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Real Option Games Between Rivals

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Highlights:

Market Share Effects on Relative Values

Real Option Games of Increasing Market Share

Real Option Games of Changing Volatility

Analytical Formulae for Game Payoffs

Actions/Reactions Between Rivals

Abstract

We build on previous solutions for mutually exclusive options in a duopoly with switching and divestment alternatives. We examine the implications of increasing the leader's market share and/or changing volatility over progressive regimes. The consequences of market share and volatility changes on the values for both the leader and follower are often surprising, because of the unique effects on the various rival and strategic option values. The leader loses with increased initial market share at low revenues, both leader and follower lose with increased middle market shares (but both gain at higher revenues). There are interesting "risk" games when the portfolio of options for the leader has differential sensitivities to volatility changes than for the follower. Sometimes the leader should prefer less volatility particularly at higher revenues (the follower more). These characteristics provide a rich context for evaluating real option games involving market shares, volatilities and eventually altering other factors.

I Introduction

A leader and follower are engaged in a real option game involving mutually exclusive options for a duopoly. There is the possibility of either party altering the market share over subsequent stages (regimes) (initial market share=IMS, or middle market share=MMS, or final market share=FMS) by paying $\$\Delta$ for a 1% increase in market share. There is also the possibility of either or both parties getting the government (or market) to alter the (price) volatility, or (eventually) other critical common parameters such as rate, yield, investment cost or salvage value.

Adkins et al. (2022, 2023) develop duopoly real option models that derive the optional switching (to a lower cost technology) or divestment threshold for the leader/follower over regimes as the market revenue changes. These models are then used in building real option games (ROG) initially developed by Smets (1993) which have been applied to a wide range of problems, including decisions on investment projects whose value is exposed to both uncertainty and competition (Dixit and Pindyck, 1994, chapter 9; Pawlina and Kort, 2006; Azevedo and Paxson, 2014).

Various real option authors assume contexts which lead to different strategies for the leader and follower. Joaquin and Butler (2000) assume a first mover leader advantage of lower operating costs. Tsekrekos (2003) allows for both temporary and pre-emptive permanent market share advantages for the leader. Paxson and Pinto (2003) focus on the partial derivatives of the value function for the leader/follower with respect to changes in the market share, market revenue and volatility. Paxson and Pinto (2005) show the partial derivatives of the value function for the leader/follower in both preemptive and non-preemptive games with respect to changes in market revenue, changing as revenue approaches the thresholds. Kong and Kwok (2007) provide standard entry thresholds for leader/follower when asymmetric in investment cost and revenue, with real option values not separately disclosed. Paxson and Melmane (2009) assume the leader starts with a larger market share, which follows a subsequent random process. Dias and Teixeira (2010) focus on the entry of a leader/follower with symmetric/asymmetric costs, and covering several game strategies.

Bobtcheff and Mariotti (2013) look at a pre-emptive game of two competitors, revealed only by a first mover investment. Bensoussan et al. (2017) study a duopoly with the possibility of regime switching. Balliauw et al. (2019) is an empirical work on the investment thresholds of leader/follower ports with capacity choices, without identifying the precise real option values. Huberts et al. (2019) examines interesting strategies where entry by competitors may be deterred, possibly in a war of attrition or pre-emption.

A key element added by some models extending the monopolistic real options literature is a factor, which Adkins et al. (2022) named "rival options", that takes into account the effect of the follower's decision on the leader's behavior and vice versa. In a typical ROG, one of the firms decides first (leader), the other firm decides after the leader (follower). The framework allows for decision games where the leader's decision enhances the follower's value, rival options.

The analytical expression that is behind most of the ROG literature and that measures the drop or the enhancement in the leaders' (or the follower's) value, caused by the follower's (leader's) decision, has the following form:

$$\frac{v_F(DL_{after}-DL_{before})}{\delta+\theta}\left(\frac{v}{v_F}\right)^{\beta_1\ (or\ \beta_2)} \tag{1}$$

where v is the state (underlying) variable such as net revenue, v_F is the follower's threshold, DL_{after} and DL_{before} are the leader's market share for after and before the follower's decision, and $\delta = (r-\mu)$ with r the interest rate and μ the drift of the geometric Brownian process associated with the underlying variable; β_1 and β_2 are the roots of the following quadratic equation, $\frac{1}{2}\sigma^2\beta(\beta-1)+(r-\delta)\beta-r$, given by: [2]

$$\beta_1 = \left(\frac{1}{2}-\frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\frac{r-\delta}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}} > 1 \tag{2}$$

$$\beta_2 = \left(\frac{1}{2}-\frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\frac{r-\delta}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}} < 0 \tag{3}$$

[2] We use β_1 if the state variable reaches the threshold from below and β_2 if the state variable reaches the threshold from above.

A key element in (1) is the so-called discounting stochastic factor $\left(\frac{v}{v_F}\right)^{\beta_2}$. Notice that, as v approaches v_F, $\left(\frac{v}{v_F}\right)^{\beta_2}$gets closer to 1; if v reaches v_F, the leader gains $\frac{v(DL_{after}-DL_{before})}{\delta+\theta}$ if $DL_{after} > DL_{before}$, or loses $\frac{v(DL_{after}-DL_{before})}{\delta+\theta}$ if $DL_{after} < DL_{before}$. These inequalities depend on the specificities of the duopoly real option game that is being modelled.

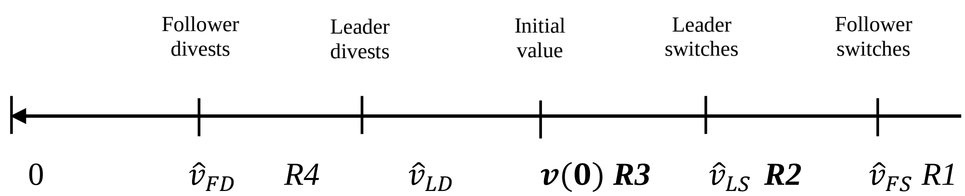
The Adkins et al. (2022) model specifies the following context: i) for some stages of the game, firms hold two mutually exclusive options, the options to switch and divest, ii) while in a standard ROG firms hold two market shares only (e.g., the leader gets 100% of the market when it is operating alone and, then, 50/50 after the follower has invested), in Adkins et al. (2022) the market share dynamic is richer since firms can hold three different market shares over time (initially 50/50, then when the leader switches 42.5/57.5, and then after the follower also switches 50/50 again).[3] The sequence by which firms exercise their options altogether with the market share each firm holds at each stage of the game affect the firms' value. This latter characteristic leads to some peculiar characteristics in these ROG, namely the existence of a possible dynamic game equilibria regarding the firms' ex-ante strategic choices on their market shares at each stage of the game. That is, while optimizing the timing of their decisions, at a given stage of the game, firms should also conjecture about whether there is an optimal market share to hold at the game stage they are at, in order to maximize the firm value considering that in the future eventually the threshold to exercise the option to pass to the next stage is reached. This is a relevant issue because, if there is an optimal market share to be achieved in the current stage, that maximizes the firm value (which might be below or above the market share the firm would hold otherwise), then the firm should act accordingly (e.g., if the optimal market share is higher than that the firm currently holds, it may should invest in advertisement campaigns to increase its market share). See the results on the

[3] In a typical ROG, each firm has two exogenously assigned market shares only: if inactive, both firms start the game with no market share (otherwise 50/50), then the leader invests first and gets 100% of the market, and finally the follower also invests and the leader's market share drops to 50% (being the drop in the leader's market share the same as the gain in the follower's market share). It is also assumed that the ex-post pre-assigned market shares will hold forever and that the real options held by each firm at the beginning of the decision game are perpetual. In our base case, there is a varied set of possible firms' market shares for both the leader and the follower: L0/F0 if both divest, L0/F100 if only the leader divests, L50/F50 if both firms operate with either the initial policy X or subsequent policy Y, and L42.5/F57.5 for when only the leader has switched to Y.

partial derivatives on the firms' value with respect to the market share of the various stages of the game.

We assume that there is a duopoly of symmetric operating firms, except the leader has an advantage of obtaining full value Z in any divestment of the existing operating facility, while the follower obtains λZ, where 0<λ<1. The follower obtains a larger market share (57.5%) after the leader has switched to a lower operating cost technology, policy Y. The order of divesting/switching thresholds divest $\{\hat{v}_{FD}, \hat{v}_{LD}\}, switch\ \{\hat{v}_{LS}\ and\ \ \hat{v}_{FS}\}$ is indicated in Figure 1. Total market revenue "v" follows a geometric Brownian motion with constant (negative) drift and volatility[4]. Each firm holds the option to divest and receive a salvage value from the initial X stage. Once the divestment option is exercised, the firm exits the market which is referred to as policy O. Since Y is the more cost efficient, the full-market operating cost $f_X > f_Y$. There is no salvage value after firms switch to policy Y. The two players in the duopoly game are designated the leader and the follower, referred to as L and F, respectively. We treat the two firms as being ex-ante symmetric, which implies that each firm has 50% of the market provided that the two firms are pursuing identical policies, so: $D_{L|X,X} = 1 - D_{F|X,X}$.

Figure 1: Leader and Follower Thresholds for a Random Revenue (v)



Regime 3 for the IMS indicates v between vLD and vLS, Regime 2 for the MMS indicates v between vLS and vFS, with Regime 1 when v>vFS, as indicated in Figure 1.

The value function for the leader is denoted by $V_L(v)$.

[4] These are also the assumptions in Adkins et al. (2022), except for the MMS proportions, along with the derived equations and solutions described in detail in Appendix A. There are many other possible configurations, with different consequences.

$$V_L(v) = \begin{cases} L/YY \left(\frac{v}{\delta+\theta} - \frac{f_Y}{r}\right) & if\ v \geq \hat{v}_{FS}\ R1 \\ L/YX \left(\frac{v}{\delta+\theta} - \frac{f_Y}{r}\right) + RO\ L\ SS\ \ v^{\beta_1} & if\ \hat{v}_{LS} \leq v < \hat{v}_{FS} \boldsymbol{R2} \\ L/XX \left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + SO\ L\ S\ \ v^{\beta_1} + SO\ L\ D\ \ v^{\beta_2} & if\ \hat{v}_{LD} < v < \hat{v}_{LS} \boldsymbol{R3} \\ Z & if\ v \leq \hat{v}_{LD} R4 \end{cases} \quad (4)$$

In (4), the first line R1 represents the expected present value of leader's net revenue once the follower has switched, when there are no further options; the second line R2 represents the expected present value of leader's net revenue plus the present value accruing to the leader when the follower switches, now denoted by $RO\ L\ SSv^{\beta_1}$; the third line R3 represents the expected present value of leader's net revenue plus the option values to switch, $SO\ L\ S\ \ v^{\beta_1} > 0,$ and to divest, $SO\ L\ D\ \ v^{\beta_2} > 0$; the fourth line R4 represents the leader's receipt from divesting the incumbent policy. The interesting regimes are R2 and R3, since once the follower has switched or the leader has divested there are no more two party moves allowed in the game.

The value function for the follower is denoted by $V_F(v)$.

$$V_F(v) = \begin{cases} F/YY \left(\frac{v}{\delta+\theta} - \frac{f_Y}{r}\right) & if\ v \geq \hat{v}_{FS}\ R1 \\ F/YX \left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + SO\ F\ S\ \ v^{\beta_1} + SO\ F\ D\ \ v^{\beta_2} & if\ \hat{v}_{LS} \leq v < \hat{v}_{FS}\ \boldsymbol{R2} \\ F/XX \left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + SO\ F\ Sv^{\beta_1} + SO\ F\ Dv^{\beta_2} & \\ +RO\ F\ SSv^{\beta_1} + RO\ F\ DD\ \ v^{\beta_2} & if\ \hat{v}_{LD} < v < \hat{v}_{LS} \boldsymbol{R3} \\ F/OX \left(\frac{v}{\delta+\theta} - \frac{f_X}{r}\right) + SO\ F\ S\ \ v^{\beta_1} + SO\ F\ D\ \ v^{\beta_2} & if\ \hat{v}_{FD} \leq v < \hat{v}_{LD} R4 \\ \lambda Z & if\ v < \hat{v}_{FD} R5 \end{cases} \quad (5)$$

In (5), the first line R1 represents the expected present value of follower's net revenue once the follower has switched; the second line R2 represents the expected present value of follower's net revenue plus the sum of the option values to switch, $SO\ F\ Sv^{\beta_1} > 0$ and to divest, $SO\ F\ S\ \ v^{\beta_2} > 0$, the third line R3 represents the expected present value of follower's net revenue plus the sum of the option values to switch, $SO\ F\ Sv^{\beta_1}$, and to divest, SO F Dv^{β_2}, and the sum of the present values (gains or losses) accruing to the follower when the leader switches, $RO\ F\ SS\ \ v^{\beta_1}$, and when the leader divests, $RO\ F\ DDv^{\beta_2}$; the fourth line R4 represents the expected present value of follower's net revenue plus the sum of the option values to switch, SO F Sv^{β_1}, and to divest, $SO\ F\ D\ \ v^{\beta_2}$; the fifth line R5 represents the follower's value on divestment.

In Table 1 A3:D11 are the assumed constant parameter values, C12:C19 are the assumed base market shares over the three regimes, B23:D26 are the derived thresholds, B27:D33 are the real option coefficients, SO denotes strategic option (exercised by the owner), and RO denotes rival option (exercised by the rival, benefits the owner).

In Table 1, column B shows the thresholds and option coefficients if the leader obtains an IMS of 51%, by spending $\$\Delta$. Column C is the base case scenario with market shares 50% (IMS, L/XX), 42.5% (MMS, L/YX) and 50% (FMS, L/YY). Column D shows the thresholds and option coefficients if the follower alone obtains an IMS of 51% by spending $\$\Delta$. Notice that the leader's divest thresholds increase and switch thresholds decrease with the higher L's IMS, and that only the last four option coefficients are affected by the IMS change.

Table 1

Parameter Values and Derived Thresholds and Option Coefficients

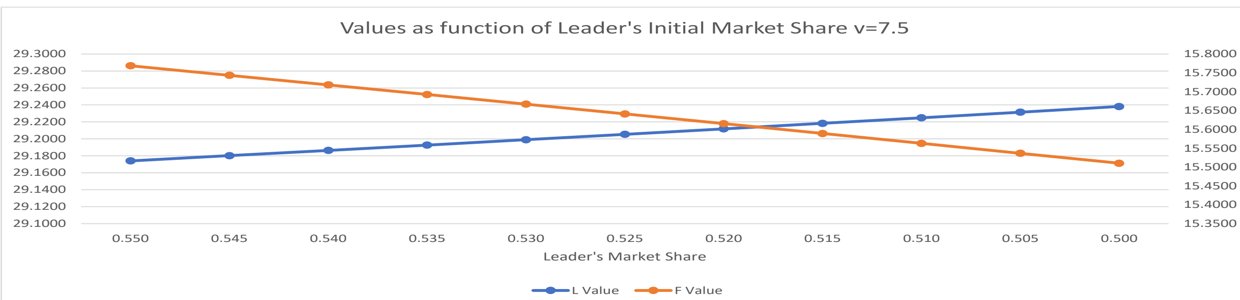
	A	B	C	D
1			***SHARE GAME R3A***	
2	*INPUT*	*L .51*	*BASE*	*F .51*
3	*r*	0.08	0.08	0.08
4	θ	0.04	0.04	0.04
5	*fX*	10	10	10
6	*fY*	2	2	2
7	*Z*	25	25	25
8	*K*	35	35	35
9	σ	0.20	0.20	0.20
10	λ	0.2	0.2	0.2
11	δ	0.03	0.03	0.03
12	*LXX*	**0.51**	0.50	0.49
13	*FXX*	0.49	0.50	**0.51**
14	*LOX*	0.00	0.00	0.00
15	*FOX*	1.00	1.00	1.00
16	*LYX*	0.425	0.425	0.425
17	*FYX*	0.575	0.575	0.575
18	*LYY*	0.500	0.500	0.500
19	*FYY*	0.500	0.500	0.500
20	*OUTPUT*			
21	β_1	2.2656	2.2656	2.2656
22	β_2	(1.7656)	(1.7656)	(1.7656)
23	*vFD*	5.7392	5.7392	5.7392
24	*vFS*	12.2631	12.2631	12.2631
25	*vLD*	6.0996	**6.0924**	6.0851
26	*vLS*	8.2470	**8.2585**	8.2701
27	*SO F S*	0.0132	0.0132	0.0132
28	*SO F D*	1034.8147	1034.8147	1034.8147
29	*RO L SS*	0.0385	0.0385	0.0384948
30	*SO L S*	0.1394	0.1412	0.1430
31	*SO L D*	874.8282	862.9820	851.1407
32	*RO F SS*	0.1280	0.1252	0.1223447
33	*RO F DD*	-657.6421	-643.7031	-629.7797

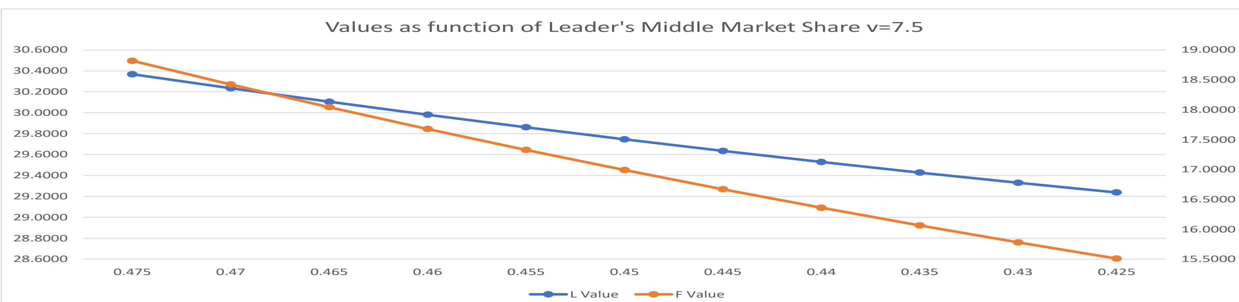
A critical first observation is that the effect of changes in market shares on values is more-or-less linear for the initial market share changes when in regime 3. Because both the leader and follower

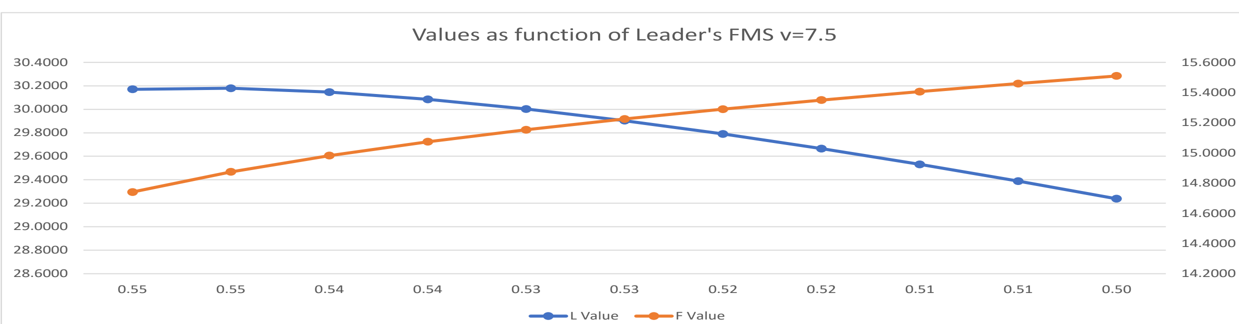
PV OPS are negative in regime 3, when v=7.5, the leader's PV OPS becomes slightly more negative as its initial market share increases, and the opposite occurs for the follower.[5] If while still in regime 3, the leader could imagine reducing market share in the middle regime, the effect on the leader and follower values is similar but not proportional. If while still in regime 3, the leader could imagine increasing market share in the final regime, the effect on the leader's value is positive, while the effect on the follower's value is negative.

Table 2

Sensitivity of Value Functions to Changes in Market Shares R 3







[5] Supplementary Appendix A shows the complete results for the three market share changes over Regime 3, and the two market share changes over Regime 2. SA B shows the partial and total derivatives for the option coefficients over Regimes 3,2 and 1.

What are lessons for the leader attempting to increase market share during any regime in a competitive Market Share Game? Game strategy is highly dependent on the level of the market revenue. As a preview, with the assumed parameter values, it is hard for the leader to benefit from increasing market share when v is low, but sometimes benefits when v is high. In a cooperative Risk Game, at the initial stage when v is low, the leader should lead a risk preferring strategy. More volatility please. At the middle and final stage, the follower benefits from more risk, the leader does not, so cooperation and collusion regarding future volatility are complex.

II Market Share Games

There are five interesting games envisioned between the parties regarding market share alterations. (1) R3A, either the L or F or both increasing IMS at the initial stage or regime 3, (2) R3B, either the L or F or both increasing MMS eventually, contemplated at the initial stage, (3) R3C, either the L or F or both increasing FMS eventually, contemplated at the initial stage, (4) R2A, either the L or F or both increasing MMS at the middle stage, or (5) R2B, either the L or F or both increasing FMS eventually, contemplated at the middle stage[6].

The duopoly game is formatted in a normal form. Notice that this is not a game on the optimal time to exercise the option to invest (as usual) but on the consequences of market share strategies, given the option of firms to maximize their value in the future when the optimal time to exercise the switch/divest options arrive. Notice that in the game, the *players* are the leader and the follower, with normally the leader being the firm that decides first; the *strategies available to each player* are the choices of changing the market share: “Initial”, “Initial L”, or “Initial F” the base market share, or the market share (reversion to 50/50) if one rival reacts equally to one player trying to increase market share. *Players’ payoffs for each strategy* are the leader’s and the follower’s value functions.

The idea behind the above game matrix is to determine, for given market conditions, what are the market shares for the leader and the follower that maximize total value functions for both firms.

(1) The game consists of the base case with v=7.5, between the leader’s base case thresholds {6.09, 8.26}, with the value function results in B83:C83 in Table 3. Then, the leader alone spends

[6] See SA C for complete ROG solutions.

$\$\Delta$=D80 the maximum expense for the leader that equalizes the total VFs in A83=A84[7], with the resulting separate value functions VF in B84:C84. An alternative is the follower alone spending $\$\Delta$ with the VFs in D83:E83, and finally both the leader and follower each spending $\$\Delta$ to alter IMS 1% with the result returning back to the base case less $\$\Delta$ for each in D84:E84, all assuming v=7.5. This is a type of Prisoner's Dilemma Game, where the best combined result is the base case (VFL + VFF= 44.7484), the worst case (VFL + VFF= 44.6298) where the leader and follower both spend $\$\Delta$, thus returning to the base case less spending $\$2\Delta$ together.

Table 3, R3A, v=7.5

	A		B	C	D	E	F	G
80			LEADER/FOLLOWER SPEND		0.0394	1% IMS		TOTAL VF
81	REGIME 3		vLD<v<vLS					
82	TOTAL VF		LEADER	FOLLOWER	LEADER	FOLLOWER		
83		44.7484	29.2381	15.5103	29.2517	15.4175	F .51	44.6692
84		44.7484	29.1854	15.5630	29.2123	15.4175	L/F .5	44.6298
85								
86			LEADER	FOLLOWER	LEADER	FOLLOWER		
87	L & F VF BASE		C90	C94	D90	D94	F .51	D83+E83
88	L .51 & F.49 VF		B90-D80	B94	D90-D80	D94-D80	L/F .5	D84+E84
89			L .51	BASE	F .51			
90	VF L		29.2248	29.2381	29.2517	SUM(D93:D95)		
91	**L 3 PV OPS**		**-9.1071**	**-8.9286**	**-8.7500**	**D12*(D35/(D4+D11)-D5/D3)**		
92	**L 3 SO L S**		**13.3891**	**13.5616**	**13.7342**	**D30*(D35^D21)**		
93	**L 3 SO L D**		**24.9428**	**24.6051**	**24.2675**	**D31*(D35^D22)**		
94	VF F		15.5630	15.5103	15.4569	SUM(D97:D101)		
95	**F 3 PV OPS**		**-8.7500**	**-8.9286**	**-9.1071**	**D13*(D35/(D4+D11)-D5/D3)**		
96	**F 3 SO F S**		**1.2644**	**1.2644**	**1.2644**	**D27*(D35^D21)**		
97	**F 3 SO F D**		**29.5043**	**29.5043**	**29.5043**	**D28*(D35^D22)**		
98	**F 3 RO F SS**		**12.2948**	**12.0232**	**11.7515**	**D32*(D35^D21)**		
99	**F 3 RO F DD**		**-18.7505**	**-18.3531**	**-17.9561**	**D33*(D35^D22)**		

Most of the benefit of the L increasing IMS would go to the follower when v=7.5, with a reduction in his negative PV OPS, and increase in RO F SS, the optional value for the follower of the leader switching (and thus giving the follower a temporary larger market share in Regime 2). The follower

[7] The leader could spend up to $.0394 to increase IMS by 1%, which would result in a total value for the L & F equal to the base case with equal IMS.

could not do better than simply encouraging the leader to increase her market share in this IMS scheme, or alternatively do nothing.

(2) Table 4 shows the leader contemplating increasing the MMS while still in the Regime 3, since the leader could spend up to \$.7445 in increasing the MMS 1%, which is the maximum expense for the leader that equates the total VF, A83=A84. The gross VFL increases slightly, indicating some potential "bang for the buck".

Table 4 R3B

	A	B	C	D	E	F	G
80		LEADER/FOLLOWER SPEND		0.7445	1% MMS		TOTAL VF
81	REGIME 3	vLD<v<vLS					
82	TOTAL VF	LEADER	FOLLOWER	LEADER	FOLLOWER		
83	44.7484	29.2381	15.5103	29.0661	14.2578	F .585	43.3239
84	44.7484	28.6833	16.0651	28.4936	14.7658	L.425/F.575	43.2594
85							
86		LEADER	FOLLOWER	LEADER	FOLLOWER		
87	L & F VF BASE	C90	C94	D90	D94	F .585	D83+E83
88	L .435 & F.565 V	B90-D80	B94	C90-D80	C94-D80	L/F .425/.575	D84+E84
89		L .435	BASE	F .585			
90	VF L	29.4278	29.2381	29.0661	SUM(D91:D93)		
91	L 3 PV OPS	-8.9286	-8.9286	-8.9286	D12*(D35/(D4+D11)-D5/D3)		
92	L 3 SO L S	13.8938	13.5616	13.2565	D30*(D35^D21)		
93	L 3 SO L D	24.4626	24.6051	24.7381	D31*(D35^D22)		
94	VF F	16.0651	15.5103	15.0023	SUM(D95:D99)		
95	F 3 PV OPS	-8.9286	-8.9286	-8.9286	D13*(D35/(D4+D11)-D5/D3)		
96	F 3 SO F S	1.4378	1.2644	1.0922	D27*(D35^D21)		
97	F 3 SO F D	29.4455	29.5043	29.5631	D28*(D35^D22)		
98	F 3 RO F SS	12.6864	12.0232	11.4171	D32*(D35^D21)		
99	F 3 RO F DD	-18.5761	-18.3531	-18.1415	D33*(D35^D22)		

(3) Table 5 shows a leader contemplating increasing the FMS while still in the Regime 3, with the leader spending up to \$.1888 in increasing the FMS 1%. While the L PV OPS in the initial regime is not affected by changing the MMS or FMS, both of the leader's strategic options to divest and switch in the initial stage are affected by the MMS and FMS. The VF net of \$$\Delta$ actually increases for the leader increases FMS, which is an effective strategy for the leader as long as the follower does not retaliate. Look to the future options, in assessing current choices.

Table 5 R3C

	A	B	C	D	E	F	G
80		LEADER/FOLLOWER SPEND		0.1888	1% FMS		TOTAL VF
81	REGIME 3C	vLD<v<vLS					
82	TOTAL VF	LEADER	FOLLOWER	LEADER	FOLLOWER		
83	44.7484	29.2381	15.5103	28.9196	15.4193	F .51	44.3389
84	44.7484	29.3423	15.4060	29.0493	15.3215	L/F .5	44.3708
85							
86		LEADER	FOLLOWER	LEADER	FOLLOWER		
87	L & F VF BASE	C90	C94	D90	D94-D80	F 0.51	D83+E83
88	L .51	B90-D80	B94	C90-D80	C94-D80	L/F .50	D84+E84
89		L .51	BASE	F .51			Change
90	VF L	29.5311	29.2381	28.9196	SUM(D91:D93)		-0.6115
91	L 3 PV OPS	-8.9286	-8.9286	-8.9286	D12*(D35/(D4+D11)-D5/D3)		0.0000
92	L 3 SO L S	14.0730	13.5616	12.9938	D30*(D35^D21)		-1.0792
93	L 3 SO L D	24.3867	24.6051	24.8544	D31*(D35^D22)		0.4676
94	VF F	15.4060	15.5103	15.6081	SUM(D95:D99)		0.2020
95	F 3 PV OPS	-8.9286	-8.9286	-8.9286	D13*(D35/(D4+D11)-D5/D3)		0.0000
96	F 3 SO F S	0.7562	1.2644	1.7903	D27*(D35^D21)		1.0340
97	F 3 SO F D	29.6788	29.5043	29.3272	D28*(D35^D22)		-0.3515
98	F 3 RO F SS	12.2629	12.0232	11.7550	D32*(D35^D21)		-0.5079
99	F 3 RO F DD	-18.3634	-18.3531	-18.3359	D33*(D35^D22)		0.0275

Table 6

Comparing Regime 3 Results

	B	C	D	E	F	G	H	I
1	P Game R3A							
2	TOTAL VF				**0.0394**	1% IMS		TOTAL VF
3	v<vLS	7.5						
4			LEADER	FOLLOWER	LEADER	FOLLOWER		
5	44.7484	BASE	29.2381	15.5103	**29.2517**	15.4175	F .51	44.6692
6								
7	44.7484	L .51	29.1854	**15.5630**	29.1987	15.4709	L/F .5	44.6696
8	P Game R3B							
9	v<vLS	7.5						
10	TOTAL VF				**0.7445**	1% MMS		TOTAL VF
11								
12			LEADER	FOLLOWER	LEADER	FOLLOWER		
13	44.7484	BASE	**29.2381**	15.5103	29.0661	14.2578	F .585	43.3239
14								
15	44.7484	L .435	28.6833	**16.0651**	28.4936	14.7658	.425/.575	43.2594
16	P Game R3C							
17	v<vLS	7.5						
18	TOTAL VF				**0.1888**	1% FMS		TOTAL VF
19								
20			LEADER	FOLLOWER	LEADER	FOLLOWER		
21	44.7484	BASE	29.2381	**15.5103**	28.9196	15.4193	F .51	44.3389
22								
23	44.7484	L .51	**29.3423**	15.4060	29.0493	15.3215	L/F .5	44.3708

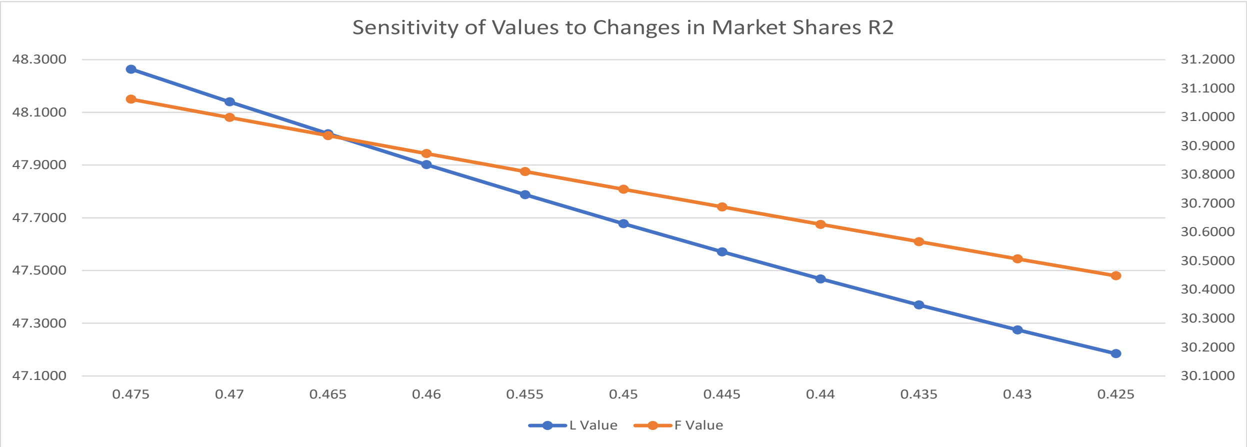
Table 6 compares regime 3 results when the PV Ops for both firms is negative, showing that for R3A the best strategy indicated in **bold** assuming the leader/follower spend .0394 is **E7** for the follower (leader spends $x to increase IMS to 51%), and **F5** for the leader. For R3B the best strategy for the leader is to do nothing **D13**, for the follower to encourage the leader to spend .7445 to increase MMS to 43.5% **E15**. For R3C the best strategy for the leader is to spend .1888 to increase the FMS to 51% **D23**, for the follower to remain in the base case **E21**. An in-depth explanation would examine the critical aspects of the effect of altering market share on the various option coefficients, involving the analytical and numerical partial derivatives. "Proofs" from the mathematical expression for each coefficient might show how an equilibrium is established as the best ultimate strategy in each of these games. One advantage of this approach using real options in game theory is the transparency in the payoff results, which are not necessarily obvious in most presentations of the Prisoners' Dilemma[8].

(4) But if v>vLS, say 10, the game is different, moving to Regime 2 (R2).

Table 7 shows the sensitivity of leader and follower value function to changes in the MMS, and FMS. If in regime 2 the leader increases market share from 42.5% to 47.5%, the value of both the leader and follower increase, but not proportionally. If while still in regime 2, the leader imagines increases FMS, the result is non-linear and opposite for the leader and follower.

Table 7

Sensitivity of Values to Changes in Market Share Regime 2



[8] An exception is Dockner et al. (2000).

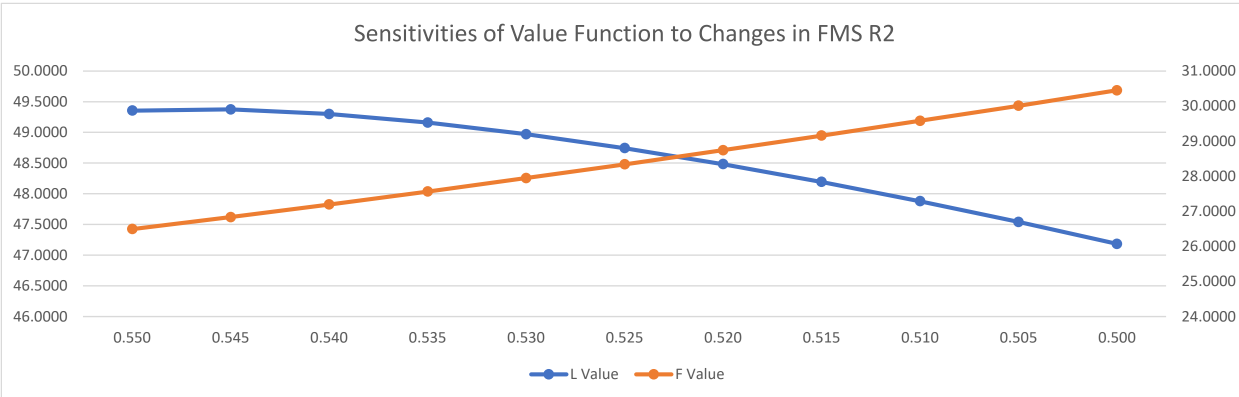


Table 8 R2A, v=10

	A	B	C	D	E	F	G
80	TOTAL VF	LEADER/FOLLOWER SPEND		0.3038	1% MMS		
81	REGIME 2A	v>vLS					
82		LEADER	FOLLOWER	LEADER	FOLLOWER		
83	77.6327	47.1845	30.4481	47.0172	30.0278	F .585	77.0450
84	77.6327	47.0657	30.5670	46.8807	30.1443	L/F .425/.575	77.0251
85							
86		LEADER	FOLLOWER	LEADER	FOLLOWER		
87	B83+C83	C90	C94	D90	D94-D80	F .585	D83+E83
88	B84+C84	B90	B94	C90-D80	C94-D80	L/F .425/.575	D84+E84
89		L .435	L .425	L .415			Change
90	VF L	47.3695	47.1845	47.0172	SUM(D91:D93)		-0.3523
91	**L 2 PV OPS**	**51.2679**	**50.0893**	**48.9107**	**D16*(D35/(D4+D11)-D6/D3)**		-2.3571
92	**L2 RO L SS**	**6.1016**	**7.0952**	**8.1064**	**D29*(D35^D21)**		2.0048
93	**L 2 K-Z**	**-10.0000**	**-10.0000**	**-10.0000**	**-(D8-D7)**		0.0000
94	VF F	30.5670	30.4481	30.3316	SUM(D95:D97)		-0.2354
95	**F 2 PV OPS**	**10.0893**	**10.2679**	**10.4464**	**D17*(D35/(D4+D11)-D5/D3)**		0.3571
96	**F 2 SO F S**	**2.7590**	**2.4262**	**2.0958**	**D27*(D35^D21)**		-0.6633
97	**F 2 SO F D**	**17.7187**	**17.7541**	**17.7894**	**D28*(D35^D22)**		0.0707

Now clearly the best strategy for the follower is for the leader to increase her MMS in Table 8 if $Δ<$.3038, which increases the follower's overall value, since no sensible follower would attempt to do the same.

Table 9 R2B, v=10

	A	B	C	D	E	F	G
80	TOTAL VF	LEADER/FOLLOWER SPEND		0.1335	1% FMS		TOTAL VF
81	REGIME 2A	v>vLS					
82		LEADER	FOLLOWER	LEADER	FOLLOWER		
83	77.6327	47.1845	30.4481	46.4154	**31.2173**	F .51	77.6327
84	77.4565	**47.8785**	29.5781	47.0510	30.3146	L/F .5	77.3657
85							
86		LEADER	FOLLOWER	LEADER	FOLLOWER		
87	B83+C83	C90	C94	D90	D94-D80	F .51	D83+E83
88	B84+C84	B90	B94	C90-D80	C94-D80	L/F .5	D84+E84
89		L .51	BASE	F .51			Change
90	VF L	47.8785	47.1845	46.4154	SUM(D91:D93)		-1.4631
91	**L 2 PV OPS**	**50.0893**	**50.0893**	**50.0893**	**D16*(D35/(D4+D11)-D6/D3)**		0.0000
92	**L2 RO L SS**	**7.7892**	**7.0952**	**6.3261**	**D29*(D35^D21)**		-1.4631
93	**L 2 K-Z**	**-10.0000**	**-10.0000**	**-10.0000**	**-(D8-D7)**		0.0000
94	VF F	29.5781	30.4481	31.3508	SUM(D95:D97)		1.7727
95	**F 2 PV OPS**	**10.2679**	**10.2679**	**10.2679**	**D17*(D35/(D4+D11)-D5/D3)**		0.0000
96	**F 2 SO F S**	**1.4512**	**2.4262**	**3.4354**	**D27*(D35^D21)**		1.9842
97	**F 2 SO F D**	**17.8591**	**17.7541**	**17.6475**	**D28*(D35^D22)**		-0.2115

(5) In this case, any increasing the FMS by the leader has a negative effect on the combined total VF of the leader and follower even at 0 cost, but increases the leader's value function. But the follower could spend $.1335 so that the combined total VF of both parties equals the base case, shown in Table 9, A83=G83. In that case, the decrease of the leader's value is due to the decrease in the L's RO L SS.

Table 10

Comparing Regime 2 Results

	A	B	C	D	E	F	G
1	SHARE GAME R2A						
2	v>vLS						
3	TOTAL VF				**0.3038**	1% MMS	TOTAL VF
4							
5		LEADER	FOLLOWER	LEADER	FOLLOWER		
6	77.6327	**47.1845**	30.4481	47.0172	30.0278	F .585	77.0450
7							
8	77.6327	47.0657	**30.5670**	46.8807	30.1443	L/F .425/.575	77.0251
9							
10	R2B						
11	v>vLS						
12	TOTAL VF		LEADER/FOLLO		**0.1335**	1% FMS	TOTAL VF
13							
14		LEADER	FOLLOWER	LEADER	FOLLOWER		
15	77.6327	47.1845	30.4481	46.4154	**31.2173**	F .51	77.6327
16							
17	77.4565	**47.8785**	29.5781	47.0510	30.3146	L/F .5	77.3657

Table 10 compares regime 2 results when the PV Ops for both firms is positive, showing that for R2A the best strategy for the follower is **C8** (leader spends $**Δ**=.3038 to increase MMS to 43.5%), and **B6** for the leader (do nothing). For R2B the best strategy for the leader is to spend .1335 on increasing the FMS 1% **B17**, for the follower to spend .1335 to increase FMS to 51% **E15, if neither retaliates as in D17:E17.**

III Price Volatility

Now we turn to a cooperative game of both the leader and follower getting the government (or another third party) to change in the "effective price volatility"[9] from 15% to 25% as indicated in Table 11. In regime 3 (v=7.5), it will pay for both parties to get the price volatility increased.

Table 11

	A	B	C	D
1			*VOLATILITY GAME R3*	
2	*INPUT*		*BASE*	
3	*r*	*0.08*	*0.08*	*0.08*
4	θ	*0.04*	*0.04*	*0.04*
5	*fX*	*10*	*10*	*10*
6	*fY*	*2*	*2*	*2*
7	*Z*	*25*	*25*	*25*
8	*K*	*35*	*35*	*35*
9	σ	***0.15***	***0.20***	***0.25***
10	λ	*0.2*	*0.2*	*0.2*
11	δ	*0.03*	*0.03*	*0.03*
12	*LXX*	*0.50*	*0.50*	*0.50*
13	*FXX*	*0.50*	*0.50*	*0.50*
14	*LOX*	*0.00*	*0.00*	*0.00*
15	*FOX*	*1.00*	*1.00*	*1.00*
16	*LYX*	*0.425*	*0.425*	*0.425*
17	*FYX*	*0.575*	*0.575*	*0.575*
18	*LYY*	*0.500*	*0.500*	*0.500*
19	*FYY*	*0.500*	*0.500*	*0.500*
20	*OUTPUT*			
21	β_1	*2.7228*	*2.2656*	*1.9757*
22	β_2	*(2.6117)*	*(1.7656)*	*(1.2957)*
23	*vFD*	*6.3599*	*5.7392*	*5.1441*
24	*vFS*	*9.9311*	*12.2631*	*16.5486*
25	*vLD*	*6.4101*	*6.0924*	*5.7394*
26	*vLS*	*7.6662*	*8.2585*	*9.0899*
27	*A1IIFS= SO F S*	0.0138	0.0132	-0.0032
28	*A2IIFD= SO F D*	4641.9220	1034.8147	472.5265
29	*A1IILSS=RO L SS*	0.0169	0.0385	0.0620
30	*A1IILS= SO L S*	0.0752	0.1412	0.1865
31	*A2IILD=SO L D*	3824.5225	862.9820	390.8268
32	*A1IIFSS=RO F SS*	0.0591	0.1252	0.2005
33	*A2IIFDD=RO F DD*	*-3330.4886*	*-643.7031*	*-267.8460*

Table 12 shows that the L's switch option SOLS decreases with an increase in p volatility -8.1665 for 15% to 25%, while the leader's divest option SOLD increases 8.8935, for a net gain of .727.

[9] Increasing the "effective market price volatility" might be achieved by removing any price caps, as established in several European countries in 2022 for natural gas and electricity. See Appendix D for real option values over high (25-35%) and low (15-25%) volatility ranges.

The F's divest option SOFD increases lots with an increase in p volatility, more than offsetting the decrease in the other three options when volatility increases from 15% to 25%.

Table 12 R3, v=7.5

	A	B	C	D	E	F
95	R3, v=7.5					
96	Volatility	**15%**	**20%**	**25%**		Change
97	VF L	29.0510	29.2381	29.7780	SUM(D98:D100)	0.7270
98	L 3 PV OPS	-8.9286	-8.9286	-8.9286	D12*(D35/(D4+D11)-D5/D3)	0.0000
99	**L 3 SO L S**	**18.1557**	**13.5616**	**9.9892**	**D30*(D35^D21)**	**-8.1665**
100	**L 3 SO L D**	**19.8239**	**24.6051**	**28.7174**	**D31*(D35^D22)**	**8.8935**
101	VF F	15.4749	15.5103	16.6836	SUM(D102:D106)	1.2087
102	F 3 PV OPS	-8.9286	-8.9286	-8.9286	D13*(D35/(D4+D11)-D5/D3)	0.0000
103	**F 3 SO F S**	**3.3416**	**1.2644**	**-0.1691**	**D27*(D35^D21)**	**-3.5108**
104	**F 3 SO F D**	**24.0608**	**29.5043**	**34.7205**	**D28*(D35^D22)**	**10.6598**
105	**F 3 RO F SS**	**14.2642**	**12.0232**	**10.7417**	**D32*(D35^D21)**	**-3.5225**
106	**F 3 RO F DD**	**-17.2631**	**-18.3531**	**-19.6809**	**D33*(D35^D22)**	**-2.4178**

This is a win-win game, since both the leader and the follower benefit if the volatility increases from 15% 25%, if v=7.5 for Regime 3, even if the L has to pay \$.727 (lobbying expense?) or the F has to pay \$1.2087 to increase volatility. Perhaps the L and F could share in the expense of promoting more p volatility.

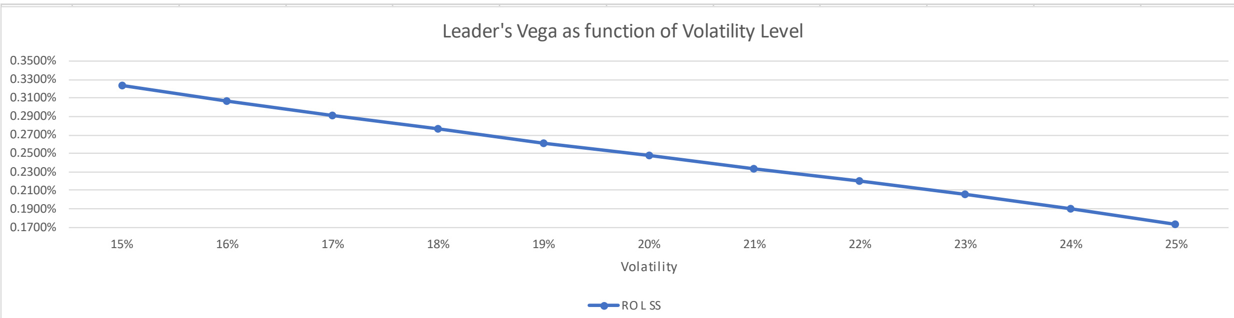
The consequences are reversed for the leader if v=9.5, above the leader's switching threshold (below the follower's) for Regime 2, as shown in Table 13.

Table 13 R2, v=9.5

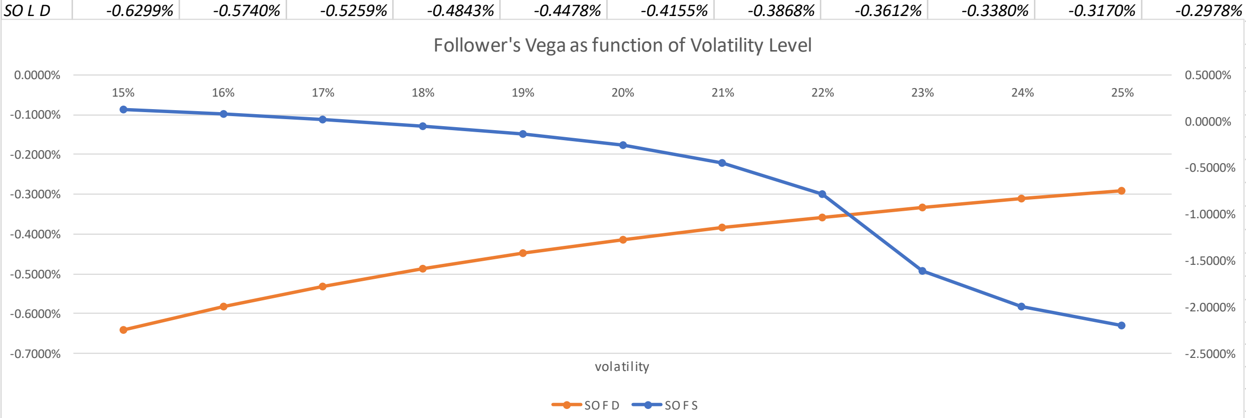
	A	B	C	D	E	F	G
80			L/F SPEND	0	5% Vol Change		
81	Regime 2, vLS<v<vFS		v=9.5				
82	TOTAL VF	LEADER	FOLLOWER	LEADER	FOLLOWER		
83	71.1281	43.3704	27.7577	42.3497	**31.4511**		
84	70.3199	**44.8213**	25.4986	44.8213	31.4511		
85							
86		LEADER	FOLLOWER	LEADER	FOLLOWER		
87	B83+C83	C90	C94	D90	**D94-D80**		
88	B84+C84	**B90-D80**	B94	B90-D80	D94-D80		
89	Volatility	**15%**	**20%**	**25%**			Change
90	VF L	44.8213	43.3704	42.3497	SUM(D100:D102)		**-2.4716**
91	L 2 PV OPS	47.0536	47.0536	47.0536	D16*(D35/(D4+D11)-D6/D3)		0.0000
92	**L2 RO L SS**	**7.7677**	**6.3168**	**5.2961**	**D29*(D35^D21)**		**-2.4716**
93	L 2 K-Z	-10.0000	-10.0000	-10.0000	-(D8-D7)		0.0000
94	VF F	25.4986	27.7577	31.4511	SUM(D104:D106)		**5.9525**
95	F 2 PV OPS	6.1607	6.1607	6.1607	D17*(D35/(D4+D11)-D5/D3)		0.0000
96	**F 2 SO F S**	**6.3605**	**2.1600**	**-0.2698**	**D27*(D35^D21)**		**-6.6303**
97	**F 2 SO F D**	**12.9774**	**19.4370**	**25.5602**	**D28*(D35^D22)**		**12.5828**
98	*vFD*	*6.3599*	*5.7392*	*5.1441*			
99	*vFS*	*9.9311*	*12.2631*	*16.5486*			
100	*vLD*	*6.4101*	*6.0924*	*5.7394*			
101	*vLS*	*7.6662*	*8.2585*	*9.0899*			

The leader would prefer less volatility, the follower more volatility. This pattern is consistent with the examination of the vegas, the sensitivity of option coefficients to change in the level of volatility (total volatility derivative), shown in Table 14. The leader has only one option left at Regime 2, the benefit of the follower switching, when then the leader's market share increases. The higher the volatility, the higher the vFS. There are two additional factors affecting RO LSS: as volatility increases the option coefficient increases but at a decreasing rate (partial); as volatility increases the power β_1 falls; so. the net effect is the value of the RO LSS rival option falls. For the follower, while the value of the SOFS option to switch falls as the volatility increases, indeed at a rate which changes sign, the value of the SOFD increases slightly at a decreasing rate. The net effect is the value function of the leader falls, of the follower increases, as volatility increases.

Table 14



σ	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%
SO F S	*0.1317%*	*0.0813%*	*0.0230%*	*-0.0472%*	*-0.1367%*	*-0.2596%*	*-0.4469%*	*-0.7835%*	*-1.6162%*	*-2.0000%*	*-2.2000%*
SO F D	*-0.6402%*	*-0.5812%*	*-0.5306%*	*-0.4867%*	*-0.4483%*	*-0.4145%*	*-0.3845%*	*-0.3577%*	*-0.3336%*	*-0.3118%*	*-0.2921%*
RO F SS	*0.2887%*	*0.2762%*	*0.2645%*	*0.2535%*	*0.2432%*	*0.2333%*	*0.2238%*	*0.2147%*	*0.2058%*	*0.1970%*	*0.1882%*
RO F DD	*0.6924%*	*0.6324%*	*0.5806%*	*0.5353%*	*0.4954%*	*0.4600%*	*0.4283%*	*0.3998%*	*0.3740%*	*0.3507%*	*0.3294%*
RO L SS	*0.3238%*	*0.3071%*	*0.2912%*	*0.2761%*	*0.2616%*	*0.2475%*	*0.2337%*	*0.2198%*	*0.2055%*	*0.1904%*	*0.1735%*
SO L S	*0.2654%*	*0.2452%*	*0.2256%*	*0.2066%*	*0.1879%*	*0.1695%*	*0.1511%*	*0.1326%*	*0.1138%*	*0.0943%*	*0.0736%*
SO L D	*-0.6299%*	*-0.5740%*	*-0.5259%*	*-0.4843%*	*-0.4478%*	*-0.4155%*	*-0.3868%*	*-0.3612%*	*-0.3380%*	*-0.3170%*	*-0.2978%*



Now, suppose the actual market volatility is 25%, but this has been reduced to 20% by the government offering fixed prices at 9.5 for 20% of the production for both parties. The leader has the opportunity to reduce volatility to 15% by hedging B84, and the follower has the chance of increasing volatility to 25% by declining the government assistance E83. The base case is B83:C83 for the leader/follower. The optimal equilibrium is for the leader to hedge and for the follower to "unhedge" (but this could change if the cost of altering "effective volatility" is not zero). In this case the follower benefits a lot, due to an increase in the SO FD (curiously, since v=9.5 is far from the vFD=5.14 at σ=25%). The leader loses from any volatility increase, due to the decline in RO L SS[10].

There are many other interesting real option games in duopolies that can be imagined, changing other parameter values. For instance, what is the effect of changing interest rates, or the percentage of Z that the follower receives on divestment, on the L/F values?

IV Summary and Conclusion

We build on previous solutions for mutually exclusive options in a duopoly with switching and divestment alternatives. We examine the implications of increasing the leader's market share and/or changing volatility at progressive regimes. The consequences of market share and volatility changes on the values for both the leader and follower are often surprising.

What are lessons for the leader attempting to increase market share during any regime? Game strategy is highly dependent on the level of the market revenue. With the assumed parameter values, it is hard for the leader to benefit from increasing market share when v is low, but sometimes benefits when v is high. Surprisingly, (i) the leader loses with increased initial market share at low revenues, but (ii) both leader and follower lose with increased middle market shares but (iii) both gain at higher revenues. (iv) The leader gains with higher volatility at low revenue levels, but loses at higher revenues.

[10] This is not an accurate calculation, since the values for the leader are based on the recalculated vFS, see (13) Appendix A. If the RO L SS are based on the vFS with 25% volatility, the value function for the leader would be at best 42.3497, or 2.4716 lower than the B84 result. But since the follower's value function is worth 31.4511, or 3.6934 more than the base case; part of this value increase might be used to compensate the leader (see Appendix E).

So, in a cooperative risk game, when v is low, the leader should lead a risk preferring strategy. When v is high, the follower benefits from more risk, the leader does not, so cooperation and collusion regarding future volatility are complex "risk" games.

There are many opportunities for future research on this topic. (1) Importantly, some of the "surprises" call for explanations and interpretations. (2) Recalculating option values as thresholds change with parameter value changes is warranted. (3) Showing that the changes in option values are consistent in sign and magnitude with the partial market share derivatives should be feasible. (4) Examining the trade-offs between the expense of changing market share and the value obtained should not be difficult. (5) Naturally, a probability of follower retaliation should be incorporated into almost all of these games.

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Supplementary Appendix

A Complete Sensitivities to Changing Market Shares

B Partial and Total Derivatives (to Market Share) across Three Regimes

C Complete Solutions for Real Option Games (Market Share)

D Real Option Values for High/Low Volatility Ranges

E Recalculation of the Risk Game Leader's Value Function

ROGs Appendix

A Formulae for Thresholds and Option Coefficients

The follower's two thresholds $\hat{v}_{FS}$ and $\hat{v}_{FD}$ are the solutions from two non-linear simultaneous equations:

$$\hat{v}_{FD}{}^{\beta_2}\left(\hat{v}_{FS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}-\frac{D_{F|Y,Y}f_Y-D_{F|Y,X}f_X}{r}-(K-\lambda Z)\right)-\hat{v}_{FS}{}^{\beta_2}\left(\lambda Z-\frac{D_{F|O,X}\hat{v}_{FD}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}+\frac{D_{F|O,X}f_X}{r}\right)=0 \quad \text{(A1)}$$

$$\hat{v}_{FD}{}^{\beta_1}\left(\hat{v}_{FS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}-\frac{D_{F|Y,Y}f_Y-D_{F|Y,X}f_X}{r}-(K-\lambda Z)\right)-\hat{v}_{FS}{}^{\beta_1}\left(\lambda Z-\frac{D_{F|O,X}\hat{v}_{FD}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}+\frac{D_{F|O,X}f_X}{r}\right)=0 \quad \text{(A2)}$$

The leader's two thresholds $\hat{v}_{LS}$ and $\hat{v}_{LD}$ are the solutions to two non-linear simultaneous equations:

$$\hat{v}_{LD}{}^{\beta_2}\left(\hat{v}_{LS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}-\frac{D_{L|Y,X}f_Y-D_{L|X,X}f_X}{r}\right)-(K-Z)-\hat{v}_{LS}{}^{\beta_2}\left(Z-\frac{D_{L|X,X}\hat{v}_{LD}}{\delta+\theta}\frac{\beta_1-1}{\beta_1}+\frac{D_{L|X,X}f_X}{r}\right)=0 \quad \text{(A3)}$$

$$\hat{v}_{LD}{}^{\beta_1}\left(\hat{v}_{LS}\frac{D_{L|Y,X}-D_{L|X,X}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}-\frac{D_{L|Y,X}f_Y-D_{L|X,X}f_X}{r}+SOLS\quad\hat{v}_{LS}{}^{\beta_1}\frac{\beta_2-\beta_1}{\beta_2}-(K-Z)\right)-\hat{v}_{LS}{}^{\beta_1}\left(Z-\frac{D_{L|X,X}\hat{v}_{LD}}{\delta+\theta}\frac{\beta_2-1}{\beta_2}+\frac{D_{L|X,X}f_X}{r}\right)=0 \quad \text{(A4)}$$

The follower's strategic switching and divestment option coefficients are:

$$\text{SOFS}=\frac{1}{\beta_1\Delta_F}\left(\hat{v}_{FS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\hat{v}_{FD}{}^{\beta_2}+\hat{v}_{FD}\frac{D_{F|O,X}}{\delta+\theta}\hat{v}_{FS}{}^{\beta_2}\right) \quad \text{(A5)}$$

$$SOFD=\frac{1}{\beta_2\Delta_F}\left(-\hat{v}_{FS}\frac{D_{F|Y,Y}-D_{F|Y,X}}{\delta+\theta}\hat{v}_{FD}{}^{\beta_1}+\hat{v}_{FD}\frac{D_{F|O,X}}{\delta+\theta}\hat{v}_{FS}{}^{\beta_1}\right) \quad \text{(A6)}$$

$$\text{where } \Delta_F=\hat{v}_{FS}{}^{\beta_1}\hat{v}_{FD}{}^{\beta_2}-\hat{v}_{FS}{}^{\beta_2}\hat{v}_{FD}{}^{\beta_1}. \quad \text{(A7)}$$

The follower's rival options (exercise determined by the leader, benefits the follower are:

$$ROFSS = \left(D_{F|Y,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{LS}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{LD}{}^{\beta_2}}{\Delta_L} - \left(D_{F|O,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{LD}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{LS}{}^{\beta_2}}{\Delta_L} \quad \text{(A8)}$$

$$ROFDD \quad = -\left(D_{F|Y,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{LS}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{LD}{}^{\beta_1}}{\Delta_L} + \left(D_{F|O,X} - D_{F|X,X}\right)\left(\frac{\hat{v}_{LD}}{\delta+\theta} - \frac{f_X}{r}\right)\frac{\hat{v}_{LS}{}^{\beta_1}}{\Delta_L} \quad \text{(A9)}$$

The leader's strategic switching and divestment option coefficients are:

$$SOLS \quad = \frac{1}{\beta_1 \Delta_L}\left(\left(\hat{v}_{LS}\frac{D_{L|Y,X} - D_{L|X,X}}{\delta+\theta} + \beta_1 ROLSS \quad \hat{v}_{LS}{}^{\beta_1}\right)\hat{v}_{LD}{}^{\beta_2} + \hat{v}_{LD}\frac{D_{L|X,X}}{\delta+\theta}\hat{v}_{LS}{}^{\beta_2}\right) \quad \text{(A10)}$$

$$SOLD \quad = -\frac{1}{\beta_2 \Delta_L}\left(-\left(\hat{v}_{LS}\frac{D_{L|Y,X} - D_{L|X,X}}{\delta+\theta} + \beta_1 ROLSS \quad \hat{v}_{LS}{}^{\beta_1}\right)\hat{v}_{LD}{}^{\beta_1} - \hat{v}_{LD}\frac{D_{L|X,X}}{\delta+\theta}\hat{v}_{LS}{}^{\beta_1}\right) \quad \text{(A11)}$$

where $\Delta_L = \hat{v}_{LS}{}^{\beta_1}\hat{v}_{LD}{}^{\beta_2} - \hat{v}_{LS}{}^{\beta_2}\hat{v}_{LD}{}^{\beta_1}$. (A12)

The leader's rival option (exercise determined by the follower, benefits the leader) is:

$$\text{ROLSS} \quad = \left(\frac{\hat{v}_{FS}}{\delta+\theta} - \frac{f_Y}{r}\right)\left(D_{L|Y,Y} - D_{L|Y,X}\right)\hat{v}_{FS}{}^{-\beta_1} \quad \text{(A13)}$$

Dias (2004) was the first to suggest that mutually exclusive options (MEO) must be treated differently than several perpetually available real options. Décamps et al. (2006) provided the essential theory for such MEO, as described in Adkins et al. (2022).