Strategic entry and market leadership in a two–player real options game

Mark B. Shackleton[*]   Andrianos E. Tsekrekos[†]

Rafał Wojakowski[‡]

February 1, 2002

Abstract

We analyse the entry decisions of competing firms in a two–player stochastic real option game, when rivals can exert different but correlated uncertain profitabilities from operating. In the presence of entry costs, decision thresholds exhibit hysteresis, the range of which is decreasing in the correlation between competing firms. The expected time of each firm being active in the market and the probability of both rivals entering in finite time are explicitly calculated. The former (latter) is found to decrease (increase) with the volatility of relative firm profitabilities implying that market leadership is shorter–lived the more uncertain the industry environment. In an application of the model to the aircraft industry, we find that Boeing's optimal response to Airbus' launch of the A3XX super carrier is to accommodate entry and supplement its current product line, as opposed to the riskier alternative of committing to the development of a corresponding super jumbo.

---

[*]Department of Accounting & Finance, Lancaster University.

[†]Corresponding author, Department of Accounting & Finance, Lancaster University, LA1 4YX, United Kingdom. Phone: +44 1524 593151. Fax: +44 1524 847321. e-mail: a.tsekrekos@lancaster.ac.uk.

[‡]Department of Accounting & Finance, Lancaster University.

1   Introduction

When a firm has the opportunity to invest under conditions of uncertainty and irreversibility (partial or complete) there is an option value of delay. By analogy to financial options, it might be optimal to delay exercising the option, (proceeding with the investment), even when it would be profitable to do so now, due to the hope of gaining a higher payoff in the future as uncertainty is resolved.

This insight, first applied to the analysis of natural resource extraction by Brennan and Schwartz [2], improves upon traditional investment appraisal approaches (decision trees or NPV-based criteria) by allowing the value of delay and the importance of flexibility to be incorporated into the assessment.[1] Subsequently, a substantial number of papers have explored this idea. Among others, McDonald and Siegel [15], [16] and Dixit [3] price option values associated with entry and exit from a productive capacity while Pindyck [20] values the operating flexibility that arises from capacity utilisation. Paddock, Siegel and Smith [19] concentrate on valuing offshore petroleum leases and Pindyck [21] values projects where different sources of cost uncertainty are involved. Majd and Pindyck [14] value projects with a substantial time–to–build element while Dixit and Pindyck [4] provide a survey of the literature as a whole.

However, real investment opportunities, unlike financial options, are rarely held by a single firm in isolation. Most investment projects (in one form or another) are open to competing firms in the same industry or line of business, subject of course to the core competencies of each firm. In some extreme cases, competing firms will be equally capable of undertaking the same project or investing in a new market. In such cases, the timing of the investment becomes a key strategic consideration which has to be optimised by taking into account the competitor's optimal responses.

In this paper we aim to elucidate the factors driving strategic entry decisions by competing firms. We do so by considering optimal entry strategies in a *two–firm, infinite–horizon stochastic game.* In a market that can only ac-

---

[1]The reconciliation of traditional investment appraisal techniques and real option valuation paradigms is an area of active research. For some recent results, see Teisberg [26] and Kasanen and Trigeorgis [12].

commodate one active firm, the idle rival has the option to claim the market by sinking an unrecoverable investment cost. Its optimal exercise decision has to incorporate the possibility that in the future, the rival firm could reclaim the market again by exercising a corresponding entry option, if optimal to do so.

What complicates the problem is that we allow competing firms to have different operating capabilities. Namely the operating profitabilities that the rival firms can exert from operating the market are assumed to follow different (but possibly correlated) diffusion processes. Each firm's optimal entry strategy, by incorporating its rival's optimal response, will ultimately depend on the parameters of both diffusions. To our knowledge, the task of solving a *two–state–variable strategic game* has not be undertaken before in the real options literature. We are able to do so by, first transforming the game into an *one–agent optimisation* problem and secondly, by exploiting the *homogeneity* of the setting, to reduce its dimensionality.

We find that the presence of fixed entry costs produces *hysteresis* in the entry decisions of competing firms in the spirit of Dixit [3].[2] In our setting this means that if the conditions that urged one firm to exercise and claim the market from its active rival completely reverse, we should not expect to see an immediate corresponding exercise strategy from the other firm. Market conditions have to change further until the rival firm finds it optimal to exercise its entry option. This hysteresis effect is found to be positively related to investment entry costs and uncertainty, but negatively related to the correlation of rival firms operations. As fixed costs are eliminated from the model, hysteresis disappears, and optimal exercise strategies of are shown to collapse to a simple, yet intuitive, current yield or profitability criterion.

We also calculate explicitly the expected and median time that each firm will spend as active in the market, as well as the probability that both competing firms will become active within a finite time horizon and we explore their dependence on uncertainty and investment costs. Interestingly, higher volatility in the market is shown to imply lower expected active times for each competing firm. The probability that both rivals will become active in a finite horizon strictly increases with volatility, implying that the more uncertain

---

[2]Hysteresis is defined as the delay in an effect reversing itself when the underlying cause is reversed.

the market environment, the more short–lived is market leadership. Highly volatile industries (e.g. biochemical, pharmaceutical) seem to conform to the model's predictions. Using market data, we then apply the model to the aircraft manufacturing industry. Our interest in this market originates from the fact that it is mainly a two–firm industry, with long periods of market domination by one of the rivals, large irreversible investment decisions, high profitability uncertainty and direct strategic competition. Moreover, this industry is currently undergoing major changes after the decision of one firm to capture the market segment for the very large aircrafts by making a huge investment. Our application in section 5 assesses the optimal rival reaction.

Turning to the relation of the paper to some recent literature on the theme, optimal real option exercise policies under duopolistic strategic competition has been the focus of work by Smets [23], Grenadier [7] and Lambrecht and Perraudin [13]. Unlike their work, we allow each competing firm's decision to be subject to a firm–specific stochastic variable, as well as its competitor's, thus making the game two–dimensional and more difficult to tackle. Like Lambrecht and Perraudin [13] who deal with the biochemical industry, our model can be applied in a real business market. Finally, our paper also draws from the seminal papers by McDonald and Siegel [16] and Dixit [3], both of which are in a single–firm context.

The rest of the paper is organised as follows: Section 2 describes the basic two–firm stochastic game and extensively presents the concepts that allow it to be transformed to an one–agent, one–variable optimisation problem much easier to tackle. Section 3 solves the problem in order to determine the equilibrium optimal strategies of rivals, while section 4 presents probability calculations and examines the sensitivity of the results to key model parameters. Section 5 applies the model to the aircraft manufacturing industry while the last section concludes.

2 The model

2.1 The basic setting

We study a market which is only big enough for one firm (a natural monopoly) and two competing firms, i and j, that can potentially monopolise it sequen-

4

tially. Each firm has its own production technology and thus can command a different operating profitability from being the active firm in the market. Let S_{it}, S_{jt} stand for the net operating profitability flow that each firm can command in the market at time t, and assume that

$$\frac{dS_{it}}{S_{it}} = (\mu_i - \delta_i)\, dt + \sigma_i dZ_{it}$$
$$\frac{dS_{jt}}{S_{jt}} = (\mu_j - \delta_j)\, dt + \sigma_j dZ_{jt} \tag{1}$$

where $\mu_{i,j}$, $\delta_{i,j}$ and $\sigma_{i,j} > 0$ are constants. The future operating profitability of each firm is uncertain and exposed to exogenous shocks, specified as the increments of standard Brownian motions dZ_{it}, dZ_{jt}. These shocks can either be firm–specific (e.g. an improvement in enterpreneurial skills, a cost–reducing innovation in one firm's production technology) or industry–wide (e.g. an unexpected shift in market demand or a change in customers' tastes etc.). To reflect the possibility of common and firm–specific economic factors driving the profitability of competing firms, we allow S_i, S_j to be exposed to correlated Brownian motions, i.e. $dZ_{it}dZ_{jt} = \rho dt$ where ρ is the increment correlation coefficient, assumed constant. Firms i, j, if idle, can enter the market by investing a fixed cost of K_i, K_j respectively.

In the absence of any considerations like severe entry barriers, we should expect competition to drive the most "efficient" of the two rivals to dominate and operate the market. We do not address operating efficiency of competing firms *per se* here, but we assume that the net operating profitabilities defined in (1) essentially represent the ability of each firm to operate the market efficiently. Thus if, for example, firm i is currently idle but S_i is higher than S_j, we would expect the underlying economic forces to shift the market from j to i once the latter decides to commit resources, at least in the long–run. In our simple economy, this shift can occur instantaneously.

Because of the uncertainty in future operating profitabilities in (1), no matter which firm is in the market currently, *both firms have a chance of being the market "monopolist" for some non–overlapping period of time* in the future, depending on their operating parameters, namely (μ, δ, σ, K). The major aims of this paper are: (a) to quantify these periods of time, as well as the probability of becoming active, for each rival firm as a function of its parameters, and (b) to study the entry decisions of rivals in this simple

5

two–player game and define optimal strategies.

2.2 Solution technique

In the presence of uncertainty (Equation (1)) and sunk costs (K_i, K_j), it has been stressed in the real options literature that there is an option value of delay. Each firm in our model, when idle, has the option to sink an investment cost and claim the market from its rival. The exercise strategy of a firm would specify the optimal stopping time for sinking this investment cost and claim the benefits from operating. What complicates the problem is that one firm's exercise strategy should take into account that its rival can claim the market back by subsequently exercising its own entry option. In our two–player game, firms' exercise strategies have to be simultaneously determined as part of an optimal equilibrium behaviour.

However, our problem of finding equilibria in our two–firm game can be converted to a much simpler one, thanks to a result in Slade [22]. Slade [22] specifies the conditions under which a general N–player game where agents act strategically is observationally identical to a central planner's optimisation problem. Her intuition is as follows: As in a perfectly competitive industry with a large (possibly infinite) number of agents, each pursuing selfish objectives, the market behaves as if an agent was maximising an objective function (social welfare), an N–player game with strategic agents can be transformed into a "fictitious" objective function, whose maxima are Nash equilibria of the game.[3,4]

In our context, assume the existence of a fictitious central planner who, at any point in time, can instantaneously delegate the market to the most profitable of the two firms. Obviously, the central planner will want firm i (firm j) to be active in the market when S_i is high (low) and/or S_j is low (high). However, every time the planner decides to change the active firm in the market, switching costs $K_{i \to j} \equiv K_j$ or $K_{j \to i} \equiv K_i$ (depending on the direction of the "switch") are incurred. The central planner is risk neu-

---

[3]Slade strengthens her argument by showing that Nash equilibria of the game that are not maxima of the function are generically unstable.

[4]See also Baldursson [1], who applies Slade's [22] result in his study of irreversible investment under uncertainty in oligopoly.

tral and wishes to maximise her expected present value of profits from the market, net of switching costs. By specifying the rules for optimal switching between firms, our planner essentially defines the exercise strategies of competing firms. The planner's optimisation problem is one of stochastic dynamic programming and we will develop the solution using the option pricing analogy.

Without loss of generality, let firm j be currently the active firm in the market and define $F_{j \to i}(S_i, S_j)$ as the planner's option to cease firm j operating, and to activate firm i in the market. Over the range of states $\Im_j \equiv \{S_i, S_j : j \text{ active}\}$ the asset of the switching opportunity must be willingly held. The return of this asset comprises of (a) the expected capital gain, $\frac{E[dF_{j \to i}(S_i, S_j)]}{dt}$, as the value of $F_{j \to i}(S_i, S_j)$ changes with S_i, S_j plus (b) a dividend, S_j, the flow of operating profit from firm j which is currently active. The total return must then equal the normal return, i.e.

$$E\left[dF_{j \to i}(S_i, S_j)\right] + S_j dt = r F_{j \to i}(S_i, S_j) dt$$

where r is the riskless interest rate, assumed constant

By applying Ito's lemma to calculate the expectation and in the light of (1), this asset equilibrium condition becomes a second–order *partial differential equation* that $F_{j \to i}(S_i, S_j)$ must satisfy

$$\frac{1}{2}\left(\sigma_i^2 S_i^2 \frac{\partial^2 F_{j \to i}(S_i, S_j)}{\partial S_i^2} + 2\rho\sigma_i\sigma_j S_i S_j \frac{\partial^2 F_{j \to i}(S_i, S_j)}{\partial S_i \partial S_j} + \sigma_j^2 S_j^2 \frac{F_{j \to i}(S_i, S_j)}{\partial S_j^2}\right)$$

$$+(r-\delta_i)S_i\frac{\partial F_{j \to i}(S_i, S_j)}{\partial S_i} + (r-\delta_j)S_j\frac{\partial F_{j \to i}(S_i, S_j)}{\partial S_j} - r F_{j \to i}(S_i, S_j) + S_j = 0$$

$$(2)$$

Correspondingly, over the range of states $\Im_i$ where it is optimal for firm i to be active in the market, the planner's option to exchange firm i with firm j in the market, $F_{i \to j}(S_i, S_j)$, will satisfy

$$\frac{1}{2}\left(\sigma_i^2 S_i^2 \frac{\partial^2 F_{i \to j}(S_i, S_j)}{\partial S_i^2} + 2\rho\sigma_i\sigma_j S_i S_j \frac{\partial^2 F_{i \to j}(S_i, S_j)}{\partial S_i \partial S_j} + \sigma_j^2 S_j^2 \frac{F_{i \to j}(S_i, S_j)}{\partial S_j^2}\right)$$

$$+(r-\delta_i)S_i\frac{\partial F_{i \to j}(S_i, S_j)}{\partial S_i} + (r-\delta_j)S_j\frac{\partial F_{i \to j}(S_i, S_j)}{\partial S_j} - r F_{i \to j}(S_i, S_j) + S_i = 0$$

$$(3)$$

7

2.3 Reducing the problem's dimensionality

Identifying the boundaries between regions $\Im_i$, $\Im_j$ essentially determines the optimal switching policy for our central planner. However, the theory of partial differential equations has little to say about such "free boundary" problems in general. Fortunately, in the present setting, the natural homogeneity of the problem allows us to reduce it to one dimension. The intuition, which is due to McDonald and Siegel [16], is that in the current setting, our central planner is not concerned about the *absolute magnitudes* of S_i, S_j. For determining the optimal switching policy, the planner *only* needs to be concerned about some measure of the *relative magnitudes* of the two firms' profitabilities. A properly defined measure will allow reduction of the dimensionality of the problem.

Letting $P \equiv \frac{S_i}{S_j}$ stand for the relative net operating profitability of competing firms and in the region $\Im_j$, write

$$F_{j \to i}(S_i, S_j) = S_j f_{j \to i}\left(\frac{S_i}{S_j}\right) = S_j f_{j \to i}(P) \tag{4}$$

where $f_{j \to i}$ is now the function to be determined. It is easy to see that a relationship like (4) can be sustained: if S_i, S_j both double in value, the boundary that determines optimal $\Im_j \to \Im_i$ switching will not change, thus the switching option $F_{j \to i}$ should be homogenous of degree 1 in (S_i, S_j). Apply successive differentiation to (4) and substitution into (2) we get

$$\frac{1}{2}\nu^2 P^2 f''_{j \to i}(P) + (\delta_j - \delta_i)P f'_{j \to i}(P) - \delta_j f_{j \to i}(P) + 1 = 0 \tag{5}$$

where $\nu^2 \equiv \sigma_i^2 - 2\rho\sigma_i\sigma_j + \sigma_j^2$ (see Appendix).

Equation (5) is an *ordinary differential equation* for the unknown function $f_{j \to i}(P)$ of the scalar independent variable P. It has to be solved subject to the boundary condition

$$\lim_{P \to 0_+} f_{j \to i}(P) = \frac{1}{\delta_j} \tag{6}$$

which essentially says that when P tends to zero ($S_i \downarrow$ and/or $S_j \uparrow$), our planner will never want to exchange active firm j for i and thus her switching option $f_{j \to i}$ should be worthless. Thus her expected present value will be

8

equal to the present value of firm j operating perpetually.[5]

Solving (5) subject to (6) yields

$$f_{j \to i}(P) = AP^a + \frac{1}{\delta_j} \tag{7}$$

where A is a constant to be determined, and a is the positive root of the characteristic quadratic function.[6]

Following similar steps for region $\Im_i$, equation (3) reduces to

$$\frac{1}{2}\nu^2 P^2 f''_{i \to j}(P) + (\delta_j - \delta_i)P f'_{i \to j}(P) - \delta_j f_{i \to j}(P) + P = 0 \tag{8}$$

subject to

$$\lim_{P \to +\infty} f_{i \to j}(P) = \frac{P}{\delta_i} \tag{9}$$

Boundary condition (9) says that with firm i active, as P gets extremely high ($S_i \uparrow$ and/or $S_j \downarrow$), the switching option $f_{i \to j}$ will be worthless and the planner will be receiving S_i perpetually (see footnote 5). The solution of (8) subject to (9) is of the form

$$f_{i \to j}(P) = BP^b + \frac{P}{\delta_i} \tag{10}$$

with B constant and b as in footnote 6.

Finally, for future reference in the paper and before proceeding to the solution of the planner's problem, one can show that the relative profitability variable P will follow

$$\frac{dP_t}{P_t} = \left(\xi + \frac{1}{2}\nu^2 \right) dt + \nu d\widetilde{W}_t \tag{11}$$

where ν as before, $\xi \equiv \mu_i - \mu_j + \delta_j - \delta_i - \frac{1}{2}\left(\sigma_i^2 - \sigma_j^2\right)$ and a standard Wiener process

$$\widetilde{W}_t = \frac{1}{\nu}\left(\sigma_i Z_{it} - \sigma_j Z_{jt}\right)$$

---

[5]Note that S_j has been factored out of $F_{j \to i}$ in the definition of $f_{j \to i}$ in (4), thus the right–hand side of boundary condition (6) should be read as $\frac{S_j}{\delta_j}$.

[6]The characteristic quadratic function $\frac{1}{2}\nu^2 x(x-1) + (\delta_j - \delta_i)x - \delta_j = 0$ has roots $a > 1$ and $b < 0$ given by

$$a, b = \frac{1}{2} - \frac{\delta_j - \delta_i}{\nu^2} \pm \sqrt{\left(\frac{\delta_j - \delta_i}{\nu^2} - \frac{1}{2}\right)^2 + \frac{2\delta_j}{\nu^2}}$$

3 Solving for the optimal switching policy

Once the planner problem is reduced to one dimension, optimal policies can be determined. These optimal switching policies, $j \to i$ and $i \to j$, are determined in the form of two time independent values of the state variable P, one an upper $\overline{P}$ *and* one a lower threshold $\underline{P}$, such that

$$\Im_i = \left\{ S_i, S_j : P \geq \overline{P} \right\} \quad \text{If } j \text{ active, switch to } i$$

$$\Im_j = \left\{ S_i, S_j : P \leq \underline{P} \right\} \quad \text{If } i \text{ active, switch to } j$$

$$\overline{P} > 1 > \underline{P} \tag{12}$$

Of course the values of $\overline{P}, \underline{P}$ have to be determined endogenously through optimality conditions that apply at the thresholds. The separation of switching thresholds in (12) is similar to the entry/exit model of Dixit [3]. To understand this in our setting, suppose firm j is currently active and $P = 1$ (both firms equally profitable). Our central planner would seem indifferent to which firm operates the market and could decide to activate firm i and cease j from operating. However, since the switching decision entails a fixed cost ($K_{j \to i}$ in this case), our planner will optimally delay this decision until $P = \overline{P} > 1$ is reached, i.e. until the option $f_{j \to i}$ is sufficiently "in–the–money". In other words, firm i does not find it optimal to commit the investment cost until its profitability is sufficiently higher than j's, so that a substantial active time period can be expected. Correspondingly, when i is active, the planner will optimally wait until P gets sufficiently below unity, so as to be optimal to sink $K_{i \to j}$ and delegate the market to firm j. This separation of thresholds for optimal $j \to i$ and $i \to j$ exchange is the manifestation of *hysteresis* (see Dixit [3]).

3.1 Hysteresis solution in four equations

The optimal policy is determined by two *value–matching*

$$A\overline{P}^a + \frac{1}{\delta_j} + K_{j \to i} = B\overline{P}^b + \frac{\overline{P}}{\delta_i}$$

(13)

$$A\underline{P}^a + \frac{1}{\delta_j} - K_{i \to j} = B\underline{P}^b + \frac{\underline{P}}{\delta_i}$$

and two high–order contact or *smooth–pasting* conditions[7]

$$Aa\overline{P}^a = Bb\overline{P}^b + \frac{\overline{P}}{\delta_i}$$

(14)

$$Aa\underline{P}^a = Bb\underline{P}^b + \frac{\underline{P}}{\delta_i}$$

that apply at the optimal boundaries, $\overline{P}, \underline{P}$, which we want to determine.

Equation (13) states what happens when the optimal boundaries are reached. When P gets sufficiently high (at the optimal level $\overline{P}$ to be determined, where $S_i \gg S_j$), our central planner exercises the (call) option to activate firm i ($A\overline{P}^a$) by ceasing firm j and incurring the switching cost ($K_{j \to i}$). In return, she gets an active firm i and the (put) option to reactivate firm j ($B\overline{P}^b$) if profitability conditions change in the future. At $\underline{P}$ ($S_i \ll S_j$) this put option is exercised, and firm's i value and the switching cost $K_{i \to j}$ are exchanged for the value of firm j and the (call) option on firm i. Equation (14) states that for $\overline{P}, \underline{P}$ to be the optimal switching policy thresholds, not only the values in (13) but also their first derivatives must meet smoothly. The intuition is that if a kink arose at $\overline{P}, \underline{P}$, perturbations in the supposedly optimal policy would make the planner better off and thus the thresholds would not be optimal.[8]

Equations (13)–(14) constitute a system that uniquely determines A, B, $\overline{P}$ and $\underline{P}$, thus completing the solution. The equations are non–linear in $\overline{P}$

---

[7]See Dumas [5] for a rigorous treatment of these conditions and Merton [17, p.171, n.60] for a discussion in the option pricing problem. Dumas and Luciano [6] also discuss these in another two sided transaction cost control problem.

[8]Note that the smooth–pasting conditions have been made homogenous by multiplying through with $\overline{P}$, $\underline{P}$.

and $\underline{P}$ and thus can only be evaluated numerically. In the next subsection we show that this system can be reduced to one, easily evaluated equation that determines the whole system and can summarise the optimal policy in one parameter.

3.2 Hysteresis solution in one equation

For reducing the system, it will be convenient to set

$$\overline{X} = \frac{1}{\delta_j} + K_{j \to i}$$

$$\underline{X} = \frac{1}{\delta_j} - K_{i \to j}$$

as the costs associated with the upper and lower switching thresholds and write the system (13)–(14) in matrix form with $\left(A, B, \overline{X}, \underline{X}\right)$ as a function of $\left(\overline{P}, \underline{P}\right)$

$$
\begin{aligned}
A\overline{P}^a + \overline{X} &= B\overline{P}^b + \frac{\overline{P}}{\delta_i} \\
A\underline{P}^a + \underline{X} &= B\underline{P}^b + \frac{\underline{P}}{\delta_i} \\
Aa\overline{P}^a &= Bb\overline{P}^b + \frac{\overline{P}}{\delta_i} \\
Aa\underline{P}^a &= Bb\underline{P}^b + \frac{\underline{P}}{\delta_i}
\end{aligned}
\iff
\begin{bmatrix}
1 & 0 & \overline{P}^a & -\overline{P}^b \\
0 & 1 & \underline{P}^a & -\underline{P}^b \\
0 & 0 & a\overline{P}^a & -b\overline{P}^b \\
0 & 0 & a\underline{P}^a & -b\underline{P}^b
\end{bmatrix}
\begin{bmatrix}
\overline{X} \\
\underline{X} \\
A \\
B
\end{bmatrix}
= \frac{1}{\delta_i}
\begin{bmatrix}
\overline{P} \\
\underline{P} \\
\overline{P} \\
\underline{P}
\end{bmatrix}
$$

Inverting the matrix to recover the vector $\left(A, B, \overline{X}, \underline{X}\right)$, the matrix product is most easily evaluated as a function of the fraction $\gamma \equiv \frac{\underline{P}}{\overline{P}}$ (a time independent ratio of the lower to the upper switching thresholds)

$$
\begin{bmatrix}
\overline{X} \\
\underline{X} \\
A \\
B
\end{bmatrix}
= \frac{\overline{P}}{ab\delta_i(\gamma^b - \gamma^a)}
\begin{bmatrix}
ab(\gamma^b - \gamma^a) - b\gamma^b + a\gamma^a - (a-b)\gamma \\
ab(\gamma^{b+1} - \gamma^{a+1}) + b\gamma^{a+1} - a\gamma^{b+1} + (a-b)\gamma^{a+b} \\
\frac{b\delta_i^a\left(\gamma^b - \gamma\right)}{\overline{P}^a} \\
\frac{a\delta_i^a\left(\gamma^a - \gamma\right)}{\overline{P}^b}
\end{bmatrix}
\tag{15}
$$

Define the variable $\pi \equiv \frac{\underline{X}}{\overline{X}}$ (the ratio of switching costs). This variable can be expressed as a function of γ by dividing the first two lines of the matrices in (15)

$$\pi(\gamma) = \frac{ab(\gamma^{b+1} - \gamma^{a+1}) + b\gamma^{a+1} - a\gamma^{b+1} + (a-b)\gamma^{a+b}}{ab(\gamma^b - \gamma^a) - b\gamma^b + a\gamma^a - (a-b)\gamma} \tag{16}$$

This is the one equation that determines the entire hysteresis system. It would be preferable to determine $\gamma(\pi)$ (the ratio of thresholds as a function of the switching cost ratio) as oppose to $\pi(\gamma)$ but a numerical solution for the inverse is easy to obtain for any particular values. This polynomial $\pi(\gamma)$ is monotonic and increasing in γ and therefore there is a unique π for every γ and vice versa. For every choice of π, once the optimal γ is numerically retrieved from (16), the thresholds and option constants can be recovered by substituting in the following equations

$$
\begin{bmatrix} \overline{P} \\ \underline{P} \end{bmatrix} = ab(\gamma^b - \gamma^a) \begin{bmatrix} \frac{\overline{X}}{ab(\gamma^b-\gamma^a)-b\gamma^b+a\gamma^a-(a-b)\gamma} \\ \frac{\underline{X}}{ab(\gamma^b-\gamma^a)+b\gamma^a-a\gamma^b+(a-b)\gamma^{a+b-1}} \end{bmatrix}
$$

$$
\begin{bmatrix} A \\ B \end{bmatrix} = \frac{\overline{P}}{ab\delta_i(\gamma^b-\gamma^a)} \begin{bmatrix} \frac{b\delta_i^a(\gamma^b-\gamma)}{\overline{P}^a} \\ \frac{a\delta_i^a(\gamma^a-\gamma)}{\overline{P}^b} \end{bmatrix}
$$

(17)

The above completes the solution by specifying the optimal switching policy for our fictitious central planner. The optimal thresholds at which our planner decides to exchange j for i $(\overline{P})$ and i for j $(\underline{P})$ are essentially the relative optimal exercise strategies of our two competing firms. In the next section we examine some of the properties of the optimal exercise policies analytically. Moreover, some interesting calculations concerning the expected active time of each rival, and the probability that one of them will be the active firm in the market are presented.

4   Results

4.1   The effects of uncertainty and investment costs

For any P in the hysteresis range $(\overline{P}, \underline{P})$, the market continues with the status quo (active firm remains the same) and that sunk costs are crucial for this inertia zone to arise.[9] We thus first turn to examine the effect of such costs.

---

[9]In this inertia zone $(\overline{P}, \underline{P})$, which firm will actually be active is history–dependent. It depends on which of the thresholds $\overline{P}, \underline{P}$ has been reached last.

It is easy to see that

$$\lim_{K_{i \to j} = K_{j \to i} \to 0} \pi\left(\gamma\right) = 1 \Leftrightarrow \gamma = 1$$

the optimal exercise thresholds collapse to a common value $\widehat{P}$ and hysteresis disappears. Using l'Hopital's rule we can evaluate this value to be

$$\widehat{P} = \frac{ab}{ab - (a + b) + 1} = \frac{\delta_2}{\delta_1} \Leftrightarrow \delta_1 S_1 = \delta_2 S_2 \qquad (18)$$

where the second equality comes from the definition of P.

Although not obvious at the outset, this result is not surprising. It means that in the absence of fixed costs, the exercise strategy for a firm is to enter whenever it has a higher profit flow than its competitor. In other words, our fictitious planner would activate an idle firm whenever the *opportunity cost* of keeping it inactive exceeds the *profitability yield* of the firm currently in the market.

Moreover, for any positive level of uncertainty, if $K_{i \to j} \to 0$, we have $\frac{d\overline{P}}{dK_{i \to j}} \to \infty$ and $\frac{d\underline{P}}{dK_{i \to j}} \to -\infty$. In other words, in the presence of uncertainty, hysteresis emerges very rapidly even for small sunk costs. Similarly when $K_{j \to i} \to 0$. This is visually confirmed by figure 1 which plots the optimal exercise thresholds as a function of sunk costs for symmetric rivals ($\mu_i = \mu_j$, $\delta_i = \delta_j$, $\sigma_i = \sigma_j$, $K_{j \to i} = K_{i \to j}$). In accordance with intuition, firms are more reluctant to enter (hysteresis increases) in the presence of higher investment costs K.

Next, turn to the effect of uncertainty. If $\sigma_i, \sigma_j \to 0$, equation (13) becomes

$$\overline{P} = \delta_i \overline{X}$$
$$\underline{P} = \delta_i \underline{X}$$

and the effect of hysteresis is only due to the sunk investment costs. It is easy to verify that for $\sigma_i, \sigma_j > 0$

$$\overline{P} > \delta_i \overline{X}$$
$$\underline{P} < \delta_i \underline{X}$$

i.e. the zone of inaction widens with uncertainty. However even for economically identical competing firms ($\mu_i = \mu_j$, $\delta_i = \delta_j$, $\sigma_i = \sigma_j = \sigma$, $K_{j \to i} = K_{i \to j}$)

14

the effect of uncertainty on the optimal thresholds is not symmetric. For any level of correlation, it can be shown that

$$\frac{d\gamma}{d\sigma} < 0 \Leftrightarrow \frac{d\underline{P}}{d\sigma} < 0 < \frac{d\overline{P}}{d\sigma}$$

the upper threshold is more sensitive to volatility than the lower threshold.

Figure 2 plots the exercise thresholds as a function of volatility for symmetric rival firms ($\delta_i = \delta_j$, $\sigma_i = \sigma_j$, $K_{j \to i} = K_{i \to j}$). The more volatile the profitabilities of competing firms, the higher the distance between the optimal exercise thresholds $(\overline{P}, \underline{P})$. The correlation of changes in rival's profitabilities also has an effect which is also apparent in figure 2. $\overline{P}$ ($\underline{P}$) is monotonically decreasing (increasing) in ρ. For $\rho = 1$, the relative profitability of rivals P is deterministic ($\nu = 0$) and the active firm in the market will be determined by the initial "yield" criterion in (18) for all time (see also figure 3).

4.2   Calculating the probability of being the active rival

Once one of the two competing firms becomes active in our model, it knows that it will operate the market only until its rival finds it optimal to claim the market back. This might well be a finite period of time. What the active firm would like to know is how long this period of time will be, at least in expectation. Moreover, for fixed time horizons, what is the probability that a firm will be active in the market by time t?

To answer these questions, assume that firm j is currently active and the relative profitability variable is at $P_0 \in (\overline{P}, \underline{P})$. Define the stopping time T_j

$$T_j = \inf \left\{ t \geq 0 : P_t \geq \overline{P} \right\}$$

as the first time that P reaches $\overline{P}$ starting from P_0. Since the future evolution of P is unknown, time T_j is essentially a random variable measuring the time firm j will be active. Using equation (1.11) in Harrison [9] and an application of a simple change in variables, the cumulative distribution function of the first passage time T_j can be written as:

$$\Pr\left[T_j \leq t\right] = \Phi\left[\frac{-\ln\left(\frac{\overline{P}}{P_0}\right) + \xi t}{\nu\sqrt{t}}\right] + \left(\frac{\overline{P}}{P_0}\right)^{\frac{2\xi}{\nu^2}} \Phi\left[\frac{-\ln\left(\frac{\overline{P}}{P_0}\right) - \xi t}{\nu\sqrt{t}}\right] \quad (19)$$

where ξ, ν as in (11) and $\Phi\left(.\right)$ is the standard normal cumulative distribution function. To provide a measure of the average time firm j will be active, one might consider the expected value of T_j. Unfortunately, for the case in which $\xi < 0$, the expectation is undefined. Essentially, if the volatility of the relative profitability measure P is large enough, $\overline{P}$ may never be reached and the expectation is infinite, i.e. firm j may remain active for ever. To overcome this limitation, we choose to measure the average active time of firm j by the median of T_j. The median active time for firm j, M_j, that divides the probability in two is the solution to the non–linear equation:

$$\Phi\left[\frac{-\ln\left(\frac{\overline{P}}{P_0}\right) + \theta M_j}{\nu\sqrt{M_j}}\right] + \left(\frac{\overline{P}}{P_0}\right)^{\frac{2\theta}{\nu^2}}\Phi\left[\frac{-\ln\left(\frac{\overline{P}}{P_0}\right) - \theta M_j}{\nu\sqrt{M_j}}\right] = \frac{1}{2} \qquad (20)$$

Figure 4 shows that the median active time of the incumbent firm j increases with the rival's investment cost. Notice however the effect of the volatility of the relative profitability, ν. The median active time M_j decreases with the volatility of the relative profitability of rivals, ν, i.e. the more volatile the industry's profitability, the less the period of time the incumbent firm can maintain its monopolistic position.

This might seem counterintuitive when viewed in conjecture with the fact that by definition $\frac{d\nu}{d\sigma_i} = \frac{d\nu}{d\sigma_j} > 0$, and $\frac{dP}{d\sigma_i} = \frac{dP}{d\sigma_j} < 0 < \frac{d\overline{P}}{d\sigma_i} = \frac{d\overline{P}}{d\sigma_j}$, the zone of hysteresis increases with rivals' volatilities. The intuition is that an increase in volatility has a two opposing effects. First, as σ_i, σ_j increase, the zone of hysteresis becomes wider (see figure 2), making it less likely that a change in the active firm will occur. On the other hand, such an increase makes the relative profitability process P in (11) more volatile (since ν increases), increasing the probability that more extreme values of P will be realised ceteris paribus. *It is this second effect that dominates, implying that the more volatile the market environments, the more short–lived market leadership by a firm will be.*

Once it loses the market to its rival, an incumbent firm would like to be able to calculate the probability of regaining the market back. In other words, firm j, if initially active, would like to know what is the probability that by time T, the relative profitability measure reaches $\underline{P}$ after having

reached $\overline{P}$ before. Towards that end, define

$$m_t^P \equiv \max_{u \in [0,t]} P_u$$

as the running maximum of the relative profitability process P up to time t. With $P_0 \in \left(\overline{P}, \underline{P}\right)$, we are interested in calculating $\Pr\left[P_t \leq \underline{P}, m_t^P \geq \overline{P}\right]$, the probability that firm j becomes active again after a finite period of firm i operating the market. It can be shown that

$$\Pr\left[P_t \leq \underline{P}, m_t^P \geq \overline{P}\right] = \exp\left[\frac{2\theta}{\nu^4} \ln\left(\frac{\overline{P}}{P_0}\right)\right] \Phi\left[\frac{\ln\left(\frac{\underline{P}}{\overline{P}}\right) - \ln\left(\frac{\overline{P}}{P_0}\right) - \theta t}{\nu^2 \sqrt{t}}\right]$$

$$(21)$$

(see for example Musiela and Rutkowski [18, Corollary B.3.1]). Obviously, this probability increases the further in time we look. For symmetric firms, it decreases with the investment entry costs (figure 5) but it increases with the volatility of firms' relative profitability ν (figure 6). In figure 5, notice that as $K_{j \to i} = K_{i \to j} \to 0$, the probability in (21) converges to a straight line at 50%[10], since in this case the thresholds collapse to the unique value $\widehat{P}$ in (18). Moreover, in figure 6, an increase in ν has again the dual effect discussed above. The ratio of entry thresholds, $\gamma \equiv \frac{\underline{P}}{\overline{P}}$, decreases, thus enhancing the gap between $\overline{P}$ and $\underline{P}$. Moreover, since the volatility of P increases, it is more probable that more extreme values will be realised. The second effect dominates to bring the probability in equation (21) up.

5   Aircraft industry application

In this section we apply the model's main results to an industry that conforms very closely to our setting: the aircraft manufacturing industry which has long been dominated by two major competing firms, Boeing Co. and Airbus Industrie. Stonier and Triantis [25] characterise the industry as a direct duopoly, report profitability and order volume uncertainty, large irreversible investments (an average of $5-$10 billion development costs per aircraft type),

---

[10]This is true because the initial value of the relative profitability variable is set at $P_0 = \frac{\overline{P} + \underline{P}}{2}$ as in all figures.

highly differentiated product lines and intense competition where strategic issues are of great importance.[11]

On June 23, 2000, the Airbus Industrie's supervisory board authorised the launching of the A3XX project, which has been under consideration since 1990. This $13-billion project ($6.125 billion in present value terms, see table 1) entails the construction of the biggest passenger carrier aircraft (550 to 990 passengers), which should guarantee market leadership for the Airbus consortium in the Very Large Aircraft (hereafter VLA) market segment.[12] For the last 30 years, the VLA market had been dominated by the flagship of the Boeing fleet, the 747-400, until 2000 when Airbus made an enormous irreversible investment in order to capture the market.

The economics of the A3XX project have been extensively reported and reviewed by the popular press in the last 3 years (Kane and Esty [11] report a long list of articles, analysts' reports and anecdotal evidence concerning the characteristics and the main uncertainties of the project). However, what has been an issue of increased speculation, is what would be the possible reactions of Boeing Co. in the fear of losing its monopolistic position in the VLA market.

Two likely responses have received much attention in the industry: one is for Boeing to develop a stretched, 520–seat version of the 747 (the 747X). The second is to commit to the construction of its own super jumbo jet to compete head on with the A3XX. In what follows we simulate the model numerically in an effort to assess the two alternatives.

We estimate the model inputs using the monthly returns of the two companies, from July 2000 (the announcement of the A3XX project) until January 2002. Market data are drawn from Datastream, while some estimates from Kane and Esty [11] are also used. We estimate Airbus' equity volatility at $\sigma_j = 0.16$ and its systematic risk at $\beta_j = 0.84$, which implies a required rate of return of approximately $\mu_j = 11\%$.[13] The correlation of returns of

---

[11]For more on the aircraft manufacturing industry, see the excellent case study by Kane and Esty [11]. Interested readers can also refer to Hallerstrom and Melgaard [8], Jordan [10] and Stonier [24].

[12]The VLA market refers to aircrafts with the capacity of seating more than 400 passengers or carrying more than 80 tons of freight.

[13]The risk–fre rate and the risk premium are assumed at 6%.

the two firms is $\rho = 0.34$ in our sample period.

For Boeing, we use two set of input parameters, representing the two alternative strategic actions. In the *"safer" alternative*, the stretch version of the 747, the investment cost will be substantially lower. Analysts estimate it at \$4 billion (see Dresdner Kleinwort Benson Research report [?]). Since this alternative will not increase the riskiness of the firm substantially, we use the historically estimated $\beta = 0.98$ for Boeing, which implies $\mu_i = 11.9\%$. The volatility of Boeing's returns is $\sigma_i = 0.15$ in our sample. For the *"risky" alternative* of building a super jumbo, we use a volatility of 0.30 and a required return of 13% (an implied beta of 1.17). We do not see any reason why the investment cost of this alternative should be different from that of Airbus thus the same investment cost is used. Table 2 summarises the inputs of our numerical example.

Using the inputs of table 2, the optimal thresholds for Boeing entry in the VLA are $\overline{P} = 1.78$ and $\overline{P} = 2.61$ for the "stretch" and "super jumbo" alternatives respectively. Boeing's expected operating profitability in the VLA market should exceed more than twice that of Airbus, for the building of a corresponding super jumbo to be optimal. This seems very unlikely even under the most optimistic scenarios concerning the demand in the VLA market. Since it takes a substantial period of time to construct such an aircraft[14], and with Airbus already locking in key customers with delivery orders, Boeing can at best expect to share the market with Airbus if a similar super jumbo is launched.

To further assess the relative attractiveness of the two alternatives, we estimate the probability that Airbus will maintain leadership of the VLA market segment under the two proposed reactions of Boeing using equation (19). Figure 7 shows that the "stretch" alternative decreases the probability of Airbus' leadership sharply. Both rivals have a 50% probability of being the market leader after $T = 11.18$ years under this alternative. This increases to $T = 18.76$ years under the more risky alternative of building a super jumbo jet.

The above seem to imply that among the two mutually–exclusive alternatives considered, Boeing might be better off waiting to build the 520–seat

[14]Delivery of the first A3XX from Airbus is expected to be in 2006.

version of 747 instead of waiting for the riskier super jumbo. Of course the input parameters used in our model could be questioned. Moreover, some of the assumptions in the model might seem inappropriate. For example, Stonier [24] reports that profitability in the aircraft manufacturing industry exhibits cyclicality, in sheer contrast with our assumption in (1). However, if the focus is on empirical applications, the discussion above can highlight that, at a minimum, the model's intuition could carry over to real business industries, probably at the expense of analytic tractability.

6   Conclusions

When firms are directly competing for the same market but can exert different and uncertain future profits from operations, investment timing becomes a strategic decision variable which can be optimised conjecturing the rival's optimal response. We solve a stochastic real option game under the assumption that only one firm can be active at any point in time, but its idle rival has the option to reclaim the market whenever optimal to do so. This is only possible because the homogeneity of the problem in the two state variables allows reduction of its dimensionality to one.

In the absence of fixed entry costs, optimal entry strategies are shown to conform to a simple criterion: invest if your opportunity cost of remaining idle exceeds your rival's operating yield, i.e. the firm with the highest current yield (opportunity cost) operates the market. When fixed costs are introduced, the optimal rival investment thresholds are shown to be separated in the two state variable space, producing a hysteresis range, and the active firm in the market is *history–dependent*. Optimal entry strategies are then determined by a simple, easily invertable, non–linear equation which uniquely describes the hysteresis system.

The range of hysteresis in entry thresholds is increasing in the investment costs but decreasing in the correlation between rivals. *The effect of volatility however is dual*: on one hand, volatility widens the gap of hysteresis driving the optimal entry times of rivals further apart. On the other hand though, the higher the volatility, the more probable big changes in the firms' profitabilities will be realised in a shorter period of time. It is this second effect

that dominates, thus making more volatile markets proner to more fierce competition and more changes in market leadership.

A   Appendix

A.1   Derivation of ODE in (5)

Successive differentiation of (4) yields

$$\frac{\partial F_{j\to i}(S_i, S_j)}{\partial S_i} = f'_{j\to i}(P)$$

$$\frac{\partial F_{j\to i}(S_i, S_j)}{\partial S_j} = f_{j\to i}(P) - P f'_{j\to i}(P)$$

$$\frac{\partial^2 F_{j\to i}(S_i, S_j)}{\partial S_i^2} = \frac{1}{S_j} f''_{j\to i}(P)$$

$$\frac{\partial^2 F_{j\to i}(S_i, S_j)}{\partial S_j^2} = \frac{1}{S_j} P^2 f''_{j\to i}(P)$$

$$\frac{\partial^2 F_{j\to i}(S_i, S_j)}{\partial S_i \partial S_j} = -\frac{1}{S_j} P f''_{j\to i}(P)$$

Substituting the above in equation (2) and simplifying yields

$$\frac{1}{2}\left[\sigma_i^2 S_i^2 \frac{1}{S_j} - 2\rho\sigma_i\sigma_j S_i S_j \frac{1}{S_j} P + \sigma_j^2 S_j^2 \frac{1}{S_j} P^2\right] f''_{j\to i}(P) + (r - \delta_i)\, S_i f'_{j\to i}(P)$$

$$+(r - \delta_j) S_j \left(f_{j\to i}(P) - P f'_{j\to i}(P)\right) - r S_j f_{j\to i}(P) + S_j = 0$$

Divide through with S_j and collect terms to get equation (5) in the text.

References

[1] F. M. Baldursson. Irreversible investment under uncertainty in oligopoly. *Journal of Economic Dynamics and Control*, 22(4):627–644, 1998.

[2] M. J. Brennan and E. S. Schwartz. Evaluating natural resource investments. *Journal of Business*, 58(2):135–157, 1985.

[3] A. K. Dixit. Entry and exit decisions under uncertainty. *Journal of Political Economy*, 97(3):620–638, 1989.

[4] A. K. Dixit and R. S. Pindyck. *Investment under Uncertainty*. Princeton University Press, New Jersey, 1994.

[5] B. Dumas. Super contact and related optimality conditions. *Journal of Economic Dynamics and Control*, 15(4):675–685, 1991.

[6] B. Dumas and E. Luciano. An exact solution to a dynamic portfolio choice problem under transaction costs. *Journal of Finance*, 46(2):577–595, 1991.

[7] S. R. Grenadier. The strategic exercise of options: Development cascades and overbuilding in Real Estate markets. *Journal of Finance*, 51(5):1653–1679, 1997.

[8] N. Hallerstrom and J. Melgaard. Going round in cycles. *Airfinance Journal*, 239(March):49–52, 1998.

[9] M. J. Harrison. *Brownian Motion and Stochastic Flow Systems*. Robert E. Krieger Publishing Company, 1985.

[10] W. S. Jordan. New aircraft orders: Still a leading indicator of airline profits. *Airfinance Journal*, 139(June):42–47, 1992.

[11] M. Kane and B. Esty. Airbus A3XX: Developing the world's largest commercial jet. Working Paper, Harvard Business School, 2001.

[12] E. Kasanen and L. Trigeorgis. Merging finance theory and decision analysis. In L. Trigeorgis, editor, *Real Options in capital investment: Models, strategies, and applications*, pages 47–68. Praeger Publishers, Westport, 1995.

[13] B. Lambrecht and W. R. Perraudin. Real options and preemption. Working Paper, IFR, 1996.

[14] S. Majd and R. S. Pindyck. Time to build, option value and investment decisions. *Journal of Financial Economics*, 18(1):7–27, 1987.

[15] R. L. McDonald and D. R. Siegel. Investment and the valuation of firms when there is an option to shut down. *International Economic Review*, 26(2):331–349, 1985.

[16] R. L. McDonald and D. R. Siegel. The value of waiting to invest. *Quarterly Journal of Economics*, 101(4):707–727, 1986.

[17] R. C. Merton. The theory of rational option pricing. *Bell Journal of Economics*, 4(1):141–183, 1973.

[18] M. Musiela and M. Rutkowski. *Martingale methods in financial modelling*. Springer–Verlag, Berlin Heidelberg New York, 36 edition, 1997.

[19] J. L. Paddock, D. R. Siegel, and J. L. Smith. Option valuation of claims on physical assets: the case of offshore petroleum leases. *Quarterly Journal of Economics*, 103(3):479–508, 1988.

[20] R. S. Pindyck. Irreversible investment, capacity choice and the value of the firm. *American Economic Review*, 78(5):969–985, 1988.

[21] R. S. Pindyck. Investments of uncertain cost. *Journal of Financial Economics*, 34(1):53–76, 1993.

[22] M. Slade. What does an oligopoly maximise? *Journal of Industrial Economics*, 42:45–61, 1994.

[23] F. R. Smets. *Essays on Foreign Direct Investment*. PhD thesis, Yale University, 1991.

[24] J. Stonier. What is an aircraft purchase option worth? Quantifying asset flexibility created through manufacturer lead-time reductions and product commonality. In Aviation Week Group, editor, *Handbook of Airline Finance*, pages 231–250. McGraw Hill, New York, 1997.

[25] J. Stonier and A. Triantis. Natural and contractual options: The case of aircraft delivery options. Working Paper, 3rd Annual International Conference on Real Options, 1999.

[26] E. O. Teisberg. Methods for evaluating capital investment decisions under uncertainty. In L. Trigeorgis, editor, *Real Options in capital investment: Models, strategies, and applications*, pages 31–46. Praeger Publishers, Westport, 1995.
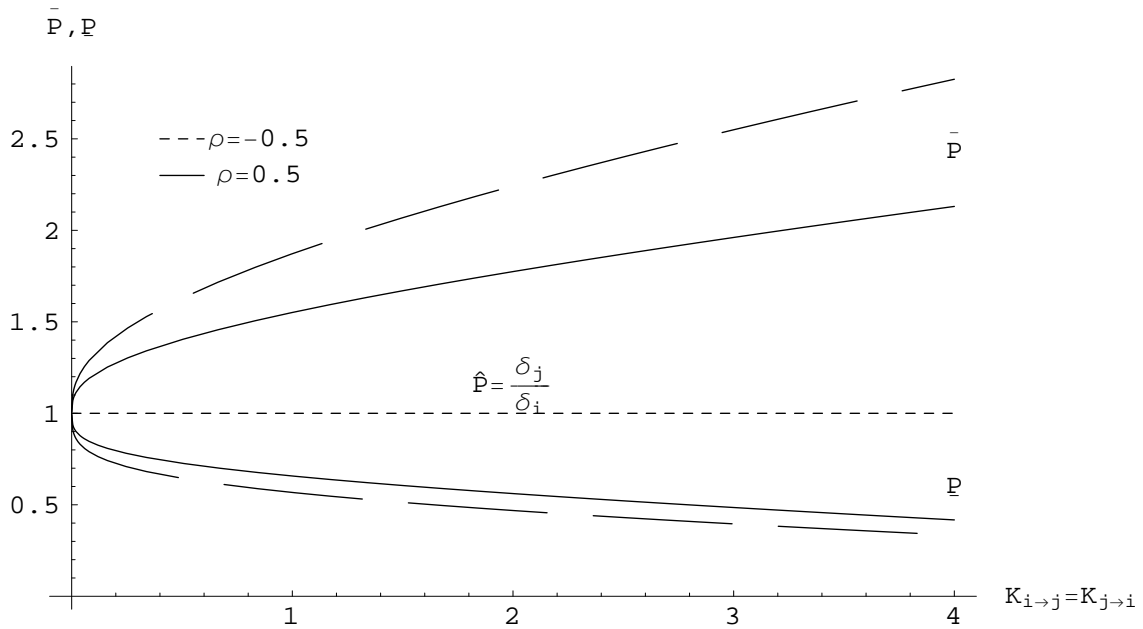
Figure 1: For *symmetric rival firms*, the figure plots the *investment thresholds* $\overline{P}$, $\underline{P}$, as a function of the *investment costs* K for *negative* (dashed line) and *positive* (solid line) *correlation* ρ between firms. The gap between thresholds increases with costs but decreases with the correlation coefficient. The horizontal dotted line corresponds to the *common threshold* in the absence of fixed costs. Even for small fixed costs, hysteresis arises very rapidly. The rest of the parameters are $\delta_i = \delta_j = 0.09$ and $\sigma_i = \sigma_j = 0.20$.
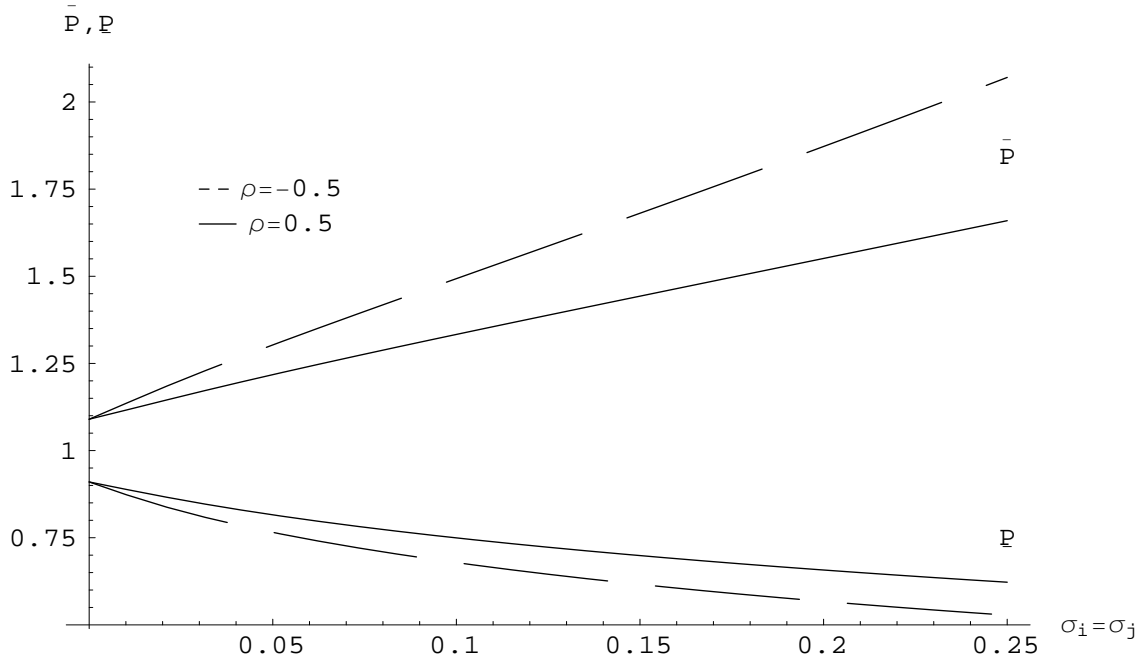
Figure 2: For *symmetric rival firms*, the figure plots the *investment thresh-olds* $\overline{P}$, $\underline{P}$, as a function of the firms' profitability *volatility* σ for *negative* (dashed line) and *positive* (solid line) *correlation* ρ between firms. The gap between thresholds increases with the volatility of each rival's profitability, but decreases with the correlation coefficient. In the absence of uncertainty, the gap between thresholds is only due to fixed investment costs. The rest of the parameters are $\delta_i = \delta_j = 0.09$ and $K_{i \to j} = K_{j \to i} = 1$.
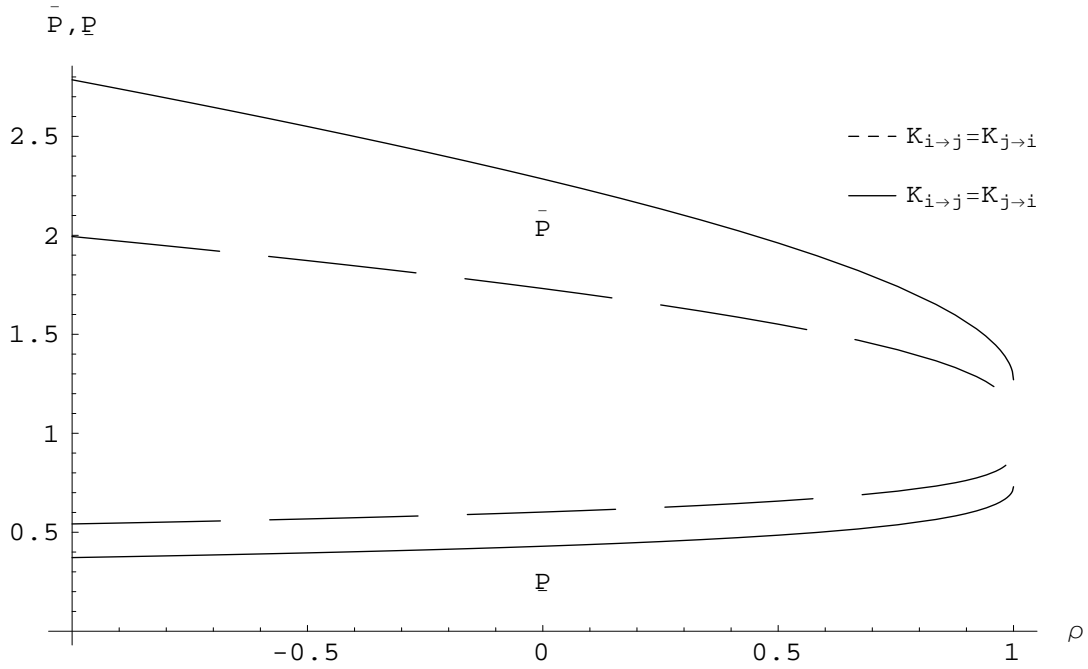
Figure 3: For *symmetric rival firms*, the figure plots the *investment thresholds* $\overline{P}$, $\underline{P}$, as a function of the *correlation* between the operating profitability of rivals ρ for high (solid line) and low (dashed line) fixed investment costs. The gap between thresholds decreases with the correlation between rivals but it increases with the investment costs. If competing firms are perfectly positively correlated ($\rho = 1$), the model degenerates since there is essentially one potential firm. The rest of the parameters are $\delta_i = \delta_j = 0.09$ and $\sigma_i = \sigma_j = 0.20$.
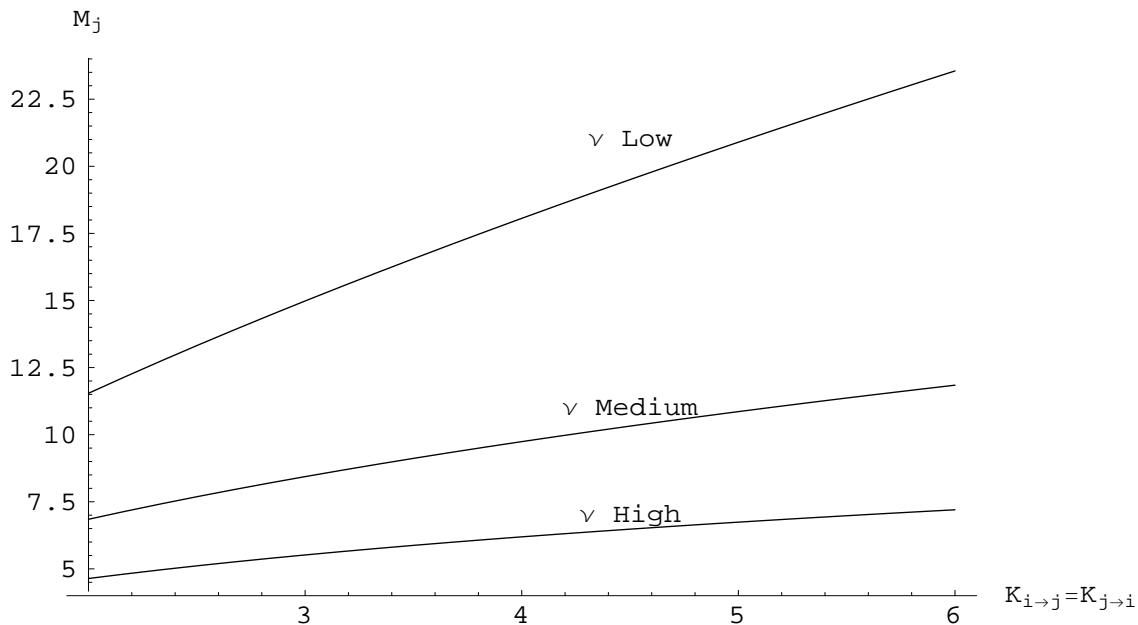
Figure 4: For *symmetric rival firms*, the figure plots the *median time* firm j will be active in the market (if active at $t = 0$) in equation (20) as a function of the *fixed investment cost* K for different values of the *relative profitability volatility* ν. The median time a firm will remain active in the market is increasing in its rival's entry cost. However as σ_i, σ_j increase, and the volatility ν of the relative profitability variable increases, the probability that the switching threshold $\overline{P}$ will be reached increases, thus bringing the median active time for firm j down. The effect of the entry costs is more pronounce the lower is the uncertainty in the relative profitability P. The rest of the parameters are $\delta_i = \delta_j = 0.09$, $\rho = -0.5$, $\sigma_i = \sigma_j = 0.20$ (ν high), $\sigma_i = \sigma_j = 0.15$ (ν medium), $\sigma_i = \sigma_j = 0.10$ (ν low) and $P_0 = \frac{\overline{P}+\underline{P}}{2}$.
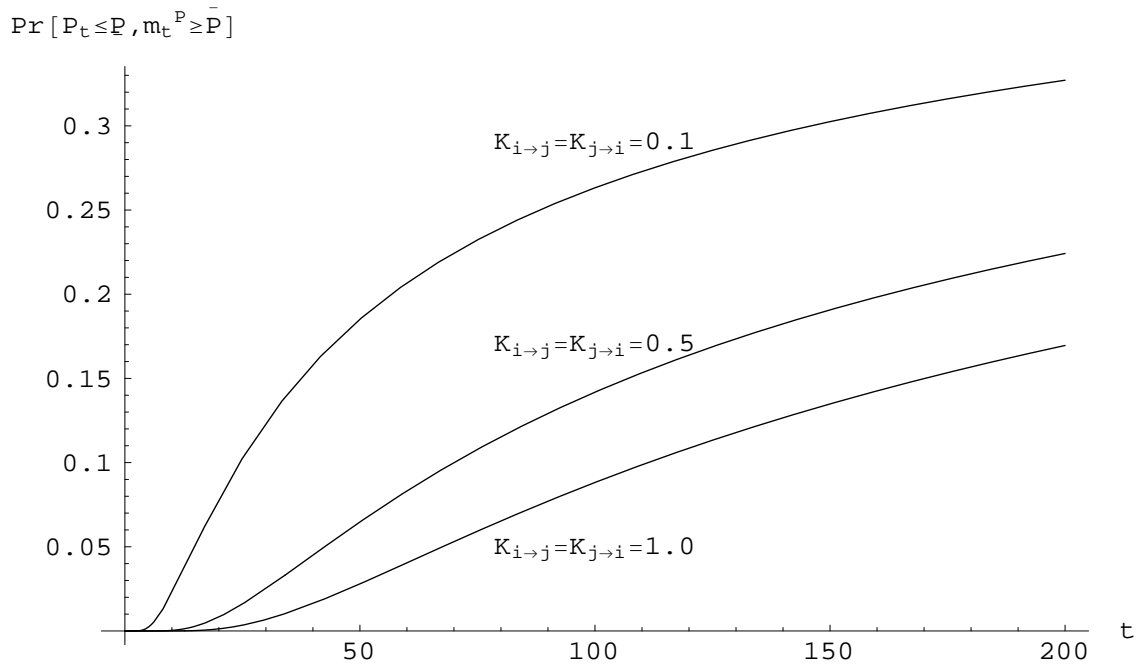
Figure 5: For *symmetric rival firms* and $P_0 = \frac{\overline{P}+\underline{P}}{2}$, the figure plots the *probability that the active firm in the market will change twice* until time t, as a function of time in years. The probability is very small but it increases rapidly the further in the future we look. The higher the investment costs that rivals have to incur to enter, the lower this probability. In the absence of entry costs, $P_0 = \widehat{P}$ in (18) and the probability in the figure collapses to a straight line at the 50% level. The rest of the parameters are $\delta_i = \delta_j = 0.09$, $\rho = -0.5$ and $\sigma_i = \sigma_j = 0.20$.
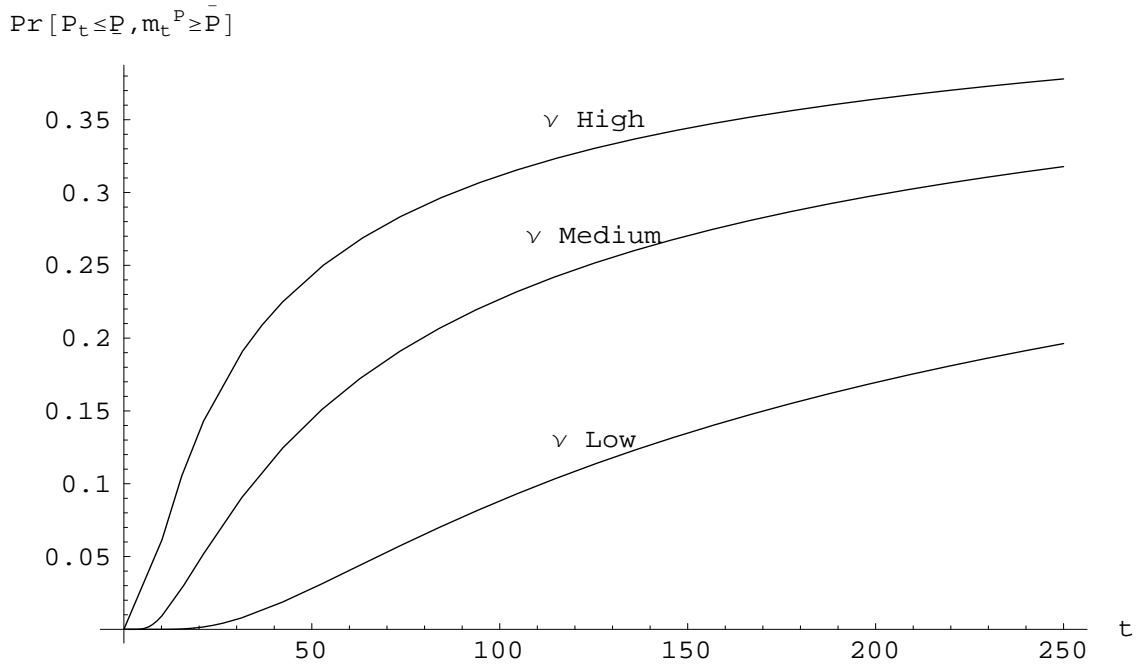
Figure 6: For *symmetric rival firms* and $P_0 = \frac{\overline{P} + \underline{P}}{2}$, the figure plots the *probability that the active firm in the market will change twice* until time t, as a function of time in years. The probability is very small but it increases rapidly the further in the future we look. The higher the volatility ν of the relative profitability of rivals, P, the more probable the switching thresholds will be reached in less time, bringing the probability of two active firm changes up. The rest of the parameters are $\delta_i = \delta_j = 0.09$, $\rho = -0.5$, $\sigma_i = \sigma_j = 0.40$ (ν high), $\sigma_i = \sigma_j = 0.30$ (ν medium), $\sigma_i = \sigma_j = 0.20$ (ν low), $K_{i \to j} = K_{j \to i} = 1$.
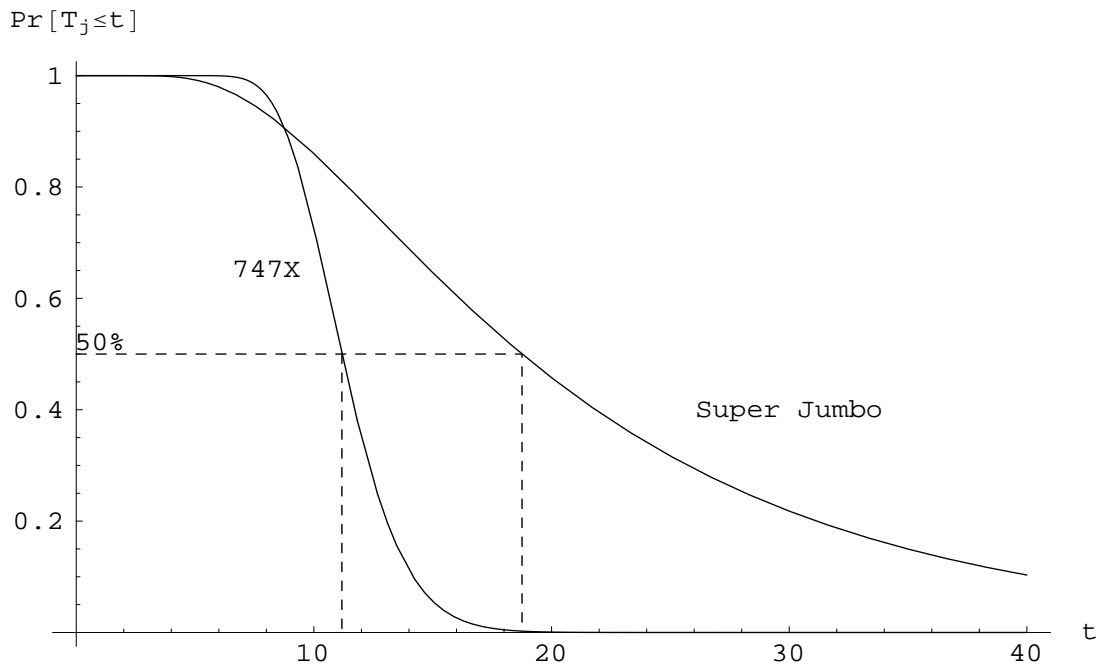
31

Figure 7: The figure plots the probability that Airbus Industrie will remain the leader in the VLA segment of the aircraft manufacturing industry as a function of time under the two possible reactions from its rival, Boeing Co: building a super jumbo or 747X, a stretch version of its existing model. The probability is shown to decrease very rapidly within the next 20 years under the second scenario. Each rival has a probability of leading the market of 50% (dotted line) after $T = 18.76$ and $T = 11.18$ years under each alternative scenario respectively.

32

Investment	2001	2002	2003	2004	2005	2006	2007	2008	**Total**
R&D Expenses	1,100	2,200	2,200	2,200	1,320	880	660	440	11,000
Capital Expenses	0	250	350	350	50	0	0	0	1,000
Working Capital	0	150	300	300	200	50	0	0	1,000
Total	1,100	2,600	2,850	2,850	1,570	930	660	440	13,000
PV on 2000	6,125								

Table 1: Airbus 3XX Expenditure by Year (in million dollars). Source: Dresdner Kleinwort Benson, Aerospace and Defense Report.

INPUTS	AIRBUS INDUSTRIE	BOEING CO.	
		Super jumbo	Stretch 747
Growth rate μ	11.0%	13.0%	11.9%
Volatility σ	0.160	0.300	0.150
Cost of delay δ	0.090	0.050	0.050
Investment cost K	6.125	6.125	4.000
Correlation ρ	0.340		

Table 2: Parameter inputs for the numerical application to the aircraft manufacturing industry.