Real Options and Adverse Incentives: Determining the Incentive Compatible Cost-of-Capital

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Abstract

In this paper, we examine the real options approach to capital budgeting in the presence of managerial adverse incentives. We show that real options have the potential to be value enhancing or value destroying depending on managerial incentives. We further examine the possibility of using a generic residual income based rule of managerial compensation to induce the proper investment incentives and we seek to determine the cost-of-capital that must be employed in such a rule.

1. Introduction

This paper is concerned with two issues. The first issue is an interesting dichotomy in the academic finance literature regarding the flexibility of management in making investment decisions. The the second issue is the use of residual income as an accounting basis for managerial compensation contracts.

With regard to the first issue, the real options literature views managerial flexibility in making investment decisions as creating value since it allows firms to capture potential benefits of future investment decisions. These potential benefits, called real options by academics and strategic value or strategic options by corporate executives, represent additional value above what the traditional Net Present Value (NPV) accounts for and proponents of the real options approach to capital budgeting argue that projects with such flexibility should be valued more that similar projects without this flexibility. This view of managerial flexibility being value enhancing is at odds with the literature on agency problems which tends to consider this very same managerial flexibility as potentially destroying value. Managers with incentives that are not aligned to the interests of outside investors such as stockholders and bondholders will use any flexibility that is present to pursue their own goals, usually to the detriment of the outside investors. If these adverse incentives cannot be controlled in some way, then the firm is better off sticking to projects that do not afford any flexibility. This paper is an attempt to combine and reconcile the above two streams of the literature by presenting a model of real options in the presence of the agency problem of adverse incentives. Thus, a key question we address is: *when will the presence of real options be value enhancing and when will it be value reducing?* This is an issue that surprisingly relatively few papers in the literature have considered. One recent paper to address this issue is Cottrell and Calistrate (2000) who examine incentive compensation contracts for optimal technological upgrades by a firm when the upgrades are supplied by an outside source.

Given the presence of real options in capital investment projects, the second issue this paper is concerned with is the use of residual income (such as Economic Value Added (EVA$^{\mathrm{T}}$)[1]) as an accounting measure on which to base managerial compensation in order to induce the right investment incentives. From Fortune Magazine to academic journals in Economics and Operations Management, EVA$^{\mathrm{T}}$ based incentive plans have been touted as a way to align managers' interest with that of shareholders when making investment decisions. Unfortunately, these incentive plans are almost always geared towards guiding the manager into taking only those projects which have traditional NPV that is positive. As Jeffrey Greene (1998) noted, when it comes to valuing real options, the EVA$^{\mathrm{T}}$ approach is deficient. One key problem with residual income based incentive plans is that implementation of such plans require the estimation of a cost of capital, and as Myers and Turnbull (1977) and Sick(1989) have pointed out, when an investment project contains real options, the relevant or so called risk adjusted cost of capital will tend to be stochastic[2]. Furthermore, Rogerson (1997) has noted that using the firm's cost-of-capital does not recognize the fact that managers may have personal hurdle rates that differ from the firm's cost-of-capital. This paper addresses this problem by first determining the incentive compatible cost-of-capital for any given manager when the agency characteristics of that manager are known. As in Rogerson, we then seek to determine if there is a particular cost-of-capital that, in a real options setting, will provide the correct incentives for all managers whose utility increases with the residual income. Contrary to Rogerson, in the prescence

[1]EVA is a registered trademark of Stern Stewart & Co.

[2]This is due to the fact that the relevant beta will now be a weighted average of two different component betas, with the weights themselves being stochastic.

of real options we do not find this to be the case. However, we find that for certain managerial characteristics which might arguably reflect most managers, there is a range of incentive compatible costs-of capital.

The rest of the paper is organized as follows. Section 2 develops the real options capital budgeting model and compares it with the traditional model that is still more widely used (and taught). Section 3 introduces the agency problem of adverse incentives which essentially leads to suboptimal exercise of the real options. It then is demonstrated that if this agency problem is severe enough, the real options NPV of a project may actually fall below the traditional NPV. Hence, real options may not always be value enhancing. Section 3 also introduces the use of residual income as an incentive measure. Section 4 develops this notion of residual income or EVA$^{\mathrm{T}}$, by assuming that one or more of the managerial agency characteristics is unknown. Using numerical examples it demonstrates a range of incentive compatible costs-of capital. Section 5 concludes. proofs are collected in the appendices.

2. The Model

The model we develop in this paper is a variation of the normally distributed cash flow model of real options discussed in Sick (1989). We consider an investment project that is to be undertaken although the analysis developed may be easily used to evaluate the existing assets of the firm. Note that Let the cash flows of the investment project follow the diffusion process:

$$dC_t = \mu dt + \sigma dW_t \tag{2.1}$$

with an initial cash flow $C_0 = C$. Here, C_t represents the operating cash flow level at time t, μ is the drift rate or expected change in the cash flow, σ is volatility of the cash flow changes, and W_t is a standard brownian motion process. The solution to this stochastic differential equation gives that C_t, the random cash flow generated by the investment project at time t, is

$$C_t = C + \mu t + \sigma\sqrt{t}z$$

where z is a standard normal random variable.

Note that the process above allows for the possibility of these cash flows becoming negative as is the case for most situations. This posibility of negative cash flows is important, because it implies that in general, there will not be a constant risk-adjusted rate of return. Valuation of these cash flows must proceed by other means. However, if we assume that financial markets are complete and we denote the market price of the risk embedded in the cash flows as θ, then it is well known that by applying risk neutral valuation, the current market value of these cash flows is given by

$$V_0 = E\left[\int_0^\infty e^{-rt}\eta_t C_t dt\right]$$

where r is the risk free rate, and η_t is the exponential martingale:

$$\eta_t = \exp\left(-\frac{1}{2}\theta t - \theta\sqrt{t}z\right)$$

Typically (for example, see Dixit and Pindyck (1994), page 115) θ will be equal to the market price of risk multiplied by the amount of (market) risk in z (the source risk of the cash flows of the project) where this amount is measured by the correlation between z and the risk embedded in the aggregate market. That is

$$\theta = Corr(z, r_m) \times \frac{(r_m - r)}{\sigma_m}$$

Implementing this model, will therefore require estimation of μ, σ, and θ from operating cash flow data and data from a proxy for the market.

2.1. The Traditional NPV Approach

Consider that the investment opportunity generating these cash flows has a current capital cost of I_0 (if the analysis is on the firm's existing assets, then I_0 represents the current book value of the assets). We assume that once the investment project is operational, its cash flows continue on forever (unless the project is sold for some salvage value which is a special case of what is discussed in the next section). The traditional approach to assessing this project is then to discount the cash flows to infinity and subtract this cost. That is

$$\begin{aligned} NPV &= V_0 - I_0 \\ &= E\left[\int_0^\infty e^{-rt}\eta_t C_t dt\right] - I_0 \\ &= E\left[\int_0^\infty e^{-rt}\eta_t \left(C + \mu t + \sigma\sqrt{t}z\right) dt\right] - I_0 \end{aligned}$$

It is shown in appendix A that this becomes

$$NPV = \frac{C}{r} + \frac{\mu - \theta\sigma}{r^2} - I_0$$

This representation is nothing more than a certainty-equivalent representation, where under certainty equivalence the expected cash flow change becomes $\mu - \theta\sigma$, and cash flows are discounted at the risk free rate. Sick (1989) arrives at a similar representation. It is straight forward to see that if the cash flows are riskless or if their risk is orthorgonal to the market, then the NPV of the project will simply be

$$NPV = \frac{C + (\mu/r)}{r} - I_0$$

2.2. The Real Options Approach

The real options approach recognizes that the investment may have the flexibility (at the discretion of whomever is managing the project) to expand or contract at some point in time in the future as uncertainty is resolved. We model this by allowing the firm to expand (or contract) its stochastic cash flows from C_t to αC_t if the existing cash flows hit an upper (lower) level B at some random time in the future $\widetilde{T}_B$. Note that for bounded expansion (contraction), α satisfies $1 < \alpha \leq M$ $(0 \leq \alpha < 1)$ for some finite number M and that the expansion (contraction) will involve a capital expenditure (recovery) of I_B.

The current market value of the investment's cash flows is given by:

$$\begin{aligned} V_0 &= E\left[\int_0^{\infty} e^{-rt}\eta_t C_t 1_{\{\widetilde{T}_B > t\}} dt\right] \\ &\quad + E\left[\int_0^{\infty} e^{-rt}\eta_t \alpha C_t 1_{\{\widetilde{T}_B \leq t\}} dt\right] \\ &\quad - E\left[e^{-r\widetilde{T}_b}\eta_{\widetilde{T}_B} I_B\right] \end{aligned}$$

The first part of the expression reflects the cash flows prior to expansion (contraction). The second part reflects cash flows after the expansion has occurred at the random time $\widetilde{T}_b$. The third part reflects the investment(recovery) made at time $\widetilde{T}_B$ in order to effect the expansion (contraction). The approach here is general enough to capture abandonment options such as in McDonald and Siegel (1985) or Dixit and Pindyck (1994) by setting α equal to zero and $-I_b$ equal to salvage value. Note, that I_B can be easily generalized to be an increasing function of α or of time. The boundary B is initially assumed to be constant and we will later verify that this assumption is valid.

Using the first passage time density $f(\acute{C}_0, B, t)$ of the cash flow process to the boundary B, we show in appendix C that the evaluation of the above representation for V_0 leads to the below closed form expression:

$$\begin{aligned} V_0 &= \frac{C}{r} + \frac{(\mu - \sigma\theta)}{r^2} \\ &+ (\alpha - 1)\left[\frac{C}{r}G(C,B) + \frac{(\mu - \sigma\theta)}{r^2}\left((1 - K(C,B)) + G(C,B) + rH(C,B)\right)\right] \\ &- I_B G(C,B) \end{aligned}$$

where

$$K(C,B) = \int_0^\infty f(t)dt = \begin{cases} 1 & \text{if } B \geq C \\ \exp\left(2\frac{\mu}{\sigma^2}(B - C)\right) & \text{if } B < C \end{cases}$$

$$G(C,B) = \int_0^\infty e^{-rt} f(t)dt = \begin{cases} \exp\left\{(B - C)\left(\frac{\mu - \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2}\right)\right\} & \text{if } B \geq C \\ \exp\left\{(B - C)\left(\frac{\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2}\right)\right\} & \text{if } B < C \end{cases}$$

and

$$H(C,B) = \int_0^\infty t e^{-rt} f(t)dt = \frac{|B - C|}{\sqrt{\mu^2 + 2r\sigma^2}} G(C,B)$$

The first two parts of the expression for V_0 is the traditional NPV value. The third and fourth parts comprise the real options value of the investment and is analogous to a financial options framework where the option value is the discounted future cash flows net of exercise price conditional on the option being "in-the-money". Note that $G(C,B)$ is the moment generating function of the random time $\widetilde{T}_b$. We can interpret, $|B - C|/(\sqrt{\mu^2 + 2r\sigma^2})$ as the expected time (under certainty-equivalence) to exercising the real option, and $G(C,B)$ as the discounted probability of exercising the option. The amount $rH(C,B)$ represents a time-weighted average of the initial cash flow generated when the real option is exercised.

For the rest of the paper we will focus on an expansion option which implies that $\alpha > 1$, and $B > C$ which implies that $K(C,B) = 1$.

2.3. Optimal Exercise of Real Options

As mentioned in the introduction, the real options approach considers managerial flexibility to be value enhancing. This is because the real options approach implicitly assumes that any real options that are present will be exercised optimally. Assuming this to be true, we can determine the optimal exercise boundary and consequently, the "first best" value of the project.

At first blush, it may appear that the optimal boundary should be that which maximizes the project's value, i.e. that B which solves $\max_B V_0$, where V_0 is as given above. However, such a boundary cannot be constant and at the same time dynamically consistent. This is because, the boundary obtained this way will in general depend on the current level of cash flows. As time passes, cash flow levels change implying that the boundary level will also change, hence the dynamic inconsistency. To obtain an optimal constant boundary that is dynamically consistent, we apply the smooth pasting condition to the real option valuation expression. This condition is the requirement that at the optimal boundary the connection between the value of the real option before expansion connects and the value of the real option after expansion is tangential which means that their derivatives (w.r.t. cash flows) equate. After expansion, the real option value is simply the NPV of the expansion which is

$$NPV_{\text{expansion}} = (\alpha - 1)\left(\frac{C}{r} + \frac{\mu - \theta\sigma}{r^2}\right) - I_B$$

The derivative of this w.r.t. cash flows is $(\alpha - 1)/r$. If we denote the real option value as ROV, then the smooth pasting condition is

$$\left.\frac{\partial ROV}{\partial C}\right|_{C=B} = \frac{(\alpha - 1)}{r}$$

In appendix C, we show that this optimal boundary, denoted as B^*, is

$$B^* = \frac{rI_B}{(\alpha - 1)} - \left(\frac{(\mu - \sigma\theta)}{r^2}\right) + \left(\frac{\mu - \theta\sigma}{r^2}\right)\left(\frac{r\sigma^2}{\mu\sqrt{\mu^2 + 2r\sigma^2} - \mu^2 + 2r\sigma^2}\right)$$

Note that under the traditional NPV rule, expansion will occur once the $NPV_{\text{expansion}}$ is positive. Using this, we get that the boundary for this rule denoted as B^{NPV}, is

$$B^{NPV} = \frac{rI_B}{(\alpha - 1)} - \left(\frac{(\mu - \sigma\theta)}{r^2}\right)$$

Comparing this with the real options boundary, we can see that it is generally not optimal to invest as soon as the NPV of expansion is positive and that the more volatile the cash flows of the project (i.e. greater σ), the longer is the optimal waiting period. We should also note that when $C = B$, then $G(C, B) = 1$

and $H(C,B) = 0$ which results in the expected conclusion that at the optimal boundary

$$ROV|_{C=B} = (\alpha - 1)\left[\frac{B}{r} + \frac{(\mu - \sigma\theta)}{r^2}\right] - I_B = NPV_{\text{expansion}}$$

Figure 1 shows how the value of the project (or firm) changes as the boundary changes given the following parameters:

- Initial cash flow, $C = 100$
- Risk free interest rate, $r = 0.05$
- Expected periodic cash flow change, $\mu = 5$
- Volatility of cash flow changes, $\sigma = 3$
- Price of risk of cash flows, $\theta = 0.12$
- Expansion factor, $\alpha = 1.5$
- Initial Capital Expenditure, $I_0 = 2500$
- Add-on Capital Expenditure, $I_B = 2500$

Using the representation above, the optimal expansion boundary is 249.2, giving a project (net) value of 1472 as compared to its traditional NPV value of 1308 and a real options value of 164.2. Note that if the project is to be expanded when cash flows are between 100 and 160, the value of the project will fall below 1308. In other words, for premature early exercise, the value of the firm may actually fall below the traditional NPV value. Thus, if managers have the incentive to exercise early, then real options may actually be value destroying. Table 1 shows how the firm value and optimal boundary is affected by I_B, μ, σ, θ, and α.

We can now see that there is a potential for real options to be advantageous or disadvantageous depending on how aligned or adverse the manager's incentives are. We address the issue of managerial incentives in detail in the next section

3. The Agency Problem of Adverse Incentives

There are many reasons why managers of firms may have different incentives to act other than maximizing firm value, and therefore may not exercise these real options optimally. One obvious reason is that although the firm is assumed to have an unlimited life span, managers generally have a limited employment horizon with any particular firm. Not only do individuals have to contend with their own mortality, but gone are the days when employment with a company was a welcome life sentence!

Jensen and Meckling (1976) along with several other authors have suggested that another reason may be that managers have the tendency to want to increase the size of firms even beyond their optimal size. Still another reason is that managers may have different levels of risk aversion than the owners. These reasons may all be summarized and captured by saying that managers have a limited horizon and a particular internal hurdle rate which they use to assess investments. As Rogerson (1997) put it, managers may invest inefficiently "because their personal cost of capital is higher than the firms or because they have a shorter time horizon than the firm".

The impact of this agency problem is that expansion boundary which the a particular manager considers personally optimal will, in general, differ from the "true" optimal boundary B^*. At one extreme, the manager's boundary may be so much larger than B^* that all real option value will effectively be destroyed. Worse, however, is the other extreme where the manager's boundary is so low that not only is all real option valued destroyed, but some traditional NPV value may also be lost.

To account for managerial incentive, we denote the manager's investment horizon as T, and personal hurdle rate as R. As the hurdle rate R captures any risk aversion on the part of the manager, it is reasonable to make the following assumption.

Assumption 1

The manager's utility (from wages and perquisite consumption) is an increasing function of the expected residual cash flows of the firm within the managers time horizon T, and with discounting done at the manager's personal hurdle rate R

Note that by time horizon, we mean a constant limited time frame beyond which the manager is either incapable or unwilling to consider. We *do not* mean a constant future date that approaches as time passes. Hence at any point in time the manager considers up to T years into the future and no more. By residual, we mean after a charge for the capital investment has been deducted. It is obvious that if operating cash flows do not have a capital cost component for the investment made, assumption 1 means that the manager will generally have the incentive to always immediately (and suboptimally) exercise expansion options. In fact, it is precisely due to such adverse incentives that firms base managerial compensation on accounting measures such as residual income or EVA$^{\mathrm{T}}$ which allocate investment expenditures over time. In the context of the model in this paper, the residual income, denoted as Π_t, is defined as:

$$\Pi_t = C_t - R^* I_t$$

where R^* is the relevant cost-of-capital and I_t is the investment level after depreciation. As in Rogerson (1997)[3], we will decompose R^* and I_t in terms of a depreciation rate δ and an imputed interest rate i. That is we assume that

Assumption 2

$$I_t = \begin{cases} I_0 e^{-\delta t} & \text{if } t < T_b \\ \{(I_0 + I_b)e^{-\delta t} & \text{if } t < T_b \end{cases}$$

$$R^* = i(1-\delta)$$

Assumption 2 say that we take the relevant cost-of-capital to be an allocated imputed charge on the undepreciated capital. The rational for this is that at any point in time, the undepreciated capital (or book value) left in the project I_t can be liquidated and employed in some alternative investment, generating a return R^*. This approach is similar to a carrying-cost approach. Note, that according to this approach, firms using the traditional operating income as the residual income will implicitly be setting $i(1-\delta) = \delta$, which implies that $i = \delta/(1-\delta)$.

Given assumptions 1 and 2 the manager will therefore view the net present value of the cash flows of any project as:

$$NPV_0^M = E\left[\int_0^T e^{-Rt}\left(C_t - R^* I_0 e^{-\delta t}\right) 1_{\{\widetilde{T}_b > t\}} dt\right]$$

[3]Rogerson (1997) actually assumed that $R^* = \delta + i(1-\delta)$

$$+E\left[\int_0^T e^{-Rt}\left(\alpha C_t - R^*(I_0+I_b)e^{-\delta t}\right)1_{\{\widetilde{T}_b\le t\}}dt\right]$$

In appendix C we show that this leads to the below closed form expression:

$$\begin{aligned} V_0^M &= \left(\frac{C}{R}+\frac{\mu}{R^2}\right)(1-e^{-RT}) - \frac{R^*I_0}{R+\delta}(1-e^{-(R+\delta)T}) - \frac{\mu T}{R}e^{-RT} \\ &+(\alpha-1)\left[\left(\frac{C}{R}+\frac{\mu}{R^2}\right)(\phi_{2T}-\phi_{1T}) + \frac{\mu}{R}\left(\frac{B-C}{\sqrt{\mu^2+2r\sigma^2}}\phi_{2T} - T\phi_{1T}\right)\right] \\ &-\frac{R^*I_b}{R+\delta}(\phi_{3T}-e^{-\delta T}\phi_{1T}) \end{aligned}$$

where

$$\begin{aligned} \phi_{1T} &= e^{-RT)}F(t,\mu) \\ \phi_{2T} &= e^{(B-C)(\mu-\sqrt{\mu^2+2R\sigma^2})}F(t,\sqrt{\mu^2+2R\sigma^2}) \\ \phi_{3T} &= e^{(B-C)(\mu-\sqrt{\mu^2+2(R+\delta)\sigma^2})}F(t,\sqrt{\mu^2+2(R+\delta)\sigma^2}) \end{aligned}$$

and where $F(t,\mu)$ represents the probability of hitting the boundary by time t, given a drift of μ. That is,

$$F(t,\mu)=F(C,B,t,\mu,\sigma)=N\left(\frac{-(B-C)+\mu t}{\sigma\sqrt{t}}\right)+e^{2\frac{\mu}{\sigma^2}(B-C)}N\left(\frac{-(B-C)-\mu t}{\sigma\sqrt{t}}\right)$$

where $N(\cdot)$ is the cumulative normal density function. It is important to remember that V_0^M represents the manager's personal NPV valuator when evaluating projects. Hence to the manager, the optimal boundary B_M^* will be the cash flow level that maximizes this valuator. By differentiating this valuator with respect to C and imposing the smooth pasting condition, it can be shown directly that B_M^* is linear w.r.t. R^* and has the representation:

$$B_M^* = A_1(R,T) + A_2(R,T,\delta)I_BR^*$$

where (denoting derivatives w.r.t. C with a "prime")

$$A_1(R,T) = \tfrac{1-e^{-RT}}{R(\phi'_{2T}-\phi'_{1T})}\left(\tfrac{2-\alpha}{\alpha-1}\right)+\tfrac{\mu}{R}\left[\left(\tfrac{\phi_{2T}/\sqrt{\mu^2+2R\sigma^2}-T\phi_{1T}}{\phi'_{2T}-\phi'_{1T}}\right)-\tfrac{1}{R}\right]$$

and

$A_2(R,T,\delta) = \frac{R^*}{R+\delta}\left(\frac{\phi'_{3T} - e^{-\delta T}\phi'_{1T}}{\phi'_{2T} - \phi'_{1T}}\right)$

Note that the intercept term $A_1(R,T)$ does not depend on the rate of depreciation. It is the boundary level of cash flows at which the manager would exercise the real option if the allocated capital charge were zero.

3.1. Project Market Values

Suppose that the firm bases managerial compensation on some arbitrary cost-of-capital (such as a CAPM based cost-of-capital) and depreciation rate rather than an incentive aligning cost-of-capital. This corresponds to some arbitrary combination of imputed interest and depreciation rates. Given the possibility of suboptimal exercise of the real options, then if outside investors know the manager's hurdle rate R and time horizon T, as well as the imputed interest and depreciation rates i and δ, they can forecast the level of B^*_M . Since B^*_M will in general differ from the true optimal B^*, the impact of adverse incentives on firm valuation will therefore be that the project will be valued at this level B^*_M and, upon announcement that the firm is taking on the project, an NPV lower than the optimal NPV will be added on to the existing market value of the firm. As pointed out in the previous section, this NPV may actually be lower than the traditional NPV. Figure 2 shows how B^*_M changes as the manager's personal discount rate (or hurdle rate) changes for various time horizons using the parameters $C = 100, I_0 = I_B = 2500, \mu = 5, \sigma = 3, T = 10, \alpha = 1.7, \delta = 0.25$ and $i = 0.15$. Note that for these values of i and δ the implied cost-of capital used to compute the residual income is $R^* = 0.15(1 - 0.25) = 0.11$. We see that managers with 5 or 10 year horizons, there is very little difference in their boundaries for levels of R below 15%. Figure 3 show how the managers boundary responds to depreciation (this relatioship can also be shown analytically). As the rate of depreciation increases, given the manager's finite horizon, there is a greater incentive to exercise the real option early. As the imputed interest charge increases, the manager has the incentive to delay exercise. Thus, the high rates of depreciation can be countered with high imputed interest rates in order to force the manager to optimally exercise the real option.

We can conclude from this that for each pair of time horizons and personal discount rates $\{T, R\}$, there is a curve, representing particular combinations of i and δ for which $B^*_M = B^*$. Along this curve, the project will achieve full market valuation. Hence, if all information is available, the selection of a combination along this curve will be incentive aligning. We describe this curve as the optimal iso-incentive curve for the given parameters $\{T, R\}$. In the next section, we examine the situation when all information is not common knowledge.

4. Incentive Compatible Cost-of-Capital

So far, we have assumed that R and T are known. However, it is likely that at least one of these will be information private to the manager. In this section, we assume that the manager's hurdle rate is known, but that the time horizon is private information. Rogerson (1997) also examines a case managerial characteristics is unobservable and concludes that efficient investment is induced if and only if the firm's existing assets' cost-of-capital is used to allocate investment expenditure. As we know, in the presence of real options, firm's existing assets' cost-of-capital, may no-longer have any meaning in the evaluation of new projects.

To determine the incentive-compatible cost of capital analytically, we would need to ask if there exists a combination of i and δ such that for this combination, $B_M^* = B^*$ for all T. Graphically, this would mean that if we plotted the optimal iso-incentive curves for various time horizons, they would all intersect (or merge) at some point. Figure 4 illustrates this point. However, since B_M^* solves an implicit function, obtaining an analytical result to this regard is difficult if not impossible. Furthermore, numerical examples indicate that this is not the case. Figure 5 plots optimal iso-incentive curves for managerial time horizons of 5 years, 10 years, and 15 years using the parameters from the previous section. As we can see, they do not intersect at a single point. Investors will then have to determine a distribution of T over all managerial types, and (assuming this distribution is independent of the cash flow risk), take the expected value and proceed from this to determining and average incentive-compatible cost-of-capital.

However, we can also see that for time horizons of 5 and 10 years, the optimal iso-incentive curves conincide for a depreciation rate of 20 percent and an imputed interest rates of 9 percent which corresponds to a cost-of-capital of 7.2%. If investors consider these time horizons to be the only probable horizons, then this implies that setting a depreciation rate at 20 percent range and a cost-of-capital at the (matching) 7.2 percentage will induce efficient investment regardless of whether the actual time horizon is known. Note that this combination of depreciation and cost-of-capital is specific to the parameters chosen. The determination of the incentive compatible combination for any particular project will be an empirical exercise depending on the project's parameters.

5. Conclusion

We have shown that having real expansion (or contraction) options in prospective investment projects may be value creating or value destroying depending on the investment incentives of the manager. Under extreme cases, it may be better if the investment project did not contain real options in which case, the manager would have no flexibility to expand (or contract) cash flows at some future date. In general however, adverse incentives in managers may simple reduce the market value of projects below their true optimal real options value which may lead to under-investment. We examined the ability of basing managerial compensation on accounting residual income or EVA$^{\mathrm{T}}$ in order to induce efficient investment when the managers time horizon in private information. We conclude that although there is not a globally efficient incentive compatible cost-of-capital, for plausible managerial time horizons there may be a combination of depreciation and cost-of-capital that will induce efficient investments. Since this rate must be determined numerically, it becomes an empirical exercise to obtain its value for any given project or firm.

Appendix A

The Certainty Equivalent Formulation

Note that η_t is a martingale, which implies that for all t, $E[\eta_t] = 1$, then:

i $E\left[\int_0^\infty e^{-rt}\eta_t C dt\right] = \int_0^\infty e^{-rt} C E[\eta_t]\, dt = C\int_0^\infty e^{-rt} dt = \frac{C}{r}$

ii $E\left[\int_0^\infty e^{-rt}\eta_t \mu t dt\right] = \mu\int_0^\infty t e^{-rt} dt$

$$= \mu\left[t e^{-rt}\Big|_0^\infty + \frac{1}{r}\int_0^\infty e^{-rt} dt\right] = \frac{\mu}{r^2}$$

iii $E\left[\int_0^\infty e^{-rt}\eta_t \sigma\sqrt{t} z dt\right] = \int_0^\infty e^{-rt}\sigma\sqrt{t} E[z\eta_t]\, dt$

Now,

$$\begin{aligned}
E[z\eta_t] &= \int_{-\infty}^{+\infty} \frac{z}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\theta t - \theta\sqrt{t}z\right)\exp\left(-\frac{1}{2}z^2\right) dz \\
&= \int_{-\infty}^{+\infty} (y - \theta\sqrt{t})\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) dy \\
&= -\theta\sqrt{t}\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) dy \\
&= -\theta\sqrt{t}
\end{aligned}$$

where we have used the substitution $y = \theta\sqrt{t} + z$. Therefore,

$$E\left[\int_0^\infty e^{-rt}\eta_t \sigma\sqrt{t} z dt\right] = -\sigma\theta\int_0^\infty t e^{-rt} dt = \frac{-\sigma\theta}{r^2}$$

and putting this all together gives

$$E\left[\int_0^\infty e^{-rt}\eta_t \left(C + \mu t + \sigma\sqrt{t}z\right) dt\right] = \frac{C}{r} + \frac{\mu - \sigma\theta}{r^2}$$

Appendix B

Integrals of First Passage Time Densities

Consider the following diffusion process

$$\begin{aligned} dX &= \mu dt + dW \\ X(0) &= a \end{aligned}$$

Let T_b denot the first passage time of this process to b. The density of the first passage time is given by:

$$f(t) = \frac{|b-a|}{\sqrt{2\pi t^3}} \exp\left\{\frac{-(b-a-\mu t)^2}{2t}\right\}$$

Let the cumulative distribution of the above density be denoted as $F(t)$. Then, following Karatzas and Shreve (1991 page 197), we obtain the following integrals of this distribution:

B(i) $\displaystyle\int_0^\infty f(t)dt = K(a,b) = \begin{cases} 1 & \text{if } b \geq a \\ e^{2\mu(b-a)} & \text{if } b < a \end{cases}$

B(ii) $\displaystyle\int_0^\infty e^{-rt} f(t)dt = G(a,b) = \begin{cases} e^{(b-a)\left(\mu-\sqrt{\mu^2+2r}\right)} & \text{if } b \geq a \\ e^{(b-a)\left(\mu+\sqrt{\mu^2+2r}\right)} & \text{if } b < a \end{cases}$

B(iii) $\displaystyle\int_0^\infty t e^{-rt} f(t)dt = H(a,b) = \int_0^\infty t\frac{|b-a|}{\sqrt{2\pi t^3}} \exp\left\{\frac{-2rt^2-(b-a-\mu t)^2}{2t}\right\} dt$

$$= e^{(b-a)\left(\mu-\sqrt{\mu^2+2r}\right)} \int_0^\infty t\frac{|b-a|}{\sqrt{2\pi t^3}} \exp\left\{\frac{-\left(b-a-t\sqrt{\mu^2+2r}\right)^2}{2t}\right\} dt$$

$= e^{(b-a)\left(\mu-\sqrt{\mu^2+2r}\right)} \displaystyle\int_0^\infty t f(t, \sqrt{\mu^2+2r}) dt$ where $f(t, \sqrt{\mu^2+2r})$ is the first passage time density for a diffusion process with the same volatility as before, but with a drift term equal to $\sqrt{\mu^2+2r}$.

Now, note that $\int_0^\infty tf(t)dt = \int_0^\infty \frac{|b-a|}{\sqrt{2\pi t}} \exp\left\{\frac{-(b-a-\mu t)^2}{2t}\right\} dt$

Multiplying both the numerator and denominator of the exponential argument by $(2\mu)^2$, and making the substitutions

$$\begin{aligned} s &= 2\mu\sqrt{t}, \\ 2b\mu &= \ln(Y_b), \text{ and} \\ 2a\mu &= \ln(Y_a) \end{aligned}$$

gives:

$$\int_0^\infty tf(t)dt = \frac{|b-a|}{Y_a\mu} \int_0^\infty \frac{Y_a}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{(\ln Y_b/Y_a) - \frac{1}{2}s^2}{s}\right)^2\right\} ds$$

Note, as in Leland and Toft(1996),

$$\begin{aligned} \frac{\partial}{\partial s}\left[Y_b N\left(\frac{(\ln Y_b/Y_a) + \frac{1}{2}s^2}{s}\right) - Y_a N\left(\frac{(\ln Y_b/Y_a) - \frac{1}{2}s^2}{s}\right)\right] &= Y_a N'\left(\frac{(\ln Y_b/Y_a) - \frac{1}{2}s^2}{s}\right) \\ &= \frac{Y_a}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{(\ln Y_b/Y_a) - \frac{1}{2}s^2}{s}\right)^2\right\} \end{aligned}$$

Therefore, from the fundamental theorem of integral calculus and :

$$\begin{aligned} \int_0^\infty tf(t)dt &= \frac{|b-a|}{Y_a\mu}\left[Y_b N\left(\frac{(\ln Y_b/Y_a) + \frac{1}{2}s^2}{s}\right) - Y_a N\left(\frac{(\ln Y_b/Y_a) - \frac{1}{2}s^2}{s}\right)\right]\Bigg|_0^\infty \\ &= \begin{cases} \frac{|b-a|}{Y_a\mu} Y_a & \text{if } b \geq a \\ \frac{|b-a|}{Y_a\mu} Y_b & \text{if } b < a \end{cases} \end{aligned}$$

Using this , and substituting $\sqrt{\mu^2+2r}$ for μ gives:

$$H(a,b) = \begin{cases} \frac{|b-a|}{\sqrt{\mu^2+2r}} \exp\left\{(b-a)\left(\mu - \sqrt{\mu^2+2r}\right)\right\} & \text{if } b \geq a \\ \frac{|b-a|}{\sqrt{\mu^2+2r}} \exp\left\{(b-a)\left(\mu + \sqrt{\mu^2+2r}\right)\right\} & \text{if } b < a \end{cases}$$

B(iv) $\int_0^\infty e^{-rt}F(t)dt = -\frac{1}{r}e^{-rt}F(t)\Big|_0^\infty + \frac{1}{r}\int_0^\infty e^{-rt}f(t)dt = \frac{1}{r}G(a,b)$

B(v) $\int_0^\infty te^{-rt}F(t)dt = F(t)\int_0^t ve^{-rv}dv\Big|_{t=0}^{\infty} + \int_0^\infty \left(\int_0^t ve^{-rv}dv\right) f(t)dt$

$$= F(t)\tfrac{1}{r^2}\left(1 - e^{-rt}(rt+1)\right)\Big|_0^\infty - \int_0^\infty \tfrac{1}{r^2}\left(1 - e^{-rt}(rt+1)\right) f(t)dt$$
$$= \tfrac{1}{r^2} - \tfrac{1}{r^2}\left[\int_0^\infty f(t)dt - \int_0^\infty te^{-rt}f(t)dt - \int_0^\infty e^{-rt}f(t)dt\right]$$

$$= \tfrac{1}{r^2}\left(1 - K(a,b)\right) + \tfrac{1}{r^2}\left(G(a,b) + rH(a,b)\right).$$

Appendix C

Real Options Value of Investment Project's Cash Flows

From the process for cash flows given equation (1), we define;

$$X_t = C_t/\sigma$$

Then,

$$\begin{aligned} dX &= \mu dt + dW \\ X_0 &= a = C/\sigma \\ \mu &= \mu/\sigma \end{aligned}$$

and the boundary b for X is

$$b = B/\sigma$$

We can thus use the first-passage time distributions and its accompanying integrals from the previous section.

The value of the cash flows can be re-expressed as

$$\begin{aligned} V_0 = & \int_0^\infty e^{-rt} E\left[\eta_t C_t | \widetilde{T}_b > t\right] P(\widetilde{T}_b > t) dt \\ & + \int_0^\infty e^{-rt} E\left[\eta_t \alpha C_t | \widetilde{T}_b \leq t\right] P(\widetilde{T}_b \leq t) dt \\ & - \int_0^\infty e^{-rt} E\left[\eta_t\right] I_b f(a, b, t) dt \end{aligned}$$

Using the first passage time distribution, this becomes

$$V_0 = \int_0^\infty e^{-rt} E\left[\eta_t C_t\right] dt + (\alpha - 1) \int_0^\infty e^{-rt} E\left[\eta_t C_t\right] F(a, b, t) dt - \int_0^\infty e^{-rt} I_b f(a, b, t) dt$$

The first part of this expression is the traditional NPV value. The second and third parts comprise the real options value of the investment and is analagous to a financial options framework where the option value is the discounted future cash flows net of exercise price conditional on the option being "in-the-money". EVA$^{\mathrm{T}}$luating the second part gives:

$$
\begin{aligned}
(\alpha-1)\int_0^\infty e^{-rt}E\left[\eta_t C_t\right]F(a,b,t)dt &= (\alpha-1)\int_0^\infty e^{-rt}E\left[\eta_t\left(C+\mu t+\sigma\sqrt{t}z\right)\right]F(a,b,t)dt \\
&= (\alpha-1)C\int_0^\infty e^{-rt}F(a,b,t)dt \\
&\quad +(\alpha-1)(\mu-\sigma\theta)\int_0^\infty te^{-rt}F(a,b,t)dt
\end{aligned}
$$

Using the integral evaluations from appendix B results in

$$
(\alpha-1)\int_0^\infty e^{-rt}E\left[\eta_t C_t\right]F(a,b,t)dt = (\alpha-1)\left[\frac{C}{r}G(a,b)+\frac{(\mu-\sigma\theta)}{r^2}\left((1-K(a,b))+G(a,b)+rH(a,b)\right)\right]
$$

Substituting for $a, b,$ and μ gives the desired result.

Optimal Cash Flow Boundary

Assuming that $B > C$ then

$$
ROV = (\alpha-1)\left[\frac{C}{r}G(C,B)+\frac{(\mu-\sigma\theta)}{r^2}\left(G(C,B)+rH(C,B)\right)\right] - I_b G(C,B)
$$

Recall from appendix B that $H(C,B) = \frac{|B-C|}{\sqrt{\mu^2+2r}}G(C,B)$, the smooth pasting condition for the optimal boundary is:

$$
(\alpha-1)\left[\frac{C}{r}\frac{\partial G}{\partial C}+\frac{(\mu-\sigma\theta)}{r^2}\left(\frac{\partial G}{\partial C}+\frac{|B-C|}{\sqrt{\mu^2+2r}}\frac{\partial G}{\partial B}+\frac{1}{\sqrt{\mu^2+2r}}G(C,B)\right)\right]\Bigg|_{C=B} - I_b\frac{\partial G}{\partial B}\Bigg|_{C=B} = \frac{(\alpha-1)}{r}
$$

Note, that $\frac{\partial G}{\partial C} = G \times \frac{(\sqrt{\mu^2+2r}-\mu)}{\sigma^2}$and that $G|_{C=B} = 1$. Substituting and reducing leads to:

$$
B^* = \frac{rI_b}{(\alpha-1)} - \left(\frac{(\mu-\sigma\theta)}{r^2}\right) + \left(\frac{\mu-\theta\sigma}{r^2}\right)\left(\frac{\sigma^2 r}{\mu\sqrt{\mu^2+2r\sigma^2}-\mu^2+2r\sigma^2}\right)
$$

The Managers Problem

The manager is assumed to use a predetermined hurdle rate R and to have a finite horizon T. Therefore, to the manager the Net Present Value of the project's cash flows is

$$\begin{aligned} NPV_0^M &= E\left[\int_0^T e^{-Rt}\left(C_t - R^* I_0 e^{-\delta t}\right) 1_{\{\widetilde{T}_b > t\}} dt\right] \\ &\quad + E\left[\int_0^T e^{-Rt}\left(\alpha C_t - R^*(I_0 + I_b) e^{-\delta t}\right) 1_{\{\widetilde{T}_b \le t\}} dt\right] \end{aligned}$$

Rewriting, this beomes

$$\begin{aligned} NPV_0^M &= E\left[\int_0^T e^{-Rt} C_t dt\right] - E\left[\int_0^T e^{-(R+\delta)t} R^* I_0 dt\right] \\ &\quad + (\alpha - 1) E\left[\int_0^T e^{-Rt} C_t 1_{\{\widetilde{T}_b \le t\}} dt\right] \\ &\quad - E\left[\int_0^T e^{-(R+\delta)t} R^* I_b 1_{\{\widetilde{T}_b \le t\}} dt\right] \end{aligned}$$

As in appendix B, we denote $F(C, B, t, \mu)$ as the cumulative distribution function of the first passage time density for the diffusion process with a drift of μ with intial point C and boundary point B. From Harrison (1990) we find the expression for $F(C, B, t, \mu, \sigma)$ to be:

$$F(C, B, t, \mu, \sigma) = N\left(\frac{-(B-C) - \mu t}{\sigma\sqrt{t}}\right) + e^{2\frac{\mu}{\sigma^2}(B-C)} N\left(\frac{-(B-C) + \mu t}{\sigma\sqrt{t}}\right)$$

where $N(.)$ is the cumulative normal distribution. Where understood, we will suppress the arguments for C, B, and σ. For further reference we note that:

$$\frac{\partial F(C, B, t, \mu, \sigma)}{\partial C} = \frac{2}{\sigma\sqrt{t}} n\left(\frac{-(B-C) - \mu t}{\sigma\sqrt{t}}\right) - 2\frac{\mu}{\sigma^2} e^{2\frac{\mu}{\sigma^2}(B-C)} N\left(\frac{-(B-C) - \mu t}{\sigma\sqrt{t}}\right) \le 0$$

where $n(.)$ is the density function for the normal distribution. Using the approach in appendix B, but integrating the probability densities up to T as opposed to over the entire positive real line, it can be shown that,

C(i) $E\left[\int_0^\infty e^{-Rt}C_t dt\right] = \left[\frac{C}{R} + \frac{\mu}{R^2}\right](1 - e^{-RT}) - \frac{\mu T}{R}e^{-RT}$

C(ii) $E\left[\int_0^T e^{-(R+\delta)t}R^* I_0 dt\right] = \frac{R^* I_0}{R+\delta}(1 - e^{-RT})$

C(iii) $E\left[\int_0^\infty e^{-Rt}C_t \middle| \widetilde{T}_b < tdt\right] = E\left[\int_0^\infty e^{-Rt}(C + \mu t + \sigma\sqrt{t}z)\middle| \widetilde{T}_b \le tdt\right]$

$$= C\int_0^\infty e^{-Rt}F(t,\mu)dt + \mu\int_0^\infty te^{-Rt}F(t,\mu)dt$$

$$= C\left[\frac{e^{(B-C)(\mu-\sqrt{\mu^2+2R})}}{R}F(t,\sqrt{\mu^2+2R}) - \frac{e^{-RT}}{R}F(t,\mu)\right]$$

$$+\mu\left[\frac{e^{(B-C)(\mu-\sqrt{\mu^2+2R})}}{R^2}F(t,\sqrt{\mu^2+2R}) - \frac{e^{-RT}}{R^2}F(t,\mu)(RT+1)\right.$$

$$\left. + \left(\frac{e^{(B-C)(\mu-\sqrt{\mu^2+2R})}}{R}\right)\frac{|B-C|}{\sqrt{\mu^2+2R}}F(t,\sqrt{\mu^2+2R})\right]$$

C(iv) $E\left[\int_0^T e^{-(R+\delta)t}R^* I_b 1_{\{\widetilde{T}_b \le t\}}dt\right] = R^* I_b E\left[\int_0^T e^{-(R+\delta)t}F(t,\mu)dt\right]$

Using the same approach as in the first part of the equation in C(iii), we get that this is equal to

$$= \frac{R^* I_b}{R+\delta}\left[e^{(B-C)(\mu-\sqrt{\mu^2+2(R+\delta)})}F(t,\sqrt{\mu^2+2(R+\delta)}) - e^{-RT}F(t,\mu)\right]$$

If we denote

$$\begin{aligned}
\phi_{1T} &= e^{-RT}F(t,\mu) \\
\phi_{2T} &= e^{(B-C)(\mu-\sqrt{\mu^2+2R})}F(t,\sqrt{\mu^2+2R}) \\
&\text{and} \\
\phi_{3T} &= e^{(B-C)(\mu-\sqrt{\mu^2+2(R+\delta)})}F(t,\sqrt{\mu^2+2(R+\delta)})
\end{aligned}$$

Then we can rewrite the manager's NPV valuator as

$$\begin{aligned}
&\left(\frac{C}{R}+\frac{\mu}{R^2}\right)(1-e^{-RT}) - \frac{R^* I_0}{R+\delta}(1-e^{-(R+\delta)T}) - \frac{\mu T}{R}e^{-RT} \\
&+(\alpha-1)\left[\left(\frac{C}{R}+\frac{\mu}{R^2}\right)(\phi_{2T}-\phi_{1T}) + \frac{\mu}{R}\left(\frac{B-C}{\sqrt{\mu^2+2r}}\phi_{2T} - T\phi_{1T}\right)\right] \\
&-\frac{R^* I_b}{R+\delta}(\phi_{3T} - e^{-\delta T}\phi_{1T})
\end{aligned}$$

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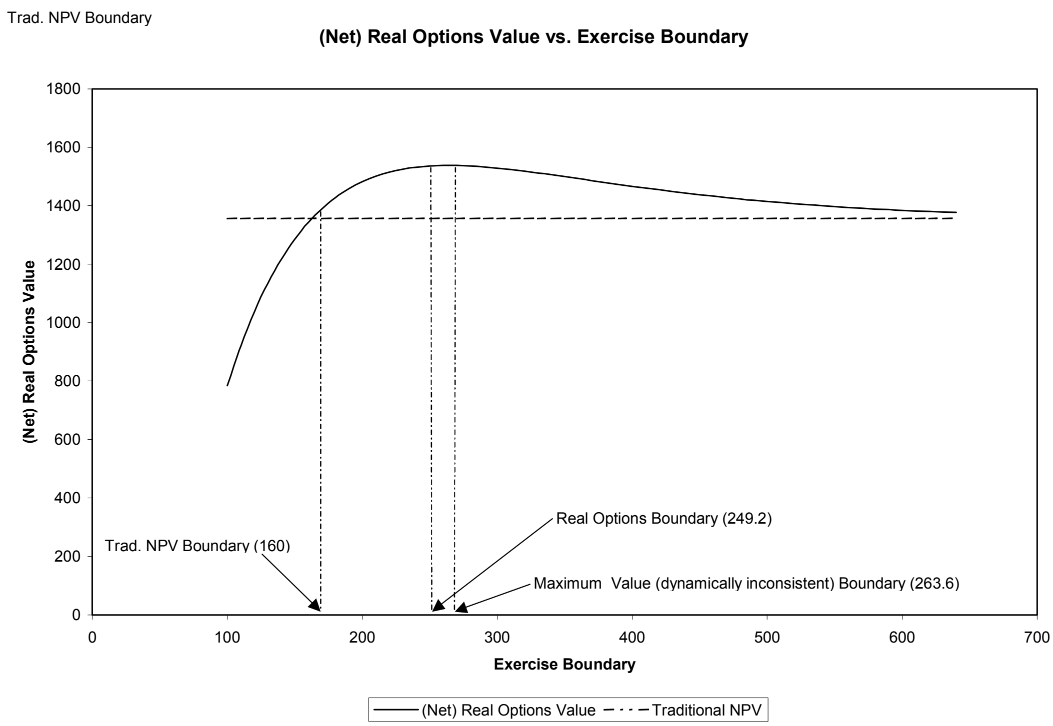

Figure .1:

Figure 1

Table 1: Comparative Statics of the Optimal Expansion Boundary

w.r.t	Direction
Investment Outlay $\left(\frac{\partial B^*}{\partial I_B}\right)$	> 0
Expansion Factor $\left(\frac{\partial B^*}{\partial \alpha}\right)$	< 0
Volatility of Cash Flows $\left(\frac{\partial B^*}{\partial \sigma}\right)$	$>, < 0$
Exp. Change in Cash Flows $\left(\frac{\partial B^*}{\partial \mu}\right)$	< 0
Price of Risk $\left(\frac{\partial B^*}{\partial \theta}\right)$	> 0
Risk free rate $\left(\frac{\partial B^*}{\partial r}\right)$	$>, < 0$

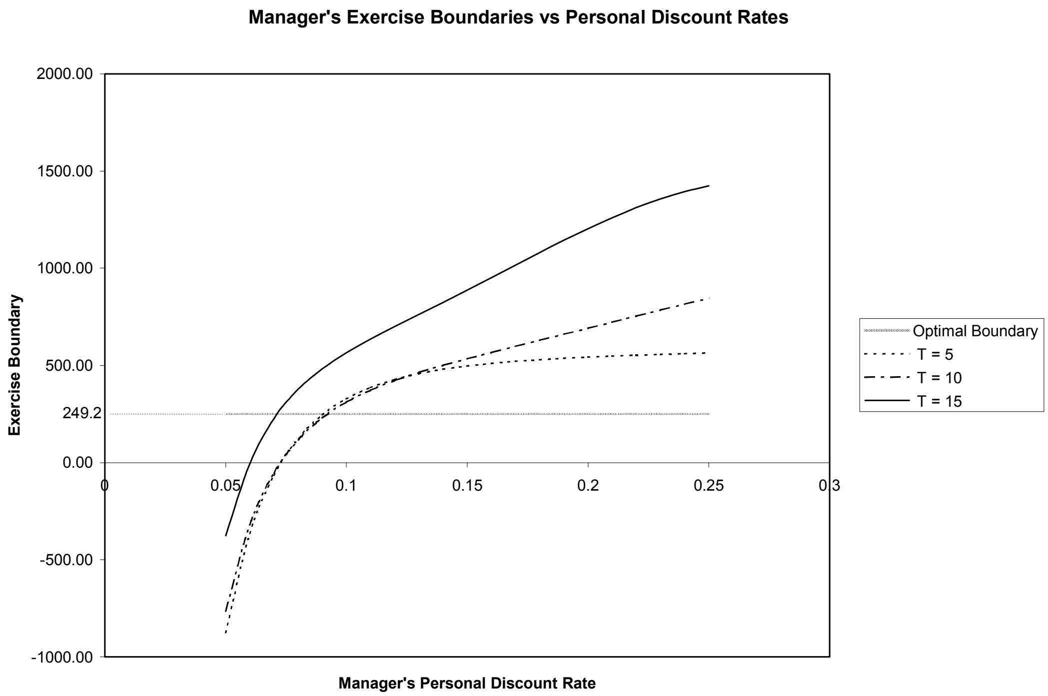

Figure .2:

Figure 2

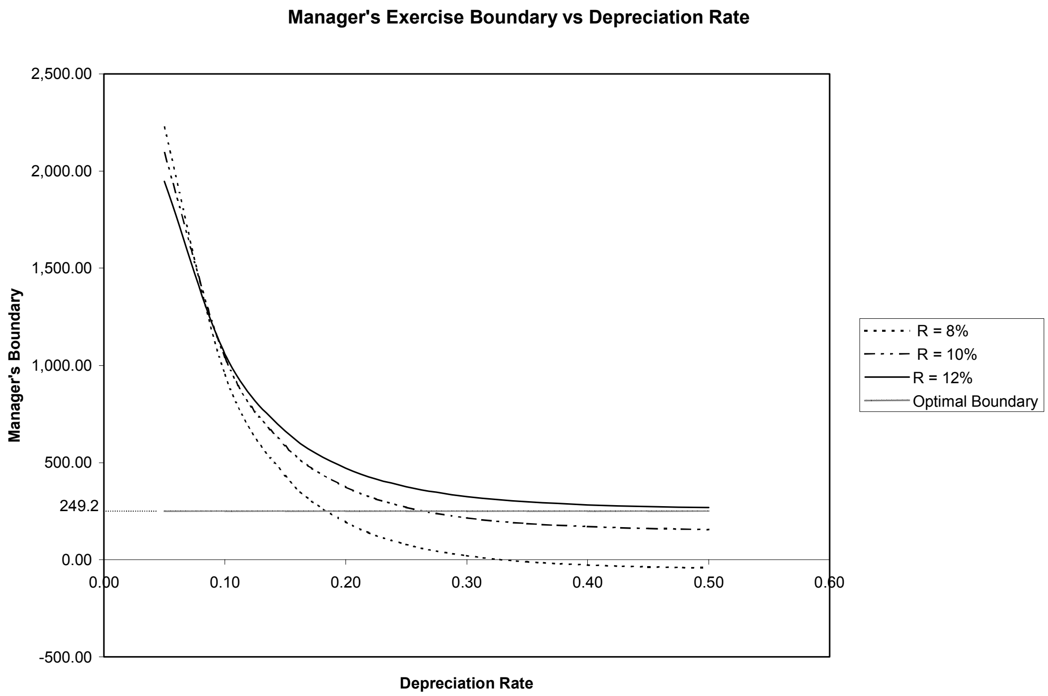

Figure .3:

Figure 3

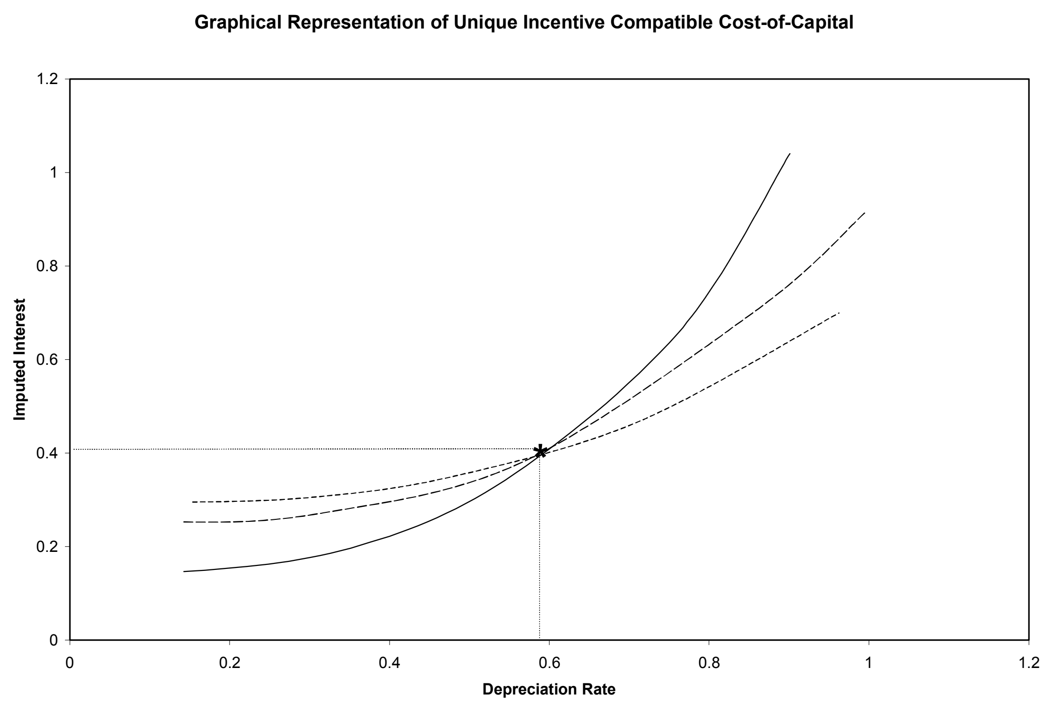

Figure .4:

Figure 4

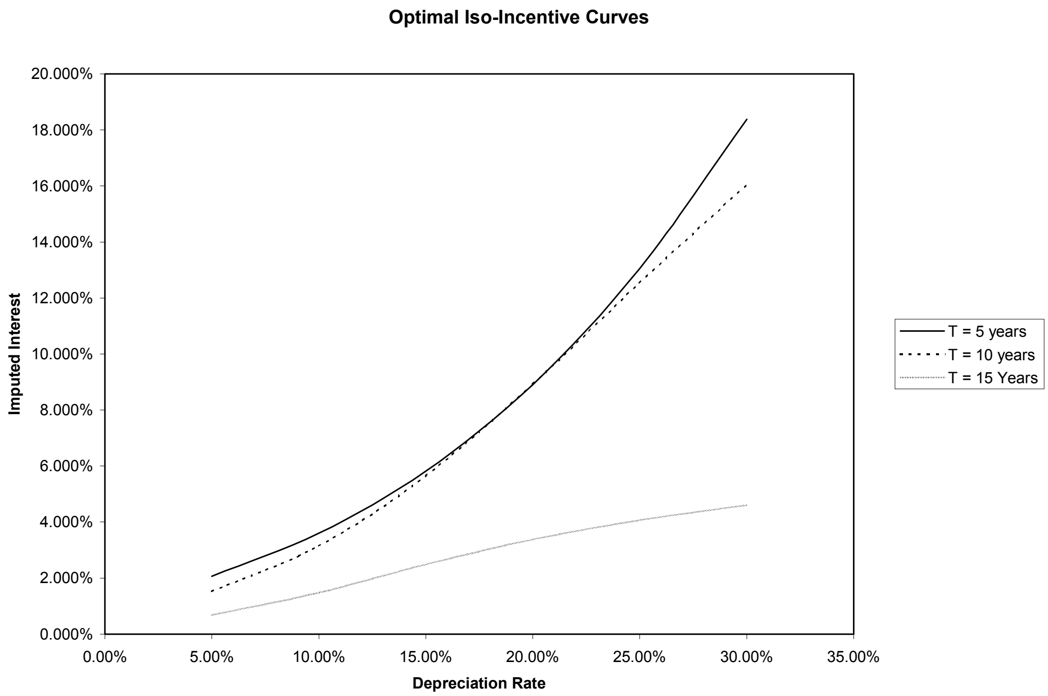

Figure .5:

Figure 5