Infrastructure investment in public transport: a strategic real options game

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Abstract: This paper investigates the investment in public transport infrastructure and service under the road capacity allocation dedicated to buses. Future demand is the main source of uncertainty and, to account for it properly, real options analysis is applied. The utilized real options model calculates the optimal bus frequency level and the optimal timing of investing by considering the monetary costs next to time costs, mainly congestion costs, of a public transport trip. Furthermore, the market structure is analyzed by investigating either a vertically integrated case, in which one decision maker holds the option to invest in the bus frequency and a dedicated bus lane at the same time or a vertically separated case, in which two decision makers are active and invest separately in the service and infrastructure. The infrastructure manager decides about investing in a dedicated bus lane first, and then the service provider invests in the bus frequency offered. A numerical example illustrates the effect of the market structure on the optimal results. The vertically integrated case is preferable, from a welfare perspective, than the vertically separated decision. Increasing congestion costs leads to earlier and smaller investment.

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1 Introduction

This paper aims to investigate the investment in public transport infrastructure and service under the allocation of the road capacity dedicated to buses. Cities face transportation problems like congestion, which causes poor public transport service quality, and high impact on local and global emissions. One answer is the provision of dedicated bus lanes, which the local authority did, for example, at the Columbia street corridor in Seattle in early 2020. Such a policy results in a smoother and faster commute by offering more reliable trips. Several groups have analyzed and modeled the effect that the implementation of bus lanes would result into. Basso and Silva (2014) investigated the effect of the implementation of a dedicated bus lane for London, UK and Santiago, Chile. Their results indicate that bus lanes improve service levels and decrease fare. Börjesson et al. (2017) have analyzed the situation in Stockholm, where bus lanes are already implemented. They concluded that, in presence of car congestion, providing the cars with more road space is welfare-improving. None of the two mentioned studies considered the time component in their analysis. Saphores and Boarnet (2006) optimized the time of congestion relief investment under population uncertainty. The outcome of their study is that the optimal timing increases with the population volatility, and that ignoring uncertainty leads to investing prematurely.

In our study, we have built a strategic real options game to analyze the effect of relieving the congestion for the public transport service. To do so, the model of CITE was used. The model consists in constructing a Stackelberg game in which two firms compete, and determining the optimal timings and investment size. In our case, two decision makers were also considered. Opposite to the case of CITE, these two agents interact positively. These decision makers are the service provider (SP), which invests in the bus frequency, and the infrastructure manager (IM), who builds a dedicated bus lane. The infrastructure manager increases the SP's current profit by relieving the congestion, while the SP's bus frequency investment increases the social surplus. The SP's objective function is to maximize profit, whereas the IM's maximizes the social surplus. The aim of this study is to find the optimal investment timing in the bus lane, which reliefs the congestion, and the optimal timing and size of the bus frequency offered under demand uncertainty.

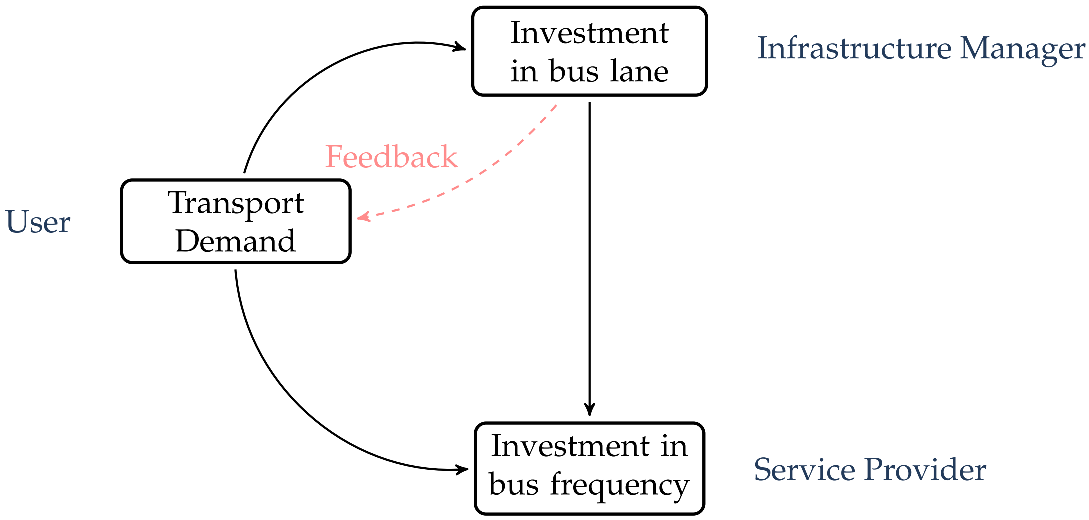
The rest of the paper is structured as follows. Section 2 presents and discusses the utilized real option model. Section 3 focuses on the analytical solution of the model and section 4 illustrates the results numerically. Section 5 concludes.

2 Model

We consider two decision markers in the transport market: the first one is the infrastructure manager (IM), who holds the option to invest in a dedicated bus lane. The second one is the public transport service provider(SP), who holds the option to invest in frequency. Both face irreversible investment costs and uncertain transport demand. They interact with each other in a Stackelberg Game. The IM, *e.g.* the government, decides about the timing of investment in a bus lane first and then the SP, *e.g.* the public transport operator, can either invest immediately or wait until she expands her bus frequency offered. Figure 1 describes the relationships between the different actors in the public transport market.

The transport demand is characterized by the monetary cost of a trip and non-monetary costs of a trip, called the generalized travel time. Here, we explicitly model the congestion costs. The government influences the transport demand positively. By introducing a dedicated bus lane, the IM dissolves the congestion problem for the bus users. This investment leads to a reduction in the cost of the trip for the

Figure 1   Overview of the relationships in the public transport market



users.

The generalized user costs of a trip equal:

$$g(p,f) = p + \frac{\alpha}{2f} + va + vb(f\theta + Q_c) \quad (1)$$

where p is the bus fare, f is the frequency, α is the Value of Waiting Time (VoWT), v is he Value of in-vehilce Time (VoT), a is the free flow time, b is the extra congestion costs per car-vehicle, θ is the car-equivalence factor, and Q_c is the fixed total car demand. The first term represents the monetary cost of a trip and the last terms stand for the travel time costs. It is assumed that the users are arriving randomly at the station and, hence, the average waiting time costs ($\frac{\alpha}{2f}$) reduces to half the headway times the VoWT. The in-vehicle travel time costs consits of two parts the first one describes the average in-vehicle travel time costs at the free-flow time (va), and the last term reflects the extra congestion costs per car of the in-vehicle travel time costs as cars interact with buses on the road ($vb(f\theta + Q_c)$) and, therefore, both, cars and buses, delay the arrivals of the service.

Taking a linear demand function $Q = A - bGP$ and the fact that in equilibrium the generalized user cost equals the generalized price ($g = GP$), one obtains the inverse demand expression

$$p = A\eta - \eta q f - \alpha\frac{1}{2f} - va - vb(f\theta + Q_c) \tag{2}$$

where $\eta = b^{-1}$.

One needs to distinguish three different demand situations: the first one, *e.g.* mixed traffic with the initial bus service, under which the government and the public transport operator have not invested yet, here, the initial buses share the road with the cars. The second one, *e.g.* dedicated bus lane with the initial bus service, in which the government has undertaken the investment, and the service provider has not invested yet, so the buses ride on their lane and congestion dissolves for them. The last one, *e.g.* dedicated bus lane with the bus service expansion, in which the service provider has undertaken his investment in the growth of the bus service.

The stochastic inverse demand for public transport in the case of shared road capacity is described in the following:

$$p_{f_0}^{MT} = X_t\left[A\eta - \eta q f_0 - \alpha\frac{1}{2f_0} - va - vb(f_0\theta + Q_c)\right] \tag{3}$$

After the infrastructure manager undertakes her investment, the congestion is relieved and the stochastic inverse demand under dedicated bus lanes can be described as:

$$p_{f_0}^{BL} = X_t\left[A\eta - \eta q f_0 - \alpha\frac{1}{2f_0} - va\right] \tag{4}$$

After the service provider undertakes her investment in the bus service expansion as well, the stochastic inverse demand under dedicated bus lanes can be described as:

$$p_{f_E}^{BL} = X_t \left[A\eta - \eta q(f_0 + f_E) - \alpha \frac{1}{2(f_0 + f_E)} - va \right] \quad (5)$$

The stochastic demand shifter (X_t) follows a geometric Brownian Motion (gBm). The gBm is modelled as follows:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad (6)$$

in which μ is the drift rate, $\sigma > 0$ is the implied volatility, and dW_t is the increment of a Wiener process.

Here, we are modelling a simple network which is characterized by a one mode bus route, a homogeneous group of users, fixed car demand and considering the peak-period.

In the following, we will obtain the analytical solution for two cases: (i) the vertically integrated case which one decision-maker has the option to invest simultaneously in bus frequency and bus lane and (ii) the vertically separated case which is characterized by the Stackelberg game. Here, the infrastructure manager (IM) is the leader and decides about the investment timing in a dedicated bus lane first. Whereas, the service provider (SP) is the follower and holds the option to invest in the bus frequency expansion.

In the former case, the decision maker has one objective function, and this is to maximize the social welfare $\left(\int_0^Q p \ \ dQ \right)$[1]. In the latter case, there are two actors. The infrastructure manager maximizes the social welfare, while the service provider maximizes the profit ($\pi = p \ qf$).

[1]Q equals qf

3 Analytical Solution

3.1 Vertically Integrated Case

Following Kort et al. (2010), the decision maker can either invest simultaneously in the bus lane and the bus frequency expansion (lumpy investment) or can split the investment, first invest in the bus lane and then in the bus frequency expansion (stepwise investment). In the former, $\{V\}_{SW}^{lumpy}$ denotes the project's value of the government's optimal investment decision in the lumpy investment case

$$V_{SW}^{lumpy} = \max_{T \geq 0; f_E \geq 0} E\Bigg[\int_{t=0}^{T} e^{-rt} X_t SW_{f_0}^{MT} \, dt + \int_{t=T}^{\infty} e^{-rt} X_t SW_{f_E}^{BL} \, dt - e^{-rT_F}(\delta_{BL} + \delta_f f_E) \Bigg| X_0 = X \Bigg] \tag{7}$$

The value consists of three terms: the first describes the social welfare under the mixed traffic condition wih the initial bus frequency, the second describes the social welfare after both investment has been taken place, and the last term summarizes both investment costs in the dedicated bus lane and the bus service expansion.

In the latter, $\{V\}_{SW}^{step}$ denotes the project's value of the government's optimal investment decision in the stepwise investment case

$$\begin{aligned} V_{SW}^{step} = \max_{T_{BL} \geq 0; T_F \geq 0; f_E \geq 0} E\Bigg[& \int_{t=0}^{T_{BL}} e^{-rt} X_t SW_{f_0}^{MT} \, dt \\ & + \int_{t=T_{BL}}^{T_F} e^{-rt} X_t SW_{f_0}^{BL} \, dt - e^{-rT_{BL}} \delta_{BL} \\ & + \int_{t=T_F}^{\infty} e^{-rt} X_t SW_{f_E}^{BL} \, dt - e^{-rT_F} \delta_f f_E \Bigg| X_0 = X \Bigg] \end{aligned} \tag{8}$$

The value consists of five terms: the first describes the social welfare under the mixed traffic condition with the initial bus frequency, the second and third describe the social welfare and their costs after the investment in the dedicated bus lane, and the last two terms describe the social welfare and their costs after the investment in the bus service expansion. The investment problem is solved as an optimal stopping problem in dynamic programming. Three conditions are applied, the optimal frequency ($\frac{\partial V}{\partial f} = 0$), smooth pasting, and value matching condition to obtain the optimal thresholds and optimal bus frequency expansion.

Proposition 1 *The optimal investment threshold X^* and the optimal bus frequency expansion f_E^* of the social welfare maximizing operator with dedicated bus lanes for the lumpy investment case are given by*

$$X^* = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)(\delta_f f_E^* + \delta_{BL})}{\left[SW_{f_E}^{BL} - SW_{f_0}^{BL}\right] + \left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right]}, \tag{9}$$

$$f_E^* = \frac{1}{\eta q}\left[A\eta - va - \frac{\delta_f (r-\mu)}{X^* q}\right] - f_0 \tag{10}$$

Proposition 2 *The optimal investment thresholds X_{BL}^*, X_F^* and the optimal bus frequency expansion f_E^* of the social welfare maximizing operator with dedicated bus lanes for the stepwise investment case are given by*

$$X_{BL}^* = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_{BL}}{\left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right]}, \tag{11}$$

$$X_F^* = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_f f_E^*}{\left[SW_{f_E}^{BL} - SW_{f_0}^{BL}\right]}, \tag{12}$$

$$f_E^* = \frac{1}{\eta q}\left[A\eta - va - \frac{\delta_f (r-\mu)}{X_F^* q}\right] - f_0 \tag{13}$$

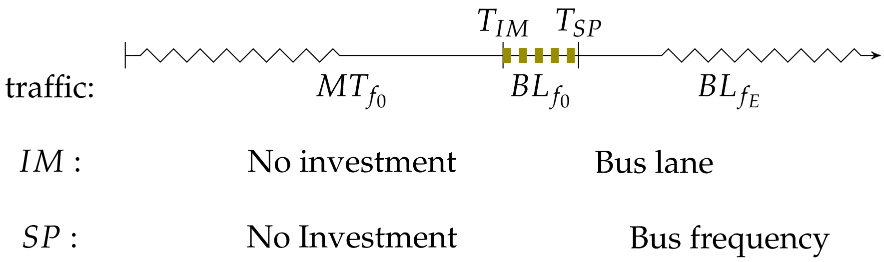
1. $\left[SW_{f_E}^{BL} - SW_{f_0}^{BL}\right]$ reflects the benefits of the bus frequency expansion, while $\delta_f f_E (r-\mu)$ reflects the associated costs.

2. $\left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right]$ reflects the benefits of the introduction of a dedicated bus

lane, while $\delta_{BL}(r-\mu)$ reflects the associated costs.

3.2 Vertically Separated Case

Figure 2 Timing decision's in the vertically separated case



In the vertically separated case the investments in either the bus lane and the bus frequency expansion are undertaking by itself by two single actors. The problem is solved as a Stackelberg game. The infrastructure manager (IM) decides about the investment timing in dedicated bus lane first, then the service provider (SP) holds the option to invest in the service expansion. The IM maximizes the social welfare, while the SP maximizes profits.

Value of the IM (leader)

$$
\begin{aligned}
V_{IM} = \max_{T_{IM}\geq 0} E\Bigg[\int_{t=0}^{T_{IM}} & e^{-rt} X_t SW_{f_0}^{MT}\, dt \\
& + \int_{t=T_{IM}}^{T_{SP}} e^{-rt} X_t SW_{f_0}^{BL}\, dt + \int_{t=T_{SP}}^{\infty} e^{-rt} X_t SW_{f_E}^{BL}\, dt \\
& - e^{-rT_{IM}} \delta_{BL} \Bigg| X_0 = X \Bigg]
\end{aligned} \tag{14}
$$

The value of the IM consists of four terms: i) the social benefit of the initial bus frequency without any investment, ii) the social benefit of the introduction of the bus lane after the investment of the IM, iii) the social benefit of the bus frequency expansion with the dedciated bus lane in place, and iv) the investment costs for the bus lane at the IM's investment timing.

Value of the SP (follower)

$$
\begin{aligned}
V_{SP} = \max_{T_{SP}\geq 0, f_E \geq 0} E\Bigg[\int_{t=0}^{T_{IM}} e^{-rt} X_t \pi_{f_0}^{MT}\, dt + \int_{t=T_{IM}}^{T_{SP}} e^{-rt} X_t \pi_{f_0}^{BL}\, dt \\
+ \int_{t=T_{SP}}^{\infty} e^{-rt} X_t \pi_{f_E}^{BL}\, dt \\
- e^{-rT_{SP}} \delta_f f \Bigg| X_0 = X \Bigg]
\end{aligned}
\tag{15}
$$

The value of the SP consists of four terms: i) the profit of the initial bus frequency without any investment, ii) the profit of the introduction of the bus lane after the investment of the IM, iii) the profit of the bus frequency expansion with the dedciated bus lane in place, and iv) the investment costs for the bus frequency expansion at the SP's investment timing.

By applying *Bellman equation* and *Ito's lemma* one gets the value in the stopping and continuation region for the IM and SP.

Value of the IM (leader)

$$
F_{IM}(X) = \begin{cases}
\frac{SW_{f_0}^{MT}}{(r-\mu)} X + A_{IM;1} X^{\beta_1} & \text{if } X_{IM}^* > X \\
\frac{SW_{f_0}^{BL}}{(r-\mu)} X - \delta_{BL} + \left[\frac{X}{X_{SP}^*}\right]^{\beta_1} \frac{\left[SW_{f_E}^{BL} - SW_{f_0}^{BL}\right] X_{SP}^*}{(r-\mu)} & \text{if } X_{SP}^* > X \geq X_{IM}^* \\
\frac{SW_{f_E}^{BL}}{(r-\mu)} X - \delta_{BL} & \text{if } X \geq X_{SP}^*.
\end{cases}
\tag{16}
$$

Three regions can be distinguished. The first one is the continuation region which is characterized by the social benefit under the mixed traffic situation with the initial bus frequency in place and the option value to invest in the dedicated bus lane. The second one is the value after the investment of the IM and before the investment of the SP. Here the value consists of three terms: i) the social benefit with the initial frequency and the bus lane, ii) the investment costs of the bus lane itself, and iii) the discounted extra social benefit of the bus frequency expansion of the SP, which positively influences the value. Two situations can accur either the public transport firm, who is here the follower, can either invest at the same time as the IM or later. The first one is referred to the simultaneous policy and the latter one to the sequential investment policy. The SP's optimal investment timing (X_{SP}^*) is the upper boundary of the sequential region and the lower boundary of the simultaneous region.

Value of the SP (follower)

$$F_{IM}(X) = \begin{cases} \frac{\pi_{f_0}^{MT}}{(r-\mu)} X & \text{if } X_{IM}^* > X \\ \frac{\pi_{f_0}^{BL}}{(r-\mu)} X + A_{SP;1} X^{\beta_1} & \text{if } X_{SP}^* > X \geq X_{IM}^*. \\ \frac{\pi_{f_E}^{BL}}{(r-\mu)} X - \delta_f f_E & \text{if } X \geq X_{SP}^*. \end{cases} \tag{17}$$

Three regions can be distinguished. The first one is the continuation region which is characterized by the profit under the mixed traffic situation with the initial bus frequency in place and the option value to invest in the dedicated bus lane. The second one is the value after the investment of the IM and before the investment of the SP. Here the value consists of two terms: i) the profit with the initial frequency and the bus lane, and ii) the option value of the bus frequency expansion of the SP. The last region is the value after investment if the SP.

Proposition 3 *The optimal investment threshold X^* and the optimal bus frequency expansion f_E^* of the service provider are given by*

$$X_{SP}^* = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_f f_E^*}{\left[\pi_{f_E}^{BL} - \pi_{f_0}^{BL}\right]}, \tag{18}$$

$$f_E^* = \frac{1}{2\eta q} \left[A\eta - va - \frac{\delta_f (r-\mu)}{X_{SP}^* q} \right] - f_0 \tag{19}$$

1. $\left[\pi_{f_E}^{BL} - \pi_{f_0}^{BL}\right]$ reflects the benefits of the bus frequency expansion, while

$\delta_f f_E(r-\mu)$ reflects the associated costs.

Proposition 4 *The optimal investment threshold X^* of the infrastructure manager under the sequential investment policy*

$$X_{OPT}^{SEQ} = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_{BL}}{\left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right]} \tag{20}$$

Proposition 5 *The optimal investment threshold X^* of the infrastructure manager under the simultaneous investment policy*

$$X_{OPT}^{SIM} = \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_{BL}}{\left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right] + \left[SW_{f_E}^{BL} - SW_{f_0}^{BL}\right]} \tag{21}$$

When does the sequential strategy occur?

$$\begin{gathered}
X_{SP}^* > X_{OPT}^{SEQ} \\
\frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_f f_E^*}{\left[\pi_{f_E}^{BL} - \pi_{f_0}^{BL}\right]} > \frac{\beta_1}{(\beta_1 - 1)} \frac{(r-\mu)\delta_{BL}}{\left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right]} \\
\frac{\left[SW_{f_0}^{BL} - SW_{f_0}^{MT}\right]}{\delta_{BL}} > \frac{\left[\pi_{f_E}^{BL} - \pi_{f_0}^{BL}\right]}{\delta_f f_E^*} \\
BCR_{BL} > BCR_{f_E}
\end{gathered}$$

Result 1 *If the Benefit-Cost-Ratio of the dedicated bus lane (BCR_{BL}) is larger than the Benefit-Cost-Ratio of the bus frequency expansion (BCR_{f_E}), than the sequential strategy occurs. Otherwise, the optimal strategy is that both decision makers invest at the same point in time. If the optimal strategy is that both invest at the same time, the largest optimal investment threshold,* e.g. *the later point in time, of the two agents under the simultaneous investment is taken.*

4 Numerical results

A numerical exercise illustrates the obtained results.

Table 1 Parameter Values

r	discount rate	0.07	
μ	drift GBM	0.015	
σ	volatility GBM	0.1	
Q_0	number of trips per hour	1000	$\frac{trips}{hour}$
q	bus capacity	100	$\frac{pax}{bus}$
Q_C		100	
Investment costs			
δ_f	frequency's investment costs	84	$\frac{\$}{h}$
δ_{BL}	bus lane's investment costs	10000	$\frac{\$}{h}$
User costs			
p_0	fare (per trip)	60	$\frac{¢}{trip}$
α	Value of the Waiting Time	15	$\frac{\$}{hour}$
v	Value of the In-vehicle Time	10	$\frac{\$}{hour}$
f_0	inital frequency (per hour)	10	$\frac{bus}{hour}$
θ	car-equivalence factor	2	$\frac{car}{bus}$
a	free flow speed time	0.5	$hour$
b	extra time per car	0.5	$\frac{min}{car}$
Demand parameters			
$\epsilon_{GP,0}$	elasticity of trip wtr GP	-2.2	
η	inverse of the demand slope	0.0074	
A	constant linear demand	3200	
β_1		2.87	
X_0	starting value	1	

4.1 Optimal Solution

Table 2 Optimal Solution

market	investment case		goal	X^*	f^*	welfare
vertically integrated	lumpy	BL & F	SW	0.72	14	319118
		BL	SW	1.21	14.53	233327
	stepwise	F	SW	0.12	7.89	-
vertically separated	IM-Seq	BL	SW	1.21	-	-
	IM-Sim	BL	SW	1.15	2.25	164111
	SP	F	profit	0.35	1.36	-

$x_0 = 1$ and $f_0 = 10$; BL - bus lane and F - frequency

4.2 changing Parameter

In the following, we will investigate the effect on the optimal results of changing the bus investment costs, the congestion costs, Value of Time (VoT), and the bus capacity. First, the vertically integrated decision is derived (Figure 3-6). It turns out that lumpy investment is always preferable. And in a second step compared to the vertically separated investment decision (Figure 7-10).

Figures 3-6 show the results for optimal frequency, optimal threshold, and welfare in the vertically integrated case for the lumpy and stepwise investment. The social welfare maximizing decision in the lumpy investment case is illustrated in the squared-solid line (lump), while for the stepwise case the investment in the bus lane by the circular-solid line (BL) and the investment decisions in the bus frequency by the triangular-dashed line (F). The lumpy investment is always preferable to the stepwise investment decision.

Figures 3-6 show the results for optimal frequency, optimal threshold, and welfare in the vertically separated case and compare them to the lumpy investment decision of the vertically integrated firm. The dashed line represents the threshold of the service provider (SP.OPT), the squared-solid line represents the optimal threshold of the infrastructure manager for the sequential strategy (IM.SeqOpt), and the triangular-solid line represents the optimal threshold of the infrastructure manager for the simultaneous strategy (IM.SimOpt). The two policy areas can be depicted: If IM's sequential investment threshold is smaller than the SP's investment threshold, then the sequential strategy (red area) occurs between these two results. Otherwise, the simultaneous policy (blue area) is optimal, of which the timing is depicted by the larger investment threshold between the IM's simultaneous or the SP's threshold. If the level of X is lower than for the two possible policies, the best what the decision makers can do is to wait. The resulting optimal bus frequency expansion is illustrated in the middle graph. Thereby, the SP's frequency (dashed line) is compared to the resulting frequency of simultaneous strategy (triangular-dashed line). The solution of the vertically integrated case is illustrated by the circular-solid line. The vertical integrated case is always preferable to the vertically separated decision.

Figure 3 VI - bus investment costs

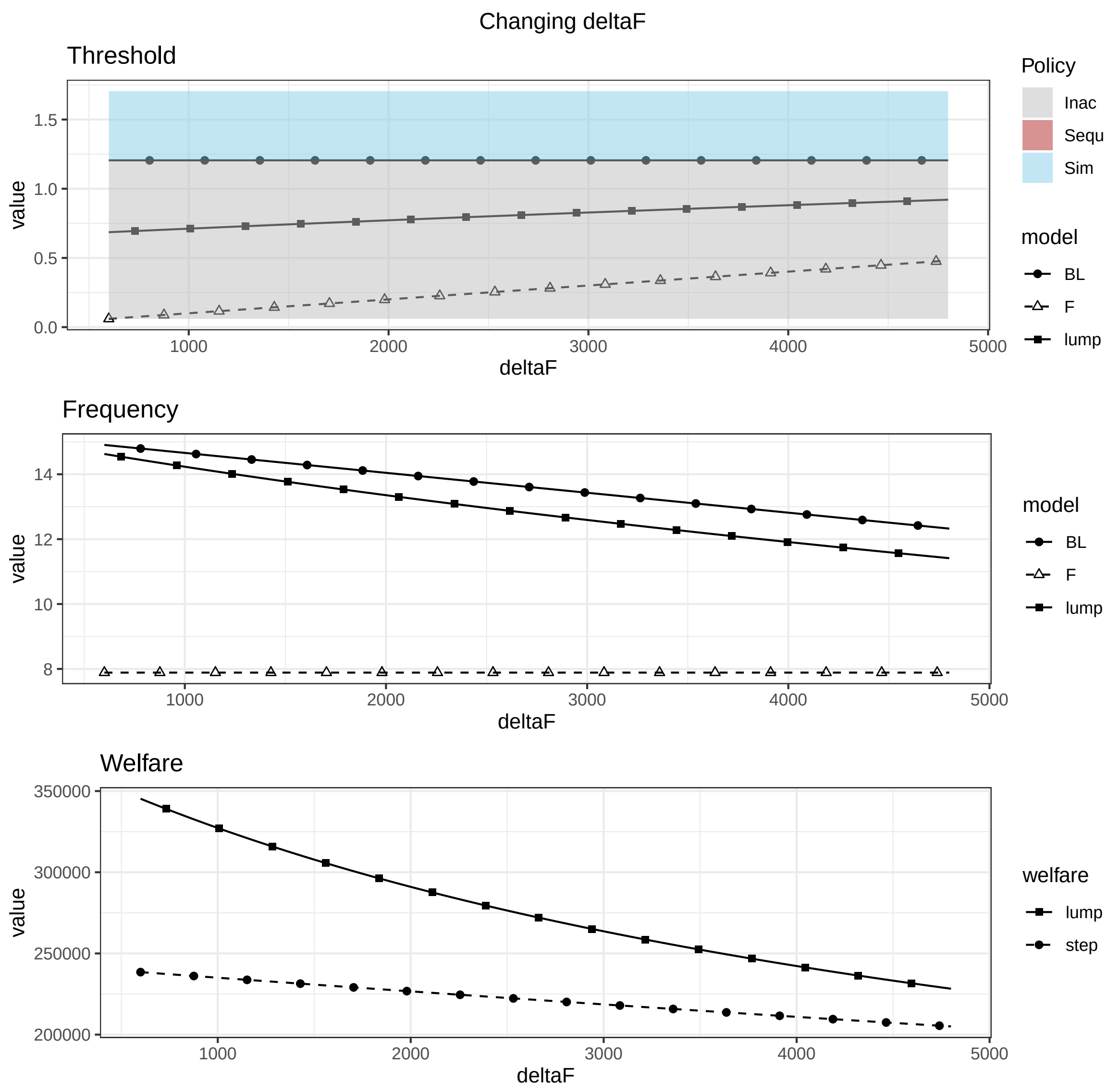


Figure 4 VI - congestion costs

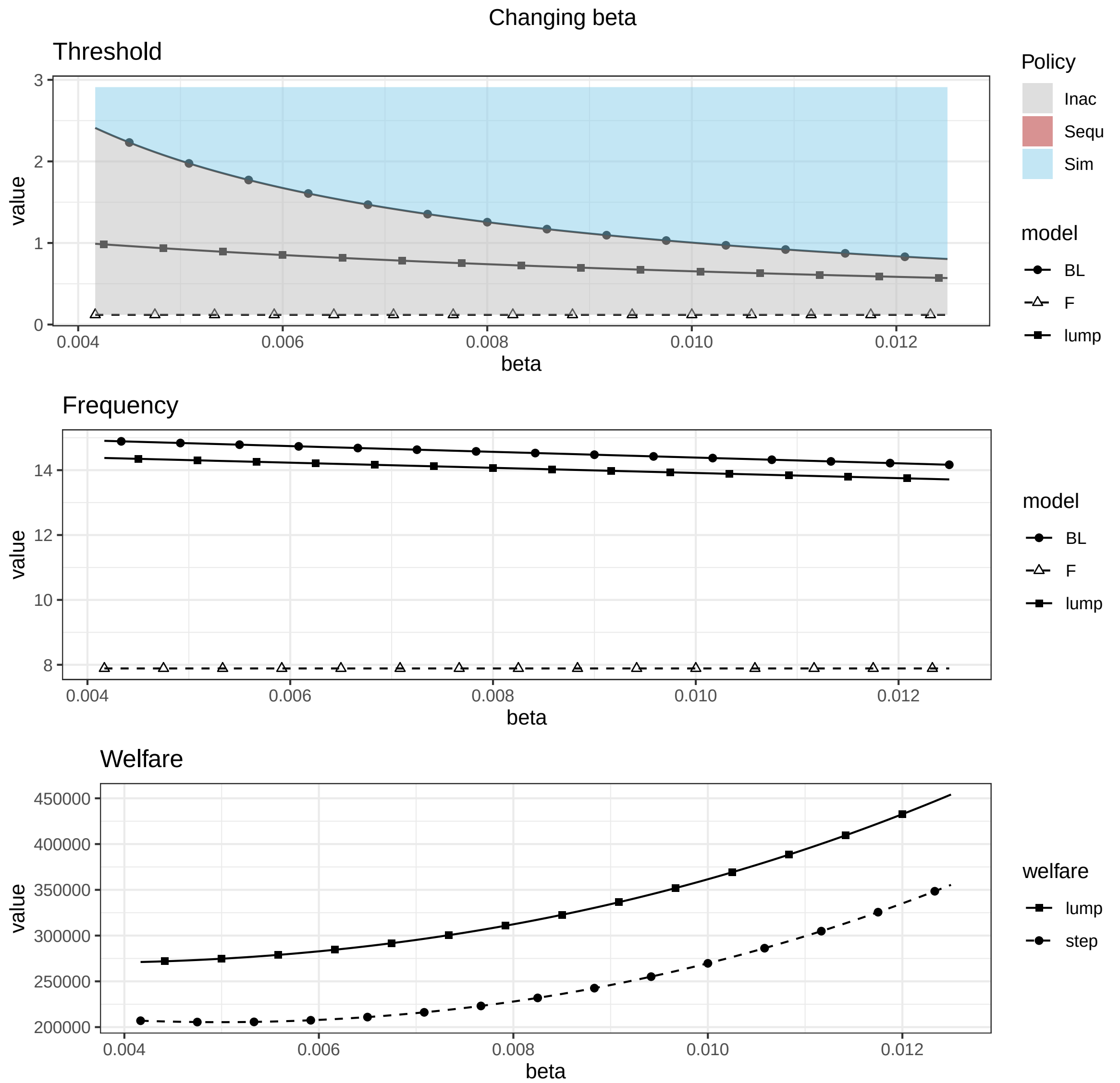


Figure 5   VI - Value of Time

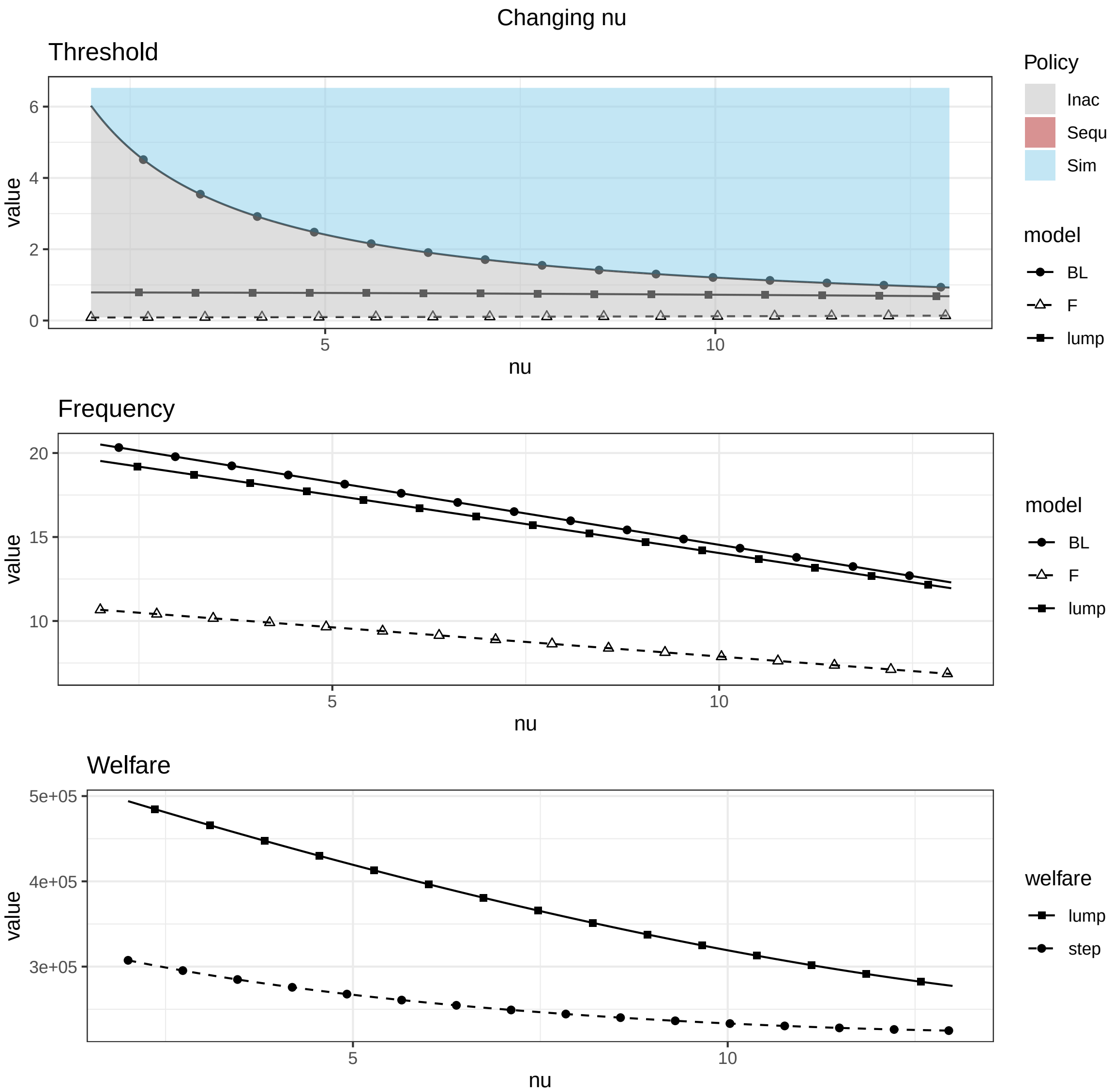


Figure 6 VI - bus capacity

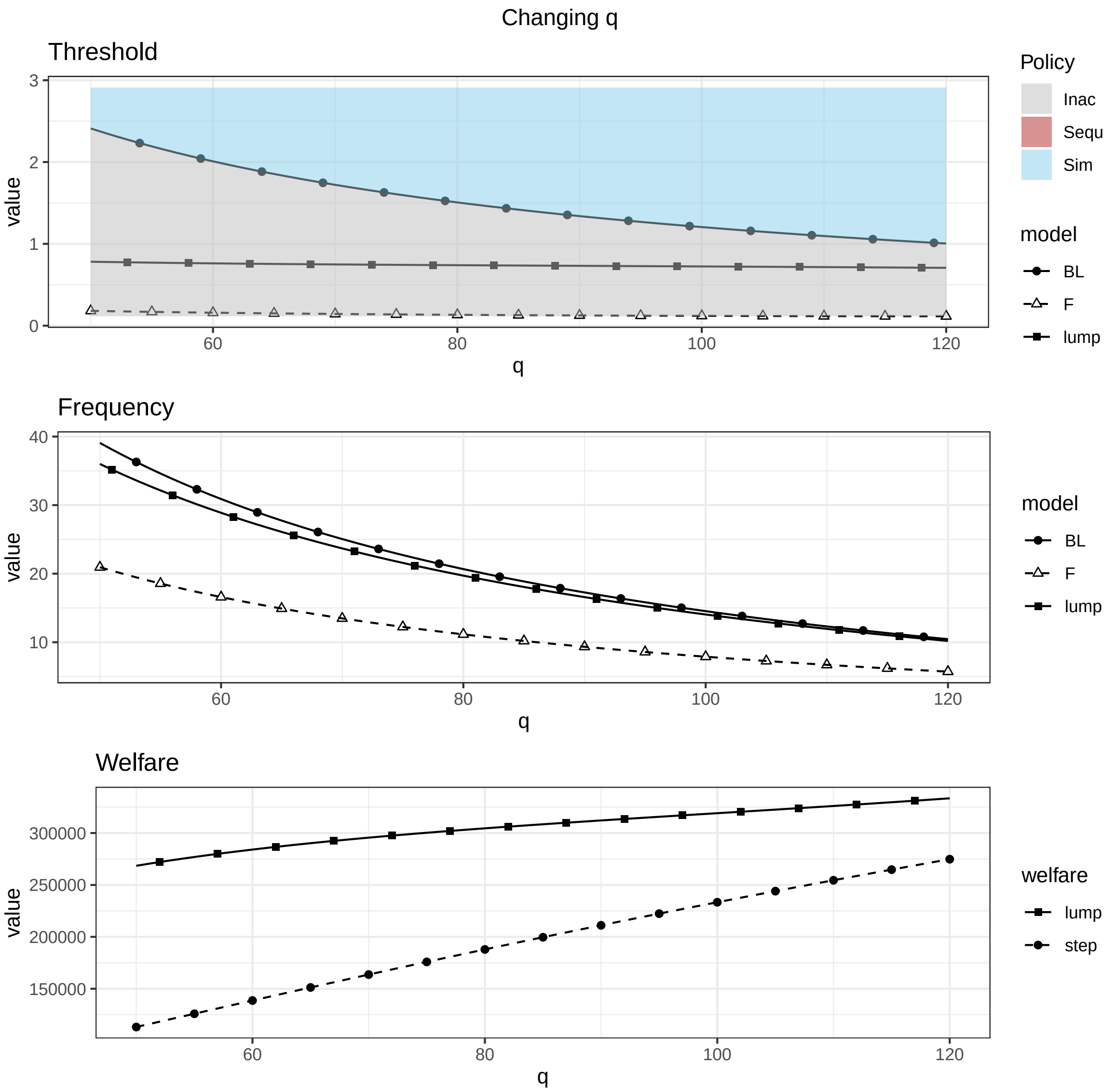
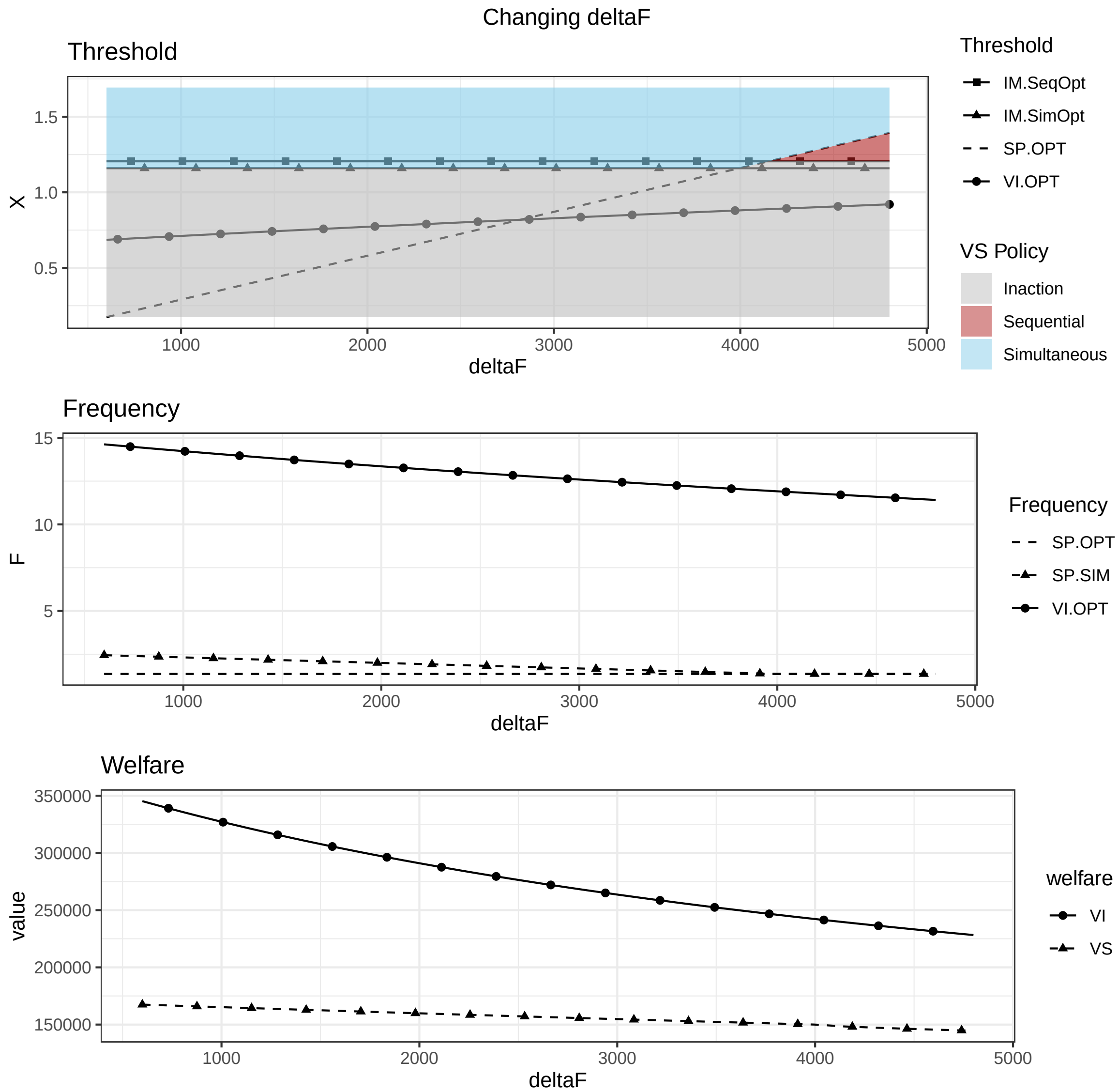
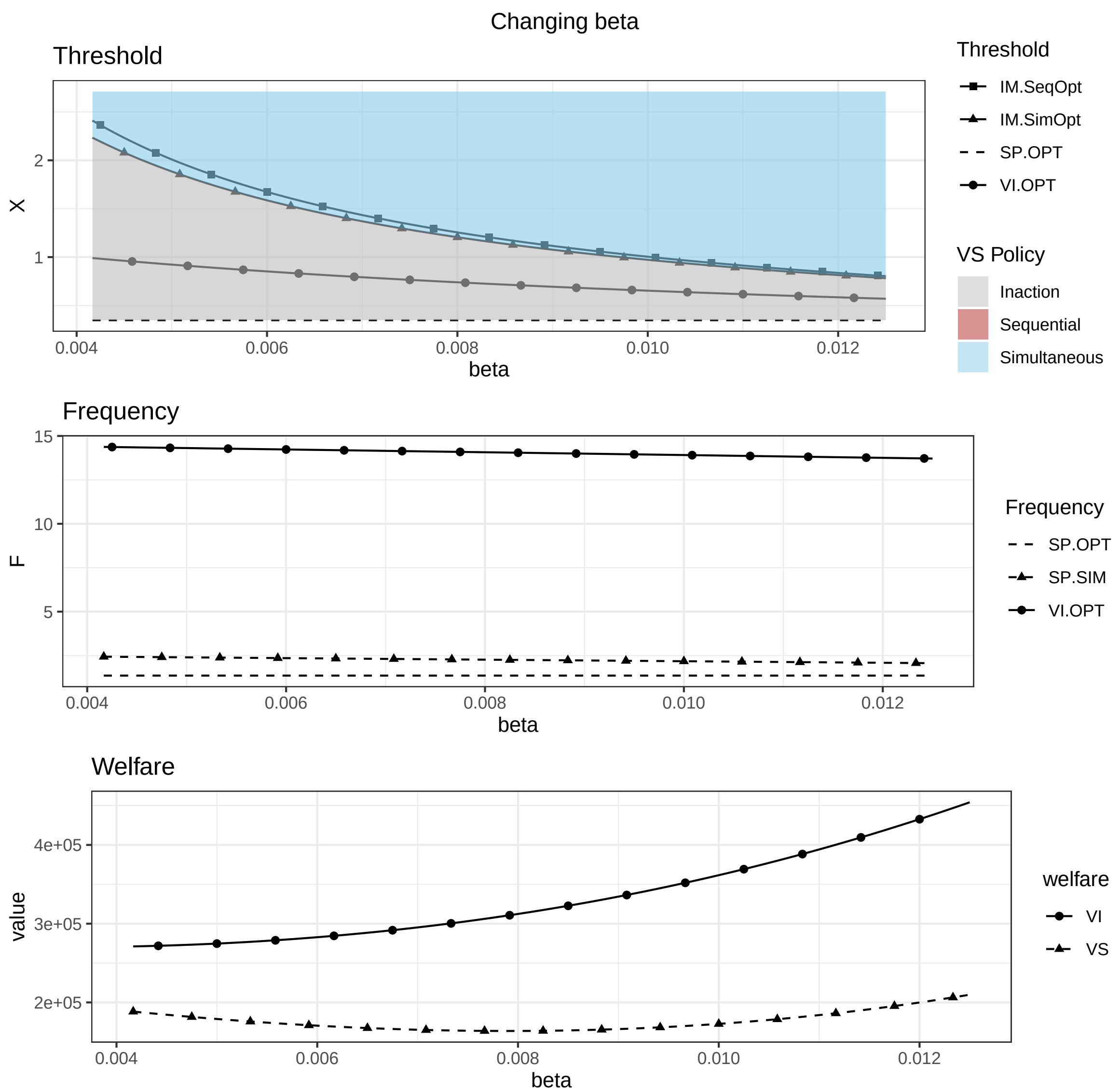


Figure 7 VS - bus investment costs



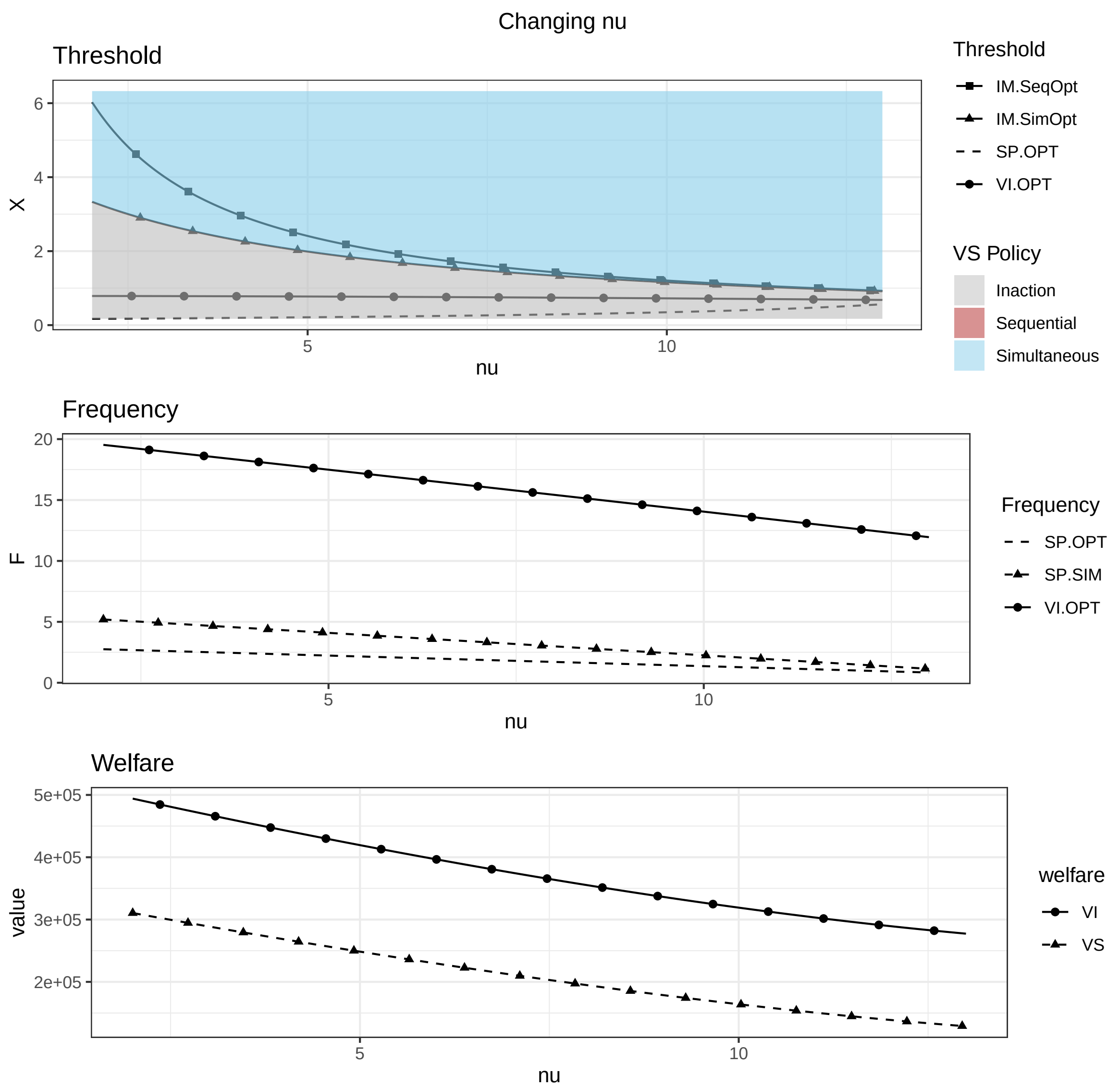
The higher the investment's cost of the SP is, the higher becomes the sequential policy region. The reason is the following, the optimal investment timing of the SP increases with higher costs in the bus frequency, while the optimal thresholds of the IM are unaffected. For low investment costs, the simultaneous strategy occurs, which implies that the service provider invests not at her optimal threshold, *e.g.* she must wait. Therefore, the optimal frequency under the simultaneous policy is higher as desired from the transit firm alone. Conversely, for high investment costs, the infrastructure manager invests earlier, and the service provider can invest at her wanted timing. This results in the optimal frequency of the transit firm.

Figure 8 VS - congestion costs



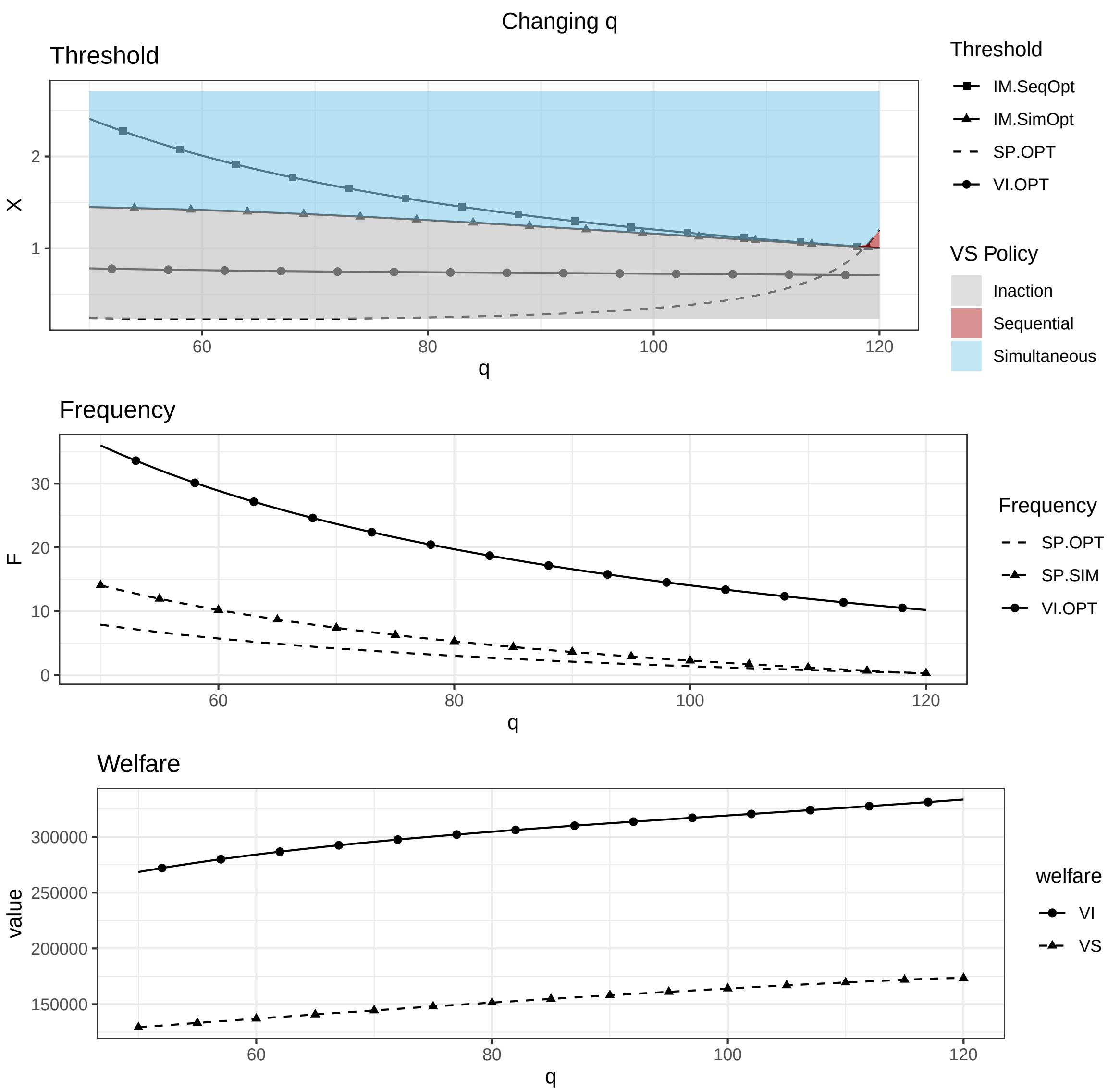
The higher the congestion cost of the users are, the higher becomes the sequential policy region. The reason is the following, the optimal investment timings of the IM decreases with higher congestions costs because the benefit of introducing the bus lane increases thereby, while the optimal threshold of the SP is unaffected. For low congestion costs, the simultaneous strategy occurs, which implies that the service provider invests not at her optimal threshold, *e.g.* she must wait. Therefore, the optimal frequency under the simultaneous policy is higher as desired from the transit firm alone. Conversely, for high congestion costs, the infrastructure manager invests earlier, and the service provider can invest at her wanted timing. This results in the optimal frequency of the transit firm.

Figure 9 VS - Value of Time



The higher the value of in-vehicle time (VoT) is, the higher becomes the sequential policy region. The reason is the following, the optimal investment timing of the SP increases with VoT as the free-flow time is valued higher. In contrast, the optimal thresholds of the IM decreases as the benefit of investing becomes larger. For low investment costs, the simultaneous strategy occurs, which implies that the service provider invests not at her optimal threshold, *e.g.* she must wait. Therefore, the optimal frequency for low VoT is higher as desired from the transit firm alone. Conversely, for high VoT, the infrastructure manager invests earlier, and the service provider can invest at her wanted timing. This results in the optimal frequency of the transit firm.

Figure 10 VS - bus capacity



5 Conclusion

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