte Carlo simulation is employed within the genetic algorithm for the purpose of simulating the possible paths of oil prices up to the expiration date, and it is assumed that oil prices follow a Geometric Brownian Motion (GBM).

Section 2 describes the problem of the optimal exercise of the development option. Section 3 describes how the problem was modeled with the use of a GA with the Monte Carlo simulation and the Real Options theory, and also the way how chromosomes were represented and evaluated. Section 4 presents the results obtained with the proposed model and finally, Section 5 contains the conclusions of this study.

2. Description of the Problem

This paper attempts to obtain an optimal decision rule for investment in an oil reserve under market uncertainty, particularly with regard to the price of oil. This decision rule is formed by three mutually exclusive alternatives which describe three exercise regions along time, up to the expiration of the concession of the field.

Each alternative presents a threshold curve, which is the maximum value of the real option and determines the optimal exercise of the real option. All the threshold curves together represent the decision rule that maximizes the value of the option of these alternatives. The threshold curve may be approximated by means of a logarithmic function in the form of: $a + b\ln(\tau)$ and a free point which is situated near the expiration of the option [3] [4] [5]. The logarithmic function is employed because it represents a good approximation to the threshold curve obtained by finite differences. For the various alternatives, there are several mutually exclusive threshold curves which are going to determine the exercise regions that are delimited by the intersections between them. The possible existence of waiting periods between the regions formed by the alternatives is also considered. Thus, for the case of three alternatives, five regions may be formed (two waiting regions and three exercise regions, one for each alternative).

Traditionally, in order to evaluate each alternative for investment in the oil field, attempts are made to maximize the *Net Present Value* (NPV) [6] [7], in other words, the best alternative is the one that presents the highest NPV. In this case, the NPV equation may be written as (equation 1):

$$NPV_t = qP_tB - D \qquad \textbf{Eq. 1}$$

where q is the economic quality of the reserve, P is the price of oil, B is the estimated size of the reserve, and D is the investment for development. This NPV formula is applicable when all its terms are known (deterministic values). However, in a real problem, all these terms may vary in time and are sources of uncertainty for the problem. For the purpose of this paper, it will be assumed that there are only market uncertainties, i.e., that the price of oil is the only source of uncertainty.

Because the price of oil varies in time, it is said that it follows a stochastic process. The Geometric Brownian Motion (GBM) is the stochastic process that is most frequently employed in the literature that addresses real options [8] for commodities, and though it is not the best choice for the case of commodities, the GBM is reasonable for the purpose of the analysis that has been undertaken in this work. The algorithm may be directly adapted to more sophisticated processes (such as mean-reversion and jumps). The equation for simulating the future price of oil P(t), which follows a GBM, given the current price P(t-1), is given by equation 2 [9] [10]:

$$P(t) = P_{t-1} \exp\left[\left(r - \delta - 0.5\sigma^2\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right] \qquad \varepsilon \approx \mathrm{N}(0,1) \qquad \textbf{Eq. 2}$$

where *r* é is the risk-free rate, δ is the convenience yield rate of the oil field, and σ is the volatility of the oil price.

The model that has been used is an extension of the one presented by Dixit (1993) [11], which was adapted to oil projects (for details regarding this adaptation see Dias, 1996 [3] [12] [13]).

The proposed model integrates a genetic algorithm, the Monte Carlo simulation and the real options theory for the purpose of obtaining an optimal decision rule for three alternatives for investment in an oil reserve, considering that the price of oil is uncertain.

In order to simulate the price of oil, this paper has considered the following parameters:

- Time to expiration (T): 2 years.
- Risk-free rate (r): 8 %
- Convenience yield rate of the oil field (δ): 8 % p.a.
- Price volatility (σ): 25 % p.a.
- Initial oil price (P_0): 20 $/bbl

The three alternatives that have been considered for this paper present the following parameters:

	Alternative 1	**Alternative 2**	**Alternative 3**
Estimated size of reserve (B):	400MM barrels	400MM barrels	400MM barrels
Quality of reserve (q):	8%	16%	22%
Investment for development (D):	400MM US$	1000MM US$	1700MM US$

In the table above, it may be observed that it only makes sense to consider higher investment alternatives if they generate an increase in the economic quality of the reserve, that is, if the investment in more wells for further drainage of the same reservoir enhances the *economic* quality of the reserve that is being developed, in other words, if such an investment represents a means by which to extract oil more quickly. For this reason, this type of alternative is worth more than the other one with few wells for draining the same reserve.

The problem of determining the optimal decision rule for these three alternatives considering the uncertainty of oil prices is difficult to compute because it is nonlinear. As a result of this feature, genetic algorithms represent a good choice for finding the optimal decision rule for oil field development.

3. Modeling of the Problem

This section describes the proposed model, which integrates the Monte Carlo simulation and the real options theory into a genetic algorithm for the purpose of obtaining an optimal decision rule.

Figure 1 presents the flow chart of the proposed model, where it may be observed how each chromosome that is generated is validated so as to satisfy the constraints that have been imposed until it completes the entire number of individuals in a population. Next, the Monte Carlo simulation is employed. For each Monte Carlo iteration (i), the price of oil is simulated every two days over a period of two years and this price sequence has been named $Path_i$. The evaluation of the chromosome is given by the average of the chromosome's evaluations for each $Path_i$ until it completes the total of the population. The evolution of the genetic algorithm continues as the operators (crossover, mutation, etc.) are applied and this same procedure is applied to the generations that follow.

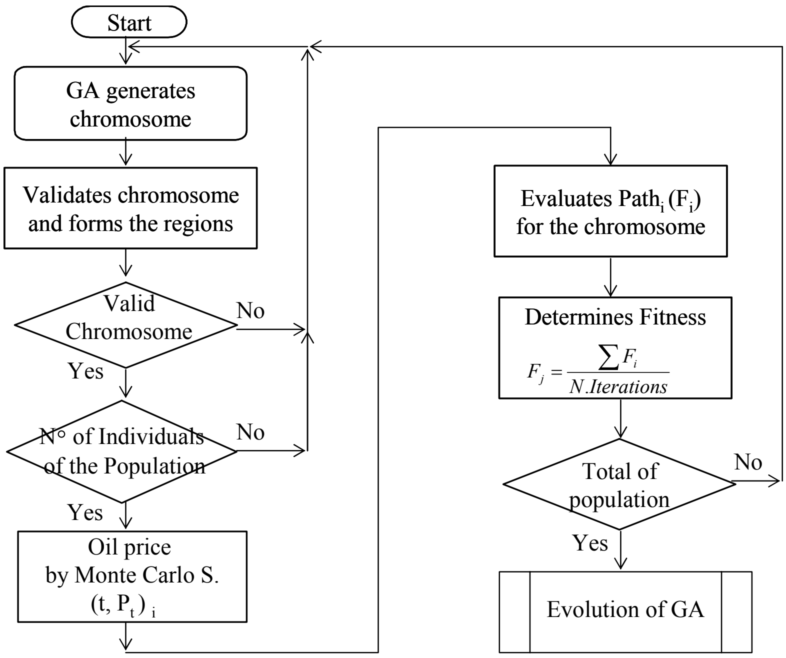

Figure 1 – Flow chart of the model

This algorithm was implemented in C++ language and *Genocop III* [2] [14] [15], which is a genetic algorithm software capable of working with real representation and with constraints (linear, nonlinear and domain). The *Genocop III* was adapted so as to work in the C++ Builder 5 environment with a Monte Carlo simulation algorithm for the purpose of estimating the price of oil for each time instant.

Representation of the Chromosome

The chromosome is made up of 15 genes which are divided into five groups of three genes. The groups of three genes of the chromosome represent the threshold curves of each alternative (variables a and b of curve $a + b\ln(\tau)$ and the free point) [3] [4] [5], as well as the possible waiting periods (curves $a - b\ln(\tau)$), according to the illustration in Figure 2. These threshold and waiting curves are subject to a set of constraints that must be satisfied in the process of generating each chromosome. These constraints ensure that the chromosomes will form exercise regions, and also introduce heuristic solutions to the problem so as to reduce the search space.

Threshold 1			Threshold 2			Threshold 3			Waiting 1			Waiting 2		
Free Point	A1	B1	Free Point	A2	B2	Free Point	A3	B3	Free Point	A4	B4	Free Point	A5	B5

Figure 2 – Chromosome

The free points are chosen in each alternative for the same time instant corresponding to 0.1 year. The logarithmic curve begins at instant 0.1 + Δt, where Δt corresponds to the time interval.

The algorithm was executed with the following parameter values:

- Population Size: 1000 individuals.
- Number of generations: 100
- Percentage of population restored (Gap): 0.25

Evaluation of the Chromosome

The objective of the Genetic Algorithm is to maximize the dynamic net present value of the real option (NPL of the oil reserve, Equation 1), where the oil price is time-dependent. To this end, the Monte Carlo simulation is employed with 10,000 iterations and at each iteration (*i*) the price of the oil is estimated for each 2-day interval (t) until expiration (2 years), based on the assumption that the price follows a GBM. In this manner, for each iteration, a "path" of the oil price, which has been named $Path_i$, is formed.

The evaluation of chromosome (j) begins with the first iteration of the Monte Carlo simulation ($i = 1$); for this iteration, there is a $Path_i$ of the oil price, and for each t of this $Path_i$, it is verified if the oil price reaches one of the exercise regions. If the oil price reaches a region, the option is exercised, the NPV for this oil price is calculated, and then one goes on to the next iteration. When the exercise is performed at an instant $t > 0$, the net present value is updated according to the risk-free discount rate and the *option value* for that iteration i (F_i) is obtained. If $Path_i$ has been completed, i.e., it is at expiration, and none of the exercise regions has been reached, then the NPV is zero. This process is repeated for each iteration (i).

The evaluation value (*fitness*) for this chromosome has been determined by the mean value of the NPVs, which was found for each iteration (Equation 3).

$$F_j = \frac{\sum_{i=1}^{10000} F_i}{N \circ Iterations} \qquad \textbf{Eq. 3}$$

The best chromosome will be the one that maximizes the value of F_j .

4. Results

The results obtained by the genetic algorithm are shown below: Figure 3 presents the optimal decision rule produced by the model for the three investment alternatives that have been considered for an option period of two years, with the exception of 10000 Monte Carlo simulations for each chromosome. The exercise regions for each alternative and the waiting regions may be observed in the graph.

The highest net present value (NPV) that the genetic algorithm obtained for the option was US $355.22 million.

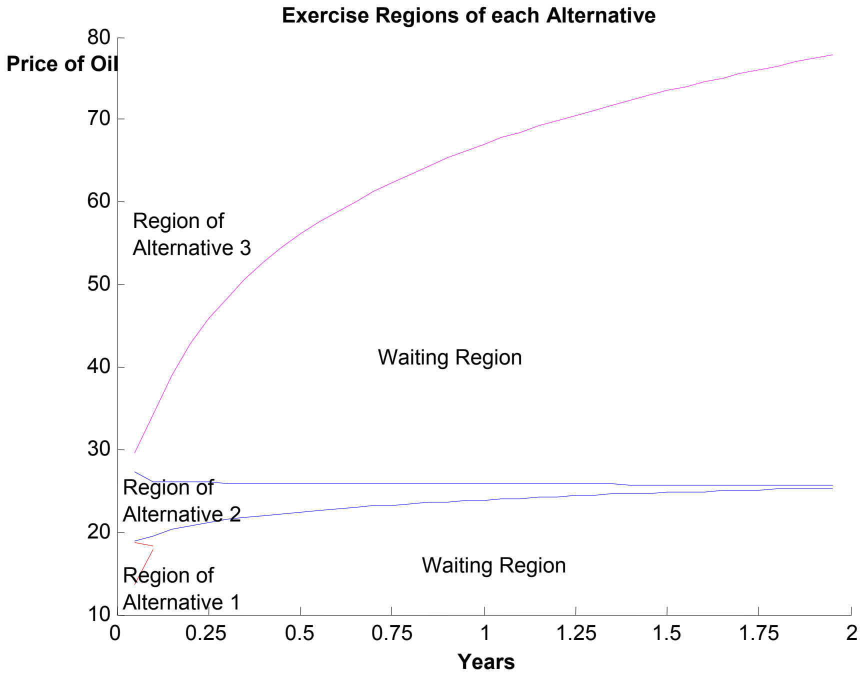

Figure 3 – Optimal exercise rule

5. Conclusions

The result obtained by the genetic algorithm proved to be similar to the result obtained in the analysis by partial differential equations.

It may be observed that the value of NPV decreases as the number of iterations in the Monte Carlo Simulation increases, which is reasonable because when the number of interactions is higher, there is a convergence to the mean value of NPV.

The advantage of using the model with genetic algorithms in the analysis of development alternatives is that it is more flexible. This makes it possible to introduce a greater number of investment alternatives, to change the stochastic process or to introduce other uncertainties with minor modifications. Such aspects represent one of the most important limitations in the case of analytical methods, where the increasingly higher number of random variables and of alternatives makes it practically impossible to solve the partial differential equations. And in addition, changing the stochastic process involves changing all the partial differential equations.

Another advantage is that the GA makes it possible to obtain optimal or suboptimal decision rules and avoids the need to solve partial differential equations (PDE).

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