Rival and Strategic Options in a Market Sharing Duopoly

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Highlights:

Decomposition of Value Functions

Market Share Sensitivities Over Three Regimes

Market Share Partial Derivatives Over Three Regimes

Analytical Solutions for Some Strategic Option Derivatives

Consequences of Increasing Market Share

Abstract

We build on previous solutions for mutually exclusive options in a duopoly with switching and divestment alternatives. We study the likely implications for increasing the leader's market share MS at successive levels of market revenue. The conventional net present value NPV thresholds for switching and divestments, ignoring rival and strategic options, are likely to be a misleading basis for making MS decisions. The consequences of MS changes on the values for both the leader and follower are often surprising. The NPV of operations for the leader is reduced by increased MS when revenue is low, with a further negative change of value in the net switch and divest option values. The NPV of operations is increased for the leader by increased MS when revenue is higher, reduced by the decrease in value of the leader's rival option value. Those strategy results are consistent with the sign and dimension of MS partial derivatives, with some novel analytical solutions.

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1. Introduction

Should the leader or follower in a duopoly attempt to increase market share when revenue is marginally less than operating cost? (i) Perhaps not if using a net present value approach, but there could be unintended consequences if rival and strategic option values are considered. (ii) What happens when revenue exceeds operating cost, but is less than the level that justifies the follower switching to lower cost technologies? (iii) What is the appropriate action in the initial regimes, for anticipating altering market share in the middle and final regimes, or in the middle regime, for anticipating altering market share in the final regime? (iv) How can competitors affect the value (and exercise) of rival options?

These are the critical questions we address in studying the real options when there are mutually exclusive **strategic** options (divest or switch to a lower cost technology) for a leader/follower in a market sharing duopoly. Following Adkins et al. (2022) we assume in the duopoly the market share is always divided only between a leader and a follower, and varies from an initial stage (or regime) to a middle stage (when the follower obtains a larger market share than initially), then to a final stage.

The first-mover advantage for the leader is dependent on only obtaining full salvage value; the second-mover follower is not immediately motivated to adopt the cost reduction technology in the second regime. But the second-mover is motivated eventually to adopt the new technology (but with a changed market share) as market revenue increases. So, there are issues about market sharing duopolies, initial and subsequent market shares, and mutually exclusive options (exercising one option cancels the other option). Also, there are examples of **rival** options (firms benefit from rivals exercising their strategic options).

There are many duopolies (or local, national near duopolies) that could be illustrations of our context. Pindyck and Rubinfeld (2018) note that Facebook faced competition from Google Plus which failed to build sufficient network externality. Airbus and Boeing, Coke and Pepsi, Uber and Lyft (or currently Twitter and Mastodon/Cohost) all have elements of duopoly with varying market shares over time, and both have strategic and rival options with marketing, technology or logistic advances.

There are several classical arguments that greater market share in a duopoly or oligopoly leads to greater profits. The frequently cited Buzzell et al. (1975) argues that a larger market share is a key to profitability, which Leontiades (1984) extends to a global context. Roberts (2003) provides proprietary evidence that increasing market share during a recession provides a competitive advantage for the leader in market upturns. These approaches appear to ignore the rival and strategic options accompanying market share rivalry over market cycles.

Kulatilaka and Perotti (1998) view the strategy of a first mover in a duopoly in terms of pre-empting (or discouraging) an entrant by investing, where there is an inverse demand function. Their discussion of "growth options" does not lead to exactly showing the value of such an anti-rival option, or how it is affected by varying market share. Paxson and Pinto (2003) focus on the partial derivatives of the value function for the leader/follower with respect to changes in the market share, market revenue and volatility, with several unusual patterns, along with some analytical expressions for deltas and vegas. Paxson and Pinto (2005) show the partial derivatives of the value function for the leader/follower in both preemptive and non-preemptive games with respect to changes in market revenue, changing as revenue approaches the thresholds. Kong and Kwok (2007) provide standard entry thresholds for leader/follower when asymmetric in investment cost and revenue, with real option values not separately disclosed. Dias and Teixeira (2010) focus on the entry of a leader/follower with symmetric/asymmetric costs, and covering several game strategies. Azevedo and Paxson (2014) review duopoly "exit options" and other "market sharing" articles.

Joaquin and Butler (2000) assume a first mover leader advantage of lower operating costs. Tsekrekos (2003) allows for both temporary and pre-emptive permanent market share advantages for the leader. In Paxson and Pinto (2003) a leader has an initial market share advantage, which changes as new customers arrive and existing customers depart. Paxson and Melmane (2009) assume the leader starts with a larger market share, which then follows a random process. Bobtcheff and Mariotti (2013) look at a pre-emptive game of two competitors, with the leadership revealed only by a first mover investment.

Bensoussan et al. (2017) study a duopoly with the possibility of regime switching. There are two investment entries for two states (good and bad {low growth, high volatility}), with the leader having 100% of the market when investing early, 50% when the follower enters, otherwise apparently symmetric firms. The solution is obtained by using the variational inequality approach. There is a non-smooth reward function for the leader at the point of the follower's entry. There are eight thresholds (two for the follower) and a simultaneous solution of 8 nonlinear equations. There is a sensitivity analysis only of the thresholds under different regimes for changes in volatility, drift and investment cost, not market shares. Balliauw et al. (2019) is an empirical work on the investment thresholds of leader/follower ports with capacity choices, without identifying the precise real option values.

Dias (2004) provides solutions for mutually exclusive options using finite differences. Décamps et al. (2006) show that when firms hold the option to switch scales, a hysteresis region between the investment region can persist even if there is uncertainty. Bobtcheff and Villeneuve (2010) conclude that uncertainties imply that payoffs are not sufficient criteria for deciding on the investment timing for mutually exclusive projects. Siddiqui and Fleten (2010) provide numerical solutions for mutually exclusive projects. There are several other applications of the theory of mutually exclusive options, such as Bakke et al. (2016), which do not develop separate valuations for the rival and strategic options.

Hagspiel et al. (2016) show that a higher potential profitability of a product market accelerates the investment timing, especially if the choice of the investment capacity is smaller, reversing an intuitive result. Huberts et al. (2019) examine interesting strategies where entry by competitors may be deterred, possibly in a war of attrition or pre-emption. Adkins and Paxson (2019) propose appropriate rescaling from an incumbent large-scale technology assuming that market revenue is declining, considering the investments both separately and jointly. Adkins et al. (2022) provide analytical and numerical solutions for the rival and strategic mutually exclusive options in a duopoly.

How important are rival and strategic options in joint formulation compared to the conventional net present value evaluation (opPV) (without options)? As a preview, with the assumed parameter values, the leader's divest joint threshold is 43% of the NPV threshold, the switch joint is 14% of the NPV threshold[2]. The follower's divest threshold is 56% of the NPV threshold, the switch joint threshold is 19% of the NPV thresholds. In the initial case, at v=5 between the leader's divest and switching thresholds, the leader's options amount to 39.9, the opPV=-14.3. In the middle case ($v = 7$) between the leader's and follower's switch thresholds, the joint value of the leader is 101% of the NPV value, but the follower's options amount to 26.6 compared to an opPV of 0. An analyst or manager looking at the effect of changing initial, middle or final market share on the value of the firms focusing just on operating PV is likely to be severely myopic.

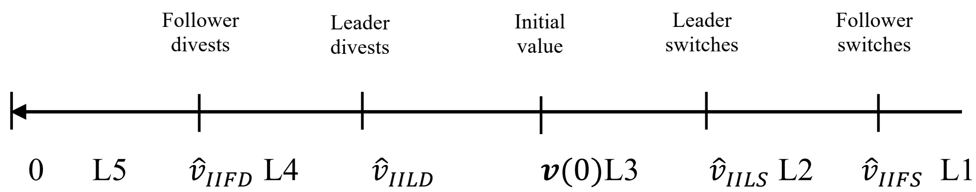
2. Joint Formulation

We assume that there is a duopoly of symmetric operating firms, except the leader has an advantage of obtaining full value Z in any divestment of the existing operating facility, while the follower obtains lZ, where 0<l<1. The follower obtains a larger market share (60%) after the leader has switched to a lower

[2] The NPV methodology and calculations are shown in Appendix A, subscript I indicates NPV, II Joint for the thresholds and option coefficients. L/XX is the leader's market share in regime 3 (line 3), L/YX is the leader's market share in regime 2 after the leader has switched, L/YY is the leader's market share in regime 1 (line 1) after both have switched, F/0X=1 is the follower's market share after the leader divests. A_1 is the option coefficient for switching when v has increased, A_2 the option coefficient for divestment when v has decreased, $\beta_{1,2} = \left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$ are the positive/negative solutions for the quadratic equation assuming v follows a geometric Brownian motion with volatility s and drift r-d, where r is the riskless interest rate and d is the convenience yield. S, D denote the strategic options for when the firm switches/divests, SS, DD denote the rival options for when the rival switches/divests with the option values. The strategic options are sometimes indicated as the option coefficients times v^{b1} or v^{b2}, such as SO FD (A_{2IIFD} v^{b2}), SO LD, SO LS, SO FS, and the rival options as RO FDD, RO FSS, and RO LSS.

operating cost technology, policy Y. The order of divesting/switching thresholds divest $\{\hat{v}_{IIFD}, \hat{v}_{IILD}\}, switch\ \{\hat{v}_{IILS}\ and\ \hat{v}_{IIFS}\}$ is indicated in Figure 1. Total market revenue “v” follows a geometric Brownian motion with constant (negative) drift and volatility[3]. Each firm holds the option to divest and receive a salvage value from the initial X stage. Once the divestment option is exercised, the firm exits the market which is referred to as policy O. Since Y is the more cost efficient, the full-market operating cost $f_X > f_Y$. There is no salvage value after firms switch to policy Y. The two players in the duopoly game are designated the leader and the follower, referred to as L and F, respectively. We treat the two firms as being ex-ante symmetric, which implies that each firm has 50% of the market provided that the two firms are pursuing identical policies, so: $D_{L|X,X} = 1 - D_{F|X,X}$.

Figure 1: Leader and Follower Thresholds for a Random Revenue (v) under the Joint Formulation



The value function under the joint formulation for the leader is denoted by $V_{IIL}(v)$.

$$V_{IIL}(v) = \begin{cases} D_{L|Y,Y}\left(\frac{v}{\delta+\theta} - \frac{f_Y}{r}\right) & if\ v \geq \hat{v}_{IIFS}\ L1 \\ D_{L|Y,X}\left(\frac{v}{\delta+\theta} - \frac{f_Y}{r}\right) + A_{1IILSS}v^{\beta_1} & if\ \hat{v}_{IILS} \leq v < \hat{v}_{IIFS}\ L2 \\ D_{L|X,X}\left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + A_{1IILS}v^{\beta_1} + A_{2IILD}v^{\beta_2} & if\ \hat{v}_{IILD} < v < \hat{v}_{IILS}\ L3 \\ Z & if\ v \leq \hat{v}_{IILD}\ L4 \end{cases} \tag{1}$$

In (1), the first line (regime 1) represents the expected present value of the leader’s net revenue opPV once the follower has switched, with no further options; the second line represents the expected present value of leader’s net revenue plus the value for the leader of the optional switching by the follower, denoted by $A_{1IILSS}v^{\beta_1}$; the third line represents the expected present value of leader’s net revenue plus

[3] These are also the assumptions in Adkins et al. (2022), along with the derived solutions described in detail in Appendix B. There are many other possible configurations, with different consequences.

the option values to switch, $A_{1IILS}v^{\beta_1} > 0$ and to divest, $A_{2IILD}v^{\beta_2} > 0$; the fourth line represents the leader's receipt from divestment.

The value function under the joint formulation for the follower is denoted by $V_{IIF}(v)$.

$$V_{IIF}(v) = \begin{cases} D_{F|Y,Y}\left(\frac{v}{\delta+\theta} - \frac{f_Y}{r}\right) & if\ v \geq \hat{v}_{IIFS} L1 \\ D_{F|Y,X}\left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + A_{1IIFS}v^{\beta_1} + A_{2IIFD}v^{\beta_2} & if\ \hat{v}_{IILS} \leq v < \hat{v}_{IIFS} L2 \\ \quad D_{F|X,X}\left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + A_{1IIFS}v^{\beta_1} + A_{2IIFD}v^{\beta_2} & \\ +A_{1IIFSS}v^{\beta_1} + A_{2IIFDD}v^{\beta_2} & if\ \hat{v}_{IILD} < v < \hat{v}_{IILS} L3 \\ D_{F|O,X}\left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + A_{1IIFS}v^{\beta_1} + A_{2IIFD}v^{\beta_2} & if\ \hat{v}_{IIFD} \leq v < \hat{v}_{IILD} L4 \\ \lambda Z & if\ v < \hat{v}_{IIFD} L5 \end{cases} \tag{2}$$

In (2), the first line represents the expected present value of follower's net revenue opPV once the follower has switched; the second line represents the expected present value of follower's net revenue plus the option values to switch, $A_{1IIFS}v^{\beta_1} > 0$ and to divest, $A_{2IIFD}v^{\beta_2} > 0$; the third line represents the expected present value of follower's net revenue plus the option values to switch, $A_{1IIFS}v^{\beta_1}$, and to divest, $A_{2IIFD}v^{\beta_2}$, and the values accruing to the follower when the leader exercises the switching option, $A_{1IIFSS}v^{\beta_1}$, or the divestment option, $A_{2IIFDD}v^{\beta_2} < 0$; the fourth line represents the expected present value of follower's net revenue plus the option values to switch, $A_{1IIFS}v^{\beta_1}$, and to divest, $A_{2IIFD}v^{\beta_2}$; the fifth line represents the follower's value on divestment.

The boundary conditions in the thresholds (value matching and smooth pasting) along with value functions (1) and (2) create a set of four equations from which the solutions to the unknown thresholds $\hat{v}_{IILS}$, $\hat{v}_{IIFS}$, $\hat{v}_{IILD}$, and $\hat{v}_{IIFD}$, are obtainable. There are four unknown strategic option coefficients associated with the leader's and follower's switching and divesting policies, A_{1IILS}, A_{2IILD}, A_{1IIFS}, and A_{2IIFD}, respectively, and three unknown coefficients associated with the rival options A_{1IILSS}, A_{1IIFSS}, and A_{2IIFDD}, which benefit or hurt the option holder when the rival chooses to switch or divest. We can obtain[4] solutions for the follower's two thresholds $\hat{v}_{IIFS}$ and $\hat{v}_{IIFD}$ from the non-linear simultaneous equations:

$$\hat{v}_{IIFD}{}^{\beta_2}\left(\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}-\frac{D_{F|Y,Y}f_Y-D_{F|Y,X}f_X}{r}-(K-\lambda Z)\right)-\hat{v}_{IIFS}{}^{\beta_2}\left(\lambda Z-\frac{D_{F|O,X}\hat{v}_{IIFD}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}+\frac{D_{F|O,X}f_X}{r}\right)=0 \quad (3)$$

$$\hat{v}_{IIFD}{}^{\beta_1}\left(\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}-\frac{D_{F|Y,Y}f_Y-D_{F|Y,X}f_X}{r}-(K-\lambda Z)\right)-\hat{v}_{IIFS}{}^{\beta_1}\left(\lambda Z-\frac{D_{F|O,X}\hat{v}_{IIFD}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}+\frac{D_{F|O,X}f_X}{r}\right)=0 \quad (4)$$

Note that these thresholds are affected only by the middle and final market shares, and the market share, assumed to be one, of the follower if the leader divests, as well as by changes in the other parameter values indicated in Table 1.

We can obtain solutions for the leader's two thresholds $\hat{v}_{IILS}$ and $\hat{v}_{IILD}$ from the non-linear simultaneous equations:

$$\hat{v}_{IILD}{}^{\beta_2}\left(\hat{v}_{IILS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}-\frac{D_{L|Y,X}f_Y-D_{L|X,X}f_X}{r}\right)-(K-Z)-\hat{v}_{IILS}{}^{\beta_2}\left(Z-\frac{D_{L|X,X}\hat{v}_{IILD}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}+\frac{D_{L|X,X}f_X}{r}\right)=0 \quad (5)$$

$$\hat{v}_{IILD}{}^{\beta_1}\left(\hat{v}_{IILS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}-\frac{D_{L|Y,X}f_Y-D_{L|X,X}f_X}{r}+A_{1IILS}\,\hat{v}_{IILS}{}^{\beta_1}\frac{\beta_2-\beta_1}{\beta_2}-(K-Z)\right)-\hat{v}_{IILS}{}^{\beta_1}\left(Z-\frac{D_{L|X,X}\hat{v}_{IILD}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}+\frac{D_{L|X,X}f_X}{r}\right)=0 \quad (6)$$

Note that these thresholds are affected by the initial and middle market shares, not by the final market share (except for A_{1IILS}) as well as by changes in the other parameter values in Table 1.

[4] Spreadsheets for the solutions for the NPV version (without options) and the joint formulation are available in the Supplementary Appendix A and B.

The follower's strategic switching and divestment option coefficients are:

$$A_{1IIFS} = \frac{1}{\beta_1 \Delta_F}\left(\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\hat{v}_{IIFD}{}^{\beta_2} + \hat{v}_{IIFD}\frac{D_{F|O,X}}{\delta+\theta}\hat{v}_{IIFS}{}^{\beta_2}\right) \tag{7}$$

$$A_{2IIFD} = \frac{1}{\beta_2 \Delta_F}\left(-\hat{v}_{IIFS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\hat{v}_{IIFD}{}^{\beta_1} + \hat{v}_{IIFD}\frac{D_{F|O,X}}{\delta+\theta}\hat{v}_{IIFS}{}^{\beta_1}\right) \tag{8}$$

where

$$\Delta_F = \hat{v}_{IIFS}{}^{\beta_1}\hat{v}_{IIFD}{}^{\beta_2} - \hat{v}_{IIFS}{}^{\beta_2}\hat{v}_{IIFD}{}^{\beta_1}. \tag{B4}$$

Note that these two option coefficients are not sensitive directly to changes in $D_{L/XX}$, but only to the difference between the market shares in the final and middle stage, since it is assumed that $D_{F/OX}=1$. It is convenient that the initial F thresholds are not sensitive to changes in the initial market shares, and are not in (7) or (8). Both the threshold and coefficient insensitivities are confirmed in Table 4 of the sensitivities to changes in MS at the initial stage. Note that this analytical expression shows the relevance of considering both the middle and final market share for the SO FD value.

The follower's rival options (exercise determined by the leader, benefits the follower are:

$$A_{1IIFSS} = \left(D_{F|Y,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{IILS}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{IILD}{}^{\beta_2}}{\Delta_L} - \left(D_{F|O,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{IILD}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{IILS}{}^{\beta_2}}{\Delta_L} \tag{9}$$

$$A_{2IIFDD} = -\left(D_{F|Y,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{IILS}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{IILD}{}^{\beta_1}}{\Delta_L} + \left(D_{F|O,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{IILD}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{IILS}{}^{\beta_1}}{\Delta_L} \tag{10}$$

Note that these two option coefficients are insensitive *directly* to changes in $D_{L/YY}$, but to the initial stage F/XX and the difference between the market share in the initial and middle stage, assuming that $D_{F/0X}$ is one (that is if the leader divests, the follower has the whole market). The RO F SS increases with increases in the L's final market share in the total derivatives sensitivities table, which must be due to the L's threshold changes.

The leader's strategic switching and divestment option coefficients are:

$$A_{1IILS} = \frac{1}{\beta_1 \Delta_L}\left(\left(\hat{v}_{IILS}\frac{D_{L|Y,X} - D_{L|X,X}}{\delta+\theta} + \beta_1 A_{1IILSS}\hat{v}_{IILS}{}^{\beta_1}\right)\hat{v}_{IILD}{}^{\beta_2} + \hat{v}_{IILD}\frac{D_{L|X,X}}{\delta+\theta}\hat{v}_{IILS}{}^{\beta_2}\right) \quad (11)$$

$$A_{2IILD} = -\frac{1}{\beta_2 \Delta_L}\left(-\left(\hat{v}_{IILS}\frac{D_{L|Y,X} - D_{L|X,X}}{\delta+\theta} + \beta_1 A_{1IILSS}\hat{v}_{IILS}{}^{\beta_1}\right)\hat{v}_{IILD}{}^{\beta_1} - \hat{v}_{IILD}\frac{D_{L|X,X}}{\delta+\theta}\hat{v}_{IILS}{}^{\beta_1}\right) \quad (12)$$

where

$$\Delta_L = \hat{v}_{IILS}{}^{\beta_1}\hat{v}_{IILD}{}^{\beta_2} - \hat{v}_{IILS}{}^{\beta_2}\hat{v}_{IILD}{}^{\beta_1}. \quad \text{(B13)}$$

Note that these two option coefficients are insensitive *directly* to changes in $D_{L/YY}$, but to the initial stage L/XX and the difference between the market share in the initial and middle stage. The SO L S and SO L D increase/decrease with increases in the L's final market share in the total derivatives sensitivities table, which must be due to the L's threshold changes.

The leader's rival options (exercise determined by the follower, benefits the leader) is:

$$A_{1IILSS} = \left(\frac{\hat{v}_{IIFS}}{\delta+\theta} - \frac{f_Y}{r}\right)\left(D_{L|Y,Y} - D_{L|Y,X}\right)\hat{v}_{IIFS}{}^{-\beta_1} \quad (13)$$

Note that the option coefficient is insensitive *directly* to changes in $D_{L/XX}$ (as confirmed in the sensitivities Table 4), but only to the difference between the market shares in the final and middle stage. Note that this analytical expression shows the relevance of considering both the middle and final market share for the RO LSS value.

How could the leader encourage the follower to switch, that is move from the middle to final stage? One answer is perhaps by reducing the L MS in the final stage, and raising the L MS in the middle stage.

3. Numerical Evaluations

For the joint formulation there is a numerical solution for the thresholds 3-4-5-6, and analytical solutions for the option coefficients, 11-12-13 for the Leader, 7-8-9-10 for the Follower. From Table 1, the values of β_1 and β_2 for the base case are 1.667 and −1.333, respectively.

Table 1: Base Case Parameter Values

Definition	Notation	Value
Risk-free rate	r	0.10
Convenience yield	δ	0.03
Market depletion rate	θ	0.04
Market price volatility	σ	0.30
Follower's divestment proportion	λ	0.40
Unadjusted periodic operating cost for policy ***X***	f_X	10.0
Unadjusted periodic operating cost for policy ***Y***	f_Y	1.0
Divestment value	Z	25.0
Switching investment cost to policy ***Y***	K	32.0
Leader's market share given both leader and follower pursue policy ***X***	$D_{L\|X,X}$	0.50
Leader's market share given both leader and follower pursue policy ***Y***	$D_{L\|Y,Y}$	0.50
Leader's market share given leader pursues policy ***Y*** and follower policy ***X***	$D_{L\|Y,X}$	0.40
Leader's market share given leader exits and follower pursues policy ***X***	$D_{L\|O,X}$	0.00

Note: The follower's market shares for the various policy assortments are obtainable from the leader's market share.

3.1 Thresholds and Coefficients

Using the base case values in Table 1, we present the numerical solutions for the leader's and follower's various thresholds and coefficients in Table 2. The thresholds in the joint formulation are always less than those under the NPV formulation, $\hat{v}_{IILD} < \hat{v}_{ILD}$, $\hat{v}_{IILS} < \hat{v}_{ILS}$, $\hat{v}_{IIFD} < \hat{v}_{IFD}$, and $\hat{v}_{IIFS} < \hat{v}_{IFS}$. Also, the leader is the first-mover since $\hat{v}_{IIFD} < \hat{v}_{IILD} < \hat{v}_{IILS} < \hat{v}_{IIFS}$. We observe that while A_{2IILSS} and A_{2IIFSS} are both positive, A_{2IIFDD} is negative. This indicates that while the leader gains when the follower switches and the follower gains when the leader switches, the follower loses when the leader divests at a low v.

Table 2: Values for the Various Thresholds and Coefficients

	Leader		Follower	
DIVEST	$\hat{v}_{IILD}$	4.524	$\hat{v}_{IIFD}$	4.328
	A_{2IILD}	258.016	A_{2IIFD}	334.144
	$\hat{v}_{ILD}$	10.500	A_{2IIFDD}	-182.405
			$\hat{v}_{IFD}$	7.700
SWITCH	$\hat{v}_{IILS}$	6.948	$\hat{v}_{IIFS}$	10.206
	A_{1IILS}	0.6628	A_{1IIFS}	0.0693
	A_{1IILSS}	0.2828	A_{1IIFSS}	0.5409
	$\hat{v}_{ILS}$	48.580	$\hat{v}_{IFS}$	53.900

3.2 How Important are the Rival and Strategic Options?

The relative importance of the option values in the value function depends on the level of v relative to the thresholds, since we assume the options prevail only over specific regimes. We assume that if an option is exercised by the firm, or its rival, the option no longer exists. For the leader value function, between the divest and switch threshold, there are only options to divest and switch. After the leader switches, the leader then obtains the value of the rival option of the follower switching only if v increases up to the follower's switching threshold.

Table 3 shows the value of the operations F Op PV and LF Op PV and each of the seven options where appropriate, over a v range from above the follower's switching threshold (hypothetically) to below the follower's divest threshold. The operating PV increases as v increases, affected by the leader's market share, 50% in the initial and final stage, 40% in the middle stage between the switching thresholds 6.94 and 10.2 (v= {7 to 10}).

At the initial regime, between the leader's divest and switch thresholds (v= {5 to 6.5}) the leader holds an option to divest SO D and an option to switch SO S. The SO D is quite large when v is low, indeed larger than the negative Op PV. Over 6.9, the rival RO L SS increases in significance until the follower switches, when then the market share reverts to 50%.

A manager or analyst looking at the value of a follower when v=8, would be misled by relying on the op PV=8.6, ignoring the two options worth an additional 23.1. When v=6, the negative opPV of -7.1 is offset by four options worth 25.9 for the follower. A leader manager assessing her firm's value when v=6, as opPV=-7.1, would be ignoring real options worth 36.8.

Table 3 shows that the strategic and rival options are quite significant over certain regimes. While a firm probably cannot influence a rival exercising the option to divest or switch, "watch the competition" can be a critical consideration.

Table 3: Decomposition of the Value Functions as Revenue (v) Changes[5]

Follower's Value Function as Function of v, Across Regimes															
v	4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000	8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000
Regime	L5	L3	L2	L2	L2	L2	L2	L2	L2	L2	L2	L2	L2	L1	L1
F Value SUM	10.0000	10.1117	12.3823	15.3715	18.8627	22.7504	26.7282	29.0370	31.6721	34.5718	37.6892	40.9878	44.4391	48.0000	51.5714
F Op PV		-35.7143	-14.2857	-10.7143	-7.1429	-3.5714	0.0000	4.2857	8.5714	12.8571	17.1429	21.4286	25.7143	70.0000	73.5714
SO S		0.8496	1.0127	1.1871	1.3723	1.5682	1.7744	1.9906	2.2166	2.4523	2.6974	2.9518	3.2152		
SO D		44.9764	39.0818	34.4179	30.6478	27.5454	24.9538	22.7607	20.8840	19.2623	17.8489	16.6074	15.5096		
RO SS			7.9077	9.2691	10.7156	12.2449									
RO DD			-21.3342	-18.7882	-16.7302	-15.0367									
InvestCost	10.0000													-22.0000	-22.0000
Leader's Value Function as Function of v, Across Regimes															
v	4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000	8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000
Regime	L4	L4	L3	L3	L3	L3	L2	L2	L2	L2	L2	L2	L2	L1	L1
L Value SUM	25.0000	25.0000	25.5820	27.2203	29.6532	32.7029	36.2438	39.9837	43.7637	47.5830	51.4408	55.3363	59.2690	70.0000	73.5714
L Op PV	0.0000	0.0000	-14.2857	-10.7143	-7.1429	-3.5714	36.0000	38.8571	41.7143	44.5714	47.4286	50.2857	53.1429	70.0000	73.5714
RO SS							7.2438	8.1265	9.0494	10.0115	11.0122	12.0506	13.1261		
SO S			9.6899	11.3581	13.1307	15.0046									
SO D			30.1778	26.5765	23.6653	21.2698									
InvestCost	25.0000	25.0000					-7.0000	-7.0000	-7.0000	-7.0000	-7.0000	-7.0000	-7.0000		

3.3 Sensitivity Analysis for Changes in the Leader's Market Share

Table 4 presents the percentage change in the thresholds and coefficients for the joint model due to a .1% increase in the leader's market share.

Table 4: Sensitivity of Rival/Strategic Options to .1% Increase in the Leader's Market Share

		Coefficient Values			Percentage Change			
	BASE	Initial	Middle	Final	Initial	Middle	Final	
β_1	*1.6667*	*1.6667*	*1.6667*	*1.6667*				
β_2	*(1.3333)*	*(1.3333)*	*(1.3333)*	*(1.3333)*				
vFD	*4.3283*	*4.3283*	*4.3279*	4.3299	0.0000%	-0.0096%	0.0356%	
vFS	*10.2062*	*10.2062*	*10.2109*	10.2177	0.0000%	0.0459%	0.1129%	
vLD	*4.5238*	*4.5242*	*4.5234*	4.5214	0.0092%	-0.0090%	-0.0539%	
vLS	*6.9480*	*6.9471*	*6.9455*	6.9446	-0.0131%	-0.0360%	-0.0497%	
A1IIFS	0.0693	0.0693	0.0697	0.0677	0.0000%	**0.5964%**	**-2.2088%**	SO F S
A2IIFD	334.1445	334.1445	334.1110	334.2686	0.0000%	-0.0100%	0.0372%	SO F D
A1IILSS	0.2828	0.2828	0.2816	0.2840	0.0000%	**-0.4271%**	**0.4327%**	RO L SS
A1IILS	0.6628	0.6623	0.6631	0.6645	-0.0792%	0.0424%	0.2550%	SO L S
A2IILD	258.0164	258.1973	257.9904	257.8600	0.0701%	-0.0101%	-0.0606%	SO L D
A1IIFSS	0.5409	0.5417	0.5415	0.5415	**0.1442%**	**0.1111%**	**0.1218%**	RO F SS
A2IIFDD	*-182.4047*	*-182.6171*	*-182.4527*	-182.4199	**-0.1164%**	-0.0263%	-0.0083%	RO F DD
Δ_F	*6.2886*	*6.2886*	*6.2951*	6.2987	0.0000%	0.1031%	0.1599%	
Δ_L	*2.4481*	*2.4467*	*2.4462*	2.4480	0.0000%	-0.0791%	-0.0062%	

[5] Investment Cost is treated as a positive cash flow when the firms divest, and negative when the firms switch, net of salvage value. See Appendix C for an alternative graphic presentation of these value functions.

Increases in the leader’s market share and the consequential decrease in the follower’s market share could be interpreted as being attractive for the leader at the detriment to the follower, if one ignores the effect on the change of market share on the thresholds, and option values over the various regimes and revenue levels. A .1% increase in the leader’s initial market share increases the RO F SS and RO F DD by more than .1 percent, but does not affect the RO L SS. An increase in the leader’s middle market share increases the RO F SS by more than .1 percent, but reduces the RO L SS by .4%. An increase in the leader’s final share increases the RO F SS by more than .1 percent, and increases the RO L SS by more than .4%. Thus, it is apparent that rival options can be affected by either the leader or the follower trying to change market share, over and beyond the effect on the PV of operations. The most significant changes are to the increases in the SO F S in the middle stage, and reduction in the final stage.

The tables below are a sample of the possible effects over v=5,7 and 12, corresponding to the initial, middle and final stages.

Table 5 shows that an increase in the leader’s **initial** market share (IMS) at low v (v=5) makes the divestment opportunity leads to an earlier exercise (higher threshold), and also an earlier exercise (lower threshold) for the switch opportunity. It has, though, no impact on the follower’s strategy since the divestment and switch opportunities only become available after the leader has divested and switched, respectively, except for the positive change in the follower’s present accrued value when the leader divests because of the greater gain in market share.

Table 5: Thresholds as Function of Initial Market Share

vFD	4.328	4.328	4.328	4.328	4.328	4.328	4.328	4.328	4.328	0.000
vLD	4.524	4.528	4.532	4.536	4.540	4.544	4.548	4.552	4.556	0.032
vLS	6.948	6.939	6.930	6.921	6.912	6.904	6.895	6.887	6.878	-0.070
vFS	10.206	10.206	10.206	10.206	10.206	10.206	10.206	10.206	10.206	0.000
D L/XX	0.500	0.505	0.510	0.515	0.520	0.525	0.530	0.535	0.540	

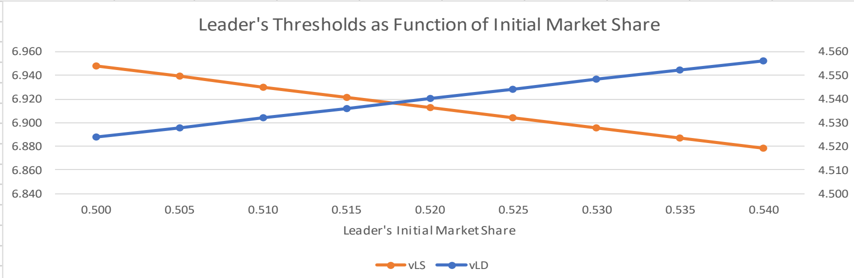
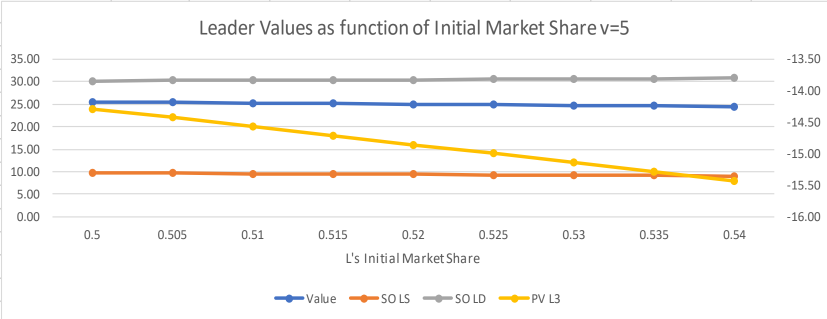


Table 6: Leader's Values as Function of Initial Market Share (v=5)

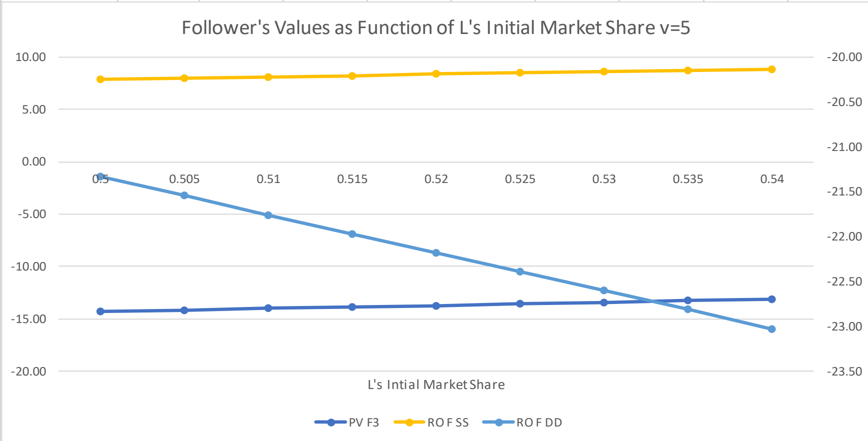
Value	25.58	25.44	25.29	25.14	25.00	24.85	24.70	24.56	24.41	-1.17
SO LS	9.69	9.61	9.54	9.46	9.38	9.31	9.23	9.15	9.08	-0.61
SO LD	30.18	30.25	30.32	30.40	30.47	30.54	30.62	30.69	30.76	0.58
PV L3	-14.29	-14.43	-14.57	-14.71	-14.86	-15.00	-15.14	-15.29	-15.43	-1.14
D LXX	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535	0.54	



Naturally, the leader's opPV decreases with increases in D L/XX at a low v. Perhaps it is less obvious that the value of the leader's strategic option SO LS declines with increases in D L/XX. When v is low, between the leader's divest and switch thresholds, and operating profit is negative, increasing market share is hardly worthwhile.

Table 7: Follower Values as Function of Leader's Initial Market Share (v=5)

Value	12.38	12.43	12.47	12.52	12.56	12.61	12.65	12.70	12.74	0.36
PV F3	**-14.29**	**-14.14**	**-14.00**	**-13.86**	**-13.71**	**-13.57**	**-13.43**	**-13.29**	**-13.14**	1.14
SO FS	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.00
SO FD	**39.08**	**39.08**	**39.08**	**39.08**	**39.08**	**39.08**	**39.08**	**39.08**	**39.08**	0.00
RO F SS	7.91	8.02	8.14	8.25	8.36	8.48	8.59	8.70	8.82	0.91
RO F DD	**-21.33**	**-21.55**	**-21.76**	**-21.97**	**-22.18**	**-22.39**	**-22.60**	**-22.82**	**-23.03**	-1.69



While the follower's divest and switch thresholds are not affected by changes in the leader's initial market share, the negative opPV is slightly decreased at v=5, while the (negative) ROF DD increases slightly. The net effect of the leader increasing initial market share when v is low and between the divest and switch thresholds is that the leader's total value slightly decreases, while the follower's total value slightly increases.

If the **middle** market share (MMS) (when v=7 after switching for the leader) $D_{L|Y,X}$ increases, then there is an increase in the present value accruing to the leader. The switching opportunity for follower also becomes more attractive with a deferred threshold because the loss in the follower's market share becomes less. Also, there is an increase in the present value accruing to the follower when the leader switches because of the gain in the follower's market share.

Table 8: Thresholds as Function of L's Middle Market Share (v=7)

vFD	4.328	4.324	4.320	4.316	4.312	4.307	4.303	4.299	4.295
vLD	4.524	4.520	4.515	4.511	4.506	4.501	4.496	4.491	4.485
vLS	6.948	6.923	6.898	6.873	6.849	6.824	6.799	6.775	6.750
vFS	10.206	10.253	10.300	10.347	10.393	10.440	10.486	10.533	10.579
D LYX	0.4	0.404	0.408	0.412	0.416	0.42	0.424	0.428	0.432

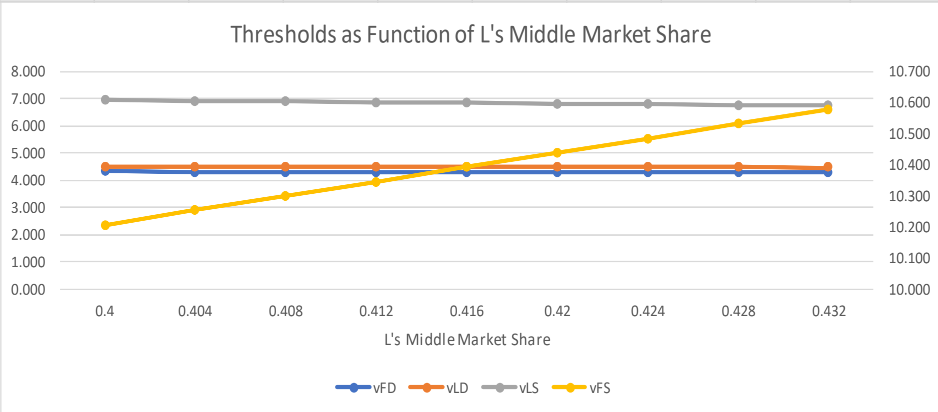
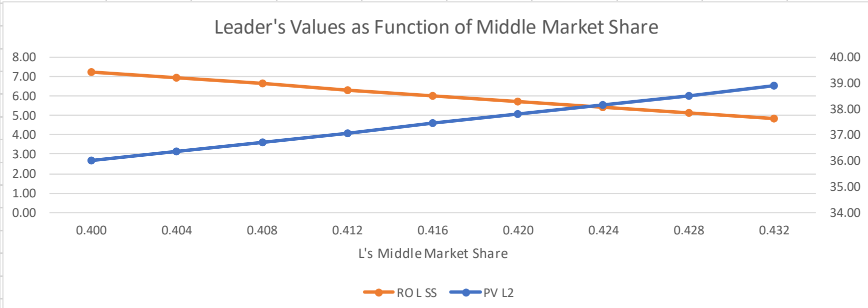


Table 9: Leader's Value as Function of L's Middle Market Share v=7

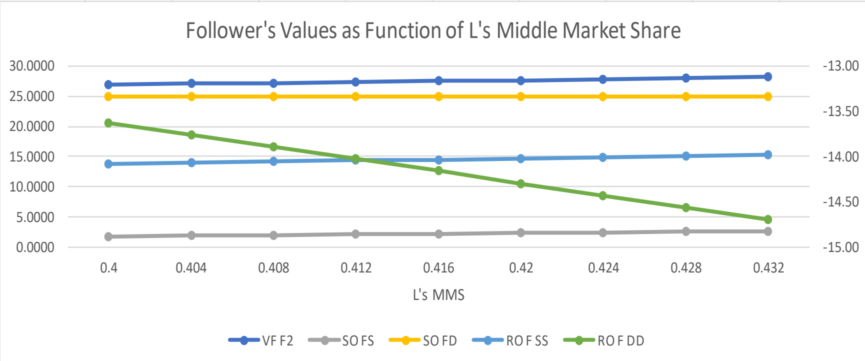
VF	36.24	36.30	36.35	36.40	36.46	36.52	36.58	36.64	36.70	0.4580
PV L2	36.00	36.36	36.72	37.08	37.44	37.80	38.16	38.52	38.88	2.8800
RO L SS	7.24	6.94	6.63	6.32	6.02	5.72	5.42	5.12	4.82	-2.4220
PV	-7.00	-7.00	-7.00	-7.00	-7.00	-7.00	-7.00	-7.00	-7.00	0.0000
D LYX	0.400	0.404	0.408	0.412	0.416	0.420	0.424	0.428	0.432	



The leader's opPV increases significantly as the L's middle market share increases (after the leader switches) but the rival ROL SS decreases, so the net value increases slightly.

Table 10: Follower Values as Function of L's Middle Market Share

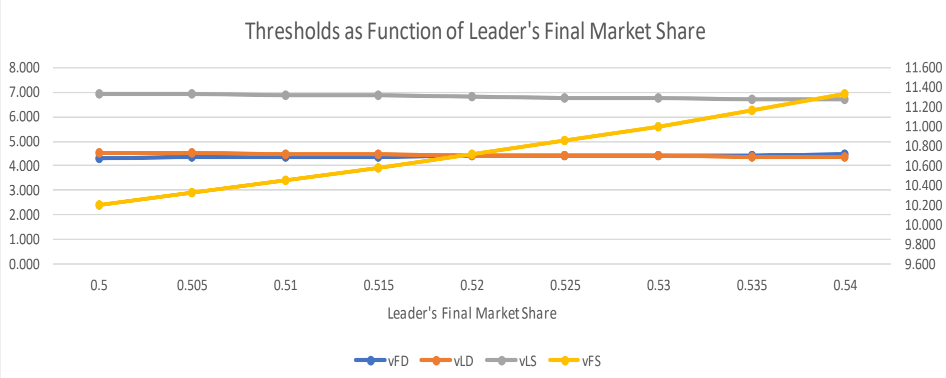
VF F2	**26.9609**	**27.0886**	**27.2233**	**27.3650**	**27.5139**	**27.6701**	**27.8337**	**28.0048**	**28.1835**	1.2226
PV F2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SO FS	**1.7744**	**1.8804**	**1.9871**	**2.0943**	**2.2021**	**2.3105**	**2.4193**	**2.5287**	**2.6386**	0.8642
SO FD	24.9538	24.9538	24.9538	24.9538	24.9538	24.9538	24.9538	24.9538	24.9538	0.0000
RO F SS	**13.85**	**14.01**	**14.17**	**14.34**	**14.52**	**14.70**	**14.89**	**15.09**	**15.29**	1.4392
RO F DD	**-13.62**	**-13.76**	**-13.89**	**-14.03**	**-14.16**	**-14.30**	**-14.43**	**-14.57**	**-14.70**	-1.0808



While the follower's opPV remains at 0 (when v=7), the SO FS and RO F SS increase somewhat as the leader's middle market share increases, so the follower's net value increases surprisingly.

Table 11: Thresholds as Function of L's Final Market Share

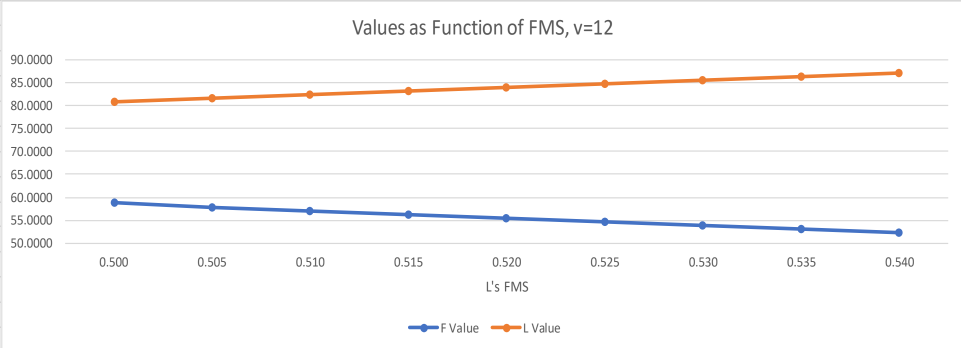
vFD	4.328	4.344	4.359	4.375	4.390	4.406	4.422	4.437	4.453
vLD	4.524	4.500	4.477	4.455	4.433	4.413	4.393	4.374	4.357
vLS	6.948	6.914	6.881	6.850	6.819	6.790	6.762	6.736	6.710
vFS	10.206	10.324	10.447	10.575	10.710	10.853	11.003	11.163	11.333
D LYY	0.5	0.505	0.51	0.515	0.52	0.525	0.53	0.535	0.54



As the leader's Final Market Share (FMS) increases, the switching threshold for the follower increases, naturally.

Table 12: Values as Function of Final Market Share (v=12)

Value Functions										
v	12	12	12	12	12	12	12	12	12	
F Value	**58.7143**	**57.9071**	**57.1000**	**56.2929**	**55.4857**	**54.6786**	**53.8714**	**53.0643**	**52.2571**	-6.4571
L Value	**80.7143**	**81.5214**	**82.3286**	**83.1357**	**83.9429**	**84.7500**	**85.5571**	**86.3643**	**87.1714**	6.4571



Once both parties have switched, an increase in the leader's **final** market share, $D_{L|Y,Y}$, makes the leader's op PV more valuable, and that of the follower (without any more options) less attractive, following the Buzzell et al. (1975) guidelines. There are many more combinations of the level of v and change of one of the three market shares that could be illustrated[6]. In general, it is not usually reasonable to focus just on the change in the relative opPVs when accessing the relative value of changing market shares[7].

4. Market Share Partial Derivatives

Some of the partial derivatives with respect to changing market share are relatively easy, others are very complex[8].

Initial Market Share: We specify the analytical change given by the partial derivative for each option coefficient value arising from a change in the leader's market share $D_{L/XX}$, when both the leader and the follower are using technology X. In the following, if the value is not zero, $|M_d| \neq 0$, then two items are presented, the analytical derivative for six option coefficients (across all of the three stages), and for

[6] Appendix F shows the effect at Stage 1 of changes in the L's Middle Market Share.
[7] Of course, this ignores the possibly irrecoverable expenditures (such as one-time advertisements) to obtain a permanent increase in the L's market share at any stage.
[8] The novel methodology for deriving these partial derivatives is described in Appendix D using the Implicit Function approach explained in Sydsaeter et al. (2005).

the rest its numerical value, since those expressions are typically long and complicated. If the determinant value is zero, $|M_d| = 0$, then only the analytical derivative is equal to zero. The analytical expressions for the partials at the initial stage are only for the leader's two strategic options, divest and switch:

$$\frac{\partial A_{1IILS}}{\partial D_{L/XX}} = -\frac{-\frac{f_X \hat{v}_{LD}{}^{\beta_2}}{r} + \frac{f_X \hat{v}_{LS}{}^{\beta_2}}{r} + \frac{\hat{v}_{LD}{}^{\beta_2} \hat{v}_{LS}}{\delta + \theta} - \frac{\hat{v}_{LD} \hat{v}_{LS}{}^{\beta_2}}{\delta + \theta}}{\Delta_L} \tag{14}$$

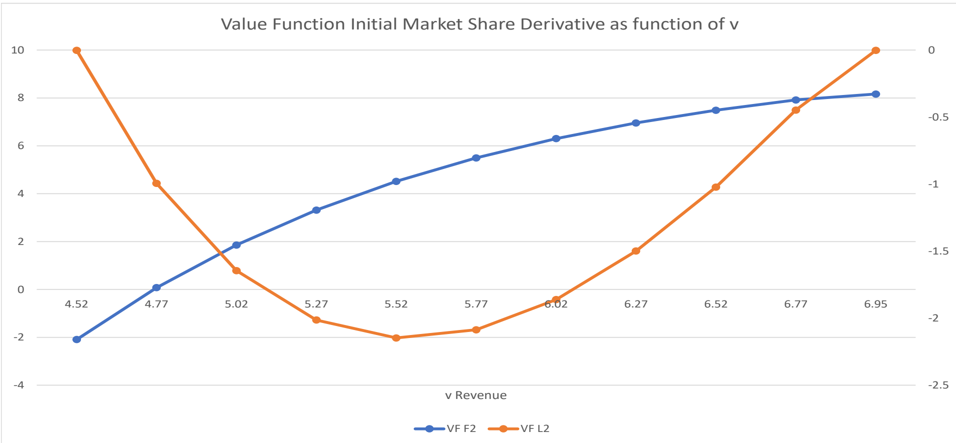
$$\frac{\partial A_{1IILD}}{\partial D_{L/XX}} = -\frac{\frac{f_X \hat{v}_{LD}{}^{\beta_1}}{r} - \frac{f_X \hat{v}_{LS}{}^{\beta_1}}{r} - \frac{\hat{v}_{LD}{}^{\beta_1} \hat{v}_{LS}}{\delta + \theta} + \frac{\hat{v}_{LD} \hat{v}_{LS}{}^{\beta_1}}{\delta + \theta}}{\Delta_L} \tag{15}$$

Other partial derivative values are calculated numerically[9].

Table 13

First Derivative of Value Function with respect to the Leader's Initial Market Share

v	*4.52*	4.77	5.02	5.27	5.52	5.77	6.02	6.27	6.52	6.77	6.95	Change
VF F2	-2.0914	0.0755	1.8546	3.3155	4.5144	5.4968	6.2999	6.9545	7.4862	7.9164	8.1642	
v-fx	35.3684	31.7970	28.2255	24.6541	21.0827	17.5112	13.9398	10.3684	6.7970	3.2255	0.7552	-34.6132
RO F SS	19.3024	21.1127	22.9872	24.9251	26.9251	28.9864	31.1081	33.2894	35.5293	37.8273	39.4503	20.1478
RO F DD	-56.7622	-52.8341	-49.3582	-46.2637	-43.4934	-41.0009	-38.7480	-36.7033	-34.8401	-33.1364	-32.0412	24.7210
VF L2	-0.0020	-0.9946	-1.6453	-2.0137	-2.1477	-2.0863	-1.8614	-1.4994	-1.0221	-0.4479	-0.0024	-0.0005
v-fx	-35.3684	-31.7970	-28.2255	-24.6541	-21.0827	-17.5112	-13.9398	-10.3684	-6.7970	-3.2255	-0.7552	34.6132
SO L S	-12.9863	-14.2043	-15.4654	-16.7692	-18.1148	-19.5016	-20.9290	-22.3965	-23.9035	-25.4495	-26.5415	-13.5551
SO L D	48.3527	45.0066	42.0457	39.4096	37.0497	34.9265	33.0074	31.2656	29.6784	28.2271	27.2942	-21.0585



[9] All of the results are shown in the Supplementary Appendix Table E4, with comparisons of the partial derivatives using Mathematica and the approximate total derivatives assuming a .1% change in the market share at each stage. Generally, all of these 27 sets of calculations are quite close, with slight differences curiously only in the middle and final stages for the SO L S and RO F DD as shown in Tables E2 and E3.

Table 13 shows clearly that the effect of increasing market share when v is low is negative for the opPV for the leader until approaching the switching threshold 6.9, and the first derivative for the switching coefficient is increasingly negative, consistent with the less obvious observation regarding Table 6 that the strategic switching option value declines, while the strategic divestment option value increases. Is there any way for the leader to avoid reducing one option without reducing the other? Table 13 confirms that the follower's strategic options are not affected at all by changes in the initial market share, consistent with Table 7 and with (14) and (15).

The analytical expressions for the middle and final market share changes are only for the follower's strategic options, divest and switch.

Middle Market Share:

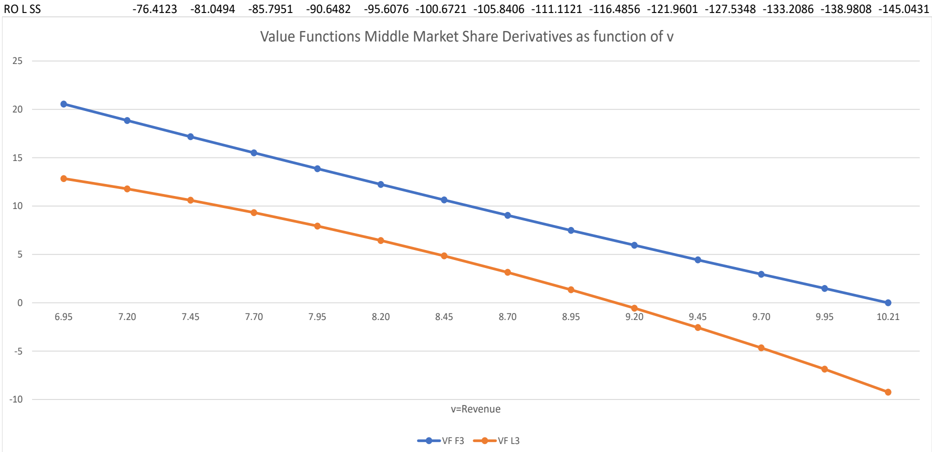
$$\frac{\partial A_{1IIFS}}{\partial D_{L/YX}} = \frac{\hat{v}_{FD}{}^{\beta_2}\left(-\frac{f_X}{r} + \frac{\hat{v}_{FS}}{\delta + \theta}\right)}{\Delta_F} \tag{16}$$

$$\frac{\partial A_{2IIFD}}{\partial D_{L/YX}} = -\frac{\hat{v}_{FD}{}^{\beta_1}\left(-\frac{f_X}{r} + \frac{\hat{v}_{FS}}{\delta + \theta}\right)}{\Delta_F} \tag{17}$$

Table 14

First Derivative of Value Function with respect to the Leader's Middle Market Share

															Change
v	*6.9480*	7.1980	7.4480	7.6980	7.9480	8.1980	8.4480	8.6980	8.9480	9.1980	9.4480	9.6980	9.9480	10.2062	3.2581
VF F3	20.5494	18.8539	17.1730	15.5085	13.8623	12.2358	10.6301	9.0464	7.4855	5.9482	4.4351	2.9468	1.4837	0.0000	-20.5494
v-fx	0.7422	-2.8292	-6.4007	-9.9721	-13.5435	-17.1149	-20.6864	-24.2578	-27.8292	-31.4007	-34.9721	-38.5435	-42.1149	-45.8025	-46.5447
SO F S	26.1223	27.7076	29.3299	30.9890	32.6845	34.4158	36.1827	37.9848	39.8218	41.6933	43.5991	45.5388	47.5120	49.5845	23.4622
SO F D	-6.3152	-6.0244	-5.7563	-5.5084	-5.2786	-5.0651	-4.8662	-4.6807	-4.5071	-4.3445	-4.1919	-4.0485	-3.9134	-3.7820	2.5332
VF L3	12.8455	11.7799	10.6056	9.3238	7.9359	6.4428	4.8458	3.1457	1.3436	-0.5595	-2.5627	-4.6651	-6.8659	-9.2406	-22.0861
v-fy	89.2578	92.8292	96.4007	99.9721	103.5435	107.1149	110.6864	114.2578	117.8292	121.4007	124.9721	128.5435	132.1149	135.8025	46.5447
RO L SS	-76.4123	-81.0494	-85.7951	-90.6482	-95.6076	-100.6721	-105.8406	-111.1121	-116.4856	-121.9601	-127.5348	-133.2086	-138.9808	-145.0431	-68.6308



In Table 14, the interesting aspects at the middle stage are regarding the leader's RO L SS (benefiting from the follower switching, whereby the L's market share returns from 40% to 50%), and the follower's SO F S. The partial for the leader's rival option is increasingly negative, but the partial for the follower's strategic option S is increasingly positive, leading to overall value gains for the follower, and losses for the leader, as the MMS increases.

Final Market Share

$$\frac{\partial A_{1IIFS}}{\partial D_{L/YY}} = \frac{\hat{v}_{FD}^{\beta_2}\left(\frac{f_Y}{r} - \frac{\hat{v}_{FS}}{\delta+\theta}\right)}{\Delta_F} \tag{18}$$

$$\frac{\partial A_{2IIFD}}{\partial D_{L/YY}} = -\frac{\hat{v}_{FD}^{\beta_2}\left(\frac{f_Y}{r} - \frac{\hat{v}_{FS}}{\delta+\theta}\right)}{\Delta_F} \tag{19}$$

Table 15

First Derivative of Value Function in the Middle Stage with respect to the Leader's Final Market Share

															Change
v	6.9446	7.1946	7.4446	7.6946	7.9446	8.1946	8.4446	8.6946	8.9446	9.1946	9.4446	9.6946	9.9446	10.2177	3.2731
VF F3	-58.6508	-64.2125	-69.8171	-75.4705	-81.1779	-86.9436	-92.7711	-98.6636	-104.6238	-110.6539	-116.7558	-122.9312	-129.1814	-136.0962	-77.4454
v-fx	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SO FS	-77.3875	-82.0861	-86.8949	-91.8126	-96.8379	-101.9698	-107.2072	-112.5489	-117.9941	-123.5417	-129.1907	-134.9404	-140.7897	-147.2927	-69.9053
SO FD	18.7366	17.8736	17.0778	16.3421	15.6600	15.0263	14.4361	13.8853	13.3703	12.8878	12.4349	12.0092	11.6084	11.1965	-7.5401
VF L3	61.8691	65.6255	69.4700	73.4015	77.4191	81.5219	85.7090	89.9796	94.3329	98.7680	103.2843	107.8809	112.5573	117.7563	55.8872
v-fy	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RO L SS	61.8691	65.6255	69.4700	73.4015	77.4191	81.5219	85.7090	89.9796	94.3329	98.7680	103.2843	107.8809	112.5573	117.7563	55.8872

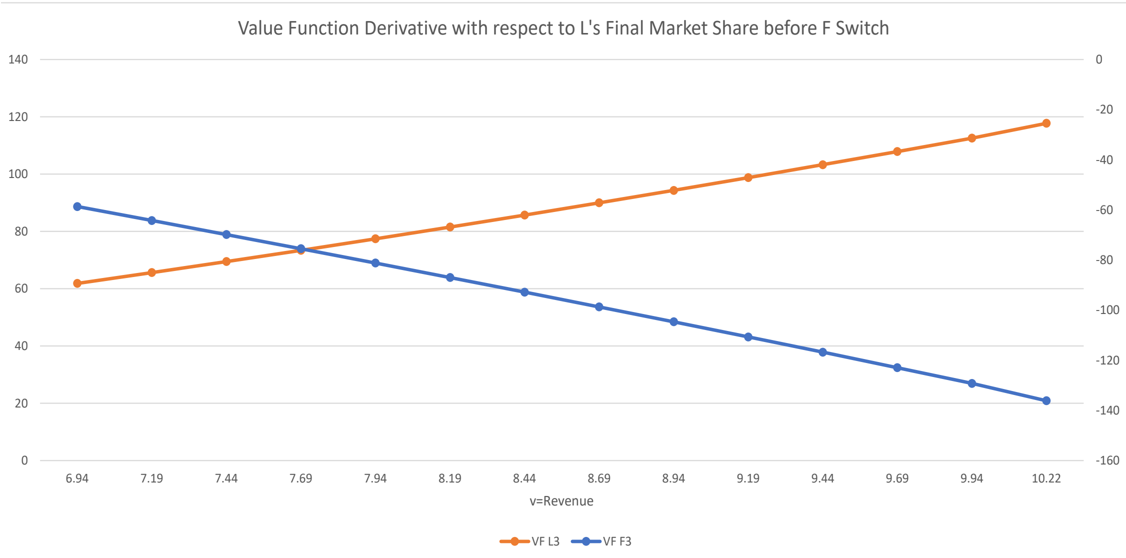


Table 15 shows that in the middle stage, if the Leader is able to increase its final market share, while the value of the immediate operating net revenue is not changed, the RO L SS (benefit to the L when the F switches, mostly due to the reversion to 50%+ MS) increases significantly, while the F's strategic option to switch becomes more negative. This is consistent with Table 12, where the L's value function increases as the L's FMS increases, when v=12.

These approaches provide a rich format for interpreting the impact of market share changes on current and prospective decisions in a duopoly, which can be reconfigured as appropriate for different contexts and parameter values.

5. Summary and Conclusions

Should the leader always attempt to increase market share? What is the appropriate action in the initial regime for anticipating altering market share in the middle and final regimes? How can competitors affect the value (and exercise) of rival options?

(i) Should the leader or follower attempt to increase market share when revenue is below operating cost? The net present value approach is increasingly negative, presenting the case for perhaps reducing market share instead. But with different parameter values it is conceivable that the strategic divest option value could increase, but at a decreasing rate.

(ii) What happens when revenue is close to over the operating cost, slightly exceeding the leader's switching threshold? Almost surely the answer is positive (increasing the opPV), but watch for the effect that it reduces the rival follower switching, whose actions may well benefit the leader.

(iii) What is the appropriate action in the initial regimes, for anticipating altering market share in the middle and final regimes, or in the middle regime, for anticipating altering market share in the final regime? Answers here depend on the relative value of most of the options given the specific parameter values. Also, what is the assurance that a leader can alter market share in subsequent stages at a reasonable cost?

(iv) How can competitors affect the value (and exercise) of rival options? The three rival options, RO L SS benefiting from the follower switching, and RO F SS and F DD, benefiting from the leader divesting or switching, have been clearly identified, along with the sensitivities for changing market share at the various stages. Even without affecting the value of these rival actions, watching the competition and quantifying the option value of potential benefits as parameter values change over the stages should demonstrate alert real option management skills.

Future research is likely to develop further configurations of this approach, empirical applications to the evolving duopolies, along with extensions to oligopolies and monopolistic competition. Hedging and trading some of these real options will be an exciting future activity. Perhaps there will be analytical or semi-analytical solutions for some more of these option coefficients.

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Supplementary Appendix

A NPV Value Functions

B Derivation of Joint Solutions

C Decomposition of Value Functions

D Methodology of the Market Share Partial Derivatives

E Coefficient Derivatives at Three Stages

F Values at Initial Stage from Changing MMS