

# Strategic Dynamic R&D Investments

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FIRST DRAFT

Abstract

In this paper we present a model, which describes firms' strategic R&D investment under technological uncertainty. It assumes two symmetric firms making strategic decisions about undertaking two-stage R&D subject to uncertainty in the outcome of the first exploratory stage.

The model concludes that using real options to evaluate the R&D investments allows the firm to undertake larger investment projects when uncertainty is large. Also using the real options creates more complex strategic interactions between the competing agents in a duopoly. If the R&D is profitable for both agents, they will invest symmetrically and compete later in production. But the technological uncertainty together with the strategic interaction between two agents can lead to the outcome when it is profitable for one agent to invest in R&D only when another agent does not. The "leader" gets a larger market share and is capable of conducting the R&D in amounts, which otherwise are nonprofitable under regular symmetric conditions or if the first-stage exploratory R&D did not succeed.

JEL Classification: C72, D21, O31

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## 1 Introduction

Two important features of R&D investments are that an R&D project takes time to complete and that the outcome of R&D investments is uncertain. This makes that "the analysis of R&D investments is surely one of the most difficult problems of investment under uncertainty" (Schwartz and Moon (2000)). Still, the aim of this paper is to provide analytical results regarding incentives for R&D investments of firms dealing with competition. To do so we design a framework as simple as possible while it still contains the specific aspects of R&D: uncertainty and time to complete. After starting out with studying the monopoly benchmark case, a duopoly framework is considered. The paper is organized as follows. The model is presented in Section 2. Section 3 treats the monopoly case, while the effects of competition in the form of a duopoly are analyzed in Section 4. Topics for further research are presented in Section 5.

## 2 The model

Consider a two-step R&D process and two identical firms. At time 0, both firms have the opportunity to make an initial irreversible R&D investment of, an amount  $\beta I$ . The outcome of this investment is stochastic in the sense that, after having carried out this initial R&D investment, at time 1 with probability  $\frac{1}{2}$  the firm needs to invest  $(1 - \beta)I - \Delta h$  to achieve the breakthrough, and with the same probability it needs to invest  $(1 - \beta)I + \Delta h$  to achieve the same breakthrough. Parameter  $\Delta h$  is known beforehand, and it can have a value between zero and  $(1 - \beta)I$ <sup>1</sup>

When the technological breakthrough is achieved due to the follow up investment at time 1, the firm is able to produce more efficiently from time 2 onwards.. In particular it is assumed that unit production costs then have reduced from  $K$  to zero. In addition, we assume that  $0 < \beta < \frac{1}{2+r}$ . This condition implies that the first-stage R&D investment is less than the net present value of second-stage R&D (see Proposition 1 below). In other words we assume that the first-stage R&D is of exploratory type (see Dixit and Pindyck (1995)), which takes less resources than the follow-up R&D investment, but contributes to resolution of uncertainty and is necessary for the later investment decision.

$P(Q)$  is the normalized inverse demand function expressing the market price as a function of total supply  $Q$ :

$$P(Q) = 1 - Q.$$

The model is related to Kulatilaka and Perotti (1998) but differs in two aspects: (i) in Kulatilaka and Perotti the firm can carry out one investment expenditure in order to reduce unit production costs in the next period, while in our framework the firm needs to go through a two stage investment procedure, and (ii) in Kulatilaka and Perotti there is demand uncertainty while we

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<sup>1</sup>The advantage of this formulation is that mean preserving spreads can be considered.

have the R&D cost uncertainty, the impact of which can be derived unambiguously. Compared to the duopoly model of D'Aspremont and Jacquemin (1988) the difference is that we assume uncertainty about the R&D costs and the research investment is conducted a two stages (an exploratory R&D and a final technology development) rather than one single research outlay.

### 3 The Monopoly R&D Investment Problem

Let us first consider behavior of an individual agent. Assume that this agent operates in the market alone and makes plans about production and R&D behavior subject to only uncertainty. Suppose that at time 1, when firm decides about its production levels the unit production costs equal  $K$ . Then the optimal output is determined by solving:

$$\max_Q [(1-Q)Q - KQ].$$

The first order condition gives

$$Q = \frac{1-K}{2}.$$

We see that in monopoly case firm will not produce if unit production cost  $K \geq 1$ . The optimal total profit is expressed as follows:

$$\tilde{\pi}(K) = \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left( \frac{1-K}{2} \right)^2 = \frac{1+r}{r} \left( \frac{1-K}{2} \right)^2.$$

In the case, when the firm already completed the two stage R&D investment process, production costs is reduced from  $K$  to zero, so that total profit is given by

$$\tilde{\pi}(0) = \frac{1+r}{4r}.$$

Now assume we are at time 1. If the first step R&D was unsuccessful, it costs  $(1-\beta)I + \Delta h$  to finish the R&D project successfully. This leads to the following investment gain (the difference between the net present value (NPV) of this investment and the NPV of not making the investment):

$$\Delta_{2u} = \tilde{\pi}(0) - \tilde{\pi}(K) - [(1-\beta)I + \Delta h].$$

We conclude that in this case the second stage investment will be made when  $\Delta_{m,2u} > 0$ , which holds when:

$$\Delta h < \tilde{\pi}(0) - \tilde{\pi}(K) - (1-\beta)I. \quad (1)$$

In case when the first step R&D investment was successful in the sense that it now only costs  $(1-\beta)I - \Delta h$  to achieve the breakthrough, the investment gain is

$$\Delta_{2s} = \tilde{\pi}(0) - \tilde{\pi}(K) - [(1-\beta)I - \Delta h].$$

Following the success in the first stage, the second stage R&D will be conducted if  $\Delta_{m,2s} > 0$ , thus:

$$\Delta h > -[\tilde{\pi}(0) - \tilde{\pi}(K) - (1 - \beta)I] \quad (2)$$

It is clear that in the case when  $\tilde{\pi}(0) - \tilde{\pi}(K) - (1 - \beta)I > 0$  inequality (2) holds automatically given inequality (1) holds too.

Having determined optimal R&D behavior in the second phase, let us now turn to the initial investment. Consider first the scenario where  $\Delta h < \tilde{\pi}(0) - \tilde{\pi}(K) - (1 - \beta)I$ . Then, given that the initial R&D investment is made, the second phase will always be carried out. The expected investment gain of the first phase investment is

$$\begin{aligned} \Delta_{1us} &= \frac{\frac{1}{2}\Delta_{m,2s} + \frac{1}{2}\Delta_{m,2u}}{1+r} - \beta I \\ \Delta_{1us} &= \frac{\tilde{\pi}(0) - \tilde{\pi}(K) - (1+r\beta)I}{1+r}. \end{aligned}$$

In this case, it is optimal to initially invest in R&D when  $\Delta_{m,1us} > 0$ , i.e.

$$I < \frac{\tilde{\pi}(0) - \tilde{\pi}(K)}{1+r\beta}$$

In the second scenario it holds that  $\Delta h > \tilde{\pi}(0) - \tilde{\pi}(K) - (1 - \beta)I$  (i.e. second-stage R&D investment is not made given unsuccessful first-stage R&D). Here the follow up investment is only carried out when the first phase was successful. This implies that the investment gain of the first phase investment is

$$\begin{aligned} \Delta_{1s} &= \frac{\frac{1}{2}\Delta_{m,2s}}{1+r} - \beta I \\ \Delta_{1s} &= \frac{\tilde{\pi}(0) - \tilde{\pi}(K) + \Delta h - (1 + \beta + 2r\beta)I}{2(1+r)}. \end{aligned}$$

In this scenario the second stage R&D investment option is exercised only when  $\Delta_{m,1s} > 0$ , which requires:

$$\Delta h > (1 + \beta + 2r\beta)I - [\tilde{\pi}(0) - \tilde{\pi}(K)]$$

Let us consider the combinations of  $\Delta h$  and  $I$ , which determine different investment strategies of our firm.

< *InsertFigure1* >

Figure 1 depicts main relationships between  $\Delta h$  and  $I$ , analyzed above. Firstly, the set of all feasible  $(\Delta h, I)$  combinations, which a monopolist faces in the model, is the area below line  $\Delta h = (1 - \beta)I$  (which is flatter than 45 degree line for all  $0 < \beta < 1$ ) and the horizontal axis. Secondly, lines  $\Delta h = \tilde{\pi}(0) - \tilde{\pi}(K) - (1 - \beta)I$  and  $\Delta h = -[\tilde{\pi}(0) - \tilde{\pi}(K) - (1 - \beta)I]$  show the critical

borders for the second-stage R&D investment decision-making at time 1. And finally, lines  $I = \frac{\tilde{\pi}(0) - \tilde{\pi}(K)}{1+r\beta}$  and  $\Delta h = (1 + \beta + 2r\beta)I - [\tilde{\pi}(0) - \tilde{\pi}(K)]$  are the critical borders for the R&D investment decisions at time 0.

As we can see, the firm decides to undertake initial R&D investment only if  $(\Delta h, I)$  falls into area **A** or area **B** in the feasible parameters set. Area **B** will always be not empty set under the assumption of exploratory first-stage R&D ( $0 < \beta < \frac{1}{2+r}$ ).

Such conclusion comes from the following proposition.

**PROPOSITION 1.** *Assume that  $0 < \beta < \frac{1}{2+r}$ . Then there exists a pair  $(\Delta h, I)$  such that:*

$$\Delta h < (1 - \beta)I \tag{3}$$

$$\Delta h > (1 + \beta + 2r\beta)I - [\pi(0) - \pi(K)], \text{ and} \tag{4}$$

$$\Delta h > [\pi(0) - \pi(K)] - (1 - \beta)I. \tag{5}$$

**PROOF.** See Appendix.

The proposition implies that if the volume of the first-stage R&D is less than the NPV of the second-stage investment, then there exists a pair of mean preserving spread  $\Delta h$  and R&D investment  $I$  (satisfying the feasibility condition (3)), where  $\Delta_{1us} < 0$  (condition (4)), and  $\Delta_{1s} > 0$  (condition (5)), thus making R&D investment profitable only when the exploratory R&D succeeds. Or in other words, the real option exists only if the first-stage R&D is of exploratory nature. If in the first stage the firm has to invest a larger chunk of resources than in the second stage ( $\beta \geq \frac{1}{2+r}$ ), then the NPV is an efficient assessment method and the firms' decisions are reduced to the simple "now or never" choice.

In area **A** the straightforward NPV criterion is positive indicating the profitable investment opportunity. In area **B** the NPV criterion is negative, but the optimal investment criterion is positive, still indicating possible success after the uncertainty is resolved. Thus using NPV we fail to see the profitable R&D investment given the success of the initial exploratory R&D. Moreover, in area **B** it is possible to obtain values of R&D investment  $I$  which are larger than the maximum investment "supported" by the NPV criterion.

The set of profitable options in area **B** also indicates that when the variance of R&D outcomes is sufficiently large, the real option decision criterion allows to start with R&D projects requiring bigger R&D investment. The large investments are more likely to be carried out when the uncertainty is higher. The reason is that the exploratory investment creates an option for the firm (in this case it is the option to produce costlessly). The option value increases with the R&D expenditures reduction uncertainty, and therefore R&D investments increase in value when variance is higher.

## 4 Duopoly

Consider now two identical firms operating in the same market. Each firm faces the same R&D investment and production decision and the same type

of technological uncertainty. The normalized inverse market demand function now is  $P(Q) = 1 - Q$ , where  $Q = q_1 + q_2$  and  $q_i$  is individual production of firm  $i$  ( $i = 1, 2$ ). Each firm produces with a fixed unit production cost  $K_i = K$ , ( $i = 1, 2$ ). Similarly to the monopoly case, if the firm undertakes R&D, unit production cost drops to 0.

We act under the assumption of simultaneous strategic entry. There are several different scenarios possible in this game: Cournot competition with no R&D, Cournot competition with symmetric simultaneous R&D (indexed with \* superscript), and asymmetric competition with firm  $i$  conducting R&D, and firm  $j$  producing with old technology ( $i, j = 1, 2, i \neq j$ ) (indexed with  $\hat{\cdot}$  for investing firm, and  $\tilde{\cdot}$  for the not investing one). At time 1 agent  $i$  solves for the optimal production level:

$$\max_{\{q_i\}} [1 - q_i - q_j]q_i - K_i q_i, \quad i, j = 1, 2, \quad i \neq j.$$

The firm  $i$ 's reaction function is:

$$q_i = \frac{1 - K_i - q_j}{2},$$

and the corresponding optimal output will be:

$$q_i = \frac{1 - 2K_i + K_j}{3} \quad i, j = 1, 2, \quad i \neq j$$

Suppose that both firms find it better not to invest in R&D and pursue production with an old technology ( $K_1 = K_2 = K$ ). Each of them reaches the Cournot game equilibrium production at:

$$q_i = \frac{1 - K}{3}.$$

Correspondingly, at this production level the firm will receive the total stream of profits:

$$\pi_i(K) = \frac{1 + r}{r} \left( \frac{1 - K}{3} \right)^2$$

If both agents decide to invest in R&D, they both obtain unit production costs  $K_i = 0, i, j = 1, 2, i \neq j$ , and optimal production output:

$$q_i^* = \frac{1}{3},$$

with profits

$$\pi_i^*(0) = \frac{1 + r}{9r}$$

Finally, if only one agent decides to invest in R&D (he will produce  $\tilde{q}_i^*$ ) and another does not (producing  $\tilde{q}_j$  correspondingly) ( $K_i = 0, K_j = K, i, j =$

1, 2,  $i \neq j$ ), we will have the following pair of optimal outputs:

$$\begin{aligned}\hat{q}_i^* &= \frac{1+K}{3} \\ \hat{q}_j(K) &= \frac{1-2K}{3},\end{aligned}$$

and their corresponding profit streams:

$$\begin{aligned}\hat{\pi}_i^*(0) &= \frac{1+r}{r} \left( \frac{1+K}{3} \right)^2 \\ \hat{\pi}_i(K) &= \frac{1+r}{r} \left( \frac{1-2K}{3} \right)^2\end{aligned}$$

It is necessary to make another observation. If unit production cost  $K$  is in the region between  $\frac{1}{2}$  and 1, then  $\hat{q}_i(K)$  is negative, and the asymmetric R&D costs game automatically degrades to the standard monopoly situation. Here we can put a difference between moderate and drastic innovation (Tirole, (1990)). The moderate innovation correspond to the case of relatively low  $K$ . If one firm decides to innovate and another does not, the innovation is not strong enough to drive the not innovating firm from out the market. On the other hand, if the innovation is strong (bringing relatively large  $K$  to zero), the innovating agent gains so much that he actually pushes the "looser" from the market and gains a monopolist position.

Now, when our firms find themselves in time 0, they observe values of  $(\Delta h, I)$ , think of the uncertainty, and combine this information with knowledge about their future profits, they can build their strategies. The logic of their competitive decision making with regards to uncertainty in R&D is the same as for the monopolist, but with different profit streams (and thus different decision making areas **A** and **B**, see Figure 1), which are dependent on other agent's strategies.

For example, agent 1 must make a decision about his investment in R&D. He knows  $(\Delta h, I)$  and corresponding probabilities of success in the first stage R&D. But in order to figure out his investment gains, he must be aware that the other agent also has two similar choices. If the competitor decides to invest in first stage R&D, then they will end up with  $K_1 = K_2 = 0$  and Cournot competitive outputs. On the other hand, if other firm does not invest (for some reasons), firm 1 will enjoy the advantages of producing with no cost and, thus, larger output and market share. Even more, if the unit production cost is high enough, the R&D investing firm can end up in an advantageous monopoly position. In general, agent's investment gains are given in Table 1.

< *InsertTable1* >

We elaborate a bit further:

$$\begin{aligned}
\pi_i^*(0) - \hat{\pi}_i(K) &= \frac{1+r}{9r} - \frac{1+r}{r} \left( \frac{1-2K}{3} \right)^2 = \frac{1+r}{9r} (4K - 4K^2) \\
\hat{\pi}_i^*(0) - \pi_i(K) &= \frac{1+r}{r} \left( \frac{1+K}{3} \right)^2 - \frac{1+r}{r} \left( \frac{1-K}{3} \right)^2 = \frac{1+r}{9r} (4K) \\
\tilde{\pi}(0) - \pi_i(K) &= \frac{1+r}{4r} - \frac{1+r}{r} \left( \frac{1-K}{3} \right)^2 = \frac{1+r}{36r} (5 + 8K - 4K^2)
\end{aligned}$$

It can be shown that  $\tilde{\pi}(0) - \pi_i(K) > \pi^*(0)$  and  $\hat{\pi}_i^*(0) - \pi_i(K) > \pi_i^*(0) - \hat{\pi}_i(K)$ .

< InsertFigure2 >

Using this information we plot another graph as shown on Figure 2. This graph illustrates individual agents decision areas with respect to different values of  $(\Delta h, I)$  combination. There are two main "decision spaces" in which our agents will have to operate. One is for the simultaneous entry Cournot game (areas **A1** and **B1**). Areas **A2** and **B2** illustrate one of the other two regimes. They correspond to either the asymmetric competition where one agent invests in R&D and another does not under conditions of moderate innovation (areas **A2** and **B2**), or the monopoly case, which arises when the drastic innovation takes place.

Assume now that the unit cost is small:  $0 < K < \frac{1}{2}$ . Then if pair  $(\Delta h, I)$  falls into areas **A1** or **B1**, both agents will find it profitable to invest in R&D in the first stage, because none of them would want to be left alone with an obsolete technology and a shrunk market share against more advanced competitor. In the case of **A1**, they will invest in second stage R&D regardless of the outcome of exploratory investment following simple NPV rule, and in the case of **B1**, they will proceed with second-stage research, only if they succeed in the first one, i.e. exercise their option.

But suppose that our firms face a situation when  $(\Delta h, I)$  is beyond **A1**, but inside areas **A2** or **B2**. This means, that one firm can still find it profitable to invest in R&D but *only if the other firm will not implement such an investment*. In this case the investing firm will obtain an advantage over its competitor in the form of larger market share, thus boosting its profits. Furthermore, if  $(\Delta h, I)$  is in the intersection of **B1** and **A2**, and two firms observe that their exploratory R&D was not successful, one of them will still be able to make profit from investing in the second stage R&D acting alone.. Note that such situation is possible only if there are no spillovers in the model, i.e. success of one agent's R&D does not provide another agent with any positive externalities.

Now look at the picture assuming that  $\frac{1}{2} \leq K < 1$ . Still, observing  $(\Delta h, I)$  in **A1** or **B1**, the firms will end up in the symmetric Cournot competition with R&D investment. Beyond **A1** and **B1**, but inside **A2** and **B2**, because the unit production cost is high, there will be no place in the market for "looser" with obsolete technology. The "winner" will take it all by using NPV or exercising

his real option. And similarly to the asymmetric competition case, the monopolist will be capable of performing the largest and riskiest of R&D projects in comparison to all other regimes.

## 5 Conclusions

The model analyzed in this paper presents several implications about firms strategic R&D investment under technological uncertainty. It assumes two symmetric firms making strategic decisions about undertaking two-stage R&D subject to uncertainty in the outcome of the first exploratory stage.

We conclude that using the real options method of assessment of R&D investment allows the firm to undertake larger investment projects when the variance is large. This supports the argument of Dixit and Pindyck (1995) about the value of exploratory investment as an "option creator" for the firm. Also the real option criterion creates more complex strategic interactions between the competing agents in a duopoly. These interactions allow us to model more realistically the R&D investment and production behavior of firms under technological uncertainty.

We observe in the model that if the R&D is profitable for both agents, they will end up in a Cournot competition with symmetric market shares. But the technological uncertainty together strategic interaction between two agents can lead to the outcome when it is profitable for one agent to invest in R&D only if another does not. The "technological leader" in this case obtains larger market share and is actually capable of conducting the R&D investment in volumes, which are not profitable under regular symmetric conditions or in cases, when the first-stage exploratory R&D was not successful. Moreover, if the unit production costs gain is substantial, it is more likely, that the "inferior" firm will leave the market and the "leader" will find himself in a monopolist position.

Clearly, under such asymmetric strategies a coordination problem arises. In this model setting it is impossible to determine which of the agents will undertake the investment and which agent will step aside. Possible resolution of this problem can come in the form of external coordination. The social planner, who has his goal of stimulating development of new technologies can simply assign the roles, granting one agent temporary advantage over others stimulating him to undertake larger (and/or more risky) research projects (much spoken about "picking winners" industrial policy). Or on the other hand, we can assume certain endogenous asymmetries between agents. The agents can be asymmetric in terms of their access to information (for example one agent will know that the other will not undertake investment given "unfavorable" combination of  $(\Delta h, I)$ ) or in terms of their production effectiveness ( $K_i \neq K_j$ ). Our model is open to all these and probably many other extensions.

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## 7 Appendix

Proof of PROPOSITION 1.

Conditions (3), (4), and (5) can be reduced to:

$$(1 - \beta)I > (1 + \beta + 2r\beta)I - [\pi(0) - \pi(K)] \implies I < \frac{\pi(0) - \pi(K)}{2\beta(1 + r)}, \text{ and}$$

$$(1 - \beta)I > [\pi(0) - \pi(K)] - (1 - \beta)I \implies I > \frac{\pi(0) - \pi(K)}{2(1 - \beta)},$$

which gives:

$$\begin{aligned} \frac{\pi(0) - \pi(K)}{2(1 - \beta)} &< \frac{\pi(0) - \pi(K)}{2\beta(1 + r)} \\ 2\beta(1 + r) &< 2(1 - \beta) \\ \beta &< \frac{1}{2 + r}. \quad \square \end{aligned}$$

		$0 < K < \frac{1}{2}$	
<b>Agent <math>j</math></b>		$(\Delta h, I) \in \mathbf{A}$	$(\Delta h, I) \in \mathbf{B}$
Invests		$\frac{\pi_i^*(0) - \bar{\pi}_i(K) - (1+r\beta)I}{1+r}$	$\frac{\pi_i^*(0) - \bar{\pi}_i(K) - (1+\beta+2r\beta)I + \Delta h}{2(1+r)}$
Does Not Invest		$\frac{\bar{\pi}_i^*(0) - \pi_i(K) - (1+r\beta)I}{1+r}$	$\frac{\bar{\pi}_i^*(0) - \pi_i(K) - (1+\beta+2r\beta)I + \Delta h}{2(1+r)}$
		$\frac{1}{2} \leq K < 1$	
Invests		$\frac{\pi_i^*(0) - (1+r\beta)I}{1+r}$	$\frac{\pi_i^*(0) - (1+\beta+2r\beta)I + \Delta h}{2(1+r)}$
Does Not Invest		$\frac{\bar{\pi}_i^*(0) - \pi_i(K) - (1+r\beta)I}{1+r}$	$\frac{\bar{\pi}_i^*(0) - \pi_i(K) - (1+\beta+2r\beta)I + \Delta h}{2(1+r)}$

Table 1. Agent  $i$ 's investment gains in different decision areas (see Figure 1) depending on parameters  $K$ ,  $(\Delta h, I)$ , and the Agent  $j$ 's strategy.

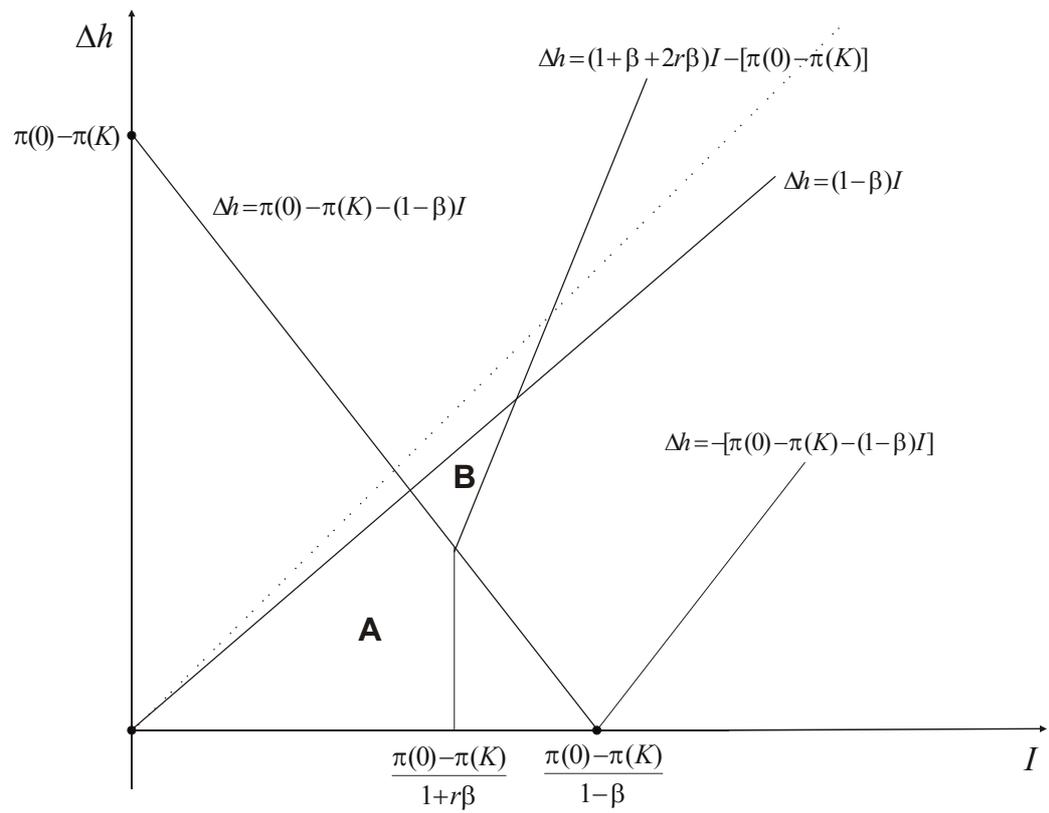


Figure 1: Investment gains for an individual firm

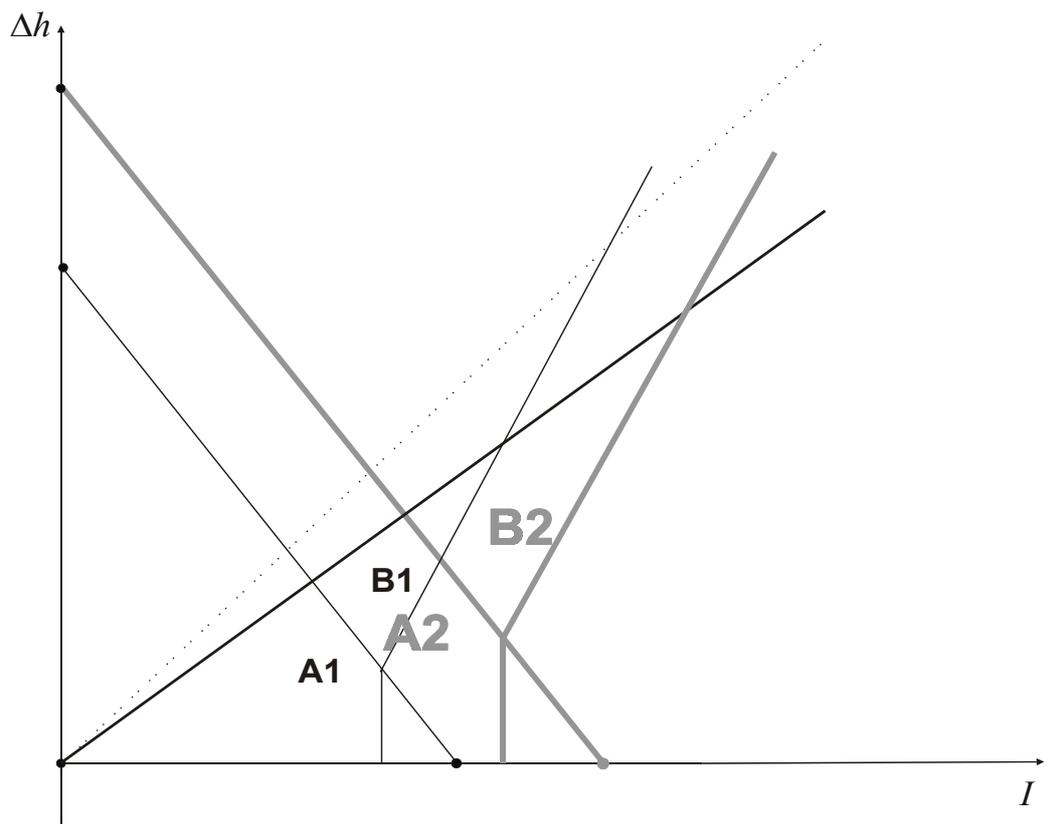


Figure 2: Investment gains in symmetric and asymmetric R&D duopoly