“RISK SHARING AND SUPPLIER SWITCHING CONTRACTS”

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ABSTRACT

Using a real options framework, we value and analyze supply contracts characterized by exchange rate uncertainty, order quantity flexibility and supplier-switching options. Analogous to the portfolio optimization framework, our framework analyzes the incentives that the suppliers face in accepting order level flexibility. The resulting tradeoff (for the supplier) is a balance between greater volatility in the supply schedule and the prices that the producer pays. In this context, we explicitly model how flexibility can be beneficial to both the producer and multiple suppliers. In other words, how a contract with quantity flexibility can be Pareto optimal, with neither the producer nor the suppliers being worse off as compared to inflexible contracts. This implies that option to switch between suppliers is not costless, thus resulting in the producer having to compensate suppliers in a manner consistent with a profit maximization objective for all parties.

I. INTRODUCTION

Recent trends in international operations have made sourcing and purchasing decisions powerful elements in the competitiveness of a multinational enterprise. In a global operating environment, multiple sourcing and global procurement have dramatically altered the traditional views of manufacturing operations. Multinational enterprises have also become aware that sourcing and procurement decisions impact other corporate functions. At the same time, multiple sourcing across international borders exposes the multinational enterprise to new dimensions of risk since the relative operating costs can vary as a result of uncertainties in economic and political factors beyond the firm's control including but not limited to exchange rates, interest rates and taxes. With the advent of greater volatility in the exchange rates in recent years, these uncertainties have taken on a greater importance. Simultaneously, multiple sourcing (or international sourcing) also presents new opportunities that allow the firm to introduce flexibility in its cost structure as well as hedge other sources of risk for the firm. For example, with sales or borrowing (requiring interest payments) in different countries, a multinational enterprise is exposed to currency risk that it could hedge with forward contracts in the financial markets. However, with multiple sourcing in different countries it could replace the financial hedging with operational hedging. Therefore, with multiple sourcing, in addition to the possibility of lower costs, designed flexibility in supply contracts can be an important component of the competitiveness of a multinational enterprise.

Previous studies have developed methodologies for evaluating supply contracts under multiple sources of uncertainty as well as analyzed optimal operating policies under exchange rate uncertainty. McDonald and Siegel (1985), Dixit (1989) and Pindyck (1991) applied the financial option pricing methodology to real assets with discretionary decisions in the presence of uncertainty. Kogut and Kulatilaka (1994) applied the real option pricing methodology to value the flexibility induced by exchange rate movements. They showed that in the case of two plants in two countries, the value of the aggregate investment with an option to switch production between plants could be positive even though each investment by itself has negative net present value. Huchzermeir and Cohen (1996) also analyze the impact of exchange rate uncertainty on profit maximization problem for the manufacturer. Dasu and Li (1997) examined the optimal operating policies of a firm with costs of production that vary over time across plants in different countries. In a related but different context, Li and Kouvelis (1999) analyze the value of flexibility for a manufacturer with multiple suppliers, in light of uncertainty in the cost of raw materials, find that risk-sharing can be beneficial.

These studies have focused on the profit-maximization problem for the manufacturer in a real-options setting. However, risk sharing in a flexible supply contract implies that the suppliers also make optimal decisions. Otherwise, the suppliers are simply absorbing the risk transferred by the manufacturer. To the best of our knowledge, the existing literature has not considered the impact of manufacturer's (or producer's) decisions on the suppliers. In this study we explicitly incorporate the suppliers' optimization problems, concurrently with that of the producer, in the real-option based valuation framework. In other words, we analyze what induces the producer and the suppliers to accept flexibility in contracts.

Thus, we explicitly model how flexibility can be beneficial to both the producer and multiple suppliers. In other words, how a contract with flexibility can be Pareto optimal, with neither the producer nor the suppliers being worse off as compared to inflexible contracts. This implies that decisions to switch between suppliers are not costless but result in the manufacturer having to compensate suppliers in one form or other. Finally, the exogenous sources of risk can be viewed as value drivers for the both producer and the suppliers, and we examine how changes in the value drivers affect the profit-maximization problems for the producer as well as the suppliers.

Li and Kouvelis (1999) analyze risk sharing in supply contracts in the presence of uncertainty as well. They analyze "time" flexibility as well as "quantity" flexibility. In their setting the manufacturer signs a contract with its suppliers for the purchase of a certain amount of a material in order to satisfy its customers' future demand. They assume that deterministic demand D at time T is given. Thus, the manufacturer needs to obtain D units of the material from its suppliers at or before time T to satisfy the demand. It is assumed that the market price of each unit follows a geometric Brownian motion. They specify a model wherein all the demand is met at the very end.

Our framework differs from Li and Kouvelis (1999) in several ways. The demand schedule faced by the manufacturer is different and their approach is a special case of our approach. In our setup the manufacturer has to meet a demand every period. It can be met partly using this period's production. The remainder of this period's production is carried over as inventory. Additionally, inventory carried over from past periods can also satisfy this period's demand. In this manner, the production/inventory policy is similar to Kamrad and Ritchken (1994).

However, the most salient difference between Li and Kouvelis (1999) and previous studies of flexible supply contracts and our approach involves the suppliers. Previous studies, including Li and Kouvelis (1999) view risk sharing purely from the manufacturer's perspectives that ignore the possibility that suppliers may change their prices in response to manufacturers' decisions (or market conditions). Our model introduces an approach for risk sharing which is very similar to the portfolio optimization problem in finance with a risk/return tradeoff. We effectively view the volatility of the supply schedule that an individual supplier faces as risk. The tradeoff that our model permits is a tradeoff between volatility of the supply schedule over time versus the prices (i.e. the supplier's revenues) that the manufacturer pays.

Organization of the paper is as follows. In the next section, we set out the assumptions, notation and the layout needed as the foundation upon which we develop the models. In section III, we develop the valuation models for the manufacturer and the suppliers. These models are a sequence of Bellman valuation equations. Since these class models typically do not yield closed form solutions, we need to depend upon numerical procedures for results. Section IV presents the numerical approach to the models' solutions. We define a stochastic dynamic program superimposed on a multinomial lattice, approximating the stochastic evolution of correlated exchange rate processes, and obtain the value maximizing policies for both the producer and the suppliers. We use these optimal policies to understand the risk-sharing characteristics of the supply contracts. We also rely upon a stylized example to illustrate these results. In this context, we establish how much value risk-sharing provides. Using comparative statics, we also examine how changes in properties of exchange rates affect the optimal policies and risk sharing. The last section presents the conclusion and possible future extensions.

II. ASSUMPTIONS, NOTATION AND THE MODEL

Consider a manufacturing firm producing a finished good item requiring a distinct raw material as an input. The firm contemplates entering a contract to furnish this product to a client according to a predetermined rate, fixed sales price and delivery schedule. Let $P(t)\ t \in [0,T]$ define the time t sales price and $D(t)\ t \in [0,T]$ the demand rate over the production horizon $[0,T]$ where T depicts the contract's termination time. To meet its demand obligations the firm produces at rate $q(t)$ and if needed maintains an inventory of its finished goods. Since production and inventory are capacitated, the producer's production and inventory plan reflects a value maximizing operating policy.

Various suppliers located in different countries can supply the required raw material for production. In that sense, the firm (producer) has an option to select from a pool of potential suppliers over the production horizon, $[0,T]$. As the medium of payments in most off-shore purchasing and supply agreements is typically currency, the risk of fluctuating exchange rates while acquiring the needed supplies, is fully borne by the producer. To capture the dynamics of this supplier selection process and to reflect on the resulting risk and reward implications for the producer and its' suppliers, an approach analogous to a mean-variance approach to portfolio analysis is adopted. In particular, given the pool of potential suppliers, the producer has the option to (optimally) create a portfolio of suppliers by separately entering into a supply contract with each supplier.

In this context, the portfolio weights, $\tilde{u}(t) \in \mathbb{R}^M$, $t \in [0,T]$ identify a fraction of the total needed input as supplied by a particular supplier. The optimal composition of this portfolio at any time, given the producer's value maximizing objective, is effectively characterized by two sets of constraints: *operational* and *market* related. The first set reflects production and inventory capacity constraints. The second constraint set affects the portfolio choice as it relates to the cost of having the option to revise the portfolio either by switching suppliers or by adjusting the weights.[1] Since the composition of this portfolio entails a separate contract with each supplier, we account for a risk sharing feature so that he upside potential from having a supplier switching option is not attained at the expense of downside loss to a particular supplier. Indeed, other related factors including sudden shifts in the size of orders also affect this particular aspect of the contract between the manufacturer and its suppliers and will be reflected upon contextually.

Let the raw material input rate be defined by $I(t)$, with $u_j(t) \,.\, I(t)$ depicting the fraction of the input supplied by supplier, $j = 1,2, \ldots, M$ where $0 \le u_j(t) \le 1.0$, with

$$\sum_{j=1}^{M} u_j(t) = 1.0 \tag{1}$$

The output rate, $q = \{q(t), t \in [0,T]\}$, which is precisely the control variable, is taken as an adapted positive real valued process. The rate of production is related to the input rate visa vi the production function,

$$q(t) = \{I(t)\}^{\beta} \tag{2}$$

with $0 < \beta \le 1.0$. This provides for a convenient characterization of technology in terms of return to scale as depicted by β. When producing at rate $q(t)$, the manufacturer incurs a production cost defined by $A(q(t))$. The producer optimally adjusts $q(t) \in [0,T]$ in a manner so as to maximize the net operating value while meeting its demand obligations. To that end, we assume that periodic adjustments to the rate of production are costly, thus resulting in a production switching cost which is captured by the function $\phi(q(s), q(t))$; $t > s \in [0,T]$. Since production is capacitated at the maximum feasible rate, $\overline{q}$, when necessary the producer may opt to produce and stockpile to meet its delivery obligations. Let $R(t)$ define the finished goods inventory at time

[1] Note that the extremities of these weights imply switching.

$t \in [0, T]$ with $\overline{R}$ as the inventory capacity. The holding cost can be assumed negligible or may be charged as an incremental change to the level of inventory, $dR(t)$. Accordingly, let $H(R(t))$ represent the inventory holding cost function. Given that the demand schedule is known, it follows that:

$$dR(t) = \{q(t) - D(t)\}\, dt \tag{3}$$

The supply contract between the producers and the suppliers effectively identifies an order-purchasing schedule for the parties involved. As remarked earlier, this contract is characterized by various risk sharing features so that no particular party's gain is at the expense of the other's demise. In this vein, as the producer adjusts its purchase quantities over time in response to exchange rate fluctuations,[2] the suppliers in turn, impose a penalty as a protective measure against unanticipated shifts in purchase quantities. Such shifts are economically interruptive as they derail previously planned schedules and due to their inherent ineffectiveness suppliers require a corresponding compensation in managing order size variability.

In our modeling approach, this penalty is manifest as an additional surcharge to the producer if the quantity change in the periodic order limits exceeds a supplier established critical amount, $\varepsilon_j(t),\ j = 1, 2, ..., M$. Since the change in periodic order quantities depend on the producer's demand schedule, the exchange rate dynamics, and the fraction of the total raw material to be purchased from each supplier, for analysis purposes $u_j = \{u_j(t),\ t \in [0, T]\}$ are also taken as an adapted positive real valued process with $0 \le u_j(t) \le 1.0$, where for any $t \in [0, T]$, condition (1) holds.

As a preliminary to developing the model, the stochastic evolution of the exchange rates must be defined. Let $Z(t) \in \mathbb{R}^M$ represent a standard Brownian motion that is a martingale with respect to the probability space $(\Omega, \mathscr{F}, \Im, \mathrm{P})$. The filtered probability space $(\Omega, \mathscr{F}, \Im, \mathrm{P})$ is defined over $[0, T]$ where the augmented filtration, $\Im = \{\mathscr{F}_t : t \in [0,T]\}$, is right continuous and increasing. Let $\{X_j(t); t \ge 0\}$ define the spot exchange rate between the producer and supplier j = 1, 2, …, M. The sample path of $\{X_j(t); t \ge 0\}$ is posited by the stochastic differential equations:

$$dX_j(t) = X_j(t)\{\alpha_j dt + \sigma_j dZ_j(t)\} \tag{4}$$

where

$$dZ_k(t) \cdot dZ_l(t) = \rho_{kl} dt \qquad k \ne l \tag{5}$$

[2] This essentially implies that the portfolio weights (fraction to be purchased from a particular supplier), $u_j(t)$ are implicitly a function of the exchange rate process involved. Thus, in addition to the output rate, $q(t)$, we also define $u_j(t)$ as a control variable.

For the above processes, the constant drift terms, α_j define the local trend of the process while the constant and instantaneous standard deviations, σ_j characterize the volatility for each (real) exchange process. Here, $dZ_j(t)$ represent an increment to the standard Guass-Weiner process: as the exchange rates are assumed correlated, their corresponding instantaneous and constant correlation coefficient is given by ρ.[3] We also assume that the constant risk-free rate of interest in the supplier's market is r_j, the producer's is r_p, and that there is a futures currency market providing an opportunity to hedge the exchange rate risks. To that end, let $F_j(X_j, T-t)$ represent the current price (in supplier *j*'s currency) for delivery of one unit of the producer's currency at

time *T* with, $F_j(X_j, 0) = X_j$ as the terminal boundary condition. Using Ito's lemma, it follows that[4]

$$F_j(X_j, T-t) = X_j(t)e^{(r_j - r_p)(T-t)} \tag{6}$$

which is the interest rate parity (IRP) relationship. For the most part, empirical tests of this no-arbitrage relationship have shown significance. Condition (4) will prove useful in developing the models as needed for evaluation of the supply contracts by relating the spot and futures exchange rates and the risk free rates of interest.

Toward characterizing the value of the supply contract, *V*(.) to the producer the cash flows accrued must be defined. Let the net cash flow rate be defined by $f(t)$. The various components to this cash flow process reflect the revenue arising from meeting the demand schedule, less the cost of purchasing, holding, production, switching and periodic order size (change) penalties. This latter component to the contract's overall cost dimension is effectively captured by the aforementioned protective measure which imposed by the suppliers in response to the manufacturer's periodic order size changes. To minimize costs, and to ensure a smooth flow of the needed input material, the producer changes periodic order levels in response to exchange rate fluctuations, where in the most extreme cases this will result in a switch to a different (set of) supplier(s). While periodic order level changes are to be anticipated in light of the defined dynamics defined, the extent of these changes have both operational and financial ramifications.

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3 From a theoretical perspective, purchasing power parity (PPP) would imply mean reversion in the (real) exchange rate behavior. Yet, the notion of what stochastic process exactly depicts real exchange rates, remains an open question. Here, we can also accommodate a mean reverting depiction in a restricted manner by allowing

$$\alpha_j = \theta\left(\frac{\overline{X} - X_j}{X_j}\right)$$

where θ defines the mean reversion constant; a parameter (or elastic force) that pulls back the X_j to its long run mean value $\overline{X}$.

4 See Ross (1978)

To the supplier(s), periodic order level changes imply unnecessary demand variability, resulting in operating inefficiencies. To protect against their implicit costs, the suppliers tolerate periodic order level changes so long as the change in the level of orders from one period to the next falls within a supplier defined "tolerance window"; else, a penalty is charged. In essence, supplier j, $j = 1$, …, M, requires a penalty charge of magnitude, $\pi_j(\varepsilon_j(t))$ at time t, if for all $t > s \in [0, T]$ and $j = 1, \ldots, M$;

$$\left| u_j(t)I(t) - u_j(s)I(s) \right| > \varepsilon_j(t)$$

The "tolerance window" or critical threshold, $\varepsilon_j(t)$, constrains the producer's flexibility for the obvious reason: larger $\varepsilon_j(t)$, $j = 1$, …, M, imply a smoother and more convenient ordering policy for the producer, albeit at the supplier's expense. As it will become evident, the penalty costs, $\pi_j(\varepsilon_j(t))$ help truncate this downside loss to the supplier(s) by constraining the manufacturer's response to a particular realization of the exchange rate at time t. Specifically, $\varepsilon_j(t)$ are the suppliers' "physical" reaction to $X_j(t)$ as are $u_j(t)$ to the producer for all $t \in [0, T]$ and $j = 1, 2, \ldots, M$. In essence, at equilibrium, the control variables, $u_j(t)$ and $\varepsilon_j(t)$, define and account for a particular risk sharing feature of the contract between the producer and the supplier(s), respectively.[5] Indeed, given the obligatory nature of the producer's demand schedule in terms of the needed raw material and in view of the aforementioned dynamics the producer may opt to forgo a switch to a different supplier(s) and retain the current supplier(s) through placing next period's orders, albeit at an additional cost of $\pi_j(\varepsilon_j(t))$. In so doing, the producer's desired order flexibility will be realized at a corresponding large penalty. This implies that,

$$\frac{d\pi_j(\varepsilon_j(t))}{d\varepsilon_j(t)} > 0 \qquad j = 1, 2, \ldots, M \tag{7}$$

The producer's response to the market conditions, $u_j(t)$ and the suppliers' corresponding reaction, $\varepsilon_j(t)$ define a certain aspect of the risk sharing feature of the contract. Another risk sharing aspect, a more subtle feature, concerns the sales prices as established by the suppliers in response to the exchange rates and the manufacturer's purchase proportions. We assume that supplier established sales prices accommodate quantity discounts but not at the expense of nullifying the critical order level change thresholds, $\varepsilon_j(t)$. Thus, in spite of economic incentives

[5] The term equilibrium is used rather loosely. In the current context, it characterizes a situation where the resulting optimal contract value for all the parties involved cannot be improved without a loss to another party. Yet, our approach to establishing such an objective does rely upon a game theoretic framework. Perhaps a more appropriate terminology would be "pareto optimal". However, the object of our paper is not to define, characterize or prove its existence.

offered for volume purchasing, suppliers remain exposed the purchased quantity volatility since in reaction to exchange rate uncertainty the producer may change order quantities. Let $C_j(\tilde{u},\tilde{\varepsilon},t) \in \mathbb{R}$ define the per unit gross margin rate for supplier j at time $t \in [0,T]$ and $j=1,\ldots,M$.[6] With $\tilde{u},\ \tilde{\varepsilon}$ as the vector of purchase weights and tolerance windows at time t. In light of the aforementioned, the sales price function, $C_j(t,.)$ is constrained such that:

$$\frac{\partial C_j(.)}{\partial u_j(t)} < 0 \quad \text{and} \quad \frac{\partial C_j(.)}{\partial \varepsilon_j(t)} > 0 \qquad j = 1,\ldots.M \tag{8}$$

We are now in a position to define and derive a sequence of interrelated models for evaluating M supply contracts, characterizing the agreement between the producer and M suppliers. The approach taken requires the construction of a requisite dynamic trading strategy that does not allow for riskless opportunities. As such, this continuous time arbitrage approach to the valuation problem results in M+1 Bellman equations formalizing the set of optimal policies that maximize the contracts' values. The optimal policies for the manufacturer and the suppliers, $\{u^*(t),\ q^*(t)\}$ and $\{C^*(t), \varepsilon^*(t)\}$, respectively, are arrived at through a numerical convergence in policy consensus. We proceed in the following manner. Specifically, conditional on a supplier defined set of inputs, an initial optimal solution for the producer is obtained. This solution is then used as an input to the suppliers' valuation models to obtain a corresponding set of supplier optimal policies and contract values. The process iterates once again in an exact manner, as described. This policy-revision, value-adjustment process continues until a convergence in policies is attained. At such a point, no one party's contract value can be improved without a corresponding loss of value to another party's contract.

In the following section, we derive the valuation models using a contingent claims framework. To that end, let $V(\tilde{X},R,t;q\ \tilde{u}) \in \mathbb{R}$ represent the value of the supply contract (to the producer) at time $t \in [0,T]$, given that the current level of exchange rate process is $\tilde{X}(t) \in \mathbb{R}^M$, the finished goods inventory is, $R(t)$ and that the current policy in place is $\{q(t),\ u(t)\}$: reflecting the production rate and purchase weights, respectively. The suppliers' contract value is fully characterized by their policies in light of the available market information. A component of the suppliers' policies reflects (sales) price whose setting, due to competition among the suppliers effectively depends on the exchange rates among the suppliers. Given the exchange rate process, $\tilde{X}(t) \in \mathbb{R}^M, t \in [0,T]$ let,

$$Y_{jk}(t) = \frac{X_j(t)}{X_k(t)} \qquad j \neq k;\ j,k = 1,2,\ldots,M \tag{9}$$

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[6] Implicit is the assumption that the suppliers' fixed costs do not enter the analysis: i.e. gross margin= price – variable costs.

The above expression (9) defines the exchange rate process between any pair of suppliers. To this end, let $\tilde{Y}_j(t) \in \mathbb{R}^{M-1}$ represent the time $t \in [0,T]$ vector of exchange rates between supplier *j* and all other suppliers. That is,

$$\tilde{Y}_j(t) = \{\tilde{Y}_{jk}(t)\} \qquad j \neq k;\ j,k = 1,2,...,M$$

Given expression (4), it is well established that $Y_{jk}(t)$ is lognormally distributed.[7] Supposing the existence of futures currency market for all feasible currency pairs, let $F_{jk}(Y_{jk}, T-t)$ define the current price (in supplier *j*'s currency) for delivering of a single unit of supplier *k*'s currency at time *T* with, $F_{jk}(Y_{jk}, 0) = Y_{jk}$. Analogous to equation (6) we have for all $j \neq k; j,k = 1,2,...,M$,

$$F_{jk}(Y_{jk}, T-t) = Y_{jk}(t) e^{(r_j - r_k)(T-t)} \tag{10}$$

In light of the above definitions, let $W_j(\tilde{Y}_j, t; C_j, \varepsilon_j) \in \mathbb{R}$ represent the time $t \in [0,T]$ value of the contract for supplier *j* given that the supplier exchange rate vector is $\tilde{Y}_j(t) \in \mathbb{R}^{M-1}$, the sales price is set at $C_j(t)$ and the critical order level change threshold is set at $\varepsilon_j(t)$, $j = 1,2,...,M$. We assume further that the functions $V(.)$ and $W_j(.)$ are Ito differentiable, for all $t \in [0,T]$ and $j = 1,2,...,M$.

III. The Valuation Models

In this section we derive a set of inter-rated valuation models characterizing the producer's and the suppliers' contract value together with their value maximizing policies. The approach we adopt requires the construction of a dynamic trading strategy. To avoid riskless arbitrage opportunities, the replicating portfolio positions signifying a particular trading strategy are chosen in such a way that the total expected return on the portfolio is the (local) riskless rate of return. In what follows we demonstrate how a dynamic trading strategy results into the valuation model for the producer by appropriately selecting and adjusting the portfolio positions. Without loss of generality, the same argument is then used in the follow-up subsection to develop the corresponding valuation models for the suppliers.

III.1 The Manufacturer's Model

Let $\psi_p(t)$ represent the value of a portfolio at time $t \in [0,T]$ consisting of a long position in $V(t,.)$, together with δ_j units short in the futures contracts on supplier *j*'s currency with $j = 1,2,...,M$. Over an instant of time, *dt* we have after adjusting for the cash flows:

[7] See Karlin and Taylor (1981)

$$d\psi_p(t) = dV(t,.) - \{\sum_{j=1}^{M} \delta_j \, dF_j(.)\} + f(t)dt \quad (11)$$

with the required condition that;

$$E(d\psi_p(t)) = r_p V(t,.)dt \quad (12)$$

To avoid arbitrage opportunities we choose δ_j such that[8],

$$\delta_j = \frac{\partial V(t,.)/\partial X_j}{\partial F_j(.)/\partial X_j} \qquad j = 1, \ldots, M \quad (13)$$

Using equations (11),(12) and (13), we have from Ito's lemma:

$$\sum_{j=1}^{M} \frac{\partial V}{\partial X_j} X_j(r_p - r_j) + \frac{\partial V}{\partial R}(D-q) + \frac{1}{2}\sum_{j=1}^{M} \frac{\partial^2 V}{\partial X_j^2} X_j^2 \sigma_j^2 + \frac{\partial V}{\partial t} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{i=1}^{M} \frac{\partial^2 V}{\partial X_i X_j} X_i X_j \sigma_i \sigma_j \rho_{ij} + f(t) - r_p V = 0 \quad (14)$$

where the net cash flow rate, $f(t)$ is given by;

$$f(t) = P(t)D(t) - \{A(q(t)) + H(R(t)) + \sum_{j=1}^{M} C_j\left(\tilde{u}, \tilde{\varepsilon}, t\right) + \sum_{j=1}^{M} \pi_j(\varepsilon_j(t))\} \quad (15)$$

The above expressions (14) and (15) fully characterize the contract's value to the producer. To that end, conditional on a predetermined supplier policy, $C_j(\tilde{u}, \tilde{\varepsilon}, t)\varepsilon_j(t)$ and a predefined $\pi_j(\varepsilon_j(t))$ the optimal value of the contract to the producer in light of a value maximizing policy $\left\{q^*(t), \tilde{u}^*(t)\right\}$ is obtained from the following Bellman valuation equation for all $t, s \in [0, T]$ and $j = 1, 2, \ldots, M$:

$$\underset{(q,u)}{Max} \left\{ \sum_{j=1}^{M} \frac{\partial V}{\partial X_j} X_j(r_p - r_j) + \frac{\partial V}{\partial R}(D-q) + \frac{1}{2}\sum_{j=1}^{M} \frac{\partial^2 V}{\partial X_j^2} X_j^2 \sigma_j^2 + \frac{\partial V}{\partial t} + \sum_{\substack{j=1 \\ (j \neq i)}}^{M} \sum_{i=1}^{M} \frac{\partial^2 V}{\partial X_i X_j} X_i X_j \sigma_i \sigma_j \rho_{ij} \right.$$

$$\left. + P(t)D(t) - \left\{ A(q(t)) + H(R(t)) + \sum_{j=1}^{M} C_j(\tilde{u}, \tilde{\varepsilon}, t) + \sum_{j=1}^{M} \pi_j(\varepsilon_j(t)) \right\} - r_P V \right\} = 0 \quad (16)$$

s.t.

$$0 \le R(t) \le \overline{R} \quad (16a)$$

$$0 \le q(t) \le \overline{q} \quad (16b)$$

$$0 \le \tilde{u}(t) \le 1.0 \quad (16c)$$

$$\pi_j(\varepsilon_j(t)) = \begin{cases} h(\varepsilon) & if \quad \left|u_j(t)I(t) - u_j(s)I(s)\right| > \varepsilon_j(t) \\ 0 & \text{otherwise} \end{cases} \quad (16d)$$

with

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8 The actual derivation of δ_j is straightforward, though tedious. For brevity sake, we have excluded it form the paper.

$$\lim_{\tilde{X}\to\infty} \frac{V(\tilde{X},R,t;q,\tilde{u})}{\tilde{X}} = 0 \tag{16e}$$

$$\lim_{\tilde{X}\to 0} \frac{V(\tilde{X},R,t;q,\tilde{u})}{\tilde{X}} < \infty \tag{16f}$$

For the above optimization problem, expressions (16a-c) are self-explanatory. In (16d), the supplier imposed penalty functions and penalty conditions are defined. For the moment, the function, $h(\varepsilon(t))$, serves as a generic characterization of the penalty incurred. We will define $h(.)$ specifically when the model is illustrated through a numerical example. Also, conditions (16e,f) ensure that the valuation function is well behaved. The above Bellman equation (16) does not yield a closed form solution and thereby, it must be solved numerically to obtain the optimal contract value $V^*(t,.)$. However, in light of known functional forms that characterize the above Bellman equation, it is possible to derive closed form expressions for the optimal policies in place: that is $\{\overset{*}{q}(t),\overset{*}{u}(t)\}$.

III.2 <u>The Suppliers' Model</u>

By taking a similar approach, as shown in the previous subsection, we can obtain supplier valuation models in the form of a sequence of Bellman equation. Recall that in the current context, suppliers' policies in place are defined by their sales price and critical order change levels, $C_j\left(\tilde{u},\tilde{\varepsilon},t\right)$ and $\varepsilon_j(t),\ j=1,2,\ldots M$, respectively. Furthermore, in this case the cash flow accrued to the each supplier over an instant, dt is defined by;

$$C_j(t,.) + \pi_j(\varepsilon_j(t)) \tag{17}$$

Given equations (9), (10) and (17) and in light of the definitions established earlier, we have, without loss of generality, for $j=1,2,...,M$:

$$\underset{C_j,\varepsilon_j}{Max}\left\{ \sum_{\substack{k=1\\k\neq j}}^{M} \frac{\partial W_j}{\partial Y_{jk}} Y_{jk}(r_j - r_k) + \frac{1}{2}\sum_{\substack{k=1\\k\neq j}}^{M} \frac{\partial^2 W_j}{\partial Y_{jk}} Y_{jk}(\sigma_j^2 + \sigma_k^2 - 2\sigma_j\sigma_k\rho_{jk}) + \frac{\partial W_j}{\partial t} + \pi_j(\varepsilon_j(t)) + C_j\left(\tilde{u},\tilde{\varepsilon},t\right) - r_j W_j \right\} = 0 \tag{18}$$

$s.t.$

$$0 \le u_j(t) \le 1.0 \tag{18a}$$

$$\lim_{Y_j\to\infty} \frac{W_j(\tilde{Y}_j,t;C_j,\varepsilon_j)}{Y_j} = 0 \tag{18b}$$

$$\lim_{Y_j \to \infty} \frac{W_j(\tilde{Y}_j, t; C_j, \varepsilon_j)}{Y_j} > \infty \tag{18c}$$

$$\pi_j(\varepsilon_j(t)) = \begin{cases} h(\varepsilon) & if \quad \left| u_j(t)I(t) - u_j(s)I(s) \right| > \varepsilon_j(t) \\ 0 & \text{otherwise} \end{cases} \tag{18d}$$

IV. <u>The Solution Procedure</u>

The Bellman Valuation equations (16) through (18) fully characterize each party's contract features. For he producer, the value maximizing policies $\left\{ q^*(t), u_j^*(t) \right\}$ effectively signify the optimal production rate and purchase weights, respectively. For the suppliers, on the other hand, their operating policies $\left\{ C_j^*(\bullet), \varepsilon_j^* \right\}$ maximize their respective net revenues. In that sense, the price of inputs purchased by the manufacturer as well as the penalties incurred for having violated the supplier established "tolerance windows" defines the revenues. The notion of net revenues maximized stems from our earlier assumption that the supplier's fixed costs are not affected by the policies in place and in that sense remain invariant.
We further note that equations (16) through (18) do not yield closed form solutions and must be solved numerically to obtain results. To that end, a dynamic programming procedure is developed to numerically solve each model in light of its corresponding setting. Our ability to invoke this recursive procedure is a direct consequence of the Feynmann-Kac results in which the solution of certain partial differential equations are known to be equivalent to the solution of an appropriately adjusted expectation.

Here this adjustment affects the drift term, α_j (the expected growth rate) of the exchange rate process as shown in equation (4). As a result of the arbitrage-free valuation approach employed herein, this drift term maybe proxied by an equivalent martingale measure: the risk neutralized expected growth rate, $(r_p - r_j),\ j = 1, 2 \ldots, M$. Hence, for valuation purposes equation (4) can be replaced by,

$$dX_j(t) = X_j(t)\left\{ (r_p - r_j)dt + \sigma_j dZ_j(t) \right\} \tag{19}$$

To implement our recursive procedure, we adapt a multinominal lattice framework to approximate the stochastic evolution of the above equation (19). Such an approximating framework provides the needed platform upon which a backward stochastic dynamic programming (SDP) algorithm can be superimposed, thus allowing for the "state" contingency of actions.

To that end, let $\mathbb{P}_n = \{0 \le t_0 < t_1 < t_2 < \dots t_n = T\}$ represent an equiwidth partition of the planning horizon $[0,T]$such that the during each production period $[t_i, t_{i+1}] = \Delta t$, the exchange rate pair $(X_1(t_i), X_2(t_i))$ $i = 0,1,2,\dots,n-1$ can take on any one of the following fine jump values:

Jump Event	Corresponding Values	Probability
(up,up)	$X_1(t_0)w_1$, $X_2(t_0)w_2$	p_1
(up, down)	$X_1(t_0)w_1$, $X_2(t_0)d_2$	p_2
(down,down)	$X_1(t_0)d_1$, $X_2(t_0)d_2$	p_3
(down, up)	$X_1(t_0)d_1$, $X_2(t_0)w_2$	p_4
(none, none)	$X_1(t_0)$, $X_2(t_0)$	p_5

With $\sum_{j=1}^{5} p_j = 1$. In the above table, $w_k = e^{\lambda\sigma_k\sqrt{\Delta t}}$ defines the sign of an up jump while $d_k = \frac{1}{w_k}$ is the down jump size with σ_k and Δt as defined earlier, $k = 1,2$. Furthermore, the sketch parameter, $\lambda \ge 1$ ensures the opportunity for a horizontal jump (i.e: a no-change state) in each period. This lattice specification is due to Kamrad and Ritchken (1991) and its computational accuracy in approximating the point lognormal distribution of the exchange rate pair is well documented. For valuation purposes the probability terms are needed and are expressed below:

$$p_1 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right) + \frac{\rho}{\lambda^2}\right\} \tag{20a}$$

$$p_2 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right) - \frac{\rho}{\lambda^2}\right\} \tag{20b}$$

$$p_3 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{-\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right) + \frac{\rho}{\lambda^2}\right\} \tag{20c}$$

$$p_4 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{-\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right) - \frac{\rho}{\lambda^2}\right\} \tag{20d}$$

$$p_5 = 1 - \frac{1}{\lambda^2} \tag{20e}$$

where

$$\mu_k = r_p - r_k - \frac{\sigma_k^2}{2} \qquad k = 1,2 \tag{21}$$

denotes the drift of the approximating process. Given this setting we can proceed with the development of the solution procedure. To that end, let $V_i(\tilde{X},R;\tilde{u}_{i-1},q_{i-1})$ Represent the time t_i value of the contract to the producer given that the current exchange rate process is $(X_1(t_i),X_2(t_i))$, the finished goods lend of inventory is R_i and that during production period $[t_{t-1},t_i]$, q_{i-1} units were produced with raw material purchase weights $(u_1(t_{i-1}),u_2(t_{i-1}))$. Given the partition $\mathbb{P}_n$ consider the time epoch t_n :

$$V_i(\tilde{X},R;\tilde{u}_{i-1},q_{i-1}) = P_n D_n - \left\{A(q_{n-1}) + \phi(q_{n-1},q_n)\right\} \tag{22}$$

with

$$A(q_n) = 0 \tag{22a}$$

$$R_n = q_n = 0 \tag{22b}$$

$$q_{n-1} = \left(I_{n-1}\right)^{\beta} = \left\{I_{n-1}\ u_1(t_{n-1}) + I_{n-1}\ u_2(t_{n-1})\right\}^{\beta} \tag{22c}$$

$$u_1\left(t_{n-1}\right) + u_2\left(t_{n-1}\right) = 1.0 \tag{22d}$$

$$\phi\left(q_{n-1},q_n\right) = \phi\left(q_{n-1},0\right) \tag{22e}$$

From the above equation (22) reflecting the termination time t_n, expressions (22a,b) are self explanatory. Equation (22c) relates the production quantity decision to total raw material purchase at time t_n. Equation (22d) identifies purchase weight's constraint, while (22e) may be consider as the switch cost function at expiration: shutdown cost. Note that the production cost $A(q_{n-1})$ is incurred at time t_n. In what follows, let $C_j(\tilde{u},\tilde{\varepsilon},t) \equiv C_j(t_i)$ and $\pi_j(\varepsilon_j(t_i)) \equiv \pi_j(t_i)$ define the per unit gross margin and penalty induced revenue to supplier j at time t_i, $j = 1,2$ and $i = 0,1,\ldots,n.$ We now consider time t_{n-1}, marking the period $[t_{n-1},t_n]$ where

$$V_{n-1}(\tilde{X},R;\tilde{u}_{n-2},q_{n-2}) = \underset{(\tilde{u}_{n-2},q_{n-2})}{Max}\ \{\tilde{E}\{P_{n-1}D_{n-1} - \{A(q_{n-2}) + H(R_{n-1}) + \sum_{j=1}^{2} C_j(t_{n-1}) + \sum_{j=1}^{2} \pi_j(t_{n-1})$$

$$+\ \phi(q_{n-2},q_{n-2})\} +\ e^{-r_p(t_n-t_{n-1})} V_{n-1}(\tilde{X},R;\tilde{u}_{n-1},q_{n-1})\}\} \tag{23}$$

s.t.

$$R_{n-1} + q_{n-1} = D_n \tag{23a}$$

$$q_{n-1} = \left\{I_{n-1}\ u_1\left(t_{n-1}\right) + I_{n-1}\ u_2\left(t_{n-1}\right)\right\}^{\beta} \tag{23b}$$

$$q_{n-2} = \left\{I_{n-2}\ u_1\left(t_{n-2}\right) + I_{n-2}\ u_2\left(t_{n-2}\right)\right\}^{\beta} \tag{23c}$$

$$u_1\left(t_{n-2}\right) + u_2\left(t_{n-2}\right) = 1.0 \tag{23d}$$

for $j = 1,2$

$$\pi_j\left(t_{n-1}\right)=\begin{cases} 0 & if \quad \left|I_{n-2}u_j\left(t_{n-2}\right)-I_{n-1}\ u_j\left(t_{n-1}\right)\right|\le\varepsilon_j\left(t_{n-1}\right) \\ h\left(\varepsilon_j\left(t_{n-1}\right)\right) & if \quad o/w \end{cases} \tag{23e}$$

$$0\le q_{n-2}\le\overline{q} \tag{23f}$$

$$0\le q_{n-1}\le\overline{q} \tag{23g}$$

$$0\le R_{n-1}\le\overline{R} \tag{23h}$$

In equation (23), $\tilde{E}(\cdot)$ defines the expectation operator with respect to equation (19). Furthermore, the total cost incurred at time t_{n-1} is comprised of production, inventory, purchase switching and penalty costs. This penalty cost is characterized by expression (23e) with $h(\cdot)$ as the functional form for the penalty cost whenever periodic change in the purchase levels violate the supplier established "tolerance windows", ε. Constraint (23a) ensures that the level of inventory on hand plus the next period's production quantity satisfies the demand during the next period. Other constraints, (23 f, g, h) account for production and inventory capacity concerns. In general for valuation purposes we have for every period $[t_i,t_{i+1}]$, $i=0,1,\ldots,n-1$ the following Bellman recursion:

$$V_{i-1}(\tilde{X},R;\tilde{u}_{i-2},q_{i-2})=\underset{(\tilde{u}_{i-2},q_{i-2})}{Max}\ \{\tilde{E}\{P_{i-1}D_{i-1}-\{A(q_{i-2})+H(R_{i-1})+\sum_{j=1}^{2}C_j(t_{i-1})+\sum_{j=1}^{2}\pi_j(t_{i-1})$$

$$+\ \phi(q_{i-2},q_{i-2})\}+\ e^{-r_p(t_i-t_{i-1})}V_{i-1}(\tilde{X},R;\tilde{u}_{i-1},q_{i-1})\}\} \tag{24}$$

For brevity's sake, we omit restating constraints (24b-h) as they echo previously stated constraints (23b-h). The only modification needed is specializing to the appropriate time epoch where the time reference is t_i. Given this SDP-based solution approach to the producer's problem, we can now address the same for the suppliers'. To this end, recall that the suppliers' objective function reflects the net revenue accrued during each purchase period. This revenue is comprised of cash flows generated through the sale of raw materials, $C_j(\tilde{u},\tilde{\varepsilon},t)\equiv C_j(t_i)$ in addition to the penalties paid by the producer, $\pi_j(\varepsilon_j(t_i))\equiv\pi_j(t_i)$. Let $W_i(Y_i,C_j,\varepsilon_j)$ represent the time t_i value of the contract to supplier j given that the supplier's exchange rate is $Y(t_i)=Y_{jk}(t_i)=\dfrac{X_j(t_i)}{X_k(t_i)}$, $j=1,2$ and $k\neq j$, the raw material sales proceeds are set to be $C_j(t_i)$ and the "tolerance window" at time t_i is established to be $\varepsilon_j(t_i)$. Consider the partition $\mathbb{P}_n$. The value of the contract to supplier j (in supplier j's currency) at time t is

$$W_n(Y; C_j, \varepsilon_j) = 0 \qquad\qquad j = 1,2 \qquad (25)$$

For all other time epochs t_i, $i = 0,1,\ldots n-1$ and the corresponding Bellman recursion is given by,

$$W_i(Y; C_j, \varepsilon_j) = \underset{c_j,\varepsilon_j}{Max}\left\{E\left\{\left\{\frac{C_j(t_i)}{X_j(t_i)} + \frac{\pi_j(t_i)}{X_j(t_i)}\right\} + e^{-r_j(t_{i+1}-t_i)} W_{i+1}(Y; C_j, \varepsilon_j)\right\}\right\} \quad (26)$$

with $\pi_j(t_i) = h(\varepsilon_j(t_i)$

The above equation (22) through (26) define our numerical solution approach as a sequence of Bellman recursions. To obtain results, the above equations are superimposed on an approximating lattice introduced earlier in this section. Each node in this lattice, such as node, (i,k,l) represents the state of the exchange rate vector at time t_i, $i = 0,1,2,\ldots n$. That is,

$$\left(X_1(t_i), X_2(t_i)\right) = \left(X_1(t_0) w_1^k, X_2(t_0) w_2^l\right)$$

With w_1 and w_2 defined earlier. Here, the set of feasible selections realizations for (k,l) at time t_i are; $(k,l) \in \left(B(i), B'(i)\right)$ with

$$B(i) = \{-i, -i+2, \ldots, i-2, i\} \text{ and } B'(i) = \{-i+1, -i+3, \ldots, i-3, i-1\}$$

In the following section, we illustrate this set up through a stylized example where the expectations as seen in equation (22) through (26) are computed on the lattice using the lattice probability terms (2a-e).

V. <u>Illustration and Results</u>

To numerically illustrate the model, various functional forms have to be defined. In what follows, the production cost function, the purchase cost functions and the penalty cost functions for each supplier *j* at time t_i is characterized, respectively:

$$A(q_i) = a_0 + a_1 q_i + a_2 q_i^2 \qquad (27)$$

$$C_j(t_i) = (C_{0j}\varepsilon_j(t_i) - C_{1j}(t_i)\varepsilon_j(t_i) Iu_j(t_i))\; u_j(t_i) \qquad (28)$$

$$\pi_j(t_i) = h(\varepsilon_j(t_i)) = (C_{0j}^2\varepsilon_j(t_i) + C_{1j}^2(t_i)\varepsilon_j(t_i) - 2C_{0j}^2 C_{1j}^2(t_i)\varepsilon_j^2(t_i)) \qquad (29)$$

Equation (27) reflects the quadratic nature of the production cost function. In equation (28), we rate the quantity discount feature of the purchase cost function. Specifically, the higher the purchase quantity (Iu_j) from a supplier, the lower the variable cost of the purchased quantity. In equation (29), reflecting the penalty cost to the producer, violating contract terms are effectively penalized as a function of current purchase costs and "tolerance windows". In this light, at each node of the approximating lattice at time t_i, $i = 0, 1, 2 \ldots, n-1$, the manufacturer is faced with choosing the optimal output quantity and the proportion of raw material to purchase from each supplier: essentially the "security weights" in a portfolio maximization context. Given the manufacturer's operating decisions $(i.e.\ q^*, u_j^*)$, the two suppliers use this policy as a given input to their corresponding optimization problem to maximize over the control variables: namely C_{1j} and ε_j, $j = 1, 2$. Implicit is the assumption that the fixed cost of purchasing, C_{0j} is not a controlled decision. Here too, the problems faced by the suppliers reflects a "portfolio maximization" concern representing conflicting choices. If C_{1j} were to be increased, the revenues also increase. However, the manufacturer can then purchase fewer units. Concurrently, the choice of a smaller ε_j results in increased penalties paid by the but here too, the manufacturer is likely to resort to purchasing smaller quantities. This trade off and the optimal equilibrium that is rendered, is ultimately achieved by imposing the constraint that an improvement in a particular party's (supplier or producer) objective function value cannot be attained at a great expense to the other's. Thus at a given node, if the supplier's value function is improved by altering ε_j and C_{1j} such that the impact of this change would result in a reduction in the manufacturer's objective function value by more than 10%, the supplier(s) cannot implement that policy. Yet, if an alteration to the suppliers' policy results in an improvement without a concurrent deterioration in the manufacturer's value function, the new policy will be acceptable as a feasible solution. The iteration then moves back to the manufacturer where the newly furnished policy is taken as an input to manufacturer's problem, subject to the above mentioned "equilibrium" constraint.[5] We carry out similar iterative optimizations at each node on the lattice. We iterate between the manufacturer and the suppliers at least 4 rounds at each node on the lattice. The values at time 0 for the manufacturer and the two suppliers are then the optimal values.

We are now in a position to illustrate the sensitivity as well as the robustness of our models in a parametric sense. The table shown below summarizes our base case parameter values.

[5] We carry out our optimization in FORTRAN and search over 1200 steps of the decision variable at each node.

Table 1: Base Case Parameters and Initial Values for Variables

Producer's Demand Schedule	D_i	$D_1 = 120; \quad D_2 = 120; \quad D_3 = 170; \quad D_A = 120$
Producer's unit Sales Price	$P(t_i)$	$P(t_i) = P = \$80$ per unit
Production Capacity	$\overline{q}$	$q_i \in (0, 300)$
Inventory Capacity	$\overline{R}$	$R_i \in (0, 60)$
Production Cost Function	$A(q_i)$	$a_0 = 1.2; \quad a_1 = 0.8; \quad a_2 = 0.2$
Elasticity	β	$\beta = 1.0$
Initial Exchange Rate	$\tilde{X}_0$	$(X_1(t_0), X_2(t_0)) = (1.0, 1.0)$
Exchange Rate Volatilities and Correlations	σ	$\sigma_1 = .30; \quad \sigma_2 = .85; \quad \rho_{12} = .25$ (per annum)
Local riskfree interest rates	r	$r_p = .08; \quad r_1 = .10; \quad r_2 = .15$ (per annum)
Supplier Furnished Initial Terms	Supplier 1	$\left(C_{01} = 3.0; \quad C_{11} = .15; \ \varepsilon_1 = 20\right)$
	Supplier 2	$\left(C_{02} = 4.0; \quad C_{12} = .20; \ \varepsilon_2 = 12\right)$

Initially, we estimate the value of the option to switch as the difference of $V^*\left(u_1, u_2\right) - Max\left[V^*\left(u_1\right), V^*\left(u_2\right)\right]$, i.e. as the difference between the maximum objective function value when the manufacturer has the option to switch between the two suppliers, versus the maximum value reached when the manufacturer is permitted to choose only a single supplier. As long as this value is positive, the option to switch is valuable for the producer even if the suppliers react to the producer's policies by altering their decisions. We also examine how this option value varies when the correlation between the two suppliers varies as well as when the volatility of a single supplier varies.

Figure 2 presents the option value when the correlation between the two suppliers' exchange rates increases. The value of the option increases when this correlation increases. Intuitively, it implies that even the correlation between the exchange rates is low (or negative), there is reduction in risk through diversification without having to switch between the suppliers. However, as the correlation increases, the benefits of switching between the suppliers become more valuable.

Figure 3 presents the variation in the value of the option to switch when the volatility of the first supplier increases. The volatility of the second supplier is fixed at 0.85%. Here we find that the option to switch is much more valuable when the volatility of the first supplier is low (thus a substantial difference exists between the two volatilities.) As the volatility of the first supplier increases, the option to switch becomes less valuable. Thus, the value of the option declines and approaches a plateau. This suggests that at a given correlation, if the volatilities of the two suppliers are similar, the ability to switch is much less valuable.

We then carry out a variety of static analyses to understand how changes in parameters affect the risk sharing between the manufacturer and the two suppliers. We first analyze the impact of changing the correlation between the two sources of risk, i.e. ρ_{12}, the correlation between the two exchange rates. For the manufacturer's costs, we should expect two possible effects from a change in ρ_{12}. The first is that with greater correlation, the gains from diversification decline. However, greater correlation can also reduce the penalties that the manufacturer has to pay the suppliers for violating the bands since if the two suppliers exchange rates move in unison, the weights should not change. At the same time, changes in correlation may also cause the manufacturer to meet more of the demands earlier, thereby increasing the value function. The total effect on the manufacturer's value function would be the sum of the three effects. For the suppliers, the value functions should decrease if the increase in correlation causes the manufacturer's costs to decrease. However, if with increasing correlation, the manufacturer purchases greater quantities from the suppliers, then their value functions can increase.

Figure 4 presents the value functions at different correlations. We find that for the manufacturer the gains resulting from the reduction in the penalties and meeting the demand earlier appear to dominate the losses from lower diversification. Additionally, the two suppliers appear to gain as well.

We next examine the impact of changing the volatilities of the two suppliers' exchange rates. We first examine the case of changing the volatility of the second supplier in Figure 5 Here again higher volatility for supplier 2 implies that supplier 1 benefits unambiguously since the manufacturer purchases more from her. The manufacturer benefits as long as the correlation between the two suppliers is positive and the second supplier's volatility is less than or equal to that of the first supplier.

However, once the volatility of the second supplier exceeds that of the first supplier, the manufacturer's value function can decrease. For the second supplier, increase in the volatility has two opposing effects. The first is that the greater volatility increases the revenues she gets from the manufacturer violating the penalty bands. However, greater volatility also results in the manufacturer buys smaller proportion of the total input from her. This reduces the revenue she gets. Figure 5 displays all these phenomena.

We then examine the impact of increasing the volatility of the first supplier on the value functions and the proportion bought from each supplier. As we expect from Figure 5, increasing the volatility of the first supplier benefits the second supplier, and the first supplier also benefits, though not as much as the second. Additionally, the manufacturer eventually is worse off as the volatility increases. Figure 6 presents these results.

V. CONCLUSIONS

Multinational enterprises have recognized that with multiple sourcing across international borders, uncertainty in exchange rates creates new risks as well as provides new opportunities to design flexibility in supply contracts. Previous studies on operating policies with exchange rate uncertainty have evaluated the profit maximization problem for the manufacturer while ignoring what would induce the suppliers to accept flexible contracts. In this paper, we provide a unified real-options (contingent-claims) based framework for valuing the flexibility in supply contracts. We jointly examine the optimization problems for the suppliers along with the manufacturer's problem. We use a portfolio-optimization like framework to analyze the incentives that the suppliers face in accepting flexibility. The tradeoff is between greater volatility in the supply schedule and the prices that the manufacturer pays. Thus, we explicitly model how flexibility can be beneficial to both the producer and multiple suppliers. In other words, how a contract with flexibility can be Pareto optimal, with neither the producer nor the suppliers being worse off as compared to inflexible contracts. This implies that decisions to switch between suppliers are not costless but result in the manufacturer having to compensate suppliers in one form or other. Finally, the exogenous sources of risk can be viewed as value drivers for the both producer and the suppliers, and we examine how changes in the value drivers affect the profit-maximization problems for the producer as well as the suppliers.

We find that the option to switch becomes more valuable with greater correlation between the exchange rates. Additionally, the option to switch is more valuable if the exchange rates have different volatilities rather than similar volatilities. Finally, we also find that increase in volatility for one supplier is beneficial for the other supplier whereas the impact on the producer is ambiguous.

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Figure 1: Lattice Path with 5 Jumps at Each Node

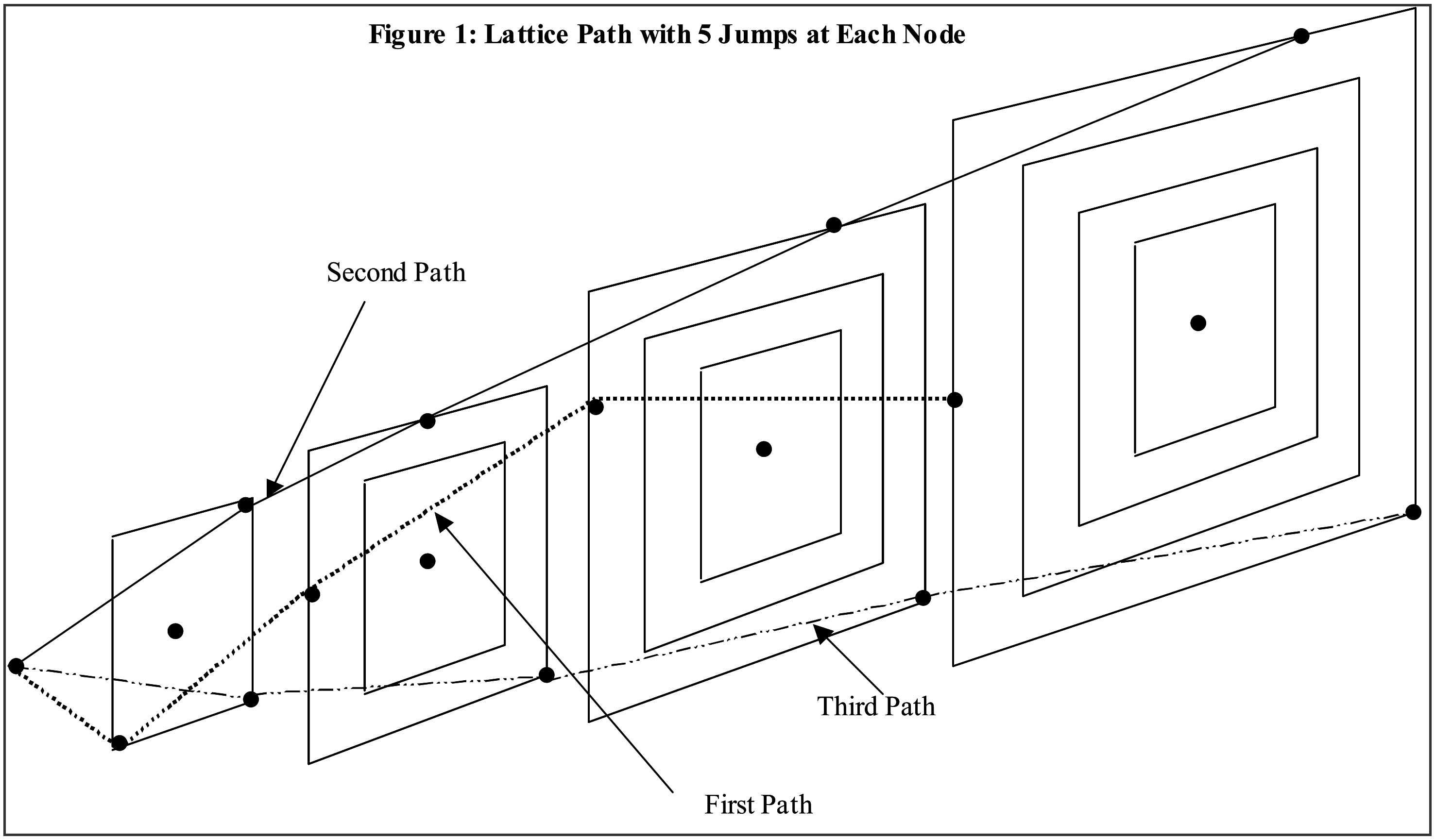


Table 2: Policies for Three Paths along the Lattice

	(i,j,k)	V	Q	R_i	u_1, u_2	W_1	C_{11}	ε_1	W_2	C_{21}	ε_2
First Path	0,0,0	781.0	165	0	0.49 0.51	217.5	0.30	15.0	622.7	0.30	15.0
	1,-1,-1	681.2	165	45	0.15 0.85	49.1	0.25	12.5	664.4	0.25	12.5
	2,-2,0	3293.3	170	45	0.68 0.32	200.7	0.25	12.5	53.4	0.25	12.5
	3,-3,+1	3760.0	75	45	0.90 0.10	464.1	0.25	12.5	10.3	0.25	12.5
	4,-4,0	8413.8	0	0							
Second Path	0,0,0	781.0	120	0	0.49 0.51	217.5	0.30	15.0	622.7	0.30	15.0
	1,+1,+1	903.0	135	0	0.35 0.65	212.0	0.25	12.5	662.8	0.20	12.0
	2,0,+2	2235.6	185	15	0.97 0.03	430.5	0.25	12.5	2.1	0.25	12.5
	3,+1,+3	2095.7	90	30	1.00 0.00	615.6	0.25	12.5	0.0	0.25	12.0
	4,+2,+4	7906.8	0	0							
Third Path	0,0,0	781.0	120	0	0.49 0.51	217.5	0.30	15.0	622.7	0.30	15.0
	1,1,-1	893.8	120	0	0.07 0.93	25.3	0.25	12.5	868.0	0.20	12.0
	2,2,-2	1515.2	185	0	0.05 0.95	20.8	0.25	12.5	370.5	0.25	12.5
	3,3,-3	1552.3	105	15	0.02 0.98	0.8	0.25	12.5	653.5	0.25	12.5
	4,4,-4	7309.8	0	0							

Figure 2: Producer's Value Function as a Switching Option

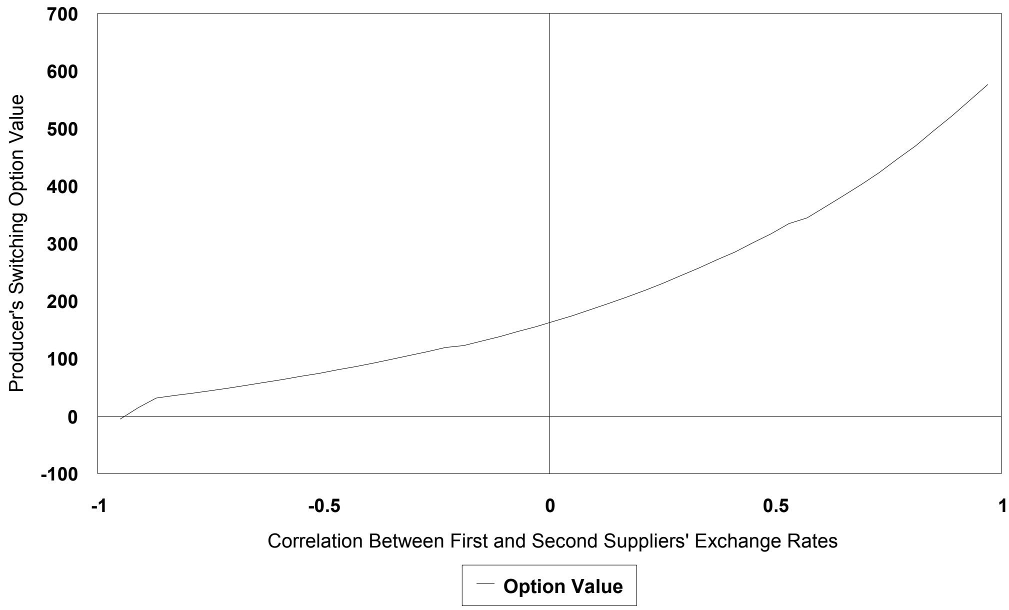
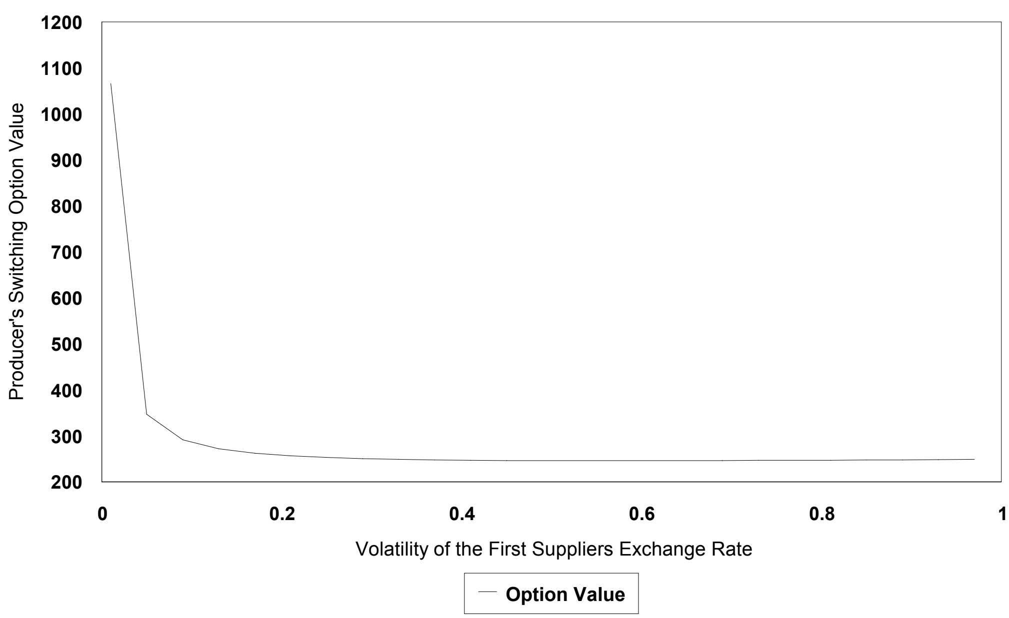


Figure 3: Producer's Value Function as a Switching Option



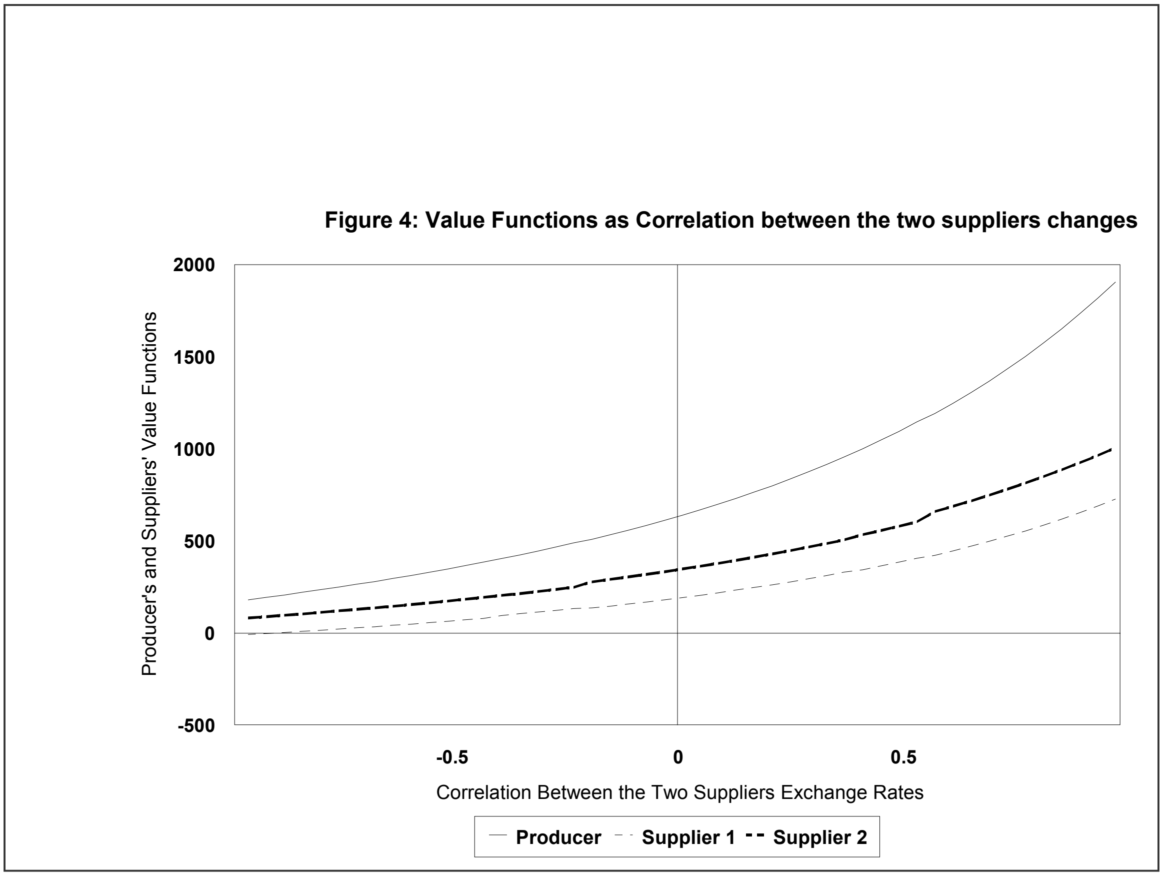


Figure 4: Value Functions as Correlation between the two suppliers changes
2000
1500
1000
500
0
-500
Producer's and Suppliers' Value Functions
-0.5
0
0.5
Correlation Between the Two Suppliers Exchange Rates
Producer
Supplier 1
Supplier 2

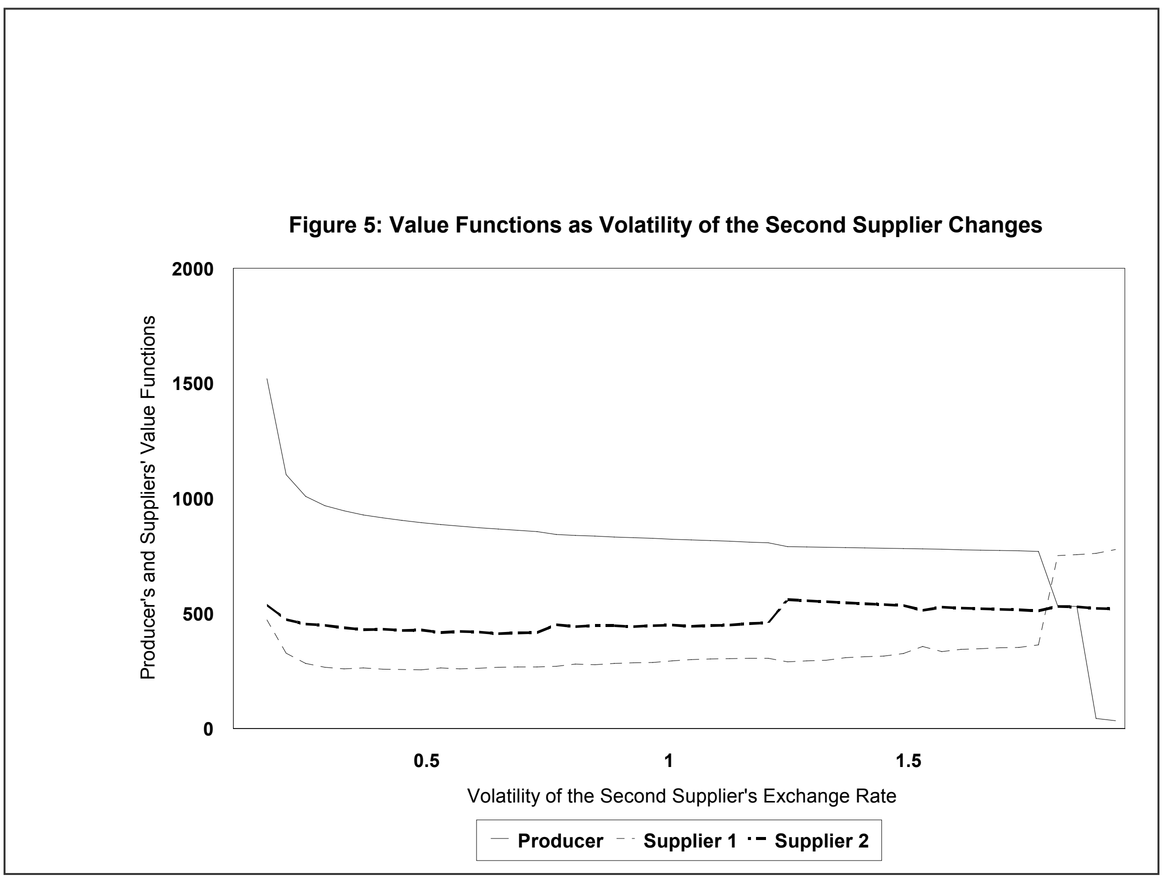


Figure 5: Value Functions as Volatility of the Second Supplier Changes
2000
1500
1000
500
0
Producer's and Suppliers' Value Functions
0.5
1
1.5
Volatility of the Second Supplier's Exchange Rate
Producer
Supplier 1
Supplier 2

Figure 6: Value Functions as Volatility of the First Supplier Changes