Extended Abstract
Strategic Investment Under Uncertainty in a Triopoly Market: Timing and Capacity Choice

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Abstract

This paper analyzes investment decisions under uncertainty in a triopoly market. We determine the investment timing and the scale of investment of firms under the condition that firms hold a heterogenous cost structure. In a sense this paper combines Huisman and Kort (2015) and Shibata (2016). The former considers a duopoly market where the firms optimize their timing and capacity size to enter the market. Shibata (2016) just concentrates on investment timing and finds for a triopoly market that the first investor is not always the lowest-cost firm in the market. Until now an economic interpretation is lacking. This paper provides an economic interpretation for Shibata's result, based on the timing that the second investor preempts the third. To derive the optimal investment timing and the scale of investment, we apply real options methods. The investment timing and the capacity size of the firms are derived backwards. The aim of this extension is to generalize Shibata's result in a model in which the firms not only determine investment timing but also the size of the investment.

Keywords: Investment under uncertainty; Strategic investment; Triopoly, Timing and capacity choice

1 Introduction

In recent decades, investment timing plays a crucial role in realizing the value of investments. More recently, the real options theory has produced many models considering the irreversible investment decisions by firms. These models are mainly focused on "investment under uncertainty". Indeed, the models investigate the timing of the investment in imperfect markets.

Some recent studies on the "strategic investment under uncertainty" examine the investment timing decisions in the triopoly market of firms. Bouis, Huisman, and Kort (2009) study investment timing decisions in the market of more than two identical firms (symmetric cost structures). Shibata (2016) investigates the triopoly market with heterogenous cost structure for firms. The paper concludes that when firms have different cost structures in the triopoly market, it is possible the firm without the lowest-cost structure to enter as the first investor in the market. However, this result is left without any economic intuition. In this paper, the economic interpretations is provided based on the timing that the second investor preempts the third.

The scale of investment is the other element which has been investigated in the literature. The latter becomes significantly important for firms to enter a competitive market. In a duopoly market, Huisman and Kort (2015) find that the entry deterrence region increases with uncertainty. Meaning that when the uncertainty increases, the second investor prefers to wait to accumulate more information before investing. This leads to the longer monopoly period for the first investor which makes an entry deterrence strategy more profitable. Huisman and Kort (2015) extends the literature by analyzing the dynamic model of the entry deterrence/accommodation strategies. The contribution of this paper is to extend the duopoly model in Huisman and Kort (2015) into the triopoly market. The aim of this extension is to generalize Shibata's result in a model in which the firms not only determine investment timing but also the size of the investment.

The set up for this paper is the following. As preliminaries, in section (2), we present Shibata (2016) triopoly model with asymmetric cost structure and the main results of his work. In that paper, Shibata finds that the firm with the lowest cost structure is not always the first investor in the market. In section (3), we give an economic interpretation to the mentioned result which is missing in Shibata (2016). Motivated by this intriguing result of an inefficient firm entering the market before the efficient firm, the ultimate aim of our paper is to check whether a similar result, e.g. scenarios exist where Firm B invests first, is also present if the firms optimize their investment decision with respect to the investment size. In section (4) and (5), we present our model and its solution which investigate the firms' optimal investment decisions based on optimal investment timing and capacity choice. In section (6), the numerical results of our model is presented. Finally, section (7) gives the main result of this paper.

2 Preliminaries

In this part, we present Shibata (2016) triopoly model with asymmetric cost structure and the main results of his work. Shibata (2016) finds that the firm with the lowest cost structure is not always the first investor in the market.

As in Shibata (2016), consider a triopoly market in which firms take their investment. Lets show the cost of investment in the market with I. Following the asymmetric cost structure, firms hold different costs in order to take an investment decision in the market. The asymmetric cost structure between Firm A, Firm B, and Firm C is imposed as $I^A < I^B < I^C$. The instant profit flow from an investment is given by

$$P = Y(t)D_n \tag{1}$$
$$dY(t) = \mu Y(t)dt + \sigma Y(t)dW(t), \tag{2}$$

where in equation (1), "D_n" stands for the competition between firms with a subscript defining the number of operating firms in the market. This implies that profit is declining as more firms enter the market ($D_1 > D_2 > D_3 > 0$). And, Y(t) shows the profit flow of an investment in the market. In equation (2), let Y(t) follow the geometric Brownian motion process, $\mu > 0$ is a constant drift rate, $\sigma > 0$ determines a constant variance and $dW(t)$ represents a Wiener process. For convergence, we assume that $r > \mu$ and $r > 0$ is a constant interest rate. Finally, it is assumed that the demand is sufficiently low such that firms do not enter the triopoly market immediately.

As in Shibata (2016), the value functions for the investors are derived. Following the dynamic programming method, first, the value function of the third investor is derived which is non-strategic. Second, the strategic investment decision of the second firm is obtained. This implies the assumption that the first investor already entered the market. Finally, the first investor's value function is represented.

2.1 Value Functions

2.1.1 Third Investor

The value of the third firm is derived assuming that the first and the second firms already entered the market. Similar to Bouis et al. (2009), the value function of the third investor is given as

$$V_{12}(Y) = \begin{cases} \left(\frac{Y}{Y_{12}^k}\right)^\beta \left(\frac{Y_{12}^k D_3}{r-\mu} - I^k\right) & \text{if } Y < Y_{12}^k, \\ \frac{Y D_3}{r-\mu} - I^k & \text{if } Y \geq Y_{12}^k. \end{cases} \tag{3}$$

The first line of equation (3) represents the option value to invest. Meaning that the firm holds the option to invest when the threshold of Y(t) is not sufficient enough to make an investment. The second line of equation (3) shows the immediate investment value of the third firm when the current demand level reaches the optimal investment threshold of the third investor.

2.1.2 Second Investor

Assuming that the first investor already entered the market, similar to Bouis et al. (2009), the value function of the second investor is given by

$$V_{11}(Y) = \begin{cases} \frac{Y D_2}{r-\mu} - I^j + \left(\frac{Y}{Y_{12}^k}\right)^\beta \frac{Y_{12}^k (D_3 - D_2)}{r-\mu} & \text{if } Y < Y_{11}^{jk}, \\ \frac{Y D_3}{r-\mu} - I^j & \text{if } Y \geq Y_{11}^{jk}. \end{cases} \tag{4}$$

The first line of equation (4) represents the option value of waiting when the current demand level (Y) is less than the optimal threshold of entry for the second investor (Y_{11}^{jk}). Whereas the second line stands for the immediate investment in the market. In the second line, the first term represents the revenue stream of investment, the second term shows the cost of investment, and the last term denotes the option value lost due to the entry of the third investor in the market.

2.1.3 First Investor

The value function of the first investor, similar to Bouis et al. (2009), is derived with respect to the best response (the optimal investment timing) of the second and the third investors.

$$V_{10}(Y) = \begin{cases} \frac{YD_1}{r-\mu} - I^i + \left(\frac{Y}{Y_{11}^{jk}}\right)^{\beta} \frac{Y_{11}^{jk}(D_2-D_1)}{r-\mu} + \left(\frac{Y}{Y_{12}^{k}}\right)^{\beta} \frac{Y_{12}^{k}(D_3-D_2)}{r-\mu} & \text{if } Y < Y_{11}^{jk}, \\ \frac{YD_2}{r-\mu} - I^i + \left(\frac{Y}{Y_{12}^{k}}\right)^{\beta} \frac{Y_{12}^{k}(D_3-D_2)}{r-\mu} & \text{if } Y_{11}^{jk} \leq Y < Y_{12}^{k} \\ \frac{YD_3}{r-\mu} - I^i & \text{if } Y \geq Y_{11}^{jk}. \end{cases} \quad (5)$$

In the first line of equation (5), the current demand level is less than the investment threshold of the second investor, all firms will wait to receive more information from the market. In the first line, $\left(\frac{Y}{Y_{11}^{jk}}\right)^{\beta}$ and $\left(\frac{Y}{Y_{12}^{k}}\right)^{\beta}$ are the stochastic discount factors. The First term represents the discounted revenue of the first firm, the Second term denotes the investment cost of entering the market as the first investor, and the third and the fourth terms denote the option value lost due to the entry of the second and the third investors in the market. Considering the second line of equation (5), it shows that the value function in range of $[Y_{11}^{jk}, Y_{12}^{k})$. If the current demand lies in this interval, the first and the second firms invest simultaneously. And the third line refers to the value function of a firm in case of joint investment in the market. Meaning that the current demand level is vary high such that all investors enter the market at the same time.

2.2 Model Solution

As in Shibata (2016), we investigate the game-theoretic model solution in the triopoly market. Moreover, we analyze the game between firms in three steps backwardly using dynamic programming. Applying the derived value functions, the first step is to find the third investor in the market. All three operating firms A, B, and C are considered to be the third investor (third node) (see Figure (1)). Then we analyze the next sub-games to find the second investor (Second node). One can see six possible scenarios when considering the competition between two firms to enter the market as the second firm. Finally, in the first investor node, we represent Shibata (2016) in which the optimal investment threshold of the first investor is derived by comparing the thresholds of possible scenarios (First node). Based on the cost asymmetry assumption, we expect firm C, the highest-cost firm, to enter the market as the last investor. The cost efficient firm (Firm A) is supposed to be the first and Firm B to be the second investors in the market. In addition, the competition between Firm B and Firm C resembles the same situation in which the cost efficient firm in the duopoly framework to be the first to enter the market.

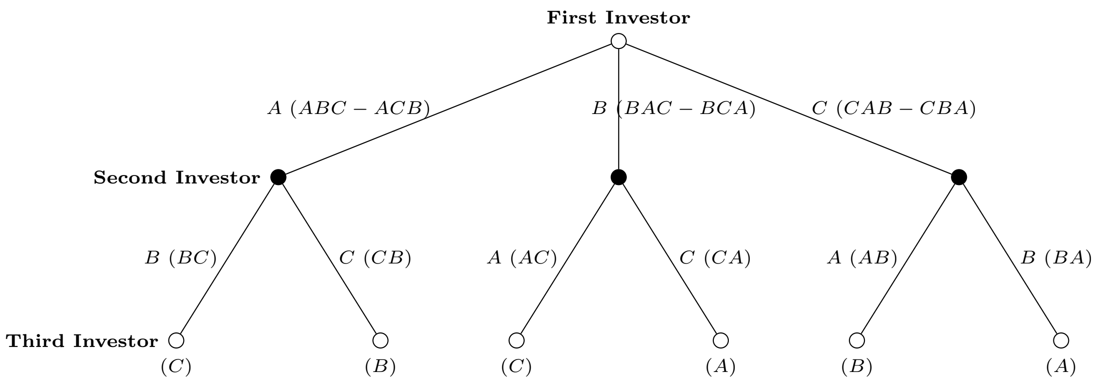

Figure 1: The tree shows the firms' possible strategies to invest (enter) in the triopoly market. The entry order of firms is showed in the parentheses.

Assuming that the first investor is A in the market (left branch of Figure (1)), there will be a competition between Firm B and Firm C to enter the market. Huisman and Kort (2015) show that in a duopoly setting, the firm with a lower cost structure will be always the leader and the firm with a higher cost structure becomes the follower. Knowing that Firm B is more cost efficient than Firm C ($I^B < I^C$), Firm B becomes the second investor in the triopoly market. And, consequently, Firm C becomes the third investor in the market. Lets consider the middle branch of the decision tree in Figure (1). Assuming that Firm B is the first investor, there exists a competition between Firm A and Firm C to enter the market as the second investor. As Firm A is a lower cost firm ($I^A < I^C$), it becomes the second investor, and Firm C is the third investor. Finally, the right branch of Figure (1) depicts the case where Firm C already entered as the first investor. Firm A, as a more cost efficient firm becomes the second investor ($I^A < I^B$). Finally, Firm B becomes the third investor in the market. As in Shibata (2016), in the following sections, we consider three scenarios $\{ABC, BAC, CAB\}$.

As in second nodes of Figure (1), Firm A can be the first or the second firm to enter the triopoly market. In Figure (2), we analyze both scenarios in more details, and analyze the game-theoretic investment decision of firms step-by-step. The following analysis is investigated in Shibata (2016). First, we consider the case that Firm A enters as the first investor in the market. Then we derive firm A's preemptive investment threshold of being the first investor. Consequently, Firm B and Firm C compete to identify the second investor in the market. Moreover, assuming that the cost of Firm B to invest in the market is less than Firm C, Firm B enters the market as the second investor. Firm A can invest either at the threshold of its competitor or its unrestricted threshold. Then Firm B applies the same procedure to find the optimal investment threshold. Meaning that Firm B invests at the preemptive threshold of the third Firm C or invests at its unrestricted threshold. Finally, Firm C invests at its non-strategic investment threshold.

Now, assume that Firm A does not enter the market as the first investor (Figure (2)). Then firm B would be the first investor due to its lower cost structure than Firm C. Consequently, Firm A and Firm C compete to enter the market as the second investor. Considering that $I^A < I^C$, Firm A becomes the second investor in the market. Moreover, Firm A invests either at the preemptive threshold of Firm C or its unrestricted investment threshold. Meaning that if the preemptive threshold does not exist, Firm A enters the market as the second investor at its unrestricted threshold. Finally, Firm C enters the market at its non-strategic investment threshold.

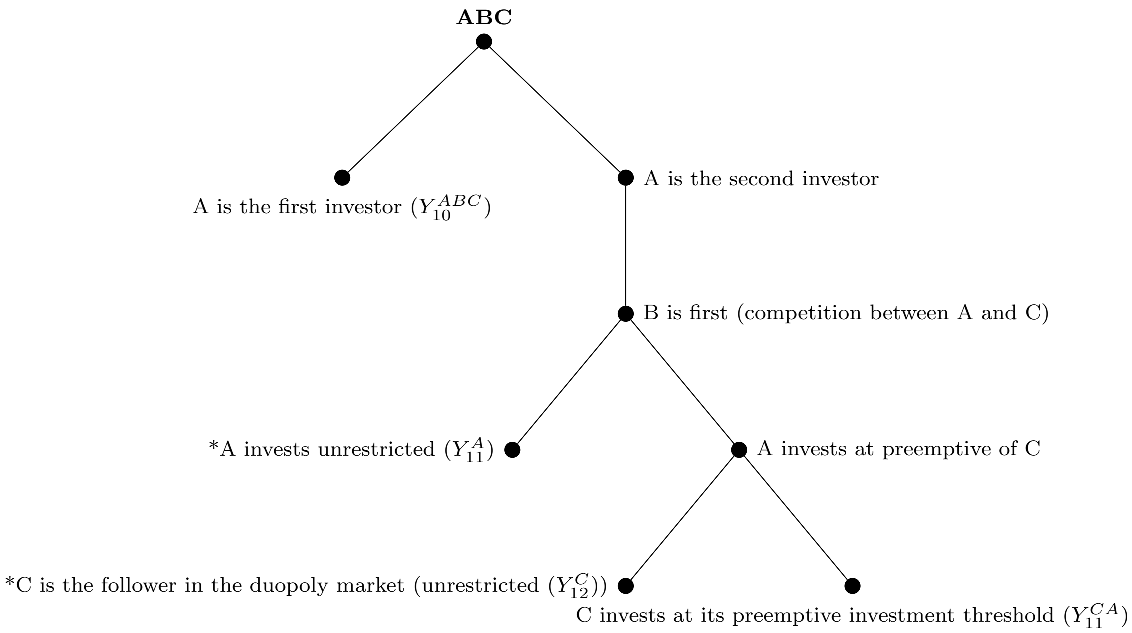

Figure 2: The figure shows the game-theoretic relation between firms. The optimal strategic investment decision of firms is represented in each node.

2.2.1 Third Investor Entry Threshold

The following results are from Shibata (2016). In a triopoly market, the optimal investment threshold of the third firm is nonstrategic. We assume that the first and the second investors already entered the market. Then the optimal nonstrategic thresholds for possible scenarios are given as

$$Y_{12}^{A} = \phi\frac{I^A}{D_3},\ Y_{12}^{B} = \phi\frac{I^B}{D_3},\ Y_{12}^{C} = \phi\frac{I^C}{D_3}, \tag{6}$$

where

$$\phi \ := \ \frac{\beta(r-\mu)}{\beta-1}. \tag{7}$$

2.2.2 Second Investor Entry Threshold

Considering the entry of the second investor, as in Shibata (2016), the preemptive threshold of being the second investor is defined as

$$Z_{11}^{jk} \ := \ \begin{cases} Y_{11}^{P} & \text{if } Y_{11}^{P} \text{ exists} \\ Y_{11}^{j} & \text{otherwise,} \end{cases} \tag{8}$$

where $j \in \{A, B, C\}$. Meaning that the second firm invests at the preemptive threshold of the competitor, otherwise it invests at its unrestricted threshold. In equation (9), the preemptive thresholds of the second investor are defined, given that the first investor entered the market.

$$\begin{cases} \begin{cases} Y_{11}^{BC} & := \inf\{Y \in [0, Y_{12}^{C}] \mid V_{11}^{BC}(Y) \geq W_{12}^{B}(Y)\} \\ Y_{11}^{CB} & := \inf\{Y \in [0, Y_{12}^{B}] \mid V_{11}^{CB}(Y) \geq W_{12}^{C}(Y)\} \end{cases} & \text{if } A \text{ invests first} \\ \\ \begin{cases} Y_{11}^{AC} & := \inf\{Y \in [0, Y_{12}^{C}] \mid V_{11}^{AC}(Y) \geq W_{12}^{A}(Y)\} \\ Y_{11}^{CA} & := \inf\{Y \in [0, Y_{12}^{A}] \mid V_{11}^{CA}(Y) \geq W_{12}^{C}(Y)\} \end{cases} & \text{if } B \text{ invests first ,} \\ \\ \begin{cases} Y_{11}^{AB} & := \inf\{Y \in [0, Y_{12}^{B}] \mid V_{11}^{AB}(Y) \geq W_{12}^{A}(Y)\} \\ Y_{11}^{BA} & := \inf\{Y \in [0, Y_{12}^{A}] \mid V_{11}^{BA}(Y) \geq W_{12}^{B}(Y)\} \end{cases} & \text{if } C \text{ invests first} \end{cases} \tag{9}$$

where "W" stands for the waiting value for firms, and is defined as

$$W_{12}^{k}(Y) = \left(\frac{Y}{Y_{12}^{k}}\right)^{\beta} V_{12}^{k}(Y_{12}^{k}). \tag{10}$$

Assuming that Firm A is the first investor, as in Shibata (2016), the set of solutions is given by

$$\Omega_{11}^{P2} := \{Z_{11}^{BC}, Y_{11}^{CB}\},\ \Omega_{11}^{P1} := \min \Omega_{11}^{P2},\ \Omega_{11}^{P} := \max \Omega_{11}^{P2} \tag{11}$$

The same scenario applies for the second investor's threshold when Firm B or Firm C are the first investors.

2.2.3 First Investor Entry Threshold

Considering the entry of the second investor, as in Shibata (2016), the preemptive threshold of being the first investor is defined as

$$Z_{10}^{P} \quad := \quad \begin{cases} Y_{10}^{P} & \text{if } Y_{10}^{P} \text{ exists} \\ Y_{10}^{i} & \text{otherwise,} \end{cases} \tag{12}$$

where $i \in \{A, B, C\}$. Meaning that the first firm invests at the preemptive threshold of the competitor, otherwise it invests at its unrestricted threshold. In equation (13), the preemptive thresholds of the first investor are defined for three scenarios $\{ABC, BAC, CAB\}$.

$$\begin{cases} Y_{10}^{ABC} & := \inf\{Y \in [0, Y_{11}^{BC}] \mid V_{10}^{ABC}(Y) \geq W_{11}^{AC}(Y)\} \\ Y_{10}^{BAC} & := \inf\{Y \in [0, Y_{11}^{AC}] \mid V_{10}^{BAC}(Y) \geq W_{11}^{BC}(Y)\} \\ Y_{10}^{CAB} & := \inf\{Y \in [0, Y_{11}^{AB}] \mid V_{10}^{CAB}(Y) \geq W_{11}^{CB}(Y)\} \end{cases} \tag{13}$$

where "W" stands for the waiting value for firms, and is defined as

$$W_{11}^{jk}(Y) = \left(\frac{Y}{Y_{11}^{jk}}\right)^{\beta} V_{11}^{jk}(Y_{11}^{jk}). \tag{14}$$

The set of solutions for equation (13), as in Shibata (2016), is given by

$$\Omega_{10}^{P2} := \{Y_{10}^{ABC}, Y_{10}^{BAC}, Y_{10}^{CAB}\},\ \Omega_{10}^{P1} := \min \Omega_{10}^{P2},\ \Omega_{10}^{P} := \min \Omega_{10}^{P2} \setminus \Omega_{10}^{P1}$$

2.2.4 Equilibrium

In this section, as in Shibata (2016), the Nash equilibrium is derived in which each firm's decision is the best response to its rivals such that there is no deviation from this strategy. The investment thresholds for the first, the second, and the third investors are given as follows

$$\begin{aligned} Y_{10}^{ijk^*} &= \min\{\Omega_{10}^{P}, Y_{10}^{i}\} \\ Y_{11}^{jk^*} &= \min\{\Omega_{11}^{P}, Y_{11}^{j}\} \\ Y_{12}^{k^*} &= Y_{12}^{k} \end{aligned} \tag{15}$$

2.3 Numerical Results

As in Shibata (2016), the numerical results in Table (1) show that the lowest-cost firm is not always the first investor in the market. In the next section, we give the economic explanation which is not included in Shibata (2016).

Panel A: First Investor

D_2	θ	Y_{10}^{ABC}	Y_{10}^{BAC}	Y_{10}^{CAB}	Y_{10}^{ACB}	Y_{10}^{BCA}	Y_{10}^{CBA}	Firm
3	0.01	0.1822	0.1833	0.1875	0.1856	0.1903	0.1910	A
3	0.05	0.1720	0.1789	0.1968	0.1871	0.2204	0.2128	A
3.25	0.01	0.2008	0.1998	0.2032	0.2075	0.2124	0.2083	B
3.25	0.05	0.1882	0.1927	0.2060	-	-	0.2162	A

Panel B: Second Investor

D_2	θ	Y_{11}^{AC}	Y_{11}^{CA}	Y_{11}^{BC}	Y_{11}^{CB}	Y_{11}^{AB}	Y_{11}^{BA}	Firm
3	0.01	0.2707	0.2821	0.2748	0.2805	0.2721	0.2778	B
3	0.05	0.2617	0.3201	0.2806	0.3092	0.2670	0.2956	B
3.25	0.01	0.2402	0.2497	0.2437	0.2485	0.2413	0.2460	A
3.25	0.05	0.2328	0.2814	0.2492	0.2732	0.2371	0.2611	B

Panel B: Third Investor

D_2	θ	Y_{12}^{C}	Y_{12}^{B}	Y_{12}^{A}	Firm
3	0.01	0.5638	0.5582	0.5527	C
3	0.05	0.6080	0.5803	0.5527	C
3.25	0.01	0.5638	0.5582	0.5527	C
3.25	0.05	0.6080	0.5803	0.5527	C

Table 1: The table shows the investment thresholds. Parameter $D_2 = 5$, $D_3 = 2$, $\sigma = 0.15$, r = 0.09, $\mu = 0.04$, and Y = 0.1.

3 Economic Interpretation of Shibata's Result

3.1 Why the lowest-cost firm is not always the first investor in the triopoly market?

In the preliminaries, the model and results of Shibata (2016) are presented. Having said that, the economic interpretation is missing from Shibata's finding. In this section, we investigate the economic reason that the firm without the lowest-cost structure enters the market as the first investor. To do so, we need to examine the game-theoretic relationship between firms. First, assume that

$$I^A < I^B < I^C$$
$$D_1 > D_2 > D_3,$$

in which the first line represents the asymmetric cost structure and the second line stands for the competition parameter. In Figure (3), we assume that Firm A enters the market as the first investor in the triopoly market. So, there is a competition between Firm B and Firm C to be the second investor. Due to the fact that $I^B < I^C$, Firm B becomes the second investor in the market. Indeed, Firm B enters the market at the preemption point of Firm C.

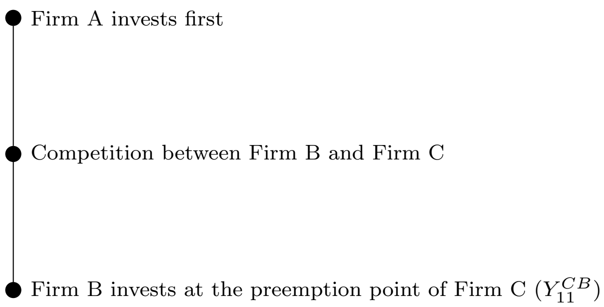

Figure 3: The figure shows the game-theoretic relation between firms when Firm A invests first. The optimal strategic investment decision of Firm B is represented in the last node.

In Figure (4), we assume that Firm B invests as the first investor in the market. Then there is a competition between Firm A and Firm C to enter the market as the second investor. Furthermore, Firm A is the second investor to invest in the market due to the cost advantage over Firm C ($I^A < I^C$). Consequently, Firm A, the second investor, invests at the preemption point of Firm C (Y_{11}^{CA}).

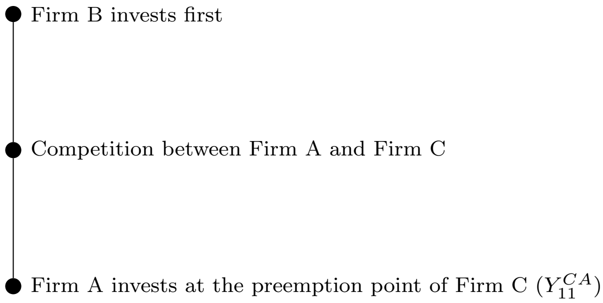

Figure 4: The figure shows the game-theoretic relation between firms when Firm B invests first. The optimal strategic investment decision of Firm A is represented in the last node.

Comparing the optimal investment timing of Firm A and Firm B (Table (1)) to enter the triopoly market as the second investor, we conclude that

$$Y_{11}^{CA} > Y_{11}^{CB}. \tag{16}$$

The reason for equation (16) is that the entry threshold of the second investor is later when Firm B is the first investor in the market. This is caused by the fact that if Firm A invests as second, this happens later in time than when Firm B would invest as second. So, Firm B is the monopolist for a longer time period in the market. Concerning the preemption point, if Firm A invests second, it invests at the preemption point of Firm C. The preemption point of Firm C is relatively late. This is due to the fact that if Firm C invests as the second investor, it knows that Firm A invests soon after firm C because the investment cost of Firm A is low. For this reason, Firm C's incentive to preempt Firm A is low.

4 Model

This section analyzes the optimal timing and capacity size in different markets. To derive the value functions and the optimal investment decisions of the firms, we apply dynamic programming. Moreover, this section investigates the model in which the order of entry is endogenously given for all firms. Assuming that the cost structure is different for firms ($\delta^A < \delta^B < \delta^C$), and that the third investor's investment decision is non-strategic, there is a competition between firms to be the first investor in the market. Assuming that the first investor already entered the market, the other two firms compete to determine the second investor in the triopoly market. Finally, the non-strategic investment decision of the last investor is made.

There are three investors to enter the triopoly market in our model. The firms consider the investment decisions as an optimization problem over the timing and the scale of investment. As in Huisman and Kort (2015), the inverse demand function is linear in quantity and influenced by a geometric Brownian motion process in the following way:

$$\begin{aligned} P =& Y(t)(1 - \eta K(t)) \\ dY(t) =& \mu Y(t)dt + \sigma Y(t)dw, \end{aligned} \tag{17}$$

where in economic terms, Y represents the profit flow of an investment, $\eta > 0$ is a positive constant which accounts for the growth rate. The firms enter the market once they make an investment and it is assumed that the firms produce up to capacity. It is assumed that the firms produce up to capacity and the asymmetric cost structure of firms to enter the triopoly market is determined as $0 < \delta^A < \delta^B < \delta^C$ (represent the unit cost of investment). For simplicity, we will refer to the unit cost as $\delta^i, \delta^j, \delta^k$ where $i, j, k \in \{A, B, C\}$. Let K show the total production in the market, i.e., in the triopoly matrket $K = k_{10}^{ijk} + k_{11}^{jk} + k_{12}^{k}$. The superscript "$ijk$" represent the order of entry in the market, $i, j, k \in \{A, B, C\}$. The subscript "mn" in k_{mn} represent the firms active in the market in which "m" stands for the number of firms competing to enter the market and "n" determines the number of firms already entered the market. k_{10}^{ijk} shows the order of entry in the market where the entry capacity threshold of the first firm to enter the market. k_{11}^{jk} represents that there is an active firm in the market (firm i) and the order of the two remaining investors. Finally, k_{12}^{k} shows the entry scale of investment for the last firm where there are two active firms in the market. If the investment option is exercised at time t, the firm receives cash flow of $P(t)K(t)$. Let $w(t)$ represent a Wiener process (or a standard Brownian motion), $\mu > 0$ a constant drift rate, and $\sigma > 0$

a constant variance. For convergence, we assume that $r > \mu$, where $r > 0$ is a constant interest rate. Indeed, if $r > \mu$ does not hold, by choosing a later point time to invest, the discounted revenue stream can be extremely large that it is always optimal for the firms to wait longer and the optimal solution can not be attained.

4.1 Value Functions

4.1.1 Third Investor

Lets assume that two firms already entered the market. Meaning that the first and the second firms took strategic investment decisions. Therefore, similar to Bouis et al. (2009), the third investor's investment decision is non-strategic and determined as in the next proposition.

Proposition 1. Given the current level of the stochastic demand process, $Y(t)$, heterogeneous cost structure, and assuming that the first and the second firms already entered the market, the optimal investment timing and capacity size are given as

$$Y_{12}^{k}(k_{10}^{ijk}, k_{11}^{jk}) = \frac{\beta+1}{\beta-1}\frac{\delta^k(r-\mu)}{1-\eta(k_{10}^{ijk}+k_{11}^{jk})}, \tag{18}$$

$$k_{12}(k_{10}^{ijk}, k_{11}^{jk}) = \frac{1-\eta(k_{10}^{ijk}+k_{11}^{jk})}{\eta(\beta+1)}. \tag{19}$$

The value function of the third investor is given by

$$V_{12}(Y, k_{10}^{ijk}, k_{11}^{jk}) = \begin{cases} \left(\frac{Y}{Y_{12}^k}\right)^\beta \left(\frac{Y_{12}^k k_{12}^k(1-\eta(k_{10}^{ijk}+k_{11}^{jk}+k_{12}^k))}{r-\mu} - \delta^k k_{12}^k\right) & \text{if } \; Y < Y_{12}^k(k_{10}^{ijk}, k_{11}^{jk}) \\ \frac{Y k_{12}^k(1-\eta(k_{10}^{ijk}+k_{11}^{jk}+k_{12}^k))}{r-\mu} - \delta^k k_{12}^k & \text{if } \; Y \geq Y_{12}^k(k_{10}^{ijk}, k_{11}^{jk}) \end{cases} \tag{20}$$

In this part, the first line of equation (20) represents the option value of waiting when the demand is less than a threshold given in equation (18). If the demand level is more than the derived threshold, $Y \geq Y_{12}^k(k_{10}^{ijk}, k_{11}^{jk})$, the investor immediately enters the market. The term $\left(\frac{Y}{Y_{12}^k}\right)^\beta$ is the stochastic discount factor. Note that the investment timing of the third investor is a function of the capacity size of the first and the second firms. Given the same argument for capacity size of the third firm, the investment decision of third investor is non-strategic.

4.1.2 Second Investor

The strategic investment threshold of the second investor is investigated, given that the first investor already entered the market. The second investor's value function is given in the following proposition.

Proposition 2. Given the current level of the stochastic demand process, $Y(t)$, heterogeneous cost structure, and assuming that the first firm already entered the market, the value function of the second investor is given by

$$V_{11}(Y, k_{10}) = \begin{cases} \frac{Y k_{11}^{jk}(1-\eta(k_{10}^{ijk}+k_{11}^{jk}))}{r-\mu} - \delta^j k_{11}^{jk} - \left(\frac{Y}{Y_{12}^k}\right)^\beta \frac{Y_{12}^k \eta k_{12}^k k_{11}^{jk}}{r-\mu} & \text{if } \; Y < Y_{12}^k(k_{10}^{ijk}, k_{11}^{jk}) \\ \frac{Y k_{11}^{jk}(1-\mu(k_{10}^{ijk}+k_{11}^{jk}+k_{12}^k))}{r-\mu} - \delta^j k_{11}^{jk} & \text{if } \; Y \geq Y_{12}^k(k_{10}^{ijk}, k_{11}^{jk}) \end{cases} \tag{21}$$

The first line of equation (21) depicts the option value of waiting and the second line shows the immediate investment value for the second investor. Considering the former, the first term shows the discounted revenue from the investment at demand size Y, the second term denotes the investment cost of entering the market for the second firm, and the last term shows the option value lost due to the entry of the third investor. Consider the second line of equation (21), the first term is the output flow from the investment and the second term is the irreversible investment cost of entering the market.

Lets assume that the first investor already entered the market. Then there exists a game-theoretic interaction between the other two firms to take the place of a second firm. The equation (21) shows that if the current investment threshold is less than that of Y_{12}^k both the second and the third investors wait to gather more information in the market. On the contrary, if the current demand (Y) is higher than the optimal investment threshold of the third investor (Y_{12}^k), investors simultaneously enter the market.

4.1.3 First Investor

Assume that there exists no investor in the market and three firms interact to enter the market as the first firm by taking an irreversible investment decision. The value function of the first investor is given in the following proposition.

proposition 3. Given the current level of the stochastic demand process (Y_t), endogenous capacity size, heterogeneous cost structure, and assuming that there is no active firm in the market, the value function of the first investor is given by

$$V_{10}(Y) = \begin{cases} \frac{Yk_{10}^{ijk}(1-\eta k_{10}^{ijk})}{r-\mu} - \delta^i k_{10}^{ijk} - \left(\frac{Y}{Y_{11}^{jk}}\right)^\beta \frac{Y_{11}^{jk}\eta k_{11}^{jk} k_{10}^{ijk}}{r-\mu} - \left(\frac{Y}{Y_{12}^{k}}\right)^\beta \frac{Y_{12}^{k}\eta k_{12}^{k} k_{10}^{ijk}}{r-\mu} & \text{if } Y < Y_{11}^{jk}(k_{10}^{ijk}) \\ \frac{Yk_{11}^{jk}(1-\eta (k_{10}^{ijk}+k_{11}^{jk}))}{r-\mu} - \delta^i k_{10}^{ijk} - \left(\frac{Y}{Y_{12}^{k}}\right)^\beta \frac{Y_{12}^{k}\eta k_{12}^{k} k_{11}^{jk}}{r-\mu} & \text{if } Y_{11}^{jk}(k_{10}^{ijk}) \le Y < Y_{12}^{k}(k_{10}^{ijk}, k_{11}^{jk}) \\ \frac{Yk_{10}^{ijk}(1-\eta (k_{10}^{ijk}+k_{11}^{jk}+k_{12}^{k}))}{r-\mu} - \delta^i k_{10}^{ijk} & \text{if } Y \ge Y_{12}^{k}(k_{10}^{ijk}, k_{11}^{jk}) \end{cases} \quad (22)$$

The first line of equation (22) represents the value function of the first investor when the current demand (Y) is less than the investment threshold $Y_{11}^{jk}(K_{10}^{ijk})$. In the latter, the first term represents the discounted revenue of the first firm, the second term denotes the investment cost of entering the market as a first investor, the third and the fourth terms denote the option value lost due to the entry of the second and the third investors in the market. The second line of the above equation shows the value function in the range $[Y_{11}^{jk}(K_{10}^{ijk}), Y_{12}^{ijk}(K_{10}^{ijk}, K_{11}^{jk}))$. If the current demand lies in this interval, the first firm and the second firm invest simultaneously. And the third line in equation (22) refers to the value function of a firm in case of joint investment in the market. Meaning that the current demand level is very high such that all investors enter the market simultaneously.

5 Model Solution

The game-theoretical interaction between firms determine the role of each investor in the triopoly market. The capacity size is not fixed in the market, and the investor can decide on the optimal scale of investment. The third investor takes non-strategic strategies as the decision by the last investor in the market imposes no strategic impact on the earlier investors. Assuming that the first investor already entered the market, the other two investors compete in a duopoly setting to enter the market as the second investor.

5.0.1 Third Investor

The investment threshold of the third investor is non-strategic and is determined as follows

$$Y_{12}^k = \frac{\beta+1}{\beta-1}\frac{\delta^k(r-\mu)}{1-\eta(k_{10}^{ijk}+k_{11}^{jk})} \quad (23)$$

$$k_{12}^k = \frac{1-\eta(k_{10}^{ijk}+k_{11}^{jk})}{\eta(\beta+1)} \quad (24)$$

5.0.2 Second Investor

THIS PART WILL BE COMPLETED

5.0.3 First Investor

THIS PART WILL BE COMPLETED

6 Numerical Results

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7 Conclusion

The first result of this paper investigates the reason why the firm with the lowest cost structure is not always the first investor in the market. Indeed, this paper concludes that firm B (the second lowest-cost firm) can invest first due to the fact that if Firm A invests as a second investor, it would happen later in time than when Firm B would invest as second. So, this situation is more attractive for Firm B to be the monopolist for a longer time period in the market. To explain, consider the scenario where Firm A invests as second. The investment will take place a the preemption point of Firm C. Regarding the fact that Firm A has the lowest cost to enter the market, the preemption point of Firm C is relatively late. The reason is that if Firm C invests second, it knows that Firm A invests soon after Firm C. So, Firm C's incentive to preempt Firm A is low. The second results come from the combination of Huisman and Kort (2015) and Shibata (2016). The aim of this extension is to generalize Shibata's result in a model in which the firms not only determine investment timing but also the size of investment.

8 Appendix

THIS PART WILL BE COMPLETED.

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