Real Switching Options and Equilibrium in Global Markets

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Abstract

This paper proposes and investigates a theoretical model in continuous time to analyze the real switching options that an economic entity in relationships with multiple external economic agents holds and the corresponding implications for equilibria between the entity and the agents if they are active. Although our basic model is generally applicable in several widely different economic scenarios, for expositional simplicity, we consider the specific problem of a firm and its global suppliers. We begin by considering the optimal dynamic policy problem for the firm where it may face different exogenously specified relationship specific *fixed* costs and random *variable* costs vis-à-vis each supplier and its goal is to dynamically choose a supplier over time so as to maximize its expected discounted cash flows. At any instant of time, the firm therefore holds compound *real options* of entering the market with a particular supplier, switching to another supplier or exiting the market. In the case where the firm has two suppliers, we derive necessary and sufficient conditions on the fixed and variable cost structures of the firm vis-à-vis the suppliers for the switching option of the firm to have strictly positive value. These also represent necessary and sufficient conditions for each supplier to have strictly positive expected cash flows. Either one of the two suppliers captures the market if these conditions do not hold. We illustrate our analytical results through several numerical simulations.

Next, we investigate the *equilibria* between the firm and its suppliers when both suppliers are in the same foreign country (or, more generally, in two countries with pegged currencies) with uncertainty driven by fluctuations in the exchange rate process. The prices quoted by the suppliers and, therefore, the variable costs of the firm are now determined endogenously in equilibrium where the suppliers and the firm respond rationally and optimally to each other's policies. We devise a procedure to derive equilibria between the firm and its suppliers where a leader-follower game between two competing suppliers allows the firm to maximize value given its bargaining power. We provide sufficient conditions for both suppliers to co-exist in any possible equilibrium with the firm. We identify equilibria between the firm and its suppliers for several different values of underlying parameters that illustrate the impact of competition in global markets.

JEL Classification Numbers: D21, D40, D80, G15, C73

Real Switching Options and Equilibrium in Global Markets

1. Introduction

In the real options literature, switching options have been analyzed in several different contexts, e.g., switching between operating and idle modes of a mine, switching production among various locations internationally for a multinational firm, etc[1]. Each switch is an exercise of an option that yields an asset with a payoff flow along with the option of switching again. However, the real switching options that implicitly arise whenever an *economic entity* (e.g. a firm, an individual, collection of individuals, etc.) faced with a *single* source of exogenous uncertainty incurs different relationship-specific costs vis-a-vis multiple *active*[2] economic agents outside the entity, have not been analyzed formally. Moreover, the implications for equilibria between the entity and the outside agents have not been explored in depth. This paper proposes and investigates a theoretical model in continuous time to analyze the real switching options that an economic entity in relationships with multiple active economic agents holds, and the implications for equilibria between the entity and the agents. We provide necessary and sufficient conditions for these switching options to be valuable and their significance for equilibria between the entity and the agents.

The model that we propose is applicable to the investigation of several interesting economic scenarios pertaining to various types of economic entities. In all the scenarios in which our framework is directly applicable, there is a single utility maximizing entity in relationships with multiple (active or passive) economic agents with whom it faces different relationship-specific *fixed* costs and random *variable* costs. The variable costs are driven by a single exogenous source of uncertainty. For concreteness, we consider the situation where there are two economic agents. The entity, faced with variable cost uncertainty, dynamically chooses to be in a relationship with one of the agents at any

[1] See e.g. Brennan and Schwarz (1985), Kogut and Kulatilaka (1994)

[2] In fact, the optimal switching problem for an economic entity in a relationship with two *passive* economic agents with whom it faces the same source of exogenous uncertainty has also not been formally analyzed in the literature. The paper also contributes a detailed analysis of this problem.

instant of time by trading off the higher variable cost due to one agent against the higher relationship-specific fixed cost due to the other.

We have deliberately chosen very general language in introducing the goal of our study since the basic parsimonious framework that we investigate is applicable in several widely different economic scenarios. We will now motivate our model by briefly outlining these different scenarios.

Production Economics

Our model is directly applicable to the consideration of a single firm with two suppliers. The suppliers of the firm may be located in the same (domestic or foreign) country or in two countries with pegged currencies. The firm faces different relationship specific costs with the suppliers for various reasons such as geographical location, infrastructure development, etc. The suppliers, who rationally anticipate the difference in the relationship specific costs the firm faces, must therefore quote different prices to the firm in order to be competitive with each other. The prices quoted by the suppliers therefore represent the variable costs of the firm.

International Trade

In a foreign investment context, the "suppliers" may really be proxies for different modes of operation for the firm in a foreign market, i.e., forming a joint venture with a foreign partner versus full ownership of a foreign subsidiary[3]. Forming a joint venture is usually associated with lower fixed costs of entry but higher variable costs due to the compensation required by the foreign partner in the venture, vis-à-vis a wholly owned subsidiary.

Labor Economics

Our framework is directly applicable to the analysis of a firm's decision to hire different types of labor, i.e. temporary labor (consultants, etc) versus permanent labor (employees). The hiring of employees typically involves higher relationship specific investments such as recruitment costs, training costs, retrenchment costs, etc. that are

[3] See e.g. Kouvelis, Axarloglou and Sinha (2001).

much lower in the case of consultants. However, the average wage rate of consultants is typically higher than those of employees. Moreover, it is reasonable to assume that the average wage rates are proportional to each other, i.e. they are driven by the same source of uncertainty in labor markets.

Financial Economics

Banks and other large financial institutions are typically faced with the decision of whether to screen and monitor[4] their customers since they are uncertain about their quality. Monitoring is costly, but allows the financial institution to potentially identify credit-risky customers thereby lowering variable costs associated with risk taking behavior by customers. In a similar vein, insurance companies are faced with a similar tradeoff between paying high costs to identify good clients with whom the companies face lower risk (and therefore, variable costs).

Our framework is also directly applicable in the problem of an investor deciding between different mutual funds with differing fee structures[5]. If the net asset values (NAV) of two different mutual funds are comparable, an investor may choose which fund to buy based on its fixed and variable cost structure.

Microeconomics

In typical markets, consumers face significant *search costs*[6] in identifying the most competitive price for a product. The search costs are by nature fixed costs, but the gains to the consumers are the lower variable costs of the product purchased. Dominant firms in turn anticipate the search costs consumers pay in obtaining the best price, and exploit it by charging higher prices to consumers compared with smaller, relatively obscure firms.

The decision of an individual (or a firm) to buy or lease a productive asset also represents the same tradeoff between fixed and variable costs. Buying an asset requires a higher initial investment but is accompanied with lower subsequent variable costs vis-à-

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[4] See e.g. Diamond (1984), Sharpe (1990)
[5] See e.g. Das and Sundaram (1998)
[6] See e.g. Klemperer (1995)

vis leasing the asset. The decision of an airline company to buy or lease airplanes is a concrete example of this tradeoff [7].

As discussed above, our basic framework is applicable in several different economic scenarios. However, purely for expositional convenience, we develop our model and results by using the terminology relevant to the *firm-suppliers* problem. We consider the situation where a single firm has two suppliers with whom it faces different relationship specific fixed costs and uncertain variable costs and the firm may use only one supplier at any time. The prices quoted by the suppliers (i.e. the variable costs of the firm) are proportional to each other (and therefore driven by the same exogenous source of uncertainty). Given variable cost uncertainty, the firm is faced with the problem of deciding when to enter the market with a particular supplier, when to switch from one supplier to another, and when to exit the market. The suppliers co-exist with each other, i.e. both obtain strictly positive expected revenues, only when the switching option of the firm has positive value so that there is a nonzero probability of the firm establishing relationships with either supplier over time.

The paper contributes several interesting results that are summarized below:

- The switching option of the firm is valuable if and only if, for given relationship specific costs, the ratio of variable costs lies in a bounded interval (that may be degenerate). Similarly, for a given ratio of variable costs, the switching option is valuable if and only if the relationship specific costs lie in bounded regions (that may be degenerate).
- We provide *analytical* characterizations of these regions and, more importantly, *sufficient* conditions for their non-degeneracy.
- These regions support *potential* equilibria between the firm and its suppliers where both suppliers co-exist and are independent of the cost structures of the suppliers.
- We solve for the optimal dynamic switching policy of the firm numerically with different choices of parameter values and present the results of simulations that illustrate the comparative statics (with respect to various parameters) of the regions where the firm's switching option has positive

[7] See e.g. March (1990).

value. In particular, we demonstrate numerically that if the volatility of the variable costs of the firm is changed ceteris paribus, there exists a *threshold value* below which the switching region is non-degenerate and above which it is degenerate. This indicates that if the volatility is too high, there can be no equilibrium where both suppliers co-exist with the firm.

- We then investigate game-theoretic equilibria between the firm and suppliers in the same foreign country (with uncertainty driven by exchange rate fluctuations) by explicitly incorporating the suppliers' cost structures that are in their domestic currency.
- In the situation where the firm has only one supplier in a foreign market we provide explicit analytical conditions for the existence of equilibrium between the firm and the supplier and the corresponding *equilibrium price* quoted by the supplier. More precisely, we show that if the volatility of the exchange rate process is below a threshold relative to its drift, an equilibrium always exists and if it is above the threshold, market failure occurs.
- We then consider the more general equilibrium problem where the firm negotiates with two suppliers in the same foreign country and provide a sufficient analytical condition for both suppliers to co-exist in any possible equilibrium with the firm.
- We derive equilibria for several different choices of underlying parameters where either of the suppliers may capture the market or both may co-exist, i.e. the firm's optimal equilibrium policy involves switching between both suppliers over time.
- We demonstrate situations where the volatility of the exchange rate process is so high that market failure would occur if only one supplier had existed at the outset, but is averted when both suppliers exist during the negotiation phase. Moreover, the equilibrium outcome may well be the capture of the market by one of the two suppliers. This result highlights the importance of considering the economic scenario where a firm entertains multiple suppliers in the same country.

Our paper follows in the tradition of two different strands in the literature. Firstly, it extends the seminal approaches developed by Brennan and Schwartz (1985) and Dixit (1988) for valuing real options. Brennan and Schwartz (1985) numerically solve a specialized model for optimal switching between operating and mothballing a mine. Dixit and Pindyck (1994) considers the optimal switching policies for operating, mothballing, and abandoning a mine[8]. In both cases, they consider optimal switching between an active asset that generates a payoff flow, and an idle asset that requires a cost flow. We extend their approaches by not only analyzing the optimal switching between two active assets (in our case, suppliers) that generate different payoff flows, but also studying the implications for game-theoretic equilibria between the firm and the suppliers.

Secondly, it extends the recently emerging finance literature on equilibrium and options on real assets (see Williams 1993, Trigeorgis 1993, Grenadier 1996) to the context of firms and multiple global suppliers. We study *three player* game-theoretic equilibria between a firm and its suppliers where a leader-follower game between two competing suppliers allows the firm to maximize value given its bargaining power.

In summary, this is one of the few papers that *integrates* real options valuation with game theoretic techniques to make inferences about equilibria in global markets between competing economic agents[9]. As discussed earlier, our model and results have implications in widely different economic scenarios.

The rest of the paper is organized as follows. **Section 2** outlines the model used in our analysis. In **Section 3**, we derive the optimal policies of the firm with exogenous fixed and variable costs. **Section 4** presents the results of numerical analyses of the region where the firm's switching option is positively valued and the two suppliers co-exist in the market. This section also provides comparative static effects of exogenously specified parameter values on the regions in which the switching option for the firm has positive value. In **Section 5**, we discuss and solve the equilibrium problem between the firm and its suppliers. **Section 6** concludes and indicates directions for future research.

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[8] The dynamics of the problem we consider is far more complicated than those in the problems considered by Brennan and Schwarz (1985) and Dixit and Pindyck (1994) due to the presence of two active assets. This complexity is similar to that discussed in footnote 46 on pg. 536 in Klemperer (1995).

[9] See e.g. Grenadier (2000).

2. The Model

The firm sells a product whose per unit output price in the domestic market is \$1 and the firm sells 1 unit of the product per unit time if it is in operation. The firm has two suppliers, supplier 1 and supplier 2 who may either supply the finished product that the firm merely sells in the domestic market or the raw materials required to manufacture the product. In other words, the firm may just be a dealer selling products manufactured by its suppliers or it may be involved with manufacturing the product itself. We assume that the costs the firm incurs in manufacturing or selling the product are accounted for in the output price of the product and that these are the same no matter which supplier the firm uses. Therefore, the only uncertainty the firm is exposed to is the uncertainty in the prices quoted by the suppliers.

The firm incurs different fixed and variable costs depending on which supplier it uses. The price (per unit) of the product $p(.)$ demanded by supplier 2 is given by

$$(2.1) \qquad dp(t) = p(t)[m dt + s\, dB(t)]$$

In the above, $B(.)$ is a Brownian motion defined on a filtered probability space (Ω, F, F_t, P). Throughout the paper, we assume that all agents have uniform beliefs about the process $p(.)$ and that they are all risk-neutral and expected utility maximizers[10].

The price uncertainty may be driven by exchange rate fluctuations if we are considering the problem of a foreign supplier or by domestic market uncertainty if it is a domestic supplier. For maximum generality, we do not make any specific assumptions about the nature of the market. We only assume that the *variable cost* faced by the firm due to supplier 2 is given by (2.1). The price per unit of the product demanded by supplier 1 is proportional to the price demanded by supplier 2 and is given by lp. We assume that the *variable cost proportion*

[10] We can easily extend the arguments of the paper to the situation where agents are risk-averse, the cash flows associated with the process p(.) are marketed and agents maximize the *market value* of their cash flows, by modeling the evolution of the process p(.) under the risk neutral measure.

(2.2) $l > 1$

so that the variable cost to the firm of using supplier 1 is greater than the variable cost of using supplier 2. However, the firm incurs different relationship specific *fixed costs* k_1, k_2 for entering into relationships with the suppliers with

(2.3) $0 \le k_1 < k_2$

Remark 1: Given this difference in relationship specific fixed costs, supplier 2 can compete with supplier 1 only by offering a lower price as expressed by (2.2).

At any time t, the firm may either be idle[11] (denoted by 0), in a relationship with supplier 1 (denoted by 1) or in a relationship with supplier 2 (denoted by 2). We assume that the firm cannot use both suppliers simultaneously. We use the variable s to denote these three possibilities so that s takes on values in the set $\{0,1,2\}$. The *feasible* policies of the firm are given by

(2.4) $\Gamma \equiv \{t_1, t_2, ...,\}$

where $\{t_n\}$ is an increasing sequence of F_t $-$stopping times representing the instants at which the firm switches between the various states. The discounted expected utility of the firm from following policy Γ is clearly given by

(2.5)
$$U_\Gamma(p, s_0) = E\sum_{i=0}^{\infty}\{ 1_{s=1}[-\exp(-b\,t_i)k_1 + \int_{t_i}^{t_{i+1}}\exp(-bs)(1-lp(s))ds] + 1_{s=2}[-\exp(-b\,t_i)k_2 + \int_{t_i}^{t_{i+1}}\exp(-bs)(1-p(s))ds]\ \}$$

[11] This clearly also encompasses the situation where there the firm has a domestic supplier who charges a constant price in the firm's currency and with whom the firm has no fixed entry costs. In other words, the idle state 0 might represent the state where the firm uses this domestic supplier.

In the above, b is the firm's discount factor (or opportunity cost), p is the initial price offered by supplier 2 and s_0 is the initial state of the firm. Each term in the summation above represents the total discounted cash flows of the firm from using either of the suppliers over the time interval (t_i, t_{i+1}). If it decides to use either supplier, it pays a fixed cost (equal to k_1 or k_2) and variable costs (given by $p(.)$ or $l p(.)$). The goal of the firm is to choose its switching policy Γ so as to maximize its discounted expected utility U_Γ.

Remark 2: Since we have assumed that the variable costs incurred by the firm with the two suppliers are proportional to each other, the "state of the firm" is represented by the price p demanded by supplier 2 and the value of the variable s. For subsequent expositional convenience we shall refer to the firm being in "state 0, state 1 or state 2" as the firm being idle, with supplier 1 or with supplier 2 respectively.

Throughout the paper, we shall be interested in the situation where the firm is initially idle and the price p offered by supplier 2 initially is greater than the output price $1 (per unit) of the firm. Since we are interested in the *long term* optimal switching policies of the firm, we can clearly make this assumption without loss of generality. From (2.5), it is clear that the optimal switching policies (if they exist[12]) of the firm must be *stationary*. At any time t, the optimal decision of the firm does not depend on time, but only on the current value of the variable s of the firm and the price p demanded by supplier 2. Therefore, it suffices to consider policies of the firm that are described as follows:

(2.6) $$\Lambda \equiv \{p_{01}, p_{10}, p_{12}, p_{21}, p_{02}, p_{20}\}$$

where p_{ij} is the *switching point* for switching from state i to state j, i.e. p_{ij} is the price of supplier 2 at which the firm will switch from state i to state j.

[12] It is not obvious at the outset that optimal policies exist. One of the contributions of the paper is a demonstration of the existence and the characterization of the optimal policies.

We now observe that it is never optimal for the firm to switch from state 2 to state 1. Intuitively, when the firm is in state 2, it has already incurred a fixed cost k_2. It would clearly not be optimal for the firm to switch to state 1 paying an additional fixed cost of k_1 and obtaining a higher variable cost in return! It therefore suffices to consider policies of the firm that are described as follows:

$$(2.7)\quad \begin{array}{l} \{p_{01}, p_{10}\} \text{, i.e. the firm only uses supplier } 1 \\ \{p_{02}, p_{20}\}\text{, i.e. the firm only uses supplier } 2 \\ \{p_{01}, p_{10}, p_{12}, p_{20}\}\text{, i.e. the firm may use both suppliers} \end{array}$$

Remark 3: The third case above clearly includes the first two cases as subsets, but we make a distinction for later expositional convenience.

Since we have assumed that the firm is initially in the idle state and the initial price $p > 1$, it also follows that in the third case above, it suffices to consider policies where

$$(2.8)\quad p_{12} \le p_{01} \le 1$$

i.e. the switching point from state 1 to state 2 is below the switching point from state 0 to state 1. If it is optimal for the firm to use both suppliers, then our argument preceding (2.7) implies that the firm will only enter state 2 via state 1. Moreover, since it is clearly never optimal for the firm to switch into state 0 from state 1 or state 2 when its variable cost is favorable, it suffices to consider policies where

$$(2.9)\quad p_{20} \ge 1,\ l p_{10} \ge 1$$

We denote the optimal *value functions* of the firm, (i.e. the firm's optimal expected utilities when it uses policies described by (2.7)) by v_1, v_2, v_{12} respectively. It follows from **Remark 3** that we must have $v_{12} \ge \max(v_1, v_2)$ so that the overall optimal value function v of the firm over all feasible policies is given by

(2.10) $v = v_{12}$

For the suppliers to co-exist, i.e. obtain positive expected revenues regardless of their cost structures, the firm's value function v_{12} must clearly be *strictly greater* than $\max(v_1, v_2)$ so that its corresponding optimal policy must involve switching between both suppliers as described by the third case in (2.7). We would like to emphasize here that, at this point, it is far from obvious that there exists some fixed cost-variable cost structure satisfying our assumptions for which the firm's optimal policy involves the use of both suppliers over time. The primary focus of our paper is the elucidation and characterization of the situations where the firm will optimally switch between *both* suppliers and the corresponding implications for equilibria between the suppliers.

Functional Forms for the Value Functions

If u is the value function of a policy (not necessarily optimal) of the firm, then it is well known (see e.g. Dixit and Pindyck 1994) that u satisfies the following system of ordinary differential equations:

$$
\begin{aligned}
& -bu + mpu_p + \frac{1}{2}s^2 p^2 u_{pp} = 0 \text{ in state 0} \\
(2.11)\quad & -bu + mpu_p + \frac{1}{2}s^2 p^2 u_{pp} + 1 - lp = 0 \text{ in state 1} \\
& -bu + mpu_p + \frac{1}{2}s^2 p^2 u_{pp} + 1 - p = 0 \text{ in state 2}
\end{aligned}
$$

with appropriate boundary conditions for the transitions between different states. Any solution to the system of equations above is of the form;

$$u(p) = Ap^{h_1^+} + Bp^{h_1^-} \text{ in state 0}$$

(2.12) $$u(p) = Cp^{h_1^+} + Dp^{h_1^-} + \frac{1}{b} - \frac{lp}{b-m} \text{ in state 1}$$

$$u(p) = Ep^{h_1^+} + Fp^{h_1^-} + \frac{1}{b} - \frac{p}{b-m} \text{ in state 2}$$

where A, B, C, D, E, F are constants determined by the boundary conditions and h_1^+, h_1^- are the positive and negative root respectively of the quadratic equation :

(2.13) $$\frac{1}{2}s^2x^2 + (m - \frac{1}{2}s^2)x - b = 0$$

We can now write down the functional forms for the value functions corresponding to the various types of policies the firm may choose:

Case 1 : The firm only uses supplier 1.

If the firm follows a policy where it only uses supplier 1, then we can use (2.12) to show that the value function of a policy defined by the switching points $\{p_{01}, p_{10}\}$ is given by

(2.14) $$\begin{aligned} u_1(p) &= A_1 p^{h_1^-}; p > p_{01} \text{ and the firm is in state 0} \\ &= B_1 p^{h_1^+} + \frac{1}{b} - \frac{lp}{b-m}; p < p_{10} \text{ and the firm is in state 1} \end{aligned}$$

with the coefficients A_1, B_1 determined by the value matching conditions at the switching points defined by

(2.15) $$A_1 p_{01}{}^{h_1^-} = B_1 p_{01}{}^{h_1^+} + \frac{1}{b} - \frac{lp_{01}}{b-m} - k_1; A_1 p_{10}{}^{h_1^-} = B_1 p_{10}{}^{h_1^+} + \frac{1}{b} - \frac{lp_{10}}{b-m}$$

If $v_1(p_0)$ is the *optimal value function* of the firm when it uses supplier 1 alone then we clearly have

(2.16) $$v_1(p_0) = \sup_{(p_{01}, p_{10})} u_1(p_0)$$

Moreover, we can use arguments that are by now well known (see e.g. Dixit and Pindyck 1994) that if the policy defined by $\{p_{01}, p_{10}\}$ is optimal within the class of policies where only supplier 1 is used, then $\{p_{01}, p_{10}\}$ are determined by the additional *smooth pasting* conditions

(2.17) $$h_1^- A_1 p_{01}{}^{h_1^- - 1} = h_1^+ B_1 p_{01}{}^{h_1^+ - 1} - \frac{l}{b - m}; h_1^- A_1 p_{10}{}^{h_1^- - 1} = h_1^+ B_1 p_{10}{}^{h_1^+ - 1} - \frac{l}{b - m}$$

Case 2 : The firm only uses supplier 2.

In this case, we similarly obtain

(2.18) $$\begin{aligned} u_2(p) &= A_2 p^{h_1^-}; p > p_{02} \text{ and the firm is in state 0} \\ &= B_2 p^{h_1^+} + \frac{1}{b} - \frac{p}{b - m}; p < p_{20} \text{ and the firm is in state 2} \end{aligned}$$

with the coefficients A_2, B_2 determined by the boundary conditions at the switching points defined by

(2.19) $$A_2 p_{02}{}^{h_1^-} = B_2 p_{02}{}^{h_1^+} + \frac{1}{b} - \frac{p_{02}}{b - m} - k_2; A_2 p_{20}{}^{h_1^-} = B_2 p_{20}{}^{h_1^+} + \frac{1}{b} - \frac{p_{20}}{b - m}$$

If $v_2(p_0)$ is the *optimal value function* of the firm when it uses supplier 2 alone then we clearly have

(2.20) $v_2(p_0) = \sup_{(p_{02}, p_{20})} u_2(p_0)$

If the policy defined by $\{p_{02}, p_{20}\}$ is optimal within the class of policies where only supplier 2 is used, then $\{p_{02}, p_{20}\}$ are determined by the additional *smooth pasting* conditions

(2.21) $h_1^- A_2 p_{02}{}^{h_1^- -1} = h_1^+ B_2 p_{02}{}^{h_1^+ -1} - \frac{1}{b-m}; h_1^- A_2 p_{20}{}^{h_1^- -1} = h_1^+ B_2 p_{20}{}^{h_1^+ -1} - \frac{1}{b-m}$

Case 3: The firm switches between the suppliers

In this case, we obtain using similar arguments and using (2.12) that

$$
\begin{aligned}
u_{12}(p) &= A_{12} p^{h_1^-}; p > p_{01} \text{ and the firm is in state } 0 \\
&= B_{12} p^{h_1^+} + C_{12} p^{h_1^-} + \frac{1}{b} - \frac{lp}{b-m}; p_{12} < p < p_{10} \text{ and the firm is in state } 1 \\
&= D_{12} p^{h_1^+} + \frac{1}{b} - \frac{p}{b-m}; p < p_{20} \text{ and the firm is in state } 2
\end{aligned}
\tag{2.22}
$$

with the coefficients $A_{12}, B_{12}, C_{12}, D_{12}$ determined by the boundary conditions at the switching points defined by

$$
\begin{aligned}
A_{12} p_{01}{}^{h_1^-} &= B_{12} p_{01}{}^{h_1^+} + C_{12} p_{01}{}^{h_1^-} + \frac{1}{b} - \frac{l p_{01}}{b-m} - k_1 \\
B_{12} p_{12}{}^{h_1^+} + C_{12} p_{12}{}^{h_1^-} + \frac{1}{b} - \frac{l p_{12}}{b-m} &= D_{12} p_{12}{}^{h_1^+} + \frac{1}{b} - \frac{p_{12}}{b-m} - k_2 \\
A_{12} p_{10}{}^{h_1^-} &= B_{12} p_{10}{}^{h_1^+} + C_{12} p_{10}{}^{h_1^-} + \frac{1}{b} - \frac{l p_{10}}{b-m} \\
A_{12} p_{20}{}^{h_1^-} &= D_{12} p_{20}{}^{h_1^+} + \frac{1}{b} - \frac{p_{20}}{b-m}
\end{aligned}
\tag{2.23}
$$

If $v_{12}(p_0)$ is the *optimal value function* of the firm when it uses suppliers 1 and 2 then we clearly have

(2.24) $v_{12}(p_0) = \sup_{(p_{01}, p_{12}, p_{10}, p_{20})} u_{12}(p_0)$

If the policy defined by $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ is optimal within the class of policies where both suppliers are used, then $\{p_{01}, p_{12}, p_{10}, p_{20}\}$ are determined by the additional *smooth pasting* conditions

$$
(2.25)\quad
\begin{aligned}
&h_1^- A_{12} p_{01}{}^{h_1^- -1} = h_1^+ B_{12} p_{01}{}^{h_1^+ -1} + h_1^- C_{12} p_{01}{}^{h_1^- -1} - \frac{l}{b-m} \\
&h_1^+ B_{12} p_{12}{}^{h_1^+ -1} + h_1^- C_{12} p_{12}{}^{h_1^- -1} - \frac{l}{b-m} = h_1^+ D_{12} p_{12}{}^{h_1^+ -1} - \frac{1}{b-m} \\
&h_1^- A_{12} p_{10}{}^{h_1^- -1} = h_1^+ B_{12} p_{10}{}^{h_1^+ -1} + h_1^- C_{12} p_{10}{}^{h_1^- -1} - \frac{l}{b-m} \\
&h_1^- A_{12} p_{20}{}^{h_1^- -1} = h_1^+ D_{12} p_{20}{}^{h_1^+ -1} - \frac{1}{b-m}
\end{aligned}
$$

The Value of the Switching Option

As stated earlier, in a real options framework, the firm clearly holds the real option of switching between the two suppliers. It is therefore interesting to determine the *value* of this switching option, i.e. what additional value does the firm obtain from having the option of using both suppliers. We can use the notation introduced above to define this value as follows:

(2.26) $\text{Value of Switching Option} = (v_{12}(p_0) - \max(v_1(p_0), v_2(p_0))$

where $p_0 > 1$ is the initial price demanded by supplier 2. In the above equation, $v_{12}(p_0)$ is the maximum value to the firm from using both suppliers and $\max(v_1(p_0), v_2(p_0))$ is the maximum value from using only one of the two suppliers[13].

Here, it is important to emphasize that it is well known in the literature (see e.g. Dixit 1989) that optimal policies exist within each of the classes described by **Cases 1** and **2** above so that solutions to the system of equations (2.14), (2.15), (2.17) or (2.18), (2.19), (2.21) exist. However, existence of optimal policies within the class defined by **Case 3** above is *not guaranteed* so that existence of a solution to the system of equations (2.22), (2.23), (2.25) is not guaranteed.

We provide *necessary* and *sufficient* conditions for existence of optimal policies in **Case 3** that are also globally optimal (i.e. optimal over all possible choices of policies by the firm) by providing corresponding *necessary* and *sufficient* conditions for existence of solutions to the system of equations (2.22), (2.23), (2.25). Alternatively, these conditions would be the conditions under which the switching option defined in (2.26) has strictly positive value. This completes the formulation of the model.

3. Optimal Policies for the Firm

In this section, we shall present our analytical results characterizing the optimal switching policies of the firm. We shall show the existence of fixed and variable cost structures for which the firm's switching option has strictly positive value. If the firm's switching option does not have strictly positive value, it is always optimal for the firm to use only one of the two suppliers so that one of the suppliers "captures the market".

As has been elaborated in the introduction, from an economic standpoint, our primary interest is in the situation where the fixed cost of using supplier 2 is larger than the fixed cost of using supplier 1 with a reverse relationship between the respective variable costs. For analytical convenience, we shall assume throughout this section that

[13] The problems of valuing the *European* option of exchanging one financial asset for another and the European option on the minimum or maximum of risky assets have been considered by Margrabe [1978] and Stulz [1982] respectively.

the fixed cost of using supplier 1 is zero, i.e. $k_1 = 0$ in the notation of the previous section[14]. Therefore, in this section, we assume that

(3.1) $k_1 = 0, k_2 > 0, l > 1$

Our major results are summarized in the following theorem.

Theorem 3.1

a) For each $k_2 > 0$*, there exists an interval of variable cost proportion values* $(l_{\min}, l_{\max})$ *such that the firm's optimal policies have the following form:*

If $l \le l_{\min}$*, the firm will use supplier 1 alone*

If $l_{\min} < l < l_{\max}$*, the firm will switch between both suppliers*

If $l \ge l_{\max}$*, the firm will use supplier 2 alone.*

In the above, we may have $l_{\min} = l_{\max}$ *in which case the firm's switching option has zero value always.*

b) For each $l > 1$*, there exists an interval of supplier 2 entry costs* $(k_{\min}, k_{\max})$ *such that the firm's optimal policies have the following form:*

If $k_2 \le k_{\min}$*, the firm will use supplier 2 alone*

If $k_{\min} < k_2 < k_{\max}$*, the firm will switch between both suppliers*

If $k_2 \ge k_{\max}$*, the firm will use supplier 1 alone.*

In the above, we may have $k_{\min} = k_{\max}$ *in which case the firm's switching option has zero value always.*

We provide *analytical* characterizations of the parameters defining the regions where the firm's switching option has strictly positive value and, more importantly, provide precise *sufficient* conditions on the market parameters for their non-degeneracy.

[14] We can relax this restriction and still maintain the qualitative features of our results, but this introduces additional analytical complexity without contributing to the intuition of the results. We have however relaxed this restriction in the numerical simulations that we present in the next section.

Since the proof of our central theorem is rather involved, we present it in the form of a series of shorter propositions.

We begin by introducing some notation. As we have commented in the previous section, it is well known that the entry exit problem for a single supplier has a solution (see e.g. Dixit 1989). In particular, in the situation where supplier 2 is the only supplier in the market, the system of equations (2.18), (2.19), (2.21) has a solution.

We denote the optimal entry and exit points for the firm vis-à-vis supplier 2 alone by p_e^2, p_q^2 respectively. It is trivial to see that in the situation where the fixed cost of using supplier 1 is zero, the optimal entry and exit points of the firm vis-a-vis supplier 1 alone are equal to $1/l$. It is also well known that

(3.2) $\quad p_e^2 < 1, p_q^2 > 1$

We shall first consider the case where the entry cost k_2 is fixed and investigate the optimal policies for the firm for each value of the variable cost proportion l.

Let the firm's value function for using supplier 2 alone, i.e. entering and exiting at the levels given by (3.2) be denoted by v_2. v_2 clearly does not depend on the variable cost proportion l. For each $l \geq 1$, we begin by considering the class of policies Π where the firm always enters the market with supplier 1 and investigate the optimal policies for the firm *within* the class Π. If $p_l \in \Pi$ is the optimal policy for the firm within the class Π, then it is easy to see that p_l is optimal over the entire set of policies of the firm, i.e. the policies where the firm may choose between both the suppliers, if and only if the firm's value function for the policy p_l exceeds the value function v_2 of using supplier 2 alone. This follows from our observation in the previous section that it is never optimal for the firm to enter a relationship with supplier 1 after entering a relationship with supplier 2[15].

[15] We have indexed the optimal policy by p_l since it depends on l, in general.

If $p_l \in \Pi$ is the optimal policy within the class Π, then it is easy to see that it must involve the firm entering the market with supplier 1 whenever the process $p(.)$ decreases to $1/l$ and exiting the market *from a relationship with supplier 1* whenever $p(.)$ increases above $1/l$. However, after entering the market with supplier 1, the firm may optimally switch to supplier 2 when the price falls further in which case the firm will continue with supplier 2 until it exits the market.

For each $l \geq 1$, let $z_1(l)$ denote the optimal value function of the firm within the class of policies Π where it always enters the market with supplier 1. We define

(3.7) $$l_{\max} = \sup\{l : z_1(l) \geq v_2\}$$

Since the policy of using supplier 1 alone is clearly optimal for the firm over all possible policies if $l = 1$, we must have $z_1(1) > v_2$. Therefore, the set $\{l : z_1(l) \geq v_2\}$ is nonempty. We also note that $z_1(l)$ is decreasing and $\lim_{l \to \infty} z_1(l) = 0$. Since $v_2 > 0$, we therefore see that $l_{\max}$ exists and is finite. We can now state the following simple proposition whose proof we omit for the sake of brevity since it follows directly from definition (3.7).

Proposition 3.1

a) *If $l > l_{\max}$, the optimal policy for the firm is to use supplier 2 throughout, i.e. it will enter the market with supplier 2 at $p(.) = p_e^2$ and exit the market from supplier 2 at $p(.) = p_q^2$.*
b) *For $l < l_{\max}$, the firm will always enter the market with supplier 1 and its optimal value function is therefore $z_1(l)$.*

The result of the above proposition says that if the proportion of variable costs $l > l_{\max}$, supplier 2 captures the market. In other words, for supplier 1 to coexist in the market with supplier 2 or capture the market, l must be less than $l_{\max}$. Since the entry cost for supplier 1 is zero, it is clear that when $l = 1$, it is optimal for the firm to use supplier 1 alone, i.e. never switch to supplier 2. We would therefore intuitively expect that as l increases from 1, the optimal policy for the firm is to use supplier 1 alone until a value l_0 where the firm is *indifferent* between the policy of using supplier 1 alone and the policy of entering the market with supplier 1 and optimally switching to supplier 2 when its variable costs fall further[16]. At the *indifference point* l_0, the firm will optimally enter the market with supplier 1 when $p(.) = 1/l_0$ and is indifferent between either continuing with supplier 1 as long as $p(.) \leq 1/l_0$ or switching to supplier 2 if $p(.)$ falls to $p_{12} \leq 1/l_0$ and continuing with supplier 2 until $p(.)$ exceeds $p_{20} > 1$. More precisely, using the notation developed in **Section 2,** we have

$$(3.8) \qquad l_0 = \inf \{ l : v_{12}(l) > v_1(l) \}$$

In the above, $v_{12}(l)$ is the value to the firm when it may switch between both suppliers over time and the variable cost proportion is l and $v_1(l)$ is the value to the firm if it may only use supplier 1. Since $v_{12}(l) \geq \max(v_1(l), v_2)$ and $\lim_{l \to \infty} v_1(l) = 0$, it is easy to see that l_0 exists and is finite. Moreover, by definition (3.8), it is clearly optimal for the firm to use supplier 1 alone if $l < l_0$.

We can now state the following result that allows us to simultaneously determine l_0, p_{12}, p_{20}.

Proposition 3.2

l_0, p_{12}, p_{20} must be given by the solutions to the following system of coupled nonlinear equations:

$$(3.9) \qquad \left[\frac{h_1^+ - 1}{b - m} - \frac{h_1^+}{b}\right](p_{20})^{h_1^-} - \frac{h_1^+ - 1}{(b - m)\, l_0^{h_1^-}}\, p_{20} = -\frac{h_1^+}{b\, l_0^{h_1^-}}$$

---

[16] We should emphasize here that although this is intuitively appealing, it is not obvious at the outset.

(3.10) $$\frac{(h_1^+ - 1)(l_0 - 1)\,p_{12}}{h_1^+(b-m)} = k_2$$

(3.11) $$M(p_{12})^{h_1^+ - 1} = \frac{1 - l_0}{h_1^+(b-m)}$$

where

(3.12) $$M = \frac{h_1^+ K(l_0)^{-h_1^+} + h_1^- L(l_0)^{-h_1^-} - \dfrac{1}{b-m}}{h_1^+(l_0)^{-h_1^+}};$$

$$L = \frac{(1-h_1^-)(p_{20})^{1-h_1^+}}{(h_1^+ - h_1^-)(b-m)} + \frac{h_1^-(p_{20})^{-h_1^+}}{(h_1^+ - h_1^-)b}; K = \frac{(l_0)^{h_1^-}}{(h_1^+ - h_1^-)}\left[\frac{h_1^+ - 1}{b-m} - \frac{h_1^+}{b}\right]$$

Proof. In the Appendix .

In the following proposition, we show that $l_0 \le l_{\max}$ (defined by (3.7)) and that for $l > l_0$ the firm's optimal policy must involve switching from supplier 1 to supplier 2.

Proposition 3.3

Let the indifference point l_0 be defined by (3.8). Then, we must have

a) $l_0 \le l_{\max}$

b) If $l > l_0$, it is never optimal to use supplier 1 alone.

Proof. In the Appendix.

From the results of **Propositions 3.1** , **3.2 and 3.3**, we can now define $l_{\min}$ as :

(3.13) $l_{\min} = l_0$

We therefore see that for $l \le l_{\min}$, supplier 1 captures the market, for $l \ge l_{\max}$, supplier 2 captures the market and for $l_{\min} < l < l_{\max}$, the firm may switch between the suppliers over time so that both suppliers coexist, i.e. they obtain strictly positive expected revenues. Hence, the firm's switching option has strictly positive value for some values of l if and only if $l_{\min} < l_{\max}$.

A natural question to ask is when this occurs. We will now provide a *sufficient condition* on the market parameters that ensures that $l_{\min} < l_{\max}$ so that the firm's switching option has strictly positive value for $l \in (l_{\min}, l_{\max})$.

Sufficient Condition for $l_{\min} < l_{\max}$

The following condition ensures that $l_{\min} < l_{\max}$.

(S) $\quad v_1(\frac{1}{p_e^2}) < v_2$

Recall that the right hand side above is the value function of the firm if supplier 2 alone exists in the market. The left hand side is the value function of the firm if supplier 1 alone exists and $l = 1/p_e^2$ where p_e^2 is the optimal entry level of the firm if supplier 2 alone exists in the market. Both these value functions may be obtained explicitly by solving equations (2.14), (2.15), (2.17) and equations (2.18), (2.19), (2.21) respectively. From the fact that the optimal entry point for the firm if supplier 1 alone exists in the market is $1/l$ for each l, we see that condition (S) states that the value function of the firm from using supplier 1 alone should be less than the value function from using supplier 2 alone when the optimal entry points in both cases are equal to p_e^2. The following two propositions show that condition (S) is sufficient for $l_{\min} < l_{\max}$.

Proposition 3.4

a) If $l < 1/p_e^2$*, it is always optimal for the firm to enter the market with supplier 1.*

b) If condition (S) holds and $l \geq \frac{1}{p_e^2}$*, i.e.* $\frac{1}{l} \leq p_e^2$*, the optimal policy for the firm is to use supplier 2 alone, i.e. the firm will never use supplier 1.*

Proofs. In the Appendix.

By the result of part b) of the above proposition, we see that $v_1(l) < v_{12}(l) = v_2$ for $l \geq 1/p_e^2$. (3.8) and (3.13) therefore imply that $l_{\min} < l_{\max}$. Moreover, the result of part a) implies that $l_{\max} = 1/p_e^2$.

From the result of part a) of **Proposition 3.4**, the intuition behind the reason why condition (S) is sufficient for non-degeneracy can be explained as follows: For $l < 1/p_e^2$, it is always optimal for the firm to enter the market with supplier 1. However, if l is sufficiently close to $1/p_e^2$, condition (S) ensures that the value of using supplier 1 alone is strictly less than the value of entering the market with supplier 1 and optimally switching to supplier 2 if the price becomes more favorable.

Remark 4: Condition (S) is an analytical relationship between the drift m and volatility s of the process $p(.)$, the firm's discount rate b and the relationship-specific costs k_1, k_2. The results of numerical simulations that we present in the next section show that as the volatility s is increased ceteris paribus, there exists a threshold value s^* below which the "switching interval" $(l_{\min}, l_{\max})$ is non-degenerate and above which it become degenerate. In other words, if the volatility of the process $p(.)$ is "too high", the two suppliers cannot co-exist with the firm and one of them, therefore, captures the market.

So far, we have investigated the nature of the optimal policies for the firm if the fixed cost of entering state 2, i.e. k_2 is kept fixed and l varies. We can use arguments[17] very similar to those used in the proofs of **Propositions 3.1 - 3.4** to obtain analogous results when the *variable cost proportion* l is kept fixed and k_2 varies.

If l is fixed the value function of the firm from using supplier 1 alone $v_1(l)$ clearly does not vary with k_2 while the value function of the firm from using supplier 2 alone $v_2(k_2)$ is explicitly a function of k_2 and is moreover a decreasing function of k_2.

[17] We omit the proofs for the sake of brevity since they are very similar. They are available from the authors upon request.

Analogous to condition (S), the following condition is sufficient for $k_{\min} < k_{\max}$, i.e. the existence of a region where the firm's switching option has strictly positive value.

(T) $v_1(l) < v_2(k^*)$

where k^* is the value of k_2 at which the optimal entry point for the firm when supplier 2 alone exists is $1/l$. Moreover, under condition (T), $k_{\min} = k^*$ and $k_{\max}$ is the value of the entry cost at which the firm is indifferent between the policy of using supplier 1 alone and the policy of entering the market with supplier 1 and optimally switching to supplier 2 when the variable costs fall further.

The results of **Propositions 3.1 - 3.4** combine to completely characterize the firm's optimal policies thereby establishing the results of **Theorem 3.1**. Moreover, conditions (S) and (T) provide precise analytical sufficient conditions on the market parameters for the switching regions described by **Theorem 3.1** to be non-degenerate. The non-degeneracy of the switching regions represents a *necessary* condition for both suppliers to co-exist in any possible equilibrium with the firm. In **Section 5**, we carry out a detailed investigation of the equilibrium problem between the firm and the suppliers.

4.Switching Options of the Firm: Numerical Results

In the previous section, we derived sufficient conditions on the market parameters that ensures the existence of regions where the firm's switching option value is strictly positive. For analytical tractability, we had assumed that the fixed cost borne by the firm for using supplier 1 is zero. We have examined the robustness of the conclusions drawn in the previous section to the more realistic situation where the firm has nonzero relationship specific costs with both suppliers, i.e. $k_1, k_2 > 0$, through the use of numerical simulations. More precisely, we solve the problems defined by (2.14), (2.15), (2.16); (2.18), (2.19), (2.20); (2.22), (2.23), (2.24) numerically to obtain the value functions $v_1(p_0), v_2(p_0), v_{12}(p_0)$ and the corresponding price triggers or switching points that define the stationary optimal policies. For the sake of brevity, we only present the results of a few simulations.[18]

The Firm's Value Functions

Figures 1a and **1b** show the variation of the firm's value functions with the variable cost proportion l for fixed k_1 and k_2. **Figure 1a** is a case where there exists a non-degenerate region $(l_{\min}, l_{\max})$ wherein the firm's switching option has strictly positive value. Consistent with our analytical results, we see that for $l \geq l_{\max}$, the firm's overall value function v_{12} is equal to the firm's value function of using supplier 2 alone, i.e. v_2. For $l \leq l_{\min}$, v_{12} is equal to the firm's value function of using supplier 1 alone, i.e. v_1 and $l_{\min}$ is the indifference point. **Figure 1b** is a case where the region $(l_{\min}, l_{\max})$ is degenerate so that the firm's switching option always has zero value. As we have discussed earlier, this is a situation where an equilibrium between both suppliers with each having nonzero expected revenues *cannot* exist.

Figures 2a and **2b** show the variation of the firm's value functions with k_2 for a fixed l. In both figures, we now see that for $k_2 \leq k_{\min}$, the firm's overall value function v_{12} is equal to the firm's value function of using supplier 2 alone, i.e. v_2. For $k_2 \geq k_{\max}$, v_{12} is equal to the firm's value function of using supplier 1 alone, i.e. v_1 and $k_{\min}$ is the indifference point. **Figure 2b** is a case where the region $(k_{\min}, k_{\max})$ is degenerate so that the firm's switching option always has zero value.

Variation of the Price Triggers

In **Figures 3a** and **3b**, we study the variation of the price triggers that define the stationary optimal policies of the firm. As described in the previous section, four price triggers, i.e. the price where the firm enters the market with supplier 1 p_{01}, switches to

[18] The details of all numerical simulations carried out are available from the authors upon request.

supplier 2 p_{12}, exits from supplier 1 p_{10} and exits from supplier 2 p_{20} come into play in the regions where the firm's switching option has strictly positive value. In the regions where either supplier 1 or supplier 2 captures the market, only the corresponding entry and exit price triggers appear.

The Firm's Switching Option

We now examine the behavior of the value of the firm's switching option defined by equation (2.26).

Variation with respect to l

Figure 4a shows the graphs of the switching option value with respect to l at different levels of the price volatility s ceteris paribus. We notice that the region $(l_{\min}, l_{\max})$ moves to the right with increasing s.

The analytical results of the previous section allow us to provide some intuition for this observation. It is well known from the investigation of the standard entry exit problem with a single supplier (Dixit and Pindyck 1994) that as s increases ceteris paribus, the value functions of the firm from using supplier 2 alone or using supplier 1 alone increase. However, due to the lower entry cost for supplier 1, the increase in the value from entering the market with supplier 1 is comparatively greater than the increase in the value of the policy of using only supplier 2. Therefore, from definition (3.7), we see that $l_{\max}$, i.e. the indifference point between the value to the firm from entering the market with supplier 1 and the value from using supplier 2 alone increases. From definition (3.14), if the interval $(l_{\min}, l_{\max})$ is not degenerate, $l_{\min}$ is the indifference point l_0 between the policy of using supplier 1 alone and the policy of entering the market with supplier 1 and optimally switching to supplier 2 later. The indifference point $l_{\min}$ also increases with s for the same reason as above.

Figure 4b shows the graphs of the switching option value with respect to l at different levels of k_2 ceteris paribus. We notice that the region $(l_{\min}, l_{\max})$ moves to the right with increasing k_2.

These observations can be understood intuitively as follows. As k_2 increases, supplier 2 becomes less competitive compared with supplier 1 for a given variable cost proportion l. Therefore, the maximum level $l_{\max}$ at which the value of the policy of entering the market with supplier 1 exceeds the value of the policy of using supplier 2 alone must increase, i.e. supplier 1 may charge a proportionally higher price than supplier 2 and still garner market share. For a similar reason, $l_{\min}$ also increases with k_2 since the indifference point l_0 where using supplier 1 alone has the same value as optimally switching to supplier 2 later, increases.

Figure 4c shows the graph of s^* versus k_2 where s^* is the volatility below which the "switching region" is non-degenerate and above which the switching region is degenerate. Therefore, for $s > s^*$, there can be no equilibrium in which both suppliers co-exist with the firm.

Variation with respect to k_2

Figure 5a shows the graphs of the switching option value with respect to k_2 at different levels of the price volatility s ceteris paribus.

These observations can be explained intuitively along the same lines as the explanations for **Figure 4a.**

Figure 5b shows the graphs of the switching option value with respect to k_2 at different levels of l ceteris paribus.

The region $(k_{\min}, k_{\max})$ moves to the right with increasing l ceteris paribus because supplier 1 becomes less competitive compared with supplier 2 so that the point $k_{\min}$ which is the indifference point between value of using supplier 2 alone and the value of entering the market with supplier 1 increases and the point $k_{\max}$ which is the indifference point between using supplier 1 alone and using both suppliers also increases.

We would like to point out here that the behavior in all cases above was uniformly observed across a wide range of choices of underlying parameter values[19].

5. Equilibria between the Firm and Suppliers

In the previous sections, we have derived the optimal policies for the firm with two suppliers with whom it has different *exogenously specified* fixed and variable costs of interaction. We derived necessary and sufficient conditions on the fixed and variable cost structures for the firm's switching option to have positive value. The positivity of the firm's switching option is a necessary condition for both suppliers to co-exist in the market, i.e. obtain strictly positive expected revenue, in any possible equilibrium with the firm. Moreover, these conditions are independent of the cost structures of the suppliers themselves that the firm is not concerned with, i.e. the firm's optimal policies are determined by the prices quoted by the suppliers and the fixed costs it incurs from establishing relationships with the suppliers.

In this section, we explicitly investigate *equilibria* between the firm and its suppliers taking into account the cost structures of the suppliers. We assume that the relationship specific costs the firm incurs with the suppliers k_1, k_2 are exogenously specified but the prices quoted by the suppliers (i.e. the firm's variable costs with either supplier) are determined competitively. In other words, our goal is the *endogenous derivation* of the variable costs of the firm in equilibria between the firm and its suppliers.

For concreteness, we consider the situation where both suppliers are in the same *foreign* country (or in two foreign countries whose currencies are pegged to each other or perfectly correlated with each other) and the firm faces *exogenously specified* relationship specific fixed costs with each supplier. The firm sells the product at a constant price per unit in its country and each supplier quotes a constant price *in its domestic currency* for

[19] The results of all simulations carried out are available from the authors upon request.

each unit of the product. Therefore, the uncertainty in the firm's variable costs is entirely driven by the uncertainty in the exchange rate[20].

We therefore investigate the game between the firm and its suppliers where the suppliers' strategies are to quote constant prices per unit of the product in their domestic currency and the firm's response is to choose its optimal switching policy given its fixed and variable costs due to the suppliers. We make the additional important assumption that the suppliers bear no additional fixed costs for entering or exiting relationships with the firm. We assume that each supplier's cost of manufacturing a single unit of the product is constant (in its domestic currency) and that each supplier follows the traditional "markup pricing" policy, i.e. each supplier offers the product to the firm at a premium to its cost. We denote the costs of the suppliers by

(5.1) $C_1, C_2 > 0$

Remark 5: We are *not* restricting the costs of the suppliers in any way other than to assume that they are constant. In other words, supplier 2's costs might well be greater than supplier 1's cost. The equilibrium is determined by the fact that the firm faces *different* relationship specific costs with the suppliers so that they must quote *different* prices to the firm in order to be competitive in the firm's market. Therefore, the markups of the suppliers and the resulting profits they generate may be very different.

Each supplier's strategy is to set a *constant* price per unit of the product at a premium to its cost. The prices set by the two suppliers are given by

(5.2) Q_1, Q_2 with $Q_1 \geq C_1, Q_2 \geq C_2$

The exchange rate process $q(.)$ between the two countries is assumed to satisfy:

(5.3) $dq(t) = q(t)[mdt + sdB(t)]$

[20] See e.g. Carbaugh and Wassink [1994] for the impact of exchange rate fluctuations on the decision of a firm to either source domestically or using a combination of domestic and foreign sourcing.

so that, in the notation of **Section 2**, the variable costs of the firm are given by $Q_1 q, Q_2 q$ respectively. Therefore, the variable costs of the firm due to the two suppliers are trivially proportional to each other in conformity with our general model (see **Section 2**). In addition, the firm faces different relationship specific costs with the suppliers given by k_1, k_2 with $k_1 < k_2$. In the notation of **Section 2**, the process $p(.)$, i.e. the variable costs of the firm due to supplier 2 is represented in this situation by the process $Q_2 q(.)$.

We assume that both suppliers are risk neutral and have the same discount rate b' for future cash flows in their domestic currency and that $b' > m$ to ensure that all value functions exist.

The Structure of the Game between the Firm and the Suppliers

The equilibria of the game between the firm and the suppliers are clearly dependent on the structure of the game. In this paper, we assume that the cost structures of both suppliers are common knowledge between the players, i.e. the firm and the two suppliers. The firm enters the market at time 0. *At its discretion*, it first elicits a price quote from either one of the two suppliers, and then obtains a price quote from the other supplier after either revealing or not revealing its first quote. Therefore, we clearly have a leader-follower game structure where one of the suppliers is chosen as the leader and the other the follower *at the behest* of the firm. The suppliers, in turn, rationally anticipate the firm's optimal switching policies (as determined in previous sections) in response to their quoted prices and that the firm chooses the leader and follower to maximize its value function.[21] Since there is no asymmetric information regarding the suppliers' cost structures, the exchange rate process, etc., any equilibrium of the game (if one exists) is fully revealing, i.e. each supplier knows whether it is the leader or the follower and rationally anticipates the price quote of the other supplier.

[21] The firm, given its optimal switching policies, chooses the better of a pair of symmetric, leader-follower equilibrium pricing strategies in which each supplier's pricing strategy, conditional upon the other's pricing strategy, is value-maximizing. Grenadier (1996) uses a leader-follower structure in a two-player game in a completely different real estate development context.

Once the price structure quoted by the suppliers is accepted by the firm, they remain in effect permanently thereafter, i.e. if the firm accepts the price quotes Q_1, Q_2 from the two suppliers, it always pays Q_i *in the suppliers' domestic currency* whenever it uses supplier i[22]. As we have seen in the previous sections, the firm's optimal policy may be to use only one of the two suppliers or to switch between the suppliers over time, i.e. the equilibrium outcome of the game may be capture of the market by either supplier or the co-existence of both suppliers in the market. Additionally, there may exist no equilibrium at all, i.e. the firm and the suppliers may never reach an agreement in which case market failure occurs. The fact that the firm chooses the leader and the follower gives it increased bargaining power as we shall see later. We shall now introduce some analytics essential to a detailed analysis of the game described above.

Let the suppliers' value functions and the firm's value function as a function of the initial prices and the initial value of the exchange rate be denoted by $V_1(Q_1, Q_2, q(0)), V_2(Q_1, Q_2, q(0)), V(Q_1, Q_2, q(0))$ respectively[23]. The firm's value function V has been derived earlier in Section 3. The suppliers' value functions will be similarly derived explicitly later in this section[24].

If supplier 1 is chosen as the leader and supplier 2 the follower, then for each price quote Q_1 of supplier 1, let $y_1(Q_1)$ be the best response of supplier 2, i.e.

(5.4) $$y_1(Q_1) = \arg\max_{Q_2} V_2(q(0), Q_1, Q_2)$$

If supplier 1 is the leader, it will clearly quote a price Q_1^* satisfying

(5.5) $$Q_1^* = \arg\max_{Q_1} V_1(q(0), Q_1, y_1(Q_1))$$

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22 The firm, in effect, writes long term non-renegotiable contracts with the suppliers.

23 We would like to emphasize here that the firm's policies are determined by the prices Q_1, Q_2 that determines the firm's and the suppliers' value functions.

24 The firm's value function is determined by its cash flows that are in its own currency. The suppliers' value functions are determined similarly by their cash flows that are in their own currency, i.e., *the foreign currency*.

Similarly, if supplier 2 is chosen as the leader and supplier 1 the follower, then for each price quote Q_2 of supplier 2, let $y_2(Q_2)$ be the best response of supplier 1, i.e.

(5.6) $\quad y_2(Q_2) = \arg\max_{Q_1} V_1(q(0), Q_1, Q_2)$

If supplier 2 is the leader, it will clearly quote a price Q_2^* satisfying

(5.7) $\quad Q_2^* = \arg\max_{Q_2} V_2(q(0), y_2(Q_2), Q_2)$

Then the firm will choose supplier 1 (supplier 2) as the leader and supplier 2 (supplier 1) as the follower in equilibrium if and only if

(5.8) $\quad V(q(0), Q_1^*, y_1(Q_1^*)) > (<) V(q(0), y_2(Q_2^*), Q_2^*)$

The Value Functions of the Suppliers

Given exogenous fixed costs k_1, k_2 and prices Q_1, Q_2 quoted by the suppliers, we have seen that both suppliers coexist in the firm's market if and only if

(5.9) $\quad l_{\min} < \dfrac{Q_1}{Q_2} < l_{\max}$

where $(l_{\min}, l_{\max})$ is the interval of variable cost proportion values where the firm's switching option has strictly positive value. In the formulation of **Sections 2** and **3**, the optimal switching policies of the firm are described by the entry and exit points

(5.10) $\quad \{p_{01}(Q_1/Q_2), p_{12}(Q_1/Q_2), p_{10}(Q_1/Q_2), p_{20}(Q_1/Q_2)\},$

The firm enters a relationship with supplier 1 from the idle state when the price of supplier 2 in the firm's domestic currency, i.e. $Q_2 q(.)$ first reaches $p_{01}(Q_1/Q_2)$. The firm switches from supplier 1 to supplier 2 when $Q_2 q(.)$ first reaches $p_{12}(Q_1/Q_2)$. The firm exits to the idle state from a relationship with supplier 1 when $Q_2 q(.)$ first reaches $p_{10}(Q_1/Q_2)$, and the firm exits to the idle state from a relationship with supplier 2 when $Q_2 q(.)$ first reaches $p_{20}(Q_1/Q_2)$.

As described in **Sections 2** and **3**, given exogenously specified fixed costs, the entry and exit points are a function of the ratio of the variable costs of the firm that is given by Q_1/Q_2. In situations where the firm only uses one of the two suppliers, i.e. supplier 1 when $Q_1/Q_2 < l_{\min}$ and supplier 2 when $Q_1/Q_2 > l_{\max}$ (**Corollary 3.1**), the corresponding entry point is zero.

We can now use standard dynamic programming arguments very similar to those used in **Sections 2** and **3** to show that the value functions $V_1(p), V_2(p)$ of the suppliers as a function of the price p quoted by supplier 2 *in the firm's currency* must satisfy the following system of differential equations:

$$
\begin{aligned}
&-b' V_i + mp\frac{dV_i}{dp} + \frac{1}{2}s^2 p^2 \frac{d^2 V_i}{dp^2} = 0; \text{ when the firm is not in state } i \\
&-b' V_i + mp\frac{dV_i}{dp} + \frac{1}{2}s^2 p^2 \frac{d^2 V_i}{dp^2} + Q_i - C_i = 0; \text{ when the firm is in state } i
\end{aligned}
\tag{5.11}
$$

where $i \in \{1,2\}$.

The first equation arises from the fact that the supplier obtains no cash flows when it is not in business with the firm and the second arises from the fact that the supplier obtains cash flows at the rate $Q_i - C_i$ when it is in business with the firm. Given the optimal switching policies of the firm, the suppliers' value functions are therefore given by

$$
\begin{aligned}
V_1(p) &= A_1 p^{r_1^-}; \; p \geq p_{01}(l) \text{ and the firm is in state 0} \\
&= B_1 p^{r_1^+} + C_1 p^{r_1^-} + \frac{Q_1 - C_1}{b'}; \; p_{01}(l) \geq p \geq p_{12}(l) \text{ and the firm is in state 1} \\
&= D_1 p^{r_1^+}; \; p \leq p_{20}(l) \text{ and the firm is in state 2}
\end{aligned}
\tag{5.12}
$$

and

$$(5.13)\quad \begin{aligned} V_2(p) &= A_2 p^{r_1^-};\ p \geq p_{02}(l) \text{ and the firm is in state 0 or state 1} \\ &= D_2 p^{r_1^+} + \frac{Q_2 - C_2}{b'};\ p \leq p_{20}(l) \text{ and the firm is in state 2} \end{aligned}$$

with r_1^+, r_1^- being the positive and negative roots of (2.13) with b replaced by b' and $l = Q_1 / Q_2$, i.e. the ratio of the prices quoted by the suppliers or, alternatively, the ratio of the variable costs of the firm. The coefficients in (5.12) and (5.13) are determined by the conditions that the value functions must be continuous.

Equilibria between the Firm and a Single Supplier

Before proceeding to the computation of equilibria between the firm and both suppliers, we first consider the situation where there is only one supplier in the foreign market available to the firm. An investigation of this case is especially relevant since it allows us to compare the bargaining power the firm derives from entertaining another supplier in the foreign market.

The notion of equilibrium we refer to in this situation is evident from the context of the problem we are considering. The firm's optimal policy problem is the standard entry and exit problem with one supplier and the supplier's optimal response is to quote a constant price (at a premium to its cost) in its domestic currency that maximizes its expected discounted cash flows in its currency.

In this situation, we can prove the following result which states that an equilibrium exists between the firm and the supplier if the volatility of the exchange rate process is below a certain threshold relative to its drift and does not exist if the volatility is above the threshold. In other words, if the volatility of the exchange rate process is too high relative to the drift, no equilibrium exists and market failure occurs, i.e. the firm will not enter the foreign market.

Proposition 5.1

If the firm has only one possible supplier in the foreign market, an equilibrium exists between the firm and the supplier if

(5.14) $b^{'} + m > s^{2}$

and no equilibrium exists, i.e market failure occurs if

(5.15) $b^{'} + m < s^{2}$

Moreover, under condition (5.14), we can provide an analytical expression for the equilibrium price quoted by the supplier.

Proof. In the Appendix.

The result of the above proposition is especially significant when compared with the results of the equilibrium analysis when both suppliers exist presented below. The proposition states that market failure occurs if the exchange rate volatility is too high when only one supplier exists. However, when both suppliers exist, we shall see that there exist equilibria even when (5.15) holds, i.e. the existence of a second supplier allows the firm to enter a foreign market in the face of a highly volatile exchange rate[25]. Further, this might occur even when the equilibrium outcome of the game with both suppliers is the capture of the market by one of the two suppliers.

It is interesting to compare the result of **Proposition 5.1** with **Remark 4** in **Section 3** and **Figure 4c** that demonstrate the existence of a critical volatility level s^{*} below which the switching region $(l_{\min}, l_{\max})$ is non-degenerate and above which it is degenerate. In the one-supplier case, there is of course no notion of a switching region, but we have a critical volatility level below which equilibria exist and above which market failure occurs.

---

[25] This is reminiscent of the results of Kulatilaka and Perotti [1998] who show that highly volatile exchange rates do not necessarily have adverse effects on investment in the context of strategic growth.

Equilibria between the Firm and both Suppliers

In the general case where both suppliers compete with each other in the firm's market, we can prove the following result that provides a *sufficient* condition for both suppliers to co-exist in any possible equilibrium with the firm.

Proposition 5.2

If $(l_{\min}, l_{\max})$ *is non-degenerate and* $l_{\min} < C_1/C_2 < l_{\max}$*, then both suppliers must co-exist in any possible equilibrium with the firm, i.e. the capture of the market by either supplier cannot be an equilibrium outcome.*

Proof. In the Appendix.

The result of the above proposition provides a precise connection between the analysis of the real switching options of the firm presented in the previous sections and the equilibrium analysis of the present section. We argued previously that the non-degeneracy of the switching region $(l_{\min}, l_{\max})$ is a *necessary* condition for both suppliers to co-exist in any possible equilibrium with the firm. The result of **Proposition 5.2** states that the condition that the ratio of the costs of the suppliers lies in the interval $(l_{\min}, l_{\max})$ is *sufficient* for both suppliers to co-exist in any possible equilibrium with the firm.

Numerical Computation of Equilibria

We can use equations (5.8)-(5.13) to devise a numerical algorithm to locate possible equilibria between the firm and one or both suppliers. We have implemented such a numerical procedure to solve the equilibrium problem and our results are displayed in the following tables. We have computed equilibria for various combinations of underlying parameter values and for different cost comparisons between the suppliers.

Equilibria with Cost Differentials

Figure 6 shows the variation of the equilibrium prices quoted by the suppliers with cost differential between the two suppliers. The figure depicts all the three types of equilibria between the firm and the suppliers: the region where supplier 1 captures the market when $C_1 < C_2$, an intermediate region where both suppliers co-exist when the costs are comparable and a third region where supplier 2 captures the market when $C_1 >> C_2$.

We see that the equilibrium shifts from supplier 1 capturing the market to both suppliers co-existing in the market to supplier 2 capturing the market as the difference between the costs of the two suppliers progressively increases. This is easy to see intuitively since supplier 1 is significantly more competitive than supplier 2 when its costs are close to those of supplier 2 because its relationship-specific costs for the firm are lower than those due to supplier 1. As the cost differential between the two suppliers increases, the competitiveness of supplier 1 versus supplier 2 declines, initially resulting in both suppliers co-existing in the market, and finally leading to market capture by supplier 2 alone.

However, if one observes the equilibrium prices quoted by the suppliers, one notices that the level of prices quoted by supplier 1 when it captures the market is higher than the level of prices quoted by supplier 2 when it captures the market. This is a dramatic illustration of the fact that equilibria are driven as much by the difference in relationship-specific costs for the firm vis-à-vis the suppliers as by the cost structures of the suppliers. As we can see supplier 1 has significantly higher latitude when it is cost-effective since it has lower relationship-specific costs vis-à-vis the firm.

We notice that when $C_1 \le C_2$, we either have supplier 1 capturing the market or both suppliers co-existing, i.e. supplier 2 can never capture the market. This is easy to see intuitively since supplier 1 can always prevent supplier 2 from attempting to capture the market since it has the advantage of lower relationship specific costs with the firm. At the other extreme where $C_1 >> C_2$, supplier 2 is much more cost effective than supplier 1 and can therefore capture the market with the firm even though its relationship specific costs with the firm are higher.

Figure 7 depicts two of the three types of equilibria between the firm and the suppliers: the region where supplier 1 captures the market when $C_1 < C_2$, and an intermediate region where both suppliers co-exist when the costs are comparable. As we saw in **Figure 6**, when the two suppliers co-exist, competition between them increases the bargaining power of the firm resulting in a lower level of prices quoted.

The figure also reveals the significant positive impact of the prevailing competition in a market with two suppliers. It is easy to see that the chosen parameter values satisfy the inequality constraint in **Proposition 5.1** for market failure when there is a single supplier in the market. The existence of equilibria with both suppliers clearly shows that the firm enters the market when both suppliers exist that it would not otherwise have entered if only one supplier had existed at the outset. Even more interestingly, the equilibrium outcome may well be the capture of the market by either supplier as the figure indicates! Thus, the presence of a second supplier in the market may prevent market failure and allow the firm to enter the market even though it may finally establish a relationship with only one of the two suppliers.

6.Conclusions and Future Research

The extant literature only considers situations where the cost structures the firm faces vis-à-vis different sources of supply are driven by multiple sources of uncertainty[26]. In this case, it is easy to see that the firm derives significant option value from switching between the sources[27]. In contrast, we show that in numerous economic scenarios the firm may derive option value even if the cost structures faced by the firm are driven by the *same* exogenous source of uncertainty. The option value is derived purely from the tradeoff between the fixed and variable costs the firm faces vis-à-vis its suppliers.

More importantly, real switching options that implicitly arise whenever firms faced with a single source of exogenous uncertainty incur different relationship-specific costs vis-à-vis multiple *active* economic agents outside the firm, have not been analyzed

[26] This encompasses the special case where the costs associated with only one of the two sources are stochastic as in Kogut and Kulatilaka (1994), Dasu and Li (1997).
[27] See e.g. Kouvelis (1999).

formally. The implications for *equilibria* between the firm and the outside agents have not been explored in depth. This paper proposes and investigates a theoretical model to analyze the real switching options that a firm with multiple suppliers in global markets holds, and the implications for equilibria between the firm and its suppliers.

We carry out a detailed investigation of the real switching options of the firm and provide *analytical* necessary and sufficient conditions for it to have positive value. We then argue that the regions where the value of the switching option is positive are the very regions supporting *potential* equilibria between the firm and the suppliers where both suppliers have positive expected revenues. We carry out detailed comparative static analyses of the regions where the switching option has positive value.

We then investigate the existence of equilibria between the firm and the suppliers where prices quoted by the suppliers (and therefore, variable costs of the firm) are *endogenously* determined in equilibrium. Assuming a leader-follower game structure we explicitly show that a situation where market failure might occur if only one supplier exists in the market becomes viable when both suppliers exist even though the equilibrium outcome might be the capture of the market by one supplier. We also provide a sufficient condition for both suppliers to co-exist in any possible equilibrium with the firm. We carry out detailed numerical analyses of equilibria between the firm and the suppliers.

Several important issues can be considered in future research. It would be interesting to consider a dynamic game between the firm and the suppliers where each supplier may *change* the price it quotes over time in response to actions by the firm and by the other supplier. In this paper the suppliers provide a price that is fixed over the infinite horizon. In a dynamic game it may be worthwhile for the economic agents to sometimes sell below cost so as to maximize their expected utility. It would not make any sense for a supplier to try this idea in our present model.

Bringing in capacity constraints may have a significant influence on the relative bargaining power of the suppliers and thus impact the existence and nature of equilibria between the firm and suppliers. Lastly, it would be important to examine the influence of information asymmetry[28] amongst the players on the existence and nature of the equilibria between them.

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[28] See e.g. Sharpe (1990)

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APPENDIX

Proof of Proposition 3.2

When the firm has entered a relationship with supplier 1, the problem of whether the firm should switch to supplier 2 when the price $p(.)$ falls further is an *optimal stopping* problem where the *reward for stopping* is the difference between the values of switching to supplier 2 at some price $p_{12} \leq 1/l$ versus using supplier 1 alone, i.e. never switching to supplier 2. Clearly, the firm may also never choose to switch to supplier 2 so that the reward for stopping is always nonnegative.

Suppose the firm is using supplier 1 and switches to supplier 2 when $p(.) = p_{12} \leq 1/l$ and uses supplier 2 till $p(.) = p_{20} \geq 1$. Then the reward for stopping denoted by w must be *nonnegative* and satisfy the following system of differential equations:

$$
\text{(A1)} \quad
\begin{aligned}
&-bw + mpw_p + \frac{1}{2}s^2 p^2 w_{pp} = 0; p > p_{12} \text{ and the firm is in states 0 or 1} \\
&-bw + mpw_p + \frac{1}{2}s^2 p^2 w_{pp} + (1-p) = 0; \frac{1}{l} < p < p_{20} \text{ and the firm is in state 2} \\
&-bw + mpw_p + \frac{1}{2}s^2 p^2 w_{pp} + (l-1)p = 0; p < \frac{1}{l} \text{ and the firm is in state 2}
\end{aligned}
$$

The first equation in the system above arises from the fact that when the firm is in states 0 or 1 the difference in the cash flows to the firm for the policy where it uses supplier 1 alone *versus* the policy where it switches to supplier 2 at $p = p_{12}$ is trivially equal to zero. The second equation arises from the fact that for $1/l < p < p_{20}$, the firm obtains no cash flows from the policy of using supplier 1 alone since it exits the market for $p > 1/l$, but obtains cash flows at the rate $(1-p)$ from using supplier 2. The third equation arises from the fact that for $p < 1/l$, the firm obtains cash flows at the rate $(1-lp)$ from the policy of using supplier 1 alone and cash flows at the rate $(1-p)$ if it were to switch to supplier 2 at $p = p_{12}$. By standard arguments very similar to those used in the previous section, we obtain the following general expressions for w:

$$
\text{(A2)} \qquad
\begin{aligned}
w &= Jp^{h_1^-}; p > p_{12} \text{ and the firm is in states 0 or 1} \\
&= Kp^{h_1^+} + Lp^{h_1^-} + \frac{1}{b} - \frac{p}{b-m}; \frac{1}{l} < p < p_{20} \text{ and the firm is in state 2} \\
&= Mp^{h_1^+} + \frac{(l-1)p}{b-m}; p < \frac{1}{l} \text{ and the firm is in state 2}
\end{aligned}
$$

where the coefficients J, K, L, M are determined by continuity conditions at the points $p_{12}, \frac{1}{l}, p_{20}$ respectively:

$$
\text{(A3)} \qquad
\begin{aligned}
&Jp_{12}{}^{h_1^-} = Mp_{12}{}^{h_1^+} + \frac{(l-1)p_{12}}{b-m} - k_2 \text{ and} \\
&w(\frac{1}{l}+) = w(\frac{1}{l}-); w(p_{20}+) = w(p_{20}-)
\end{aligned}
$$

The policy of switching to supplier 2 at $p = p_{12}$ and exiting at $p = p_{20}$ is optimal if and only if the following additional *smooth pasting* conditions are satisfied:

$$
\text{(A4)} \qquad \frac{dw}{dp}(p_{12}+) = \frac{dw}{dp}(p_{12}-); \frac{dw}{dp}(\frac{1}{l}+) = \frac{dw}{dp}(\frac{1}{l}-); \frac{dw}{dp}(p_{20}+) = \frac{dw}{dp}(p_{20}-)
$$

If l_0 is the *indifference point*, i.e. it is the variable cost proportion at which the firm is indifferent between the policy of using supplier 1 alone and the policy of entering the market with supplier 1 and optimally switching to supplier 2 later, then it must be determined by the condition that the reward for switching, w, is zero. Therefore, we must have

$$\text{(A5)}\quad \begin{aligned} w &= 0;\ p > p_{12} \text{ and the firm is in state 0 or state 1} \\ &= Kp^{h_1^+} + Lp^{h_1^-} + \frac{1}{b} - \frac{p}{b-m};\ \frac{1}{l_0} < p < p_{20} \text{ and the firm is in state 2} \\ &= Mp^{h_1^+} + \frac{(l_0 - 1)p}{b-m};\ p < \frac{1}{l_0} \text{ and the firm is in state 2} \end{aligned}$$

with the coefficients K, L, M and l_0, p_{12}, p_{20} determined by the *value matching* and *smooth pasting* conditions

$$\text{(A6)}\quad \begin{aligned} & w(p_{12}+) = w(p_{12}-), \frac{dw}{dp}(p_{12}+) = \frac{dw}{dp}(p_{12}-) \\ & w(\frac{1}{l_0}+) = w(\frac{1}{l_0}-), \frac{dw}{dp}(\frac{1}{l_0}+) = \frac{dw}{dp}(\frac{1}{l_0}-) \\ & w(p_{20}+) = w(p_{20}-), \frac{dw}{dp}(p_{20}+) = \frac{dw}{dp}(p_{20}-) \end{aligned}$$

The equations (A5) arise from the fact that the reward function w is the difference between the value function of the policy of using supplier 1 alone and the value function of the policy of using both suppliers. If l_0 is the *indifference point*, then at any $p > p_{12}$ where the firm is either in the idle state or in state 1, the reward function for optimally switching to state 2 must be zero. From equations (A5), (A6) and some tedious algebra, we can deduce that l_0, p_{12}, p_{20} are determined by the system of coupled nonlinear equations (3.9), (3.10), (3.11), (3.12). This completes the proof. ♦

<u>Proof of Proposition 3.3</u>

a) By definition, the function $z_1(l)$ is the optimal value to the firm if it were to always enter the market with supplier 1 and have the option of switching to supplier 2 later. On the other hand, the function $v_1(l)$ is the optimal value to the firm if it were to only use supplier 1. Therefore, $v_1(l) \le z_1(l)$ for all l. By the definition (3.7) of $l_{\max}$, we therefore see that $v_1(l) \le z_1(l) < v_2$ for $l > l_{\max}$. Since $v_{12}(l) \ge \max(v_1(l), v_2)$, we see that $v_{12}(l) > v_1(l)$ for $l > l_{\max}$. The definition (3.8) of l_0 therefore implies that $l_0 \le l_{\max}$.

b) For $l = l_0$, the firm is indifferent between the policy of using supplier 1 alone and the policy of entering the market with supplier 1 and optimally switching to supplier 2 if the price falls further. Let the optimal entry and exit points for state 2 be given by p_{12}, p_{20} respectively. Then l_0 is the indifference point only if

(A7) $$E_{p_{12}} \int_0^{t_{p_2}} \exp(-bs)(1-p(s))ds - E_{p_{12}} \int_0^{t_{1/l_0}} \exp(-bs)(1-l_0 p(s))ds = k_2$$

In the above, the first term on the left is the expected value (conditional on the current price being p_{12}) of switching to supplier 2 and continuing with supplier 2 until the exit trigger p_2 is reached and the second term on the left hand side is the corresponding expected value if the firm were to continue with supplier 1 until the exit trigger for supplier 1 , $1/l_0$, is reached.

Suppose it were optimal to use supplier 1 alone for some $l > l_0$. Then (A7) implies that

(A8) $$E_{p_{12}} \int_0^{t_{p_{20}}} \exp(-bs)(1-p(s))ds - E_{p_{12}} \int_0^{t_{1/l}} \exp(-bs)(1-l\ p(s))ds > k_2,$$

since $l > l_0, 1/l < 1/l_0$. Therefore, the policy of switching to supplier 2 at p_{12} and exiting at p_{20} has strictly greater value than the policy of using supplier 1 alone. Therefore, the policy of using supplier 1 alone cannot be optimal. This completes the proof. ♦

Proof of Proposition 3.4

a) Suppose, to the contrary, that using supplier 2 alone is optimal for the firm. The supposed optimality of the policy therefore implies that any deviation of the policy leads to a lower expected utility of the firm. Consider the policy where the dealer uses supplier 1 when $p \in [p_e^2, \frac{1}{l}]$ before switching to supplier 2 at $p = p_e^2$. Note that $\frac{1}{l} > p_e^2$ by hypothesis. It is very easy to see that the value function of this policy is strictly greater than the value function of using supplier 2 alone since

$$\text{(A9)} \qquad E_{p_0}\left[\int_{t_{1/l}}^{t_{p_e^2}} \exp(-bs)(1 - lp(s))ds\right] > 0$$

where $p_0 > 1$ is the initial price demanded by supplier 2 and $t_{1/l}, t_{p_e^2}$ are the first times the price $p(.)$ demanded by supplier 2 reaches $1/l$ and p_e^2 respectively. Therefore, using supplier 1 when $p \in [p_e^2, \frac{1}{l}]$ has a strictly greater value than using supplier 2 alone and this completes the proof.

b) We shall prove this part of the proposition in two steps.

Step 1 : *If* $l = \frac{1}{p_e^2}$, *it is optimal for the firm to use supplier 2 alone.*

In the hypothetical situation where supplier 1 is the only supplier in the market, the optimal policy for the firm is clearly to enter and exit supplier 1 whenever the price process $p(.) = \frac{1}{l} = p_e^2$. In the optimal stopping framework introduced in the proof of **Proposition 3.2**, it therefore clearly suffices to show that the optimal policy for the firm is to switch to supplier 2 as soon as it enters supplier 1, i.e. it is optimal for the firm to "stop immediately".

Since the optimal policy for the firm when supplier 1 alone exists is to enter and exit at $p(.) = p_e^2$, then by the results of the previous section, $p_{01} = p_{10} = p_e^2$ solves equations (2.14), (2.15), (2.17) with the value function v_1 described by (2.14). Similarly, since the optimal policy for the firm when supplier 2 alone exists is to enter and exit at

p_e^2, p_q^2 respectively, $p_{02} = p_e^2, p_{20} = p_q^2$ solves equations (2.18), (2.19), (2.21) with the value function v_2 described by (2.18).

In the optimal stopping framework, we clearly need to show that $p_{12} = p_e^2, p_{20} = p_q^2$ solves equations (A2), (A3), (A4). This basically describes the fact that after the firm enters supplier 1 at $p(.) = p_e^2$, it is optimal for the firm to *immediately* switch to supplier 2 and continue with supplier 2 until $p(.) = p_q^2$.

Define $w = v_2 - v_1$. By condition (S), we see that $w > 0$. By the definitions (2.14), (2.18) of v_1, v_2 respectively, we see that

$$
\begin{aligned}
& w = (A_2 - A_1)\, p^{h_1^-}; p > p_e^2 \text{ and the firm is in state 0} \\
\text{(A10)} \qquad & \quad = B_2 p^{h_1^+} - A_1 p^{h_1^-} + \frac{1}{b} - \frac{p}{b-m}; p_e^2 < p < p_q^2 \text{ and the firm is in state 2} \\
& \quad = (B_2 - B_1) p^{h_1^+} + \frac{(l-1)p}{b-m}; p < p_e^2 \text{ and the firm is in state 2}
\end{aligned}
$$

Recall that the coefficients A_1, A_2 and B_1, B_2 are determined so that the system of equations (2.14), (2.15), (2.17) and (2.18), (2.19), (2.21) are satisfied. Hence, in particular, both v_1, v_2 satisfy the *smooth pasting* conditions at $p = p_e^2$ and v_2 satisfies the *smooth pasting* condition at $p = p_q^2$. From (A10), it is now easy to see that w is nonnegative and satisfies equations (A2), (A3), (A4) with $p_{12} = p_e^2, p_{20} = p_q^2$ and the coefficients J, K, L, M given by

(A11) $\quad J = A_2 - A_1, K = B_2, L = -A_1, M = B_2 - B_1$

Therefore, it is optimal for the firm to enter state 1 at $p(.) = p_e^2$ and switch immediately to state 2, i.e. it is optimal for the firm to use supplier 2 alone.

Step 2 *If* $l > 1/p_e^2$*, it is optimal for the firm to use supplier 2 alone.*

Suppose, to the contrary, that it were optimal for the firm to use both suppliers, i.e. it were optimal for the firm to enter state 1 at $p(.) = \frac{1}{l} < p_e^2$ and switch to supplier 2 when $p(.) \leq \frac{1}{l}$. We arrive at a contradiction by the following arguments.

As l increases, the firm's value function is clearly monotonically decreasing. If we denote the firm's value function as a function of l by $v(l)$, we must therefore have

(A12) $v(l) \leq v(\frac{1}{p_e^2})$ for $l > \frac{1}{p_e^2}$

By the arguments of **Step 1**, we have seen that when $l = \frac{1}{p_e^2}$, the optimal policy for the firm is to use supplier 2 alone. Since the value function corresponding to this policy is obviously independent of l and the policy is always feasible, (A12) clearly implies that $v(l) = v(\frac{1}{p_e^2})$ for $l > \frac{1}{p_e^2}$ and the policy of using supplier 2 alone is optimal.

This completes the proof. ♦

Proof of Proposition 5.1

Without loss of generality, let us assume that the supplier in the foreign market is supplier 2. By our previous analysis, given a price quote $Q_2 \geq C_2$ of supplier 2, the firm's long term stationary switching policies are to enter a relationship with supplier 2 when the price process $Q_2 q(.)$ in the firm's domestic currency hits a level p_e^2 and to exit a relationship with supplier 2 when the price process hits a level p_q^2 where p_e^2 and p_q^2 depend only on the fixed cost of using supplier 2, i.e. k_2, the drift and volatility of the exchange rate process and the firm's discount parameter b.

By specializing equation (5.13) to the case where only supplier 2 exists, it is easy to see that supplier 2's initial value function is given by

$$\text{(A13)}\quad \begin{aligned} V_2(Q_2 q(0)) &= A_{Q_2}(Q_2 q(0))^{r_1^-} \text{ if } Q_2 q(0) \geq p_e^2 \\ &= D_{Q_2}(Q_2 q(0))^{r_1^+} + \frac{Q_2 - C_2}{b^{'}} \text{ if } Q_2 q(0) < p_e^2 \end{aligned}$$

where the coefficients above are determined by the following value matching conditions:

$$\text{(A14)}\quad \begin{aligned} A_{Q_2}(p_e^2)^{r_1^-} &= D_{Q_2}(p_e^2)^{r_1^+} + \frac{Q_2 - C_2}{b^{'}} \\ A_{Q_2}(p_q^2)^{r_1^-} &= D_{Q_2}(p_q^2)^{r_1^+} + \frac{Q_2 - C_2}{b^{'}} \end{aligned}$$

From (A14), we easily see that

$$\text{(A15)}\quad A_{Q_2} = \frac{Q_2 - C_2}{b^{'}}\left[\frac{(p_q^2)^{r_1^+} - (p_e^2)^{r_1^+}}{(p_q^2)^{r_1^+}(p_e^2)^{r_1^-} - (p_e^2)^{r_1^+}(p_q^2)^{r_1^-}}\right] > 0 \text{ since } p_q^2 > p_e^2$$

The price quote Q_2^* is an equilibrium price if and only if supplier 2's value function is maximized at Q_2^*, i.e.

$$\text{(A16)}\quad Q_2^* = \arg\max_{Q_2} V_2(Q_2 q(0))$$

From (A13), (A15), we see that

$$\text{(A17)}\quad \begin{aligned} &\lim_{Q_2 \to \infty} V_2(Q_2 q(0)) = \lim_{Q_2 \to \infty} A_{Q_2}(Q_2 q(0))^{r_1^-} \\ &= \lim_{Q_2 \to \infty} \left(\frac{Q_2 - C_2}{b^{'}}\right)(Q_2)^{r_1^-}(q(0))^{r_1^-}\left[\frac{(p_q^2)^{r_1^+} - (p_e^2)^{r_1^+}}{(p_q^2)^{r_1^+}(p_e^2)^{r_1^-} - (p_e^2)^{r_1^+}(p_q^2)^{r_1^-}}\right] \\ &= \lim_{Q_2 \to \infty} \left(\frac{(Q_2)^{1+r_1^-} - C_2(Q_2)^{r_1^-}}{b^{'}}\right)(q(0))^{r_1^-}\left[\frac{(p_q^2)^{r_1^+} - (p_e^2)^{r_1^+}}{(p_q^2)^{r_1^+}(p_e^2)^{r_1^-} - (p_e^2)^{r_1^+}(p_q^2)^{r_1^-}}\right] \end{aligned}$$

We now recall that r_1^- is the negative root of the quadratic equation

$$-b' x + (m - \frac{1}{2}s^2)x + \frac{1}{2}s^2 x^2 = 0$$

We easily see that $r_1^- > -1$, i.e. –1 lies to the left of the roots of the above equation, if and only if the left hand side above is positive when evaluated at $x = -1$ and this is exactly condition (5.15) of the proposition. We therefore see that if (5.15) holds, $r_1^- > -1$ and we easily see from (A17) that

(A18) $\lim_{Q_2 \to \infty} V_2(Q_2 q(0)) = \infty$

(A18) clearly implies that an equilibrium price Q_2^* does not exist and therefore no equilibrium exists.

On the other hand, condition (5.14) is equivalent to $r_1^- < -1$. In this case, it is easy to see that equations (A13), (A14), (A17) imply that

(A19) $\lim_{Q_2 \to \infty} V_2(Q_2 q(0)) = \lim_{Q_2 \to C} V_2(Q_2 q(0)) = 0$

Since $V_2(.)$ is clearly a continuous function, we see that its maximum exists and is attained at some Q_2^* which is the required equilibrium price. *Moreover, we can analytically determine* the equilibrium price Q_2^* from the explicit analytical expressions for $V_2(.)$. Therefore, an equilibrium between the firm and the supplier exists. This completes the proof. ♦

Proof of Proposition 5.2

The proof proceeds by contradiction. Suppose supplier 2 captures the market in some equilibrium and let the equilibrium price quoted by supplier 2 be $Q_2 \geq C_2$. Since, $C_1 < l_{\max} C_2$ by hypothesis, in response to the price Q_2 quoted by supplier 2, supplier 1 may always guarantee itself strictly positive expected profits by quoting a price

$Q_1 = l_{\max} Q_2 \geq l_{\max} C_2 > C_1$. Therefore, capture of the market by supplier 2 cannot be an equilibrium outcome.

On the other hand, suppose supplier 1 captures the market in some equilibrium and let the equilibrium price quoted by supplier 1 be $Q_1 \geq C_1$. Since $C_1 > l_{\min} C_2$ by hypothesis, in response to the price Q_1 quoted by supplier 1, supplier 2 may always guarantee itself strictly positive expected profits by quoting a price $Q_2 = Q_1 / l_{\min} \geq C_1 / l_{\min} > C_2$. Therefore, capture of the market by supplier 1 cannot be an equilibrium outcome.

Therefore, both suppliers must co-exist, i.e. both suppliers must obtain strictly positive expected profits, in any possible equilibrium of the game between the firm and the suppliers. This completes the proof. ♦

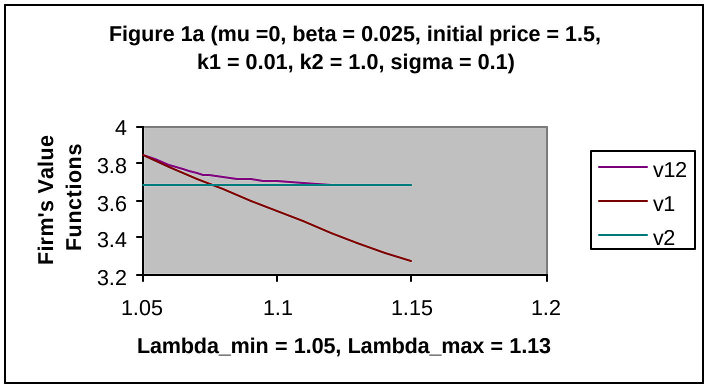


Figure 1a (mu =0, beta = 0.025, initial price = 1.5, k1 = 0.01, k2 = 1.0, sigma = 0.1)

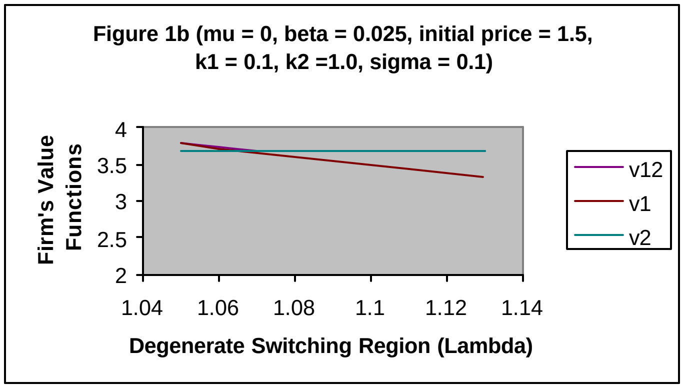


Figure 1b (mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.1, k2 =1.0, sigma = 0.1)

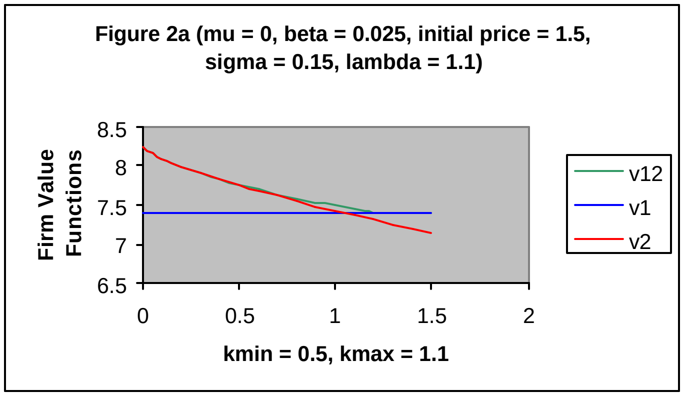


Figure 2a (mu = 0, beta = 0.025, initial price = 1.5, sigma = 0.15, lambda = 1.1)

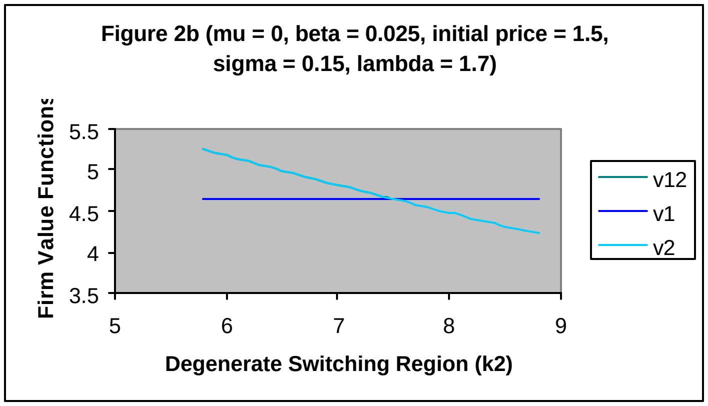


Figure 2b (mu = 0, beta = 0.025, initial price = 1.5, sigma = 0.15, lambda = 1.7)

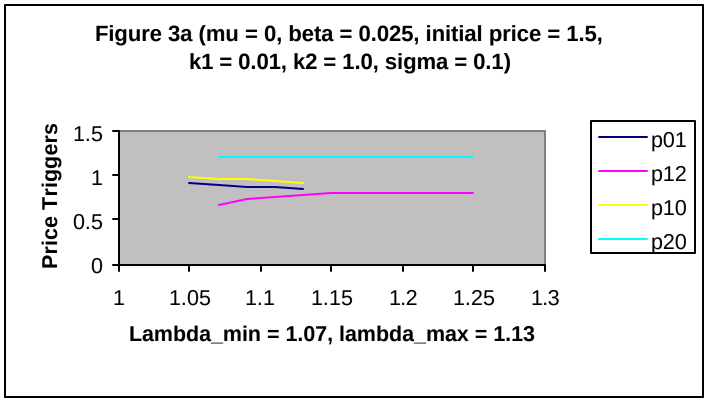


Figure 3a (mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.01, k2 = 1.0, sigma = 0.1)

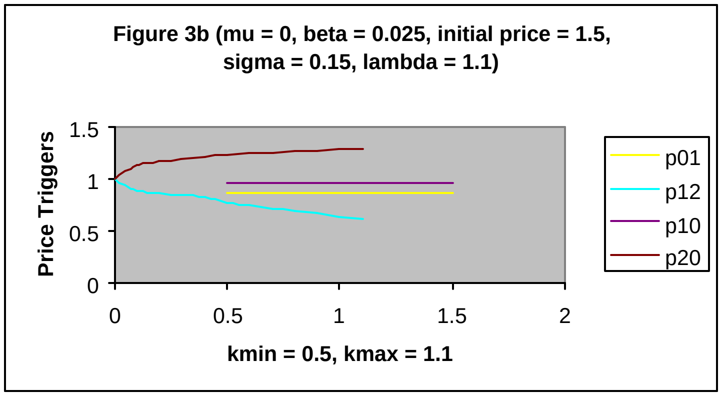


Figure 3b (mu = 0, beta = 0.025, initial price = 1.5, sigma = 0.15, lambda = 1.1)

Figure 4a (mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.01)

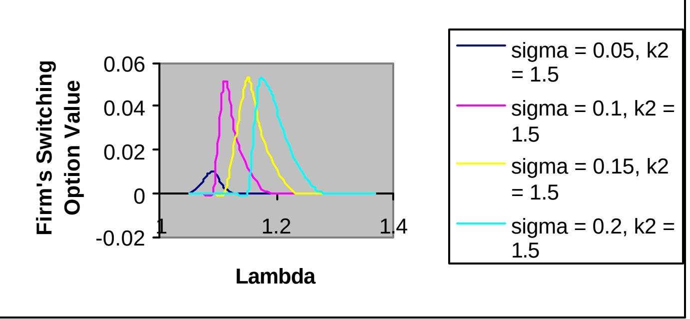


Figure 4b (mu = 0. beta = 0.025, initial price = 1.5, k1 = 0.01)

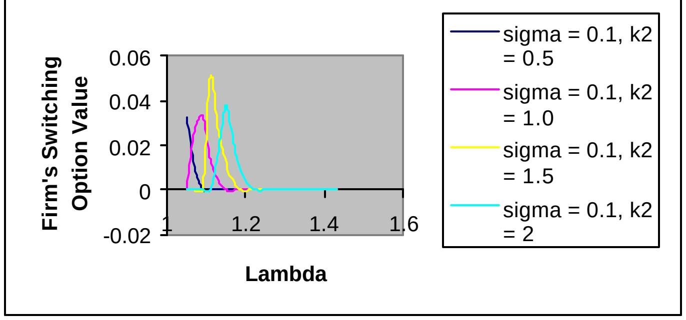


Figure 4c: Variation of sigma* with k2(mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.01)

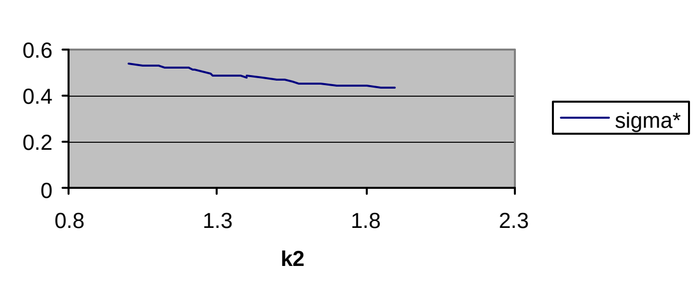


Figure 5a (mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.01)

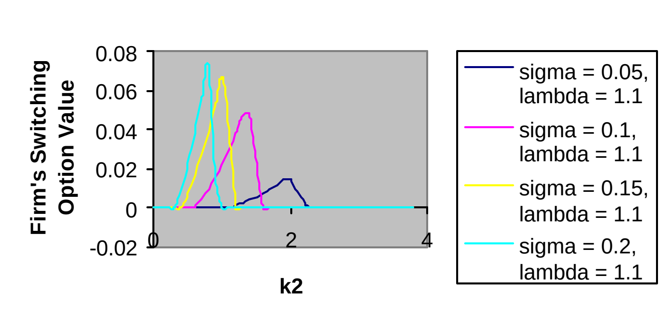
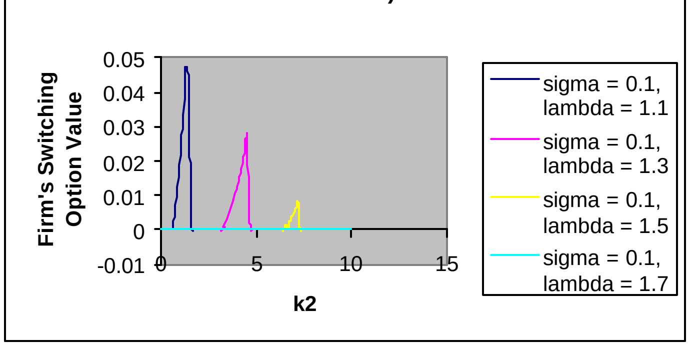


Figure 5b (mu = 0, beta = 0.025, initial price = 1.5, k1 = 0.01)



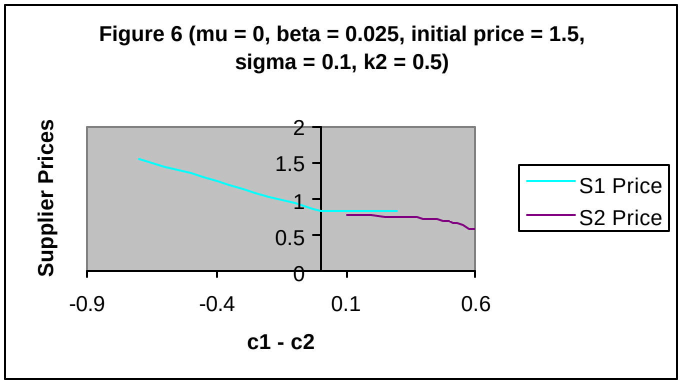


Figure 6 (mu = 0, beta = 0.025, initial price = 1.5, sigma = 0.1, k2 = 0.5)
Supplier Prices
2
1.5
1
0.5
0
-0.9
-0.4
0.1
0.6
c1 - c2
S1 Price
S2 Price

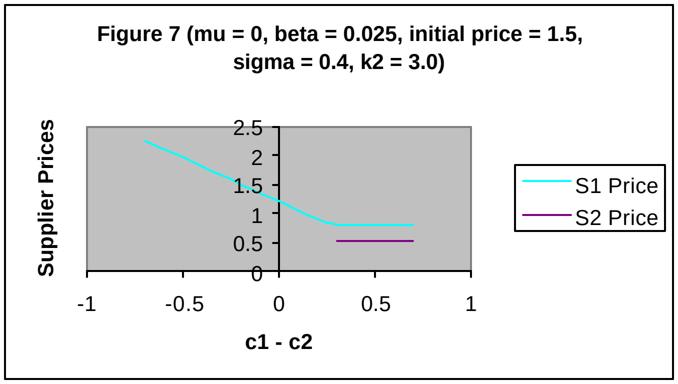


Figure 7 (mu = 0, beta = 0.025, initial price = 1.5, sigma = 0.4, k2 = 3.0)
Supplier Prices
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1
c1 - c2
S1 Price
S2 Price