Valuation and Information Acquisition Policy for Claims Written on Noisy Real Assets*

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Abstract

This paper studies the effects of noise on contingent-claim values, option exercise policies and the incentives to acquire information to improve irreversible exercise decisions. We determine distributional parameters for the conditional expected asset value in which the noise and the underlying asset value dynamics follow normal, lognormal and mean-reverting processes. Option prices are found to depend on the revealed variance of asset price, suggesting that only information that can be acted upon are useful in formulating option exercise policy. To study incentives to acquire costly information, we examine the case of a borrower who holds the default (put) option inherent in a risky discount debt contract. Consistent with classical option pricing results, the value of (purposefully acquired) information is found to be highest at the point in which the claimholder is indifferent between exercising or not exercising an irreversible (default) option. Under a more general setting where there are multiple opportunities to gather information, it is optimal to acquire information in smaller increments to reduce the potential of ex-post overinvestment and underinvestment in information acquisition. Finally, we examine incentives to share information and regulation that can be used to encourage transparency and liquidity in real asset markets when tendencies toward opacity and illiquidity are high.

I Introduction

In contrast to exchange-traded financial asset markets, real asset markets are often decentralized (Williams (1995)) due to the unique physical, contractual-relational or locational characteristics of real assets. Decentralization can be expected to introduce noise into the asset valuation process, in the sense that asset values cannot be continuously and precisely determined. Noise persists because real assets are infrequently traded, because market incompleteness prevents using combinations of other assets to accurately reveal true asset value, and because information acquisition technology is costly and imperfect. If the exact value of the real asset is not known with certainty, both valuation and any exercise decisions based on the asset value must reflect the imprecise value estimate. In order to reduce the likelihood of error and hence to increase the value of a claim, the claimholder may have an incentive to more precisely determine asset value by paying to acquire additional information about the underlying asset value.

Uncertainty regarding the precise value of real assets and incentives to acquire information to resolve that uncertainty are commonplace in many economic settings. An important example is that of new products that result from purposeful research and development activity. New product markets can be interpreted as pure growth options. Because revenue estimates must be made for products that do not currently exist, any exercise decision on these options will be based on noisy estimates of product value. As a result the product developer may wish to acquire additional information through experimentation or related R&D activity. Further, the firm has several information acquisition strategies (with different precision and costs) to choose from. Precision may be moderately increased by a relatively low cost technique like focus groups. Alternatively, higher precision may be obtained by test marketing the new product, but at a potentially much higher expense.

Other important classes of noisy real assets are residential and commercial real estate.

Many properties are unique and hence generate service or cash flows that may be difficult to replicate using existing assets. Moreover, they trade infrequently, which makes value estimation difficult. To estimate value, owners/investors typically rely on self-generated or third party appraisals, the value signals from which are notoriously noisy estimates of true value (see, e.g., Geltner, Graff, and Young (1994)). Goetzmann (1993) has found that even more sophisticated repeat sales price estimates are noisy, and become increasingly so as time from the last sales date increases. Claims that depend on real estate price include the pure growth option inherent in raw land (e.g., Titman (1985)), the redevelopment opportunity for many improved properties (e.g., Williams (1997)), and debt financing (e.g., Titman and Torous (1989)). Exercise decisions for these options must reflect imprecision in the measurement of the underlying asset value, implying that it may be prudent for the claimholder to acquire additional information. For example, prior to making exercise decisions, the real option-holder may pay for a market analysis (for development or redevelopment options) or one or more independent appraisals of property value (for debt financing and repayment decisions).

A third important class of noisy real assets is human capital. At any point in time the precise value of an individual's economic worth is uncertain. As a consequence, the individual may choose to acquire information as to the value of her human capital by occasionally searching the labor market. If a new offer of employment appears, accepting that offer may require irreversibly terminating the current employment situation. As a result, there may be an incentive to gather additional information to resolve uncertainty as to the benefits and costs of terminating the old position and accepting the new one (e.g., to determine more precisely the value of fringe benefits, the cost of housing, the quality of local schools, etc.).

This paper studies the effects of noise on contingent-claim values, option exercise policies and the incentive to acquire information to improve irreversible exercise decisions. We consider noisy real asset markets in which information arrives in two possible ways: continuously with the arrival of imperfect but costless value signals and discretely with the application of a costly information acquisition technology. The discrete acquisition of information often occurs at an asset sale or financing date, or at other points in time when important financial decisions must be made (such as at an option exercise date). However, in between these dates it is often the case that market participants rely on low-cost public information to generate imperfect value estimates. Thus, consistent with the descriptions of Black (1986) and others, the economic setting we consider can be characterized as one in which noise is pervasive and often cumulative in its impact on real asset valuation, but where costly information acquisition occasionally happens to partially erase cumulative noise and hence increase the precision of the asset value signal.

In our analysis we allow for two types of noise in the determination of real asset value signals: an initial level of noise present when the underlying asset value is originally observed or estimated, and a dynamic process that accumulates noise after the initial observation. We apply the optimal filtering techniques of Liptser and Shiryayev (1978) to determine the distributional parameters for the conditional expected asset value (the value used as a best estimate of the true underlying asset value). To fully develop the economic implications that can be derived from these filtering methods, specific assumptions must be made as to the underlying stochastic processes. Consequently, we determine the distributional characteristics of the conditional expected value under three distinct 'noise propagation' regimes in which, i) outside noise steadily accumulates over time, ii) there is no outside noise, but a portion of asset value is unobservable, and iii) noise mean reverts and past errors tend to dissipate.

We find that history may or may not matter in the determination of the conditional expected asset value. When the true and observed asset values are lognormally or normally distributed, and when the asset value is partially revealed, only current information is required to determine expected asset value. In contrast, under similar structural assumptions with the exception that noise increasingly hinders accurate observation of the true asset value, the expected asset value is a weighted average of the initial value estimate and the most recent noisy value signal. The initial value estimate is useful, since it contains the lowest level of accumulating noise. Finally, when noise follows a mean-reverting Ornstein-Uhlenbeck process with normally distributed true asset value, the entire historical observed value path is useful in estimating the expected asset value. Since the noise and the true value have different distributions, a path of historical values that behaves more like the true asset distribution (noise distribution) suggests that the noise component is relatively small (large).

Because noise suppresses the information content of the observed value signal, the revealed variance of asset price (the variance of the expected asset value over time) is useful in the determination of contingent-claim valuation and optimal exercise policy. In general, the revealed variance of real asset price will be less than the true (full information) variance, suggesting that information that can be acted upon in the determination of option exercise decisions arrives at a slower rate than it does in a full information economy. Noise therefore reduces the value of call and put options below their full information values. It also effects option exercise policy on many types of options. For example, noise may increase the rate at which proprietary American or compound options are exercised, since the value of waiting to resolve additional uncertainty decreases relative to the full information case. Alternatively, exercise of strategic American options may be delayed significantly if costly information acquired by the first-mover is publicly observable or partially revealed to other optionholders.

The existence of noise impairs a claimholder's ability to make good exercise decisions (relative to when the asset value is known with certainty). This will create incentives to acquire additional information as to the true asset value. When a costly information acquisition (IA) technology exists, the claimholder must determine how much, if any, information to acquire. Further, the claimholder may be able to repeatedly gather and process information, and must determine an optimal sequence of information acquisition. To focus on these issues, we examine information acquisition for the case of a borrower who holds the default option inherent in a risky discount debt contract. We model the *IA* cost as an increasing function of precision, and then calculate the optimal level of information acquisition, the effects of *IA* on the arrival rate of information and its impact on claim valuation.

Costly information acquisition can be interpreted as an option to resolve residual asset value uncertainty at a discrete point in time. *IA* will be utilized to reduce errors at the debt payoff date if the technology is sufficiently inexpensive and if the noisy signal of asset value is sufficiently close to the critical value (face value of the debt) at which the default option is exercised. The option premium on *IA* is analogous to the time premium found in the classical (complete information) options. The major distinction between the two is that information arrives continuously (and relatively slowly) in the classical case whereas information arrives in a discrete 'package' in the case of *IA*. If noise of both types (initial or accumulating) can be reduced by acquiring a more precise value estimate, exercise decisions, and therefore claim value, are functions of all noise, independent of the source. Comparative static results reveal that when asset volatility is greater than the accumulating noise volatility, the optimal level of information acquisition is most sensitive to noise volatility. Initial noise volatility has the greatest impact for shorter-lived claims while accumulating noise volatility has the greatest impact for longer-lived claims. Finally, we demonstrate that, under the more general setting when there are multiple opportunities to gather information, it is optimal to acquire information in smaller increments to reduce the potential of ex-post overinvestment and underinvestment in information acquisition.

Our model of debt contracting and information acquisition extends the costly state verification approach of Townsend (1978) and Gale and Hellwig (1985) in several ways. Because noise degrades precision of the real asset value estimate in our model, the borrower has an *a priori* incentive to acquire information prior to making a debt repayment decision. The inspection range extends to either side of the debt payoff amount as opposed to *IA* taking place only at sufficiently low asset values in the costly state verification model. Moreover, tradeoffs between cost and precision of *IA* may result in (optimally chosen) imperfect *ex post* value estimates. The lender anticipates the acquisition of information and this expectation will be priced. To reduce the costly duplication of *IA*, incentives will exist for the borrower to credibly commit to share information that is acquired with the lender.

The remainder of the paper will be organized as follows. In Section II optimal filtering techniques are utilized to determine the distribution of the conditional expected value used to value contingent claims. Section III contains an application of a risky debt contract (with its embedded put option) in which the optimal levels and sequence of information acquisition are determined. Section IV considers further implications and extensions to the model. In particular, we address incentives for banks to share information in a world with and without forced liquidation regulation as well as consider *IA* incentives on other types of claims such as compound, American and strategic options. Section V summarizes our major findings.

II Optimal Filtering Results and Special Cases

The general setting described below considers a real asset for which the true value of the asset cannot be perfectly observed. Instead, a noisy observation of value is available and is used to estimate the true asset value which can be used in contingent-claim pricing. We consider a general class of true value/noisy observation relationships for which standard filtering theory provides a set of differential equations. Solutions to the differential equations and economic implications are developed in several special cases. Initially we assume that it is not possible to increase the accuracy of estimates of true value through the application of a costly information acquisition technology. This assumption is relaxed in Section III to study the impact of costly information acquisition on value estimates and option exercise decisions.

Earlier work has addressed valuation and optimal exercise policy for contingent claims when information is symmetric across investors but imperfectly observed. Flesaker (1991) and Gauthier and Morellec (1997), develop models where the claimholder observes only a noisy signal of the underlying asset value (the unobservable state variable). They derive the expectation of the underlying asset value conditional on the noisy signal and show that the presence of noise reduces the value of the claim and alters exercise policies. In this section we generalize and extend the the earlier work on contingent claims on noisy assets. We allow for two types of noise: an initial level of noise present when the underlying asset value is originally observed or estimated and a dynamic process that accumulates noise after the initial observation. Further, we apply general optimal filtering techniques to find conditional expected values when the accumulating noise and the underlying asset value dynamics follow normal, lognormal and mean reverting processes.

A Asset Value, Observed Value and the Conditional Expected Value

Consider an economy in which the true real asset value, $X(t)$, is imperfectly observable to all market participants, in the sense that it can neither be directly observed nor perfectly inferred by payoffs of any existing assets. Implications of this assumption include: (i) markets are incomplete, and (ii) cash flows generated by the real asset cannot be used to perfectly determine true asset value. We also assume there exists a continuous stream of information, publicly available at no cost, that can be used to partially resolve uncertainty as to contemporaneous asset value. For example, information regarding market activity of comparable real assets or the continuous release of relevant macroeconomic data may provide partial information as to the true asset value. Thus, for $t \geq 0$, the real asset holder observes an estimate of true value, $Z(t) = f(X(t), Y(t))$, which is a function of both the true value of a noise term, $Y(t)$.

For $t \geq 0$, let the dynamics of the true value and the observed value be

$$dX(t) = [a_0 + a_1 X(t)]\, dt + b_1 dW_1 + b_2 dW_2, \tag{1}$$

$$dZ(t) = [A_0 + A_1 X(t)]\, dt + B_1 dW_1 + B_2 dW_2, \tag{2}$$

where dW_1 and dW_2 are increments of standard Wiener processes, and all a_i, b_i, A_i, and B_i can be functions of time and the observed value. We assume the dynamics for $X(t)$ and $Z(t)$ are risk-neutral processes so that valuation is determined by discounting expected cash flows at the risk-free rate of interest.[1]

Since the true asset value is not available, investors must form an expectation of true value given the σ-field of information available at time t, $I(t)$. Note that $I(t)$ contains the observed values, $Z(s)$, for $0 \leq s \leq t$. Further, $\{I(t)\}$ is a filtration; i.e.,

$$I(0) \subseteq I(s) \subseteq I(t) \qquad \forall\ 0 \leq s \leq t.$$

Now define the conditional expected value

$$m(t) = E\left[X(t)\,|I(t)\right].$$

The level of uncertainty about the current true value given the current observed value is

$$\gamma(t) = \mathrm{Var}\left[X(t)|\, I(t)\right].$$

That is, $\gamma(t)$ is the *residual variance* that remains after optimally estimating the true value

[1]For example, if investors are risk-neutral, the dynamics that describe the processes $X(t)$ and $Z(t)$ in Equations (1) and (2) will be non-adjusted (or real) processes. A weaker assumption could be, with appropriate qualification, that there exists an equivalent martingale, so that after a risk-adjustment the cash flows can be discounted back at the risk-free rate of interest. In this case, Equations (1) and (2) would represent the risk-adjusted dynamics. See Rubinstein (1976) for a utility-based application of equivalent martingale pricing in potentially incomplete markets.

using the most current information. In the absence of noise, $\gamma(t) = 0$, $m(t) = X(t)$ and the current true value is known with certainty. When noise is present, $\gamma(t) > 0$ so that even conditioning on current information does not completely reveal the current true value.

In this paper we are particularly interested in the effects of noise on contingent-claim valuation and exercise policy. Option valuation with risk-neutral processes involves taking the present value of the conditional expected cash flows. Thus, another variance that will be important is the variance of $m(t)$ given the information available at some earlier date, $\text{Var}\left[m(t)\,|I(s)\right]$, $s \leq t$. We refer to the variance rate of $m(t)$ from the initial date of an option to its terminal date as the *revealed variance* rate. This statistic summarizes the rate of arrival of costless information that can be acted upon when making option exercise decisions, and in general will differ from the true asset variance.

Given the dynamics in Equations (1) and (2), Liptser and Shiryayev (1978) provide conditions such that the conditional expected value, $m(t)$, and the residual variance, $\gamma(t)$, are unique solutions of

$$dm(t) = \left[a_0 + a_1 m(t)\right] dt + \frac{b_1 B_1 + b_2 B_2 + \gamma(t) A_1}{B_1^2 + B_2^2} \left[dZ(t) - \left(A_0 + A_1 m(t)\right) dt\right], \qquad (3)$$

$$\frac{d\gamma(t)}{dt} = 2\gamma(t) a_1 + b_1^2 + b_2^2 - \frac{\left(b_1 B_1 + b_2 B_2 + \gamma(t) A_1\right)^2}{B_1^2 + B_2^2}, \qquad (4)$$

with initial conditions $m(0) = E\left[X(0)|\,I(0)\right]$ and $\gamma(0) = \text{Var}\left[X(0)|\,I(0)\right]$.[2,3] In a persistently noisy economy the initial observation contains uncertainty, that is $\gamma(0) = \sigma_{yo}^2 > 0$. For example, one could view $m(0)$ as an asset price derived from a market transaction or, alternatively, some (potentially imperfect) asset pricing model or appraisal process where residual uncertainty about the true current value may exist. Note also that $\gamma(t)$ is the solution to an ordinary differential equation, while $m(t)$ is the solution of a stochastic differential

[2]Equations (3) and (4) are a direct application of Theorem 12.7 in Liptser and Shiryayev (1978).

[3]Discrete time versions of the results in Equations (3) and (4) are presented as recursive equations in Theorem 13.4 of Liptser and Shiryayev (1978).

equation which depends on $\gamma(t)$. Lastly, $m(t)$ is an optimal estimate (i.e., it is efficient in a mean square sense) of the true value.

To illustrate the usefulness of Equations (3) and (4) in an economic setting, it is necessary to make specific assumptions about the distributions of $X(t)$ and $Z(t)$. Four special cases are considered in the next sub-section, and are chosen to illustrate a range of frequently encountered stochastic processes.[4]

B Special Cases

B.1 Lognormally Distributed Variables with Accumulating Noise

Consider the case where the true value, the noise and the observed value are all lognormally distributed and $Z(t) = X(t)Y(t)$. The dynamics (after taking logs to transform the variables to normal variables) are presented in Equation (5) of Table 1. In this example, additional noise accumulates at the rate of σ_y^2 which, along with the initial uncertainty, causes the observed value to differ from the true value. The noise has an expected value of 1 and is persistent in the sense that future changes in noise levels, $\ln(Y)$, are completely independent of past values. Black (1986) suggests that noise is typically cumulative in the absence of an ability to acquire and trade on proprietary information regarding the true asset value. Consequently, the lognormal case may best describe projects in which it is difficult to infer value from other existing assets and in which continuous acquisition of more precise information of true asset value is prohibitively expensive.

Let $m^*(t) = E\left[\ln(X(t))\,|I(t)\right]$ and $\gamma^*(t) = \text{Var}\left[\ln(X(t))\,|I(t)\right]$. Applying Equations (3) and (4) to Equation (5) generates the differential equations for $m^*(t)$ and $\gamma^*(t)$ in Equation (6). The solution is provided in Equation (7) of Table 1. The conditional expected value,

[4]The examples that follow are linear filtering problems, special cases of the non-linear filtering results presented earlier. Theorem 10.3 in Liptser and Shiryayev (1977) is a special case of Theorem 12.7 in Liptser and Shiryayev (1978) that applies to linear filtering problems. The solution to the linear differential equations is the generalized Kalman-Bucy filter.

$m(t)$, is the geometric weighted average of the time 0 conditional expected value (or forward price) of $Z(t)$ (i.e., $Z(0)e^{\mu_x t}$), and the convexity adjusted current observed value, $Z(t)e^{\frac{1}{2}\sigma_y^2 t}$.[5] The weight is

$$\rho^2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_y^2},$$

or the true variance to total accumulating variance ratio.

For any $\rho < 1$ ($\sigma_y^2 > 0$), the initial observation is used in determining $m(t)$; i.e, the conditional expected value is not Markov. However, only a very limited set of historical observations are required to determine the conditional expected value: the initial observation and the most current observation. The original observation has no accumulating noise (and hence the least total noise), but does not contain current information about the true value. The most recent observed value contains the most current information about the true value, but also has the most accumulated noise. The weighting scheme places more weight on the current value when there is less accumulating noise. More weight is put on the time 0 conditional expected value when more accumulating noise makes the current observed value less precise. Also note that, while ρ and $m(t)$ depend on the variance rate at which noise accumulates, σ_y^2, they do not depend on the level of initial noise, σ_{yo}^2. Without being able to acquire additional information the initial noise cannot be acted on, implying that σ_{yo}^2 has a symmetric or neutral effect on the conditional expected value.

The residual variance, $\gamma(t)$ in Equation (7), is the sum of the initial variance, σ_{yo}^2, and further errors which accumulate at the rate of $\rho^2\sigma_y^2$. The revealed variance rate of $m(t)$ is $\rho^2\sigma_x^2$. Thus, to value a European (American) call or put option written on a noisy asset in this setting, simply use the Black-Scholes formula (numerical techniques) where the underlying asset price is $m(t)$ and the variance rate is $\rho^2\sigma_x^2$. When the noise variance is positive, In

[5] The convexity adjustment is required due to the lognormal prices. This adjustment is required irrespective of whether payoffs to a claim, are linear or not.

other words, the value of options with convex payoff functions is inversely related to the level of accumulating noise.

B.2 Lognormally Distributed Variables with Partial Observation

An alternative specification when $X(t)$, $Y(t)$, and $Z(t)$ are lognormally distributed is presented in Equation (8). Again, the noise term has an expected value of 1, and the observed value is the product of the true value and the noise. While in Section B.1 observation was continuously hindered by outside noise, the dynamics in Equation (8) allow for perfect observation of some portion of the true variance, $\sigma_{x_1}^2$, and complete unobservability for the remaining portion of true variance, $\sigma_{x_2}^2$, where $\sigma_{x_1}^2 + \sigma_{x_2}^2 = \sigma_x^2$.

Applying Equations (3) and (4) and solving the differential equations give the solutions found in Equation (10). With partial observation, the conditional expected value is the (convexity-adjusted) most recently observed value. Historical observations provide no incremental value since the current observation accurately reveals a portion of current asset value. All historical observations are simply stale values of the current revealed component and provide no information on the current unrevealed component. This is in contrast to the previous model in which noise steadily increases over time resulting in a role for the initial noisy asset value estimate in the determination of an updated conditional asset value.[6]

One way to view the differences in the models is through different types of model misspecification. For example, the accumulating noise model might correspond to using a multifactor CAPM model that assumes constant prices of risk in a world where the prices of risk are non-constant.[7] At some initial date the model is used to calibrate the prices of risk. At later

[6]The accumulating noise model is analogous to a person with deteriorating eyesight and the partially observable model is analogous to a person with tunnel vision each trying to establish the characteristics of an unpredictably changing object. To best determine the current state of the object, the individual with deteriorating eyesight will rely not only on his current view of the object, but also on a historical view of the object when his eyesight was better. The individual with tunnel vision can see only a portion of the object and relies only on his current view of the object since he couldn't see any more of the object in the past and past observations would not contain any recent changes in the object.

Table 1. Dynamics, Stochastic Differential Equations and Solutions for Lognormal Variables. This table contains the dynamics, differential equations, conditional expected value, residual variance and revealed variance when the true value, noise and observed value are lognormally distributed. In Panel A, the true value is obscured by additional noise. In Panel B, the observed value reveals only a portion of the true value variance.

Panel A: Lognormal Variables with Additional Noise

$$Z(t) = X(t)Y(t)$$

$$\text{Dynamics:}\begin{cases} d\ln(X(t)) &= \left(\mu_x - \frac{1}{2}\sigma_x^2\right)dt + \sigma_x dW_1 \\ d\ln(Y(t)) &= \left(-\frac{1}{2}\sigma_y^2\right)dt + \sigma_y dW_2 \\ d\ln(Z(t)) &= \left(\mu_x - \frac{1}{2}\sigma_z^2\right)dt + \sigma_x dW_1 + \sigma_y dW_2 \end{cases} \tag{5}$$

$$\text{Differential Equations:}\begin{cases} dm^*(t) &= \left(1-\rho^2\right)\mu_x dt + \rho^2 d\ln(Z(t)) \\ \frac{d\gamma^*(t)}{dt} &= \rho^2\sigma_y^2 \end{cases} \tag{6}$$

$$\text{Solution:}\begin{cases} m(t) &= \left(Z(0)e^{\mu_x t}\right)^{1-\rho^2}\left(Z(t)e^{\frac{1}{2}\sigma_y^2 t}\right)^{\rho^2} \\ \gamma(t) &= \rho^2\sigma_y^2 t + \sigma_{yo}^2 \end{cases} \tag{7}$$

$$\text{Revealed Variance:}\ \rho^2\sigma_x^2$$

Panel B: Lognormal Variables with Partial Observation

$$Z(t) = X(t)Y(t)$$

$$\text{Dynamics:}\begin{cases} d\ln(X(t)) &= \left(\mu_x - \frac{1}{2}\sigma_x^2\right)dt + \sigma_{x_1} dW_1 + \sigma_{x_2} dW_2 \\ d\ln(Y(t)) &= \left(-\frac{1}{2}\sigma_{x_2}^2\right)dt - \sigma_{x_2} dW_2 \\ d\ln(Z(t)) &= \left(\mu_x - \frac{1}{2}\sigma_{x_1}^2\right)dt + \sigma_{x_1} dW_1 \end{cases} \tag{8}$$

$$\text{Differential Equations:}\begin{cases} dm^*(t) &= d\ln(Z(t)) \\ \frac{d\gamma^*(t)}{dt} &= \sigma_{x_2}^2 \end{cases} \tag{9}$$

$$\text{Solution:}\begin{cases} m(t) &= Z(t)e^{\frac{1}{2}\sigma_{x_2}^2 t} \\ \gamma(t) &= \sigma_{x_2}^2 t + \sigma_{yo}^2 \end{cases} \tag{10}$$

$$\text{Revealed Variance:}\ \sigma_{x_1}^2$$

dates, new sensitivities to the risks are calculated, but the new βs are combined with the fixed model prices of risk. The user of the model in this case recognizes that it is mispecified in some way, but is unaware of the nature of the mispecification (an hence is unable to fix it). The partially observable model might correspond to a multifactor CAPM world where some of the factors (and the price of risk of these factors) are known, but there is one or more factor that is consistently unknown. Again, the user recognizes that the model is not perfectly specified but is not capable of determining the remaining factors.

Note that standard European (American) puts and calls can be valued by using the Black-Scholes formula (numerical techniques) with underlying asset value equal to $m(t)$ and variance rate equal to the revealed variance rate, $\sigma_{x_1}^2$. Again, option value decreases as noise volatility increases. This volatility-reducing property of noise may partially explain why attempts to establish exchange-traded options on indices of real asset values have proven difficult (e.g., options and futures on house prices as indexed by Case-Shiller-Weiss). Because the model and the data used to compute real asset prices are often proprietary, very little information is typically available in-between index value announcement dates to allow traders to update the index in a reliable way. This decreases revealed volatility to very low levels to decrease option contract prices and volume, and hence the profitability of the contracts to the exchanges. It may also introduce an additional source of basis risk for those interested in using these options contracts to hedge natural long or short positions in the real assets.

B.3 Normally Distributed Variables with Cumulative Noise

Panel A of Table 2 contains the dynamics, differential equations and solution when (i) $X(t)$, $Y(t)$, and $Z(t)$ are normally distributed, (ii) the noise term has an expected value of 0, and (iii) the observed value is the sum of the true value and the noise. The conditional expected

[7] The examples of model mispecification provided here are familiar, risk-based models. For these examples, the stochastic processes of this paper must be viewed as risk-neutralized processes (see Footnote 1).

value is the arithmetic weighted average of the time 0 conditional expected value, $Z(t)+\mu_x t$, and the current observed value, $Z(t)$. The revealed variance rate is $\rho^2\sigma_x^2$, which is strictly less than the true variance for positive levels of accumulating noise. European calls and puts on these noisy assets can be valued by using the formulas developed in Brennan (1979) using $m(t)$ as the value of the underlying asset and $\rho^2\sigma_x^2$ as the variance rate. Options with convex payoffs are again less valuable when noise is present.

A relevant alternative expression for the conditional expected value is

$$m(t) = Z(0) + \mu_x t \left(1 - \frac{1}{t}\int_0^t \rho^2 ds\right) + \int_0^t \rho^2 dZ(s). \tag{11}$$

The conditional expected value is the original observed value scaled up by two terms. The first is the total drift, $\mu_x t$, times one minus the average ρ^2. The second term is the cumulative weighted change in the observed value where the weight is ρ^2. This expression clarifies the role for intermediate value signals in the determination of the conditional expected value. When ρ is constant, $Z(t)$ and $Z(0)$ are the only observed values relevant in determining the conditional expected value. However, when ρ is time varying (see Section B.4), the entire path of value signals may be relevant when attempting to disentangle noise effects from actual value in the determination of the expected asset value.

B.4 Normally Distributed True Value with Mean-Reverting Noise

In the final case (Panel B of Table 2), the true value is normally distributed, the noise term is mean reverting and the observed value is the sum of the true value and the noise term. When the observed value is below (above) the true value, the noise term has a positive (negative) drift. Thus, past errors have a tendency to dissipate, where κ is the rate of dissipation. This case suggests that the actions of certain agents may keep observed prices from wandering too far from their fundamental values (Black (1986)). Purposeful research or the application of

Table 2. Dynamics, Stochastic Differential Equations and Solutions for Normal Variables. This table contains the dynamics, differential equations, conditional expected value, residual variance and revealed variance when the true value, noise and observed value are normally distributed. In Panel A, the true value is obscured by additional noise, and the noise is persistent. In Panel B, the additional noise is mean-reverting.

Panel A: Normal Variables with Additional Noise

$$Z(t) = X(t) + Y(t)$$

$$\text{Dynamics:}\begin{cases} dX(t) &= \mu_x dt + \sigma_x dW_1 \\ dY(t) &= \sigma_y dW_2 \\ dZ(t) &= \mu_x dt + \sigma_x dW_1 + \sigma_y dW_2 \end{cases} \tag{12}$$

$$\begin{matrix}\text{Differential}\\ \text{Equations}\end{matrix}:\begin{cases} dm(t) &= \left(1-\rho^2\right)\mu_x dt + \rho^2 dZ(t) \\ \dfrac{d\gamma(t)}{dt} &= \rho^2\sigma_y^2 \end{cases} \tag{13}$$

$$\text{Solution:}\begin{cases} m(t) &= (Z(0)+\mu_x t)(1-\rho^2) + \rho^2 Z(t) \\ \gamma(t) &= \rho^2\sigma_y^2 t + \sigma_{yo}^2 \end{cases} \tag{14}$$

$$\begin{matrix}\text{Revealed}\\ \text{Variance}\end{matrix}: \rho^2\sigma_x^2$$

Panel B: Normal Variables with Mean-Reverting Noise

$$Z(t) = X(t) + Y(t)$$

$$\text{Dynamics:}\begin{cases} dX(t) &= \mu_x dt + \sigma_x dW_1 \\ dY(t) &= \kappa(X(t)-Z(t))dt + \sigma_y dW_2 \\ dZ(t) &= (\mu_x + \kappa(X(t)-Z(t)))\,dt + \sigma_x dW_1 + \sigma_y dW_2 \end{cases} \tag{15}$$

$$\begin{matrix}\text{Differential}\\ \text{Equations}\end{matrix}:\begin{cases} dm(t) &= \mu_x dt + \dfrac{\sigma_x^2+\gamma(t)\kappa}{\sigma_x^2+\sigma_y^2}\left(dZ(t) - (\mu_x - \kappa Z(t) + \kappa m(t))dt\right) \\ \dfrac{d\gamma(t)}{dt} &= \sigma_x^2 - \dfrac{\left(\sigma_x^2+\gamma(t)\kappa\right)^2}{\sigma_x^2+\sigma_y^2} \end{cases} \tag{16}$$

$$\text{Solution:}\begin{cases} m(t) &= Z(0) + \mu_x t\left(1 - \dfrac{1}{t}\displaystyle\int_0^t \rho^2(s,\kappa)ds\right) + \displaystyle\int_0^t \rho^2(s,\kappa)dZ(s) \\ & \qquad + \kappa\displaystyle\int_0^t \rho^2(s,\kappa)(Z(s)-m(s))ds \\ \gamma(t) &= \dfrac{1}{\left(\frac{1}{\gamma(0)-c} + \frac{\phi}{2\sigma_x}\right)e^{2\phi\sigma_x t} - \frac{\phi}{2\sigma_x}} + c \\ c &= \dfrac{-\sigma_x^2 + \sigma_x\sqrt{\sigma_x^2+\sigma_y^2}}{\kappa} \\ \phi &= \dfrac{\kappa}{\sqrt{\sigma_x^2+\sigma_y^2}} \\ \rho^2(t,\kappa) &= \dfrac{\sigma_x^2+\kappa\gamma(t)}{\sigma_x^2+\sigma_y^2} \end{cases} \tag{17}$$

an information acquisition technology may ultimately be responsible for this mean reverting tendency, although the precise structure underlying this phenomenon remains unmodeled in this specification.

The conditional expected value in Equation (17) is the initial observation, $Z(0)$, plus three terms. The first term is the unconditional drift of the observed value times one minus the average $\rho^2(s,\kappa)$.[8] The second term is a weighted sum of increments of the observed value, where the weights now depend on time. These first two terms are similar to the terms in Equation (11) for the normal case except that the weights, $\rho^2(s,\kappa)$, are now time dependent. The third term is related to the past conditional noise terms (note that $Z(s) - m(s) = E\left[Y(s)\left|I(s)\right.\right]$). As opposed to the first three cases, the mean-reverting model has a true value and a noise value with fundamentally different distributions. As a result, more historical observed values are used to determine the current conditional expected value. Since the noise is mean reverting, persistently positive (negative) levels of conditional noise suggests that past estimates of the true value were too low (too high), implying that the current estimate of value should be increased (decreased) to correct for ex post estimation error. Hence, the entire time path of realized observed prices is utilized here, whereas the initial observed value was the only historical value of importance in the accumulating noise models and no historical values played a role in determining the conditional expected value in the partially observable model.

Also note that, in contrast with previous results, the initial level of noise, σ_{yo}^2 (as well as all intermediate values of $\gamma(s)$), is relevant in the determination of noisy asset and contingent-claim values (as seen in the calculation of $\gamma(t)$). For example, a high initial σ_{yo}^2 combined with a sample path of observed values that closely track true values suggests that the arrival of price information is highly informative relative to initial conditions. This, in turn, suggests that the initial $Z(0)$ should be discounted as an informed estimate of the time t asset value.

[8] The weight $\rho^2(t,\kappa)$ when $\kappa = 0$ is equal to the weight from the first three special cases, ρ^2.

C Implications for the Exercise of Real Options

The inability to observe the underlying asset value may result in a contingent-claim exercise policy that differs dramatically from exercise policy under perfect observability. First, the optionholder will use the conditional expected asset value when calculating the costs and benefits of option exercise. This introduces error into the process, which, in many cases, decreases option value. The potential for committing exercise mistakes thus creates an incentive to acquire additional information regarding the true asset value, an issue we address in detail in the next section.

Second, the degradation of low quality asset value information may affect decision making when discretion exists as to the timing of option exercise. For many proprietary real options, uncertainty as to the true asset value may result in a more aggressive exercise policy than when asset value is perfectly observable. Since information quality has been dulled by noise, the value of waiting is lessened. This effect may explain empirically documented cases in which individuals apparently exercise their options earlier than theoretical model predictions would suggest (see, e.g., Vandell (1992), Quigley and Van Order (1995)).

In other cases there may be the opposite incentive to delay exercise for much longer than would be the case with complete information. For example, in an imperfectly competitive new product market a potential market leader may have a strategic incentive to significantly delay investment. This follows because contemporaneous product demand may not be known with certainty and because initial investment generates a price information externality that may be difficult to internalize. As a result, competitors may wait extremely long periods of time to exercise if first-mover benefits are small relative to the costs (see, e.g., Rob (1991) and Banerjee (1992)).

III Debt Contracting and Incentives to Acquire Information

The existence of noise in determining the underlying asset value impairs a claimholder's ability to make precise exercise decisions. As a result, the claimholder has an incentive to more accurately determine asset value in order to improve exercise decisions and increase option value. Given the existence of a costly information acquisition (IA) technology that leads to more precise estimates of true asset value, the claimholder can determine how much (if any) information to acquire. Further, the claimholder may be able to repeatedly gather and process information, and thus must determine an optimal sequence of IA.

This section explores incentives to optimally acquire and distribute costly information by examining the case of an equityholder with a default (put) option inherent in risky discount debt used to finance an imperfectly observable real asset. We choose this setting because it aptly illustrates information acquisition incentives in a contingent claims context and because it directly relates to a rich literature on debt contracting and financial intermediation. Indeed, the issues we address in this section cut across numerous topic areas, including the valuation of corporate liabilities (e.g., Black and Scholes (1973), Merton (1974)), optimal contract design and costly state verification (e.g., Townsend (1978), Gale and Hellwig (1985)), incentives to share as opposed to hoard information (e.g., Ross (1979), Diamond (1985)), delegated monitoring and scale in the production of information (e.g., Diamond (1984), Ramakrishnan and Thakor (1984)), and the verification role of courts in bankruptcy (e.g., Roe (1996)).

The layout for this section is as follows. To provide a foundation for further analysis, Section A develops a model for debt valuation and default when the acquisition of additional information regarding the asset's value is infeasible. We extend the model in Section B to consider the case where one or more opportunities exist to acquire additional information.

A *Imperfect Observability without Costly Information Acquisition*

Consider a liquidity constrained investor who uses outside debt to finance ownership of a noisy real asset. In particular, suppose that at time 0 an equityholder issues non-callable, non-recourse discount debt with face value F, and maturity date $T > 0$ that is secured by the non-dividend paying asset. Suppose also that when the debt is issued, the equityholder and lender both believe the asset value is $m(0) = Z(0)$, which may differ from its true value, $X(0)$. This observed value may be derived from a market transaction or an imperfect screening process, where residual uncertainty can exist about the true current value. Assume that the initial uncertainty as to the true value of the asset is known to be $\gamma(0) = \sigma_{yo}^2$. After the debt is issued, additional uncertainty may accrue as to the true asset value. Specifically, assume that at time $t > 0$, the observed signal, $Z(t)$, is the product of a lognormally distributed true asset value, $X(t)$, and a lognormally distributed noise process, $Y(t)$, as in Section II.B.1. The value of the conditional expected value, $m(t)$, the residual variance, $\gamma(t)$, and the revealed variance rate were presented in Panel A of Table 1.

To provide a benchmark for further analysis and discussion, consider optimal exercise policy and debt valuation when information acquisition is infeasible. In this case the equityholder will default on the debt at time T if and only if $m(T) < F$. The time $t \in [0, T)$ debt value when noise is present, $D_0(m(t), F, r, T-t, \rho^2\sigma_x^2)$, can be stated as

$$D_0(m, F, r, T-t, \rho^2\sigma_x^2) =$$
$$Fe^{-r(T-t)}\Phi(d_2(m, Fe^{-r(T-t)}, \rho^2\sigma_x^2(T-t))) + m\Phi(-d_1(m, Fe^{-r(T-t)}, \rho^2\sigma_x^2(T-t))),$$

where

$$d_i(x, y, z) = \frac{\ln\left(\frac{x}{y}\right) + (\frac{3}{2} - i)z}{\sqrt{z}},$$

and where r is the risk-free rate of interest and Φ is the normal distribution function.

Debt value is simply the Merton (1974) debt value with (i) underlying asset price equal to the conditional expected value, m, and (ii) variance rate equal to revealed variance rate, $\rho^2\sigma_x^2$. Without the ability to acquire additional information, the exercise policy and debt value depend on the conditional expected value and not on the initial level of noise. When there is incomplete information (*II*) about the true asset value, the lender and equityholder recognize that default or repayment decisions may differ from the case where full information (*FI*) about the true asset value is known. Sub-*FI* optimal outcomes will be anticipated to result in debt value (yield-to-maturity) that is higher (lower) than the debt value that obtains from the classical noiseless case of Merton (1974).

Consider more specifically the differences between the *II* optimal exercise policy and the *FI* optimal exercise policy. Define a Type I (Type II) error as *II* optimally exercising (not exercising) when it is not *FI* optimal to do so. The potential for a Type I error exists when $m(T) < F$. Without additional information as to the true asset value, the expected loss is

$$\begin{aligned} EL_I(Z(T)) &= \int_F^{\infty} (X(T) - F) g(X(T)|I(T)) dX(T) \\ &= m(T)\Phi\left(e_1(m(T), 1, T)\right) - F\Phi\left(e_2(m(T), 1, T)\right) \end{aligned} \tag{18}$$

where g is the lognormal conditional density function and

$$e_i(m, \beta, T) = d_i\left(m, F, \beta\left(\rho^2\sigma_y^2 T + \sigma_{yo}^2\right)\right). \tag{19}$$

Note that the residual variance, $(\rho^2\sigma_y^2 T + \sigma_{yo}^2)$, is used in the calculation of the expected loss. Similarly, the potential for a Type II error exists when $m(T) > F$, and the expected loss is

$$EL_{II}(Z(T)) = \int_0^F (F - X(T)) g(X(T)|I(T)) dX(T) \tag{20}$$

$$= F\Phi\left(-e_2(m(T),1,T)\right) - m(T)\Phi\left(-e_1(m(T),1,T)\right).$$

The difference between debt value under full information and partial information is equal to the discounted present value of Type I and Type II errors. This difference is also equal to the discounted expected benefits of information acquisition in which asset value is fully revealed. Thus the equityholder may be willing to acquire costly information in an effort to reduce or eliminate *FI* sub-optimal exercise decisions.

B Costly Acquisition of Information

We now examine the strategic acquisition of information when real asset value is imperfectly observable to the borrower as well as to outsiders. Results are developed under the assumption that, if costly information is acquired as to true asset value, it is freely available to the debtholder upon default of the debt.[9] In the course of our analysis we justify and expand upon the shared information assumption in the context of hold-up and *IA* cost minimization.

Our model extends the costly state verification (CSV) approach of Townsend (1978), Gale and Hellwig (1985) and others, who assume that the borrower can perfectly observe asset value but that outsiders cannot. In many scenarios it is unrealistic to assume that the owners of real assets can costlessly infer the true asset price over time or even that their information is necessarily better than it is for certain types of outsiders (such as investors, lenders, appraisers and auditors who specialize in valuing particular kinds of real assets). As a result, it is the borrower who has an incentive to acquire information as to the true asset value before making an irreversible default or repayment decision. In further contrast to the CSV approach, we utilize a continuous range of *IA* precision. Intermediate precision

[9] Liquidity constraints at the loan payoff date may provide incentives to share not only 'bad' state-related information that is used to justify default and bankruptcy decisions, but also to share 'good' information in order to enhance the availability of outside resources with which to refinance the original debt. The anticipation of liquidity needs in the future thus serves to further harden *ex ante* incentives to acquire and share information as efficiently and credibly as possible.

levels allow for multiple IA opportunities and a study of the optimal sequence of information collection.

B.1 Single-Opportunity Information Acquisition

Consider an equityholder who has a one-time opportunity to acquire additional information as to the true underlying asset value. When this new information is acquired it can be combined with any existing information to determine a new conditional expected value, where the conditioning takes place over a new, finer σ-field of information, $I^{IA}(\beta, T)$, where $I(T) = I^{IA}(0, T) \subseteq I^{IA}(\beta, T)$ for $\beta \in [0, 1]$. The information that is acquired generates a new conditional expected value

$$m_1^{IA}(\beta, T) = E\left[X(T)\,\middle|\,I^{IA}(\beta, T)\right]$$

which reduces residual uncertainty about the value of $X(T)$ to

$$\gamma^{IA}(\beta, T) = (1-\beta)\gamma(T) = (1-\beta)\left(\rho^2\sigma_y^2 T + \sigma_{yo}^2\right). \tag{21}$$

That is, acquiring information reduces the residual uncertainty by a factor of β. Thus, consider β the level of IA precision, where $\beta = 0$ corresponds to a completely uninformative signal (IA does not reduce noise at all) and $\beta = 1$ corresponds to a fully revealing signal (IA completely eliminates noise).[10]

[10] One way to conceptualize the process of acquiring higher precision information is to envision variable effort by an individual/firm capable of generating higher precision asset value information. An alternative view is to assemble a diverse panel or focus group in which expert opinion is solicited and information is pooled together. In this case a direct relationship between precision level and panel size can be established. For example, suppose that an IA technology is employed to generate $(n-1)$ i.i.d. value signals, $\{W_i\}$, where W_i has the same distribution as $Z(T)$. The combined noisy estimate would be

$$Z^{IA}(T) = \sqrt[n]{Z(T)\prod_{i=1}^{n-1} W_i}.$$

We now examine IA incentives at the loan payoff date. As noted above, when asset value information is incomplete, it is the equityholder (borrower) who has the incentive to gather additional information prior to exercising an irreversible repayment or default option. Define

$$B(m,\beta,T) = \begin{cases} m\Phi\left(e_1(m,\beta,T)\right) - F\Phi\left(e_2(m,\beta,T)\right) & \text{if } 0 < m < F \text{ and } \beta > 0, \\ F\Phi\left(-e_2(m,\beta,T)\right) - m\Phi\left(-e_1(m,\beta,T)\right) & \text{if } m \geq F \text{ and } \beta > 0, \\ 0 & \text{if } \beta \leq 0 \text{ or } m = 0. \end{cases} \tag{22}$$

with e_i as defined in Equation (19). For a given precision, β, the expected benefit of single opportunity IA is $B(m(T),\beta,T)$.[11] Note that the benefits to information acquisition are greatest at the point where the equityholder is indifferent between debt payoff and default (i.e., when $m(T) = F$), and benefits decline monotonically as $|m(T) - F|$ increases. In other words, the acquisition of additional information is most valuable when one is indifferent between exercise alternatives and decreases as one choice increasingly dominates other alternatives.

The expected benefits to discretionary IA are an option value, where the total variance of the option equals the variance reduction due to IA. When $m(T) < F$ and no costly information is acquired, the equityholder defaults on the debt. The advantage of IA in this case is that the better informed equityholder will not default if the revised estimate of asset value, $m_1^{IA}(\beta,T)$, exceeds F, realizing expected benefits (at exercise) of $m_1^{IA}(\beta,T) - F$. Thus, acquiring information corresponds to a call option to 'repurchase' the assets (with pre-IA

The standard deviation of the new estimate is $\frac{\sigma_y}{\sqrt{n}}$. Thus, the relationship between n and β is $\beta = \frac{(n-1)\sigma_x^2}{n\sigma_x^2 + \sigma_y^2}$ and the post- IA conditional expected value is

$$m_I^{IA} = \left(Z^{IA}(T)e^{(1-\beta)\rho^2(\beta)\sigma_y^2 T}\right)^{\frac{\rho^2(1)}{\rho^2(\beta)}} \left(Z(0)e^{\mu_y T}\right)^{1-\frac{\rho^2(1)}{\rho^2(\beta)}},$$

where $\rho^2(\beta) = \frac{\sigma_x^2}{\sigma_x^2 + \beta\sigma_y^2}$. A similar relationship holds when the $\{W_i\}$ are correlated.

[11] It is straightforward to check that the benefit function is continuous for $m(T) \in [0,\infty)$, $\beta \in [0,1]$, and $T \in [0,\infty)$.

expected value of $m(T)$) for the face value of the loan, F. Similarly, when $m(T) > F$ and no costly information is acquired, the equityholder would make the final payment on the debt. In this case, acquiring information corresponds to a put option, where the equityholder creates an opportunity to put the asset for F, realizing expected benefits (at exercise) of $F - m_1^{IA}(\beta, T)$ if $m_1^{IA}(\beta, T)$ is less than F.[12]

The option premium for IA is analogous to the option time premium in the classical (complete information) setting. In the classical option case the time premium is due to the continuous (and relatively slow) arrival of new information as to the underlying asset value. With τ time left until the option exercise date, the total uncertainty to be resolved equals $\tau\sigma_x^2$. In our case, information due to IA arrives discretely at the option exercise date and corresponds to the reduction of uncertainty of the current asset value by $(1 - \beta)\left(\rho^2\sigma_y^2 T + \sigma_{yo}^2\right)$.[13]

Next, consider the cost of acquiring information. Let the cost be

$$C_1(\beta) = \eta\beta^{\alpha}, \tag{23}$$

where η and α are positive constants. The cost function is increasing in precision with convexity depending on α. It is independent of expected asset value, implying that the cost structure corresponds to a competitive market for information production.[14]

Some level of IA will take place when benefits exceed costs. Assuming that IA is feasible

[12] Information acquisition and subsequent default on the debt can thus result in a discontinuous drop in estimated debt value. Sudden changes in debt value are consistent with evidence on significant declines in corporate bond prices as a result of default (e.g., Altman (1989)) and with sudden drops in commercial real estate loan values once borrower default and foreclosure has occurred (e.g., Ciochetti and Riddiough (1999)). Also see Duffie and Lando (1998) for further discussion of this issue.

[13] In a related application, Ross (1989) analyzes effects on asset and option prices when changes occur in the (continuous) arrival rate of information. His focus, however, is on the rate of uncertainty resolution (whether intentionally effected or not) in the absence of being able to act on the new information. In contrast, a change in the rate of information flow is intentional in our model and can be acted on immediately.

[14] An alternative characterization might be to assume monopolistic information production, in which price is determined based on the marginal benefits gained from production of such information (e.g., Allen (1990)). In this case, costs would be a function of both β and $m(T)$.

for some $m(T)$ (i.e., there exists a β such that $B(F,\beta,T)-C_1(\beta)>0$), there will exist a region containing F in which it is optimal for the equityholder to acquire additional information. Figure 1 illustrates the net benefits to IA across various conditional expected asset values, $m(T)$. The figure compares four different levels of precision and demonstrates that the optimal level of precision varies across different conditional expected values. For example, when $m(T)$ is near $F=100$, an intermediate precision of $\beta=0.6$ is best (given only these four βs). As conditional expected values move away from face value in either direction, less precision is optimal (i.e., $\beta=0.4$ is optimal), and eventually it is no longer profitable to acquire any information. Note that there are high ($\beta=0.8$) and low ($\beta=0.2$) levels of precision that are never optimal given this chosen set of parameters. Thus, different conditional expected values will yield different optimal precision levels, but not all precision levels will be optimal.

If the equityholder can choose any $\beta\in[0,1]$, the expression for the optimal, β^* is

$$\beta^*(m(T),T)=\arg\max_{\beta\in[0,1]}\left[B\left(m(T),\beta,T\right)-C_1(\beta)\right].$$

If the optimal precision is not a corner solution, then β^* must equate marginal benefits and marginal costs, and implicit differentiation can be used to calculate the partial derivatives of optimal precision with respect to σ_y^2, σ_{yo}^2, and σ_x^2.[15] The pairwise relationships between these partials are

$$\frac{\partial\beta^*}{\partial\sigma_y^2}=\rho^4T\frac{\partial\beta^*}{\partial\sigma_{yo}^2}, \tag{24}$$

$$\frac{\partial\beta^*}{\partial\sigma_x^2}=\left(1-\rho^2\right)^2T\frac{\partial\beta^*}{\partial\sigma_{yo}^2}, \tag{25}$$

$$\frac{\partial\beta^*}{\partial\sigma_y^2}=\frac{\rho^4}{\left(1-\rho^2\right)^2}\frac{\partial\beta^*}{\partial\sigma_x^2}. \tag{26}$$

[15] See the appendix for these derivations.

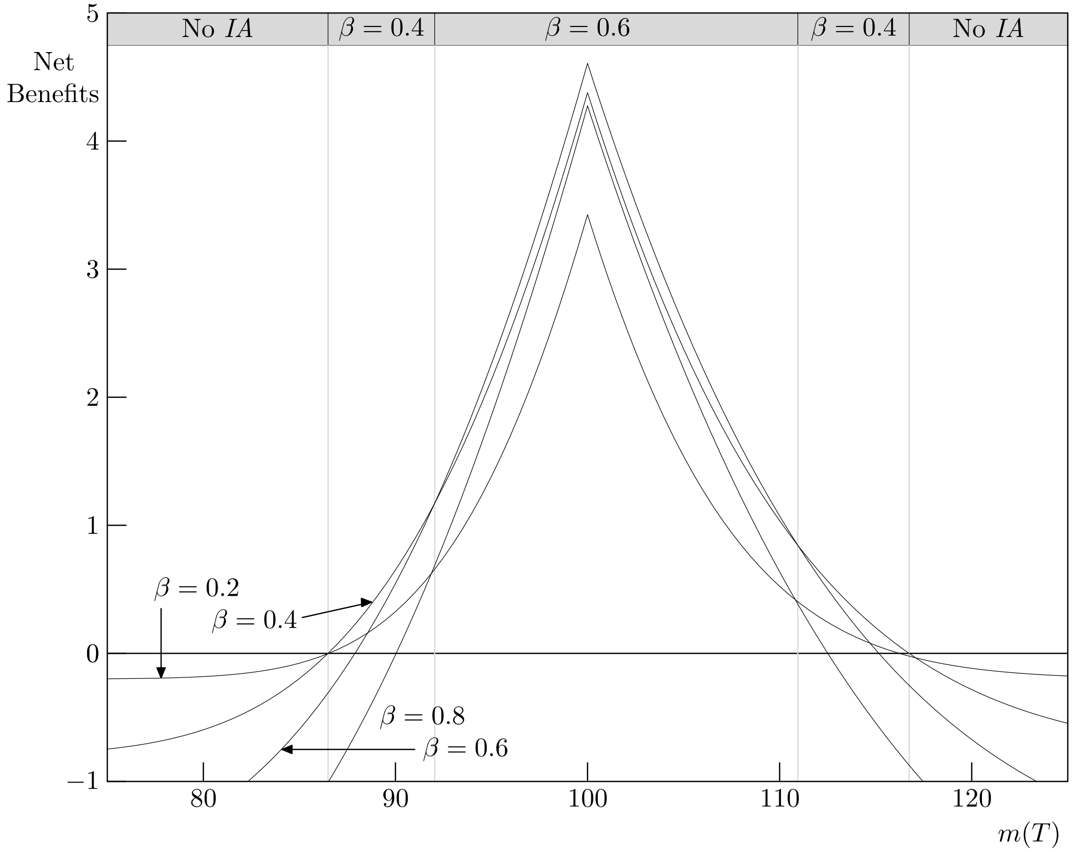

Figure 1. Net benefits to *IA* as a function of expected asset value, m(T). This figure displays the net benefits (benefits minus costs) of IA at a given level of the conditional expected value, $m(T)$. Net benefits from IA are greatest at $m(T) = F$ and decline as $m(T)$ moves away from F in either direction. Eventually, net benefits from IA are negative, and it is not optimal to acquire any information. Parameter values used to create this figure are: $r = 0.05$, $Z_0 = 125$, $F = 100$, $T = 5$, $\sigma_x = 0.30$, $\sigma_y = 0.10$, $\sigma_{yo} = 0.10$, $\alpha = 2$, and $\eta = 5$.

Consider the likely case in which true asset volatility is larger than the accumulating noise volatility (i.e., $\rho^2 > 0.5$). For shorter term options ($T < \rho^{-4}$), the optimal precision level is most sensitive to changes in initial noise level and least sensitive to changes in the true asset variance rate. For longer term options $\left(T > \rho^{-4}\right)$, noise accumulates over a sufficiently long time so that optimal precision is most sensitive to changes in the accumulating noise variance rate.

Figures 2A, 2B and 2C graphically display these comparative static results for the optimal

IA precision, β^*.[16] Figure 2A shows that as the standard deviation of the noise process increases, so does the optimal level of *IA* since the benefits to *IA* increase with the amount of noise.[17] The increase in *IA* precision is quite dramatic. For example, at $m(T) = F$, the optimal precision increases from 0.445 to 0.731 as the accumulating noise volatility rate goes from 0.05 to 0.15. As seen in Figure 2B, optimal precision is not as sensitive to changes in the initial noise volatility for the intermediate term option. For $m(T) = F$, optimal precision increases from 0.573 to 0.643 as initial noise volatility goes from 0.05 to 0.15. When noise volatility is less than true asset volatility, optimal precision is least sensitive to changes in true volatility (Figure 2C). For $m(T) = F$, the optimal level of *IA* increases only slightly (from 0.582 to 0.614) as true asset volatility increases from 0.20 to 0.60. Note that, because benefits to *IA* increase with increases in volatility, the range of conditional expected values for which it is optimal to gather information also increases as any of the volatilities increase.

Our model differs sharply from the standard CSV model of debt contracting along several dimensions. In the CSV model the borrower has perfect information regarding asset value. *IA* (monitoring) by the lender occurs at asset values below the loan payoff amount, since the lender desires to maintain a credible threat of liquidation, given borrower default. Further, the *IA* technology is perfectly precise in the CSV model, and debt is the optimal outside financing arrangement since it minimizes expected transaction costs. In contrast, *IA* in our model is undertaken by the borrower at intermediate expected asset values (i.e., in an interval containing F). Moreover, the *IA* range depends on the tradeoff between the cost and precision of information, and may result in (optimally chosen) imperfect value estimates. Efficiency of the debt contract in our model will in part depend on incentives for the borrower

[16] The expected benefits will be unchanged if the maturity increases while the true asset variance rate and accumulating noise variance rate decrease by the same multiplicative factor. Thus, the base parameters for Figure 2 ($\sigma_x = 0.30$, $\sigma_y = 0.10$, and $T = 5$) yield the same optimal precision levels as $\sigma_x = 0.15$, $\sigma_y = 0.05$, and $T = 20$.

[17] Although the net benefits to *IA* are greatest at $m(T) = F$, the maximum level of *IA* occurs at an $m(T) > F$ due to the lognormality of asset values (see the appendix for further detail).

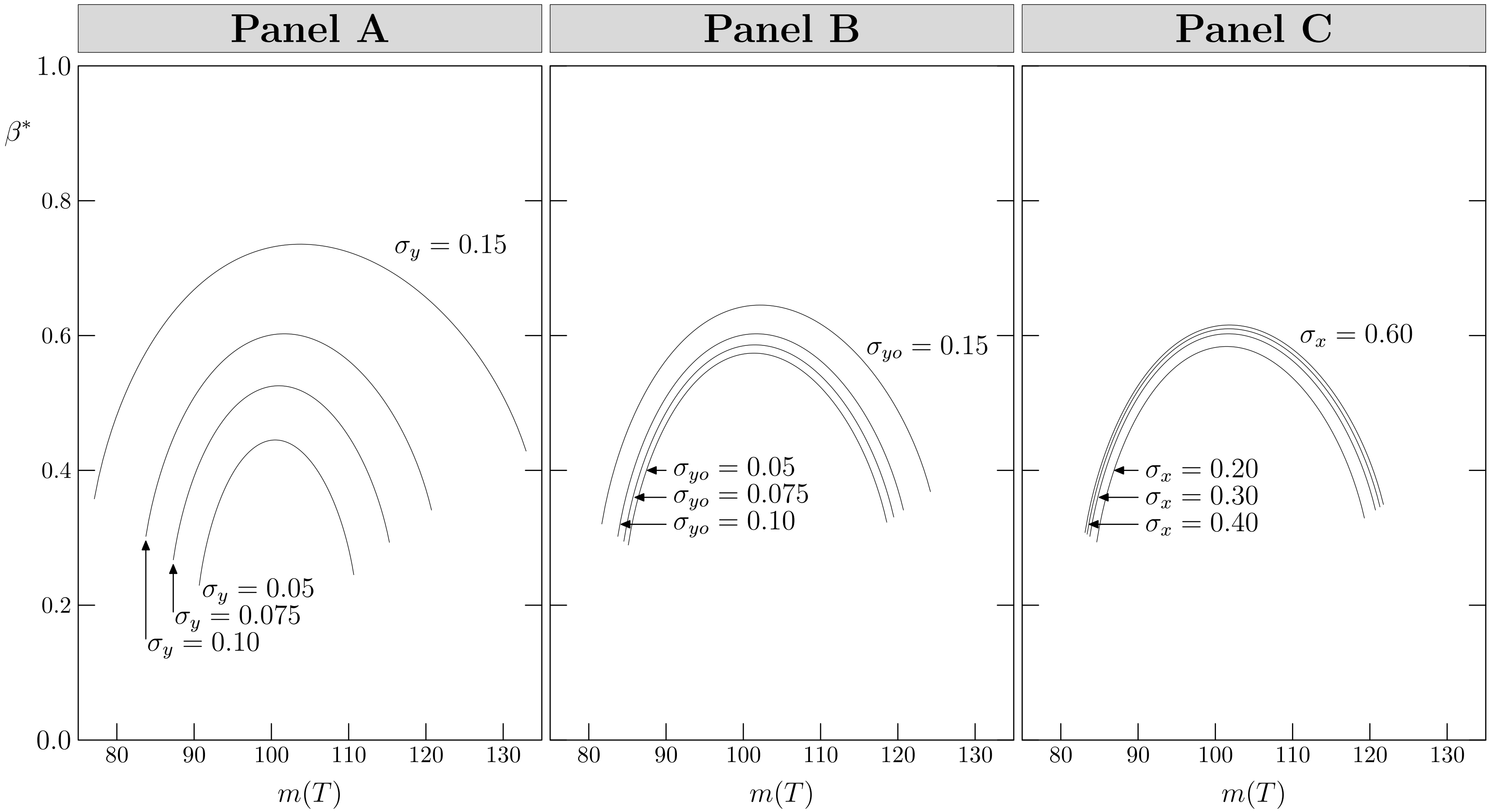

Figure 2. Optimal *IA* precision as a function of volatility. This figure displays the optimal precision of IA as a function of the pre-IA conditional expected value, $m(T)$. In Figure A, $\sigma_x = 0.30$, $\sigma_{yo} = 0.10$ and four different σ_y values are graphed. In Figure B, $\sigma_y = 0.10$, $\sigma_x = 0.30$ and four different σ_{yo} values are graphed. In Figure C, $\sigma_y = 0.10$, $\sigma_{yo} = 0.10$ and four different σ_x are values graphed. Parameter values common to all figures are: $r = 0.05$, $Z(0) = 125$, $F = 100$, $T = 5$, $\alpha = 2$, and $\eta = 5$.

to share information in order to minimize the expected costs of *IA*. That is, if the borrower has an incentive to reveal information with the lender when default occurs, it may be that *IA* by the lender is not required, hence lowering debt costs.

Given the existence of a known *IA* technology, the potential acquisition of additional asset value information will be anticipated at the time the debt is issued. Debt value in this case is simply the value of the debt without *IA*, less the expected present value benefits to *IA*. The equityholder expects to incur non-reimbursable costs associated with *IA* when $m(T)$ is in the feasible monitoring range. Because of these costs the equityholder would prefer to avoid outside finance altogether, but is unable to do so since she is liquidity constrained. As an alternative, the equityholder would prefer to credibly commit *ex ante* to never acquire information *ex post* in order to save the expected costs of *IA*. This sentiment is unlikely to be enforceable, however.

If costs of *IA* by the lender were to be priced into the debt, and there are no other opportunities to use private information strategically, a credible commitment to share information would be preferred by the borrower.[18] A credible information sharing commitment could involve a third-party *IA* specialist whose cost schedule is known in advance and who agrees to share the results of *IA* with the lender, should default occur. Incentives to acquire and then share information thus resembles the Diamond (1984) concept of delegated monitoring, with the critical difference that the *IA* specialist works directly for the borrower instead of the lender. Numerous other papers have addressed incentives to share financial information. Perhaps closest to our approach is Diamond (1985), who argues that voluntary information disclosure helps avoid costly duplication of effort by outside investors. Diamond and Verrecchia (1991) extend this idea to show that disclosure reduces information asymmetries

[18] The costs to hoarding as opposed to sharing information are at a minimum equal to the costs of duplicated monitoring. If information asymmetries are unresolvable through additional information acquisition, further costs will accrue ex ante due to adverse selection problems (Akerlof (1970), Grossman (1978), Stiglitz and Weiss (1981)).

to increase demand from large investors. Ross (1979) and others have employed 'unraveling' arguments that suggest outsiders infer bad news from the lack of disclosure, which forces the informed investor to reveal her information in equilibrium. A crucial difference between our model and these other approaches is that the 'market' specific and bilateral in our case whereas it is non-specific and multi-lateral in the other models. A specific bilateral relationship requires the explicit introduction of ex ante contracting and a third party *IA* specialist to minimize opportunistic behavior ex post, whereas a non-specific relationship with a large number of (anonymous) outsiders is sufficient to provide truth-telling incentives for the informed investor/seller.

B.2 Multiple-Opportunity Information Acquisition

One-time *IA* may be a reasonable approach when there is only a short time in which to acquire information to improve an irreversible exercise decision. There are numerous other circumstances, however, in which multiple, as opposed to all-at-once *IA*, is both a possible and an optimal policy. Indeed, the option to acquire additional information may be quite valuable, even when *IA* has been undertaken numerous times in the past.

The intuition behind multiple-time *IA* is that there will be an incentive to acquire information in smaller increments in an attempt to reduce *ex post* over- or under-investment in information. To see this, suppose that a one-time *IA* opportunity is available to the equityholder. If $m(T)$ is near the exercise boundary, significant *IA* may be desirable to reduce the probability of error. With multiple-time *IA*, the equityholder initially chooses lower precision (and hence lower cost) *IA*. If the revised $m_1^{IA}(\beta, T)$ turns out to be significantly different than F, the equityholder will be sufficiently confident in making an exercise decision with little or no additional costly *IA*. If, however, $m_1^{IA}(\beta, T)$ is still near F, the equityholder may aggressively acquire additional information, in this way targeting high, costly levels of *IA* only when it is most needed.

To develop this intuition more formally, consider the case in which a total of two *IA* opportunities are available to the equityholder. Denote the precision realized from the first *IA* opportunity as β_1, the precision realized from the second *IA* opportunity as β_2, and the revised expected asset value from the first *IA* opportunity as $m_1^{IA}(\beta_1, T)$. The second precision, β_2, is the cumulative or total precision, suggesting that $\beta_1 \leq \beta_2 \leq 1$ and that higher precision leads to finer information sets, $\left(I^{IA}(\beta_1, t) \subset I^{IA}(\beta_2, t)\right)$.[19] The information sets are invariant to the order in which information arrives. For example, acquiring information once at a precision of $\beta = 0.7$ generates the same information set as a sequence where initial precision is $\beta_1 = 0.4$ and the cumulative precision is $\beta_2 = 0.7$.

Multiple-opportunity *IA* can be interpreted as a compound option that resolves the asset's residual variance. To develop the optimal multiple-opportunity *IA* policy, consider the second (final) round of *IA*. The variance that is reduced from acquiring information a second time is the difference in the residual variance in the first and second rounds of *IA*,

$$\gamma^{IA}(\beta_1, T) - \gamma^{IA}(\beta_2, T) = (\beta_2 - \beta_1)\left(\rho^2 \sigma_y^2 T + \sigma_{yo}\right).$$

Conditional on first-time *IA* occurring, calculating the optimal *IA* region for the second time is similar to the one-time case except that $m_1^{IA}(\beta_1, T)$ is the revised conditional expected asset value and precision level increases from β_1 to β_2. The benefits for the second round of *IA* are simply the benefits for the incremental level of precision, $B(m_1^{IA}(\beta_1, T), \beta_2 - \beta_1, T)$.

We assume that the cost function for increasing precision from β_1 to β_2 is

$$C(\beta_1, \beta_2) = \begin{cases} \eta\beta_2^\alpha - \theta\eta\beta_1^\alpha & \text{if } \beta_2 > \beta_1, \\ 0 & \text{Otherwise.} \end{cases} \tag{27}$$

[19] It is straightforward to show that a multiple-time approach to *IA* is at least as desirable as the one-time approach. To see this, consider the *IA* policy where β_1 equals the optimal precision resulting from the one-time *IA* opportunity and β_2 is always zero. This policy is feasible but generally not optimal *IA* for the multiple-time case.

The parameter $\theta \in [0,1]$, indicates the degree of second-time cost savings derived from first-time IA. When $\theta = 1$, second-time IA costs are based solely on the incremental amount of precision obtained. Conversely, when $\theta = 0$, first-time IA costs must be duplicated for any second-time IA. Intermediate values of θ indicate that partial duplication of cost occurs in the second round. The optimal precision level for the second round of IA (given a precision from the first round) maximizes the net benefits, $B(m_1^{IA}(\beta_1,T),\beta_2-\beta_1,T)-C(\beta_1,\beta_2)$. Thus, the optimal precision level, $\beta_2^*(m_1^{IA}(\beta_1,T),\beta_1)$, depends on the precision and the conditional expected value that results from the first round of IA. If a second round of IA is not optimal, then $\beta_2^*(m_1^{IA}(\beta_1,T),\beta_1) = \beta_1$.

Determining the optimal precision for the first IA opportunity is more complicated because the precision from the first round affects optimal precision for the second round and because the benefits from first round IA include the option value associated with a second-time IA opportunity. The value of the option to acquire information the second time is the expected net incremental benefits obtained from the second round of IA for a given round one precision, β_1,

$$E\left[NB_2(\beta_1)\left|I^{IA}(0,T)\right.\right] = \int\limits_{R_2(\beta_1)} \left[B\left(m,\beta_2^*-\beta_1,T\right)-C(\beta_1,\beta_2^*)\right] g\left(m\left|I^{IA}(\beta_1,T)\right.\right) dm,$$

where

$$R_2(\beta_1) = \left\{m\left|B(m,\beta_2^*-\beta_1,T)-C(\beta_1,\beta_2^*)\geq 0\right.\right\},$$

is the interval containing F for which a second round of IA is desirable. Therefore, the total net benefits from a first round of IA are the direct net benefits from the first round plus the expected incremental net benefits from the second round

$$B(m(T), \beta_1, T) - C(0, \beta_1) + E\left[NB_2(\beta_1)\,\middle|\,I^{IA}(0,T)\right]. \tag{28}$$

The optimal first round precision, β_1^*, maximizes the total net benefits in Equation (28).[20]

Figure 3A displays endogenously determined optimal precision levels under a two-time *IA* approach with $\theta = 0$ (i.e., first-time costs must be duplicated upon second-time *IA*). Precision levels in this figure are determined prior to the initial acquisition of information, in which optimal precisions given two-time *IA* are compared to those obtained in the one-time *IA* case. Also shown is the expected increment to first-round *IA*, $E[\beta_2^* - \beta_1^* | \, I^{IA}(0,T)]$, which equals the difference between the expected cumulative precision from second-round *IA* and the optimal precision level from first-round *IA*.

Total expected precision with multiple-time *IA* in this figure is less than the optimal precision realized from one-time *IA*. Multiple-opportunity *IA* allows information to be gathered in smaller increments to reduce the potential for ex post over-investment in information. As β_1^* moves away from it maximum value, the expected second-round incremental *IA* increases and total expected *IA* decreases. Indeed, *incremental* second-round *IA* is expected to be more intense than first-round *IA* for some $m(T)$ in the *IA* range that are sufficiently far from F. Thus, an advantage of multiple-time *IA* is that the equityholder can perform a low precision 'test' for the first-time *IA*when $m(T)$, is not near F. If the outcome of the test, $m_1^{IA}(\beta, T)$, is still sufficiently far from F, no further *IA* is warranted. If $m_1^{IA}(\beta, T)$ is closer to F the equityholder invests in *IA* at a much higher level the second time, but since the first round was a low precision level there is little replication of costs.

Figure 3B shows optimal precision levels for $\theta = 1$, (i.e., when first-time costs are *not*

[20] If precision levels were set exogenously in advance, one could develop closed-form expressions for the compound *IA* options that would resemble those first derived by Geske (1979). Because we determine precision levels endogenously, numerical techniques must be applied to solve for the optimal precision levels and associated option values.

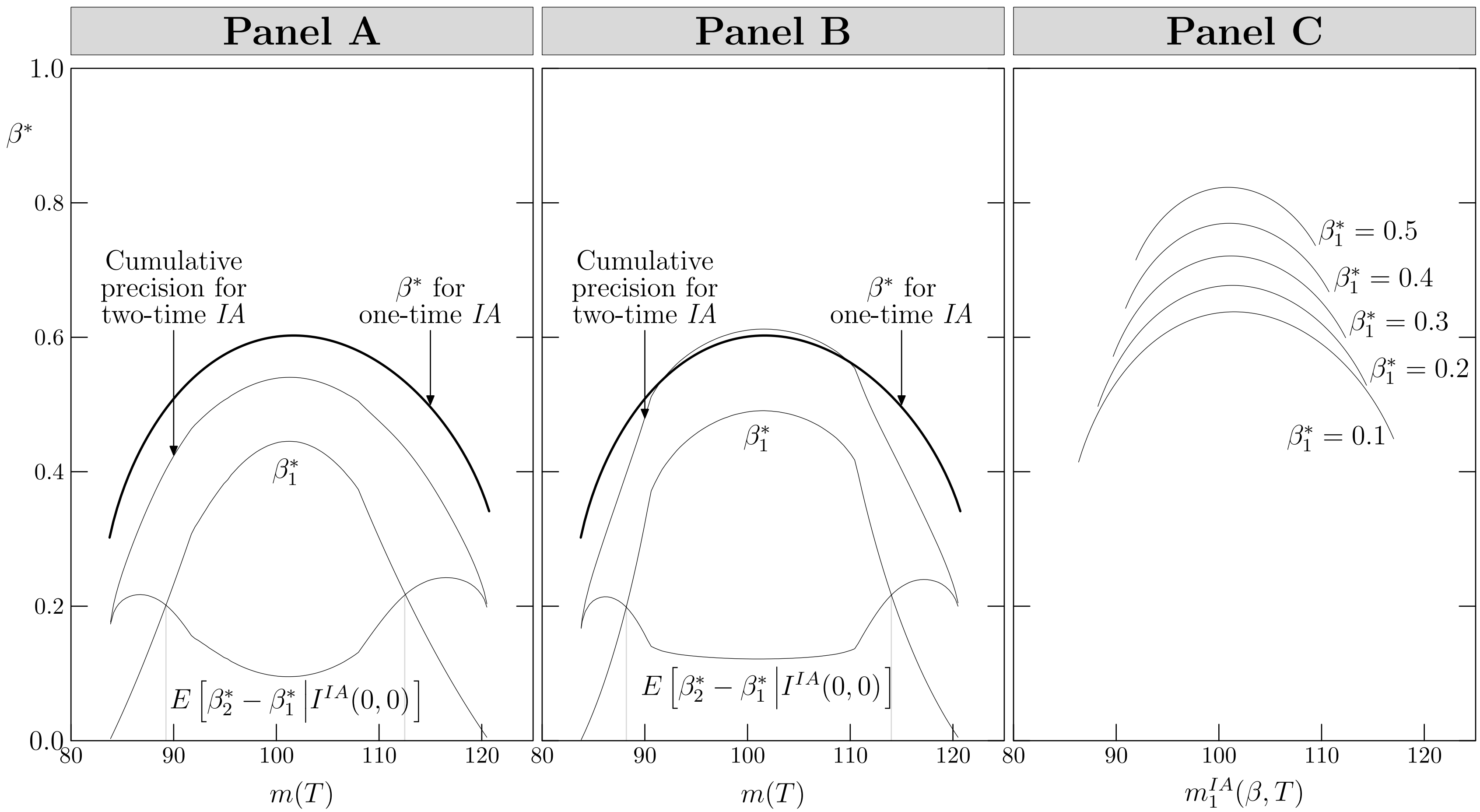

Figure 3. *IA* precision as a function of conditional expected value. This figure displays the optimal (both actual and expected) precision levels of IA as a function of the conditional expected value. In panels A and B the optimal precision level for the one-time case is shown. Additionally, the optimal first-time precision, expected incremental second-time precision, and the expected cumulative precision for the two-time case are shown. In Panel C, the optimal cumulative second-time precision is shown for various levels of first-time precision. In Panel A, $\theta = 0$, while in Panels B and C, $\theta = 1$. Parameter values common to all figures are $r = 0.05$, $Z(0) = 125$, $F = 100$, $\sigma_x = 0.30$, $\sigma_{yo} = 0.10$ $T = 5$, $\alpha = 2$, and $\eta = 5$.

duplicated upon second-time IA). In this case, the first round of IA is more intense than when $\theta = 0$, but still less intense than the one-time case. Moreover, because it is not as costly to break up IA into two parts, the equityholder is expected to do more IA in total (including more first-time IA) as compared to the $\theta = 0$ case. In fact, when the benefits to IA are highest (i.e, for $m_0(T)$ near F), the cumulative IA levels for the two-time case are greater than the optimal level for the one-time case. These results occur because expected net benefits from second round IA are higher in the $\theta = 1$ case due to lower second round costs. Thus, total net and marginal benefits from a first round of IA are also higher, creating an incentive to initially acquire more information than when $\theta = 0$. Also note that the range over which first-round IA is expected to exceed incremental second-round IA is wider in the $\theta = 1$ case, due to the low cost of second-round IA feeding back to result in more intense first-round IA.

Figure 3C shows the amount of cumulative precision conditional on various levels of first-time precision when $\theta = 1$.[21] Note that second-time IA occurs at high levels over a narrow range of conditional expected values centered near $m_0(T) = F$ again illustrating the advantage of multiple-opportunity monitoring (i.e., to have lower initial precision while retaining the option to pursue aggressive IA the second-time only when it is necessary). Also note that second-round IA occurs over increasingly narrower ranges and that the incremental level of IA (i.e., $\beta_2^* - \beta_1^*$) decreases as first-round IA is more intense. This follows because second-round IA is relatively more expensive at higher initial IA levels, to result in diminishing marginal returns to increasing levels of IA are realized.

Finally, it is again worth noting that the debtholder and lender will anticipate the benefits of IA and build that into the debt price at issuance. For the general problem where

[21] In determining second-time optimal precision, β_1 has already been chosen. Thus, the only difference in the choice of β_2^* for $\theta = 0$ versus $\theta = 1$ is in the cost function. Since β_1 is fixed at the time β_2 is chosen, the optimal second-time precision levels are the same for $\theta = 1$ and $\theta = 0$, but the range over which a second round of IA is optimal is smaller when $\theta = 0$.

information can be acquired n times, the expected incremental benefits for the ith instance of IA is

$$\zeta_i(Z(0)) =$$
$$\int\limits_{R_1}\int\limits_{R_2}\cdots\int\limits_{R_i} B(m_{i-1}^{IA}(\beta_{i-1}^*,T),\beta_i^*-\beta_{i-1}^*,T)\prod_{j=0}^{i} g\left(m_j^{IA}(\beta_j^*,T)\,\Big|\,I_j^{IA}(\beta_j^*,T)\right) dm_{i-1}^{IA}\cdots dm_0^{IA},$$

where

$$R_i = \left\{m_{i-1}\,\Big|\,B(m_{i-1}^{IA}(\beta_{i-1}^*,T),\beta_i^*-\beta_{i-1}^*) - C(\beta_{i-1}^*,\beta_i^*) > 0\right\},$$

is the optimal IA region for the ith time information is acquired, g is the lognormal density function and $\beta_0^* = 0$. The value of the debt with n IA opportunities is

$$D_n = D_0 - e^{-rT}\sum_{i=1}^{n}\zeta_i(Z(0)).$$

which is the debt value without IA less the present value of the benefits gained from multiple IA opportunities.

IV Further Implications and Extensions to the Model

A Information Sharing Incentives and Forced Liquidation Regulation

We have previously argued that borrowers will have incentives to agree to share asset value information that is acquired in the determination of a default option exercise decision. When information is acquired and default on the debt is realized, the lender will be better informed than outsiders as to the true asset value. This information asymmetry leads us to ask, will the lender have an incentive to freely share its information with outsiders? Or, stated differently, will private incentives exist for financial intermediaries to increase the transparency of their distressed asset portfolios?

The answer to the information sharing question partially depends on whether the intermediary is better off retaining ownership of the asset or liquidating it. If the asset is non-specific and if liquidity needs are not overriding, the lender can simply choose to hold the asset. In this case information sharing considerations are less immediate. However liquidity needs may overwhelm asset retention intentions so that immediate liquidation is often preferred (for social as well as private reasons) on assets obtained as a result of borrower default.

When bargaining and asset sales occur under 'steady state' market conditions, transaction cost minimization or unraveling arguments can be applied, suggesting that private incentives will exist to truthfully share information in order to accomplish liquidation objectives. Moreover, when asset and loan sales are anticipated to occur frequently in the course of normal business operations, reputational considerations provide additional incentives to truthfully disseminate information to outsiders (Mester (1992)). It can therefore be argued that transparency is incentive compatible and hence sustainable in equilibrium.

Steady state arguments ignore the disruptive effects of systemic risk, however. Although infrequent and often unpredictable, systematic waves of loan default and bankruptcy do occur, and in the extreme can tip the financial intermediary (and the entire financial system) towards financial distress. Systemic financial distress, in turn, can compromise steady state equilibria and result in incentives for financial intermediaries to undertake inefficient actions in the near term. In particular, in order to increase its chances for economic survival, there may be incentives for the distressed financial intermediary to attempt to lever its informational advantage and discretion to retain or sell assets against outside investors. These incentives increase opacity and illiquidity at a time when transparency and liquidity are most needed.

As a consequence, there may exist a role for market intervention to counteract the tendency towards institutional opacity in cyclical downturns. This may explain regulation in the US that limits bank holding periods on real assets acquired from loan default and bankruptcy.[22] By limiting discretion in the asset retention/liquidation decision, the short-term value of transparency increases to restore incentives to truthfully share information on acquired assets. It is important to note that this outcome is accomplished without the requirement of financial disclosure regulation per se, since voluntary disclosure by the financial intermediary is the spontaneous result of liquidation requirements.

This argument, with its emphasis on systemic risk and transparency in financial institutions, rationalizes Glass-Steagall style legislation and other scope-reducing regulation that emerged from the depression era in the US. It is also relevant to difficulties encountered in Japan and certain other East Asian countries with prominent banking systems, but without explicit liquidation regulation. Facilitating a transition from exchange relationships that are primarily bilateral/specific and for which opacity has value (e.g., Rajan (1992)) to ones that are multilateral/non-specific and for which transparency is advantageous has been a crucial policy issue in these countries (see, e.g., Rajan and Zingales (1999) for further discussion). In contrast, a pre-announced and credibly executed policy of forced loan and asset liquidation by the Resolution Trust Corporation (RTC) significantly shortened the US banking/S&L crisis of the late 1980's and early 1990's, keeping clean-up costs from spiraling out of control.[23] A policy of speedy liquidation also contributed to the development of an asset-backed securities market for commercial real estate loans.

[22]Federal bank regulation limits the holding period for real assets acquired as a result of foreclosure or bankruptcy to approximately one year. However, reserve requirements steadily increase from the time a loan is initially classified as non-performing to provide a strong incentive to liquidate earlier than the stated deadline. Individual states often provide an additional layer of regulation that further limits the period over which a bank can assume an equity position in an acquired asset.

[23]In addition to creating incentives to share information, asset specifities were also a factor in this policy choice of forced liquidation. Although many of the most productive owners/operators were sidelined due to the cyclical downturn in commercial real estate, it is probably safe to conclude that the RTC was not the second or even third-most productive owner/operator of problem assets. Forced liquidation regulation in this case was attractive because it increased the rate at which assets passed into the control of those who could more productively operate them.

B IA Policy for Other Option Types

In our analysis of the risky discount debt contract, the default option is effectively European since no dividends are paid on the asset and because no action (payment or default) is required until the debt payoff date. As a consequence, it is optimal to wait until the scheduled debt payoff date to acquire additional information as to the true underlying asset price. Many options, such as coupon debt, are compound as opposed to European in the sense that a cash payment must be made to keep the option alive for another period. Compound options on noisy real assets require a dynamic *IA* policy. This policy will have characteristics similar to the multiple-opportunity *IA* analysis undertaken previously, with the additional concerns of accounting for interim cash flows and the time value of money when calculating the value of delay. We would expect intermediate *IA* ranges to be relatively narrow (when interim cash flows are small as compared to the final payoff) and depend on the time to maturity.

Perpetual options arise in project development and investment. Prior to investment there is often a considerable amount of uncertainty as to the precise location of the product demand curve. However, once investment occurs and the revenue stream begins to be realized, residual uncertainty as to product demand is more or less resolved. The investment option in this case is a perpetual American call on a dividend-paying asset whose exercise price is the initial cost of investment. In contrast to the compound option case discussed above in which an explicit payment is required to keep the option alive, in this case the periodic dividend flow resulting from investment (whose value is uncertain prior to investment) is an opportunity cost of delaying investment for another time period.

Because the costs to fully develop new products are often quite substantial as well as irreversible, there are obvious incentives to conduct lower-cost experiments or undertake related *IA* activities in an effort resolve residual uncertainty as to the actual product value. In general, there will often exist opportunities to conduct a series of experiments prior to determining whether full product investment is warranted or not. It is interesting to note that, even in the case of a one-time only *IA* opportunity, the investor will often acquire information at expected asset values below (but typically in the neighborhood of) the investment hurdle value. When the timing of option exercise is discretionary, acquiring information at expected asset values somewhat below the hurdle value helps reduce the probability of waiting too long to invest (i.e., noise hinders recognition that the investment option may be well into the money) as well as increases the accuracy of the asset value estimate when investment is delayed further. *IA* policy of this type is also consistent with findings derived from a search model approach to R&D like Roberts and Weitzman (1981).

Product market structure can effect private incentives to acquire and disseminate information as well as the social value of that information. For example, Admati and Pfleiderer (1998) find that, from a social standpoint, individual firms may have an incentive to under-disclose private information when disclosure also results in more precise competitor valuations. How product price information is correlated and communicated can have significant real effects in the context of innovation and competitive new product market development. Consider, for example, an emerging market in a homogeneous good in which lead investment fully reveals the product demand curve. As described earlier, the inability to hinder communication or otherwise internalize the information externality from lead investment results in inefficient delay. Underinvestment incentives, in turn, suggest a (perhaps public) role in the creation of pre-investment *IA* mechanisms that can used to mitigate the first-mover disadvantage and to result in more accurately timed investment decisions.

V Summary

Many real assets are infrequently traded so that asset values cannot be continuously and precisely observed. If the value of the asset that underlies the contingent claim is not known with certainty, both the valuation and any exercise decision must be made with an imperfect (noisy) estimate of real asset value. We model the value of underlying assets as partially obscured by noise. Optimal filtering techniques are used to determine distributional parameters for the conditional expected asset value when the noise and the underlying asset value dynamics follow normal, lognormal and mean-reverting processes. We also examine the effects of two types of noise: an initial level of noise present when the underlying asset value is originally observed or estimated, and a dynamic process that accumulates noise after the initial observation. Because noise hinders the claimholder's ability to make quality exercise decisions, there is an incentive to more precisely determine asset value by acquiring costly information. We determine optimal information acquisition policy for the case of a borrower who holds the default (put) option inherent in risky discount debt.

We find that when a costly information acquisition technology does not exist, the initial level of noise volatility does not affect option value or exercise policy, while noise that accumulates results in lower option values. In contrast, when costly information acquisition technology is available, any noise that can be resolved is important in determining claim value and exercise policy. In our debt example, information will be acquired to reduce potential errors in exercise policy at the debt payoff date when the technology is sufficiently inexpensive and the conditional expected asset value is sufficiently close to the face value of the debt. When asset volatility is greater than the accumulating noise volatility, the optimal level of information acquisition is most sensitive to noise volatility. Initial noise volatility has the greatest impact for short-lived claims, while accumulating noise volatility has the greatest impact for longer-lived claims. When there are multiple opportunities to gather information, it is optimal to acquire information in smaller increments to reduce the potential of ex-post overinvestment and underinvestment in information acquisition. Nevertheless, the cumulative level of information acquisition is often higher relative to the case when information can only be gathered once. Finally, we find that there are incentives to share information in order to increase market liquidity and to reduce the costs of information acquisition.

Appendix

This appendix contains the derivations. It will be convenient to suppress some of the notation. So, for example, $d_i = d_i\left(\frac{m}{F}, \sqrt{\beta}s\right)$ where $s = \sqrt{\rho^2\sigma_y^2 T + \sigma_{yo}^2}$. MB and MC are the marginal benefits and marginal cost functions with arguments suppressed. In addition, a starred superscript signifies that these functions are evaluated using the optimal precision, β^* (e.g., d_i^*, MB^* and MC^*).

The partials for optimal precision are

$$\frac{\partial\beta^*}{\partial\sigma_y^2} = \frac{\rho^4\beta^* T\left(1 + d_1^* d_2^*\right)}{s^2\left(2\alpha - \left(1 + d_1^* d_2^*\right)\right)}, \tag{29}$$

$$\frac{\partial\beta^*}{\partial\sigma_{yo}^2} = \frac{\beta^*\left(1 + d_1^* d_2^*\right)}{s^2\left(2\alpha - \left(1 + d_1^* d_2^*\right)\right)}, \tag{30}$$

$$\frac{\partial\beta^*}{\partial\sigma_x^2} = \frac{(1-\rho^2)^2\beta^* T\left(1 + d_1^* d_2^*\right)}{s^2\left(2\alpha - \left(1 + d_1^* d_2^*\right)\right)}, \tag{31}$$

$$\frac{\partial\beta^*}{\partial T} = \frac{\beta^*\rho^2\sigma_y^2\left(1 + d_1^* d_2^*\right)}{s^2\left(2\alpha - \left(1 + d_1^* d_2^*\right)\right)}, \tag{32}$$

$$\frac{\partial\beta^*}{\partial m} = \frac{-2\sqrt{\beta^*}d_2^*}{ms\left(2\alpha - \left(1 + d_1^* d_2^*\right)\right)}, \tag{33}$$

$$\frac{\partial\beta^*}{\partial F} = \frac{2\sqrt{\beta^*}d_1^*}{Fs\left(2\alpha - \left(1 + d_1^* d_2^*\right)\right)}. \tag{34}$$

Proof: The derivation for Equations (29)-(31) is below. Derivations for Equations (32)-(34) are similar and are left to the reader.

The partials of total standard deviation are

$$\frac{\partial s}{\partial\sigma_{yo}^2} = \frac{1}{2s}, \tag{35}$$

$$\frac{\partial s}{\partial\sigma_y^2} = \frac{\rho^4 T}{2s}, \tag{36}$$

$$\frac{\partial s}{\partial\sigma_x^2} = \frac{\left(1-\rho^2\right)^2 T}{2s}. \tag{37}$$

The partial of d_2^* with respect to variance is (where σ^2 is σ_{yo}^2, σ_y^2 or σ_x^2)

$$\begin{aligned}\frac{\partial d_2^*}{\partial \sigma^2} &= -\frac{\partial s}{\partial \sigma^2}\left(\frac{\ln\frac{\xi}{F}}{\beta^{*\frac{1}{2}}s^2}+\frac{1}{2}\beta^{*\frac{1}{2}}\right)-\frac{1}{2}\frac{\partial \beta^*}{\partial \sigma^2}\left(\frac{\ln\frac{\xi}{F}}{\beta^{*\frac{3}{2}}s}+\frac{1}{2}\frac{s}{\beta^{*\frac{1}{2}}}\right)\\ &= -\frac{d_1^*}{s}\frac{\partial s}{\partial \sigma^2}-\frac{d_1^*}{2\beta^*}\frac{\partial \beta^*}{\partial \sigma^2}. \end{aligned} \tag{38}$$

The optimal precision is defined implicitly by where marginal benefits equal marginal costs (for $\beta^* \in (0,1)$)

$$\frac{sFn(d_2^*)}{2\beta^{*\frac{1}{2}}} = \alpha\gamma\beta^{*(\alpha-1)}. \tag{39}$$

Implicit differentiation can be used to find the partials of optimal precision. First, take the partial of Equation (39) with respect to variance.

$$\frac{Fn(d_2^*)}{2\beta^{*\frac{1}{2}}}\frac{\partial s}{\partial \sigma^2}-\frac{sFn(d_2^*)}{4\beta^{*\frac{3}{2}}}\frac{\partial \beta^*}{\partial \sigma^2}-\frac{sd_2^*Fn(d_2^*)}{2\beta^{*\frac{1}{2}}}\frac{\partial d_2^*}{\partial \sigma^2}=\alpha\gamma(\alpha-1)\beta^{*(\alpha-2)}\frac{\partial \beta^*}{\partial \sigma^2}.$$

Next, substitute in MB^*, MC^*, and Equation (38).

$$\frac{MB^*}{s}\frac{\partial s}{\partial \sigma^2}-\frac{MB^*}{2\beta^*}\frac{\partial \beta^*}{\partial \sigma^2}+d_2^*MB^*\left(\frac{d_1^*}{s}\frac{\partial s}{\partial \sigma^2}+\frac{d_1^*}{2\beta^*}\frac{\partial \beta^*}{\partial \sigma^2}\right)=\frac{\alpha-1}{\beta^*}MC^*\frac{\partial \beta^*}{\partial \sigma^2}.$$

Finally, recognize that $MB^* = MC^*$, multiply through by $\frac{2\beta^*}{MB^*}$ and collect terms

$$\left((1-d_1^*d_2^*)+2(\alpha-1)\right)\frac{\partial \beta^*}{\partial \sigma^2}=\frac{2\beta^*\left(1+d_1^*d_2^*\right)}{s}\frac{\partial s}{\partial \sigma^2}.$$

This is easily solved for $\frac{\partial \beta^*}{\partial \sigma^2}$

$$\frac{\partial \beta^*}{\partial \sigma^2}=\frac{\partial s}{\partial \sigma^2}\frac{2\beta^*\left(1+d_1^*d_2^*\right)}{s\left(2\alpha-(1+d_1^*d_2^*)\right)}. \tag{40}$$

Combining Equation (40) with Equations (35) - (37) provides the partial of optimal precision level with respect to the three variance rates, σ_{yo}^2, σ_y^2 and σ_x^2 □

Derivation of Equations (24)-(26):

Equations (24)-(26) are a direct result of Equation (40) and Equations (35) - (37). □

Proposition: Let m_{max} be the conditional expected value which produces the highest level of information acquisition (i.e., $\beta^*(m_{max}) \geq \beta^*(m), \forall m \in (0, \infty)$). If $0 < \beta^*(m_{max}) < 1$, then $m_{max} > F$.

Proof: If $0 < \beta^*(m_{max}) < 1$, then $\left.\frac{\partial \beta^*}{\partial m}\right|_{m=m_{max}} = 0$. This partial is zero only if

$$d_2\left(\frac{m_{max}}{F}, \sqrt{\beta^*(m_{max})}s^2T\right) = 0.$$

Rearranging yields

$$m_{max} = Fe^{\frac{1}{2}\beta^*(m_{max})s^2T}.$$

If $\beta^*(m_{max}) > 0$ then $s^2T > 0$ and $m_{max} > F$. □

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