Crude Oil Industry Dynamics:

A Leader/Follower Game between the OPEC Cartel and Non-OPEC Producers

Jostein Tvedt*

DnB Markets, Economic Research

(Work in progress, April 1999, please do not quote)

Short abstract

This paper studies the dynamics in the crude oil industry. At any point of time the equilibrium in the oil market is model as a leader/follower game. The basic model derives the price dynamics of an industry for which fixed capital is produced by another industry with costs of structural changes. Due to the cost of increasing and decreasing capacity in the capital producing industry together with a stochastic demand, output prices in the industry that apply the capital will follow a long run mean-reverting pattern. A stochastic partial differential equation for this price process is derived. The relevance for the oil market is outlined and possible extensions regarding the leader's market power is discussed.

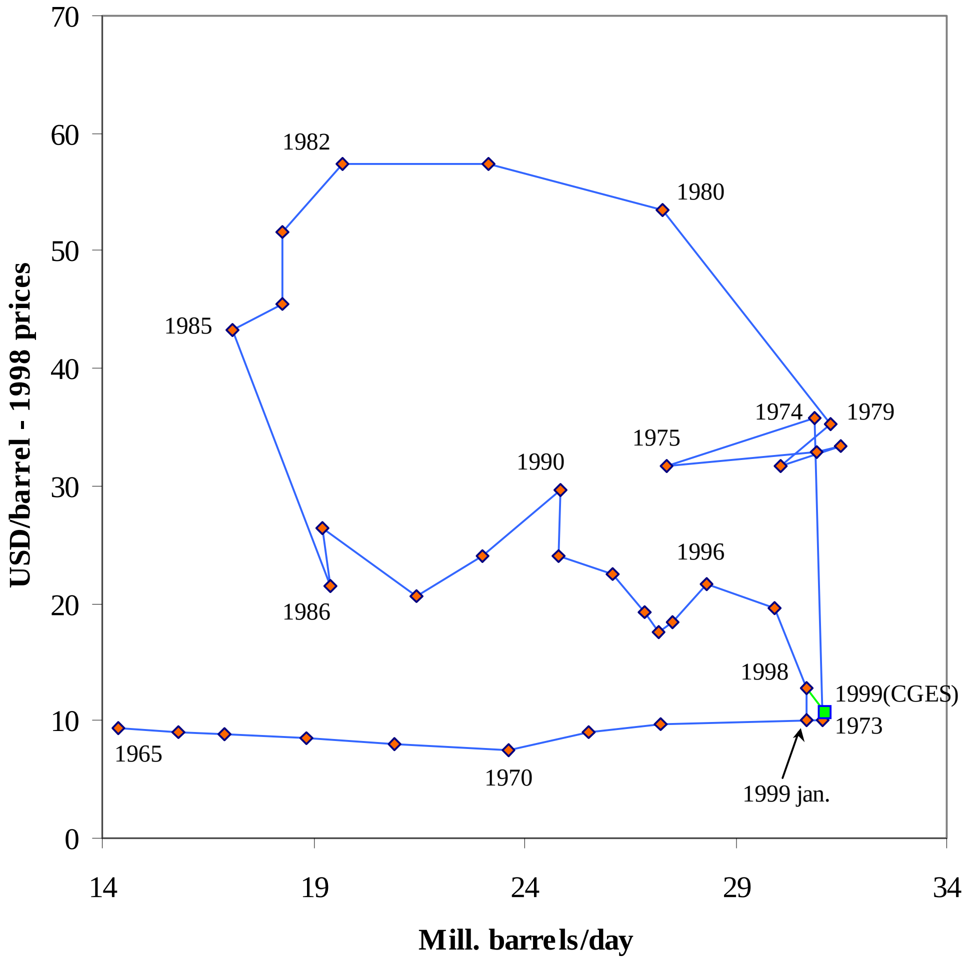
* Address of correspondence: DnB Markets,
P.B. 1171 Sentrum,
0107 OSLO
Norway
tel: + 47 22 01 76 64
fax: + 47 22 48 29 83
e-mail: jostein.tvedt@dnb.no

Crude Oil Industry Dynamics: A Leader/Follower Game between the OPEC Cartel and Non-OPEC Producers

In long periods the oil market has been close to a perfectly competitive market. However, from the 1970's the oil price has been strongly influenced by the production policy of the OPEC cartel. See figure 1 below. The price increase 1973/1974 stopped the continuous growth in oil consumption that had lasted for decades. Price and volume stabilised from 1974 until 1980. Then OPEC attempted to increase prices further, but this triggered a reduction of total world consumption of oil and a dramatic fall in the OPEC market share followed. Hugh investments in oil production were made outside OPEC during 1980 - 1985. Finally, in 1986 OPEC had to give up the high-price regime, and a substantial fall in oil prices followed. Outside OPEC this turned focus towards cost saving research and development. The new technology proved to be successful and marginal costs for producers outside OPEC started to fall to levels closer to the low levels of the efficient Arabian producers. This technology has gradually been implemented during the second half of the 1990s. Consequently, the marginal cost of producing non-OPEC oil has been reduced and the global commercially available oil resources have increased. Recently, OPEC seems to have changed policy once again, and oil prices and OPEC production are back to the same levels as in 1973. There are signs that this will have significant negative effects on oil investments outside OPEC and, hence, this will reduce future oil production in non-OPEC countries. After almost a year with oil prices at almost "perfectly competitive" levels, in March 1999 OPEC agreed on substantial production cuts, and other non-OPEC countries indicated that they too would reduce production. Hence, oil prices started to rise.

At present OPEC produces slightly above 40 percent of the total world consumption of crude oil. Back in the mid-1980's the OPEC market share was only marginally above 30 percent, which was down from above 50 percent in the first part of the 1970's. The market share of OPEC is in strong contrast to OPEC's share of known oil reserves. OPEC's share of known oil reserves is about 77% and the Middle East OPEC countries have alone 65% of known reserves. The production cost of OPEC and in particular that of the Middle East countries is significantly below that of the rest of the world. According to rough estimates (Economist 1999) production cost of oil, including costs related to finding and field development, is about US$2.- per barrel in the Middle-East in contrast to US$10,- in the US-Gulf and US$11.- in the North Sea. Hence, OPEC operates as a leader in the crude oil market and restricts production to a significant degree in order to keep prices at a preferred level from an OPEC perspective.

Figure 1, Crude oil price and the OPEC oil production 1965 – 1998, "The OPEC wheel comes full circle"



Data source: BP Amoco Statistics and Centre of Global Energy Studies, primo 1999

Overview of the paper

This paper studies the dynamics of the crude oil industry. The model presented is an application and an extension of the stochastic partial equilibrium model in Tvedt (1996). The basic model derives the price dynamics of an industry where fixed capital is produces by another industry with costs of structural changes. Due to the cost of increasing and decreasing capacity in the capital producing industry, output prices in the industry that apply the capital will follow a long run mean-reverting pattern. A stochastic partial differential equation for this price process is derived.

At any point of time the equilibrium in the oil market is model as a leader/follower model. The leader, OPEC, has the lowest marginal costs whereas the marginal costs of the non-OPEC producers, the followers, depend on past and present investments. Present investments are restricted by the size of the investment goods producing industry. To increase or reduce the capacity of this industry entail costs. The dynamic of the market is, therefore, modelled as a stochastic optimal control problem. Hence, uncertainty in the demand for oil, via the demand for oil industry investments, implies hysterisis effects in the adjustment of the capacity of the investment goods industry.

In a perfectly competitive market changes in investments are solely triggered by the current oil price, where the oil price is determined by current demand and production capital. However, the cartel members know that they influence the oil price by their production strategy. Hence, when choosing the optimal production path the cartel may take into account the effect this path may have on the investment decision of the non-cartel producers. Hence, the cartel production strategy is a trade-off between high oil prices and the risk of triggering non-cartel oil production investments.

Model of the crude oil market

The static equilibrium of the oil market is described by a fairly simple model . It is assumed that OPEC operates as a cartel without internal rivalry, whereas the rest of the world's oil producers take market prices as given, that is, they operate as if the market was perfectly competitive. The non-OPEC producers, or fringe producers, have an aggregated total variable production cost function given by

$$C = \varepsilon k^{-\alpha} Q_f^{\ 2} \tag{1}$$

where k is the total capital employed by the fringe producers in oil production and Q_f is the total oil production of the fringe producers, at a given point of time. $\varepsilon > 0$, $\alpha > 0$ are constants. Given the cost function (1) short term marginal cost equal the oil market price gives the linear inverse supply function of the fringe producers

$$cQ_f = X \tag{2}$$

where $c = 2\varepsilon k^{-\alpha}$ and X is the price of oil. Total oil demand, Y, is assumed totally inelastic to the oil price. This is a simplification that will be discussed in some detailed in the empirical part of the paper. Further demand is assumed stochastic, with incremental change given by a geometric Brownian motion

$$dY_t = \hat{\mu} Y_t dt + \hat{\sigma} Y_t dB_t \tag{3}$$

where $\hat{\mu}$ and $\hat{\sigma}$ are constants and have the standard interpretations, and dB_t is the increment of a standard Brownian motion. In the model there is no change in the oil stocks and, therefore, total supply of oil, Q, must at any time be equal demand. Therefore, the cartel will at any time produce the residual demand, and the cartel is assumed to have unlimited production capacity. The cartel's oil production, Q_l, will then be

$$Q_l = Y - \frac{X}{c} \tag{4}$$

The cartel is aware of how their oil production influences the oil price and that the fringe producers take the oil price as given. For simplicity assume that the cartel has zero production costs. The profit of the cartel in then given by

$$\pi_l = XQ_l = c\left(YQ_l - Q_l^2\right) \tag{5}$$

Maximising instantaneous profit gives the optimal oil production of the cartel

$$Q_l = \frac{Y}{2} \tag{6}$$

That is, given the zero cost assumption for the cartel, the totally inelastic demand and the linear supply function of the fringe producers, the cartel simply supply half the demand for oil. Historically, OPEC has produced about 30% to 50% of the total world consumption of oil. Given the 50/50 split of the oil market between the fringe producers and OPEC the model gives an equilibrium oil price given by

$$X = \frac{1}{2} cY \tag{7}$$

A competitive market implies that the present value of the instantaneous total market surpluses less the investment costs is maximises. See Lucas and Prescott (1971). This model describes an imperfectly competitive market. However, the fringe producers and the consumers are assumed to behave as if they were operating in a perfectly competitive market. Hence, investment in the oil industry is consistent with the competitive nature of the non-OPEC part of the market if investments are made such that the total consumer surplus and the producer profit of the fringe producers are maximised. This follows since the cartel is assumed to have an infinite production capacity and therefore has no incentives to make additional investments.

Given the oil price (7), the production split (6) and the production cost of the fringe (1), the instantaneous profit of the fringe producers and the cartel is $\pi_f = \frac{1}{8} cY^2$ and $\pi_l = \frac{1}{4} cY^2$ respectively. Given totally inelastic demand consumer surplus is infinite. To make consumer surplus finite, assume a upper price level, P, at which oil is substituted for other types of energy sources. Then the consumer surplus plus the profit of the fringe producers are given by

$$S = PY - \frac{3}{8} cY^2 \tag{8}$$

Observe that the negative part of this relation is simply given by the instantaneous profit of the cartel plus the variable production costs of the fringe producers, g

$$g = \frac{3}{8} cY^2 \tag{9}$$

By changing the capital stock, k, of the fringe producers, future consumer and fringe producer surpluses are influenced via changes in c, i.e. via changes in the slope of the supply function of the fringe producers. Hence, to maximise the surplus S in (8) is equivalent to minimising the level of the cartel's surplus and the production cost of the fringe producers via an optimal path of the production capital.

Capital, *k*, is produced by an oil construction industry. Let the incremental change in the capital stock of the oil industry be linear in the total level of the existing capital stock of the fringe producers, i.e.

$$dk = (\eta_t - \delta)kdt = a_t kdt \tag{10}$$

where a_t is the marginal relative change of the capital stock at time *t*. Let the production of new capital be given by the marginal relative net addition of capital, η_t. Hence, η_t represents the activity level of the construction industry. Let the cost of producing new capacity, at time *t*, be given by $\kappa_t = p\eta_t$ where *p* is a constant. The activity level of the construction industry can be changed, but at a cost. Let there be to alternative levels of capital production, η_1 and η_2, and let the cost of moving from the low activity level, η_1, to the high activity level, η_2, be given by q_2 and let the cost of moving from the high to the low activity level given by q_1. Existing capital deteriorates at a fixed rate δ. This may either be due to general depreciation of the capital or it may be interpreted as a depletion rate of oil wells.

Given the dynamics of the capital stock the dynamics of the slope of the supply curve, $c = 2\varepsilon k^{-\alpha}$, will be given by

$$dc_t = -a_t \alpha c_t dt \tag{11}$$

That is, an increase in the capital stock will make the supply function more price elastic, i.e. *c* falls, and the equilibrium price will fall as well. Consequently, the profit of the cartel is reduced as *k* increases. However, the 50/50 split of production will remain unchanged given the specification of the model.

From (3), (9) and (11) it follows by Ito's lemma that the dynamics of the production cost of the fringe and the profit of the cartel, *g*, is given by a geometric Brownian motion

$$dg_t = \mu g_t dt + \sigma g_t dB_t \tag{12}$$

where $\mu = \frac{3}{8}\left(-(\eta-\delta)\alpha + 2\hat{\mu} + \hat{\sigma}^2\right)$ and $\sigma = \frac{3}{4}\hat{\sigma}$.

Dynamic equilibrium of the crude oil market

To derive the dynamic equilibrium is restricted to regulate the capital stock in such a manner that the present value of the cartel's profit and the production cost, plus the cost of producing capital and regulating the construction level, is minimised.

Let $e^{-\rho t}$ be a discount factor where ρ is constant and let $\overline{X}_t$ be the state of the economic system, $\overline{X}_t = [s+t, k_t, Y_t, a_t]^{-1}$. Then it follows from the discussion above that the present value, at time zero, of the cartels profit and the production cost, at time *t*, is given by

$$F(\overline{X}_t) = e^{-\rho t}\frac{3}{8}cY^2 = e^{-\rho t}g \tag{13}$$

and that the present value, at time zero, of the cost of adjustment, at time *t*, is

$$\overline{K}(\overline{X}_{\theta_j}, \xi_j) = e^{-\rho t}\begin{cases} q_i k_t & ; \quad \xi_j > 0 \\ q_r k_t & ; \quad \xi_j < 0 \\ 0 & ; \quad \xi_j = 0 \end{cases} \tag{14}$$

where ξ_j is the jump in a_t at time θ_j. The minimum of the present value of the cartel's profit, the production cost and the costs of keeping an optimal investment path is given by the value function $\Phi(\bar{x})$.

$$\Phi(\bar{x}) = \inf_{\omega} E^{\bar{x}}\left[\int_0^{\infty} F(\overline{X}_t)dt - \sum_{j=1}^{N}\overline{K}(\overline{X}_{\theta_j}, \xi_j)\right] \tag{15}$$

where the controls are given by ω, where $\omega = (\theta_1, \theta_2, \ldots, \theta_N; \xi_1, \xi_2, \ldots \xi_N)$, $N < \infty$, and θ_1 is the time of the first control and ξ_1 is the size of the first jump in a_t, and so forth. The incremental change in the state of the system, $\overline{X}_t$, between each change in *a*, is given by

$$d\overline{X}_s = \begin{bmatrix} 1 \\ (\eta_s - \delta) \\ \mu Y_s \\ 0 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ \sigma Y_s \\ 0 \end{bmatrix} dB_s \tag{16}$$

It follows from the assumptions above that the process $\{\bar{X}_t ; t \geq 0\}$ is a Markov process, and hence has an infinitesimal generator A ,

$$\mathrm{A} = \frac{\partial}{\partial t} + \mu g_t \frac{\partial}{\partial g} + \frac{1}{2}\sigma^2 g_t^2 \frac{\partial^2}{\partial g^2} \tag{17}$$

Our optimal control problem may be handled by applying the approach of formulating the quasi-variational inequalities

$$\mathrm{A}\Phi + F \geq 0 \tag{18}$$

$$\Phi(\bar{x}) \leq \mathrm{M}\Phi(\bar{x}) \tag{19}$$

$$\{\mathrm{A}\Phi + F(\bar{x})\}\{\Phi(\bar{x}) - \mathrm{M}\Phi(\bar{x})\} = 0 \tag{20}$$

where M is the shift operator, defined by

$$\mathrm{MH}(\bar{x}) = \inf_{\xi} \{\mathrm{H}(\bar{x},\xi) - \bar{K}(\bar{x},\xi)\} \tag{21}$$

Informally in words, if the value function in the present state is smaller then what it would have been after a control, i.e. (19) does not hold with equality, then a control cannot be optimal. The value function is equal to the discounted value of future cartel's profits and the production costs of the fringe plus the net value of the controls, given an optimal control strategy. If there is no control the change in the value should be equal to the cartel's profits and the production costs. Hence, if (18) does not hold with equality then a control is optimal. But in that case the value function after the control must be at least as small as before the control. Therefore, (19) holds with equality at the optimal time of a control, and (18) holds with equality between the controls. In our case we have $\mathrm{H}(\bar{x}, \xi) = \Phi(s,k,y,a+ \xi)$.

We try a solution of the form $\Phi(\bar{x}) = e^{-\rho t}\Psi(\bar{x})$ for the value function, where $\Psi(\bar{x})$ is a time homogeneous function. Consequently, we may write the relations (18) and (19) as

$$-\rho\Psi + \mu g_t \frac{\partial\Psi}{\partial g} + \frac{1}{2} g_t^2 \frac{\partial^2\Psi}{\partial g^2} + g_t + p\eta \geq 0 \tag{22}$$

$$\psi(\bar{x}) \leq \inf_{\xi} \{\Psi(k,y,a+\xi) + (q_2\chi_{\xi>0} + q_1\chi_{\xi<0})\} \tag{23}$$

where χ_A is the indicator function of the event *A*. Between each point of time of adjustment, relation (22) must hold with equality. Trying the functional form $\Psi = g^{\gamma}$ for the homogenous part of the relation gives the following solution

$$v_i = \begin{cases} v_1 = A_1 g^{\gamma_1^1} + B_1 g^{\gamma_2^1} + \dfrac{g}{\rho - \mu} + \dfrac{p\eta_1}{\rho} \\ v_2 = A_2 g^{\gamma_1^2} + B_2 g^{\gamma_2^2} + \dfrac{g}{\rho - \mu} + \dfrac{p\eta_2}{\rho} \end{cases} \tag{24}$$

where

$$\gamma_i^j = \frac{\frac{1}{2}\sigma^2 - \mu(a_j) \pm \sqrt{\left(\frac{1}{2}\sigma^2 - \mu(a_j)\right)^2 + 2\rho\sigma^2}}{\sigma^2}; \quad j = 1,2 \tag{25}$$

Let $\gamma_1^j > 0$ and $\gamma_2^j < 0$. This assumption makes it easier to derive the optimal controls. Observe that γ_1 and γ_2 depend on *a*. The optimal level of capital increase, a^*, will then be a function, though in most cases not continuous, of *g*.

For very low values of *g*, i.e. low demand or a very large capital stock, it is optimal to keep production of new capital at a low level. The value function is then given by v_1. In the case that *g* approaches zero then the value of v_1 should be finite. Hence, it follows that $B_1 = 0$. For very high demand or low levels of existing capital, production of new capital is kept at a high level and the value function is given by v_2. If *g* becomes very high the option to reduce the production of new capacity becomes small. From this it follows that $A_2 = 0$. Consequently, the value function is reduced to

$$v_i = \begin{cases} v_1 = A_1 g^{\gamma_1^1} + \dfrac{g}{\rho - \mu} + \dfrac{p\eta_1}{\rho} \\ v_2 = +B_2 g^{\gamma_2^2} + \dfrac{g}{\rho - \mu} + \dfrac{p\eta_2}{\rho} \end{cases} \tag{26}$$

In order to satisfy relation (21) of the optimisation problem, the value matching relations must hold, that is

$$v_1(g_i) = v_2(g_i) + q_i \tag{27}$$

and

$$v_1(g_r) = v_2(g_r) - q_r \tag{28}$$

Further, for the optimal controls to be optimal the high contact principle must be satisfied. That is,

$$\frac{dv_1(g_i)}{dg} = \frac{dv_2(g_i)}{dg} \tag{29}$$

$$\frac{dv_1(g_r)}{dg} = \frac{dv_2(g_r)}{dg} \tag{30}$$

Hence, relations (27) to (30) gives four equations to determine the four unknowns, g_i, g_r, *A*, and *B*.

Oil price dynamics

Empirically, there seems to be a long run mean revering pattern in the oil price when observations of the crude oil price for the last 100 years are studied. See e.g. Pindyck & Rubinfeld (1991).

From (3), (7) and (11) it follows from Ito's lemma that the dynamics of the price of oil, *X*, is given by a stochastic differential equation

$$dx_t = (\hat{\mu} - \eta_t^* + \delta)x_t dt + \hat{\sigma} x_t dB_t \tag{31}$$

where η^* is the optimal percentage increase of new capital in the oil industry. Between any change in η^*relation (31) is a geometrical Brownian motion. However, if the production of new capacity is low, i.e. $\hat{\mu} > \eta^* - \delta$, then the trend of the process is positive. If the production of new capital is high, i.e. $\hat{\mu} < \eta^* - \delta$ then the trend of the process is negative. High demand and high oil prices trigger high oil investments and vice versa. Consequently, the overall dynamics of the oil price, given by (31) and the optimality conditions for η^* is a mean reverting pattern.

The standard deviation of the relative change in the oil price is identical to the standard deviation of the relative change in demand, $\hat{\sigma}$. This property is related to the chosen shape of the supply and demand functions and is not a general result. Generally, volatility in prices compared to the volatility of demand depends on the price elasticity of supply and demand.

Final comments – OPEC production and the effect on investments

As discussed in the first part of this paper OPEC controls almost 80% of the world oil reserves but OPEC production varies from about 30 percent to slightly above 50 percent of the world oil production. The model gives a 50/50 sharing of production between OPEC and the other oil producers. However, it may be optimal for the cartel to deviate from the 50/50 sharing of the market. OPEC prefers low oil investments in non-OPEC countries. Hence, it may be optimal for the cartel in the model to produce more than 50 percent of the world demand such that the trigger level g_r, and the corresponding $x_r(g_r)$, is reached earlier than what is the case in the basic model. If the fringe producers were totally myopic, that is, they mistake the fall in the oil price from an increase in the cartel's production for a fall in demand, a short-term OPEC production increase will trigger a reduction in oil investments. However, since production statistics are easily available, such myopic behaviour is a fairly unrealistic assumption. Nevertheless, there will be a trade off between the period of high OPEC production that is necessary to trigger lower development of new fields in non-OPEC areas, and the future higher profit from higher oil prices due to a different path of η^*.

This industry level model indicates that the oil price process in the crude oil market will be mean reverting. Hence, when valuing the option to invest in an oil field development, the market power of the efficient OPEC oil producers should be taken into account. Uncertainty is not only due to shifts in demand and technological shocks, but also due to strategic changes in OPEC production policy.

References

Bensoussan, A. and J.-L. Lions (1984). *Impulse control and quasi-variational inequalities*: Gauthier-Villars

Brekke, K. A. and B. Øksendal (1994). "Optimal switching in an economic activity under uncertainty", *SIAM Journal of Control and Optimization*, 7: 1021-1036

Dixit, A. (1989). "Entry and Exit Decisions under Uncertainty", *Journal of Political Economy*, 97, 3: 620-638

Dixit, A. (1993). “The Art of Smooth Pasting”, Vol. 55 *in Fundamentals of Pure and Applied Economics,* edited by J. Lesourne and H. Sonnenschein: Harwood Academic Publishers

Dixit, A. and R. S. Pindyck (1994). *Investment under Uncertainty*, Princeton University Press

Dixit, A. (1991). "Irreversible Investment with Price Ceiling", *Journal of Political Economy*, vol. 99, no. 3

Harrison, J. M., T. M. Sellke and A. J. Taylor (1983). “Impulse control of Brownian motion”, *Mathematics of Operations Research*, 8: 454-466

Lucas, Robert E. Jr. and Edward C. Prescott (1971). "Investment Under Uncertainty", *Econometrica*, Volume 39, number 5, September

Øksendal, B. (1992). *Stochastic Differential Equations - An Introduction with Applications*, third edition: Springer-Verlag

Pindyck R. and D. Rubinfeld (1991). *Econometric Models & Economic Forecasts*, third edition. McGraw-Hill

Tvedt, Jostein (1996). *The Structure of the Freight Rate, A Stochastic Partial Equilibrium Model of the VLCC Market*, Working Paper, 17/96, Institute of Economics, Norwegian School of Economics and Business Administration