Universitá degli Studi - L'Aquila
Facoltá di Economia
Dip. di Sistemi ed Istituzioni per l'Economia

*****

Controlling CFaR with Real Options. A Univariate Case Study

Giuseppe Alesii*

5th March 2003

Abstract

Cash Flow at Risk (CFaR) can be controlled using real options. In this normative paper, we derive numerically in a univariate discrete time model, extension of (Kulatilaka, 1988), the expanded NPV of an industrial investment and, simultaneously, state variable thresholds to optimally exercise real options for the whole life of the project. In this framework, we model total variability in expanded NPV using a Markov chain Monte Carlo method. A number of original results are derived for an all equity financed firm. Cash Flow distribution and CFaR is derived for each epoch in the life of the project. A VaR for the expanded NPV at time 0 is derived. These new methods have been applied to two case studies in shipping finance, namely a very large crude carrier and a Panamax.

JEL classification code:G34, G32, G10.

*Ricercatore a v. di Finanza Aziendale, **ADDRESS**: Universitá de L'Aquila, Facoltá di Economia, Piazza del Santuario 19, Roio Poggio 67040 AQ, Italy, Phone: (intl-code-for Italy) + 0862+434877; Fax idem+0862+434803; e-mail: galesii@luiss.it.
The author wishes to thank for helpful hints and suggestions Andrea Berardi, Andrea Gamba, Olivier Erpelding and Martin Stopford. The usual disclaimer applies.

Contents

List of Figures

List of Tables

Introduction

The control and reduction of volatility of profits from operations is beneficial to shareholders' wealth which should seek it through appropriate hedging operations. This has been well proven both from a theoretical and an empirical point of view.[1]

In a firm viewed as a nexus of exposures (MacMinn, 2002), enterprise risk management (ERM) can be implemented in a transaction by transaction way or in a holistic integrated way hedging overall profitability of the firm against marketable risks. In both these approaches to ERM, the role of real options is neglected, overlooking the "in house" hedging effect of managing an industrial plant according to real options.

Capital budgeting for intrinsically illiquid assets like industrial plants is interwined with risk management. Hence there is the need to value investment projects not only within the usual risk return framework but also taking into account their diversifiable risk dimensions (Stultz, 1999).

In this normative paper, we add a new dimension to capital budgeting with real options: in a univariate framework, (Kulatilaka, 1988), we model total variability in *expanded* NPV and in CF in each period of the investment project life when this is managed exercising optimally real options. This allows us to tackle downside risk from two different perspectives. From a static point of view, we provide a measure of what is usually called the "project at risk" or a VaR of expanded NPV. From a dynamic point of view, instead, we provide a measure of the downside risk in each epoch of the investment both from a timeless and a path dependent perspective. This, in turn, ends up in modeling the survival probabilities of the investment project.

Following this double perspective, the choice of real investments with real options can be taken trading off absolute NPV loss or default probability with the expected increase in shareholders' wealth. Moreover, our model gives market hedging operations a benchmark of comparison provided by the "in house" hedging properties of real options embedded in the investment project. We have chosen to apply our normative model to a shipping finance case.[2]

This paper is organized as follows. In section 1, we provide a brief review of literature about CFaR in order to position our paper in the existing literature. In section 2, we motivate the choice of the (Kulatilaka, 1988) model and report its extension to control cash flow from operations. The procedure followed to compute VaR for NPV at time zero and CFaR in each epoch is described. In section 3 two case studies in

[1] Both strands of literature are vast and two dedicated papers would not suffice to summarize them. For just to give the reader a red herring to follow, we quote the article by (Froot and Stein, 1997) and (Froot et al., 1993) among the stream of literature that justifies risk management from a theoretical point of view. Among the empirical stream of literature, we quote (Allayannis and Weston, 2001). For a textbook like approach see chapter 20 in (Jorion, 2001).

[2] See, for instance the studies of a leading shipping consultants firm (Drewry and Jupe, 2001) and (Drewry and Kellock & Co, 1999). For a more general view about capital budgeting practices in the shipping industry see (Cullinane and Panayides, 2000).

shipping finance are reported focusing on the valuation of a very large crude carrier (VLCC) and of a dry bulk ship. Section 4 draws the conclusions and sets the blueprint for several extensions and applications this model may have. Three appendixes follow references, the first one about the time series used to estimate the Ornstein Uhlenbeck process parameters, a second one about the econometric methods followed and the results obtained, a third one about convergence tests of the Markov chain Montecarlo simulations.

1 A Brief Review about CFaR

The aim of this section is to give the coordinates of our model in the existing literature which deals with CFaR. We start defining CFaR and review briefly the two mainstreams in the Cash flow at risk literature. This review is useful to single out the differences and the analogies between existing models and ours.

Cash Flow at Risk (CFaR) is the flow equivalent of VaR. While VaR is the worst loss over a target horizon with a given level of confidence on a *stock* represented by a financial asset, see (Jorion, 2001) page 22, CFaR is the worst loss, out-*flow* of cash, over a given accounting period that an industrial investment project can cause to a non-financial firm. A bank's VaR is also its CFaR (Shimko, 1998). As a matter of fact a bank's portfolio is constantly marked to market hence any changes in asset value (VaR) is reflected in an immediate change in earnings and, depending on the actual asset sale, in cash flows. Instead, non financial firms assets are seldom and to a little extent marked to market, being mostly tangible and intangible assets whose prices are not listed on a stock exchange (Hayt and Song, 1995). Those are mostly information intensive assets which cannot be frictionlessly traded in capital markets (Froot and Stein, 1997). Their value can be deducted in a very dubious way from thin second hand markets in which several trading anecdotes can be spotted instead of a price regular time series, (Asplund, 2000).

For these reasons, the only way to tackle variability in operating profitability is to focus on projected cash flows from operations over a multi year planning horizon (Hayt and Song, 1995). CFaR gives a single consolidated measure of risk of an investment project which is easy to communicate within the firm to senior management and board of directors and to the financial analysts community outside the firm.[3] CFaR is considered by the business community as a sufficient statistic of the default risk of an investment project which can help in determining its credit rating.[4]

CFaR has been derived both endogenously and exogenously to the firm. Several prominent consulting firms have derived their version of CFaR mostly exogenously, i.e. bottom up.[5] For instance, NERA,

[3]For instance several firms state their CFaR target in their briefing reports drafted for the road show towards a financial analysts audience, e.g. BHP Billinton

[4]See for instance the lecture given by Dr. Arlie Sterling, President of Marsoft, primary US shipping consultants firm, at the German Ship Finance Forum held in Hamburg in April 2002.

[5]Following the layout of a Profit and Loss Statement, a bottom up perspective starts from the net income (or cash flow)

see (Stein et al., 2000), has derived a comparables approach to CFaR using variability of net income of public listed firms which are similar to privately owned in the same industry. In a similar procedure, but for different purposes, KMV, see (Nyberg et al., 2001), has derived a model which estimates the asset value and volatility for private firms in order to define the expected default frequency.

In one way or the other, both these methods aim to define the default point with respect to the individual period in the life of a non financial firm investment project or the point below which cash flow produced by operations is not enough to cover outflows due to financial structure and/or to strategic expenditures. Both these methods share the same shortcomings. They rely mostly on financial markets data. It has been proved thoroughly that financial markets, expecially stock exchanges, are more volatile than the underlying fundamental value.[6] Therefore, asset, like in KMV model, or cash flow, like in NERA model, volatility estimate is certainly biased upward. In conclusion, it would be irrational to rely on the inefficient messages that the stock market sends us.[7]

On the other side, a number of consulting firms and academic authors have derived endogenous, i.e. top down, versions of CFaR or other variations on the theme of VaR similar to the cash flow version. Among the consulting firms, the Riskmetrics Group has derived the Corporatemetrics models, see (Lee et al., 1999) and (Kim et al., 1999), the Risk Capital Management Partners propose a similar model, see (Shimko, 1998) and (Shimko, 2001). Among the academic authors that have derived CFaR endogenously, we remember (Hayt and Song, 1995), (Turner, 1996), (Godfrey and Espinosa, 1998), (Dorris and Dunn, 2001) and (Ku, 2001).

In these models the perspective is top down. Following the Corporatemetrics terminology, a business model of the firm is sketched and an exposures mapping is constructed, e.g. an equation linking market sources of uncertainty with profitability figures, Earnings or Cash Flows. Market price and rate scenarios are derived from an econometric description of their time series, see (Kim et al., 1999) or from an historical simulation, see (Godfrey and Espinosa, 1998). The exposure mapping equation is evaluated on each market price and rate scenario [8] deriving a distribution of the profitability figure for each epoch within the horizon covered recursively by the econometric model forecasting, page 102 (Kim et al., 1999). CFaR or EaR is derived in the way usually adopted for the computation of a financial assets portfolio VaR.

figure to single out economic factor that contributed to its derivation. Instead, top down perspective starts from economic factors to derive their influence on the (economic or financial) profitability figure.

[6] See for instance the vast literature about variance bounds a' la (Shiller, 1981).

[7] In a blatant contradiction, Jeremy Stein stands as a prominent representative of both parties in this *querelle* . On one side he states the caveats for a rational capital budgeting in an irrational world, (Stein, 1996). On the other side, he is one of the co-authors of the comparables approach to the derivation of C-FaR for NERA, (Stein et al., 2000). The late Stein seems to have forgotten what the early one wrote on the Journal of Business, i.e. that the stock market is inefficient and a rational manager should not rely on its message for capital budgeting purposes.

[8] A scenario corresponds to a path for one or more market variables over a specific horizon in (Lee et al., 1999), page 51.

Lurking in the background of this endogenous derivation of CFaR literature is the need for an active management of the business model which should respond pro-actively to an ever changing competitive arena, see for instance page 38 in (Turner, 1996), or page 110 (Godfrey and Espinosa, 1998) or page 370 in (Jorion, 2001). As a matter of fact, from this point of view, both streams of literature share the same weakness, i.e. they assume a passive management of the business model. On one side, the NERA like models presuppose that the firm behaves like a given group of comparables competitors. In this way the firm is specified as a Marshallian black box, more or less a neo classic production function. On the other side, the Corporatemetrics like models postulate that the exposures mapping stays the same for very long periods while the management watches passively its firm going bankrupt due to changed business environment.

Feature	C-FaR (NERA, KMV et al)	CFaR (Corporatemetrics, Risk Capital Management Partners et al)
Accounting definition	overall firm's profitability	earnings derived component by component
Accounting figure	financial with accrual conventions $\frac{EBITDA}{Assets}$, pure number figure	accrual basis (EaR), financial, fair value mark to market basis (CFaR), hedging accounting basis, absolute number figure
Horizon of the model	3-12 months	2-24 months
Econometric model to describe time series and to create CFaR distribution	AR(4)	VAR, VECM non parametric scenario construction
Benchmark for CFaR derivation	Arbitrarily stated, expectations from AR(4)	strategic or financial as stated in the budget
Distribution of CF	Normal, Classical OLS Gauss-Markov conditions; stationary over the estimation period	Not necessarily Normal, not necessarily stationary resulting from paths on the derivation period
Types of Risk	C-FaR as a sufficient statistics of all the risks of a firm, market and private	CFaR in a market risks framework as a result of an analytical mapping of the exposures
Implied management	passive, the same as in the industry comparables. Firm as a Marshallian Black Box	passive, the same as in the firm itself in the last months. Firm specified as a map of exposures with respect to a business plan
Suggested practical uses	Mainly security analysis and investor relations. Also debt and risk management	Mainly individual risks hedging
End Users	both inside and outside the firm	mostly inside the firm

Table 1: Summary Comparison of Top down and Bottom Up derivations of Cash Flow at Risk

Both kind of models have some advantages and disadvantages, see table 1. To begin with, the accounting figure chosen in NERA model is C-FaR= $\frac{EBIT}{Assets}$, see page 8 in (Stein et al., 2000). It is a pure number suitable for comparison exercises in security analysis. On the other hand, this value is less appropriate for capital budgeting or risk management purposes than a Corporatemetrics figure, see chapter 6 in (Lee et al., 1999). In the latter model, profitability can be derived in the most suitable way for the problem to be tackled. In conclusion, Corporatemetrics model is more flexible in the choice of the profitability figure.

Like VaR, CFaR is also defined as the maximum shortfall at a given probability level with respect to a defined benchmark. In NERA model this benchmark is defined as the level forecasted through an AR(4) model. The errors are then pooled and sorted according to size, profitability, profit variability and stock volatility. The distribution of errors for the sorted groups, or comparables, is used to derive C-FaR. To sum up, C-FaR is derived cross sectionally building up a comparable virtual firm. Because of this, C-FaR looks like a dress cut for the average customer but not for firms which are outliers within their industries.

On the other hand, in Corporatemetrics model market exposure mappings allows to tailor made a dress for the individual firm. Although that is true, there is a high degree of subjectivity in the specification of the business model. Within these business models, the end user can specify the benchmark she considers most appropriate, strategic or financial as stated in her budget.

Like VaR computed for a portfolio of assets, CFaR is computed with respect to a portfolio of long (revenues) and short (cost) positions influenced by several sources of uncertainty. According to NERA model, relative EBIT is a sufficient statistic for all these risks. In this way the influence of each source of uncertainty is not considered separately. This does not allow to specify hedging policies dedicated to specific risks, as it happens in Corporatemetrics. In conclusion, while NERA model entails all non-financial risks "in bulk", including pure operational and private risks, Corporatemetrics entails only market risks that are actually mapped.

Like VaR, CFaR is also defined on a definite horizon of time. NERA model has a shorter horizon than the Corporatemetrics one. Because of this, the former is less appropriate for the description of the volatility of firm profitability over multi year planning horizons.[9]

To summarize, existing cash flow at risk models have been mostly proposed by consulting firms as a solution to risk management and investor relations problems. They are standard products that at most assume a very simple structure of the firm, risk exposure mapping. Moreover, in any existing model, management is postulated as passive, although, lurking in the background some authors feel the need to specify and quantify the effect on CF risk of an active management.

2 The Control of CFaR with Real Options

Both families of CFaR derivations do not consider the fact that the active management of the firm can control CFaR, increasing its upside potential and decreasing the downside risk. This active risk management can be performed through the optimal exercise of real options, see (Trigeorgis, 1996) page 123. We claim that the original contribution of this paper is to derive the distribution of *controlled* cash flows in each period through the whole life of the project.[10] In addiction to the flow dimension of risk, we derive its *stock* dimension

[9] This distinction resembles asset and earnings based VaR, see page 391 (Jorion, 2001).

[10] This allows us to compute not only CFaR in the presence of real options but also many widely (mis)-used capital budgeting criteria, such as Pay back and IRR. We save this extension for another paper.

computing a sort of VaR of the project expanded NPV.

It is possible to find something similar to this in the literature, see (Godfrey and Espinosa, 1998) and (Chen et al., 2000), but, still, NPV-Project at Risk analysis is derived under a passive management hypothesis.[11] In the same way, the so called consolidated approach to project volatility estimation in chapter 9 of (Copeland and Antikarov, 2001) derived the distribution of the project NPV under a passive management. Between passive and actually dynamic active management models, we place (Luciano et al., 2003) in which VaR is computed in a static multiperiod framework for an inventory management problem. One of the few authors that investigates the effects of real options on risk management is Gordon Sick (Calistrate et al., 2001). Although that is true, even in the last reference, this effect is tackled with respect to stock variables, i.e. NPV at time zero, and not with respect to flow, i.e. cash flow from operations. Finally, (Mun, 2002) on page 326 proposes the computation of VaR for the expanded NPV. In this case simulation and real option analysis are interwined although in a way much different from ours.[12] A similar approach is followed by (Cruz, 2002) page 323. Our model intuition resembles that of (Chorn and Carr, 1999) in (Trigeorgis, 1999), although there *intelligent decision pathways* are derived for irreversible options only and risk reduction is not quantified.

Finally, the procedure that mostly resembles to ours is given in (Li and Chiu, 2003) page 624, a case study in a book recently edited by (Ronn, 2003). There from historical data they formulate a set of (heuristic) rules for exercising real options. Then they run a Montecarlo simulation taking into account of these rules for the management of power generation assets. To our knowledge, theirs is the only active management framework in which a VaR of NPV is derived.

The procedure we have followed is numerical and it can be easily adapted by practitioners to any kind of real business investment project. The model of (Kulatilaka and Trigeorgis, 1994) has been used in the version developed by Kulatilaka, see (Kulatilaka, 1988), on a grid discretizing a univariate Ornstein Uhlenbeck (OU). The Kulatilaka Trigeorgis model has been used to derive in a Bellman Dynamic Programming (DP) method both the net present values of the investment project at time zero and the real options optimal exercise thresholds throughout the whole life of the project. Then, the same Ornstein Uhlenbeck process

[11] An application of the classical risk analysis a' la Hertz to shipping finance problems can be found in (Haralambides, 1993) in (Gwilliam, 1993). There IRR is assumed as objective function in a passive management framework.

[12] To summarize briefly (Mun, 2002) procedure, he derives the distribution of the value at time zero of the underlying asset, passively managed project, or the present value of the free cash flow of the discounted cash flow model simulated using a uni/multivariate distribution of profitability factors, e.g. prices, costs, quantities, etc. From this distribution he takes the expected value and the standard deviation to be used respectively at the starting node of a binomial lattice as underlying asset to value european and american real options and as the volatility needed to compute binomial discrete movements. His simulation of the actively managed plant is basically a Montecarlo simulation of input variables, see page 334 in (Mun, 2002), taking into account parameters point estimates risk. In conclusion, Mun uses a stock variable in the real option model and does not take into account of the optimal exercise of real options on cash flows which, instead, are simulated in the passive management project only.

is simulated with the same discretization used in the grid. On each simulated path observation, the profit and loss equation of the firm is computed taking into account the optimal policy derived according to the Bellman Principle of Optimality. Hence, to each path of the OU, it corresponds a simulated cash flow history which has been optimally managed according to Bellman DP. On these cash flows histories, it is possible to compute not only expanded net present values but also, for each epoch, it is possible to value variability in CF both from a timeless and a path dependent perspective.

In the remaining part of this section, the Kulatilaka Trigeorgis real option model is briefly reviewed in order to motivate its choice among the vast variety of real options models. Moreover, the scenario construction method is described giving a graphic portrayal of the CF computation according to optimal exercise real options thresholds. Finally, the procedures used to compute downside risk measure on expanded NPV and Cash Flow are motivated merging two quite different frameworks of analysis, Fisherian capital budgeting and risk management.

2.1 A review of the Kulatilaka Trigeorgis model

The most important difference between Kulatilaka Trigeorgis (henceforth KT) model and most of the real options models stays in the computation of the value of the capital budgeting project with real options. While most of the authors in real options base this valuation on a *stock* variable represented by the value of the project depending on one or more stochastic state variables, [13] KT are among the few [14] that base their valuation on the *flow* which is optimally produced in each epoch of the project technical life. In other words, they used a *running present value* derivation of the expanded NPV. For these reasons, KT's is the most suitable model to assess the variability of cash flow in each epoch of the investment project and at the same time to value the downside risk of capital budgeting decision criteria at the beginning of the life of the investment, time zero.

The version of the KT model we have used here is univariate, with a stochastic state variable specified as an arithmetic Ornstein-Uhlenbeck process, discretized in a grid (Kulatilaka, 1988), (Kulatilaka, 1993). The choice of this specification is instrumental to the kind of case study we have developed in section 3 where the state variable we have chosen evolves like a mean reverting process. Moreover, this choice was motivated by the nice properties of a grid discretization which avoids mesh ratio problems that a binomial lattice has. Finally, extending the Kulatilaka model allows us to ignore the distinction between risk neutral and natural/historical probabilities since in this model under certain, restrictive, conditions, they happen

[13] See for instance (Calistrate et al., 2001) for a (Cox et al., 1979) like model where the state variable is the value, i.e. *stock* of the project.

[14] Among the others, we quote (Luenberger, 1998).

to be the same.

In essence, Kulatilaka general real option model (GROPM),[15] starts with the observation that an industrial plant has several operating modes which can be chosen optimally in order to maximize the value at time zero of the investment project, expanded net present value NPV_e. In detail, an investment opportunity can be left fallow, option to wait, or after it is implemented, it can be abandoned, option to abandon, or its scale can be changed, option to expand/contract. In addition to these irreversible options, GROPM accommodates a number of reversible options such as the possibility to suspend production, mothballing option. Therefore, after choosing a basic cost volume profit equation like (1) as our exposure mapping according to Corporatemetrics terminology, we can compute in each epoch of the life of the project and for each level of the state variable the profits we would get, choosing optimally those that correspond to the best operating mode.

$$\Pi\left(\theta_t, \ell', t\right) = Q_{t,\ell'} \cdot \left(P_{t,\ell'} - Uvc_{t,\ell'}\right) - F_{t,\ell'} \tag{1}$$

where:

θ_t	:=	state variable level at time t, $\forall t = 0, \ldots, T$ being T the end of the technical life of the project. Any of the primitive variables in a CVP equation can be specified as the stochastic state variable, i.e. Q, volume produced and sold; P, unit price; Uvc, unit variable cost; F, fixed costs.
$l = 1, \ldots, m$	:=	production modes of an investment project: wait, operate, suspended, expanded, contracted, abandoned;

Blatantly enough, the choice of the operating mode in each epoch and for each level of the state variable is dependent on the operating mode held in the previous epoch, since profits are net of mode transition costs. Because of this, the problem of the optimal management of the flexible plant is resolved choosing NPV as criterion function and maximizing it applying Bellman Dynamic Programming, see equation (2).

$$F\left(\theta_t, \ell', \mathrm{t}\right) = \overset{max}{\ell'} \left\{\Pi\left(\theta_t, \ell', t\right) - c_{\ell,\ell'} + \rho \cdot E_t^{\theta^*_{t+1}}\left[F\left(\theta_{t+1},, \ell', \mathrm{t}+1\right)\right]\right\} \tag{2}$$

where, in addition to previously exposed notation,

$F\left(\theta_t, \ell, \mathrm{t}\right)$	:=	value of the plant for the level of the state variable θ_t, for an optimizing operating mode ℓ at time t;
$E_t^{\theta^*_{t+1}}[\,]$	:=	expectation operator on equivalent martingale measure, hence starred, of the process θ_t;
$\rho = 1/\left(1 + i_{1/m}\right)$	:=	present value factor, in which $i_{1/m} = (1 + r_f)^{1/m} - 1$.
$c_{\ell,\ell'}$	:=	operating mode transition cost, being l the beginning mode and l' the ending mode;

Boundary conditions for equation (2) are given by salvage values of the plant at the end of its technical life. Recursions are performed in a simple backward induction process being this problem a finite horizon DP one.

---

[15] See several extension and applications in (Kulatilaka, 1988), (Kulatilaka, 1993), (Kulatilaka, 1995) and (Amram and Kulatilaka, 1999).

Simultaneously, at each backward iteration, for any level of the state variable θ_t, the DP procedure gives us one or more than one optimal operating modes see equation (3). In the cases in which $\theta_{j,t} \Longleftrightarrow \widehat{\ell'}_{j,t}$, we have one mode regions. In the case in which $\theta_{j,t} \Longrightarrow \widehat{\ell'}_{j,t}$, we have hysteresis regions, i.e. levels of the state variable in which it is optimal within the DP procedure to maintain the same operating mode in which the dinamic system entered the region.

$$\left.\begin{array}{l} \theta_{j,t} \Longrightarrow \\ \\ \theta_{j,t} \Longleftrightarrow \end{array}\right\} \widehat{\ell'}_{j,t} = \underset{\ell'}{\text{argmax}} \ \left\{ \mathrm{F}\left(\theta_{\mathrm{t}}, \ell', \mathrm{t}\right) \right\} \tag{3}$$

The exogenous uncertainty faced by the firm is summarized by a state variable θ_t which follows an arithmetic Ornstein-Uhlenbeck diffusion process, equation (4), see (Dixit and Pindyck, 1994) page 74. This process can have, in theory, negative values [16]. Although this is true, for a small level of σ_θ when compared to η and $\overline{\theta}$, negative values are quite unlikely, see page 664 (Sick, 1995) in (Jarrow et al., 1995).

$$d\,\theta_t = \eta \cdot \left(\overline{\theta} - \theta_t\right)\, d\,t + \sigma_\theta\, d\,Z \tag{4}$$

where, in addition to the previous notation:

η	:	the speed of reversion, e.g. for $\eta = 0$ the process becomes a geometric brownian motion while for $0 < \eta < 1$ the process tends to be mean reverting, negative levels are excluded to avoid mean aversion, one is excluded to avoid overshooting ;
$\overline{\theta}$	:	the normal level of θ, i.e. the level at which θ tends to revert;
σ_θ^2	:	instantaneous variance rate;
$d\,t$	:	time differential;
$d\,Z$	:	standard Wiener process, normally distributed with $E\,(d\,Z) = 0$ and $Var\,(d\,Z) = E\left((d\,Z)^2\right) = d\,t$.

The stochastic process governing θ_t is discretized in an equally spaced grid. In this discretized space, the state variable moves according to a one step transition probability matrix, (Kulatilaka, 1993) page. 279 Appendix, which is constructed discretizing a normal distribution for each of the levels of θ_t, see equation (5).

$$\Delta\,\theta_t \sim N\left(\eta\left(\overline{\theta} - \theta_t\right) \cdot \Delta\,t, \sigma_\theta^2 \cdot \Delta\,t\right) \tag{5}$$

It is possible to show, following (Cox et al., 1985) (Lemma 4), that expected values on these discretized probabilities can be considered following a certainty equivalent drift rate, (Kulatilaka, 1993). Hence, any cash flow dependent on this θ_t can be discounted at the risk free rate, under the restrictive condition that systematic risk of the investment project is null.

2.2 Scenario Construction

As a too often neglected sideproduct of the KT model, the optimal exercise thresholds have been derived for the whole life of the project. They partition the discretized space of the state variable in regions in which

[16]We wish to thank Andrea Berardi for pointing this out to us.

different operating modes are optimal according to a Bellman DP procedure, see upper graph in figure 1.

We run a Markov chain Montecarlo simulation of the solution of the PDE equation in expression (4) and we obtain recursively a path of the levels of θ_t $\forall t = 0, \ldots, T$, see equation (6). This path meanders on the grid going through the thresholds, passing from an hysteresis to a one mode region and the other way around, see upper graph in figure 1.

$$\theta_t = \theta_{t-1} \cdot e^{-\eta \Delta t} + \overline{\theta} \cdot \left(1 - e^{-\eta \Delta t}\right) + \epsilon_t \tag{6}$$

where, in addition to the previous notation:

$\epsilon_t \sim N\left(0, \frac{\sigma_\theta^2}{2 \cdot \eta} \cdot \left(1 - e^{-\eta \Delta t}\right)\right)$: noise term distributed normally with mean zero and variance as a fraction of σ_θ^2.

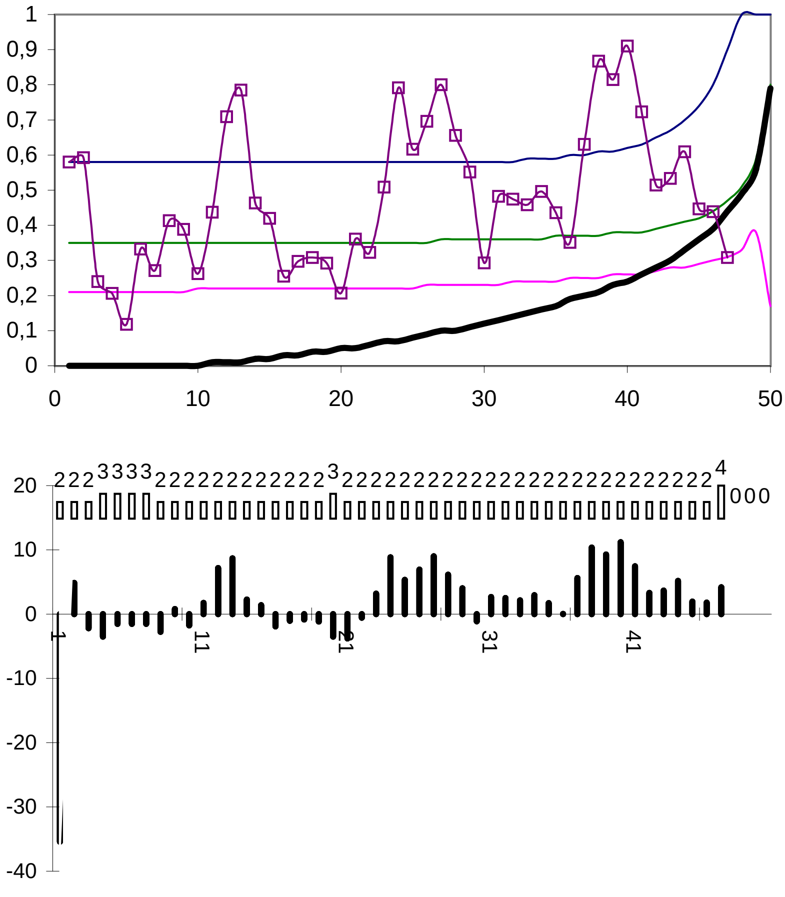


Figure 1: Path of θ and CF computation

Legend:

The upper graph in the figure represents the real options optimal exercise thresholds. To be specific, the highest represents the investment trigger threshold, the lowest, in bold, represents the abandonment threshold. In the middle, the higher is the restarting threshold while the lower is the mothballing threshold. An indicative path of θ_t is represented on the same graph. In the lower graph in the figure the corresponding time series of CF_t is represented in bold, together with an indicator function, in bars, representing the operating mode in which cash flows at that epoch were generated.

For each θ_t in each path we were able to compute $\Pi(\theta_t, \ell, t)$ as in equation (1). In the case study, the price was specified as the state variable. Computation of cash flows in each period takes place taking into due account the region in which the observation within the path happens to fall. Hence, exposure mapping equation not only is non linear in the variable θ_t, but also it is path dependent. As a matter of fact, cash flows will be computed taking into account the nature – reflecting, absorbing, transient – of the operating modes the investment project may have. In other words, equation (1) is specified for each of the modes where the dynamic system happens to operate, see expressions (7)-(10).

$$\Pi(\theta_t, \ell = W, t) = 0 \tag{7}$$
$$\Pi(\theta_t, \ell = O_i, t) = Q_{O_i} \cdot \left(\tilde{P}_t - Uvc\right) - F \;\forall i = 1, \ldots, m \tag{8}$$
$$\Pi(\theta_t, \ell = M, t) = -M_t \tag{9}$$
$$\Pi(\theta_t, \ell = A, t) = 0 \tag{10}$$

where, in addition to previous notation,

$\ell = W$	:	Waiting to invest mode, payoff is zero under the hypothesis that there are no opportunity costs to wait. It is a reflecting state, meaning that once the dynamic system has come out of it, it never comes back;
$\ell = O_i \;\forall i = 1, \ldots, m_O$	:	Operating modes, payoff is computed normally using the CVP equation for the observation of the state variable on that epoch of the simulated path. These operating modes are transient, meaning that the plant can be mothballed if DP principle of optimality requires it;
$\ell = M$	:	Mothballed mode, operating activity temporarily suspended. This mode is transient, meaning that production can be restarted when the DP principle of optimality requires so;
$\ell = A$	:	Abandoned mode, operating activity definitively terminated, plant sold for salvage value. This mode is absorbing, meaning that abandonment of operations and their sale for salvage value is irreversible.

Therefore, a θ_t path corresponds to a time series of cash flows optimally managed according to Bellman Optimum Principle, see lower graph in figure 1. In order to have the cash flows that the firm actually earns we have to consider transition costs that must be afforded to move the dynamic system from one operating mode to the other, see expression (11) for an example, beginning modes are on the rows, ending modes on the columns. Modes are ordered as Wait, Operate, Suspended, Abandoned.

$$\delta = \begin{pmatrix} 0 & c_{1,2} & +\infty & +\infty \\ +\infty & 0 & c_{2,3} & +\infty \\ +\infty & c_{3,2} & 0 & c_{3,4} \\ +\infty & +\infty & +\infty & 0 \end{pmatrix} \tag{11}$$

where:

$c_{1,2}$	:=	Lump sum invested in the project, transition cost from the wait to invest mode to the operating mode;
$c_{2,3}$	:=	Mothballing costs, transition cost from the operating mode to the mothballed mode;
$c_{3,2}$	:=	Restarting costs, transition cost from the mothballed mode to the operating mode;
$c_{3,4}$	:=	Salvage Value, transition (negative) cost from the mothballed mode to the abandoned mode;

Hence, cash flows actually used in the computation of the ensuing capital budgeting decision rules are the payoff of the operating mode, as represented in equations (7)-(10) net of mode transition costs reported in matrix (11), see equation (12), when and if they are actually afforded.

$$\Pi^* \left(\theta_t, \ell', t\right)_n = \Pi \left(\theta_t, \ell', t\right) - c_{l,l'} \; \forall t = 0, \ldots, T \tag{12}$$

To give a graphic portrayal of the CF computation, we have reported in figure 1 a simulated path of the time charter and the corresponding cash flow that the investment project would yield when managed according to dynamic programming thresholds. In the example represented, the investment project is implemented at time $t = 0$ since the series starts at a level higher than the investment threshold, (operating mode=2). The very low CF at the beginning is the result of the lump sum initially invested and the first operating cash flow. After three epochs the time charter level goes below the mothballing threshold. Hence the ship is laid up for four periods (operating mode=3) until the time charter level reaches the restarting threshold. Even if in the following epoch it goes below it, the project is kept in operating mode, hysteresis situation. The same kind of situation take place just before epoch 20 and 30. At period 47 the time charter price goes below the abandonment threshold and the ship is scrapped for its salvage value (operating mode=4).

2.3 NPV Forward Computation: Who pays for Successive Outflows?

NPV in capital budgeting and VaR and related measures are based on two very different set of hypotheses. Hence, merging these two frameworks together requires some *caveats* in order to interpret results correctly.[17]

Therefore, in order to compute expanded net present values going forward on each of the time series given by equation (12), it is convenient to delve into the hypotheses implicit in the computation of NPV in traditional capital budgeting and of VaR and CFaR in risk management.[18]

NPV in traditional capital budgeting stands as a generalization of the Fisher's Separation principle (FSP).[19] Among its hypotheses, the most relevant for our model are that capital markets are efficient and the representative agent's utility is time separable and linear in the CF consumed. In a multiperiod generalization of FSP, the Fisherian NPV, this means that markets are perfect foresight efficient in discounting CF produced in each epoch, offsetting the present value of negative and positive outflows paid in each epoch, making any

[17] We wish to thank Andrea Gamba for drawing our attention on this crucial issue.

[18] In classic capital budgeting, the question who pays for negative cash flows following positive ones is simply ignored in the computation of NPV or it is solved with a double rate discounting in the computation of IRR (Teichroew, 1964). Moreover, textbook like investments are usually Point Input Continuous Output (PICO), meaning an outflow at the very beginning and some inflows in the following periods, see (Dean, 1950). Because of this, NPV computation does not have to take into account the economic intuition of negative cash flows in epochs following the first. Here, instead, cash flows change sign frequently.

[19] See page 518 (Hirshleifer, 1992) in (Newman et al., 1992).

capital constraint irrelevant.[20] As a matter of fact any project is financed if only it has a positive expected NPV. Moreover, implicit in this multiperiod generalization of the FSP, the representative agent has a utility function which is time separable in the strong sense $U(\sum_{t=0}^{T} c_t) = \sum_{t=0}^{T} U(c_t)$ and linear in the CF.

In a world like the one just sketched, it is irrelevant to compute downside risk, like VaR and CFaR.[21] In other words, if $E(NPV) > 0$, an efficient market will always finance any outflow due to the multiperiod production frontier or technology. The net present value expanded for real options stays as the latest and most sophisticated version of the traditional Fisherian NPV. As a matter of fact, in the DP algorithm just described above, we implicitly assume a representative agent who spends $c_{1,2}$ when the investment threshold is reached while the funding for the successive outflows, due to negative operating cash flows and further transition costs, is simply provided by a FSP like efficient market. Considering that on the average the market will finance positive $E(NPV)$ projects, it would be rational for her to finance these outflows following the first lump sums. In this framework, $NPV = -c_{1,2}$ has the intuitive meaning of an investment project which destroys completely the endowment of the representative agent; $NPV < -c_{1,2}$ means a loss in excess of the lump sum initially poured into the project, implicitly financed by an efficient market.[22] This, in turn, will recoup on the average the funds provided together with the required interest rate, used in expected net present value computation.

Hence, considering the DP valuation as a generalization of the FSP, we simply compute the NPV in each of the time paths like equation (12) ignoring the fact that in some intermediate epochs negative cumulative cash flows can be well in excess of the initial endowment. Some of these project realization will be financed at a loss, $NPV < 0$, some will be financed with a profit, $NPV > 0$, being this imbalance between outflows and inflows a temporary insolvency. At each epoch, cross sectionally on each project we have a distribution of cash flows which allows us to evaluate the downside risk in a timeless framework (*timeless CFaR*) as a matter of fact in this framework it is not relevant through which path the current level of the state variable was reached.

On the other hand, if we remove the two main hypotheses of the FSP[23], bankruptcy and insolvency become relevant issues being the agent capital constrained. This means that we have to dedicate an endowment not only to the initial payment $c_{1,2}$ but also to successive outflows.[24] In this framework, $NPV < -c_{1,2}$

[20] The preference for timing of payments can be determined by discounting at a constant rate since the payments can be shifted backwards and forwards at the market rate of interest (Borch, 1968).

[21] As a matter of fact those two concepts are linked to bankruptcy and insolvency respectively.

[22] If we remove the hypothesis that the representative agent has limited endowment or limited liability this would simply mean that she would end up with negative wealth, or debts, after implementing the investment project. In one way or the other this involves a role for a capital market or for the agent acting as her own banker.

[23] To recap, efficient markets and strong separability and linearity of the utility function of the representative agent.

[24] As an alternative, we could have specified a one project firm retaining cash flows and paying a dividend. This would have

has no sense, there is no possibility of losing more than the initial endowment. Going forward on the project realization, as soon as the endowment is exhausted by outflows, the one project limited liability firm goes bankrupt and the abandonment option is exercised, even if subsequent CF pattern would have produced a positive NPV. At each epoch, in this case, we evaluate the probability that cash flows deplete completely the initial endowment. Moreover, we have a distribution of cash flows which is path dependent, taking only those contingencies in which the firm survived (*path dependent CFaR*). In this framework the firm has an additional decision variable: overfunding the project i.e. providing funds in excess of the initial expense $c_{1,2}$, trading off bankruptcy risk for NPV.[25]

In conclusion, in the Montecarlo simulations, we end up with two versions of expanded NPV. In one we have a "Fisherian NPV" whose derivation hinges on two quite strong hypotheses, in the other we have a "workable NPV" which takes into account the insolvency occurrences of a capital constrained representative agent. In the former, expected value is computed on all the project realizations, time paths, including those in which one or more insolvency episodes occurred. In the latter, expected value is computed only on the project realizations that survived, discarding time paths that, although with a positive NPV, lead the firm to bankruptcy. A "workable NPV" derivation would require a much different application of optimal control theory. Therefore, we decided not to implement it in the ensuing case studies.

3 Two case studies in shipping finance

Among the others, in the real options literature it is possible to find two "realistic" cases in ship valuation, (Dixit and Pindyck, 1994) on page 237, (Goncalves de Oliveira, 1999) page 185. These differ from those developed below in a number of relevant details. The first one is the data generating process which is respectively a driftless GBM and a GBM with drift. We have thoroughly shown in Appendix B that freight rates follow a mean reverting process and that the estimation of an arithmetic Ornstein Uhlenbeck process fits well the data available. Moveover, both these models are derived in a infinite time horizon, being based on stationary dynamic programming. Being a ship a finite lived asset, we have preferred to study the opportunity to invest in a ship over a finite time horizon, T=25 years. Because of these specific features, we were able to derive exercise thresholds for the whole life of the project while this was not the case for the two papers previously mentioned in which only individual levels of the state variable are given as thresholds. Finally, while (Dixit and Pindyck, 1994) on page 237 evaluates the vessel with all the real options we have considered here, (Goncalves de Oliveira, 1999) page 185 studies only the switching options to mothball and

decoupled cash flow to equity from operating cash flow on which the DP algorithm is based in this model.

[25] Funds in excess of the initial expense are put in an escrow account yielding the risk free rate.

to restart.

Although the methods and the main purposes of the case studies just mentioned are very different from ours, we have used them as base cases respectively for a very large crude carries (VLCC) 280.000 dwt, double hull (DH),[26] and a dry bulk cargo, namely a Panamax 80-90.000 dwt, taking from those two references, when they were not available elsewhere, data necessary to our model. A summary of the data actually used and of their sources is reported in table 3.

3.1 Value and Risk in Real Option Analysis: A Provisional Case Study

In this paragraph a provisional case study is reported. In the forthcoming draft the shipping finance cases will be included. The initial investment $c_{1,2} = 40$. Costs to mothball the project are $c_{2,3} = 2$. Costs to restart the project are $c_{3,2} = 4$. If the project is abandoned it yields $c_{3,4} = 5$.

The project has an expected technical life of 10 years and its operating mode can be revised every six months exercising the options to start the project, to mothball, to restart or to abandon it. In operating mode the profit is $\pi_O = 20 \cdot \theta_s - 7$ while in mothballing mode it is $\pi_M = -1.5$. In both waiting mode and abandonment mode cash flows are nil.

The state variable has been specified as an arithmetic Ornstein Uhlenbeck with the following parameters: $\eta = .125$, $\overline{\theta} = .5$, $\sigma_\theta = .125$, in a grid with $\theta_{min} = 0$ and $\theta_{max} = 1$ with $\Delta\,\theta_t = 1\%$. This process has been choosen after estimating the process parameters on 48 years of monthly time series reported in appendix A. The proportions between the normal value and volatility are equivalent to those of dry bulk time charter computed in appendix B. Reversion speed resembles that of the same time series.

We have derived the value of the investment project at time $t = 0$ in a backward induction procedure applied to equation (1). Results are represented by the smoothed lines without markers in figure 2. The same procedure has been run both for dynamic active management and passive management. From the same procedure we have derived the real options exercise thresholds for the whole life of the project as represented in figure 1.

We have performed 10,000 Markov Chain Montecarlo simulations of the state variable θ_t in equation (6) in the same discretization used in the backward induction. Then, on each of these time series we have computed CF_t net of transition costs taking into due account the optimal operating modes indicated by real options exercise thresholds. Results are reported in figure 1 as smoothed lines with markers. As a matter of fact, while the RPVs have been derived for 100 different initial values simultaneously, Montecarlo

[26] As (Tolofari et al., 1987) proved, most of the economies of scale in shipping take place for levels of output, ton-miles, lower than 50.000. Because of this, even a VLCC is representative of wide range of tankers.

A. Transition costs matrix (in millions of US$):

$$\delta_{\mathbf{OIL}} = \begin{pmatrix} 0 & 70.3 & +\infty & +\infty \\ +\infty & 0 & 0.27 & +\infty \\ +\infty & 1.062 & 0 & -60.7/5.47 \\ +\infty & +\infty & +\infty & 0 \end{pmatrix}$$

B. Cash Flows for each operating mode (in US$):

$$\begin{aligned} \Pi(\theta_t, \ell = W, t) &= 0 \\ \Pi(\theta_t, \ell = O_i, t) &= \left\{ \begin{array}{c} \text{Daily} \\ \text{Time} \\ \text{Charter} \end{array} \right\} - \left\{ \begin{array}{c} \text{Operating} \\ \text{Costs} \end{array} \right\} = \theta_t - 7955 \\ \Pi(\theta_t, \ell = M, t) &= 692,000/360 \\ \Pi(\theta_t, \ell = A, t) &= 0 \end{aligned}$$

C. Rescaling of the parameters of the OUP θ_t (in US$):

$Parameters$	Stopford	Rescaled
$\theta_{base=2Q2000}$	48.00	23,378.00
$\overline{\theta}$	32.66	15,904.38
σ_θ	25.39	12,366.20
$min(\theta)$	7.63	3,714.76
$max(\theta)$	104.57	50,929.98

A. Transition costs matrix (in millions of US$):

$$\delta_{\mathbf{DRY}} = \begin{pmatrix} 0 & 23 & +\infty & +\infty \\ +\infty & 0 & 0.757 & +\infty \\ +\infty & 2.276 & 0 & -14.8/2.38 \\ +\infty & +\infty & +\infty & 0 \end{pmatrix}$$

B. Cash Flows for each operating mode (in US$):

$$\begin{aligned} \Pi(\theta_t, \ell = W, t) &= 0 \\ \Pi(\theta_t, \ell = O_i, t) &= \left\{ \begin{array}{c} \text{Daily} \\ \text{Time} \\ \text{Charter} \end{array} \right\} - \left\{ \begin{array}{c} \text{Operating} \\ \text{Costs} \end{array} \right\} = \theta_t - 4190 \\ \Pi(\theta_t, \ell = M, t) &= 379,000/360 \\ \Pi(\theta_t, \ell = A, t) &= 0 \end{aligned}$$

C. Rescaling of the parameters of the OUP θ_t (in US$):

$Parameters$	Stopford	Rescaled
$\theta_{base=2Q2000}$	7000.00	7060.00
$\overline{\theta}$	6729.1461	6786.82
σ_θ	1671.1472	1685.47
$min(\theta)$	2762.66	2786.34
$max(\theta)$	15000.00	15128.57

Table 2: Summary of the data used in the case studies

Legend:

cost matrix refers to expression (11)

$c_{1,2}$:= newbuilding price (Source (Drewry and Jupe, 2001))

$c_{3,4}$:= secondhand price / Scrap value (Source (Drewry and Kellock & Co, 1999))

$c_{2,3}$:= for OIL revaluation of $E_M = .200$ as suggested by (Dixit and Pindyck, 1994) page 238; for DRY revaluation of $K_1 = .564$ as suggested by (Goncalves de Oliveira, 1999) page 185.

$c_{3,2}$:= for OIL revaluation of R=.790 as suggested by (Dixit and Pindyck, 1994) page 238; for DRY revaluation of $K_2 = 1.694$ as suggested by (Goncalves de Oliveira, 1999) page 185.

cash flow equations refer to expression (7)-(10)

$\Pi(\theta_t, \ell = O_i, t)$:= Daily Time Charter minus operating costs not due by the charterer (Source (Drewry and Kellock & Co, 2002))

M := annual figure, for OIL revaluation of M=.515 as suggested by (Dixit and Pindyck, 1994) page 238; for DRY revaluation of M=.282 as suggested by (Goncalves de Oliveira, 1999) page 185.

simulations have been performed for 10 initial values $\theta_0 = 0.0, 0.1, 0.2, \ldots, 0.8, 0.9, 1.0$.

At a first glance on figure 2 expected values seem to be the same. This is confirmed by convergence tests performed in Appendix C. Expected values differ at most 2%. This assures that both backward induction and forward computation are modeling the same optimal dynamic behaviour.

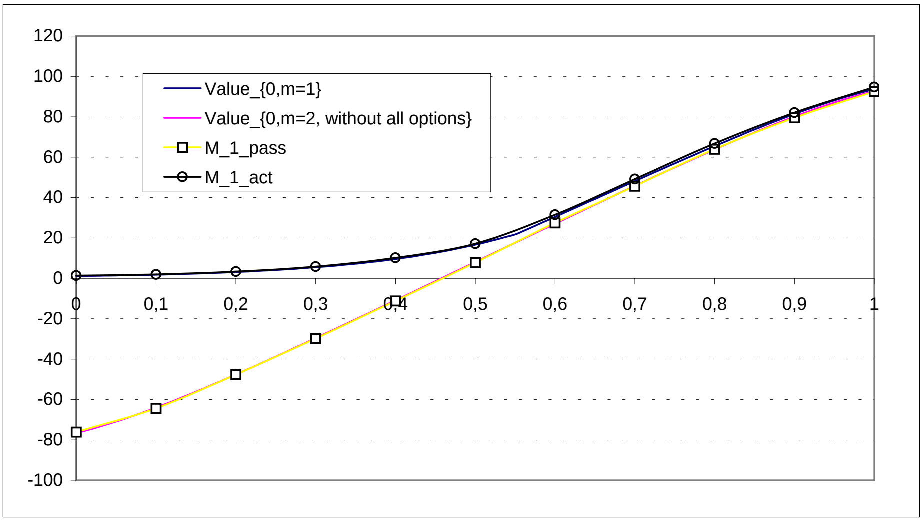
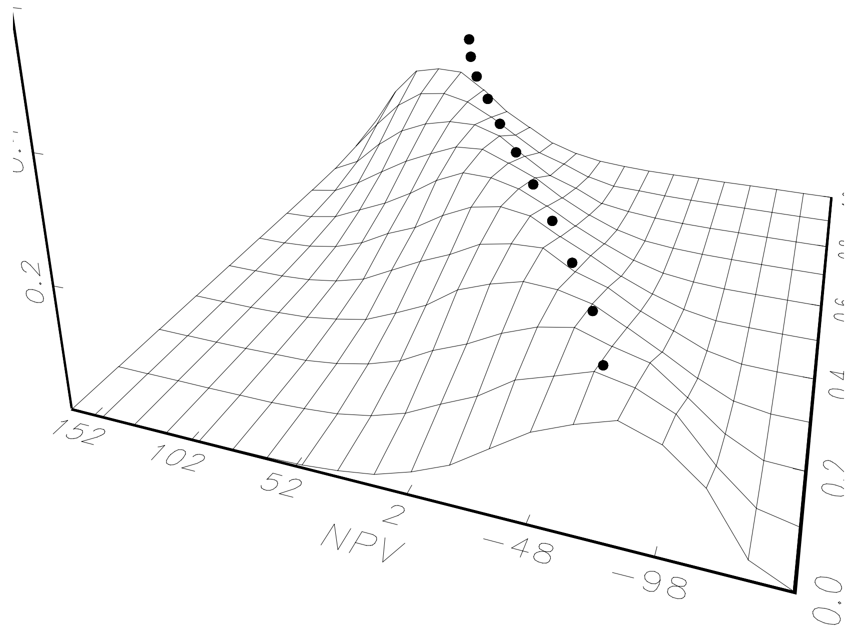


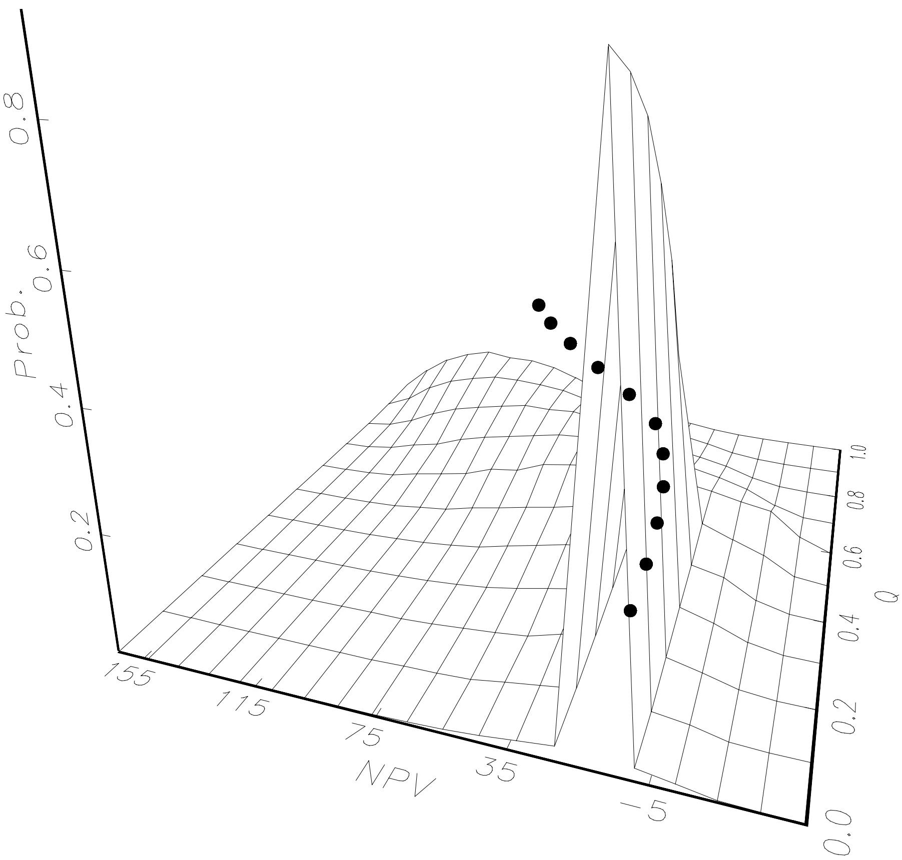
Figure 2: RPV and Montecarlo Markow Chain Expected Expanded NPVs

Legend: The graph reports RPVs for both the active and passive management of the project, with and without real options, smoothed line without markers. Together with these values, the corresponding averages from the Montecarlo Markov Chain for selected levels of $\theta_{t=0}$ are reported, namely $\theta_{t=0} = 0, .1, .2, \ldots, .9, 1$. These are represented with markers, circles and squares respectively.

Apart from average values, Markov chain Montecarlo simulations provide us with the whole distribution of values of expanded NPVs at time $t = 0$, see figure 3. In the upper graph of this figure, the distributions of passive management NPV are represented together with the average values plotted previously in figure 2. These distributions resemble to a normal, and this is confirmed also by standardized kurtosis coefficient reported in the last column of panel A in table 3 which is not significantly different from 3. This does not happen for active management NPVs reported in the lower graph of the same figure. There, expecially for low initial levels of θ_t, distributions are markedly leptokurtic and they almost collapse to a spike. This is due to the presence of the option to wait, being the investment threshold at time $t = 0$ at $\theta_0 = .59$. For levels higher than the threshold just mentioned distributions become again similar to a normal although rather platikurtic, see last column of panel B in table 3.



Panel A: NPV distributions and expected values under passive management.



Panel B: NPV distributions and expected values under dynamic active management.

Figure 3: Expanded NPVs distributions at time $t = 0$

Legend: NPV distributions are reported for each of the simulated initial levels of θ_0. Dots along the "ridge" of the distributions represent mean values. Those are the expected values that converge to dynamic programming results.

A. Passive management:

θ_0	Prob(loss)	$Prob(loss) > I$	min	VaR_{99}	VaR_{95}	$q_{.5}$	Ave. sim.	E(RPV)	Std	Max	Sk_{co}	Ku_{co}
0.0	96.82%	83.74%	-148.0	-137.6	-127.2	-80.2	-76.2	-76.6	35.9	67.3	0.56	2.95
0.1	94.15%	74.43%	-148.5	-131.6	-120.6	-68.3	-64.4	-64.0	38.2	87.1	0.49	2.93
0.2	86.81%	58.72%	-137.9	-124.5	-110.0	-49.8	-47.7	-47.7	41.0	94.4	0.32	2.68
0.3	75.11%	42.37%	-133.7	-115.4	-97.1	-31.0	-29.8	-29.6	42.6	110.4	0.19	2.58
0.4	60.63%	26.57%	-123.3	-102.1	-82.2	-12.2	-11.2	-10.9	43.4	129.8	0.12	2.64
0.5	43.60%	14.05%	-118.4	-89.1	-64.3	7.5	7.8	8.1	43.4	131.9	0.00	2.61
0.6	26.63%	6.36%	-112.3	-72.4	-45.4	28.2	27.4	27.0	43.1	142.7	-0.10	2.62
0.7	14.72%	2.72%	-103.4	-56.9	-27.3	47.0	45.6	45.8	42.6	155.7	-0.22	2.65
0.8	6.74%	0.76%	-100.5	-36.1	-6.8	66.4	64.0	63.8	41.0	157.6	-0.31	2.63
0.9	2.97%	0.22%	-79.8	-19.4	10.3	82.8	79.5	80.1	38.3	159.3	-0.47	2.88
1.0	1.03%	0.07%	-53.2	-0.6	29.6	96.0	92.5	92.8	35.2	165.3	-0.53	3.00

A. Active management:

θ_0	Prob(loss)	$Prob(loss) > I$	min	VaR_{99}	VaR_{95}	$q_{.5}$	Ave. sim.	E(RPV)	Std	Max	Sk_{co}	Ku_{co}
0.0	3.38%	0.01%	-31.1	-14.5	0.0	0.0	1.4	1.1	7.5	80.5	4.18	27.34
0.1	4.40%	0.01%	-30.0	-16.6	0.0	0.0	1.9	1.8	9.4	95.4	4.07	25.73
0.2	7.00%	0.01%	-37.3	-21.1	-5.6	0.0	3.3	3.1	12.8	98.7	2.94	14.34
0.3	9.74%	0.01%	-37.1	-23.2	-9.7	0.0	5.8	5.5	17.0	111.6	2.25	9.20
0.4	13.77%	0.03%	-43.2	-28.3	-15.6	0.0	10.1	9.6	23.2	132.9	1.61	5.72
0.5	18.29%	0.05%	-44.3	-33.2	-21.5	2.3	17.1	16.7	30.0	129.9	0.96	3.43
0.6	24.67%	0.32%	-46.7	-37.0	-28.3	29.8	31.5	30.4	39.2	142.7	0.24	2.30
0.7	12.09%	0.02%	-46.7	-27.4	-14.8	48.6	49.2	48.2	39.3	155.7	0.06	2.22
0.8	4.74%	0.01%	-31.1	-15.0	0.7	68.1	66.8	65.5	38.8	157.6	-0.12	2.23
0.9	1.50%	0.01%	-22.0	-3.6	17.8	84.5	82.1	81.4	36.5	159.3	-0.30	2.42
1.0	0.23%	0.01%	-13.4	12.9	34.8	97.7	94.8	93.7	33.8	165.3	-0.38	2.54

Table 3: Risk and Value

Legend: VaR_{99}:= 99th quantile; VaR_{95}:= 95th quantile; Ave.Sim:= average of the simulated NPVs; E(RPV):= expected running present values on the DP procedure at time $t = 0$; Sk_{co}: standardized skewness coefficient according to I. Fisher; Ku_{co}:= standardized curtosis coefficient;

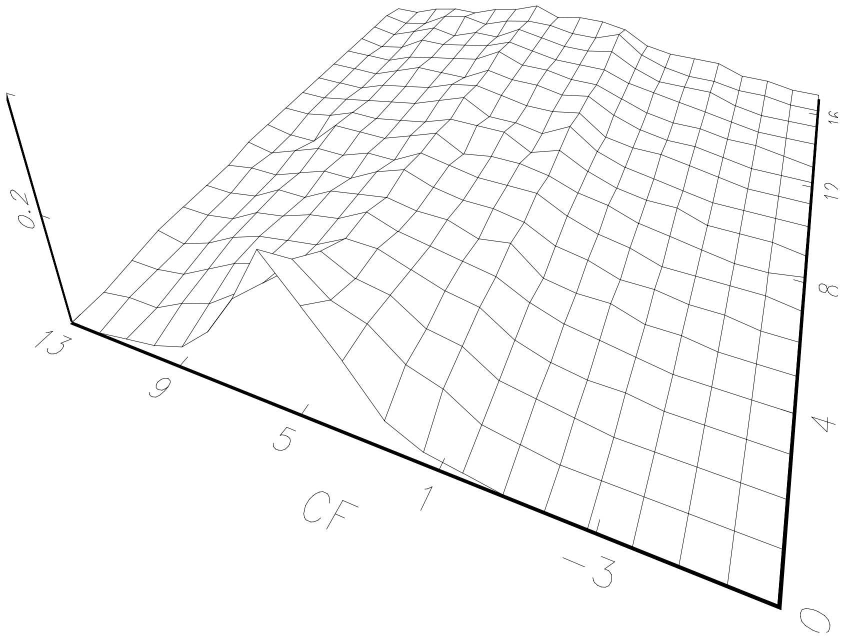
For levels of $\theta_0 > .59$ both distributions are almost symmetric, as shown by the skewness standardized coefficient not significantly different from zero in table 3. This is due to the fact that this investment project has only downside risk real options.[27] Instead, for levels of $\theta_0 < .59$ while passive NPVs distribution is still symmetric, the active NPVs distributions show a marked positive asymmetry as (Trigeorgis, 1996) on page 123 suggested. This is confirmed by the difference between the median $q_{.5}$ and the expected values reported in panel B of table 3. This difference becomes not significantly different from zero for levels of θ_0 greater than the investment threshold.

The effect on risk management of real options are evident from a comparison of the first two columns of the panels in table 3. While the probability of a negative NPV is highly and monotonically dependent on the initial values of θ_t for passively managed projects, in the presence of real options these percentages are drastically reduced and are lower also for levels of θ_0 greater than the investment threshold. To the same token, active dynamic management is very effective in controlling losses which exceed the initial lump sum invested in the project, see column three in table 3. These observations dovetail with VaR computations at 99% and 95% confidence level. For all the initial levels of θ_t value at risk of NPVs is much lower for actively managed projects than for passively managed ones. On these same distributions we have also tested the applicability of stochastic dominance criteria. To sum up results that are not reported, first order stochastic dominance is sometimes applicable while second order is always applicable, meaning, *il va sans dir*, that dynamic active managed projects dominate passively managed ones.

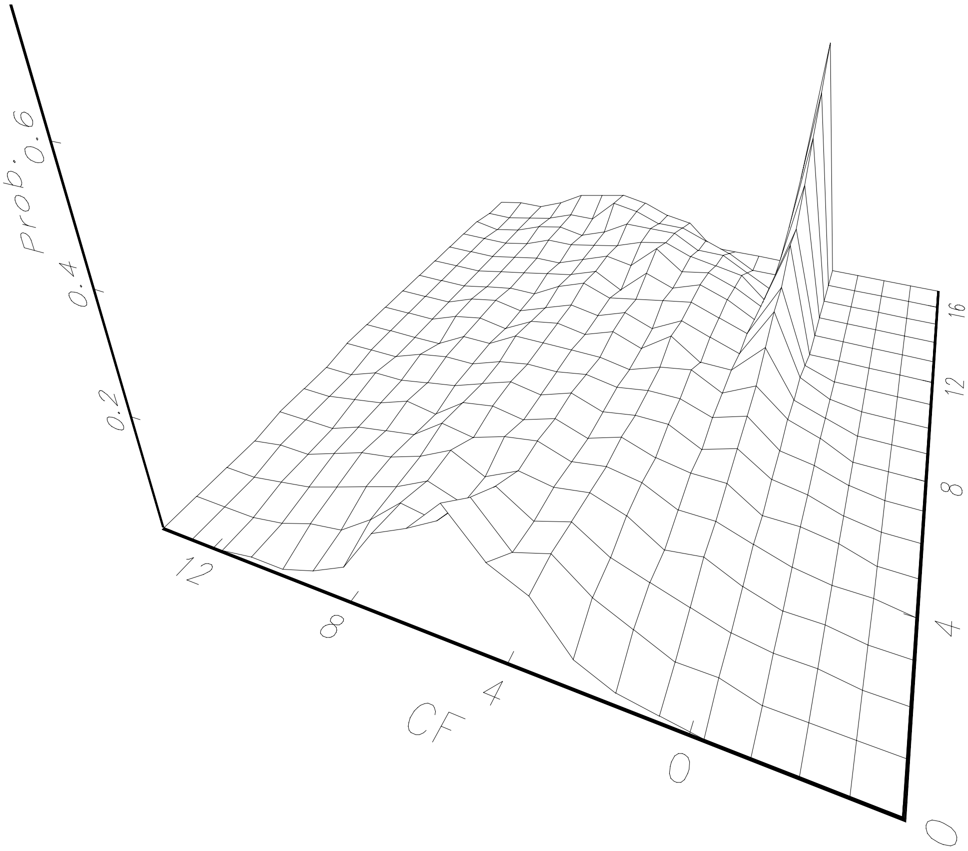
Together with these results on a stock variable like NPV which, as shown above, is rather elusive, we have computed cash flow at risk (CFaR) for each of the epochs, but the first, of the investment project, see distributions in figure 4. Unlike NPV distributions, those for CF_t are less immediate to interpret. The exercise of the option to abandon produces a spike like behaviour for the last periods of the investment project life. Instead, in this case too, the computation of VaR on cash flows help us to tackle the effect of the exercise of real options on operational risk. As shown in table 4 CFaR at the 99% and 95% confidence level is always much lower for actively than for passively managed projects. This dovetails with the probability of incurring a loss reported in the second column in table 4. While a passive management leads to double digit probabilties of incurring losses, these are less than half when a dynamic active management is followed.

In conclusion, this case study shows that real options are really effective in taming operational risk both at the stock, VaR on NPVs, and at the flow level, VaR on CF or CFaR. This effect has been quantified using value at risk and verifying the applicability of stochastic dominance criteria. The intuition of (Trigeorgis, 1996) page 123 is definetly verified and quantified.

[27] To recall, option to wait, option to mothball and option to abandon.



Panel A: CF distributions and expected values under passive management.



Panel B: CF distributions and expected values under dynamic active management.

Figure 4: CF distributions in each epoch of the project life.

Legend: Cash Flows distributions are reported for each of the epochs of the life of the project for an initial level of $\theta_0 = .6$. Dots along the "ridge" of the distributions represent mean values.

A. Passive Management Results:

t	Prob(loss)	min	VaR_{99}	VaR_{95}	$q_{.5}$	Ave. sim.	Std	Max
1	0.25%	-0.8	0.6	2.0	4.8	4.9	1.8	11.8
2	2.55%	-3.0	-1.0	0.8	4.8	4.9	2.5	13.0
3	4.45%	-4.0	-1.8	0.0	4.8	4.8	2.9	13.0
4	6.30%	-5.2	-2.6	-0.6	4.6	4.7	3.2	13.0
5	8.80%	-7.0	-3.4	-1.2	4.6	4.6	3.5	13.0
6	10.20%	-7.0	-4.0	-1.8	4.6	4.5	3.7	13.0
7	12.10%	-7.0	-4.8	-2.0	4.4	4.5	3.9	13.0
8	13.55%	-7.0	-5.6	-2.4	4.4	4.3	4.0	13.0
9	15.45%	-7.0	-5.6	-2.6	4.2	4.2	4.1	13.0
10	16.15%	-7.0	-5.8	-3.0	4.2	4.2	4.2	13.0
11	15.90%	-7.0	-6.0	-2.8	4.2	4.1	4.2	13.0
12	17.35%	-7.0	-6.8	-3.4	4.0	4.0	4.3	13.0
13	17.45%	-7.0	-6.2	-3.6	4.0	4.0	4.3	13.0
14	18.55%	-7.0	-6.2	-3.8	3.8	3.8	4.3	13.0
15	20.20%	-7.0	-6.8	-3.6	3.8	3.8	4.4	13.0
16	19.75%	-7.0	-7.0	-4.0	3.8	3.7	4.4	13.0
17	19.95%	-7.0	-6.8	-4.2	3.8	3.7	4.4	13.0
18	20.15%	-7.0	-6.8	-4.2	3.7	3.6	4.5	13.0
19	21.55%	-7.0	-7.0	-4.2	3.8	3.5	4.5	13.0

B. Active Management Results:

t	Prob(loss)	min	VaR_{99}	VaR_{95}	$q_{.5}$	Ave. sim.	Std	Max
1	0.25%	-0.8	0.6	2.0	4.8	4.9	1.8	11.8
2	2.55%	-3.0	-1.0	0.8	4.8	4.9	2.5	13.0
3	4.45%	-3.5	-1.8	0.0	4.8	4.8	2.9	13.0
4	6.10%	-3.5	-2.4	-0.4	4.6	4.7	3.2	13.0
5	8.05%	-3.5	-2.6	-0.8	4.6	4.6	3.4	13.0
6	8.40%	-3.5	-2.6	-1.2	4.6	4.6	3.6	13.0
7	9.15%	-4.0	-2.4	-1.0	4.4	4.6	3.6	13.0
8	8.95%	-2.6	-2.4	-1.2	4.4	4.5	3.7	13.0
9	8.90%	-3.6	-2.0	-0.8	4.2	4.5	3.7	13.0
10	7.35%	-3.8	-1.6	-0.4	4.2	4.5	3.7	13.0
11	5.55%	-1.5	-1.2	-0.2	4.2	4.5	3.7	13.0
12	2.95%	-2.6	-0.6	0.0	4.0	4.5	3.6	13.0
13	0.70%	-0.8	0.0	0.0	4.2	4.5	3.6	13.0
14	0.05%	0.0	0.0	0.0	4.2	4.5	3.6	13.0
15	0.05%	0.0	0.0	0.0	4.2	4.4	3.7	13.0
16	0.05%	0.0	0.0	0.0	4.4	4.2	3.8	13.0
17	0.05%	0.0	0.0	0.0	4.4	4.1	3.9	13.0
18	0.05%	0.0	0.0	0.0	4.2	4.0	4.2	13.0
19	0.05%	0.0	0.0	0.0	0.0	4.2	5.4	13.6

Table 4: The Effect of Real Options on Cash Flow at Risk

Legend: Results have been derived for $\theta_0 = .6.$ on the cross section of 2,000 simulations.

4 Conclusions

In this normative model we model total variability in NPV_0 and CF_t $\forall t = 0, \ldots, T$ for an industrial investment project in the presence of real options. To begin with, with a backward induction process, we derive expanded NPVs and real options optimal exercise thresholds in an extension of (Kulatilaka, 1988). Then we perform a Markov chain Montecarlo simulation of the state variable θ_t $\forall t = 0, \ldots, T$, specified as an arithmetic Ornstein Uhlenbeck process. Going forward on these simulated time series, we compute cash flows, taking into due account the optimal exercise of a whole string of real options, both reversible and irreversible. With these controlled cash flows series, expanded net present values have been computed.

The expected value of these simulated expanded NPVs converges to that computed recursively in the Bellman DP procedure which provided us with the optimal exercise thresholds previously used to control CF_t. In addition to the expected value, our model provides the whole distribution of expanded NPVs at time zero and of cash flows at each epoch of the investment project. We have described total variability applying Value at Risk (VaR) concepts to these distribution. Hence, we derived an actively managed project at risk or VaR of the expanded NPV. Moreover, we computed a cash flow at risk CFaR for each epoch of the investment project. As a complement to this local measure of risk, we have verified applicability of stochastic dominance criteria in the comparison of actively and passively managed investment projects.

The results of this paper confirm what (Trigeorgis, 1996) page 123 observes but does not quantify: real options reduce variability in expanded NPV, leading to a positive asymmetry in its distribution. In this paper it is shown what is the effect on downside risk of the options to wait, to mothball and to abandon.[28] As a matter of fact, VaR on NPV decreases dramatically with the exercise of real options previously mentioned. Therefore, we conclude that real options are effective not only in enhancing value but also in taming operational risk. The same kind of effect is observed on CF from operations through the computation of Cash Flow at Risk (CFaR).

The results of this paper can be considered as a base to value credit risk and to decide financial structure to finance the industrial project. Under the assumption of no interaction between financial structure and industrial strategy, having assessed analytically risk from operations it is easy to set up a normative model of financial leverage and debt duration based on a trade off between bankruptcy risk and shareholders value increase. Moreover, bankers would find in this model a way to comply with regulations, as Basel II, which require an analytical determination of the borrower credit risk.

A side product of this paper is the computation of IRR and PBP corresponding to a NPV optimizing

[28] The exercise of upside potential real options would lead to an even more positively skewed distribution. We leave the study of the latter for another paper.

dynamic strategy in the presence of real options. As a matter of fact, while all the models in real options literature compute only expected expanded present values, practitioners often require an internal rate of return and a payback period for the investment. Our model can be easily extended to provide such investment criteria which are intrinsically path dependent. Finally, this model can be easily extended to other data generating processes: e.g. GBM univariate and multivariate; geometric and arithmetic OU processes discretized on a binomial lattice.

In conclusion, this paper adds a new dimension to capital budgeting with real options, i.e. total variability. This dimension is usually ignored in the real options literature [29] as it is ignored in the financial options one. Other normative models should be devised to make workable choices among different investment projects in this new risk return framework. Here VaR for $E(NPV_e)$ and CFaR for cash flows are just sketched. Instead, a whole trade off theory between the downside risk measures and expected increase in shareholder values should be applied in this framework.[30] As an alternative, a useful tool for analyzing the distribution of NPV_e would be a three moment CAPM which takes into account not only mean and variance but also skewness.[31]

[29] Being (Calistrate et al., 2001) the only notable exception to our knowledge.

[30] See for instance, (Baumol, 1963) and all the other bankruptcy minimization criteria such as (Telser, 1955), (Kataoka, 1963) and (Roy, 1952).

[31] See for instance (Jurczenko and Maillet, 2001) and reference contained therein.

References

Allayannis, G. and Weston, J. (2001). The use of foreign currency derivatives and firm market value. *Review of Financial Studies*, 14:243–276.

Amram, M. and Kulatilaka, N. (1999). *Real Options: Managing Strategic Investment in an Uncertain World*. Financial Management Ass. Survey and Synthesis. Harvard Business School Press, Boston, Massachussets.

Asplund, M. (2000). What fraction of a capital investment is sunk costs? *Journal of Industrial Economics*, XLVIII(3):287–304.

Baumol, W. (1963). An expected gain confidence limit criterion for portfolio selection. *Management Science*, 11:174–182.

Bjerksund, P. and Steinar, E. (1995). *Contingent Claims Evaluation of Mean-Reverting Cash Flows in Shipping*, chapter 12, pages 207–221. Volume 1 of (Trigeorgis, 1995), first edition.

Borch, K. (1968). Decision rules depending on the probability of ruin. *Oxford Economic Papers*, 20:1–10.

Brennan, M. and Trigeorgis, L. (2000). *Project Flexibility, Agency and Competition: New Developments in the Theory and Applications of Real Options*. Oxford University Press, New York, Oxford, England, first edition.

Calistrate, D., Paulhus, M., and Sick, G. (2001). Real options for managing risk: Using simulation to characterize gain in value. Working paper, Department of Mathematics, University of Calgary, Calgary, Alberta, Canada T2N 1N4.

Chan, K., Karolyi, A., Longstaff, F. A., and Sanders, A. B. (1992). An empirical comparison of alternative models of the short-term interest rate. *Journal of Finance*, XLVII(3):1209–1227.

Chen, D.-H., Lin, Y.-Y., Lu, Y.-C., and Wu, S. (2000). Bot projects in taiwan: Financial modelling risk, term structure of net cash flows, and project at risk analysis. *Journal of Structured and Project Finance*, 5(4):53–63.

Chorn, L. and Carr, P. (1999). *The Value of Purchasing Information to Reduce Risk in Capital Investment Projects*, chapter 12, pages 279–294. Volume 1 of (Trigeorgis, 1999), first edition.

Copeland, T. and Antikarov, V. (2001). *Real Options: A Practitioner's Guide*. Texere, New York, first edition.

Cox, J., Ingersoll, J., and Ross, S. (1985). An inter-temporal general equilibrium model of asset prices. *Econometrica*, 53:363–384.

Cox, J., Ross, S., and Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 12(1):229–263.

Cruz, M. G. (2002). *Modeling, Measuring and Hedging Operational Risk*. Wiley Finance Series. John Wiley & Sons, Ltd, Baffins Lane, Chichester, West Sussex PO19 1UD, UK, first edition.

Cullinane, K. and Panayides, P. M. (2000). The use of capital budgeting techniques among uk-based ship operators. *International Journal of Maritime Economics*, 2(4):313–330.

Dean, J. (1950). *Capital Budgeting*. Columbia University Press, New York, first edition.

Dixit, A. and Pindyck, R. S. (1994). *Investments under Uncertainty.* Oxford University Press, Oxford, first edition.

Dorris, G. and Dunn, A. (2001). Energy enterprise risk management. *Energy and Power Risk Management*, pages 5–7.

Drewry, S. C. and Jupe, D. C. (2001). Ship finance & and investment. Internal report, Drewry Shipping Consultants Ltd, Drewry House, Meridian Gate - South Quay, 213 Marsh Wall, Lonfon E14 9FJ, England, UK.

Drewry, S. C. and Kellock & Co, C. (1999). Second hand ships: Investment potential and value for money. Internal report, Drewry Shipping Consultants Ltd, Drewry House, Meridian Gate - South Quay, 213 Marsh Wall, Lonfon E14 9FJ, England, UK.

Drewry, S. C. and Kellock & Co, C. (2002). Ship operating costs annual review and forecast 2002. Internal report, Drewry Shipping Consultants Ltd, Drewry House, Meridian Gate - South Quay, 213 Marsh Wall, Lonfon E14 9FJ, England, UK.

Froot, K. A., Sharfstein, D. S., and Stein, J. C. (1993). Risk management: Coordinating corporate investment and financing policies. *Journal of Finance*, 48(5):1629–1658.

Froot, K. A. and Stein, J. C. (1997). Risk management, capital budgeting, and capital structure policy for financial institutions: An integrated approach. *Journal of Financial Economics*, 47:55–82.

Godfrey, S. and Espinosa, R. (1998). Value at risk and corporate valuation. *Journal of Applied Corporate Finance*, 10(4):108–115.

Goncalves de Oliveira, F. (1999). Strategic decision in ocean shipping with contingent claims. *Advances in Futures and Options Research*, 10:169–195.

Gourieroux, C. and Jasiak, J. (2001). *Financial Econometrics: Problems, Models and Methods.* Princeton Series in Finance. Princeton University Press, Princeton, NJ, first edition.

Gwilliam, K. (1993). *Current Issues in Maritime Economics.* Norwell and Kluwer Academics, Massachussets and Dordrecht.

Haralambides, H. E. (1993). *Sensitivity Analysis of Risk in Shipping Finance*, chapter 11, pages 168–177. Volume 1 of (Gwilliam, 1993).

Hayt, G. and Song, S. (1995). Handle with sensitivity. *Risk*, 8(9):94–99.

Hirshleifer, J. (1992). *Investment Decision Criteria: Private Decisions*, chapter Dictionary Entry, pages 517–525. Volume 1 of (Newman et al., 1992), first edition. 3 voll.

Jarrow, R., V., M., and Ziemba, W. (1995). *Finance*, volume 9 of *Handbooks in Operations Research and Management.* North Holland Elsevier, Amsterdam, first edition. Informs Institute for Operations Research and the Management Sciences.

Jorion, P. (2001). *Value at Risk: The New Benchmark for Controlling Derivarives Risk.* McGrawHill, New York, second edition. First Edition 1997.

Jurczenko, E. and Maillet, B. (2001). The three-moment capm: Theoretical foundations and an asset pricing models comparison in an unified framework. working paper, TEAM-ESA 8059 du CNRS - Universitè de Paris 1 Panthéon Sorbonne, Bd de L'Hôpital 75647 Paris Cedex 13 France.

Kataoka, S. (1963). A stochastic programming modes. *Econometrica*, 31:181–196.

Kim, J., Malz, A. M., and Mina, J. (1999). Longrun. Technical document, Riskmetrics Group, formerly Risk Management Products and Research Group at J.P. Morgan, New York, London.

Ku, A. (2001). Value at risk: Variations on a theme. *Global Energy Business*, pages 12–19.

Kulatilaka, N. (1988). Valuing the flexibility of flexible manufacturing systems. *IEEE Transactions on Engineering Management*, 35(4):250–257.

Kulatilaka, N. (1993). The value of flexibility: The case of a dual-fuel industrial steam boiler. *Financial Management*, 22(3):271–280.

Kulatilaka, N. (1995). *The Value of Flexibility: A General Model of Real Options*, chapter 5, pages 89–109. Volume 1 of (Trigeorgis, 1995), first edition.

Kulatilaka, N. and Trigeorgis, L. (1994). The general flexibility to switch: Real options revisited. *International Journal of Finance*, 6(2):778–796.

Lee, A. Y., Kim, J., Malz, A. M., and Mina, J. (1999). Corporatemetrics: The benchmark for the corporate risk management. Technical document, Riskmetrics Group, formerly Risk Management Products and Research Group at J.P. Morgan, New York, London.

Li, F. S. and Chiu, W. (2003). *Valuing Generation Assets Using Real Option Competitive Price Analysis: A Step by Step Example of Portfolio of Generation Assets*, chapter 20, pages 597–634. Volume 1 of (Ronn, 2003), first edition.

Luciano, E., Peccati, L., and Cifarelli, D. M. (2003). Var as a risk measure for multiperiod static inventory models. *International Journal of Production Economics*, 81-82:375–384.

Luenberger, D. G. (1998). *Investment Science*. Oxford University Press, New York, Oxford, first edition.

MacMinn, R. D. (2002). Value and risk. *Journal of Banking & Finance*, 26:297–301.

Mun, J. (2002). *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*. Wiley Finance. John Wiley & Sons, Hoboken, New Jersey, first edition. with Crystal ball CD-ROM.

Newman, P., Millgate, M., and Eatwell, J. (1992). *The New Palgrave Dictionary of Money & Finance*. MacMillan, London, first edition. 3 voll.

Nyberg, M., Sellers, M., and Zhang, J. (2001). Private firm model: Introduction to the modeling methodology. Working paper series, KMV LLC, 1620 Montgomery Street, Suite 140, San Francisco, CA 941111 USA.

Roncalli, T. (1998). Financial application with gauss. Technical report, Financial Econometric Research Centre, City University Business School, City University Business School, Froibsher Crescent, Barbican Center, London, UK. web: http://www.city.as.uk./cubs/ferc/theirry.

Roncalli, T. (1999). *TSM: Advanced Time Series Estimation*. GAUSS^{TM} Application. Ritme Informatique, Paris, France, second edition.

Ronn, E. I. (2003). *Real Options and Energy Management: Using Options Methodology to Enhance Capital Budgeting Decisions*. Risk Books. RiskWaters Publishers, London, first edition.

Roy, A. (1952). Safety first and the holding of assets. *Econometrica*, 20:431–449.

Shiller, R. J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, 71(3):421–436.

Shimko, D. (1998). Cash before value. *Risk*, 11(7):45.

Shimko, D. C. (2001). Npv no more: Rpv for risk based valuation. Working paper series, Risk Capital Management Partners, 767 Fifth Avenue, 50th Floor, New York, NY 10153.

Sick, G. (1995). *Real Options*, chapter 21, pages 631–691. Volume 9 of (Jarrow et al., 1995), first edition. Informs Institute for Operations Research and the Management Sciences.

Stein, J., Usher, S. E., LaGattuta, D., and Youngen, J. (2000). A comparables approach to measuring cashflow-at-risk for non-financial firms. Working Paper Series 39, NERA National Economic Research Associates, 50 Main Street White Plains, New York 10606, USA.

Stein, J. C. (1996). Rational capital budgeting in an irrational world. *Journal of Business*, 69(4):429–455.

Stopford, M. (1988). *Maritime Economics*. Unwin Hyman, London, first edition.

Stultz, R. (1999). What's wrong with modern capital budgeting. www.ssrn.com paper, Ohio State University. Address Eastern Finance Association Meeting 1999.

Teichroew, D. (1964). *An Introduction to Management Science: Deterministic Models*. Wiley, New York, first edition.

Telser, L. (1955). Safety first and hedging. *Review of Economic Studies*, 23:1–16.

Tolofari, S. R., Button, K. J., and Pitfield, D. E. (1987). An econometric analysis of the cost structure of the tanker sector of the shipping industry. *International Journal of Transport Economics*, 14(1):71–84.

Trigeorgis, L. (1995). *Real Options in Capital Investments, Models, Strategies and Applications*. Praeger, Westport, Conn, first edition.

Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. MIT Press, Cambridge, Mass, first edition.

Trigeorgis, L. (1999). *Real Options and Business Strategy*. Risk Books. RiskWaters Publishers, London, first edition.

Turner, C. (1996). Var as an industrial tool. *Risk*, 9(3):38–40.

Tvedt, J. (2000). *The Ship Lay-Up Option and Equilibrium Freight Rates*, chapter 17, pages 340–348. Volume 1 of (Brennan and Trigeorgis, 2000), first edition.

.

A Data description

The monthly time series used in the shipping finance case study are taken from (Stopford, 1988) figure 2.1 page 57-58.[32] Several primitive time series have been spliced together from different sources, see table 5, in order to make up the series reported in figure 5. The resulting spliced series spans the period 1/1947|−|7/2000 for both a dry bulk cargo and an oil tanker.

	Start	End	Denomination
1	1/'47	12/'65	Trip Index 1947=100
2	1/'60	12/'90	Time Charter Index 1966=100
3	1/'91	8/'91	Baltic Freight Index BFI
4	1/'47	9/'49	British Ministry of Transportation = US Maritime commission
5	9/'49	12/69	British Ministry of Transportation, SCALE INTASCALE
6	9/'69	12/'74	World Scale
7	1/'75	7/'99	VLCC/ULCC
8	1/'75	6/'85	MED SIZE CRUDE
9	1/'71	7/'99	1 year Time Charter $ 000 Per Day 60.000 DWT Bulker

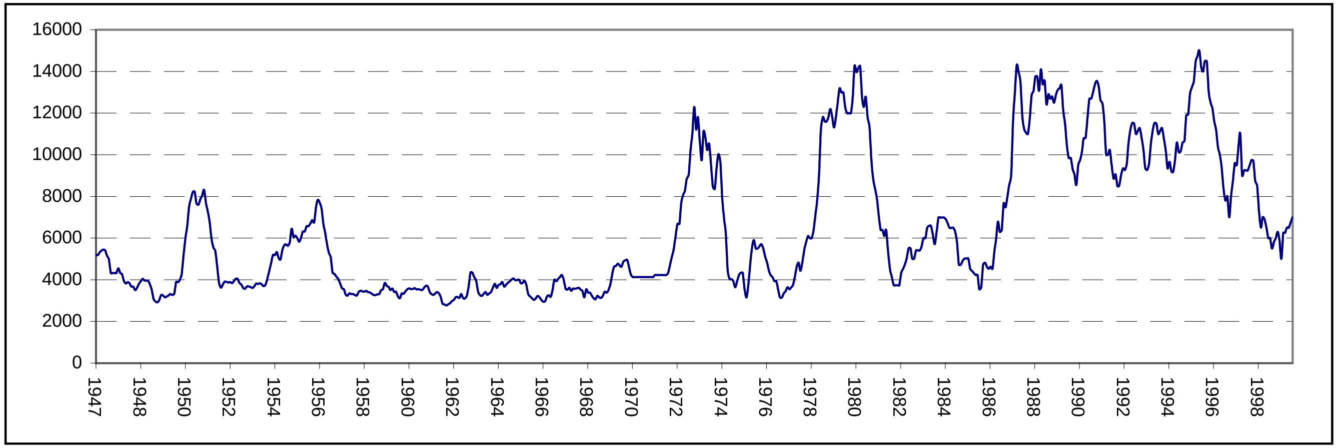
Table 5: Stopford Time Series of Dry and Bulk Time Charter 1947| − |2000

For a dry bulk cargo 60.000 DWT Bulker, older series have been rescaled to the most recent ones, namely series 1, 2 and 3 with series 9. To the same token, for an oil tanker, series 4 has been spliced together with series 5, 6 and finally series 7. Both series have been converted into time charter rates (TCR, $/day).

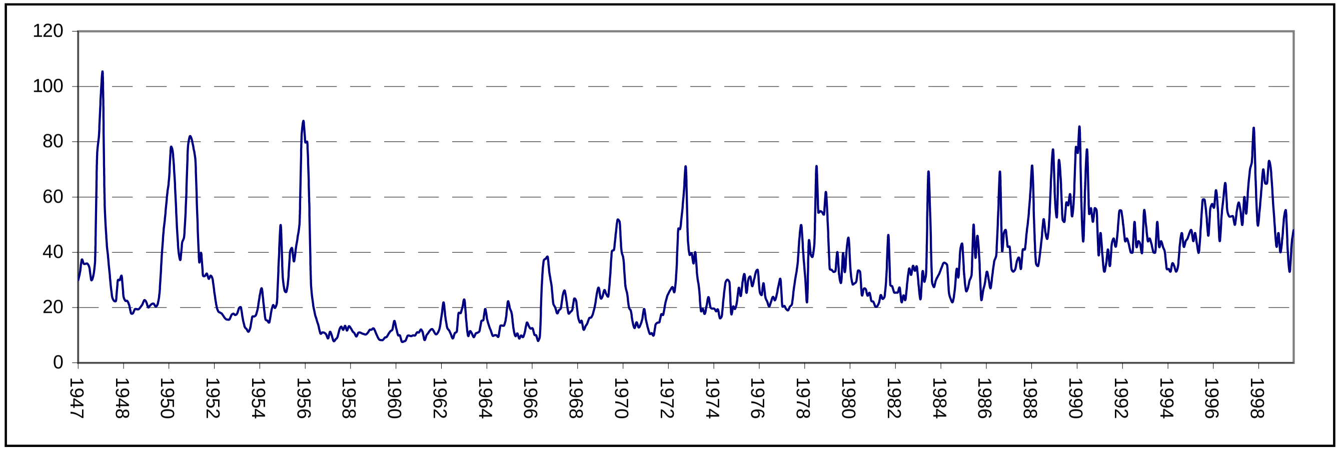
In principle, Stopford's series provide an accurate account of the year by year movements in rates, while they do not necessarily reflect movements in rates over long periods, see legend of figure 2.1 page 57-58 (Stopford, 1988). Hence, in order to convert these series into TCR, we consider the series constructed above as an index which allows us to rescale any TCR for the same kind of ship, bulk or oil tanker. This is instrumental to getting the statistical properties of the series and feed them into valuation algorithms.

For example, for oil tanker, Jan/'99=45 in Stopford's series, while in Drewry's series 1Q'99 it is 11,900 for an 80,000 DWT. Hence series has been re-scaled dividing it for 45 and multipling for 11,900. The major shortcoming of this procedure is that a point rescaling may not take into account of the discrepancies in construction of the two series. On the other hand, rescaling the series on two averages, e.g. 12 months averages, may iron out these time specific discrepancies towards structural differences in series construction.

[32] We wish to thank professor Martin Stopford for providing us with these time series. Any inappropriate use of these series is our fault.



A. Dry Bulk Carrier Time Charter Index.



B. Oil Tanker Time Charter Index.

Figure 5: Time Charter for Oil and Dry Bulk Carriers

Legend: Monthly index number for the oil and dry bulk carriers constructed as reported in table 5.

B Ornstein Uhlenbeck Parameters Estimation.

Econometric Methods

The arithmetic Ornstein Uhlenbeck process parameters have been estimated using several econometric methods in order to cross check their results being our model so sensitive to any change in the estimates. Two of these methods use OLS, while the other two use a GMM and a MLE respectively. The first two methods have a linear specification of the corresponding discrete time model while the last two have a non linear specification which is closer to the continuos time counterpart. Inference on the first two hinges on the asymptotic properties of the estimators and of their s.e. Instead, finite sample inference is used to test GMM and MLE estimates.

Being the results amazingly the same while the methods and their hypotheses are so different, we could be quite adamant in concluding that the two time series of Dry Bulk Carrier Time Charter Index and Oil Tanker Time Charter Index constructed by Martin Stopford, see figure 5, are effectively described by two Arithmetic Ornstein Uhlenbeck processes, with, respectively a low and high reversion speed.

The simplest among the econometric methods used is the one by (Dixit and Pindyck, 1994), page 76-77.[33] They rearrange the terms of the solution to the stochastic differential equation of the Arithmetic Ornstein Uhlenbeck process, see equation (6), in a form that can be considered the limiting case as $\Delta t \rightarrow 0$ of the OUP equation, (13).

$$
\begin{aligned}
\theta_t - \theta_{t-1} &= \overline{\theta} \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + \left(e^{-\eta \cdot \Delta t} - 1\right) \cdot \theta_{t-1} + \epsilon_t \\
\theta_t - \theta_{t-1} &= \widehat{\alpha} + \widehat{\beta} \cdot \theta_{t-1} + \epsilon_t \qquad (13)
\end{aligned}
$$

From expression (13), imposing just the ordinary OLS conditions,[34] (Dixit and Pindyck, 1994) derive the OUP parameters, see (14). Hence, their model hinges on a linear model which does not imply any non linear relationship between η and σ. Moreover, they do not provide standard errors for the nonlinear functions of the regression estimates. Finally, regression (13) resembles a traditional Dickey Fuller regression with all the implications that this has about correct inference.

$$
\overline{\theta} = -\frac{\widehat{\alpha}}{\widehat{\beta}} \qquad \eta = -\frac{\ln\left(1+\widehat{\beta}\right)}{\Delta t} \qquad \widehat{\sigma} = \widehat{\sigma}_\epsilon \cdot \sqrt{\frac{2 \cdot \eta}{1 - e^{-2 \cdot \eta \Delta t}}} \qquad (14)
$$

[33] The reader should be aware that in the original presentation of Dixit and Pindyck there where two typos. One omitting Δt in the exponent of expression (19) and the other omitting a 2 in the expression for the computation of σ on page 77.

[34] The same discrete time model is used in other articles, see for instance (Chan et al., 1992) page 1218 table III. The crucial difference is that here it is only a linear relationship between the level and the variation of the time series, there in addition to the usual condition on the residuals, respected by construction, other conditions are posed on the proportionality of the variance with respect to the current level of the process. Including this nonlinear relation requires a GMM method to get estimates.

(Gourieroux and Jasiak, 2001), page 290, propose a variant of the (Dixit and Pindyck, 1994) method. They do not rearrange equation (6) and suggest a simple Maximum Likelihood estimation of the three parameters of the OUP, see expression (16).

$$
\begin{aligned}
\theta_t &= \overline{\theta} \cdot \left(1 - e^{-\eta \cdot \Delta\, t}\right) + e^{-\eta \cdot \Delta\, t} \cdot \theta_{t-1} + \epsilon_t \\
\theta_t &= \overline{\theta} \cdot \left(1 - \widehat{\beta}\right) + \widehat{\beta} \cdot \theta_{t-1} + \epsilon_t
\end{aligned}
\tag{15}
$$

The estimates and their asymptotic variances are given by expression (16).

$$
\begin{array}{rclrclrcl}
\overline{\theta} &=& \frac{1}{T} \cdot \sum_{t=1}^{T} \theta_t & \widehat{\beta} &=& \frac{Cov(\theta_t, \theta_{t-1})}{Var(\theta_t)} & \widehat{\sigma}_\epsilon^2 &=& \frac{1}{T} \cdot \sum_{t=1}^{T} \epsilon_t^2 \\
V_{asy}(\overline{\theta}) &=& \frac{\widehat{\sigma}_\epsilon^2}{T \cdot \left(1 - \widehat{\beta}\right)} & V_{asy}(\widehat{\beta}) &=& \frac{1}{T} \cdot \left(1 - \widehat{\beta}^2\right) & V_{asy}(\widehat{\sigma}_\epsilon^2) &=& \frac{2 \cdot \widehat{\sigma}_\epsilon^4}{T}
\end{array}
\tag{16}
$$

where[35]: $\epsilon_t \quad = \quad \theta_t - \overline{\theta} - \widehat{\beta} \cdot \left(\theta_{t-1} - \overline{\theta}\right)$

From these estimates we get the remaining two parameters of the OUP, namely the reversion speed and the volatility of the process together with their asymptotic variances derived through the δ method, see expression (17).

$$
\begin{array}{rclrcl}
\widehat{\eta} &=& -log(\widehat{\beta}) & \widehat{\sigma}^2 &=& -\frac{2 \cdot log(\widehat{\beta})}{1 - \widehat{\beta}^2} \cdot \widehat{\sigma}_\epsilon^2 \\
V_{asy}(\widehat{\eta}) &=& \left[\frac{\partial\left(-log(\widehat{\beta})\right)}{\widehat{\beta}}\right]^2 \cdot V_{asy}(\widehat{\beta}) & V_{asy}(\widehat{\sigma}^2) &=& \left[\frac{\partial(\widehat{\sigma}^2)}{\partial \boldsymbol{\theta}}\right] \cdot \boldsymbol{\Sigma}_{\widehat{\beta}, \widehat{\sigma}_\epsilon^2} \cdot \left[\frac{\partial(\widehat{\sigma}^2)}{\partial \boldsymbol{\theta}}\right]'
\end{array}
\tag{17}
$$

where:

$$
\begin{aligned}
\left[\frac{\partial(\widehat{\sigma}^2)}{\partial \boldsymbol{\theta}}\right] &= \left[\left(\frac{\partial(\widehat{\sigma}^2)}{\partial \widehat{\beta}}\right) \left(\frac{\partial(\widehat{\sigma}^2)}{\partial \widehat{\sigma}_\epsilon}\right)\right]; \\
\boldsymbol{\Sigma}_{\widehat{\beta}, \widehat{\sigma}_\epsilon} &= \begin{bmatrix} V_{asy}(\widehat{\beta}) & 0 \\ 0 & V_{asy}(\widehat{\sigma}_\epsilon^2) \end{bmatrix}
\end{aligned}
$$

Apart from $\overline{\theta}$, (Gourieroux and Jasiak, 2001) estimates are by construction identical to those of (Dixit and Pindyck, 1994), the only difference being that the former give asymptotic standard errors for the estimates of the OUP parameters being a non linear function of the parameters estimated in expression (15).[36] In both of the previous methods estimation does not involve non linear relations between estimated parameters.

[35] The reader should be aware that the expression for $V_{asy}(\widehat{\sigma}_\epsilon^2)$ contains a typo, having been omitted the exponent 2. Because of this, in order to get the s.e. of the standard deviation the expression should be raised to 1/2 twice. Once to get the s.e. of the variance estimate, another to get the s.e. of the corresponding standard deviation estimate.

[36] The asymptotic properties of these estimator would require a Monte Carlo investigation. We save this task for a dedicated paper.

Instead, in order to estimate *simultaneously* all three OUP parameters in a non linear function like equation (6) it is necessary to adopt GMM or MLE estimation methods, as suggested in (Roncalli, 1998) page 79, (Roncalli, 1999) page 196. The solution of the Ornstein Uhlenbeck process PDE can be explicited with respect to ϵ_t, see expression (18).

$$\epsilon_t = \theta_t - e^{-\eta\, h}\, \theta_{t-1} - \overline{\theta} \cdot \left(1 - e^{-\eta\, h}\right) \tag{18}$$

Imposing moment conditions on ϵ_t, see expression (19), it is possible to estimate OUP parameters $\left[\eta, \overline{\theta}, \sigma_\theta\right]$ using a perfectly identified GMM.

$$\begin{cases} E_{t-1}\left[\epsilon_t\right] &= 0 \\ E_{t-1}\left[\epsilon_t^2 - \sigma_\theta^2 \left(\frac{1-e^{-2\eta\, h}}{2\,\eta}\right)\right] &= 0 \\ E_{t-1}\left[\epsilon_t\, x_{t-1}\right] &= 0 \end{cases} \tag{19}$$

Blatantly enough, moment conditions are constructed taking into account the non-linear relations between the three OUP parameters. Another method that takes fully into account non-linearities in the solution of the OUP partial differential equation is MLE. From expression (18) the expression (20) of the log-likelihood can be derived and maximized on the OUP three unknown parameters:

$$\ell_t = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\left(\sigma^2\left(\frac{1-e^{-2\alpha h}}{2a}\right)\right) - \frac{1}{2}\frac{\epsilon_t^2}{\sigma^2\left(\frac{1-e^{-2ah}}{2a}\right)} \tag{20}$$

Estimation Results

Applying the four econometric methods listed above on the 53 years period, 643 monthly observations, the results reported in table 6 for the linear methods and in table 7 for the non linear methods have been obtained. All four methods give quite similar and significant estimates for all three parameters.

Dry Bulk Dry Bulk ships time charter index seems to have a lower reversion speed that Oil Tanker ships time charter. These estimates are generally very significant, size less than 1%, but for the MLLE and GMM for Dry bulk for which the level of significance is more than 5% and 10% respectively.

Asymptotic standard errors given by (Gourieroux and Jasiak, 2001) allow us to perform a specification test comparing the variances of the residuals and of the OU process itself, see panel C in table 6. These tests indicate that $\sigma_\theta \neq \sigma_\epsilon$ at a high significance level. This allows us to conclude that the two time series DGP is very well described by an arithmetic Ornstein Uhlenbeck process.

This result confirm previous conjectures by other authors that do not prove them empirically. For instance (Tvedt, 2000) in (Brennan and Trigeorgis, 2000) who concludes that while demand for shipping follows a GBM, the freight rate follows an Ornstein Uhlenbeck process although his specification is different from ours; moreover, (Bjerksund and Steinar, 1995) in (Trigeorgis, 1995) uses an arithmetic OUP in order to derive the value of several shipping contracts. On the other hand our result contradicts the empirical hypotheses of another strand of literature. For instance, (Dixit and Pindyck, 1994) on page 237 set up a case study in ship valuation base on a driftless GBM generating gross revenue. In the same way (Goncalves de Oliveira, 1999) bases its model on a GBM with drift. Both these models have been heavily conditioned by the choice of continuous time/simbolic stochastic calculus framework which does not provide very tractable general solutions for a OU process. Because of this they preferred to adapt reality to their method.

A.Ornstein Uhlenbeck Parameters for Dry Bulk ships time charter A.1 Regression Estimates

Parameters	Dixit Pindyck		Gourieroux Jasiak	
b	64.65901	-0.01044	64.65901	0.98956
sd	41.57789	0.00572	41.57789	0.00572
t	1.55513	-1.82397	1.55513	172.93280
Asy s.e.				0.00569

A.2 Parameters Estimates

Method Parameters	Dixit Pindyck Estimates	Gourieroux Jasiak Estimates	(Asy s.e.)	[Asy z H_0: = 0]
$\overline{\theta}$	6195.062	6462.879	(131.537310)	[49.133427]
η	0.125904	0.125904	(0.021119)	[5.9615585]
σ_ϵ	480.272	480.272	(113.465)	[4.2327855]
σ_θ	1672.445	1672.445	(331.879)	[5.0393301]

B. Ornstein Uhlenbeck Parameters for Oil Tanker ships time charter B.1 Regression Estimates:

Parameters	Dixit Pindyck		Gourieroux Jasiak	
b	02.50112	-0.07827	02.50112	0.92173
sd	0.56663	0.01527	0.56663	0.01527
t	4.41406	-5.12462	4.41406	60.35319
Asy s.e.				0.01531

B.2 Parameters Estimates:

Est Parameters	Dixit Pindyck Estimates	Gourieroux Jasiak Estimates	(Asy s.e.)	[Asy z H_0: = 0]
$\overline{\theta}$	31.957	32.290	(0.717062)	[45.030313]
η	0.977970	0.977970	(0.097240)	[10.057234]
σ_ϵ	7.046	7.046	(1.665)	[4.2327855]
σ_θ	25.410	25.410	(5.044)	[5.0380435]

C. Test of the specification [Asy z H_0: $= \sigma_\epsilon = \sigma_\theta$]:

$z < Z$	DRY	OIL
$\frac{(\sigma_\theta - \sigma_\epsilon)}{s.e._{\sigma_\theta}}$	[3.5921991]	[3.6409848]
$\frac{(\sigma_\epsilon - \sigma_\theta)}{s.e._{\sigma_\epsilon}}$	[-10.507002]	[-11.031396]

Table 6: Linear Estimates for Ornstein - Uhlenbeck Parameters

Legend: (Dixit and Pindyck, 1994) method is based on the following regression

$$\begin{aligned}\theta_t - \theta_{t-1} &= \overline{\theta} \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + \left(e^{-\eta \cdot \Delta t} - 1\right) \cdot \theta_{t-1} + \epsilon_t \\ \theta_t - \theta_{t-1} &= \widehat{\alpha} + \widehat{\beta} \cdot \theta_{t-1} + \epsilon_t\end{aligned}$$

from which it is easy to derive point estimates of the OUP parameters

$$\overline{\theta} = -\frac{\widehat{\alpha}}{\widehat{\beta}} \qquad \eta = -\frac{\ln\left(1+\widehat{\beta}\right)}{\Delta t} \qquad \widehat{\sigma} = \widehat{\sigma}_\epsilon \cdot \sqrt{\frac{2 \cdot \eta}{1 - e^{-2 \cdot \eta \Delta t}}}$$

(Gourieroux and Jasiak, 2001) method is a variant of the (Dixit and Pindyck, 1994) method based on the following regression

$$\begin{aligned}\theta_t &= \overline{\theta} \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + e^{-\eta \cdot \Delta t} \cdot \theta_{t-1} + \epsilon_t \\ \theta_t &= \overline{\theta} \cdot \left(1 - \widehat{\beta}\right) + \widehat{\beta} \cdot \theta_{t-1} + \epsilon_t\end{aligned}$$

where estimates and their asymptotic variances are given as follows

$$\overline{\theta} = \frac{1}{T} \cdot \sum_{t=1}^{T} \theta_t \qquad \widehat{\beta} = \frac{Cov\left(\theta_t, \theta_{t-1}\right)}{Var(\theta_t)} \qquad \widehat{\sigma}_\epsilon^2 = \frac{1}{T} \cdot \sum_{t=1}^{T} \epsilon_t^2$$

$$V_{asy}(\overline{\theta}) = \frac{\widehat{\sigma}_\epsilon^2}{T \cdot \left(1 - \widehat{\beta}\right)} \qquad V_{asy}(\widehat{\beta}) = \frac{1}{T} \cdot \left(1 - \widehat{\beta}^2\right) \qquad V_{asy}(\widehat{\sigma}_\epsilon^2) = \frac{2 \cdot \widehat{\sigma}_\epsilon^4}{T}$$

where $\epsilon_t = \theta_t - \overline{\theta} - \widehat{\beta} \cdot \left(\theta_{t-1} - \overline{\theta}\right)$ The remaining two parameters of the OUP, namely the reversion speed and the volatility of the process together with their asymptotic variances are derived through the δ method.

A) MLL estimates:

	DRY			OIL		
MLLE	η	$\overline{\theta}$	σ_ϵ	η	$\overline{\theta}$	σ_θ
estimates	0.128193	6728.7175	1671.1405	0.964414	32.655753	25.390472
std.err.	0.069053	1789.1332	46.880135	0.198357	3.602852	0.737393
t-statistic	1.856455	3.760881	35.647093	4.862021	9.063861	34.432765
p-value	0.063849	0.000185	0.	0.000001	0.	0.
Value of the maximized						
LogLikelihood Function			-668.53732			-2164.31778
Total observations			642			642
Usable observations			642			642
Number of parameters to be estimated			3			3
Degrees of freedom			639			639

B) GMM estimates:

	DRY			OIL		
GMM	η	$\overline{\theta}$	σ_ϵ	η	$\overline{\theta}$	σ_θ
estimates	0.128184	6729.1461	1671.1472	0.964426	32.655069	25.390436
std.err.	0.083463	1900.1472	77.425405	0.309515	3.675637	1.522903
t-statistic	1.535818	3.541382	21.583965	3.115932	8.884192	16.672391
p-value	0.125078	0.000427	0.	0.001916	0.	0.
Value of the minimized						
Criterion Function			0			0
Total observations			642			642
Usable observations			642			642
Number of parameters to be estimated			3			3
Degrees of freedom			639			639
Number of moment conditions			3			3

Table 7: Non Linear Estimates for Ornstein - Uhlenbeck Parameters

Legend: MLL and GMM estimates have been computed from the following discrete time equation

$$\epsilon_t = \theta_t - e^{-\eta\, h}\,\theta_{t-1} - \overline{\theta}\cdot\left(1 - e^{-\eta\, h}\right)$$

imposing, respectively the minimization of the following log-likelihood for MLLE

$$\ell_t = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\left(\sigma^2\left(\frac{1-e^{-2\alpha h}}{2a}\right)\right) - \frac{1}{2}\frac{\epsilon_t^2}{\sigma^2\left(\frac{1-e^{-2ah}}{2a}\right)}$$

and the following moment condition for GMM estimates

$$\begin{cases} E_{t-1}\left[\epsilon_t\right] = 0 \\ E_{t-1}\left[\epsilon_t^2 - \sigma_\theta^2\left(\frac{1-e^{-2\eta\, h}}{2\,\eta}\right)\right] = 0 \\ E_{t-1}\left[\epsilon_t\, x_{t-1}\right] = 0 \end{cases}$$

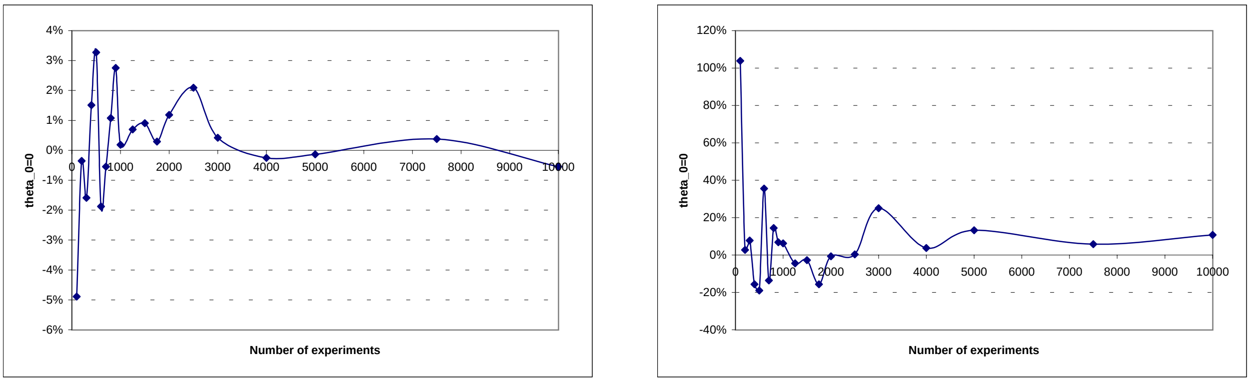
C Convergence Tests

The gist of the present paper stays in the convergence of the expected Running Present Value in the DP procedure and the average of the Markov chains Montecarlo simulations. This fact guarantees that these two procedures model the same dinamic optimal behaviour although the former is based on backward induction while the latter is based on a forward computation of the expanded net present value.

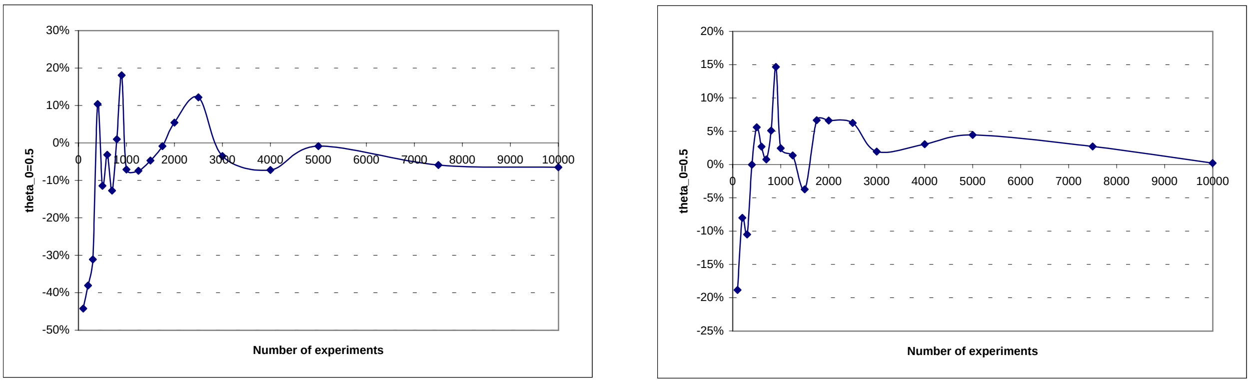
Therefore, this appendix reports some results about convergence of expected value of Markov chains Montecarlo simulations towards the values produced by the Dynamic Programming procedure. We have checked both the results based on active and passive management. Moreover, we have checked whether the grid discretization has some influence on convergence choosing a level in the middle of the discretized state variable space $\theta = .5$ together with the minimum and the maximum $\theta = 0$ and $\theta = 1$.

To test convergence we have computed expected values of the Markov chains Montecarlo simulations for several number of experiments n, namely 100-1,000 (step 100), 1000-2000 (step 250), 2,000-3000 (step 500), 3000-5000 (step 1000), 5000-10,000 (step 2500). Changing the step allowed us to test the variability of the results in different neighborhoods of n. Because of this experiments have a higher degree of granularity in the hundreds while this decreases as the number of experiments increases. This experiment design was choosen to show that results variability decreases as the number of experiments increases. We have consideres the relative difference between the averages as computed above and the corresponding RPV.

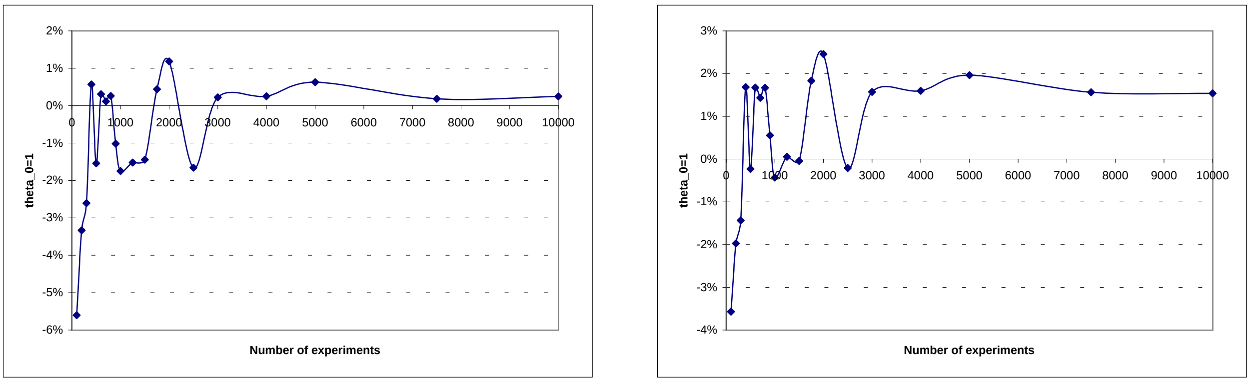
Results reported in figure 6 show that averages of Montecarlo simulations converge to the corresponding RPVs. This is true for both the active and passive management expanded present values. Convergence seems to be quite fast. As a matter of fact results become quite stable for 5000 simulations experiments. Reminding the economic intuition of these values and the high transaction costs to buy and sell investment projects like those valued in this paper, relative differences in the order of 1-2% can be considered quite good.



Panel A: Convergence of results on $\theta_{t=0} = 0$.



Panel B: Convergence of results on $\theta_{t=0} = .5$.



Panel C: Convergence of results on $\theta_{t=0} = 1$.

Figure 6: Convergence results of Montecarlo towards RPV Results

Legend: The graphs report convergence results for three different relevant initial values of the Ornstein Uhlenbeck process as the number of experiments increases. Left column reports results for the Passive Management RPVs while right column reports results for the Active Management RPVs. The dashed line represents the difference between the average of the experiments and the the Running Present Value at time $t = 0$ in the DP procedure divided by the latter.