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Option markets and the stochastic behavior of commodity prices[1]

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1.- Introduction

Pricing and risk management of commodity-contingent assets requires an adequate specification and estimation of the risk-adjusted underlying stochastic commodity prices. Recent efforts include Gibson, R, Schwartz, E. S. (1990), Schwartz, E. S. (1997), Schwartz, E. S., Smith, J.E. (2001), and Cortazar et al (2000) among many others. A shared attribute of all of them is their reliance only on linear payout assets (futures and sometimes swaps) for estimation purposes. The benefit of using futures prices is that they trade in a relatively deep market. On the other hand the drawback of this approach is that some process parameters (i.e. volatility) may be poorly estimated, because they do not have a strong effect on futures prices. This paper explores the use of option prices (in addition to futures prices) to estimate commodity stochastic prices and discuses preliminary evidence on the behavior of the proposed models for valuing option-like assets.

2.- Stochastic Models and Futures Prices

2.1 Two-Factor Model and Futures Prices

We use the two and the three-factor models described in Cortazar et al (2000):

The state variables are:

S Spot price
y Deviations from long term expected-price-return
t Time

The parameters are:

μ_B Expected price return
μ_B^* Expected risk-adjusted price return
κ Mean-reverting parameter
λ Market price of deviations from long term expected-price-return
$\sigma 1$ Volatility of price-returns
$\sigma 2$ Volatility of deviations from long term expected-price-return
ρ Correlation between prices and deviations from long term expected-price-return

The risk-adjusted process is:

$$dS = \left(\mu_B^* - y\right)Sdt + \sigma_1 Sdz_1^*$$

$$dy = (-\kappa y - \lambda)dt + \sigma_2 dz_2^*$$

$$dz_1^* dz_2^* = \rho dt$$

Under this model, Future prices are:

$$F(S, y, T) = S \cdot \exp\left[-y \frac{1-e^{-\kappa T}}{\kappa} + \left(\mu_B^* + \frac{\lambda}{\kappa} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2} - \frac{\sigma_1 \sigma_2 \rho}{\kappa} \right) T + \frac{1}{4}\sigma_2^2 \frac{1-e^{-2\kappa T}}{\kappa^3} + \left(-\lambda + \sigma_1 \sigma_2 \rho - \frac{\sigma_2^2}{\kappa} \right) \frac{1-e^{-\kappa T}}{\kappa^2} \right]$$

2.2 Three-Factor Model and Futures Prices

The state variables are:

S	Spot price
y	Deviations from long term expected-price-return
μ	Long term expected-price-return
t	Time

The parameters are:

$\bar{\mu}$	Average long term expected-price-return
κ	Mean reverting parameter for y
a	Mean reverting parameter for μ
λ_1	Market price of spot price risk
λ_2	Market price of deviations from long term expected-price-return
λ_3	Market price of deviations from average long term expected-price-return
$\sigma 1$	Volatility of price-returns
$\sigma 2$	Volatility of deviations from long term expected-price-return
$\sigma 3$	Volatility of deviations from average long term expected-price-return
ρ_{12}	Correlation between prices and deviations from long term expected-price-return
ρ_{13}	Correlation between prices and long term expected-price-return
ρ_{23}	Correlation between deviations from long term expected-price-return and deviations from average long term expected-price-return

The risk-adjusted process is:

$$dS = (\mu - y - \lambda_1)Sdt + \sigma_1 S dz_1^*$$

$$dy = (-\kappa y - \lambda_2)dt + \sigma_2 dz_2^*$$

$$d\mu = a\left((\overline{\mu} - \mu) - \lambda_3\right)dt + \sigma_3 dz_3^{\;*}$$

$$dz_1^{\;*} dz_2^{\;*} = \rho_{12} dt$$

$$dz_1^{\;*} dz_3^{\;*} = \rho_{13} dt$$

$$dz_2^{\;*} dz_3^{\;*} = \rho_{23} dt$$

Futures prices under this model are:

$$F(S, y, \mu, T) = S \exp\begin{bmatrix} -y\frac{1-e^{-\kappa T}}{\kappa} + \mu\frac{1-e^{aT}}{a} + -(\lambda_1)T + \frac{\lambda_2 - \rho_{12}\sigma_1\sigma_2}{\kappa^2}\left(\kappa T + e^{-\kappa T} - 1\right) \\ + \frac{\sigma_2^2}{4\kappa^3}\left(-e^{-2\kappa T} + 4e^{-\kappa T} + 2\kappa T - 3\right) + \frac{a\overline{\mu} - \lambda_3 + \rho_{13}\sigma_1\sigma_3}{a^2}\left(aT + e^{-aT} - 1\right) \\ -\frac{\sigma_3^2}{4a^3}\left(e^{-2aT} - 4e^{-aT} - 2aT + 3\right) - \frac{\rho_{23}\sigma_2\sigma_3}{\kappa^2 a^2(\kappa + a)}\begin{pmatrix} \kappa^2 e^{-aT} + \kappa a e^{-aT} + \kappa a^2 T \\ + \kappa a e^{-\kappa T} + a^2 e^{-\kappa T} \\ - \kappa a e^{-(\kappa + a)T} - \kappa^2 \\ - \kappa a - a^2 + \kappa^2 a T \end{pmatrix} \end{bmatrix}$$

3.- European and American Option Prices

In order to use option market price information to calibrate the processes we must have an expression for the theoretical option prices under both models. Even though traded options are of the American type, we start by stating the analytical expression for the European call, *c*, and the European put, *p*, options written on a futures contract:

$$c = e^{-rT}\left\{FN[d] - XN\left[d - \sqrt{v(T)}\right]\right\}$$

$$p = e^{-rT}\left\{-FN[-d] + KN\left[-d + \sqrt{v(T)}\right]\right\}$$

with

$$d = \frac{\ln\frac{F}{X}}{\sqrt{v(T)}} + \frac{1}{2}\sqrt{v(T)}$$

F is the underlying futures, *X* is the exercise price and *v(T)* is the accumulated variance on futures returns.

The above expression holds for both the two and the three-factor models presented in the last section, requiring only an adjustment for the value of the variance, *v* the only parameter which is model-contingent.

For the two-factor model, v(T) is:

$$v(t) = \int_0^T \sigma_1^2 + \sigma_2^2 \frac{(1-e^{-\kappa t})^2}{k^2} + 2\rho_{12}\sigma_1\sigma_2 \frac{(1-e^{-\kappa t})}{k} dt$$

$$v(T) = \sigma_1^2 T + \frac{\sigma_2^2}{k^2}\left[T + \frac{1}{2k}(1-e^{-2\kappa T})\right] - \frac{2\rho_{12}\sigma_1\sigma_2}{k}\left[T - \frac{1}{k}(1-e^{-\kappa T})\right]$$
$$-\frac{2\sigma_2^2}{k^3}\left[(1-e^{-\kappa T})\right]$$

For the three-factor model:

$$v(t) = \int_0^T \begin{matrix} \sigma_1^2 + \sigma_2^2 \frac{(1-e^{-\kappa t})^2}{k^2} + \sigma_3^2 \frac{(1-e^{-at})^2}{a^2} - 2\rho_{12}\sigma_1\sigma_2 \frac{(1-e^{-\kappa t})}{k} \\ + 2\rho_{13}\sigma_1\sigma_3 \frac{(1-e^{-at})}{a} - 2\rho_{23}\sigma_2\sigma_3 \frac{(1-e^{-at})(1-e^{-\kappa t})}{ak} \end{matrix} dt$$

$$v(T) = \sigma_1^2 T + \frac{\sigma_2^2}{k^2}\left[T - \frac{2}{k}(1-e^{-\kappa T}) + \frac{1}{2k}(1-e^{-2\kappa T})\right]$$
$$+\frac{\sigma_3^2}{a^2}\left[T - \frac{2}{a}(1-e^{-aT}) + \frac{1}{2a}(1-e^{-2\kappa T})\right]$$
$$-\frac{2\rho_{12}\sigma_1\sigma_2}{k}\left[T - \frac{1}{k}(1-e^{-\kappa T}) + \frac{2\rho_{13}\sigma_1\sigma_3}{a}(T - \frac{(1-e^{-aT})}{a})\right]$$
$$-\frac{2\rho_{23}\sigma_2\sigma_3}{ak}\left[T - \frac{1}{k}(1-e^{-\kappa T}) - \frac{1}{a}(1-e^{-aT}) + \frac{1}{(a+k)}(1-e^{-(a+\kappa)T}))\right]$$

Traded commodity options are mainly American, so we cannot directly use the above analytic expressions. Thus we must resort to some numerical procedure to incorporate market option price information. There are many procedures to approximate the value of an American option, one of the best-known ones being Geske and Shatri (1985). We use in this paper a recent improvement proposed in Huang et al (1996) that values European option values for several exercise dates, and estimates the value of the American put option (p) using Richardson Extrapolation, as:

$$P(0) = \frac{32}{3}P(4) - \frac{27}{2}P(3) + 4P(2) + \frac{1}{6}P(1)$$

with

$$P_1 = p_0$$

$$P_2 = p_0 + \frac{rXT}{2}e^{-rT/2}N(-d_2(F_0, \mathrm{B}_{\frac{T}{2}}, T/2)) - \frac{rF_0T}{3}e^{-rT/2}N(-d_1(F_0, \mathrm{B}_{\frac{T}{2}}, T/2))$$

$$P_3 = p_0 + \frac{rXT}{3}\left[e^{-rT/3}N(-d_2(F_0, \mathrm{B}_{\frac{T}{3}}, T/3)) + e^{-2rT/3}N(-d_2(F_0, \mathrm{B}_{\frac{2T}{3}}, 2T/3))\right] - \frac{rF_0T}{3}\left[e^{-rT/3}N(-d_1(F_0, \mathrm{B}_{\frac{T}{3}}, T/3)) + e^{-2rT/3}N(-d_1(F_0, \mathrm{B}_{\frac{2T}{3}}, 2T/3))\right]$$

$$P_4 = p_0 + \frac{rXT}{4}\left[\begin{array}{l} e^{-rT/4}N(-d_2(F_0, \mathrm{B}_{\frac{T}{4}}, T/4)) + e^{-2rT/4}N(-d_2(F_0, \mathrm{B}_{\frac{2T}{4}}, 2T/4)) \\ + e^{-3rT/4}N(-d_2(F_0, \mathrm{B}_{\frac{3T}{4}}, 3T/4)) \end{array}\right] - \frac{rF_0T}{4}\left[\begin{array}{l} e^{-rT/4}N(-d_1(F_0, \mathrm{B}_{\frac{T}{4}}, T/4)) + e^{-2rT/4}N(-d_1(F_0, \mathrm{B}_{\frac{2T}{4}}, 2T/4)) \\ + e^{-3rT/4}N(-d_1(F_0, \mathrm{B}_{\frac{3T}{4}}, 3T/4)) \end{array}\right]$$

4.- Model Estimation and Results

4.1 Procedure

To estimate model parameters we extend the procedure in Cortazar et al (2000), now including option market prices. For both the two an the three-factor models we follow the same steps:

i.- Compute initial option values assuming that market options are European.

ii- Estimate state variables and parameter values which minimize errors in market prices using analytic expressions for option values.

iii.- Re-compute option values as the analytic expression for the European option plus an adjustment using Huang et al (1996) for American options

iv.- Repeat steps ii and iii until errors converge.

We estimate parameter and state variable values using as our objective function the sum of weighted squared errors in futures and option prices. By changing weights we can adjust the model to fit better futures, options or a linear combination of them.

4.2 Data

We use daily prices on futures and options (on futures) on Light, Sweet Crude Oil traded on NYMEX between January 1997 and September 1999. There are both call and put options with maturities for the next 12 months and 18, 24 and 36 June and December contracts, for several exercise prices.

4.3 Results for the two-factor and the three-factor model.

The following table presents values for the two and the three-factor models when we use as data only futures, only options, or both futures and options.

Parameter	Futures	Options	Futures and Options
μ_2	0.046	0.061	0.049
κ	0.939	1.483	1.042
σ_1	0.365	0.407	0.372
σ_2	0.374	0.574	0.405
ρ	0.849	0.884	0.859
λ	0.001	0.043	0.006
μ_2^*	0.053	0.048	0.053
$\hat{\mu}^*$	0.054	0.077	0.059
MSE of Futures	0.000161	0.000242	0.000164
MSE of Options	0.1858	0.1641	0.1777

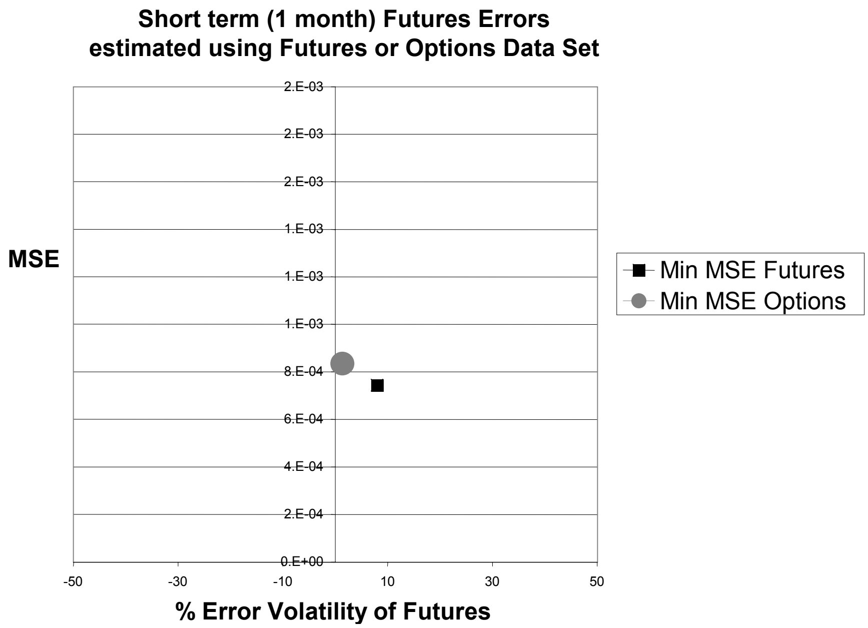
Two-Factor Model

Parámetros	Futuros	Opciones	Fut. Y Opc.
a	0.679	0.672	0.641
κ	1.596	2.813	1.911
$\hat{\mu}^*$	0.010	-0.010	0.022
σ_1	0.391	0.419	0.398
σ_2	0.849	0.952	0.765
σ_3	0.387	0.239	0.266
ρ_{12}	0.580	0.673	0.645
ρ_{23}	0.805	0.397	0.621
ρ_{31}	0.051	-0.312	-0.107
λ_1	-0.013	-0.129	-0.018
λ_2	-0.068	-0.549	-0.080
λ_3	-0.037	-0.069	-0.032
$\overline{\mu}$	0.039	0.056	0.046
MSE of Futures	2.0E-5	6.2E-5	2.1E-5
MSE of Options	0.1646	0.1584	0.1617

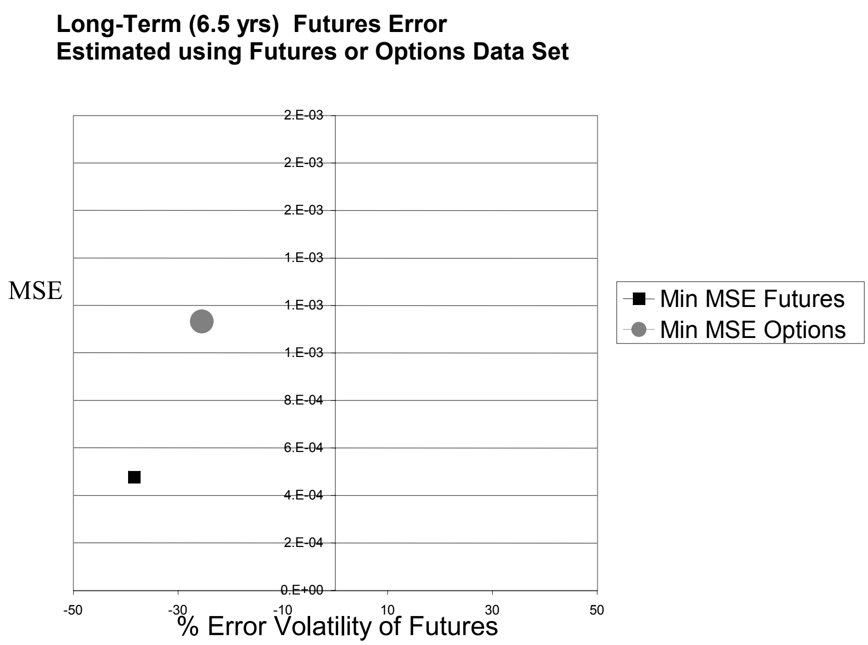
Three-Factor Model

We can see that by using only futures data (as other papers have) we obtain a good fit for futures prices but as a trade-off we get a worse fit on options data.

Next, we analyze the effect on futures estimation of not using options data. The next figure compares errors on futures value and volatility implied by the futures and the options data sets:

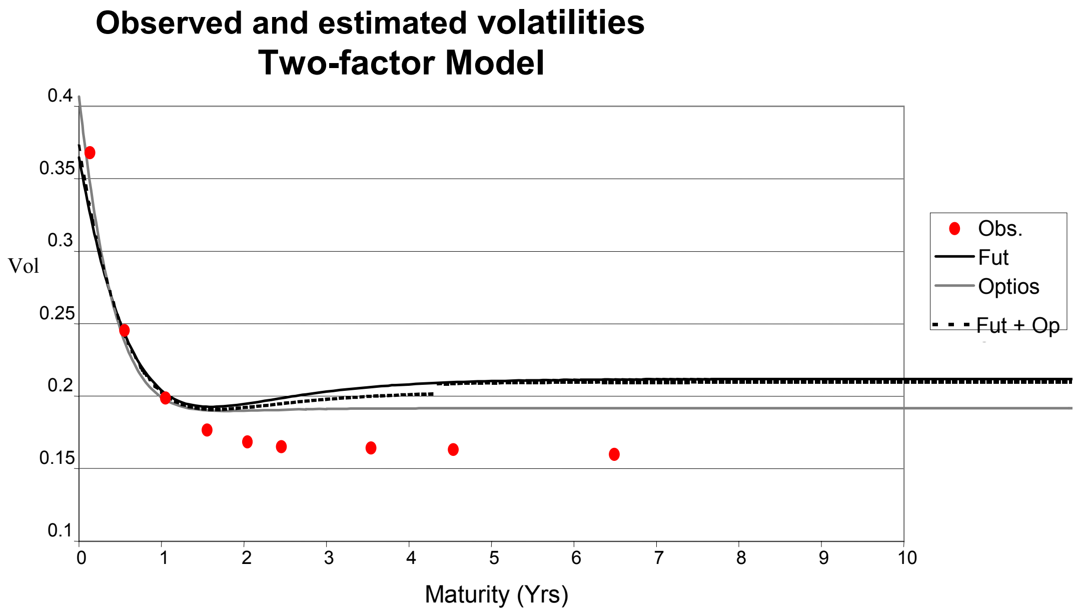


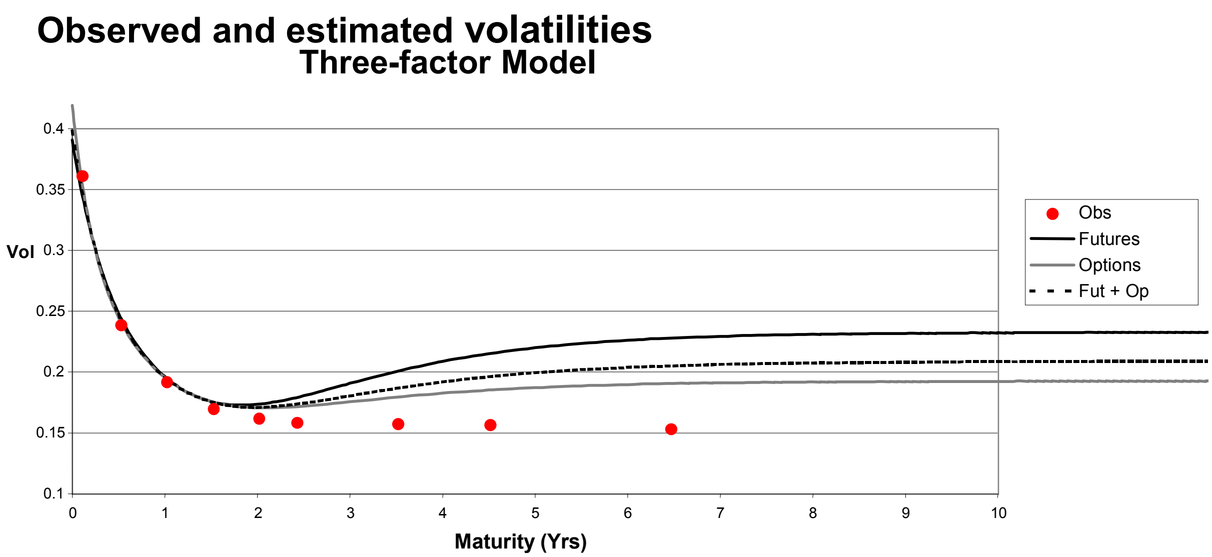
Two-Factor Model



Two-Factor Model

We can see from the above figures that minimizing MSE on a futures data set actually reduces the MSE, but at the expense of poorly estimating volatility of futures. On the other hand if we use options data we actually obtaining better estimates on futures volatility, while reducing the fit on the futures level. This error on estimating the volatility of futures depends on the maturity of futures, as can be seen in the following figure:





5.- Conclusion

Two main conclusions can be drawn from the above results. First, when estimating the stochastic process for commodity prices, options data seems to be non-redundant information and may help to adequately estimate some parameters of the process, in particular volatility. A second conclusion that may be obtained is that some adjustments to the price model may be explored to take into account that long-term model volatility seems to be overestimated by the model, and that using options data reduces, but does not eliminate, this problem.

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