

**EXPORTS AND PRODUCTION TECHNOLOGY  
UNDER VOLATILE EXCHANGE RATES**

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There are a variety of sources of uncertainty in the business environment. To accommodate uncertainty, businesses may choose to invest in flexibility rather than the cost advantages that could be offered by specialization. The costs of operating flexible production technologies, for instance, may be relatively insensitive to the mix of inputs or outputs° consider chemical processing plants that can easily switch between naphtha and natural gas feedstocks or plastics extrusion facilities which can readily produce a wide variety of products. Investments in modular design can lower the costs of future capacity expansion at electrical generating plants or enhance the adaptability of a basic product across a variety of differentiated markets as with electronic components. Advances in the design of organization structures and procedures work to decentralize information, authority and responsibility to reduce response times and improve decision quality in an uncertain environment. Flexibility, however, does not come cheaply. In

each of the above instances additional costs are incurred to provide flexibility in a volatile environment as compared to specialization in a certain environment.<sup>1</sup>

Perhaps the simplest characterization of flexibility uses the average cost curve. Stigler (1939) initially relates flexibility to the curvature of the average cost function. Marschak and Nelson (1962), assuming quadratic costs, measure flexibility as the reciprocal of the second derivative of the average cost function. The less flexible the technology, the more steeply sloped the average cost curve, and hence the greater the cost penalty for deviating from the minimum efficient scale of production. Mills and Schumann (1985) relate flexibility to the elasticity of supply at the expected market price. Empirically they find an inverse relationship between firm size or capital intensity and sales variability, and between firm size and employment variability. Large, capital-intensive firms invest in specialized equipment which conveys a cost advantage over smaller rivals. To survive, smaller, less capital-intensive firms utilize proportionately more variable factors, such as labor. This difference in production technology allows the smaller firms to be more responsive to fluctuations in demand.

This paper investigates the link between the volatility of exchange rates and the production technology of exporting firms. I define a technology's flexibility, or dynamic leverage, as the ratio of sunk cost per unit of capacity to the marginal cost of production.<sup>2</sup> Highly specialized, inflexible technologies thus possess high dynamic leverage. Once sunk, these technologies tend to be very cost efficient° specialized investments in skills and equipment lead to low marginal

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<sup>1</sup>An alternative response to uncertainty is insulation. Caves, et al. (1979) consider the role of a firm's buffer stocks in volatile environments.

<sup>2</sup>Dynamic leverage is to sunk costs as operating leverage is to fixed costs, both affect the value of a firm in a volatile environment. Refer to Edleson (1989).

operating costs. In an uncertain market, however, a firm may choose to be less specialized and invest in a more flexible technology capable of responding to volatility. The higher marginal operating costs of the more flexible technology are tolerated for a smaller sunk investment.<sup>3</sup>

This paper is divided into four sections. Section 1 sets out the basics of the model. Section 2 assumes that production technology is exogenously determined and considers the effect of exchange rate volatility on capacity and capacity utilization upon entry. I show that technologies with less dynamic leverage, a lower ratio of sunk costs to marginal operating costs, enter the export market with a larger scale and at a less favorable exchange rate than a technology with higher dynamic leverage. Section 3 considers the optimal choice of technology in an uncertain environment. I show that the more volatile the exchange rate, the smaller is the firm's investment in specialization and the lower is the chosen degree of dynamic leverage. Section 4 summarizes the findings and discusses some extensions to this line of research.

## **1 Basic Structure**

Consider a monopolist, based at Home, contemplating the construction of a facility to produce widgets for export to a Foreign market.<sup>4</sup> Costs are denominated in Home Currency, HC, and revenues are denominated in Foreign Currency, FC. The real exchange rate,  $R$ , is the HC price of FC,  $R=HC/FC$ . The Foreign inverse demand curve is linear,  $P(Q)=a-bQ$ . The HC revenue from the sale of  $Q$  units in the Foreign market is thus  $RP(Q)Q$ .

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<sup>3</sup>I do not suppose that the more flexible technology requires a smaller initial investment, only that the sunk costs of the investment are smaller. Indeed, with the advent of flexible manufacturing systems, the increase in general skills required to be productive in a more flexible technology, and the development of more sophisticated organizational structures, a more flexible facility may well entail a larger initial investment than a more specialized facility.

The real exchange rate,  $R$ , is assumed to follow a geometric Brownian motion. The instantaneous percentage change in the exchange rate is given by the following expression:  $dR/R = \sigma dz$ .<sup>5</sup> The variance of the percentage change is  $\sigma^2$  and  $dz = \omega(t)dt$ , where  $\omega(t)$  is a serially uncorrelated standard normal random variable. The real exchange rate is the only stochastic parameter in the model.

The firm must decide when to build capacity, how much capacity to build, and, in Section 3, in which technology to invest. For simplicity, assume that plant construction is instantaneous upon the firm's decision to enter the Foreign market.<sup>6</sup> Further, assume that once the firm has built capacity,  $K$ , there can be no adjustments to capacity; upgrades, reductions, and exit, are all not permitted. There is, however, flexibility in the scale of production. Given the capacity constraint, the firm determines the optimal quantity of widgets to produce and export each period as a function of the exchange rate. Neither the firm nor the consumer can accumulate inventory; widgets must be sold and consumed in the period produced.

Figure 1

### Optimal Production with a Volatile Exchange Rate

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<sup>4</sup> I require that the Home country have a monopoly on the supply of widgets; the analysis is invariant to the firm's status as a monopolist or Cournot oligopolist.

<sup>5</sup> To focus on the effect of volatility and render the analysis more tractable, I do not assume any drift term in the exchange rate. Future reference to a depreciation or appreciation in the HC implies only that  $R$  has moved above or below a particular level.

<sup>6</sup> Issues regarding construction lags are analyzed in Majd and Pindyck (1987) and Teisberg (1988).

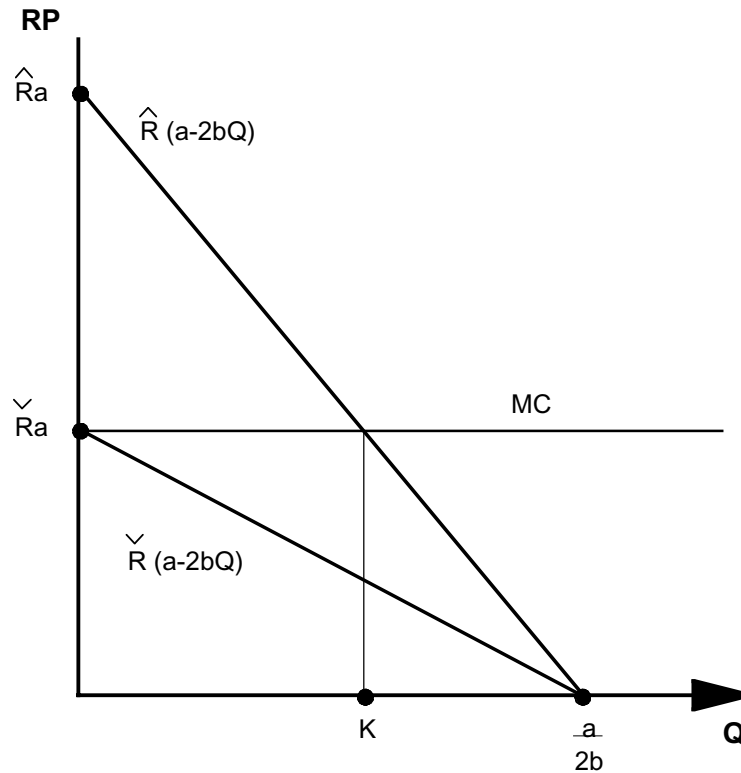


Figure 1 shows the effect of exchange rate fluctuations on the optimal production quantity,  $Q^*(R)$ . Changes in the exchange rate rotate the marginal revenue curve about the intercept on the Q axis. Optimal production is such that the marginal revenue, expressed in HC, is just equal to the marginal cost, MC, incurred in HC. The figure shows the HC marginal revenue curve under two particular exchange rates. At  $R(K) = MC/(a - 2bK)$ , the optimum production quantity is just equal to capacity,  $Q^*(R) = K$ . Depreciation of the HC,  $R > R(K)$ , would force production to be capacity constrained. At  $\check{R} = MC/a$ , the optimum production quantity is 0. Appreciation of the HC,  $R < \check{R}$ , would require the monopolist to remain inactive. For  $\check{R} < R < R(K)$ , the monopolist produces but is not capacity constrained. The optimal quantity in these instances is  $Q^*(R) = (Ra - MC)/(2Rb)$ .

## 2 Exogenous Technology

This section assumes that technology is exogenously determined and parameterized by  $\gamma$ , measuring the sunk capital intensity of production,  $0 \leq \gamma \leq 1$ . Production of each widget requires  $\gamma$  units of sunk capital and  $1-\gamma$  units of labor. For  $\gamma=0$ , the costs of production are 100% variable. For  $\gamma=1$ , the costs of production are 100% sunk. The sunk cost per unit of infinitely lived capital is  $k$ , the discount rate is  $\rho$ . The variable cost per unit of direct labor is  $v$ . The per unit full cost of production, when operating at 100% of capacity, is  $\gamma\rho k+(1-\gamma)v$ . To focus on the implications of dynamic leverage, assume that sunk capital and labor are equally efficient in production,  $\rho k=v$ .

The decision to enter the Foreign market depends upon the exchange rate at the time the opportunity to invest becomes available. This opportunity may arise due to a number of factors: the results of a research and development program, the reduction of barriers to trade, or the willingness of firm management to pursue international markets. Once the opportunity arises, the firm holds an option.<sup>7</sup> The firm may invest immediately, at the optimal capacity given the exchange rate, or wait and invest in greater capacity if the HC depreciates sufficiently.

Immediate investment earns the firm instantaneous operating profits; waiting economizes on the expected cost of regret. The profit from immediate investment increases in  $R$  and the expected benefit from waiting decreases in  $R$ . Thus there exists some exchange rate,  $R^*(\sigma, \gamma)$ , where the firm is indifferent between immediate investment and waiting. At  $R^*(\sigma, \gamma)$ , the firm optimally invests capacity  $K^*(\sigma, \gamma)$ . If the opportunity to invest arises when  $R < R^*(\sigma, \gamma)$ , the firm waits until the HC depreciates to  $R^*(\sigma, \gamma)$  before investing  $K^*(\sigma, \gamma)$ . If the opportunity to invest arises when

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<sup>7</sup>The option is assumed to be infinitely lived. There is no time limit on the investment opportunity, such as there might be if the option were due to a patent which would expire or an innovation which could be copied and entry pre-empted by a competitor.

$R > R^*(\sigma, \gamma)$ , the firm invests immediately  $\tilde{K}(R; \sigma, \gamma) > K^*(\sigma, \gamma)$ . The firm may not utilize all the capacity it builds at entry. The construction of unutilized, or excess, capacity represents the purchase of an option to produce widgets in the future, if the HC depreciates sufficiently. The greater the uncertainty and the lower the cost of sunk capacity, the more likely the firm will build excess capacity upon entry.<sup>8</sup>

Given  $\sigma$  and  $\gamma$ , the firm's problem is to determine the trigger point,  $(K^*; R^*)$ , and the schedule of capacity investments,  $\tilde{K}(R; \sigma, \gamma)$ , for opportunities that arise at an exchange rate which exceeds the trigger level,  $R > R^*(\sigma, \gamma)$ . Much of the solution procedure follows Dixit (1989), except the addition of variable costs allows the firm to produce at less than 100% of capacity. For details refer to the Solution Appendix.

Comparative statics concerning the solution are straightforward. An increase in  $R$  raises the desired capacity at entry; the higher is  $R$ , the less likely is appreciation of the HC such that the firm would rationally choose to exercise its option to limit production. An increase in  $\sigma$  raises the value of the option to reduce production since it increases the likelihood that the HC will appreciate significantly in the future. Accordingly, the firm buys more capacity. An increase in  $\gamma$  reduces the marginal operating cost and thus lowers the exchange rate at which the firm chooses to constrain production. This lowers the value of the option to reduce production since it is less likely to be exercised. As a consequence, the firm invests in less capacity; more irreversible capacity is less valuable.

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<sup>8</sup>Investment in excess capacity does not require a drift term in the stochastic process governing demand. Pindyck's (1988) model does not yield a deliberate investment in excess capacity since the firm may freely add capacity in the future.

The solutions for optimal capacity,  $\tilde{K}(R; \sigma, \gamma)$  for  $R \geq R^*(\sigma, \gamma)$ , and the trigger point,  $(K^*; R^*)$ , must be determined numerically. In general, for  $R \geq R^*(\sigma, \gamma)$ , more capacity is built under conditions of exchange rate uncertainty than is built under conditions of certainty. The difference in capacity is due to the excess capacity which a firm operating under uncertainty might acquire and the value of the option to curtail production. As with all options, the value of the option to curtail production increases with a mean preserving increase in volatility, and consequently the difference in capacity, between operations based in regimes of exchange rate uncertainty and those based in regimes of certainty, increases. As the exchange rate at entry increases, however, the difference in capacity decreases. The likelihood of the HC appreciating sufficiently to encourage a reduction in production is reduced and thus the value of the option to reduce production declines.

An increase in  $\gamma$  also reduces the difference in capacity. The larger is  $\gamma$ , the greater is the sunk capital required per widget produced and the lower the marginal cost of production. This reduces the exchange rate at which production is curtailed,  $R_\gamma(K; \gamma) < 0$ . Consequently the value of the option to curtail output is reduced.

Figures 2 to 4 offer some insight regarding the optimal actions of the firm. Numerical results are graphed based upon the following parameter values:  $a=1$ ,  $b=0.1$ ,  $\rho=0.05$ ,  $k=10$ , and  $v=0.5$ . The general shapes of the graphs are not sensitive to the parameter values.



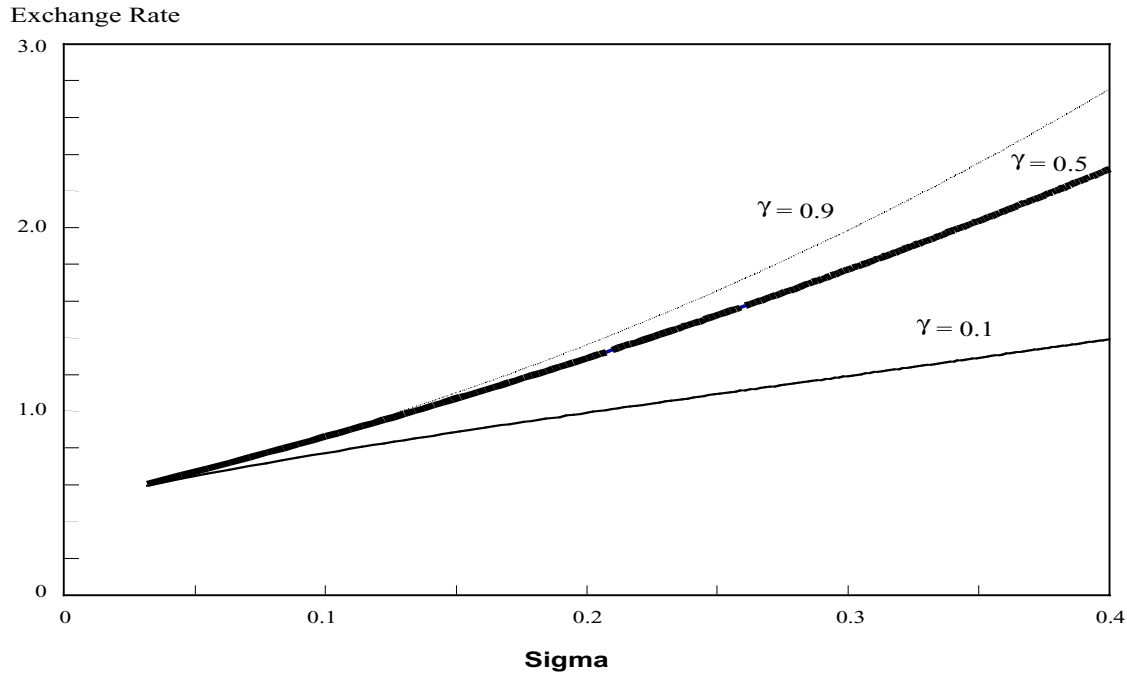


Figure 2  
Exchange Rate Entry Trigger,  $R^*(\sigma, \gamma)$

Figure 2 plots the exchange rate which triggers entry with minimum scale as a function of volatility, for values of  $\gamma$  equal to 0.1, 0.5, and 0.9. Under conditions of certainty, the value of  $\gamma$  is irrelevant and the firm is willing to build capacity for any  $R \geq 0.5$ . As uncertainty increases,  $R^*(\sigma, \gamma)$  increases. The more sunk capital intensive is the technology, the faster is the rate of increase for  $R^*(\sigma, \gamma)$ . For  $\sigma=0.4$ , a firm which would have only 10% of its costs sunk in capacity investments is willing to build for any  $R \geq 1.39$ . A firm which would have 90% of its costs sunk in capacity investments, is not willing to build until  $R$  rises to at least 2.75.

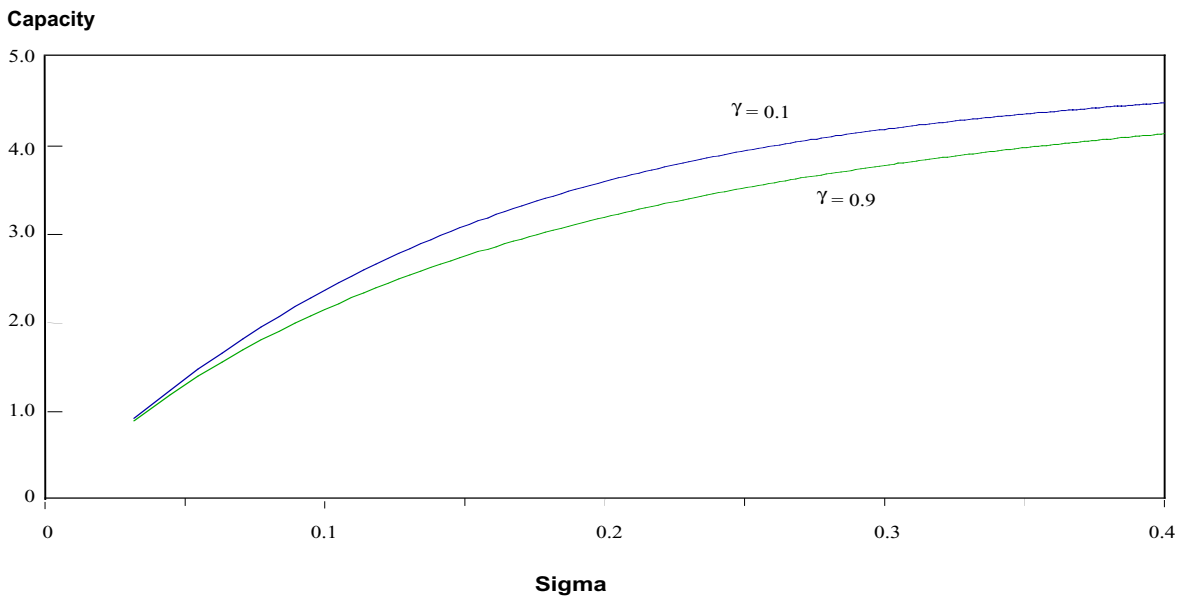


Figure 3  
Minimum Scale,  $K^*(\sigma, \gamma)$

Figure 3 plots the minimum scale as a function of volatility, for values of  $\gamma$  equal to 0.1 and 0.9. As uncertainty rises, minimum scale increases, initially at a faster rate for the less sunk capital intensive technology. For  $\sigma=0.4$  and  $\gamma=0.1$ , the trigger point,  $(K^*; R^*)$  is (4.46; 1.39); for  $\sigma=0.4$  and  $\gamma=0.9$ , the trigger point is (4.11; 2.75). The more flexible technology enters the market at a lower exchange rate and with greater capacity.

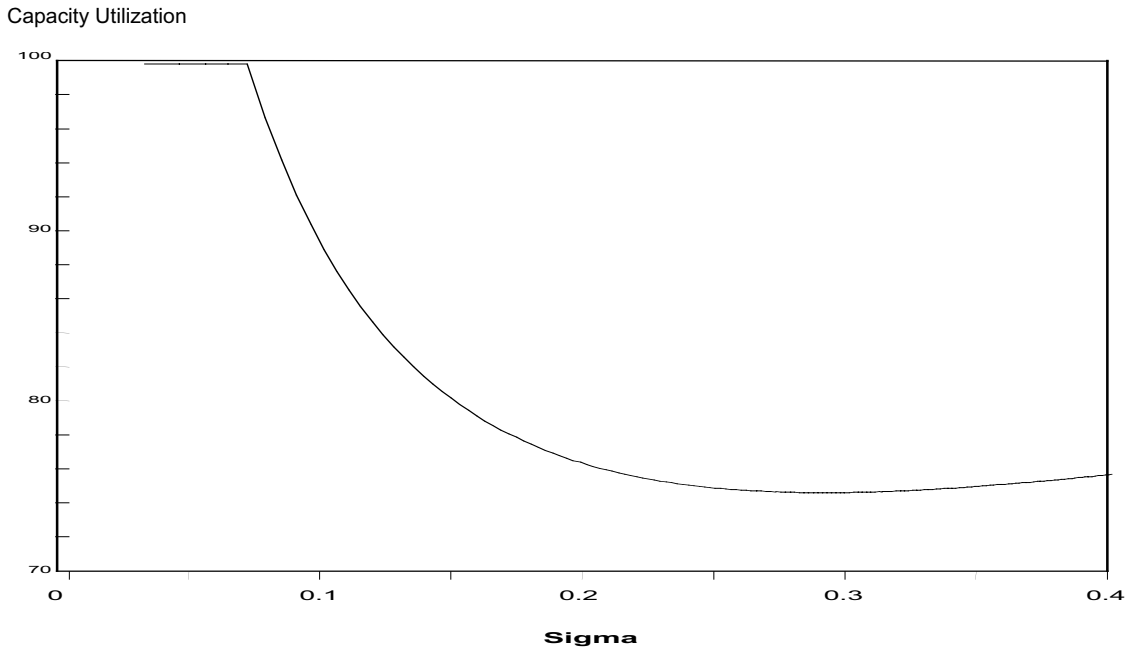


Figure 4  
Capacity Utilization upon Entry,  $\gamma=0.1$

Finally, Figure 4 plots capacity utilization, at the trigger point for  $\gamma=0.1$ , as a function of volatility. For more sunk capital intensive technologies,  $\gamma$  equal to 0.5 and 0.9, capacity utilization upon entry is always 100%. For  $\gamma=0.1$ , capacity utilization is 100% from  $\sigma=0$  to  $\sigma=0.07$ ; thereafter, as volatility continues to rise, capacity utilization drops to 74% around  $\sigma=0.3$ . Beyond this point, capacity utilization begins to increase with  $\sigma$  since the increase in minimum scale,  $K^*(\sigma, \gamma)$ , is less than the increase in the optimal quantity of production,  $Q^*(R^*)$ .

Providing capacity with some salvage value and thus permitting the firm an option to exit has little qualitative effect on the results discussed above. The greater the required return on recoverable investments in capital and the lower the volatility of the exchange rate, the higher the  $R$ , call it  $R_L$ , at which the option to exit would be exercised. There would be three possible scenarios: the firm could exit at some exchange rate below that at which it becomes inactive,  $R_L(K; \sigma, \gamma) < \check{R}(\gamma)$ ; the firm could exit before it would choose to halt production,

$\dot{R}(\gamma) < R_L(K; \sigma, \gamma) < R(K; \gamma)$ ; or the firm could exit even before it would choose to curtail production,  $R_L(K; \sigma, \gamma) > R(K; \gamma)$ . In any event, the presence of the option to exit raises the option value of capacity in place, by raising the value of the firm's opportunities in the event of a HC appreciation. Consequently, for all  $\sigma$  and  $\gamma$  greater than 0, the presence of an option to exit reduces the exchange rate which triggers investment in minimum capacity and increases the capacity invested at each value for  $R$  above the trigger point.

Permitting the firm more than one opportunity to build capacity would have the opposite effect on the capacity invested. The exercise price of the option to build would exceed the marginal cost of capacity due to the presence of capacity adjustment costs. Nevertheless, the existence of additional options to build permits the firm to expand capacity if the HC depreciates sufficiently; accordingly, initial investment is curtailed to conserve on the cost of regret.<sup>9</sup> Of course, permitting the firm the additional option of downsizing, as opposed to wholesale exit, works to mitigate the effects of options to add capacity.

The principal results of Section 2 are not surprising. The more sunk capital intensive a technology, the greater the exchange rate triggering entry with minimum scale and the smaller the investment in sunk costs. One would therefore expect export oriented investments to be made in more labor intensive technologies when the HC is volatile and appreciating. Once investments are made, sunk capital intensive industries produce output that is relatively inelastic with respect to changes in the exchange rate.

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<sup>9</sup>Pindyck (1988) presents a model of costless capacity adjustment. He finds that uncertainty reduces investment levels below those which would obtain in an environment of certainty.

### 3 Endogenous Technology

This section considers the choice of an optimal technology to produce exports in an environment of volatile exchange rates. The circumstances are similar to those which prevailed in Section 2. A monopolist based at Home is contemplating the construction of a facility to manufacture products designed for the Foreign market. The technology of production, however, is not predetermined. The firm can choose to invest in flexibility, opting for capital less specialized and less sunk, in exchange for higher labor costs.<sup>10</sup> The degree of dynamic leverage becomes a choice variable.

This section continues to use  $\gamma$  to parameterize technology. One unit of capital and  $\lambda$  units of labor are required to produce one widget each period. The parameter  $\lambda$  is used to characterize the relative productivity of labor, a lower value of  $\lambda$  corresponds to more productive labor. The cost of each unit of infinitely lived capital is  $e^\gamma$ , the associated labor cost per unit of production is  $\lambda e^{1-\gamma}$ . More specialized capital, corresponding to a higher value for  $\gamma$ , is more expensive but is combined with less costly labor to produce each widget. More specialized capital also offers the firm less flexibility; supply is less elastic with respect to changes in the exchange rate.

Initially consider the problem of the firm operating under expectations of certain future exchange rates. To choose the optimal scale and technology of investment,  $\bar{K}$  and  $\bar{\gamma}$ , the firm solves the following maximization problem:

$$\max_{K, \gamma} R(a - bK)K - \lambda e^{1-\gamma}K - \rho e^\gamma K \quad (1)$$

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<sup>10</sup>This issue was suggested in Dixit (1989).

The standard first order conditions lead to the following solutions for optimal scale and technology:

$$\bar{K} = \frac{Ra - \rho e^\gamma - \lambda e^{1-\gamma}}{2Rb} \quad (2)$$

$$\bar{\gamma} = \frac{1 + \ln\lambda - \ln\rho}{2}$$

Note that while optimal scale is a function of the exchange rate, optimal technology is only a function of the relative efficiency of capital and labor.<sup>11</sup> For  $\rho=\lambda$ , capital and labor are equally efficient;  $\bar{\gamma} = 0.5$ , and dynamic leverage, the ratio of sunk costs to marginal production costs, is 1. The less efficient is labor, the higher is  $\bar{\gamma}$  and the greater is dynamic leverage.

Now I introduce volatility to the exchange rate process. Once the opportunity to export to the Foreign market becomes available, the firm must choose between investing immediately and waiting. At each value for  $R$  the firm may invest immediately, with the optimal scale and related technology,  $\tilde{K}(R; \sigma, \lambda)$  and  $\tilde{\gamma}(R; \sigma, \lambda)$ , or choose to wait and invest in a larger scale and a less flexible, lower cost technology once the HC depreciates sufficiently. Again a trigger point exists,  $(K^*, \gamma^*; R^*)$ , such that if the investment opportunity arises for  $R < R^*(\sigma, \lambda)$ , the firm optimally chooses to wait until the HC depreciates to  $R^*(\sigma, \lambda)$ . If, however, the opportunity to invest arises at  $R \geq R^*(\sigma, \lambda)$ , then the firm invests immediately with capacity

$\tilde{K}(R; \sigma, \lambda) \geq K^*(\sigma, \lambda)$  and technology  $\tilde{\gamma}(R; \sigma, \lambda) \geq \gamma^*(R; \sigma, \lambda)$ . Where technology is a choice parameter, the firm does not choose to enter the market with unutilized or excess capacity.

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<sup>11</sup>In this case optimal technology is independent of scale. More realistically,  $\lambda$  could be an increasing function of  $K$ , so that larger scale facilities are more capital intensive.

Lowering  $\gamma$  slightly raises the marginal cost of production such that the firm optimally uses all capacity upon entry.

The solution procedure is similar to the approach taken in Section 2; for details, the reader is referred to the Solution Appendix.

The solution for the schedule of optimal technology,  $\tilde{\gamma}(R; \sigma, \lambda)$  for  $R \geq R^*(\sigma, \lambda)$ , cannot be analytically defined. What I find is that the optimal technology under expectations of volatile future exchange rates is less specialized than the optimal technology under expectations of certain future exchange rates,  $\tilde{\gamma} < \bar{\gamma}$ . The expectation of volatile future exchange rates adds an extra term to the first order condition for  $\tilde{\gamma}$ . This term is the change in the value of the option to reduce production as  $\gamma$  increases. This term is always negative. An increase in  $\gamma$  lowers  $R(K, \gamma) = \lambda e^{1-\gamma} / (a - 2bK)$ , the exchange rate which triggers a reduction in production. The less flexible the technology is, the less valuable is the option to reduce production. As the exchange rate at entry increases, specialization increases, and  $\tilde{\gamma}$  asymptotically approaches  $\bar{\gamma}$ . Increases in  $\lambda$ , an improvement in the relative efficiency of capital, results in an increase in  $\tilde{\gamma}$ , and the difference between  $\tilde{\gamma}$  and  $\bar{\gamma}$  decreases as capital becomes relatively more efficient. An increase in uncertainty results in a reduction of dynamic leverage, less specialization and a lower  $\tilde{\gamma}$ . The more uncertain the environment, the more valuable is flexibility.

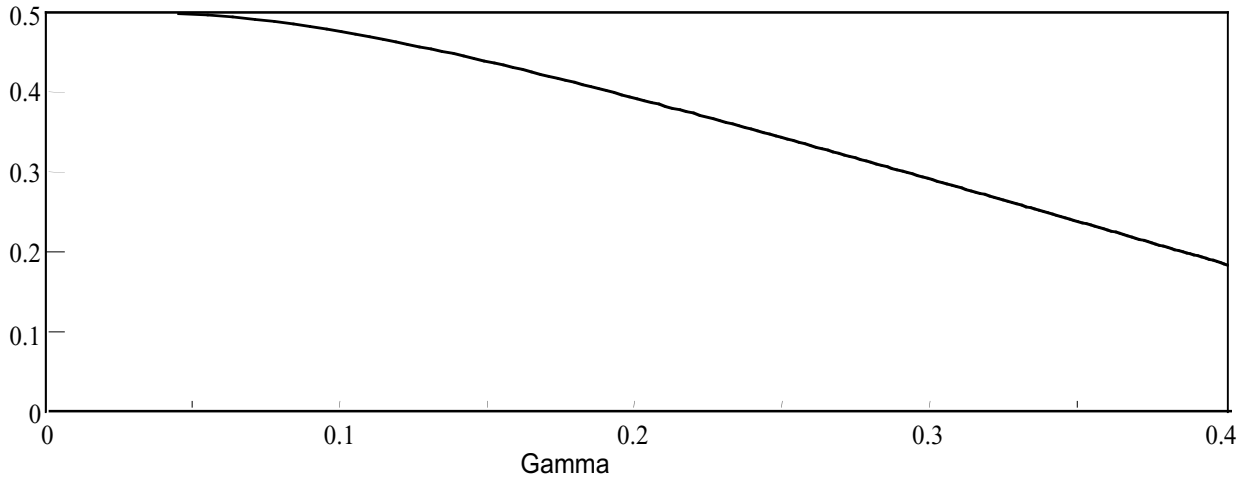


Figure 5  
Optimal Technology,  $\gamma^*(\sigma, \lambda)$

Figure 5 plots optimal technology at the trigger point,  $\gamma^*(\sigma, \lambda)$ , as a function of volatility, assuming  $\lambda = \rho$ . As the volatility of the exchange rate increases, specialization decreases, lowering dynamic leverage. Less dynamic leverage offers the firm more flexibility for the price of higher production costs. At  $\sigma = 0.4$ , the trigger point,  $(K^*, \gamma^*; R^*)$  is  $(4.21, 0.18; 0.71)$ . The optimal technology is 0.18, down from 0.5 under certainty; dynamic leverage is 0.53, down from 1 under certainty; and the per unit full cost of production at capacity is 5% higher than under certainty.

The results of Section 3 are as expected. The higher the dynamic leverage of a technology, the more significant are the effects of volatility. Hence, the greater the uncertainty, the greater the investment in flexibility and the lower the optimal dynamic leverage. The higher the exchange rate, the less likely that the firm will need to utilize investments in flexibility; consequently, the greater the firm's investment in specialization.



## 4 Summary

This paper considers the role of production technology in export oriented, irreversible investments made in a regime of volatile exchange rates. Technologies are characterized by their flexibility. More specialized technologies incur larger sunk costs in return for a cost advantage at the expected level of output. By reducing sunk costs and offering the firm a broader array of responses to changes in the exchange rate, flexibility reduces the cost of regret incurred by an investment. Accordingly, more flexible technologies enter the Foreign market at lower exchange rates and with higher scale. Where the firm has a choice of technology, increases in volatility lead to the selection of less specialized technologies, the sacrifice of cost advantages for flexibility.

One implication of these findings is yet another explanation for the Leontief paradox.<sup>12</sup>

Traditional Heckscher-Ohlin trade theory holds that a nation exports products which make intensive use of its relatively abundant factor. Post WWII, it was generally agreed that the US was capital rich in comparison to its trading partners. Yet the industries in which the US was a net exporter were less capital intensive than import competing industries. If exports require investments in sunk costs and export markets are more volatile than domestic markets, then one might expect export technologies to be less specialized and more labor intensive than the technologies of import competing industries.

One significant issue in the choice between flexible and specialized investments, which I do not discuss, is the strategic advantage of sunk costs. Specialized, irreversible investments are a

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<sup>12</sup>The Leontief paradox is discussed in Caves and Jones (1985), pp. 130-134.

credible commitment to a market and can be important deterrents to rival entry or expansion (Dixit, 1980). The higher the dynamic leverage, the more powerful is the deterrent. This strategic advantage of specialized investments will mitigate the advantages of flexibility in an oligopolistic market. The issue of strategic technology choice and the value of commitment in a volatile environment is a topic for further research.

## Solution Appendix

The solution procedure for the exogenously determined technology of Section 2 is as follows.

Let  $G(K^*;R)$  represent the value of the monopolist's option to invest in production for the Foreign market.<sup>13</sup> This asset must earn the required rate of return.<sup>14</sup> The only component to the return is the expected appreciation in the value of the option. Thus:

$$E[dG(K^*;R)/dt] = \rho G(K^*;R) \quad (1)$$

Using Ito's Lemma and taking expectations yields:

$$1/2\sigma^2 R^2 G_{RR}(K^*;R) = \rho G(K^*;R) \quad (2)$$

The solution for  $G(K^*;R)$  is:

$$G(K^*;R) = A(K^*)R^{1-\beta} + D(K^*)R^\beta \text{ for } R \leq R^*(\sigma,\gamma) \quad (3)$$

where:

$$\beta = \frac{1 + \left[1 + \frac{8\rho}{\sigma^2}\right]^{1/2}}{2} > 1 \quad \frac{\partial\beta}{\partial\sigma} < 0$$

$\lim_{\sigma \rightarrow 0} \beta = \infty$                        $\lim_{\sigma \rightarrow \infty} \beta = 1$

and  $A(K^*)$  and  $D(K^*)$  are constants to be determined.

Once the monopolist invests, there exist three alternatives: the monopolist is inactive, the monopolist operates at less than full capacity, or the monopolist operates at full capacity. For

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<sup>13</sup>There are a number of options to invest, each with a different  $K$ . Since the process for  $R$  is continuous, though, only the minimum capacity will ever be built for  $R < R^*(\sigma,\gamma)$ . Consequently, only the option of investing  $K^*(\sigma,\gamma)$  is of interest.

<sup>14</sup>Pindyck (1991) discusses the issues regarding an appropriate rate of return. If spanning holds, such that there is a portfolio of traded assets whose price is perfectly correlated with the value of the firm's investment opportunity, then the return on the portfolio of assets anchors the required return on the investment opportunity. If spanning does not hold, however, then there is no correct rate of return on the investment opportunity and valuation is subject to an arbitrary choice of the discount rate. The ability to sell the investment opportunity or spin it off into a traded entity should be sufficient to anchor the discount rate.

$R > R(K; \gamma) = (1 - \gamma)v / (a - 2bK)$ , the monopolist operates at full capacity and owns an option to reduce production if the HC appreciates sufficiently. For  $R < \check{R}(\gamma) = (1 - \gamma)v / a$ , the monopolist is inactive and owns an option to produce if the HC depreciates sufficiently. For  $\check{R}(\gamma) < R < R(K; \gamma)$  the monopolist operates at less than 100% of capacity and owns options to reduce or increase production in response to changes in the exchange rate. For each of these three alternatives, there is a similar equation for the instantaneous required rate of return on the asset.

The resulting system of four equations completely describes the value of the firm's position, either in or out of the market, at any exchange rate. There are 10 unknowns to be determined: eight constants which are functions of scale; the trigger point,  $(K^*; R^*)$ ; and the schedule of optimal capacity,  $\tilde{K}(R; \sigma, \gamma)$ , for  $R \geq R^*(\sigma, \gamma)$ . To solve for these 10 unknowns, there exist 10 standard boundary conditions. The first two boundary conditions state that if the FC becomes worthless, then the firm's position in the Foreign market, either as an opportunity to invest or as capacity in place, also becomes worthless. The third condition shows that as the HC becomes worthless, it becomes increasingly unlikely that the HC will ever appreciate sufficiently to force the firm to reduce production below capacity. Accordingly, the option to reduce production becomes worthless and the value of the firm becomes the expected present value of operations at capacity. Six of the conditions are the standard value matching and smooth pasting conditions governing the change in the value of the firm as the exchange rate approaches  $\check{R}(\gamma)$ , where the firm chooses to become inactive, and as the exchange rate approaches  $R(K; \gamma)$  where production becomes capacity constrained. The last condition governs the optimal choice of scale upon entry; this is a standard marginal cost equals marginal benefit requirement.

The solutions for optimal capacity,  $\tilde{K}(R; \sigma, \gamma)$  for  $R \geq R^*(\sigma, \gamma)$ , and the trigger point,  $(K^*; R^*)$ , depend upon the capacity utilization at the time of entry and must be determined numerically.

For the endogenously determined technology of Section 3, the solution procedure is no different; simply replace all occurrences of  $\gamma k$  with  $e^\gamma$  and all occurrences of  $(1-\gamma)v$  with  $\lambda e^{1-\gamma}$ . There is one additional unknown, the schedule of optimal technologies,  $\tilde{\gamma}(R; \sigma, \lambda)$  for  $R \geq R^*(\sigma, \lambda)$ .

Accordingly, there is one additional boundary condition which is a standard marginal cost equals marginal benefit condition governing technology choice. The solution for the schedule of optimal technology cannot be analytically defined and must be determined numerically.

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