Real Options Valuation of Companies Run by Theory of Constraints

Ricardo Ratner Rochman*
FGV-EAESP, Brazil

This version: April 29, 2002

Abstract

The valuation of companies managed by theory of constraints is difficult due to the flexibilities inherent to the system, like the exploration of new markets, the expansion or shrinkage of production, the modification made in the products, etc. Traditional valuation models like the net present value or discounted cash flow do not work suitably because they ignore the flexibility management has to revise its decisions. So we present a framework using real options valuation, more specifically the binomial model, to value the company because it considers the flexibilities of the system and has the assumption that management is proactive. The correct use of both theories together result in optimization of decision making in the short and long run and the consequent creation of value for the shareholders.

---

* The author gratefully acknowledges for the helpful support of the Finance Research Center of the Getúlio Vargas Foundation (CEF-GV). Comments can be addressed to the author at : rrochman@fgvsp.br

Introduction

The valuation of companies managed by theory of constraints is difficult due to the flexibilities inherent to the system, like the exploration of new markets, the expansion or shrinkage of production, the modification made in the products, etc. Traditional valuation models like the net present value or discounted cash flow do not work suitably because they ignore the flexibility management has to revise its decisions. So we employ real options valuation to value the company because it considers the flexibilities of the system and has the assumption that management is proactive. We begin this article with an discussion of the theory of constraints and its managerial system called throughput accounting. Then we present real options and how to value the company managed by the theory of constraints using the real options vauation model.

Theory of Constraints

The theory of constraints (TOC) views a company as a system, whose goal (Goldratt and Cox, 1993) is to make money now and in the future, subject to the conditions (Goldratt, 1994): security and satisfaction for employees, and customer satisfaction. To accomplish this goal the company must recognize and manage its bottlenecks - or constraints - in order to maximize its throughput, that is defined by Goldratt and Cox (1993) as the rate at which the organization makes money through sales. TOC considers a constraint anything that might prevent a system from achieving its goal.

The theory of constraints should be used as a dynamic process and managers can improve throughput by following the steps:

(1) Identify the system's constraint(s);
(2) Decide how to exploit the system's constraint(s);
(3) Subordinate everything else to the above decision;
(4) Elevate the system's constraint(s);
(5) If, in the previous steps, a constraint has been broken, go back to Step 1. But do not allow inertia to cause a system constraint.

The constraints might be external like a market, a limit imposed by the government; or internal like a machine, a company policy, a control or accounting system, for instance.
To put in practice the theory of constraints concepts the company needs a management accounting system, which is called throughput accounting in the TOC world.

Throughput Accounting

Throughput accounting is a very simple system that employs only five measures defined by Corbett (1998):

- Throughput (T): the rate at which the system generates money through sales, or all the money that enters the company minus what it paid to its vendors. Another way to view this definition is the money that the company generated minus the money generated by other companies (vendors). Throughput per unit of each product can be easily calculated by the subtraction of the Totally Variable Cost (TVC) from its selling price. We can understand TVC as the cost that varies for every extra unit produced. For manufacturing and other companies it is composed only by the raw material employed in the product.
- Investment or Inventory (I): all the money the system invests in purchasing things the system intends to sell. The total investment is constituted of the company's assets: buildings, machines, and inventory, for instance. The main discrepancy between throughput and conventional accounting is the way we value work in process and finished goods inventory. According to throughput accounting these items should assign just the price – called totally variable cost - that we paid to the vendors for the material and purchased parts that were added or assembled in the product. Unlike conventional accounting systems there is no added value in the inventory, not even direct labor.
- Operating Expenses (OE): all the money the system spends in turning investment into throughput. The total operating expenses comprises total wages and benefits paid to the company's employees (including direct labor and management), interest paid to creditors, depreciation, fuel, electricity, rentals, etc. Corbett (2000) remarks that operating expenses, theirs increases and decreases should be analyzed on a case by case basis, and its impact in the bottom line taken into account. Operating expenses are not fixed costs because there is no such classification or others (variable, indirect, direct, etc.) in the theory of constraints and throughput accounting.

- Net Profit (NP): simply defined as total throughput minus total operating expenses (NP = T – OE). This is a measure of profitability used by management in decision-making, and can be compared – with some adjustments – to the free cash flow generated by the system minus the interest paid to creditors (here we consider that there are no changes in the working capital and the all capital expenditures in the period are equal to the depreciation).
- Return on Investment (ROI): is defined as the net profit (NP) divided by the total investment (I). As stated by Corbett (2000), any decision that has a positive impact on ROI moves the company toward its goal. Moreover, the one who decides if it is a good decision or not, is ROI. The reason for this is that ROI reflects the interdependencies between throughput, operating expenses and inventory. The return on investment can be expressed by the formulas: ROI = NP / I; or ROI = (T - OE) / I.

The fact that throughput accounting has only these five measures and performs no cost allocation, makes it very simple and attractive not only for managerial use but also for the whole company in general.

To illustrate the use of these measures and how throughput accounting works in practice we will develop the following examples.

Consider a company that manufactures only two products, P1 and P2, and has two machines, M1 and M2. The weekly capacity available of machines M1 and M2 is 2.400 minutes (5 days/week x 8 hours/day x 60 minutes/hour).

Each unit of product P1 spends 2 minutes in machine M1 and 2 minutes in machine M2 to be ready to deliver. Product P2 is simpler than P1 meaning it is manufactured only in machine M1 where each unit of P2 spends 4 minutes to be finished. The market where the company is inserted demands 500 units of P1 and 400 units of P2 per week. Price and totally variable cost of each product is shown on table 1:

Products	P1	P2
Market Demand (units)	500	400
Price per Unit ($)	100	80
TVC per Unit ($)	60	40
Throughput per Unit ($)	40	40

Table 1. Products P1 and P2 market data.

We can observe in the table 1 that both P1 and P2 have the same throughput per unit. This fact might lead someone to be indifferent on the number of units to be produced of each product.

To meet the whole market demands the company will need more capacity of machine M1 (table 2) than it has available, so in our example M1 is the constraint of the system.

Capacity Demanded by the Market	M1	M2
P1	1000	1000
P2	1600	0
Total (minutes / week)	2600	1000

Table 2. Capacity demanded to supply everything the market demands.

Being the system's bottleneck M1 dictates the flow of the whole system and the throughput (T) as well. Therefore to determine the production and sales mix of products P1 and P2 we have firstly to manufacture the product that generates the highest throughput per minute on the constraint, and after that we manufacture the product with the second highest throughput per minute on the constraint, and so on. Table 3 shows the calculation of throughput per time spent on the constraint for P1 and P2:

Product	Throughput per unit ($)	Time spent on the constraint (minutes)	Throughput / Time spent on the constraint ($ / minutes)
P1	40	2	20
P2	40	4	10

Table 3. Throughput per time spent on the constraint by each product.

Notice that we do not have to worry about the other machine (M2) because it is not a constraint so it has capacity available to meet the market demands. Based on table 3, we should produce all we can and the market demands of product P1 because it provides a greater throughput per time on the constraint than product P2. Considering that and the capacity available, the sales and production mix is defined below in table 4:

Product	Market Demand (units)	Capacity required on the constraint (minutes/week)	Production Capacity Available (minutes/week)	Sales Mix (units)
P1	500	1000	1000	500
P2	400	1600	1400	350

Table 4. Sales mix calculation.

Once the sales mix is defined we have just to multiply the product units (table 4) by the respective throughput per unit (table 3) to obtain the total throughput. Suppose that the operating expenses per week are $4.000 and the investment per week is $100.000. With this additional data we can calculate the net profit and return on investment of the system, as shown on table 5:

Product	Sales Mix (units)	Throughput per Unit ($)	Total ($)
P1	500	40	20000
P2	350	40	14000
		Throughput (T)	34000
		Operating Expenses (OE)	4000
		Net Profit (NP = T - OE)	30000
		Investment (I)	100000
		Return on Investment (ROI = NP / I)	30.0%

Table 5. Throughput, Net Profit and ROI calculation.

Following with our example, now management knows how much they have to return to the shareholders. Therefore they have to search new ways to increase the system's total throughput.

One sales manager realizes that it is possible to increase the price of the product P1 by 10% if minor changes were made to the product. So he has met the production manager who explained him that these minor changes include an additional part to the product, which increases the totally variable cost by $4 per unit, and hiring quality assurance personnel, which increases the operating expenses by $500 per week.

This new scenario does not change the constraint of the system (machine M1) but changes the basic product data as seen on table 6:

Products	P1	P2
Market Demand (units)	500	400
Price per Unit ($)	110	80
TVC per Unit ($)	64	40
Throughput per Unit ($)	46	40

Table 6. Adjusted products P1 and P2 data.

Since throughput per unit was changed, we have to recalculate the throughput per time spent on the constraint (table 7), because it is the key to define the company's sales mix.

Product	Throughput per unit ($)	Time spent on the constraint (minutes)	Throughput / Time spent on the constraint ($ / minutes)
P1	46	2	23
P2	40	4	10

Table 7. Throughput per time spent on the constraint by each product.

Again product P1 is the first to be produced because it has the greatest throughput per time spent on the constraint so our sales mix remains unchanged (see table 4). Notice that in this case any other mix of products P1 and P2, respecting the system's capacity available, will not provide the maximum total throughput.

We proceed then to the calculation of net profit and return on investment:

Product	Sales Mix (units)	Throughput per Unit ($)	Total ($)
P1	500	46	23000
P2	350	40	14000
		Throughput (T)	37000
		Operating Expenses (OE)	4500
		Net Profit (NP = T - OE)	32500
		Investment (I)	100000
		Return on Investment (ROI = NP / I)	32.5%

Table 8. Throughput, Net Profit and ROI calculation.

Despite the fact that operating expensed increased by $500 and a new part was added to each product P1, the net profit and ROI increased in comparison to the previous example.

Real Options

The flexibility that managers have to adapt, revise and change capital budgeting decisions in the future based on the arrival of new information that resolve things, which are uncertain today, are called real options. They called this to distinguish them from option contracts traded in the financial markets. An option is the right, but not the obligation, to take an action at a predetermined cost (the exercise price), for a predetermined period of time (the life of the option), as an example we have an option to expand a factory if market conditions turn to be favorable.

In the presence of uncertainty, real options can significantly increase the value of a project because they eliminate unfavorable outcomes and the project becomes less risky. For instance, an option to close a mine of copper if the metal's price is unattractive, or an option to abandon a pharmaceutical research if the outcomes from tests are unfavorable.

The advantage of real options, and real option valuation that will be discussed later, is that they capture the value of flexibility. Real options capture also synergies and interdependencies between the company and markets, providing insights on how management can act to improve and preserve value. As put by Luehrman (1998b), a business strategy is much more like a series of options than it is like a series of static cash flows.

According to Copeland and Keenan (1998) we can classify individual real options into three categories: growth/investment options, deferral/learning options, and abandonment/shrinkage options. Growth options are those that allow the company to expand, switch up, or scale up the rate of production by incurring a follow-on cost if market conditions turn out more favorable. Deferral options enables management to delay investments until more information or skill is acquired. Abandonment and shrinkage options allow management to contract, switch down, or scale down a project if new information changes the expected payoffs limiting losses. The individual options can be compounded sequentially or simultaneously so that the value of the compound option is contingent on the value of other options. Options that have many sources of uncertainty like for instance the price and demand of oil are called rainbow options.

Once recognized the existence of real options in the project or capital budgeting opportunity it is need to choose a method to evaluate its value. The discounted cash flow (DCF) or net present value (NPV) method forecasts the project's expected future cash flows and discounts these cash by an appropriate rate that reflects the time value of money and project's risk. The main problem with DCF method is that it ignores flexibility. With DCF, once decision was made management becomes passive to the arrival of new information. In other words, DCF sterilizes future prospects of all problems of timing and risk to deliver a single measure of value. By mapping out all feasible managerial actions based on the possible states of nature, decision tree analysis (DTA) attempts to capture the value of flexibility but the main problem of DTA is that it does not offer an easy way to estimate the appropriate discount rate to be used in the calculation of the value of the project.

On the other hand, real options valuation (ROV) benefits from the structure of DTA and solves the problem of estimation of the discount rate by using the no-arbitrage principle (or law of one price) and techniques like risk-neutral valuation borrowed from the financial options theory. Therefore ROV is the right method to employ when valuing the flexibility of modifying future investment decisions. ROV can reflect the interactions among competitors in a given industry. To value an option (Luehrman, 1998a) we require the following information:

- The present value of a project's operating assets to be acquired (current value of the underlying asset);
- The value lost over duration of option (cash flows and/or dividends);
- The length of time the decision may be deferred (time to expiration or maturity date);
- The expenditure required to acquire the project assets (exercise price or strike price);
- The time value of money (risk-free rate of interest);
- The uncertainty of the expected cash flows (volatility of the underlying asset).

The variables above affect the value of an option and the proper management of their value can improve the project's value. Dixit and Pindyck (1994), Trigeorgis (1996), Copeland and Antikarov (2001) present many types and models of valuation of real options including the compound and rainbow ones.

As stated by Leslie and Michaels (1997), real options provide certain reactive flexibilities on its holder meaning the options to grow, learn or abandon the project in response to new information. The reactive flexibilities are only related to the advantages of the real options valuation over other methods. When management recognizes an option it observes a reactive flexibility. More important are the proactive flexibilities, or the flexibility to take action in ways that will improve the value of an option once acquired. These proactive flexibilities are the reflection of the active and strategic role management develops in the company. By altering the variables that affect the value of an option management can enhance the value of the project positively or not. Leslie and Michaels (1997) suggest some actions that influence positively the value of an option:

- Increase the present value of the project's operating assets by, for instance, develop alliances with low-cost suppliers;
- Reduce value lost by waiting to exercise by, for instance, create barriers to competition;
- Extend the length of time of decision making by, for example, innovate to hold technology lead;
- Reduce the expenditures to acquire the project assets by, for example, leverage economies of scale, scope or learning;
- Increase uncertainty of the expected cash flows by, for instance, product innovation/bundling.

Notice that when we increase uncertainty of expected cash flows we increase the value of the option. The cause of that is the presence of the option that benefits from the higher positive outcomes and avoids the negative outcomes.

Valuation of a Company Managed by Theory of Constraints

One of the main features a company managed by the theory of constraints has it is the flexibility to change its operations in order to maximize the throughput system. We interpret this flexibility as real options available to the company, as in the example above management had the option to change the characteristics of product P1 to accomplish the goal of the system according to the theory of constraints. Other real options available to the company are expand the market, switch the technology, alter the suppliers, etc. These options must be exercised if and only if they increase the throughput of the system as a whole.

Since it is not possible to model flexibility using the net present value, or with decision tree analysis unless we proceed with the adequate adjustments as proposed by Smith and Nau (1995), real options valuation is basically the most appropriate method to value a company managed in consonance with the theory of constraints. Prior to the valuation of the company it is necessary to identify and model the uncertainties related to the system. These could be the price of a specific product, the sales volume, the amount of investment, etc. The uncertainties affect the behavior of the constraints of the system forcing management to act and making the exercise of the real options more likely to happen.

Let's show how to perform the valuation of a company that produce only two products named A and B. The system (company) has only one constraint, machine M, and both products are processed in the constraint once during the process flow. The quantity of products A and B demanded by the market are Q_A and Q_B, the total capacity allocated on the constraint is CA_A and CA_B, the total capacity required on the constraint is CR_A and CR_B respectively. The throughput per unit is defined as

Equation 1. $$T_i = P_i - TVC_i$$

where T_i is the throughput per unit of product i, P_i is the net price per unit of product i, and TVC_i is the totally variable costs of product i.

The net profit is obtained through the use of the following formula

Equation 2. $$NP = \left(\sum_{i \in \Omega} \frac{T_i . CA_i}{CR_i} \right) - OE$$

where NP is the net profit and OE are the operating expenses of the system according to the theory of constraints. Ω is the set of products of the system, here this set has two elements, products A and B. The ratio CA_i / CR_i defines the quantity of product i that will be produced. In this paper we assume this ratio will be always inferior or the same as the quantity Q_i of the product i demanded by the market meaning there is always buyers to the products offered by the company.

TOC defines the return on investment (ROI) as

Equation 3. $$ROI = \frac{NP - OE}{I}$$

where I is the total investment of the system.

The goal of the system is to make money now and in the future so it is necessary to maximize ROI, subject to the constraints of the system. At each time, t, management has a set of options, Ψ, which allows the company to achieve its goal. These real options could be expand the constraint, or in other words, replace the machine M by a new one, or export the products to new markets, modify the products by adding features, etc. In this manner we can write the goal as

Equation 4. $$\begin{cases} \text{Maximize ROI} \\ \text{subject to } \Psi \end{cases}$$

So at each time we solve the optimization problem above given the set the options Ψ available to the company. The solution of this problem is represented by the total capacity allocated on the constraint for each product i (CA_i).

Suppose that the net price of product A follows a stochastic diffusion Wiener process of the form

Equation 5. $$dP_A = P_A.\alpha.dt + P_A.\sigma.dW$$

where α is the instantaneous expected return on the net price of A, σ is the instantaneous standard deviation of the net price returns of A, and dW is the differential of a standard Wiener process, with mean 0 and variance 1.

It is possible to model the diffusion process of equation (5) through the use of a multiplicative binomial process or random walk. For what purpose we can use the Cox, Ross and Rubinstein (1979) binomial method or some variation like the Trigeorgis (1991) log-transformed binomial. The net price can also be modeled by a mean-reverting process if the decision maker thinks it suits better the behavior of the variable.

In our example we consider the net price of A as the only uncertainty of the whole system so the performance of the system, as well as the sales mix, is defined by the comportment only by the changes of the net price of the product A and options management can take to maximize the return on investment. Based on the net price of A management defines the sales mix, because with an increase in the price of A the company wishes to produce and sell more units of A than B, because the throughput of the system increases. On the other hand, with a decrease in the price of A the company is inclined to produce and sell more units of B than A. Once the sales mix is defined, management assess the set options available to improve the return on investment. The assessment of the real options may force management to reevaluate the sales mix, because one or more of the options may result in changes on the total investment, the operating expenses or the totally variable costs. This procedure should be followed until we reach the solution of the optimization problem described in equation (4).

To proceed with the valuation of the company it is necessary to translate the solution of the optimization problem into free cash flows this can be achieved by using the theory of constraints measures. The value of the company at time t is given by

Equation 6.
$$V_t = \frac{\left(\sum_{i \in \Omega} \frac{T_i . CA_i^{optimal}}{CR_i} \right) - OE^{optimal}}{k} - I_{adjusted}^{optimal}$$

where Vt is the value of the company at time t, k is the target cost of capital of the company, $CA_i^{optimal}$ is total capacity allocated to product i on the constraint that generates the optimal solution of the problem in equation (4), $OE^{optimal}$ is the operating expenses of the system resultant of the choice of the real option that generates the optimal solution of the problem in equation (4), and $I_{adjusted}^{optimal}$ is the investment resultant of the choice of the real option that generates the optimal solution of the problem in equation (4). Note that $I_{adjusted}^{optimal}$ is an incremental part of the total investment I of the system. The target cost of capital k can be estimated by models like the capital asset pricing model (CAPM) or the arbitrage pricing theory (APT) for instance. In addition to that note that in our example the value of the company is stochastic because the net price of A follows a stochastic process.

Implied in equation (6) we have that the company has an infinite life and the growth of the value at a given time to the infinite is zero.

With equation (6) and a horizon period delimited by the decision maker, we can employ the binomial method to advance with the valuation of the company. The value of the company at each node of the tree is given by equation (6), which is defined by the solution of the maximization problem (4) that evaluates the whole set of real options Ψ available to the company. Once the value of each node is calculated, we start to roll back the tree from the final time (T) until we reach the value of the company at the initial time (0).

At the end of the binomial tree the value of the company is given by equation (6), for instance at the node $V_{t+1,j+1}$ in figure 1, we have

Equation 7. $$V_{t+1,j+1} = \frac{\left(\sum_{i\in\Omega} \frac{T_i^{t+1,j+1}.CA_i^{optimal}}{CR_i}\right) - OE^{optimal}}{k} - I_{adjusted}^{optimal}$$

Equation 8. $$T_A^{t+1,j+1} = P_A^{t+1,j+1} - TVC_A^{t+1,j+1} = P_A^0.u.u.d - TVC_A^{t+1,j+1}$$

where P_A^0 is the net price of product A at the initial date, u and d are the up and down movements of the net price of A respectively. The parameters u and d can be calculated by the method developed by Cox, Ross and Rubinstein (1979), for instance.

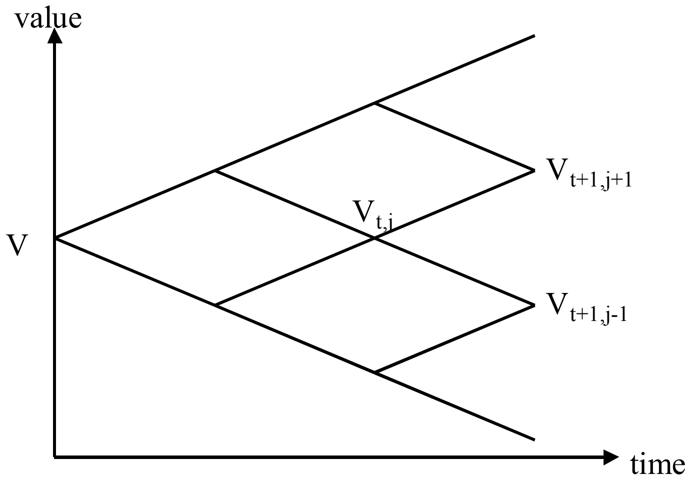

Figure 1. Binomial tree of three periods.

According to the binomial method, node $V_{t,j}$ in figure 1 is obtained by the formula

Equation 9. $$V_{t,j} = \frac{V_{t+1,j+1}.p + V_{t+1,j-1}.(1-p)}{(1+r)^{T/n}}$$

where p is the risk-neutral probability calculated by the Cox, Ross and Rubinstein (1979) binomial method, r is the risk free rate of return, T is the end of period of analysis or investment horizon, and n is the number of periods of the binomial tree, in the figure 1 the binomial tree has n equals to 3.

The value of node $V_{t,j}$ is the maximum between the result of equation (9) and the result of equation (7) adjusted for the parameters t and j, as we can see below

Equation 10. $$V_{t,j} = \text{Max}\left\{ \frac{\left(\sum_{i \in \Omega} \frac{T_i^{t,j}.CA_i^{optimal}}{CR_i}\right) - OE^{optimal}}{k} - I_{adjusted}^{optimal}; \frac{V_{t+1,j+1}.p + V_{t+1,j-1}.(1-p)}{(1+r)^{T/n}} \right\}$$

Equation 11. $$T_A^{t,j} = P_A^{t,j} - TVC_A^{t,j} = P_A^0.u.d - TVC_A^{t,j}$$

This procedure is repeated until we reach the initial node at time 0 and we have the value of the company.

Final Comments

In this article we presented a method to value a company managed by the theory of constraints with the application of the real options theory. While theory of constraints shows how to maximize the throughput of the system and at the same time the return on investment, real options shows how to value companies with flexibilities, which is something inherent and necessary to the use of the theory of constraints. Both theories have the assumption that management must be proactive and know how and when make use of the flexibility existent in the system. The correct use of both theories together result in optimization of decision making in the short and long run. The framework presented here can be extended for cases where there is more than one uncertainty like the quantity demanded by the market or the behavior of the constraints, and be used as a daily tool by decision makers in strategic and tactical sense.

References

COPELAND, T.E., ANTIKAROV, V. *Real Options: A Practitioner's Guide*. New York, NY: Texere, 2001.

COPELAND, T.E., KEENAN, P. T. "How Much Is Flexibility Worth?". *The McKinsey Quarterly*. n.2, p.38-49, 1998.

CORBETT, T. *Throughput accounting*. 1.ed. Great Barrington : North River Press, 1998.

CORBETT, T. Throughput accounting and activity-based costing: the driving factors behind each methodology. *Journal of Cost Management*, New York, v.14, n.1, p.37-45, 2000.

COX, J., ROSS, S., RUBINSTEIN, M. "Option Pricing: A simplified approach". *Journal of Financial Economics*, v. 7, n.3, p.229-263, 1979.

DIXIT, A.K., PINDYCK, R. S. *Investment under Uncertainty*. Princeton, NJ: Princeton University Press, 1994.

GOLDRATT, E.M., COX, J. *The goal*: a process of ongoing improvement. 2.ed. London: Gower, 1993.

GOLDRATT, E.M. *It's not luck*. 1.ed. London: Gower, 1994.

LESLIE, K.J., MICHAELS, M.P. "The Real Power of Real Options". *The McKinsey Quarterly*. n.3, p.4-22, 1997.

LUEHRMAN, T.A. "Investment Opportunities as Real Options: Getting Started on the Numbers". *Harvard Business Review*. v.76, n.4, p.51-62, 1998a.

LUEHRMAN, T.A. "Strategy as a Portfolio of Real Options". *Harvard Business Review*. v.76, n.5, p.89-99, 1998b.

SMITH, J., NAU, R. "Valuing Risky Projects: Option Pricing Theory and Decision Analysis". *Management Science*. v.41, n.5, p.795-816, 1995.

TRIGEORGIS, L. "A log-transformed binomial numerical analysis method for valuing complex multi-option investments". Journal of Financial and Quantitative Analysis. v.26, n.3, p.309-326, 1991.

TRIGEORGIS, L. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. Cambridge, MA: The MIT Press, 1996.