Asymmetric Information and Irreversible Investments: Competing Agents

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In this paper real option theory and auction theory is combined. A decision maker has a real option consisting of a right to implement an investment project by paying an investment cost. Two or more agents have private information about the constant investment cost. The owner of the project organizes auction, where the privately informed agents participate. The investment strategy, formulated as an optimal stopping problem, is delegated to the winner of the contract. An optimal compensation function is found, which induces the winning agent to follow the investment strategy preferred by the project owner. It is shown that asymmetric information causes an additional wedge between affecting the critical price of implementation, with the inverse hazard rate being a key component.

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1 Introduction

The benchmark model is a classic real option problem: an investor owns a right to invest in a project, and his optimization problem is to find the optimal time to invest, given uncertainty about project cash flows[1]. The purpose of this paper is to analyze what happens if some agents have private information about the project's investment. Thus, an incentive problem is introduced by assuming arises because some agents have private information about the (constant) investment cost, and the project owner needs an expert to manage the investment. The project owner organizes an auction, where the privately informed agents participate. The project owner's problem is to find a contract which optimizes his value of the investment project, given the agents' private information.

In this paper the model in Mæland (2002) is extended. In Mæland (2002) a similar investment problem is studied, but where only one agent has private information. Hence, the problem is analyzed within a principal-agent framework. In this case an optimal contract is found, where the investment decision is delegated to the privately informed agent. Furthermore, a result is that private information may lead to under-investment, compared to the situation of full information.

The problem applies to all types of real options where there is private information. Thus, applications can be found both within corporate valuation and government regulation.

In most cases it is more reasonable to assume that there are more than one agent having private information, than the problem of only one agent with private information discussed in Mæland (2002). One application is a situation where a company owns some oil resources, and needs to rely on a privately informed supplier of technical solutions, in order to exploit the resources. Typically there exists more than one supplier having private information about technical solutions for producing the resources.

In the auction model below we assume that n agents competing about the management of the investment project. The investment cost of each agent may be different, reflecting that the agents' qualifications may not be identical.

The incorporation of competition follows an approach similar to Laffont and Tirole (1987). Laffont and Tirole (1987) assume that the respective agents' private

[1]Such a model is analyzed in Dixit and Pindyck (1994), Bjerksund and Ekern (1990), Paddock, Siegel, and Smith (1988), among others.

information is constant and formulate their model as a *second-price sealed-bid private-values auction*, also called a *Vickrey auction*. In such an auction, each bidder simultaneously submits a bid, without seeing others' bids, and the contract is given to the bidder who makes the best bid. However, the contract is priced according to the second-best bidder. Although we apply a Vickrey auction in the presentation below, it can be shown by the *revenue equivalence theorem* that under the assumptions we use the results does not depend on the organization of the auction[2].

2 Model assumptions

We assume that n agents compete about a contract that gives the winner the right to manage the investment strategy (or more specifically, gives the winner the right to decide on an optimal stopping strategy), and to receive a pre-determined compensation.

Each agent i has private information about his own cost of the investment, K^i, but has no private information about the competitors' costs. We define the competitors' costs by the vector $K^{-i} = (K^1, ..., K^{i-1}, K^{i+1}, ..., K^n)$. The investor is now called the auctioneer (and he is identical to the principal in the principal-agent models). The auctioneer does not observe any of the n agents' investment cost parameter values, but it is common knowledge that the values are drawn independently from the same distribution, having a cumulative distribution function $F(\cdot)$ on the interval $[\underline{K}, \overline{K}]$.[3] We assume that $F(\cdot)$ is absolutely continuous.

[2]The revenue equivalence theorem says that by any auction mechanism in which (*i*) the contract always goes to the buyer with the best bid, and (*ii*) any bidder with the worst bid expects zero surplus, yields the same expected revenue, and results in the same compensation as a function of his report. Thus, when the revenue equivalence compensation is satisfied, the expected outcome from the auction is the same no matter how the auction is organized. See Klemperer (1999), Myerson (1981), Riley and Samuelson (1981) and Vickrey (1961).

[3]The assumptions that the cost parameters are different for the agents, and that the parameter values are independently drawn from the same distribution, are important for the results. An alternative assumption we could make about the agents' information, is that the true value is the same for everyone, but that the agents' have different information about the true value. In this case one agent learns about the true value if he observes another agent's signal. If these assumptions are made, the game is analyzed in a *pure common-value* model, whereas our assumptions about the agents' information above yield a *private-value* model, see an overview of auction theory by Klemperer (1999). We can also assume models where both kinds of information is present, i.e., where the value of an object differs from agent to agent (for example because of subjective valuation), and where at the same time each agent learns more about the

As $F(\cdot)$ is common knowledge, agent i's knowledge about the competitors' true investment cost is identical to the auctioneer's knowledge. We assume that the fraction $F(\cdot)/f(\cdot)$ is non-decreasing.

The option to invest in the project is perpetual. The output value (the value of the "asset in place") from the investment project is denoted S_t, and is known by all the participants in the auction, including the auctioneer. The output value S_t is a stochastic process, defined by a complete probability space $(\Omega, \mathcal{F}, P)$ and state space $(0, \infty)$. Under the equivalent martingale measure Q the stochastic process is given by

$$dS_t = (rS_t - \delta(S_t))dt + \sigma(S_t)dB_t^S, \quad s \equiv S_0. \tag{1}$$

The parameter r denotes the risk free rate, $\delta(\cdot)$ denotes the convenience yield function, $\sigma(\cdot)$ is the volatility function, and B_t^S is a standard Brownian motion with respect to the equivalent martingale measure. The functions $\delta(\cdot) > 0$ and $\sigma(\cdot) > 0$ are Lipschitz continuous. Moreover, the stochastic process in (1) is a linear diffusion.

It is assumed that the investor's information at time t is given by $\mathcal{F}_t^S$, generated by $\{S_\xi, \xi \leq t\}$. Each agent i's information at time t is given by $\mathcal{F}_t^{S,K^i}$ generated by $\{S_\xi, K^i, \xi \leq t\}$.

Define the vector of reports by $\hat{K} = (\hat{K}^1, ..., \hat{K}^n)$. Each agent i's expected compensation $X^i(S_t, \hat{K})$ is received at the time the investment is exercised. Observe that the compensation function may be dependent on the vector of all reports $\hat{K} = (\hat{K}^1, ..., \hat{K}^n)$, in addition to all the observable quantities.

The investment strategy, if agent i wins the contract, is given by the optimal stopping time $\tau_{\hat{K}}^i$, and based on the reports given by the agents, as well as the value of S_t. Moreover, the investment strategy is time independent, as the option to invest is perpetual and S_t is driven by a time-homogeneous stochastic process. We denote the critical price by $S^i(\hat{K})$. When $S_t > S^i(\hat{K})$ the strategy prescribes immediate investment, whereas the investment is postponed if $S_t \leq S^i(\hat{K})$. Note that as the investment strategy $S^i(\hat{K})$ may be dependent on all the cost reports, the investment strategy is stochastic to each agent i.

value from others' signals. Klemperer (1999) refers to any model in which the value depends on some extent on others' bids, as *common-value* models.

The revenue equivalence theorem applies only in the case of a private-value model, or if the bidders' signals are independent.

The auction is organized such that the agents simultaneously report their investment cost $\hat{K} \equiv (\hat{K}^1, ..., \hat{K}^n)$ to the auctioneer. The agents do not know the other agents' reports.

We introduce a control variable $y^i(\cdot)$ that depends on the vector of the agents' reports $\hat{K}$. By $y^i(\cdot)$ the auctioneer decides on the winner of the contract. Thus, the variable can be interpreted as a probability, where $y^i(\hat{K})$ is the probability that agent i wins the contract. We make the following restrictions:

$$\textstyle\sum_{i=1}^n y^i(\hat{K}) \leq 1 \quad \text{for any } \hat{K}, \tag{2}$$

i.e., the sum of each agent's probability of winning the contract cannot exceed one. In addition, as probabilities are always negative, we assume that

$$y^i(\hat{K}) \geq 0 \quad \text{for any } \hat{K}. \tag{3}$$

Incentive mechanisms. We shall see that the results of the auction lead to the same outcome whether it is the auctioneer or the winning agent who decides on the investment strategy. However, in order to solve the problem, we now assume that the auctioneer decides on the investment strategy (i.e., on the optimal stopping time) based on the winning agent's cost report. Thus, the incentive scheme is given by $(X^i(\hat{K}), \tau^i_{\hat{K}}, y^i(\hat{K}))$. As we look for truth telling equilibria, we approach the problem in the same way as for an analogous principal-agent problem. More specifically, we look for mechanisms $(X^i(\hat{K}), \tau^i_{\hat{K}}, y^i(\hat{K}))$ that induce truth telling Bayesian Nash equilibria[4].

For simplicity we assume that the investment cost is not correlated to capital markets.

Agent i's value function $v^i(\cdot)$ is given by the value of the compensation function reduced by the expected investment cost, where the expected investment cost is adjusted for the probability of winning the contract, i.e.,

$$v^i(s, K^i; \hat{K}^i) = E\left[e^{-r\tau^i_{\hat{K}}} \left(X^i(S_{\tau^i_{\hat{K}}}, \hat{K}) - y^i(\hat{K})K^i \right)^+ \middle| \mathcal{F}_0^{S,K^i} \right], \tag{4}$$

[4]In a Bayesian Nash equilibrium each agent's reporting strategy is a function of his own information, and each agent maximizes his value function given the other agents' strategies, and given his beliefs about the other agents' information. In our model the agents' beliefs about the others' private information is given by the probability density $f(\cdot)$ together with the limits $\underline{\theta}$ and $\overline{\theta}$. A Bayesian Nash equilibrium is the appropriate equilibrium concept in auctions because of the presence of asymmetric information.

where $\tau^i_{\hat{K}}$ is a stopping time for agent i. We may note that agent i's value function is identical to the no competition case in in Mæland (2002) when $n = 1$. In the no competition case $y^i(K)$ equals 1. However, when $n > 1$ the investment cost is corrected for the probability that the agent obtains the contract. This implies that the expected cost is lower than under no competition. Furthermore, when $n > 1$, equation (4) shows that the compensation $X^i(\cdot)$ and the investment strategy $\tau^i_{\hat{K}}$ may depend on the competitors' reports as well as the report of each agent i. Hence, when two or more agents compete about a contract the investment strategy and the value of the compensation may be stochastic to the winning agent.

The auctioneer's value function is given by

$$v^P(s;\hat{K}) = E\left[\sum_{i=1}^{n} e^{-r\tau^i_{\hat{K}}} \left(y^i(\hat{K}) S_{\tau^i_{\hat{K}}} - X^i(S_{\tau^i_{\hat{K}}};\hat{K}) \right) \middle| \mathcal{F}^S_0 \right]. \tag{5}$$

The auctioneer's value of the investment depends on the net present value of future cash flows, reduced by the sum of the transfer functions $X^i(\cdot)$. The term $y(\hat{K})S_{\tau^i_{\hat{K}}}$ is the output value the auctioneer obtains at the investment time, adjusted for the probability that agent i wins the contract. To find the auctioneer's expected value of the output from the project, we need to sum up over all the agents participating in the contract, as done in (5). The compensation X^i is the amount paid to each agent i.

The optimization problem. We are now ready to state the auctioneer's optimization problem:

$$V^P(s;\hat{K}) = \sup_{X^i(\cdot),\tau^i,y^i(\cdot)} v^P(s;\hat{K}), \tag{6}$$

subject to each agent i's optimization problem

$$V^i(s,K^i;\hat{K}^i) = \sup_{\hat{K}^i} v^i(s,K^i;\hat{K}^i). \tag{7}$$

Our aim is to find an optimal contract where the winner's investment strategy is delegated to the contract winner, whereas in the above formulation of the optimization problem, each agent only optimizes his value function with respect to the report $\hat{K}^i$. However, this is just a device in order to solve the problem. In section 5 we find an implementable, optimal compensation where the investment decision is delegated to the privately informed winner. In addition to the stopping problems τ^i, the auctioneer optimizes his value function with respect to the compensations $X^i(\cdot)$ and each agent i's probability of winning the auction, $y^i(\cdot)$.

Valuation of the expected, future cash flows. Define $\mathcal{F}_t^{S,K}$ as the information set at time t under full information, generated by $\{S_\xi, K, \xi \leq t\}$. By a result from the theory of linear diffusions[5], the value of the "discounting factor" of agent i is expressed as[6]

$$E\left[e^{-r\tau^i_{\hat{K}}}|\mathcal{F}_0^{S,K}\right] = \begin{cases} \frac{\phi(s)}{\phi(S^i(\hat{K}))} & \text{if } s \leq S^i(\hat{K}) \\ \\ 1 & \text{if } s > S^i(\hat{K}) \end{cases} \tag{8}$$

where $\phi(\cdot)$ is a strictly positive and increasing function. Defining $u(s) = E\left[e^{-r\tau^i_{\hat{K}}}|\mathcal{F}_0^{S,K}\right]$, the value of the discounting factor satisfies the ordinary differential equation

$$\frac{1}{2}(\sigma(s))^2\frac{\partial^2 u}{\partial s^2} + (rs - \delta(s))\frac{\partial u}{\partial s} - ru(s) = 0,$$

with boundaries $\lim_{s\downarrow S^i(\hat{K})} u(s) = 0$ and $\lim_{s\uparrow S^i(\hat{K})} u(s) = 1$. We interpret equation (8) as the value of the discounting factor given that the vector of investment cost reports is known.

Using the result in equation (8), agent i's value function may be formulated as (computed in appendix A.1),

$$\begin{aligned} v^i(s, K^i; \hat{K}^i) \quad = \quad & E\left[\frac{\phi(s)}{\phi(S^i(\hat{K}))}\left(X^i(S^i(\hat{K}), \hat{K}) - y^i(\hat{K})K^i\right)\mathrm{I}_{\left\{s\leq S^i(\hat{K})\right\}}\right. \\ & \left. + \left(X^i(s, \hat{K}) - y^i(\hat{K})K^i\right)\mathrm{I}_{\left\{s>S^i(\hat{K})\right\}}\middle|\,\mathcal{F}_0^{S,K^i}\right]. \end{aligned} \tag{9}$$

As long as $n > 1$, i.e., when we have competition, the direct mechanism now may be stochastic as agent i only observes his own report, and not the others. This means that agent i's value function in the auction model does not consist only of "deterministic" functions, as is the case if $n = 1$.

If the auctioneer does not observe the agents' cost parameter, his value function is given by,

$$\begin{aligned} v^P(s; \hat{K}) \quad = \quad & E\left[\sum_{i=1}^n\left\{\frac{\phi(s)}{\phi(S^i(\hat{K}))}\left(y^i(\hat{K})S^i(\hat{K}) - X^i(S^i(\hat{K}), \hat{K})\right)\mathrm{I}_{\left\{s\leq S^i(\hat{K})\right\}}\right.\right. \\ & \left.\left. + \left(y^i(\hat{K})s - X^i(s, \hat{K})\right)\mathrm{I}_{\left\{s>S^i(\hat{K})\right\}}\right\}\middle|\,\mathcal{F}_0^{S}\right], \end{aligned} \tag{10}$$

[5] A linear diffusion is a one-dimensional, strong Markov process with continuous value paths taking values on an interval, see Borodin and Salminen (1996), ch. II.

[6] Confer Itô and McKean (1965), section 4.6 and Borodin and Salminen (1996), section II.10.

derived in appendix A.2.

The reformulations of the auctioneer's and the agents' respective value functions simplify the optimization problem given by (6) to (7), as the value functions no longer are stochastic with respect to the value of the variable S_t. However, the value functions are still uncertain with respect to the auctioneer's and the agents' respective vectors of unobservable investment cost parameters.

3 The agents' reporting behavior

Similarly to the approach in the principal-agent models we find a truth telling equilibrium, implying that the first-order condition for the report $\hat{K}$ must be satisfied for each agent i at the point where $\hat{K}^i = K^i$, i.e.,

$$\left.\frac{\partial v^i(s, K^i; \hat{K}^i)}{\partial \hat{K}^i}\right|_{\hat{K}^i = K^i} = 0. \tag{11}$$

Hence, for the truth telling condition to hold, reporting the true cost is optimal for each agent i when the condition in (11) is satisfied.

Let now $v^i(s, K^i)$ be each agent i's value function given truth telling. The value function of agent i under truth telling is written as

$$\begin{aligned} v^i(s, K^i) &= E\left[\tfrac{\phi(s)}{\phi(S^i(K))}\left(X^i(S^i(K), K) - y^i(K)K^i\right)\mathrm{I}_{\{s \le S^i(K)\}}\right. \\ &\quad \left.+\left(X^i(s, K) - y^i(K)K^i\right)\mathrm{I}_{\{s > S^i(K)\}}\right| \mathcal{F}_0^{S,K^i}\Big], \end{aligned} \tag{12}$$

which is equal to equation (9) with the exception that the vector $\hat{K}$ is replaced by the vector K.

By the envelope theorem, the first-order condition in (11) is found as

$$\frac{dv^i(s, K^i)}{dK^i} = E\left[-\frac{\phi(s)}{\phi(S^i(K))} y^i(K)\mathrm{I}_{\{s \le S^i(K)\}} - y^i(K)\mathrm{I}_{\{s > S^i(K)\}}\right| \mathcal{F}_0^{S,K^i}\Big]. \tag{13}$$

The second-order condition mimics to the second-order condition for truth telling in Mæland (2002).

Integration of both sides of the first-order condition in (13) leads to an expression of agent i's value of private information,

$$\begin{aligned} & v^i(s,K^i) \\ = \;& E\left[\int_{K^i}^{\overline{K}} \frac{\phi(s)}{\phi(S^i(K^{-i},u))} y^i(K^{-i},u)du\mathrm{I}_{\{s\leq S^i(K)\}} + \left(\int_{K^i}^{\vartheta(s,K^{-i})} y^i(K^{-i},u)du\right.\right. \\ & \left.\left.+\int_{\vartheta(s,K^{-i})}^{\overline{K}} \frac{\phi^i(s)}{\phi^i(S^i(K^{-i},u))} y^i(K^{-i},u)du\right)\mathrm{I}_{\{s>S^i(K)\}}\right| \mathcal{F}_0^{S,K^i}\Big]. \end{aligned} \tag{14}$$

In equation (14) we have formulated agent i's value of private information without including the unknown compensation function $X^i(\cdot)$. Agent i's value of private information differs from the agent's value in the principal-agent model in Mæland (2002) because the auction model adjusts each agent's value of private information for the probability of winning the contract. Also, the value of private information is stochastic as each agent does not observe the other agents' private information.

4 The auctioneer's optimization problem

In this section we solve the auctioneer's optimization problem, i.e., we choose the winner of the auction and find the optimal investment strategy. In order to do so, we approach the problem in the same way as earlier: we substitute the compensation function, $X^i(\cdot)$, by agent i's value function in equation (12). Then the auctioneer's optimization problem in (10) is reformulated as

$$\begin{aligned} & V^P(s,K) \\ = \;& \sup_{S^i(\cdot),y^i(\cdot)} E\left[\sum_{i=1}^n \left\{\frac{\phi(s)}{\phi(S^i(K))} y^i(K)\left(S^i(K)-K^i\right)\mathrm{I}_{\{s\leq S^i(K)\}}\right.\right. \\ & \left.\left. +y^i(K)\left(s-K^i\right)\mathrm{I}_{\{s>S^i(K)\}} - v^i(s,K^i)\right\}\right| \mathcal{F}_0^S\Big], \end{aligned} \tag{15}$$

where $v^i(s,K^i)$ is given by (14).

Observe that the optimization problem could be simplified if the trigger price S^i were dependent only on agent i's cost level K^i, instead the vector of all costs, K. The reason is that if $S^i(K^i)$ equals $S^i(K)$ we can optimize the auctioneer's value with respect to each agent i separately. In appendix A.3, it is shown that this is the optimal solution indeed, i.e., $S^{i*}(K^i) = S^{i*}(K)$, where $S^{i*}(\cdot)$ is defined as the optimal entry threshold of agent i. The idea of this simplification is based on Laffont and Tirole (1987), where a similar argument is used to show that a random incentive scheme is not optimal in the solution of their problem.

The auctioneer's value function is linearly dependent upon the probability that agent i is the winner of the contract, $y^i(K)$. Thus, we can substitute $y^i(K)$ by defining $Y^i(K^i) = E\left[y^i(K)|\mathcal{F}_0^{S,K^i}\right]$ in the optimization problem (15), where the function $Y^i(K^i)$ is interpreted as agent i's probability of winning the contract.

Define $\hat{V}^P(s;K^i) = \sup_{S^i(\cdot),y^i(\cdot)} \hat{v}^P(s;K^i)$ as the auctioneer's optimization problem when $S^i(K)$ is replaced by $S^i(K^i)$. For given $y^i(\cdot)$, and hence for given $Y^i(\cdot)$, the auctioneer's optimization problem (derived in appendix A.4), is given by

$$\begin{aligned} &\hat{V}^P(s;K^i) \\ &= \sup_{S^i(\cdot)} \sum_{i=1}^n \left\{ \int_{\underline{K}}^{\overline{K}} \left[\frac{\phi(s)}{\phi(S^i(K^i))} Y^i(K^i) \left(S^i(K^i) - K^i - \frac{F(K^i)}{f(K^i)} \right) \mathrm{I}_{\{s \le S^i(K^i)\}} \right. \right. \\ &\quad \left. \left. + \left(Y^i(K^i) \left(s - K^i - \frac{F(K^i)}{f(K^i)} \right) \right) \mathrm{I}_{\{s > S^i(K^i)\}} \right] f(K^i) dK^i \right\}. \end{aligned} \tag{16}$$

Observe that we now can separate the problem of finding the optimal critical price $S^{i*}(K^i)$, and the problem of choosing a winner of the contract. This means that the optimal investment strategy is identical to the optimal investment strategy in the principal-agent model in Mæland (2002), as will be seen by optimization of the auctioneer's simplified optimization problem in (16), with respect to $S^i(K^i)$, i.e.,

$$S^{i*}(K^i) - K^i - \frac{F(K^i)}{f(K^i)} = \frac{\phi(S^{i*}(K^i))}{\phi'(S^{i*}(K^i))}. \tag{17}$$

The function $\phi'(S^{i*}(K^i))$ denotes the derivative of $\phi(\cdot)$ with respect to the optimal investment strategy S^{i*}. The left-hand side of equality (17) represents the net value of the auctioneer's payoff at the time when the investment is exercised. The right-hand side is interpreted as the opportunity cost of exercising the option with payoff value equal to $S^{i*}(K^i) - K^i - \frac{F(K^i)}{f(K^i)}$.

The control variable $y^i(K)$ is linear in the auctioneer's problem of finding the investment strategy of agent i. Therefore, we choose an optimal $y^{i*}(K)$ such that

$$y^{i*}(K) = \begin{cases} 1 & \text{if } K^i < \min_{j \neq i} K^j \\ 0 & \text{if } K^i > \min_{j \neq i} K^j. \end{cases} \tag{18}$$

Thus, the agent with the lowest cost wins the contract, provided it is sufficiently low. If $K^i = \min_{j \neq i} K^j$ the auctioneer is indifferent between which agent to choose as a winner of the contract.

As the optimal investment strategy given by (17) equals the optimal investment strategy in the one-agent case, the efficiency is not improved when competition is introduced. However, the winner of the contract in the competition probably has a lower investment cost than the agent in a principal-agent model, and thereby the investment will probably take place at a lower cost. Moreover, if the number of competing agents gets large, the winner's cost level gets close to the lowest possible cost, $\underline{K}$. When the winner's cost level converges to $\underline{K}$, the cumulative distribution $F(\cdot)$ converges to zero, which leads to no inefficiency in the investment strategy.

5 Implementation of the contract

Using (14), (17) and (18), agent i's value of private information is found as

$$V^i(s;K^i) = \begin{cases} \int_{K^i}^{\overline{K}} \frac{\phi(s)}{\phi(S^{i*}(u))} Y^{i*}(u)du & \text{if } s \leq S^{i*}(K^i) \\ \\ \int_{K^i}^{\vartheta^*(s)} Y^{i*}(u)du + \int_{\vartheta^*(s)}^{\overline{K}} \frac{\phi(s)}{\phi(S^{i*}(u))} Y^{i*}(u)du & \text{if } s > S^{i*}(K^i) \end{cases} \tag{19}$$

Hence, we find that agent i's optimal value of the compensation $X^{i*}(s, K^i)$ is given by

$$X^{i*}(s,K^i) = K^i Y^{i*}(K^i) + \int_{K^i}^{\vartheta^{i*}(s)} Y^{i*}(u)du + \int_{\vartheta^{i*}(s)}^{\overline{K}} \frac{\phi(s)}{\phi(S^{i*}(u))} Y^{i*}(u)du, \tag{20}$$

when $s > S^{i*}(K^i)$. Otherwise, $X^{i*}(s,K^i) = 0$. Equation (20) represents the expected compensation of each agent participating in the auction. The main difference between each agent's value of the compensation in the auction and the agent's compensation in the principal-agent model in Mæland (2002), is that the compensation function in the auction model is adjusted for the probability of winning the contract, $Y^{i*}(K^i)$. As the probability is lower than one, each agent's expected compensation in the auction model is lower than in the principal-agent model.

In the above expression of the optimal compensation function, each agent's strategy is optimal based on "average quantities", i.e., the strategy depends on $Y^i(K^i) = E\left[y^i(K)|\mathcal{F}_0^{S,K^i}\right]$ and $S^i(K^i) = E\left[S^i(K)\mathrm{I}_{\{y^i(K)=1\}}|\mathcal{F}_0^{S,K^i}\right]$.

Now, construct a dominant strategy auction[7] where each agent has a reporting strategy that is optimal for any reports by the other agents. We formulate a second-price sealed-bid private values auction (or a Vickrey auction) that implements the optimal investment strategy, and selects the agent with the lowest cost. We denote the compensation function $\tilde{X}^i$, and its value is given by

$$\tilde{X}^i(s,K) = \begin{cases} \vartheta^{i*}(s) + \int_{\vartheta^{i*}(s)}^{K^j} \frac{\phi(s)}{\phi(S^{i*}(u))} du & \text{if } S^{i*}(K^i) < s \le S^{i*}(K^j) \\ \\ K^j & \text{if } s > S^{i*}(K^j), \end{cases} \tag{21}$$

if $K^i = \min_h K^h$ and $K^j = \min_{h \neq i} K^h$. If $s \le S^{i*}(K^i)$, $\tilde{X}^i(s,K) = 0$. Thus, $\tilde{X}^i$ is the optimal and implementable compensation to agent i, given that agent i is the winner of the contract. Note that $\tilde{X}^i$ is the optimal compensation to the winner of the contract, whereas X^{i*} is each agent i's expected compensation of participating in the auction. In appendix A.5 it is shown that each agent's expected value of the compensation function in (21), $E\left[\tilde{X}^i(s,K)|\mathcal{F}_0^{S,K^i}\right]$, equals the value in (20), i.e., $X^{i*}(s,K^i) = E\left[\tilde{X}^i(s,K)|\mathcal{F}_0^{S,K^i}\right]$.

The implementable compensation $\tilde{X}^i$ ensures that the agent having the lowest investment cost obtains the contract. When agent i wins the contract, the agent's compensation equals the value of his private information when the distribution is truncated at K^j. Thus, competition for the best agent amounts to a truncation of the interval $(\underline{K}, \overline{K})$ to $(\underline{K}, K^j)$, where K^j is the second-lowest report.

To sum up, we see that the optimal compensation in (21) is formally identical to the optimal compensation when there is only one agent, given in Mæland (2002), with the exception that the truncation is changed from $\overline{K}$ to the second-lowest report K^j. Truth telling is an optimal strategy, whether there are competing agents or not. The only difference between the principal-agent model and the auction is that the upper level of possible reports is changed, leading to a lower value of the agent's private information.

[7] A dominant strategy auction is an auction in which each agent has a strategy that is optimal for any strategies of its competitors.

6 Numerical illustration of the effect of competition

Under the assumptions that the value of the asset in place is driven by a geometric Brownian motion,

$$dS_t = (r - \delta_S)S_t dt + \sigma_S S_t dW_t,$$

and the unobservable investment cost parameters K^i are uniformly distributed, with

$$f(K^i) = \frac{1}{\overline{K} - \underline{K}},$$

we illustrate some effects of competition. The parameter values used are as follows:

Base case:	The investment cost:	$K^i = 100$
	The lower limit of the investment cost:	$\underline{K} = 50$
	The upper limit of the investment cost:	$\overline{K} = 200$
	The risk-free rate:	$r = 0.05$
	The proportional convenience yield:	$\delta_S = 0.03$
	Volatility of asset in place:	$\sigma_S = 0.10$

The parameter values lead to the following pre-computed constants in the base case:

The probability density, $\underline{K} \leq K^i \leq \overline{K}$:	$f(K^i) = 1/150$
The distribution:	$F(K^i) = 50/150$
The inverse hazard rate:	$F(K^i)/f(K^i) = 50$
The positive root satisfying the ODE:	$\beta = 2$
The entry threshold, full information:	$S^*_{sym}(K^i) = 200$
The entry threshold, asymmetric information:	$S^{i*}(K^i) = 300$
The entry threshold, asymmetric info., $K^i = \overline{K}$:	$S^{i*}(\overline{K}) = 700$

In Figure 1 the winner's compensation function $\tilde{X}^i$ is drawn for different levels of the second-lowest cost report K_j. We assume that agent i is the winner, and that agent j gives the second-lowest cost report. In the case where the cost of the agent with the second-lowest report equals 200, i.e., $K_j = 200$, the winner's compensation is equal to the compensation when we have no competition, i.e., to the principal-agent model in Mæland (2002). The compensation functions are equal in the two models in this case because agent j's cost level coincide

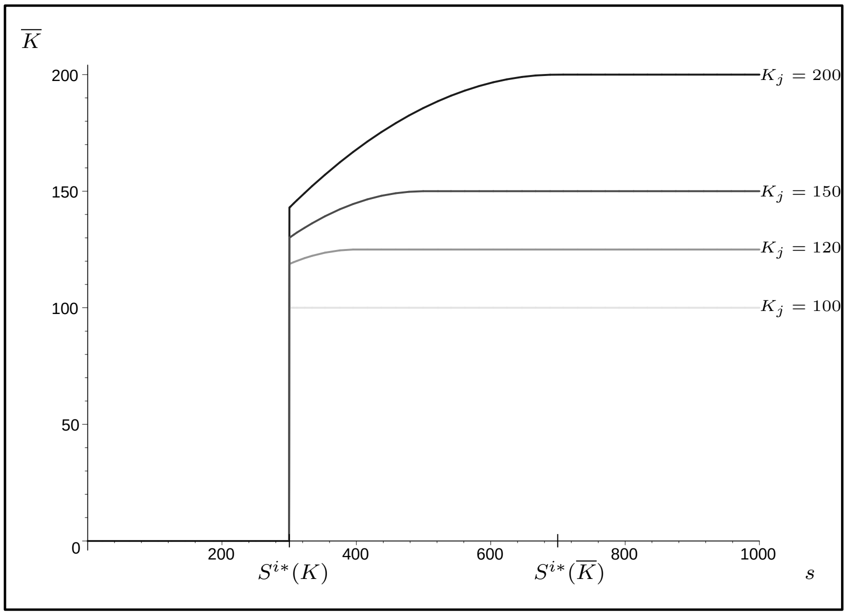

Figure 1: The compensation $\tilde{X}^i$ as a function of the asset value s for different values of second-lowest cost report K_j.

with the upper level cost $\overline{K}$. As agent j's cost level gets closer to the winner's investment cost $K^i = 100$, the value of the agent's private information decreases. Moreover, as agent j's cost level decreases, the interval where the compensation is independent of the asset value s gets larger. This is the effect from reducing the possible cost reports from $[\underline{K}, \overline{K}]$ to $[\underline{K}, K_j]$. In the limiting case, where $K_j = 100$, the winner's value of the contract is zero, as the winner only obtains a compensation equal to his cost level for all asset values s. This situation is illustrated in the lower curve in Figure 1. Observe that although the agent's value is zero, the situation does not necessarily coincide with the full information case (except when $K_j = \underline{K}$) as the optimal investment strategy is not the same as under full information.

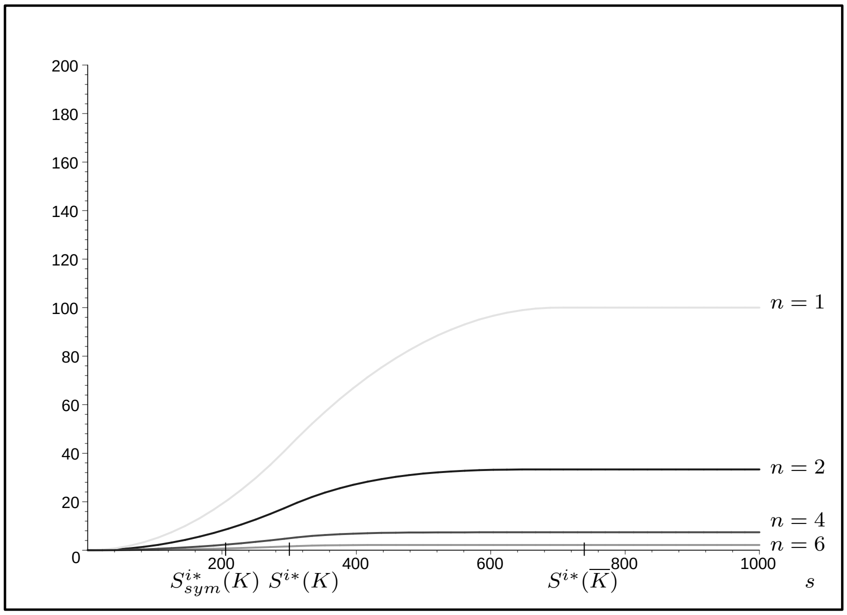

Figure 2: The winner's value V^i as a function of the asset value s. The number of competitors is denoted by n, $K_i = 100$.

Figure 2 illustrates the effect from competition on the agent's value of private information. In the figure we draw four curves representing the winner's contract value when there is no competition ($n = 1$), and when there are 2,4 and 6 competitors, respectively. The value function represented by the upper curve, showing the case of no competition, is identical to the agent's value in the principal-agent model. As the number of competitors increases, the winner's value of the contract falls rapidly. In our example, the winner's value falls by about two thirds when we go from no competition to two competitors. When there are six competitors the value of each auction participant is close to zero.

Figure 3 illustrates the investor's value of the contract under competition. The upper curve is the full information case, whereas the lower curve is the value when there is only one agent having private information. Thus, the lower curve

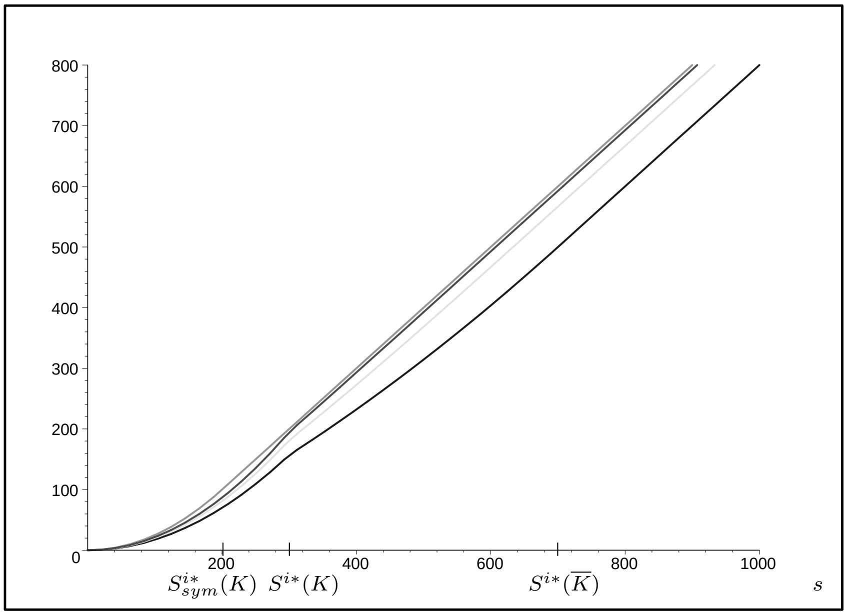

Figure 3: The investor's value $\tilde{V}^P$ as a function of the asset value s. The number of participants in the auction is denoted by n. The upper curve: full information. The lower curve: no competitors. The second-lower curve: $n = 2$. The second-upper curve: $n = 4$. Base case, i.e., $K_i = 100$.

is identical to the investor's value under asymmetric information in the case of no competition. The second-lower curve and the second-upper curve are the investor's values in the case of asymmetric information and when there are two and four competitors, respectively. From Figure 3 we see that as the number of competitors increases the agent's value gets close to zero, implying that the auctioneer's value gets closer to the full information value. However, even when the winner's value is close to zero because of competition, the optimal investment strategy is not efficient as long as the winner's cost is above the lower limit $\underline{K}$. The effect is illustrated in Figure 3. When there are four competitors (corresponding to the second-upper curve) the investor's value almost coincides with the value under full information in the interval where it is optimal to invest immediately, i.e., when $s > S^{i*}(K^i) = 300$. However, in the interval where $s \leq S^{i*}(K^i)$, the difference between the full information case and the auctioneer's value when $n = 4$ is larger.

7 Concluding remarks

In this paper we have extended the principal-agent model in Mæland (2002) to the case of n agents having private information. Similarly to the solutions of the principal-agent models, we find optimal contracts via direct, truthful mechanisms. We have found that the (second-best) optimal investment strategy is not improved because of competition: it is identical whether we have competition or not. The compensation, however, is lower in the case of competition, which implies a higher value of the uninformed owner of the real option.

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A Appendix

A.1 Derivation of agent i's value function in equation (9)

The value function in (9) is found as follows. Equation (4) equals

$$v^i(s, K^i; \hat{K}^i) = E\left[e^{-r\tau^i_{\hat{K}}} \left(X^i(S_{\tau^i_{\hat{K}}}, \hat{K}) - y^i(\hat{K})K^i \right)^+ \middle| \mathcal{F}_0^{S,K^i} \right].$$

By conditional expectations, the value function is formulated as,

$$v^i(s, K^i) = E\left[E\left[e^{-r\tau^i_{\hat{K}}} \left(X^i(S_{\tau^i_{\hat{K}}}, \hat{K}) - y^i(\hat{K})K^i \right)^+ \middle| \mathcal{F}_0^{S,K} \right] \middle| \mathcal{F}_0^{S,K^i} \right].$$

Time-homogeneity implies that the value of the discounting factor can be written independently of the value of the options' payoff, i.e.,

$$v^i(s, K^i) = E\left[E\left[e^{-r\tau^i_{\hat{K}}} \middle| \mathcal{F}_0^{S,K} \right] E\left[\left(X^i(S_{\tau^i_{\hat{K}}}, \hat{K}) - y^i(\hat{K})K^i \right)^+ \middle| \mathcal{F}_0^{S,K} \right] \middle| \mathcal{F}_0^{S,K^i} \right].$$

From equation (8) we know that

$$E\left[e^{-r\tau^i_{\hat{K}}}|\mathcal{F}_0^{S,K}\right] = \frac{\phi(s)}{\phi(S^i(\hat{K}))}\mathrm{I}_{\{s\leq S^i(\hat{K})\}} + 1\mathrm{I}_{\{s>S^i(\hat{K})\}}.$$

We exploit the relationship above, and replace $S_{\tau^i_{\hat{K}}}$ by the critical price $S^i(\hat{K})$, leading to

$$\begin{aligned} v^i(s,K^i) &= E\left[E\left[\tfrac{\phi(s)}{\phi(S^i(\hat{K}))}\left(X^i(S^i(\hat{K}),\hat{K}) - y^i(\hat{K})K^i\right)\mathrm{I}_{\{s\leq S^i(\hat{K})\}}\right.\right. \\ &\qquad \left.\left.+\left(X^i(s,\hat{K}) - y^i(\hat{K})K^i\right)\mathrm{I}_{\{s>S^i(\hat{K})\}}\right|\mathcal{F}_0^{S,K}\right]\left|\mathcal{F}_0^{S,K^i}\right]. \end{aligned}$$

By conditional expectations we obtain,

$$\begin{aligned} v^i(s,K^i) &= E\left[\tfrac{\phi(s)}{\phi(S^i(\hat{K}))}\left(X^i(S^i(\hat{K}),\hat{K}) - y^i(\hat{K})K^i\right)\mathrm{I}_{\{s\leq S^i(\hat{K})\}}\right. \\ &\qquad \left.+\left(X^i(s,\hat{K}) - y^i(\hat{K})K^i\right)\mathrm{I}_{\{s>S^i(\hat{K})\}}\right|\mathcal{F}_0^{S,K^i}\right], \end{aligned}$$

identical to equation (9) in the text.

A.2 Deriving the auctioneer's value function in equation (10)

The auctioneer's value function is in (5) given by

$$v^P(s;\hat{K}) = E\left[\sum_{i=1}^n e^{-r\tau^i_{\hat{K}}}\left(y^i(\hat{K})S_{\tau^i_{\hat{K}}} - X^i(S_{\tau^i_{\hat{K}}},\hat{K})\right)^+\right|\mathcal{F}_0^S\right].$$

Conditional expectations lead to

$$v^P(s;\hat{K}) = E\left[E\left[\sum_{i=1}^n e^{-r\tau^i_{\hat{K}}}\left(y^i(\hat{K})S(\tau^i_{\hat{K}}) - X^i(S(\tau^i_{\hat{K}}),\hat{K})\right)^+\right|\mathcal{F}_0^{S,K}\right]\left|\mathcal{F}_0^S\right].$$

Because of time-homogeneity we are allowed to separate the expression of the discounting term from the option's payoff as follows,

$$\begin{aligned} &v^P(s;\hat{K}) \\ &= E\left[E\left[\textstyle\sum_{i=1}^n e^{-r\tau^i_{\hat{K}}}\right|\mathcal{F}_0^{S,K}\right]E\left[y^i(\hat{K})S_{\tau^i_{\hat{K}}} - X^i(S_{\tau^i_{\hat{K}}},\hat{K})\right|\mathcal{F}_0^{S,K}\right]\left|\mathcal{F}_0^S\right]. \end{aligned}$$

Next, we insert the value of the discounting factor as expressed in equation (8), given an (arbitrary) value of the investment trigger, $S^i(\hat{K})$,

$$\begin{aligned} v^P(s;\hat{K}) \;\; = \;\; & E\left[E\left[\textstyle\sum_{i=1}^n \left\{ \tfrac{\phi(s)}{\phi(S^i(\hat{K}))}\left(y^i(\hat{K})S^i(\hat{K}) - X^i(S^i(\hat{K}),\hat{K})\right) \mathrm{I}_{\{s\leq S^i(\hat{K})\}} \right.\right.\right. \\ & \left.\left.\left. + \left(y^i(\hat{K})s - X^i(s,\hat{K})\right)\mathrm{I}_{\{s>S^i(\hat{K})\}}\right\} \right| \mathcal{F}_0^{S,K}\right] \right| \mathcal{F}_0^S\Big]. \end{aligned}$$

This expression is equivalent to

$$\begin{aligned} v^P(s;\hat{K}) \;\; = \;\; & E\left[\textstyle\sum_{i=1}^n \left\{ \tfrac{\phi(s)}{\phi(S^i(\hat{K}))}\left(y^i(\hat{K})S^i(\hat{K}) - X^i(S^i(\hat{K}),\hat{K})\right) \mathrm{I}_{\{s\leq S^i(\hat{K})\}} \right.\right. \\ & \left.\left. + \left(y^i(\hat{K})s - X^i(s,\hat{K})\right)\mathrm{I}_{\{s>S^i(\hat{K})\}}\right\} \right| \mathcal{F}_0^S\Big], \end{aligned}$$

which is identical to equation (10).

A.3 Properties of the optimal investment strategy

We now prove that in optimum we have $S^{i*}(K) = S^{i*}(K^i)$.

Suppose that agent i is the winner of the contract, i.e., $y^i(K) = 1$. Define agent i's expected critical price as $S^i(K^i) = E\left[S^i(K)\mathrm{I}_{\{y^i(K)=1\}}|\mathcal{F}_0^{S,K^i}\right]$. For $s \leq S^i(K)$, the principal's value if agent i wins the contract can be written as (from (15))

$$E\left[\frac{\phi(s)}{\phi(S^i(K))}\left(S^i(K) - K^i\right)\mathrm{I}_{\{s\leq S^i(K)\}} + (s-K^i)\mathrm{I}_{\{s>S^i(K)\}} - v^i(s,K^i)|\mathcal{F}_0^S\right].$$

Observe that, by Jensen's inequality,

$$E\left[\phi(S^i(K))\mathrm{I}_{\{y^i(K)=1\}}|\mathcal{F}_0^S\right] \geq \phi(S^i(K^i)),$$

under the assumption that $\phi(\cdot)$ is a convex function and

$$\phi(S^i(K^i)) = \phi(E[S^i(K)\mathrm{I}_{\{y^i(K)=1\}}|\mathcal{F}_0^S]).$$

This implies that

$$\begin{aligned} & \frac{\phi(s)}{\phi(S^i(K^i))}\left(S^i(K^i) - K^i\right) - v^i(s,K^i) \\ & \geq E\left[\left(\tfrac{\phi(s)}{\phi(S^i(K))}\left(S^i(K) - K^i\right) - v^i(s,K^i)\right)\mathrm{I}_{\{y^i(K)=1\}}|\mathcal{F}_0^S\right]. \end{aligned}$$

Thus, the auctioneer's value function can be replaced by a larger quantity, substituting $S^i(K)$ by $S^i(K^i)$. From this result we see that a stochastic mechanism as given by $S^i(K)$ is not optimal.

A.4 The auctioneer's simplified optimization problem

Define $\hat{v}^P$ as the auctioneer's arbitrary value function when $S^i(K^i) = S^i(K)$. Replace the investment triggers $S^i(K)$ by $S^i(K^i)$, in the principal's value function specified by equation (15), leading to

$$\begin{aligned}\hat{v}^P(s,K) &= E\left[\sum_{i=1}^n \left\{\frac{\phi(s)}{\phi(S^i(K^i))} y^i(K)\left(S^i(K^i) - K^i\right) \mathrm{I}_{\{s\le S^i(K^i)\}}\right.\right. \\ &\qquad \left.\left. + y^i(K)\left(s - K^i\right) \mathrm{I}_{\{s> S^i(K^i)\}} - v^i(s_0, K^i)\right\}\Big|\, \mathcal{F}_0^S\right].\end{aligned}$$

Furthermore, conditional expectations yield

$$\begin{aligned}\hat{v}^P(s,K) &= E\left[\sum_{i=1}^n E\left[\left\{\frac{\phi(s)}{\phi(S^i(K^i))} y^i(K)\left(S^i(K^i) - K^i\right) \mathrm{I}_{\{s\le S^i(K^i)\}}\right.\right.\right. \\ &\qquad \left.\left.\left. + y^i(K)\left(s - K^i\right) \mathrm{I}_{\{s> S^i(K^i)\}} - v^i(s_0, K^i)\right\}\Big|\, \mathcal{F}_0^{S,K^i}\right]\Big|\, \mathcal{F}_0^S\right],\end{aligned}$$

which, by exploiting the definition $Y^i(K^i) = E\left[y^i(K)|\mathcal{F}_0^{S,K^i}\right]$, is written as

$$\begin{aligned}\hat{v}^P(s,K) &= E\left[\sum_{i=1}^n E\left[\left\{\frac{\phi(s)}{\phi(S^i(K^i))} Y^i(K^i)\left(S^i(K^i) - K^i\right) \mathrm{I}_{\{s\le S^i(K^i)\}}\right.\right.\right. \\ &\qquad \left.\left.\left. + Y^i(K^i)\left(s - K^i\right) \mathrm{I}_{\{s> S^i(K^i)\}} - v^i(s_0, K^i)\right\}\Big|\, \mathcal{F}_0^{S,K^i}\right]\Big|\, \mathcal{F}_0^S\right].\end{aligned}$$

Each agent's "contribution" to the auctioneer's value is an expression that depends only on each agent's report K^i (i.e., the direct mechanism is not stochastic), which means that the outer expectation operator is superfluous, resulting in

$$\begin{aligned}\hat{v}^P(s,K^i) &= \sum_{i=1}^n E\left[\frac{\phi(s)}{\phi(S^i(K^i))} Y^i(K^i)\left(S^i(K^i) - K^i\right) \mathrm{I}_{\{s\le S^i(K^i)\}}\right. \\ &\qquad \left. + Y^i(K^i)\left(s - K^i\right) \mathrm{I}_{\{s> S^i(K^i)\}} - v^i(s, K^i)|\mathcal{F}_0^{S,K^i}\right].\end{aligned}$$

The above expression can equivalently be written

$$\begin{aligned}\hat{v}^P(s,K^i) &= \sum_{i=1}^n \left\{\int_{\underline{K}}^{\overline{K}} \left[\frac{\phi(s)}{\phi(S^i(K^i))} Y^i(K^i)\left(S^i(K^i) - K^i\right) \mathrm{I}_{\{s\le S^i(K^i)\}}\right.\right. \\ &\qquad \left.\left. + Y^i(K^i)\left(s - K^i\right) \mathrm{I}_{\{s> S^i(K^i)\}} - v^i(s, K^i)\right] f(K^i)dK^i\right\}.\end{aligned}$$

Substituting the expression in (14) into the value above, and replacing $S^i(K)$ by $S^i(K^i)$, leads to

$$
\begin{aligned}
&\hat{v}^P(s,K)\\
=\ & \textstyle\sum_{i=1}^n \left\{ \int_{\underline{K}}^{\overline{K}} \left[\left(\frac{\phi(s)}{\phi(S^i(K^i))} Y^i(K^i)\left(S^i(K^i)-K^i\right) - \int_{K^i}^{\overline{K}} \frac{\phi(s)}{\phi(S^i(u))} Y^i(u)du \right) \mathrm{I}_{\{s\leq S^i(K^i)\}} \right.\right.\\
& \left.\left. + \left(Y^i(K^i)\left(s-K^i\right) - \int_{K^i}^{\vartheta^i(s)} Y^i(u)du + \int_{\vartheta(s)}^{\overline{K}} \frac{\phi(s)}{\phi(S^i(u))} Y^i(u)du \right) \mathrm{I}_{\{s>S^i(K^i)\}} \right] f(K^i)dK^i \right\}\\
=\ & \textstyle\sum_{i=1}^n \left\{ \int_{\underline{K}}^{\overline{K}} \left[\frac{\phi(s)}{\phi(S^i(K^i))} Y^i(K^i) \left(S^i(K^i) - K^i - \frac{F(K^i)}{f(K^i)} \right) \mathrm{I}_{\{s\leq S^i(K^i)\}} \right.\right.\\
& \left.\left. + \left(Y^i(K^i)\left(s-K^i-\frac{F(K^i)}{f(K^i)}\right)\right) \mathrm{I}_{\{s>S^i(K^i)\}} \right] f(K^i)dK^i \right\},
\end{aligned}
$$

where the last equality follows from partial integration of

$$\int_{\underline{K}}^{\overline{K}} \int_{K^i}^{\overline{K}} \frac{\phi(s)}{\phi(S^i(u))} Y^i(u)\mathrm{I}_{\{s\leq S^i(K^i)\}} du f(K^i)dK^i$$

and

$$\int_{\underline{K}}^{\overline{K}} \left[\int_{K^i}^{\overline{K}} Y^i(u) + \int_{\vartheta^i(s)}^{\overline{K}} \frac{\phi^i(s)}{\phi^i(S^i(u))} Y^i(u)du \right] \mathrm{I}_{\{s>S^i(K^i)\}} du f(K^i)dK^i,$$

respectively.

A.5 Equality between the two approaches of finding the optimal compensation function

The probability $Y^{i*}(K^i)$ given the principal's optimal choice of the winner of the contract, equals $\left[1-F(K^i)\right]^{n-1}$, which is understood as the probability of having the lowest cost in a sample of n. Substitution of $Y^{i*}(K^i) = \left[1-F(K^i)\right]^{n-1}$ in (20), leads to

$$
\begin{aligned}
&X^{i*}(s,K^i)\\
&= K^i\left[1-F(K^i)\right]^{n-1} + \textstyle\int_{K^i}^{\vartheta^{i*}(s)} \left[1-F(u)\right]^{n-1} du + \int_{\vartheta^{i*}(s)}^{\overline{K}} \frac{\phi^i(s)}{\phi^i(S^{i*}(u))} \left[1-F(u)\right]^{n-1} du
\end{aligned}
\tag{22}
$$

if $s > S^i(K^{i*})$.

We will now show that $X^{i*}(s,K^i) = E\left[\tilde{X}^i(s,K)|\mathcal{F}_0^{S,K^i}\right]$. We treat K^j as the first-order statistic in a sample of size $n-1$, which means that we assume that K_j is the lowest cost parameter in a sample of $n-1$ parameters. We find $E\left[\tilde{X}^i(s,K)|\mathcal{F}_0^{S,K^i}\right]$ as follows

$$\begin{aligned} E\left[\tilde{X}^i(s,K)|\mathcal{F}_0^{S,K^i}\right] &= \int_{K^i}^{\vartheta^{i*}(s)} K^j d\left(-[1-F(K^j)]^{n-1}\right) \\ &\quad + \int_{\vartheta^{i*}(s)}^{\overline{K}} \left(\vartheta^{i*}(s) + \int_{\vartheta^*(s)}^{K^j} \frac{\phi(s)}{\phi(S^{i*}(u))} du\right) d\left(-[1-F(K^j)]^{n-1}\right) \end{aligned} \tag{23}$$

when $s > S^{i*}(K^i)$. Partial integration of equation (23) leads to

$$\begin{aligned} & E\left[\tilde{X}^i(s,K)|\mathcal{F}_0^{S,K^i}\right] \\ &= K^i[1-F(K^i)]^{n-1} + \int_{K^i}^{\vartheta^{i*}(s)} [1-F(u)]^{n-1} du + \int_{\vartheta^{i*}(s)}^{\overline{K}} \frac{\phi(s)}{\phi(S^{i*}(u))} [1-F(u)]^{n-1} du \end{aligned} \tag{24}$$

if $s > S^{i*}(K^i)$. We see that equation (24) equals equation (22), and thus equals equation (20).

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