Is the myopic investor right? Numerical evidence for systematic overestimation of investment reluctance for real options

Oliver Mußhoff [*], **Norbert Hirschauer** [**], **Alfons Balmann** [***], **Martin Odening** [****]

Abstract

Empirical applications of real options models in competitive environments implicitly exploit the optimality of myopic planning. In a seminal paper Leahy (1993) shows that the optimal investment strategy of a myopic planner, who ignores market entries and exits of competitors as well as the resulting price effects, also constitutes a market equilibrium under rather general conditions. As a result, the calculation of optimal investment strategies is simplified considerably because competition does not have to be taken into account. In this paper, however, we demonstrate that myopic planning may lead to non-optimal investment strategies. This is due to the fact that it is difficult, or even impossible, to specify the correct or equivalent price process for the myopic investor using real world data. The myopic investor acts on the assumption of an unregulated (exogenous) price process. But what we observe in the real (competitive) world is indeed the outcome of a regulated (endogenous) price process. Hence, an estimation of parameters which is based on the unregulated form of stochastic process is inconsistent. This misconception, whose outcome we call "competitive bias", has been widely ignored in the literature. Our paper quantifies this bias, analyses its determinants and shows the outcome of alternative estimation procedures which could be used to get around it. It turns out that due to the "competitive bias" the widely acknowledged "reluctance to invest" is overestimated. The suitability of alternative estimation methods depends on their respective specifications.

Keywords: investment, uncertainty, competition, myopic planning

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A. Introduction

The analysis of investment decisions is important from a prescriptive as well as from a descriptive viewpoint. Investments pin down a firm's capital for a long time period and represent an important factor for the economic development both on firm and sector level. In the last two decades, investment theory has largely focused on the simultaneous consideration of uncertainty, irreversibility and managerial flexibility when asking for the optimal timing of capital stock adjustments, i.e. investments and disinvestments. In particular real options models are used to address these aspects[1]. Real options models exploit the analogy between a financial option and an investment project. The following statement applies: Irreversible investments under uncertainty should only be realized if the expected (and discounted) investment returns exceed the initial investment cost by a significant amount. In other words, investment triggers derived from real options models deviate from traditional investment criteria like net present value or internal rate of return. Real options theory thereby offers a potential explanation for economic hysteresis. In recent history, the real options approach has become increasingly popular for analyzing investment problems in general (see e.g. Carey & Zilberman (2002), Dias (2001) and Pinches (1998)). Almost all of these applications emphasize the potential of real options for explaining an empirically observed reluctance to (dis)invest.

However, there is a difference between financial options and investment projects that seems to hamper a transfer of option pricing methods. Whereas financial options constitute exclusive rights for their owners, real investment opportunities are rarely unique. Due to the non-exclusiveness of investment options likewise responses of competitors can be expected when they are faced with aggregate uncertainty[2]. Their combined reactions change sectoral supply and hence equilibrium prices. As a consequence, the price process, which determines option values and optimal investment triggers, can no longer be considered to be exogenous. This reasoning has given rise to real options models which consider competition explicitly[3]. Leahy (1993) demonstrates in a seminal paper that under general conditions perfect competition need not to be considered in the producer's investment decision in terms of trigger prices[4]. The implications of Leahy's theorem are interesting: Provided that the correct parameters of the price process can be determined, the burdensome and iterative determination of an endogenous equilibrium price process can be avoided, when dealing with competitive markets. Accordingly, an investor can decide myopically and ignore market entries of competitors. Although the decision of such a myopic planner is based on wrong expectations concerning the stochastic price process, he will find the correct investment strategy in terms of a trigger price which constitutes a competitive equilibrium. In other words: The myopic planner is right for the wrong reasons. This statement is the starting point of the present contribution. Our paper addresses problems which may arise when applying real options models to competitive markets. More concretely, we quantify estimation errors and biases that may occur when a myopic planner estimates parameters of the (wrongly assumed) unregulated price process from real world data. This problem will necessarily occur because the price process assumed by the myopic planner does not exist and hence, cannot be observed in reality. In fact, empirical price data are always the outcome of an endogenous equilibrium process. Caballero und Pindyck (1996) mention possible biases when estimating the price dynamics assumed by the myopic investor from real world data. However, they do not quantify the magnitude of this error.

Our paper has the following structure: The subsequent section B defines and compares the decision problem of a competitive investor (small investor) with the decision problem of a myopic investor. Following Leahy (1993) the equivalence of both problems is stated. Section C assesses the performance of different procedures to estimate the parameters of the price process from time series of prices. Additionally it illustrates the magnitude of the random error emerging in practical applications whatever method used. Section D completes the paper with conclusions concerning the specification and interpretation of applications of real options models.

B. Problem Statement

We consider a perfectly competitive industry consisting of homogenous risk neutral firms. All firms are small and hence price takers. They all produce with the same constant-returns-to-scale technology at constant variable costs c per unit. Capital is the only input. One unit of capital allows producing one unit of output. The capital stock can be increased by investments I_t. Investments are irreversible and infinitely divisible. Furthermore, the capital stock is subject to depreciation with rate γ. Under these assumptions the capital stock of the industry at time t which equals the market supply and is denoted by q_t. The relation between output price p_t at time t on the one hand and the current supply and an exogenous demand shock x_t on the other hand is defined by a time invariant inverse demand function D. Without loss of generality, we subsequently assume an isoeleastic demand function.

(1) $$p_t = D(q_t, x_t) = \left(\frac{x_t}{q_t}\right)^{\alpha} \quad \text{with } \alpha = -\frac{1}{\eta}$$

η denotes the price elasticity of demand. The demand shock x_t follows a diffusion process:

(2) $$dx = \mu \cdot x \cdot dt + \sigma \cdot x \cdot dz$$

μ and σ are the drift rate and the volatility, respectively; dz stands for a Wiener-process. According to Ito's Lemma the stochastic demand process (2) translates into the stochastic price process:

$$(3)\qquad dp = \frac{\partial D}{\partial q} dq + \frac{\partial D}{\partial x} dx + \frac{1}{2}\sigma^2 \cdot x^2 \cdot \frac{\partial^2 D}{\partial x^2} dt$$

Replacing x by $x = q \cdot p^{-\eta}$ and substituting dx by (2) allows to express the price process (3) in terms of prices and quantities. After some algebraic manipulation we get

$$(4)\qquad dp = \hat{\delta}(p,q) \cdot dq + \hat{\mu} \cdot p \cdot dt + \hat{\sigma} \cdot p \cdot dz$$

with

$$\hat{\delta} = -\alpha \cdot q^{-1} \cdot p$$
$$\hat{\mu} = \alpha \cdot \mu + \frac{1}{2}\sigma^2 \cdot \left(\alpha^2 - \alpha\right) + \gamma \cdot \alpha$$
$$\hat{\sigma} = \alpha \cdot \sigma$$

(4) describes a regulated stochastic process. The first term in (4) reflects the price changes which are induced by market entries and exits of competitors. Since all firms will react in the same way the price process will be truncated when the stochastic price reaches either a specific entry threshold $\overline{p}$ or an exit threshold $\underline{p}$. A myopic investor, however, ignores such effects. His investment strategy is based on the assumption of an exogenous unregulated price process:

$$(5)\qquad d\tilde{p} = \hat{\mu} \cdot \tilde{p} \cdot dt + \hat{\sigma} \cdot \tilde{p} \cdot dz$$

Figure 1 illustrates the difference between the regulated endogenous price process (4) and the unregulated exogenous price process (5) for the case of a geometric Brownian motion (GBM), where parameters refer to relative price changes. Both price simulations utilize identical parameters $\hat{\mu}$ and $\hat{\sigma}$. Nevertheless, the sample paths look completely different.

Here Figure 1

The myopic investor faces the problem to determine an optimal (profit maximizing) adjustment strategy for his capital stock, i.e. an optimal (dis)investment policy:

(6) $$E\int_0^\infty \left(\tilde{p}_t(\overline{q}) - c - I_t\right)\cdot e^{-r\cdot t}dt \rightarrow \max$$

$\overline{q}$ indicates that price expectations of the myopic planner are based on the assumption of a constant market supply and r is a discount rate. For this setting, the solution of the investment problem is given by a pair of constant price thresholds $\overline{p}$ and $\underline{p}$. An explicit solution for the trigger prices is presented in the next section.

A small competitive investor faces a decision problem which is quite similar to (6). The only difference is that the exogenous price process (5) is replaced by (4). Surprisingly, the competitive investor and the myopic planner find identical optimal trigger prices which represent the competitive equilibrium (Leahy 1993). The reason is that the myopic planner commits two errors which offset each other exactly. On the one hand, he ignores the truncation of the price process. He therefore overestimates the profitability when contemplating an investment. On the other hand, he wrongly assumes to have an exclusive option to defer the investment. The value of waiting which comes along with the latter makes it less attractive to invest immediately.

However, empirical applications of real options models dealing with competitive markets cannot proceed in the way outlined above. Neither the diffusion process of demand shocks (2) nor the parameters of the demand function $D(q_t, x_t)$ (1) are known and data to estimate them are not available. Therefore, it is virtually impossible to derive the parameters $\hat{\mu}$ and $\hat{\sigma}$ of the price process (5) from the underlying parameters σ, μ, γ and η using (4). One way out is to estimate the parameters $\hat{\mu}$ and $\hat{\sigma}$ directly from empirical price data and to use these estimates to calculate the sought-after trigger price for the myopic planner. However, such a procedure ignores the following fact: Even accepting necessary simplifications and assuming e.g. perfect competition, empirical price data originating from a competitive market are necessarily realizations of the regulated price process (4) and not of the unregulated process (5). Hence, the estimation procedure using standard GBM-estimators is not consistent, since the first term in (4), which actually takes into account the price effects induced by competitive entry and exit, is disregarded. Estimates of the parameters $\hat{\mu}$ and $\hat{\sigma}$ will be biased, since they spuriously capture this effect. Obviously, the resulting trigger price will also be affected by this bias. The wrong estimates are designated $\tilde{\hat{\mu}}$, $\tilde{\hat{\sigma}}$, and $\tilde{\overline{p}}$, respectively. We emphasize that this bias differs from commonly known estimation or specification errors insofar as it occurs “by construction". It is an immediate consequence of competition which causes regulated price processes.

C. Assessment of estimating errors

In this section we will asses the magnitude of errors caused by different procedures to estimate the parameters $\hat{\mu}$ and $\hat{\sigma}$ directly from price data which are, in fact, realizations of the regulated price process (4). Such an examination cannot be accomplished with real world data since the true parameters are unknown. Therefore, we have to resort to simulation experiments. Starting from the aforementioned bias we examine in detail the errors resulting from the following procedures:

Experiment 1 assumes an unregulated GBM and is based on all simulated price data of a regulated GBM.

Experiment 2 also uses standard GBM-estimators for an unregulated process, but is based on simulated price data from non-capped periods of a regulated GBM only.

Experiment 3 estimates a mean reverting process (MRP) and is based on simulated price data of a regulated GBM.

Experiment 4 estimates an unregulated GBM, but is based on only one simulated random realization (i.e. one price path) of a regulated GBM.

The implementation of "Experiment 1" is motivated by the fact that, due to mathematical convenience, many empirical applications of the real options approach seem to rely on the unquestioned assumption that parameters of the price process may be smoothly estimated using standard estimators although they require in fact a GBM. "Experiment 2" is implemented because it seems highly probable that the impact of such an erroneous use of standard GBM-estimators for price data originating from a regulated process depends on the frequency and length of regulated (capped) periods. Subsequently, we abandon the a priori assumption of GBM altogether and use statistical tests for an unprejudiced determination of the type of stochastic process. According to the results, we estimate in "Experiment 3" a MRP and assess the performance of such a procedure. "Experiments 1 to 3 have one feature in common: The suitability of different estimators is tested by isolating the bias "by construction" which is due to the unavoidable fact that these estimators are not completely consistent with the underlying data. In contrast, we assess with "Experiment 4" the magnitude of random errors which in practical applications add to the bias "by construction" whatever estimators used.

I Experiment 1: Standard GBM-estimation of process parameters based on data stemming from the regulated price process

Design of experiment 1:
Our first experiment aims at isolating and measuring the bias induced by a direct estimation of the parameters $\hat{\mu}$ and $\hat{\sigma}$ from price data using standard GBM-estimators. The simulation experiment is based on the following train of thought:

(1) We use an artificially generated test bed of price data; i.e. we know the "true parameters". We assume a highly simplified and perfect competitive market with homogenous, small and risk neutral firms which all produce with the same constant-returns-to-scale-technology. We also suppose that stochastic demand shocks follow a GBM according to (2) with known parameters μ and σ. We also predefine the elasticity of demand η and the depreciation γ.

(2) In order to obtain a simple closed form solution for the dynamic investment problem (6) we additionally assume variable costs $c = 0$. In that case the exit barrier $\underline{p}$ equals the absorbing barrier of the geometric Brownian motion $p = 0$. The optimal entry barrier (investment trigger) for a myopic planner is given by (Dixit and Pindyck 1994, p. 201):

$$(7) \qquad \bar{p} = \frac{\beta_1}{\beta_1 - 1}(r - \hat{\mu} + \gamma)I$$

β_1 is the positive root of the quadratic equation:

$$(8) \qquad \frac{1}{2}\hat{\sigma}^2\beta(\beta_1 - 1) + \hat{\mu}\beta - (r + \gamma) = 0$$

A myopic planner who ignores the price effects of competition (i.e. the time variable market supply) will find the correct strategy in terms of a trigger price, *if* he succeeds in defining the equivalent price process with correct parameters $\hat{\mu}$ and $\hat{\sigma}$ according to (5).

(3) In the real world, however, he has limited knowledge concerning the determinants of the stochastic price process mentioned above (i.e. composition of the market and characteristics of competitors, type and parameters of the stochastic demand process, elasticity of demand, depreciation). Therefore, he has to resort to an estimation of the parameters of the stochastic price process which is based on empirically observed price data. The attempt to estimate the parameters of the price process from empirically observed price data which are in fact realizations of a regulated process causes a general problem: No estimators for such a process are known. Therefore, we have to approximate the parameters even if we were fully aware of competition. In order to evaluate the performance of such an approximation we have to compare, firstly, the true values $\hat{\mu}$ and $\hat{\sigma}$ with the wrong estimates $\tilde{\hat{\mu}}$ and $\tilde{\hat{\sigma}}$. Secondly, the true trigger price $\overline{p}$ which is computed according to (7) is compared to the wrong $\tilde{\overline{p}}$. The wrong trigger price $\tilde{\overline{p}}$ is computed analogous to $\overline{p}$, but with $\hat{\mu}$ and $\hat{\sigma}$ replaced by $\tilde{\hat{\mu}}$ and $\tilde{\hat{\sigma}}$.

We implement the simulation experiment by the steps given below:

Step 1: We first derive the correct parameters $\hat{\mu}$ and $\hat{\sigma}$ of the regulated and the unregulated price process according to the transformation given in (4) from the exogenous parameters σ, μ, γ and η. We subsequently calculate $\overline{p}$ according to (7).

Step 2: We generate a discrete sample path of the regulated price process using the parameters μ, σ, λ, α and r with time increments $\Delta t = 0.1$. The simulation of regulated price process is based on Balmann and Mußhoff (2002) who determine the regulated price process as

$$(9) \qquad p_t = \begin{cases} \overline{p} \cdot \exp\left[\alpha \cdot \left(-\frac{\sigma^2}{2} \cdot \Delta t + \sigma \cdot \varepsilon_t \cdot \sqrt{\Delta t}\right)\right] & \text{if } p_{t-\Delta t} \geq \exp\left[-\alpha \cdot (\mu - \log(1-\gamma)) \cdot \Delta t\right] \cdot \overline{p} \\ p_{t-\Delta t} \cdot \exp\left[\alpha \cdot \left(\left(\mu - \log(1-\gamma) - \frac{\sigma^2}{2}\right) \cdot \Delta t + \sigma \cdot \varepsilon_t \cdot \sqrt{\Delta t}\right)\right] & \text{otherwise} \end{cases}$$

with $\overline{p}$ as the trigger price which fulfils the zero-profit condition.[5] The simulation is repeated 5 000 times. This ensures that sampling errors will average out. Because the data we use really represents the reality we assume, the observed differences between the true and the estimated parameters indeed reflect the omission of the first term in (4).

Step 3: We use the data stemming from the regulated price process which we generated in step 2. Nevertheless, we estimate the drift and volatility parameter of the *unregulated* price process (5) assumed by the myopic planner by using standard GBM-estimators (Luenberger 1998, p. 310 and Hull 2000, p. 242):

$$(10) \qquad \tilde{\hat{\mu}} = \frac{1}{\Delta t} \cdot \ln\left[\left(\sum_{t=1}^{N} \frac{p_t}{p_{t-\Delta t}}\right) \cdot \frac{1}{N}\right] \text{ and}$$

$$(11) \qquad \tilde{\hat{\sigma}} = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N} y_t^2 - \frac{1}{N \cdot (N-1)}\left(\sum_{t=1}^{N} y_t\right)^2} \cdot \frac{1}{\sqrt{\Delta t}}, \text{with } y_t = \ln\left(\frac{p_t}{p_{t-\Delta t}}\right)$$

N is the total number of observations during a time span T ($N = T / \Delta t$). We choose $N = 1\ 000$ and $T = 100$. The asymptotic standard errors of these estimators are (Campbell 1997, p. 364):

$$(12) \qquad Var\left[\tilde{\hat{\alpha}}\right] = \frac{\hat{\sigma}^2}{T}$$

$$(13) \qquad Var\left[\tilde{\hat{\sigma}}^2\right] = \frac{2\hat{\sigma}^4}{N}$$

Results of experiment 1

Table 1 depicts the results. First, the effects of the transformation stated in (4) are illustrated. For instance, the volatility of the stochastic demand (i.e. the original source of uncertainty) is enhanced (dampened) by a low (high) absolute elasticity of demand. Furthermore, depreciation leads ceteris paribus to a reduction of aggregate supply and causes prices to rise. This effect is expressed by an increase of the drift parameter of the endogenous price process $\hat{\mu}$. The parameters μ and σ of the stochastic demand process transform only into identical parameters $\hat{\mu}$ and $\hat{\sigma}$ of the price process if $\eta = -1$ and $\gamma = 0$.

Here Table 1

Second, with respect to the estimation bias we find following results: $\tilde{\bar{p}}$ exceeds $\bar{p}$ in all cases; i.e. the myopic planner systematically overestimates the optimal trigger price. An increase of the drift rate μ of the stochastic demand x magnifies the difference between $\bar{p}$ and $\tilde{\bar{p}}$ in absolute and in relative terms. An increase of the (absolute) price elasticity has an opposite impact. The effects induced by a change of the volatility depend on the level of the other parameters. Finally, increasing depreciation enlarges the gap between the correct and the estimated investment trigger. The magnitude of the estimation error for different constellations of parameters is also remarkable. While e.g. the bias is negligible for an elasticity $\eta = -2$, a drift rate $\mu = -2.5\%$, a volatility $\sigma = 10\%$ and a depreciation rate $\gamma = 0$ it amounts to more than 15 percent for $\eta = -1$, $\mu = 0\%$, $\sigma = 10\%$, and $\gamma = 5$. In markets (e.g. for agricultural products) that are characterized by an inelastic demand ($|\eta| < 1$) and depreciation rates of 5 % and more, significant estimation errors have to be expected in general.

It is also interesting to trace back deviations between $\bar{p}$ and $\tilde{\bar{p}}$ to biases of the parameters $\hat{\mu}$ and $\hat{\sigma}$. Deviations of the trigger prices mainly go back to estimation errors in the drift parameter which is systematically biased downwards ($\tilde{\hat{\mu}} < \hat{\mu}$). Moreover, our results show that the estimation errors of the volatility parameter are quite small. Both a slight over- and a slight underestimation may occur. This finding contradicts Caballero and Pindyck (1996, p. 654) who conjecture that the volatility parameter of the regulated price process will be smaller than that of the unregulated price process.

II Experiment 2: Standard GBM-estimation of process parameters based on data stemming from non-capped periods of the regulated price process

Design of experiment 2

The disregard of the fact that our data are indeed stemming from a regulated GBM was the cause of the bias demonstrated in experiment 1. The impact of the erroneous use of standard GBM-estimators and, for this reason, the magnitude of the estimation error ultimately depends on the frequency and length of price-capped periods. This suggests a correction of the estimation procedure in such a way that all capped periods, when prices match the entry barrier ($p_t = \overline{p}$), are excluded. A corresponding approach was already made by Caballero and Pindyck (1996). However, since they were using real world data, they could not assess the performance of such a method.

Our second experiment therefore explicitly considers this effect investment effect. Starting with the price rule (9), it is quite straightforward to consider only those values for the estimation of $\hat{\mu}$ and $\hat{\sigma}$ which are not affected by investment responses, i.e. those values generated by the lower rule. In other words: We resort to a modified price series where all periods are excluded in which the price is larger than a certain level. Ideally, we should consider only those values for which holds

$$(14) \qquad p_{t-\Delta t} \leq \exp\left[-\alpha \cdot (\mu - \log(1-\gamma)) \cdot \Delta t\right] \cdot \overline{p}$$

Knowing the correct parameters (see step 1) and having truncated the price paths which were originally generated in step 2, we could estimate the drift and volatility parameter of the *unregulated* price process (5) as it is assumed by the myopic planner by using standard GBM-estimators (see step 3). In practice, however, this knowledge is not given. Therefore, we use a simplified rule, which excludes a certain upper percentiles.

Results of experiment 2

Table 2 depicts the results for two simulations using drift rates of $\hat{\mu}$ = 2.5 % and $\hat{\mu}$ = 5 % respectively. It is evident that an increase of the truncation level reduces the initial underestimation of the drift ($\tilde{\hat{\mu}} < \hat{\mu}$) caused by the use of standard GBM-estimators. Equally, the estimation of the volatility improves. However, if we exclude to high a percentage of upper prices, the results reverse and the drift will e.g. be overestimated. Accordingly, there is, at first, a positive, and then a negative bias of the trigger price, which is calculated respectively.

Figure 2 illustrates the fact that the percentage of upper prices which should indeed be omitted before using standard GBM-estimators depends on the constellation of parameters of the underlying price process. In reality, these (true) parameters are not known. On the contrary, they are exactly the parameters which are to be estimated from an observed price series. Therefore, they can by no means be used for a specification of the estimation method intended to reduce the initial bias "by construction". However, in terms of a cautious conclusion one might propose to omit a small percentage of upper prices; at least in cases where it seems to be realistic to assume generally that prices follow a regulated GBM with a positive drift rate $\hat{\mu}$. However, it needs further investigation to determine the practical consequences such a conclusion might have for applications, since, in reality, one is in possession of one price path only.

Here Table 2

Here Figure 2

III Experiment 3: Estimation of a MRP based on data stemming from the regulated price process

Design of experiment 3

Experiment 1 and 2 demonstrated the effects of an erroneous use of standard GBM-estimators (which are valid for an unregulated GBM only) for price data stemming from a regulated GBM. In order to isolate this estimation error "by construction" we used data of a price simulation which was repeated 5000 times. Common sampling errors therefore averaged out.

Still averaging out sampling errors we examine in experiment 3 a bias by construction caused by an alternative estimator. Such a bias will e.g. arise if we assume that the price process represents a MRP, namely an Ornstein-Uhlenbeck process. MRP are plausible because of two reasons: (i) their characteristic propensity to revert to equilibrium in the long run which is the reason why they are often proposed in the context of real options (e.g. Dixit and Pindyck 1994, p. 74). (ii) the results of the Dickey-Fuller-Test according to which nearly 80 % of simulated paths of a regulated GBM are deemed to be stationary, whereas the respective result for an unregulated GBM is only 4 %. In both cases 5 000 simulated price paths (each consisting of 1000 price data) with a drift rate of 5 % and a standard deviation of 20 % were used as data base. The error probability used for the Dickey-Fuller Test was 5 %.

The time continuous version of a MRP is given by Vasicek (1977):

$$(15) \qquad dp = \kappa \cdot \left(p^* - p\right) \cdot dt + \sigma_{MRP} \cdot dz$$

κ describes the speed or inclination of p to revert to the equilibrium level p^*. σ_{MRP} is the standard deviation of the MRP. The time discrete version is defined by:

$$(16) \qquad p_t = p^* \cdot \left(1 - e^{-\kappa \cdot \Delta t}\right) + e^{-\kappa \cdot \Delta t} \cdot p_{t-\Delta t} + \sigma_{MRP} \cdot \sqrt{\frac{1 - e^{-2 \cdot \kappa \cdot \Delta t}}{2 \cdot \kappa}} \cdot \varepsilon_t$$

ε_t is a standard normally distributed random variable.

For the sake of convenience, we quote the complete sequence of steps:

Step 1: We derive the correct parameters $\hat{\mu}$ and $\hat{\sigma}$ according to (4). We subsequently calculate $\overline{p}$ according to (7).

Step 2: We generate 5000 discrete sample paths of the regulated price process for 100 periods using the parameters We generate a discrete sample path of the regulated price process using the parameters μ, σ, λ, α and r with time increments $\Delta t = 0.1$.

Step 3: We use the price data originating from the regulated process which were generated in step 2. Nevertheless, we estimate the drift and volatility parameter assuming that they are realizations of a MRP as defined in (14). Estimators for the MRP can be derived by running a regression for the following first-order autoregressive process:

$$(17) \qquad p_t = a_0 + a_1 \cdot p_{t-\Delta t} + \sigma_{Reg} \cdot \varepsilon_t \text{ , with}$$

$$a_0 = p^* \cdot \left(1 - e^{-\kappa \cdot \Delta t}\right),\ a_1 = e^{-\kappa \cdot \Delta t} \text{ and } \sigma_{Reg} = \sigma \cdot \sqrt{\frac{1 - e^{-2 \cdot \kappa \cdot \Delta t}}{2 \cdot \kappa}}$$

σ_{Reg} describes the standard deviation of the regression and Δt the time interval between two observations. The parameters p^*, κ and σ_{MRP} are calculated as follows:

$$p^* = \frac{a_0}{1 - a_1},\ \kappa = -\frac{1}{\Delta t} \cdot \ln\left(a_1\right) \text{ and } \sigma_{MRP} = \sigma_{Reg} \cdot \sqrt{\frac{2 \cdot \kappa}{1 - e^{-2 \cdot \kappa \cdot \Delta t}}}$$

Results of experiment 3

Table 3 depicts the results. In contrast to experiment 1 and 2, where process parameters were estimated from simulated price paths, but trigger prices were determined analytically, the assumption of a MRP requires a numerical determination of the trigger price. In the present case, we only consider a positive drift rate of 5 % and a volatility of demand of 20 %. First, the correct corresponding parameters of the unregulated stochastic price process according to (4) as well as the correct trigger price are given. For the sake of convenient comparison, we also restate the results of the procedure using standard GBM-estimators which were demonstrated in simulation experiment 1.

Here Table 3

The results show that using the estimates of a MRP may lead to an even more pronounced overestimation of the trigger price (24.15 %) than using standard GBM-estimators (19.34 %). This is interesting because according to statistical tests we would deem nearly 77.9 % of our simulated paths to be stationary if we had no prior knowledge concerning the type of stochastic process.

However, it should be noted that so far we have been trying to isolate the error "by construction" (i.e. the error caused by using wrong estimators). Accordingly, we based the estimation of parameters on the information of all 5000 simulated sample paths in order to ensure that sampling errors would average out. Knowing which paths would statistically be deemed stationary we re-examine the error "by construction" induced by competition by using only stationary paths for the estimation of MRP-parameters. We find that in this case, the relative bias of the trigger price is reduced to 10.56 %. Using estimators which are consistent with statistical tests therefore seems to yield more reliable results, especially if there is no theoretically assured prior knowledge concerning the type of process in question.

IV Experiment 4: The effects of random errors

The design of the simulation experiments described so far enabled us to analyze the bias "by construction", i.e. the bias caused by an erroneous assumption concerning the underlying stochastic process when we estimate the parameters of the process. Apart from that, there may be a deficient estimation of parameters due to the unavoidable fact that one given past time series is always only one random realization of the stochastic process. We call this effect simply "random error". This error will emerge in practical applications whatever estimation procedures used. We explore the relevance of this error by giving standard statistics such as standard deviation, maximum, minimum etc. for the conventional procedure which uses standard GBM-estimators (cf. experiment 1).

Here Table 4

Table 4 shows that apart from the general bias problem, there always is additional uncertainty concerning the trigger price. This is true even though we use a very long and well frequented price series consisting of 1000 price data. With realistic price data the importance of this obstacle increases.

D. Conclusions

Our results are important for the interpretation of models which attempt to analyze the impact of real options on competitive markets under aggregate uncertainty. Such models seem to be more appropriate for applications than models which postulate the exclusiveness of real options or reduce uncertainty to firm specific shocks. However, our analysis reveals that most of the empirical results of real options models might be wrong even assuming that prices follow a regulated GBM (competitive environment). To be specific, empirical applications tend to overestimate the reluctance to invest if standard GBM-estimators are used to determine the parameters of the price process, although they are inconsistent with the underlying data stemming from a regulated process.

To our knowledge, this bias has been widely ignored so far. The effect we pinpoint here might be interpreted as a bias "by construction". It is, by its nature, quite different from most modeling errors that have been highlighted by other authors[7]. It occurs under rather general conditions and is difficult to avoid. In other words: Having to rely on real world data, it is very difficult or even impossible to specify an investment problem of the myopic planner which is exactly equivalent to the small investor's decision problem.

The relative bias of the trigger price induced by a standard GBM-estimation of parameters may seem to be negligible for drift rates beneath or close to 0 %, since it amounts to approximate 5 % only. Such drift rates, however, are not often realistic. Many markets are characterized by an inelastic demand and depreciation rates of 5 % and more. The joint effect of elasticity and depreciation results in drift levels well above 5 % which, in turn, cause a considerable bias of the deduced trigger prices.

According to our findings, this bias "by construction" can be reduced in two ways: (1) conforming the underlying price data, which indeed originate from a regulated process, to the standard GBM-estimators by excluding a small percentage of upper prices; (2) using alternative estimators according to statistical tests. In terms of a cautious conclusion one might say that a small percentage of upper prices of approx. 2 percent to 3 percent## should be excluded in cases where it is realistic to assume that prices follow a regulated GBM with a positive drift rate $\hat{\mu}$. Using unprejudiced statistical tests in order to determine appropriate estimators, in contrast, is preferential in all cases where there is no theoretically assured prior knowledge concerning the type of process in question. However, it needs further investigation to determine the practical consequences of such a conclusion for applications, since, in reality, one is in possession of one price path (i.e. one random realization) only.

In addition to this bias "by construction", we face the obstacle that we cannot avoid the effect of random errors which biases the resulting trigger price even if we can rely on a very long and well frequented price series. According to circumstances, both problems should be at least taken into account, when interpreting the results derived from empirical applications.

E. Notes

1 There are other models coping with risk, irreversibility, and flexibility in the context of investments (e.g. adjustment-cost-models or the q-theory). Caballero (1997) and Abel et al. (1996) describe these models and their relation to real options models.

2 In addition to aggregate uncertainty producers may also face idiosyncratic shocks. However, the problems discussed below arise from aggregate uncertainty which we focus on.

3 Dixit and Pindyck (1994, ch. 8 and 9) and Trigeorgis (1996, ch. 9) provide an overview of procedures to incorporate different types of competition into real options models.

4 A generalization of the myopic principle can be found in Baldursson und Karatzas (1997).

5 Balmann and Mußhoff (2002) show that alternatively, the same trigger price and the identical price dynamics can be identified by an agent-based approach in which a number of competing agents invest and produce by using Genetic Algorithms to identify equilibrium investment strategies.

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competing agents invest and produce by using Genetic Algorithms to identify equilibrium investment strategies.

7 For example Laughton and Jacoby (1995) show that a wrong choice of the functional form of the stochastic process (2) may also lead to an overestimation of the irreversibility effect.

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Figure 1: Price dynamics with and without competition

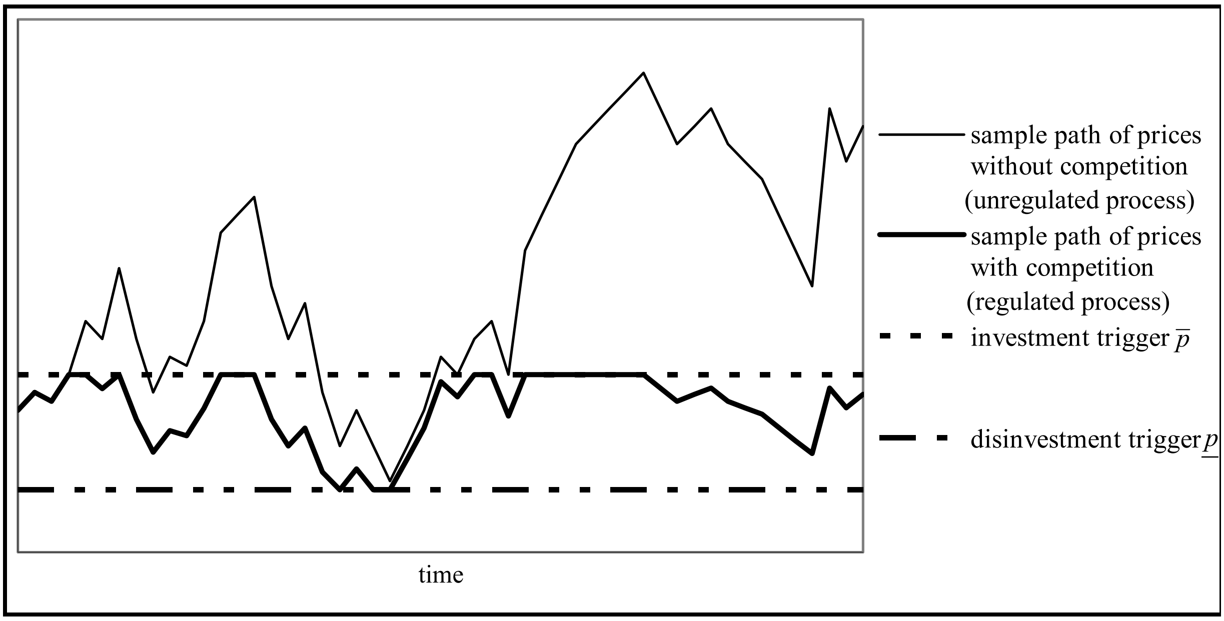

Figure 2: Investment triggers depending on drift and volatility based on estimates of selected percentiles

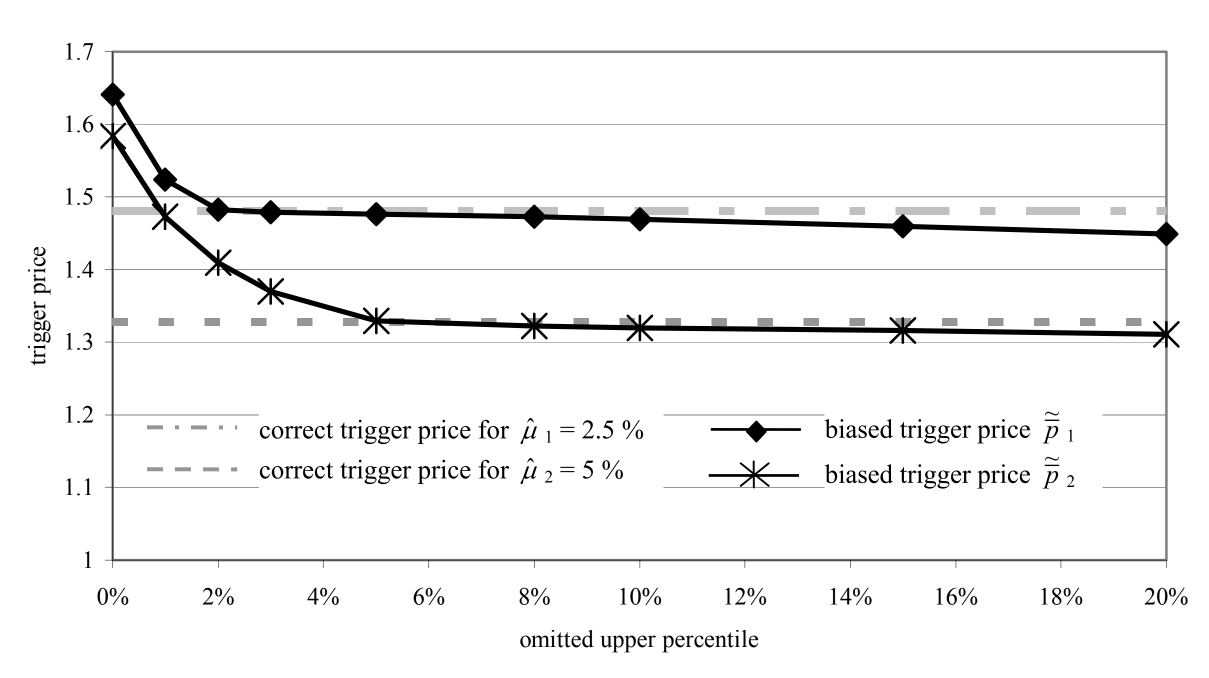

Table 1: The effect of a standard GBM-estimation of process parameters from price data stemming from a regulated GBM

a) Variation of drift and volatility [a]

parameters of stochastic demand	μ (%)	-2.5	0	2.5	-2.5	0	2.5
	σ (%)	10			20		
correct parameters of stochastic price process	$\hat{\mu}$ (%)	-2.5	0	2.5	-2.5	0	2.5
	$\hat{\sigma}$ (%)	10			20		
correct trigger price	$\overline{p}$	1.5767	1.3054	1.1456	1.9700	1.6961	1.4812
biased estimates of stochastic price process	$\tilde{\hat{\mu}}$ (%)	-2.6464	-0.5920	0.3077	-2.9667	-0.8090	0.6895
	$\tilde{\hat{\sigma}}$ (%)	10.0219	10.0921	10.3874	20.0721	20.1308	20.3208
biased trigger price	$\tilde{\overline{p}}$	1.5959	1.3637	1.2927	2.0298	1.7847	1.6444
absolute bias	$\tilde{\overline{p}} - \overline{p}$	0.0192	0.0583	0.1470	0.0599	0.0885	0.1632
relative bias	$\left(\tilde{\overline{p}} - \overline{p}\right) / \overline{p}$	0.0122	0.0447	0.1283	0.0304	0.0522	0.1102

[a] $\Delta t = 0.1$, $\eta = -1$, $\gamma = 0$ %, $r = 7$ %

b) Variation of elasticity [a]

elasticity of demand	η	-0.5			-2		
parameters of stochastic demand	μ (%)	-2.5	0	2.5	-2.5	0	2.5
	σ (%)	10					
correct parameters of stochastic price process	$\hat{\mu}$ (%)	-4	1	6	-1.375	-0.125	1.125
	$\hat{\sigma}$ (%)	20			5		
correct trigger price	$\overline{p}$	2.1570	1.6026	1.2811	1.2874	1.1534	1.0740
biased estimates of stochastic price process	$\tilde{\hat{\mu}}$ (%)	-4.2245	-0.0757	1.7004	-1.3787	-0.3986	0.0199
	$\tilde{\hat{\sigma}}$ (%)	20.0384	20.2194	20.7875	4.9986	5.0493	5.1941
biased trigger price	$\tilde{\overline{p}}$	2.1880	1.7133	1.5758	1.2878	1.1800	1.1472
absolute bias	$\tilde{\overline{p}} - \overline{p}$	0.0309	0.1107	0.2947	0.0004	0.0266	0.0732
relative bias	$\left(\tilde{\overline{p}} - \overline{p}\right) / \overline{p}$	0.0143	0.0691	0.2301	0.0003	0.0231	0.0681

[a] $\Delta t = 0.1$, $\gamma = 0$ %, $r = 7$ %

c) Variation of depreciation [a]

parameters of stochastic demand	μ (%)	0			
	σ (%)	10			
depreciation	γ (%)	0	2.5	3.5	5
correct parameters of stochastic price process	$\hat{\mu}$ (%)	0	2.5	3.5	5
	$\hat{\sigma}$ (%)	10			
correct trigger price	$\overline{p}$	1.3054	1.1355	1.1049	1.0784
biased estimates of stochastic price process	$\tilde{\hat{\mu}}$ (%)	-0.5920	0.4907	0.5536	0.6538
	$\tilde{\hat{\sigma}}$ (%)	10.0921	10.7982	11.0502	12.0388
biased trigger price	$\tilde{\overline{p}}$	1.3637	1.2504	1.2419	1.2473
absolute bias	$\tilde{\overline{p}} - \overline{p}$	0.0583	0.1149	0.1370	0.1689
relative bias	$\left(\tilde{\overline{p}} - \overline{p}\right) / \overline{p}$	0.0447	0.1012	0.1240	0.1566

[a] $\Delta t = 0.1$, $\eta = -1$, $r = 7$ %

Table 2: The effect of a standard GBM-estimation of process parameters from price data omitting upper percentiles [a]

a) … for drift $\hat{\mu}$ = 2.5 %

correct values		$\hat{\mu}$ **= 2.5 %,** $\hat{\sigma}$ **= 20 %,** $\bar{p}$ **= 1.4812**							
omitted upper percentile	(%)	0	1	3	5	7.5	10	15	20
truncation level		∞	1.5492	1.4523	1.3894	1.3256	1.2708	1.1751	1.0920
biased estimates of stochastic price process	$\tilde{\hat{\mu}}$ (%)	0.6895	1.9638	2.5275	2.5677	2.6248	2.6748	2.8030	2.9639
	$\tilde{\hat{\sigma}}$ (%)	20.3208	20.0462	20.0063	20.0045	20.0072	20.0093	20.0142	20.0199
biased trigger price	$\tilde{\tilde{\bar{p}}}$	1.6444	1.5238	1.4794	1.4764	1.4724	1.4689	1.4600	1.4490
absolute bias	$\tilde{\tilde{\bar{p}}} - \bar{p}$	0.1632	0.0426	-0.0018	-0.0048	-0.0088	-0.0123	-0.0212	-0.0322
relative bias	$\left(\tilde{\tilde{\bar{p}}} - \bar{p}\right)/\,\bar{p}$	0.1102	0.0288	-0.0012	-0.0032	-0.0059	-0.0083	-0.0143	-0.0217

b) … for drift $\hat{\mu}$ = 5 %

correct values		$\hat{\mu}$ **= 5 %,** $\hat{\sigma}$ **= 20 %,** $\bar{p}$ **= 1.3277**							
omitted upper percentile	(%)	0	1	3	5	7.5	10	15	20
truncation level		∞	1.4384	1.3749	1.3380	1.3026	1.2724	1.2191	1.1703
biased estimates of stochastic price process	$\tilde{\hat{\mu}}$ (%)	1.5206	2.7640	4.2511	4.9636	5.1076	5.1511	5.2333	5.3451
	$\tilde{\hat{\sigma}}$ (%)	20.6365	20.2782	20.0496	19.9986	19.9965	19.9956	19.9959	19.9940
biased trigger price	$\tilde{\tilde{\bar{p}}}$	1.5845	1.4730	1.3695	1.3295	1.3222	1.3201	1.3160	1.3106
absolute bias	$\tilde{\tilde{\bar{p}}} - \bar{p}$	0.2568	0.1453	0.0418	0.0018	-0.0055	-0.0077	-0.0117	-0.0171
relative bias	$\left(\tilde{\tilde{\bar{p}}} - \bar{p}\right)/\,\bar{p}$	0.1934	0.1095	0.0315	0.0014	-0.0041	-0.0058	-0.0088	-0.0129

[a] $\Delta t = 0.1$, $\eta = -1$, $r = 7$ %, $\gamma = 0$ %

Table 3: The effect of estimating a MRP from price data stemming from a regulated GBM[a]

correct values		**$\hat{\mu}$ = 5 %, $\hat{\sigma}$ = 20 %, $\overline{p}$ = 1.3277**		
		estimates using standard GBM-estimators	estimates using MRP-estimators on … …all paths	…stationary paths
estimates of the parameters of the GBM	$\tilde{\hat{\mu}}$ (%)	1.5206	–	–
	$\tilde{\hat{\sigma}}$ (%)	20.6365	–	–
estimates of the parameters of the MRP	p^*	–	0.8651	0.9309
	κ	–	0.3195	0.3772
	σ_{MRP}	–	0.2021	0.2118
biased trigger price	$\tilde{\overline{p}}$	1.5845	1.6483 [c]	1.4679 [c]
absolute bias	$\tilde{\overline{p}} - \overline{p}$	0.2568	0.3206	0.1402
relative bias	$\left(\tilde{\overline{p}} - \overline{p}\right) / \overline{p}$	0.1934	0.2415	0.1056

[a] $\Delta t = 0.1$, $\eta = -1$, $r = 7$ %, $\gamma = 0$ %.

[b] 77.9 % of the paths are stationary with a probability of error of 5 %.

[c] For lack of an analytical solution the results for the MRP are calculated by means of a numerical GA-based simulation model (cf. Balmann et al. 2001).

Table 4: The effect of random errors within a procedure based on a standard GBM-estimation of process parameters [a]

	trigger price	estimated parameters	
		drift (%)	**volatility (%)**
true values ($\overline{p}$, $\hat{\mu}$, $\hat{\sigma}$)	1.4812	2.5000	20.0000
biased values ($\tilde{\overline{p}}$, $\tilde{\hat{\mu}}$, $\tilde{\hat{\sigma}}$): expected value	1.6444	0.6895	20.3208
standard deviation	0.1032	1.0998 [b]	0.5138 [b]
maximum trigger price	2.4103	-5.9759 [b]	19.7030 [b]
minimum trigger price	1.4856	1.7860 [b]	18.7494 [b]
10% percentile	1.7877	0.3968 [b]	21.1560 [b]
25% percentile	1.6951	0.5402 [b]	21.1607 [b]
75% percentile	1.5669	1.8304 [b]	20.8294 [b]
90% percentile	1.5419	1.9710 [b]	20.5012 [b]

[a] $\Delta t = 0.1$, $\eta = -1$, $r = 7$ %, $\gamma = 0$ %. [b] [b]

[b] values corresponding to the respective trigger price.