Uncertain Commodity Prices and Informed Sensitivity Analyses

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Abstract

From corporate budgeting to public planning, we hear that commodity prices are uncertain and that when they vary, key investment measures sway with them. However, claiming that commodity prices are outside a firm's domain of control, corporate decision makers mostly neglect this uncertainty or at best reflect it in naïve sensitivity analyses. Yet, to avoid inferior decisions and loss of value, firms should make decisions that take in the understanding about key uncertain factors. In this paper, we show that the customary practice of analysis with arbitrary "high" and "low" prices is inconsistent with the general understanding about commodity price dynamics and the financial theory. To alleviate this, we suggest a consistent and project-specific sensitivity analysis method that supports valuations and decision making.

1. Introduction

For most project appraisals, a slight change in product price assumptions could strikingly sway the investment decision measures. An upstream petroleum project that is highly appealing at, e.g., 70 US/bbl crude oil price could be marginal at 50 USD/bbl or disastrous at 30 USD/bbl. Price forecasts are central to commodity asset valuations but with their inherent uncertainty, how should the investors decide? The answer has traditionally been along the lines: "of course prices are uncertain, we evaluate projects at expected prices and run sensitivity analyses to see how value measures change". While it is common (and recommended) to examine changes in value by changing the parameters of the valuation model, this begs another, yet unanswered, question: "How much of a change in prices?" As for most projects, an unrealistic range of prices could lead to confusion rather than insight.[1]

The uncertainty we perceive should be consistent with our information and understanding. Yet in practice, most price ranges are either too wide, too narrow, or inconsistent with the general understanding about the markets. Commodity prices do not have an arbitrary range. In principle, they reflect the rates of production and consumption, constantly changing to keep the fine balance of supply and demand. This makes some price scenarios more likely, and others outright impossible. The scenarios we consider in sensitivity analyses should reflect this economically reasonable range of prices.

Traditional sensitivity analysis defines the "normal" range of prices as an arbitrary gap between extremes, leaving the analysis inconsistent and personal, with no basis in theory. Extreme bounds of a variable do not necessarily show a useful range and are commonly mistaken as absolute minimum or maximum (Leamer, 1985). Perhaps if valuations were studies of *fragility*, then assessing the models with extreme prices would have made sense. Or, it would have been informative to see if a prescribed configuration in a project stands the test of extreme prices and is still robust across the conceivable extremes. But project appraisals are not about robustness.[2] In practice, sensitivity analyses should support the formulation of an investment decisions by showing a reasonable range of value drivers and then screening off the inferior courses of action. Next, it should show how project values (and

[1] Often corporate analyses assume "high" and "low" prices as compared with an "expected" price scenario—also known as the corporate planning price curve. Here, the "high" and "low" are arbitrary deviations from the base case, say 40% lower or higher than the expected prices. It is often unclear why such prices represent a reasonable range and why should this lead to useful decision insights.

[2] We believe valuations are integral to decision-making and should contribute to a project design by finding or constructing value-maximizing alternatives. Yet, recommending a course of action based on its robustness (rather than value creation potentials) is also destructive.

managerial decisions) change by changing value drivers. With this description, the range of prices for an investment decision should be economically informed and project specific.

In this paper, we devise a sensitivity analysis model that is consistent with the market dynamics and adjusts to a project's features. Like pieces of puzzle that fit only in a certain way to show a picture, we believe the components of valuation—such as price forecasts and discounting method—should also fit together. This unified view of valuations (e.g. Myers and Turnbull, 1977) concludes that project risks depend on at least the project length. Then in the context of sensitivity analysis, price ranges should be a function of project life. Using the scheme in Jafarizadeh and Bratvold (2019) in this paper we formulate this effect in sensitivity analysis. In addition, we describe the market dynamics of prices using the economics of supply and demand. Consistent with this description of price uncertainty, we use the Schwartz and Smith (2000) two-factor price model to estimate price ranges.

We use a simulation procedure to devise uncertainty bounds for future spot prices—within an interval corresponding to a project's length. Then, we apply an optimization model to generate the "high" and "low" price curves (planning prices) for an informed sensitivity analysis. Through such analysis, we assess price scenarios that are simple and informative, yet consistent with the valuation scheme and economic dynamics. The next section discusses the uncertain commodity prices using economic dynamics and the general understanding about the markets. We suggest a two-factor process to describe these dynamics. Then in section 3 we show how we can devise planning price bounds for sensitivity analysis in a project based on the earlier description of price dynamics. In section 4 we apply the method to an example and further discuss the applications of sensitivity analyses. In section 5, we summarize the discussions and conclude. The appendices show parameter estimation for our price model and the results for single factor processes.

2. A Description of Price Uncertainty

2.1 General Understanding

Early attempts in modeling commodity investments used a random walk process to describe the dynamics of the prices. Commodity prices were thought to behave like the prices of financial stocks—random independent ticks building up to a Brownian motion. The assumption was easy and convenient. It also drew similarities to financial investments, allowing the spread of financial valuation techniques to real assets (e.g. Brennan and Schwartz, 1985).

Admittedly, commodity prices are random and unpredictable. Statistical Studies of the history of the commodity prices, for example the crude oil prices over the past decades, reveal randomness. The studies also show mean reversion in historical prices (Pindyck, 1999, Geman, 2007, Xu et al, 2012). Yet in general, the past is not a predictor of the future. With uncertain prices, more could have happened than happened. The past is but one realization of the uncertain prices. Therefore, in addition to studying the past, we could also see whether our perceived behavior of prices agrees with the general belief in the commodity markets. Here, the random walk assumption is in odds with the general understanding about commodity markets (Lund, 1993). [3]

Unlike financial assets, commodities are physically produced and consumed. Their prices reflect the balance of supply and demand. In global markets (like that of crude oil), the balance is between worldwide production and consumption. Regional bottlenecks are common and could upset the local balance, resulting in temporary price mismatches—e.g. the recent storage shortages that led to negative West Texas Intermediate (WTI) crude oil prices. Natural gas markets are inherently regional with

[3] Similarly, we build geological subsurface models based on limited available information. The data offers only a glimpse of the subsurface geological structures. We must therefore consider the available information within the context of geological principles (like e.g., deeper depositions are older, or, lower density liquids migrate upwards). The models we build should be consistent with both field data and such general beliefs.

fundamentally independent prices—leading to e.g. different prices in Northwest Europe vs. North America. Still, within market boundaries, the forces of supply and demand primarily shape the commodity markets and the dynamics of prices.

While it is difficult to describe the intricate factors of supply and demand to model prices, we could still make useful models that follow such dynamics. When prices are higher than the equilibrium level, we expect higher cost producers to come online and existing asset produce longer. This results in more barrels of oil in the market pushing the prices down towards the equilibrium. On the other hand, when prices are below the equilibrium level, then expensive producers should go offline, and existing production should cease faster and earlier. This results in less barrels of oil in the market pushing the prices up towards the equilibrium. This supports the "mean-reverting" assumption about prices. We mostly expect the deviations from the equilibrium to dissipate and disappear. Prices move randomly but tend to revert towards an average. Beyond these dynamics, changes in technology could also cause random but permanent changes in prices.

The collective market belief—the market expectation about prices' trend and uncertainty—supports mean reverting behavior of prices. The traded derivative contracts imply that the volatility of futures prices is different from that of spot prices, unlike how the Geometric Brownian motion behaves (e.g. Schwartz, 1997), and as if prices were mean reverting. But this is not all. The market also admits of random and permanent price moves (caused by changes in technology), like those of the Geometric Brownian motion. Eventually, we expect to see both effects, yet with differing levels of importance, in the dynamics of prices.

The above discussion naturally leads to models that accounts for both random permanent moves and transitory deviations—at least two stochastic factors describing the dynamics of prices (e.g. Schwartz and Smith, 2000). We could model the random moves with a geometric Brownian motion process and assume the transitory deviations follow a mean-reverting stochastic process. We could further consider a third factor describing the mean-reverting long-term trend to refine the model further (e.g. model 3 in Schwartz, 1997, or Schwartz and Smith, 1998). Still, as Carlson et al (2007) discuss, there are inherent problems about extrapolation of such models to long-term price forecasts for project valuations.

Additionally, a price model cannot ignore the frame of investment decision. The context of the decisions should lead the modelling. For example, in an upstream petroleum development project with a long lead-time, the short-term transitory dynamics may not affect the decision at all while for a tight oil project with a rapidly declining production, value creation is in short-term dynamics. The added refinements in the price model, may be completely unnecessary, leading to identical courses of action with or without the added model features. A price model should be relevant and useful in the context of decisions, in addition to conforming to the general market understanding and economic reasoning.

2.2 Model Description

The Schwartz and Smith (2000) two-factor price model assumes the price S_t at time t has a short- and long-term component, $\log S_t = \chi_t + \xi_t$. The long-term factor ξ_t follows a Brownian motion and reflects the persisting changes in equilibrium prices while the short-term factor χ_t follows a mean-reverting process and stands for the transitory deviations from the equilibrium.

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \tag{1}$$

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \tag{2}$$

Here κ reflects the speed of mean-reversion in the short-term factor, μ_ξ is the drift of the long-term factor, and σ_χ and σ_ξ are the standard-deviations for, respectively, the short- and long-term factor. In addition, dz_χ and dz_ξ are increments of the standard Brownian motion correlated with $\rho_{\chi\xi}dt = dz_\chi dz_\xi$.

The log of prices in the future will be normally distributed. Depending on the current price factors χ_0 and ξ_0, they will have the following expectation and variance

$$\mathrm{E}(\log S_t) = e^{-\kappa t}\chi_0 + \xi_0 + \mu_\xi t \tag{3}$$

$$\mathrm{Var}(\log S_t) = (1 - e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \tag{4}$$

The prices in the future will be lognormally distributed and, for a set of parameters, Figure 1 shows their confidence intervals.

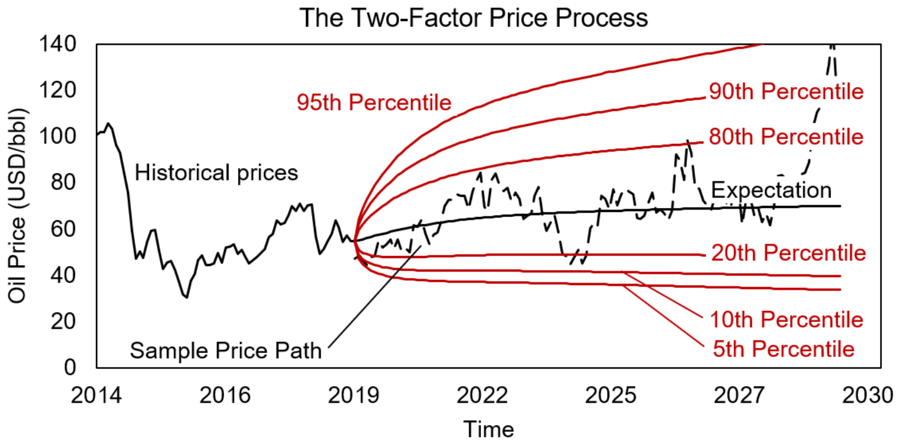

Figure 1 The two-factor price model and the uncertainty in future prices

The two-factor price process is simple and versatile. It reflects both the permanent and transitory dynamics of prices, and in addition, leads to explicit formulations for prices of derivative contracts (futures and options) that are consistent with market observations. The model can also easily reduce to its constituent single-factor price models if valuations call for more simple models at the expense of losing realism. Appendix A discusses the single factor price models.

3. Informed Sensitivity Analysis

3.1 Sum-Discounted Prices

The stochastic models describe the uncertainty of spot prices in the future. They show the future expectation and a probability distribution of prices. Yet, most sensitivity analyses do not fit such a description. They mostly define a range of planning prices from the most optimistic to the most pessimistic and are completely detached from the stochastic models. Such sensitivity analyses do not show, for example, if the value of a project at pessimistic planning prices is the lowest possible project value, the least likely value, or even a reasonable value at all.

This section discusses a general approach to building sensitivity ranges for planning prices from stochastic price models. Like the confidence intervals of spot prices in the future, there should also be confidence intervals for planning prices. Such price scenarios should be consistent with the stochastic properties of spot prices and be still useful in the detailed industrial cash flow models. To achieve this, we study the distribution of *sum of discounted prices* over the planning horizon as a relevant measure of investment decisions. Discussed in e.g. Dixit (1993), the discounted prices reflect the uncertainty over a specific period and considers the decision maker's time preference. The expected planning prices, for example, will be a price path S_t^*, $0 < t < T$, so that

$$\int_0^T S_t^* e^{-rt} dt = \mathrm{E}\left(\int_0^T S_t e^{-rt} dt\right) \quad (5)$$

Here, T is the planning horizon, r is the discount rate, and e^{-rt} is the continuous discounting factor.

To numerically estimate the expected (or any of the *n*th percentiles) for the planning price paths, we use a numerical approximation of the above. We simulate the future spot prices within the planning time-horizon and estimate the distribution of present value of prices. Then we ask: what planning price scenarios stand for the *n*th percentile of such a distribution? Within an optimization model, we next calculate the planning price scenarios representing this distribution.

3.2 Numerical Procedure

We use a discrete version of equations (1) and (2) to simulate prices. Assuming the time-steps Δt are one month[4], we calculate the log spot price

$$\log S_{t+\Delta t} = \xi_{t+\Delta t} + \chi_{t+\Delta t} \quad (6)$$

$$\xi_{t+\Delta t} = \xi_t + \mu_\xi \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t} \quad (7)$$

$$\chi_{t+\Delta t} = e^{-\kappa \Delta t} \chi_t - \left(1 - e^{-\kappa \Delta t}\right) \frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \quad (8)$$

We reflect the correlation between the simulated factors by assuming ε_ξ is a standard normal distribution and ε_χ is a function of ε_ξ and ε (another independent normal distribution)

$$\varepsilon_\chi = \varepsilon_\xi \rho_{\chi\xi} + \varepsilon \sqrt{1 - \rho_{\chi\xi}^2} \quad (9)$$

Using these equations, we simulate price paths for the planning horizon $0 < t < T$, and then use the following Riemann approximation to the sum discounted prices

$$\int_0^T S_t e^{-rt} dt \cong \sum_{t=0}^{T} S_t e^{-rt} \Delta t \quad (10)$$

Note that in the discrete approximation, $t \in \{0, \Delta t, 2\Delta t, \dots, (n-1)\Delta t, T\}$ and n is the number of time steps.

To simplify the calculations further, we assumed one-month time steps. In other words, the prices are for one barrel of crude oil monthly produced and that we intend to estimate monthly planning prices. Using a set of price parameters, Figure 2 shows the distribution of sum discounted prices.

[4] Instead of continuous price changes, in this numerical procedure we assume prices are constant during the length of the time step. Thus, the choice of Δt is key to our numerical procedure. A too long time-step may not be realistic and a too short time-step increases computational details.

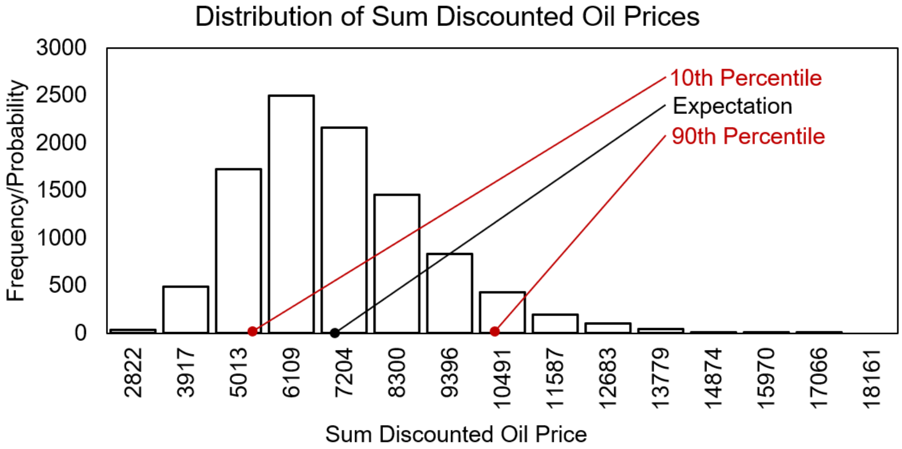

Figure 2 Distribution of sum discounted prices

We selected the tenth and ninetieth percentiles from the distribution of sum discounted prices and devised corresponding planning price curves. For example, to generate the "low" planning prices, we use an optimization model that varies the state variables of the price curve until it has the same discounted value as the tenth percentile of the sum-discounted simulated prices.

To calculate planning price curve S_t^* corresponding to the pth percentile of the discounted prices, we use the optimization procedure below

$$\begin{aligned}
&\min_{\xi_0} \left(\sum_{t=0}^{T} S_t^* e^{-rt} - X \right)^2 \\
&\text{s.t.} \\
&P\left(\sum_{t=0}^{T} S_t^* e^{-rt} < X \right) \leq \frac{p}{100} \\
&P\left(\sum_{t=0}^{T} S_t^* e^{-rt} \leq X \right) \geq \frac{p}{100}
\end{aligned} \tag{10}$$

Here, $t \in \{0, \Delta t, 2\Delta t, \dots, (n-1)\Delta t, T\}$ for n time steps. In addition, X is an internal variable we defined to calculate the percentiles.

A spreadsheet implementation of the above optimization problem is simple and efficient. Using MS Excel and Solver, we calculated the 10$^{\text{th}}$ and 90$^{\text{th}}$ percentile planning prices by changing the long-term initial state variable ξ_0 so that the discounted monthly planning price converges to respectively the 10$^{\text{th}}$ and 90$^{\text{th}}$ percentiles of the discounted sum of simulated prices. Figure 3 shows the results. We have also implemented the procedure in the accompanying spreadsheet model.

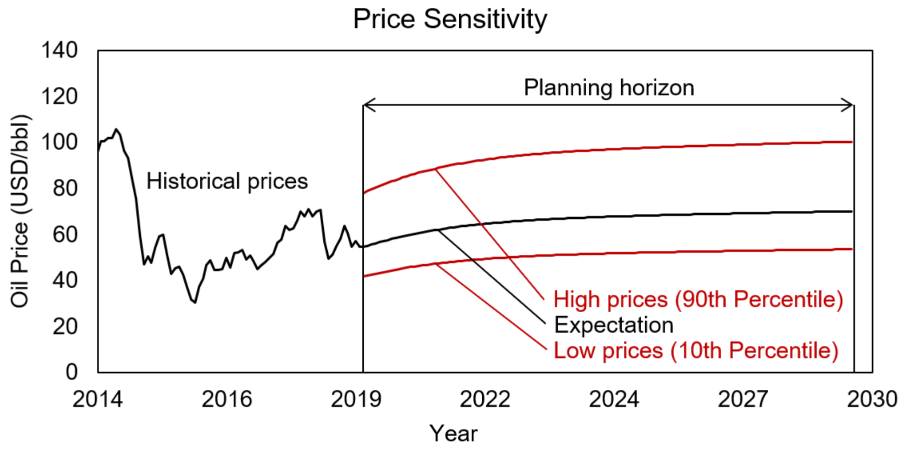

Figure 3 the “high” and “low” planning prices corresponding to the percentiles of the sum discounted simulated prices.

While we believe the Schwartz and Smith (2000) two-factor price process is useful in valuations as it is both simple and realistic, the above numerical procedure is specific to this model. If we can discretize and simulate the price paths, we could apply the procedure to any other stochastic price model.

3.3 The Choice of Discount Rate

Discounting prices conveniently converts a price path to a single value. This key feature makes our sensitivity analysis simple and useful. Yet, discounting also reveals a preference for time and risk and should be consistent with the valuation principles.

In this paper, we use an implied method to calibrate the parameters of the two-factor price model. We estimate parameters of the price model by fitting the model’s futures and option volatility curves to the market observations. This implied method of parameter estimation, discussed in Jafarizadeh and Bratvold (2012) and Jafarizadeh (2019), calibrates the risk-free price expectation of the model with the futures contracts of varying maturities (which are themselves risk-free quotes of prices).

This market-calibrated model applies to risk-neutral valuations—i.e. calculating certain-equivalent cash flows and discounting them with a risk-free rate—but for most industrial valuations that apply a risk-adjusted discount rate to risky cash flows, the price expectations need to also include a risk premium. In general, we should integrate the two separate tasks of estimation of a project cash flows and discounting. In other words, discounting physical prices (estimated with risk-premiums) with a risk-free rate, or, discounting risk-free prices with a risk-adjusted rate, are both inconsistent.

4. Applications and Discussion

4.1 Example: An Asymmetric Fiscal Regime

Governments apply tax systems to generate value from extraction of their natural resources while in addition, encouraging exploration and development activities. However, the best all-round tax system is hard to achieve. Business uncertainties are so diverse that no tax system could account for everything. As a result, governments have devised distinct tax regimes specific to their economic resource conditions. For example, the Norwegian tax encourages exploration of even marginal opportunities by covering their downsides; but, a resource-rich country in the middle east, less concerned with minor assets, could devise a tax regime that capitalizes on the oil price spikes. For the latter, a consistent appreciation of oil price uncertainty is key to sound investment decisions.

However, disparity in appreciation of price uncertainty could lead to losses. Any model the governments use to describe price dynamics, within that context they also predict the industry’s investment decisions.

If a firm is using a completely different price model with inconsistent value metrics, then they could make confused decisions and lose value.

To simplify the discussions, we only show the effect of asymmetric deductions. Here, if oil prices go beyond a threshold, the government absorbs all the added income. The highest price that the oil company should expect is this threshold level. Table 1 shows the cash flow elements of an upstream petroleum project and six different price scenarios. Assuming the government set threshold to 75 USD/bbl, we compare the results from applying the two sets of price scenarios—the corporate planning prices and our price model.

Table 1 Cash flow element of the project and six price scenarios

Year	0	1	2	3	4	5	6	7	8	9	10
Corporate Prices (USD/bbl)	70	70	70	70	70	70	70	70	70	70	70
Corporate Prices +40%	98	98	98	98	98	98	98	98	98	98	98
Corporate Prices -40%	42	42	42	42	42	42	42	42	42	42	42
Expected Prices (USD/bbl)	59	63	65	67	68	68	69	69	70	70	70
High Prices (P90)	85	90	93	95	97	98	98	99	100	100	101
Low Prices (P10)	45	48	50	51	52	52	53	53	53	54	54
Production (MMbbl)	0	0.5	0.4	0.3	0.3	0.2	0.2	0.2	0.1	0.1	0.1
Costs (Million USD)	50	10	5	5	5	5	5	5	5	5	5

Decision makers use a range of measures and analytics to make value-maximizing decision. Here, the analysis shows a break-even oil price of 47 USD/bbl, and using corporate price scenarios, the net present value of the project could be as low as -10 USD million or as high as 52 USD million. Shown in Table 2, these results may well lead to the conclusion that “with high break-even price, there is a reasonable chance of negative net present value, while the upside potentials of the project are taken by the government”.

Table 2 Project values, using corporate planning assumptions compared with price curves from our model

Price Assumptions	Low	Expected	High
Project value using corporate prices (Million USD)	-10	43	52
Project value, using our model's prices (Million USD)	6	36	52

However, our model and its analytics depict a different picture. First, within the high- and low- price scenarios (making up 80% of the possible price curves) the net present value is positive. Second, the break-even flat price of 47 USD/bbl for the duration of the project, corresponds to just the 6th percentile of the sum-discounted prices. Any flat price assumption above 47 USD/bbl leads to a positive net present value and the sum-discounted price distribution (figure 2) shows that such price scenarios make up 94% of the outcomes. Third, the government threshold limit of 75 USD/bbl corresponds to the 65th percentile of the sum-discounted prices. It is correct that the government cuts the upside potentials, but it also leaves a significant potion to the oil company.

Overall, an analysis that reflects the probabilistic properties of the oil prices would normally be more informative than any arbitrary set of price assumptions. To support valuations, in this paper we discuss two sets of analytics: 1) the sum-discounted distribution of prices for the planning horizon, and 2) the “high” and “low” price curves, calculated from this distribution, and reflecting the confidence bounds for uncertain prices. Together they create decision insight that traditional economic analyses usually neglect.

4.2 Caveats of Sensitivity Analysis

Any sensitivity analysis is based on a model of uncertainty. All models are abstractions, they are never true to reality. There are no ways to prove if a model is the correct reflection of the reality, and neither this should not be the goal of any analysis.[5] The benefits of models are not in their veracity but in how useful they are in decision-making. In this paper, while we discuss that a stochastic two-factor process is more consistent with the general economic understanding, there is no way to confirm that this is the "best" model. We discuss that it is a more *consistent* model compared to other simple price processes and arbitrary uncertainty bands that are common in practice. We also show that through simple analytical steps, we can generate useful decision insights that are not available in traditional analysis.

Yet, sensitivity analysis is a helpful, but limited, decision insight. When formulating decisions, sensitivity analysis helps in finding key uncertain factors. Later, it also helps to show how *fragile* the decision insights are with respect to changes in the uncertain variables. But even at its most consistent form, a one-factor-at-a-time sensitivity analysis studies the change in value as a single variable varies. It mostly ignores the co-movements of input variables. For example, in the upstream oil and gas business the average capital costs drift with oil prices, usually with a lag time. In a one-way sensitivity analysis, it would be inconsistent to keep costs unchanged when assuming high- or low-price scenarios. Correlated factors may be key features of a decision model.

In addition, most projects have flexibilities to change course. Managers can capitalize on economic upswings or mitigate the downswings, or they can apply their learnings to make better upcoming decisions. In the oil and gas exploration business, a sudden drop in prices usually prompts managers to defer or even cancel drilling wildcats. These flexibilities sometimes significantly affect value. Yet, sensitivity analysis is a static view of investment decision and generally overlooks value creation from these managerial flexibilities.

Despite its shortcomings, sensitivity analysis is a long-established technique in the oil and gas industry for analysis of investments (e.g. Titman and Martin, 2015, page 74). It is a technique used in analysis, among multiple other decision support tools. Our method goes a long-way in building cross-project investment decision insight as with a change of time horizon, we can apply the same price model and uncertainty reasoning to any project.

5. Conclusions

Most valuation models are complex. With multiple cash flow elements and nonlinear relationships, it is hard to intuitively draw insights about the effect of uncertain factors on value. Sensitivity analysis is a tool to assess this effect. In practice, we analyze the changes in value measures (like a project's Net Present Value or its Internal Rate of Return) by changing the variables, modifying the model structure, or correcting for dependence among variables. In this context, the key to assess the effect of uncertain oil prices on value is to use an informed range of changes in planning prices.

In this paper, we estimate such price ranges by considering the market dynamics of commodity prices. We describe the dynamics using a two-factor process, and based on simulated prices, estimate the confidence bounds for planning prices. Compared to an arbitrary range of prices, our method is more informed. It is consistent with economic dynamics and shows a confidence bound for planning prices. Yet, compared to a comprehensive simulation analysis of a project's outcomes, our method is simple and easy to communicate. With an adjustment for the length, our model applies to any project, making

[5] It is theoretically impossible to assess the veracity of a real-world model; or to prove it is right (Oreskes et. al, 1994). For a price model, validation by checking if a model is consistent with historical data leads to the fallacy of affirming the constituent. In addition, even checking the model's consistency with both history and the market expectations (as implied in derivatives contracts) is not validation; it is just collecting agreeable evidence. Overall, the acceptance of a price model should be based on its consistency with our market understanding and its usefulness in decision-making.

it suitable for comparative analysis across an organization. Overall, this method brings consistency and transparency to valuations.

We implemented the analysis method of this paper in an MS Excel spreadsheet. It is available in the link below:

https://www.dropbox.com/s/1tuh39pvtxacw4u/Data%20and%20Models.xlsx?dl=0

References

Brennan, M. J., & Schwartz, E. S. (1985). Evaluating natural resource investments. *Journal of business*, 135-157.

Carlson, M., Khokher, Z., & Titman, S. (2007). Equilibrium exhaustible resource price dynamics. *The Journal of Finance*, *62*(4), 1663-1703.

Dixit, A. K. (1993). *The art of smooth pasting* (Vol. 55). Taylor & Francis.

Geman, H. (2007). Mean reversion versus random walk in oil and natural gas prices. In *Advances in Mathematical finance* (pp. 219-228). Birkhäuser Boston.

Jafarizadeh, B. (2019, June). Discount rates and price forecasts for upstream petroleum valuations. In *23rd Annual International Real Options Conference 2019*.

Jafarizadeh, B., & Bratvold, R. (2012). Two-factor oil-price model and real option valuation: an example of oilfield abandonment. *SPE Economics & Management*, *4*(03), 158-170.

Jafarizadeh, B., & Bratvold, R. B. (2019). Exploration economics: taking opportunities and the risk of double-counting risk. *Mineral Economics*, *32*(3), 323-335.

Leamer, E. E. (1985). Sensitivity analyses would help. *The American Economic Review*, *75*(3), 308-313.

Lund, D. (1993). The lognormal diffusion is hardly an equilibrium price process for exhaustible resources. *Journal of Environmental Economics and Management*, *25*(3), 235-241.

Myers, S. C., & Turnbull, S. M. (1977). Capital budgeting and the capital asset pricing model: Good news and bad news. *The Journal of Finance*, *32*(2), 321-333.

Oreskes, N., Shrader-Frechette, K., & Belitz, K. (1994). Verification, validation, and confirmation of numerical models in the earth sciences. *Science*, *263*(5147), 641-646.

Pindyck, R. S. (1999). The long-run evolutions of energy prices. *The energy journal*, *20*(2).

Schwartz, E. S. (1997). The stochastic behavior of commodity prices: Implications for valuation and hedging. *The Journal of finance*, *52*(3), 923-973.

Schwartz, E., & Smith, J. E. (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science*, *46*(7), 893-911.

Titman, S., & Martin, J. D. (2014). *Valuation: The Art and Science of Corporate Investment Decisions*. Pearson Higher Ed.

Xu, L., Sepehrnoori, K., Dyer, J., & Van Rensburg, W. C. (2012, January). Application of real options to valuation and decision-making in the petroleum E&P industry. In *SPE Hydrocarbon Economics and Evaluation Symposium*. Society of Petroleum Engineers.

Appendix A: Single-Factor Price Processes

The two-factor price process of Schwartz and Smith (2000) is so versatile that it could reduce to single factor processes simply by "turning off" the other factor. For example, when $\chi_t = 0$ the two-factor process reduces to the geometric Brownian motion, describing prices as random walk with a drift. The price S evolves in time t following the equation

$$dS = \mu S dt + \sigma_1 S dz \tag{A-1}$$

Here μ is the drift rate, σ_1 is the volatility, and dz is the increment of the Brownian motion. The log of prices $x = \log S$ follows a simple Brownian motion.

$$dx = \left(\mu - \frac{\sigma_1^2}{2}\right) dt + \sigma_1 dz \tag{A-2}$$

If the current price is S_0, then the expected value $\mathrm{E}(x)$ and variance $\mathrm{Var}(x)$ of x will be

$$\mathrm{E}(x) = \log S_0 + \left(\mu - \frac{\sigma_1^2}{2}\right) t \tag{A-3}$$

$$\mathrm{Var}(x) = \sigma_1^2 t \tag{A-4}$$

Alternatively, by setting set $\xi_t = \xi$ we describe price S as mean-reverting towards the equilibrium price $\bar{S}$ following the equation

$$dS = \kappa(\bar{S} - \log S) S dt + \sigma_2 S dt \tag{A-5}$$

Here κ is the speed of mean-reversion, σ_2 is the volatility. The log of prices $x = \log S$ follows a simple Ornstein-Uhlenbeck process

$$dx = \kappa(\alpha - x) dt + \sigma_2 dz \tag{A-6}$$

$$\alpha = \log \bar{S} - \frac{\sigma_2^2}{2\kappa} \tag{A-7}$$

If the current price is S_0, then the log of prices is normally distributed and its expected value $\mathrm{E}(x)$ and variance $\mathrm{Var}(x)$ of x will be

$$\mathrm{E}(x) = e^{-\kappa t} \log S_0 + (1 - e^{-\kappa t}) \alpha \tag{A-8}$$

$$\mathrm{Var}(x) = \frac{\sigma_2^2}{2\kappa} (1 - e^{-2\kappa t}) \tag{A-9}$$

Again, the prices are lognormally distributed. Figure 4 shows the confidence bands for these single factor price models.

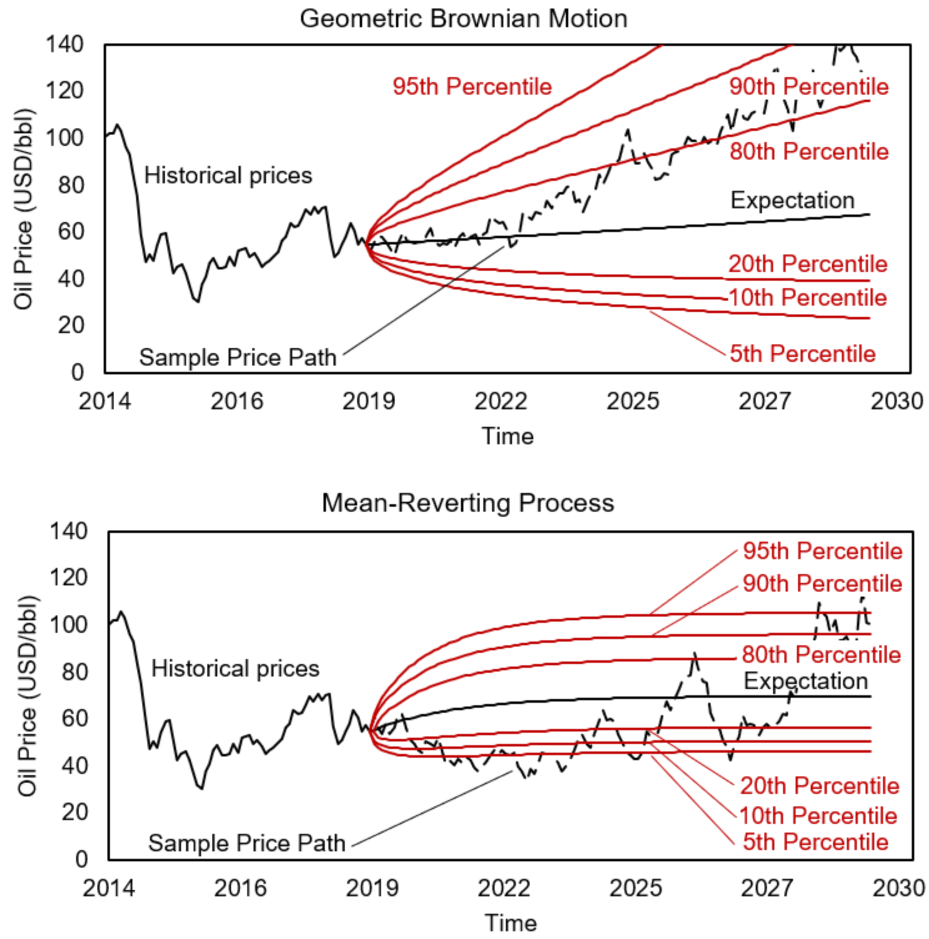

Figure 4 Single-factor price models and their range of uncertainty

Appendix B: Parameter Estimation for the Price Model

We estimate parameters of the price model by fitting the futures and option volatility curves to observed market information. The calibrated parameters are the solution to the following minimization problem

$$\begin{aligned}\{\chi_t, \xi_t\} \in \underset{\chi_t, \xi_t}{\operatorname{argmin}} \sum_{i=1}^{N} & w_i \left(\ln \hat{F}_{0,i}(\chi_t, \xi_t, T_i, \mathbf{\Omega}) - \ln F_{0,i}\right)^2 \\ & + w_i \left(Var\left(\ln \hat{F}_{0,i}(\chi_t, \xi_t, T_i, \mathbf{\Omega})\right) - Var\left(\ln F_{0,i}\right)\right)^2 \\ \text{with } \mathbf{\Omega}&: \{\mu, \kappa, \sigma_\chi, \sigma_\xi, \rho\}\end{aligned} \quad \text{B-1}$$

Here, $\hat{F}_{0,i}$ is the model's price for futures contract with maturity i, and $F_{0,i}$ is the observed market price at this same maturity. In addition, $Var\left(\ln \hat{F}_{0,i}\right)$ is the model's variance of the futures contract and $Var\left(\ln \hat{F}_{0,i}\right)$ is the implied variance of the options on the futures as observed in the market. We assume there are N market observations across varying maturities. We define importance weights w_i to enable customized fits. Note that we do not estimate the price risk premiums, λ_χ and λ_ξ. Hedging instruments generally do not inform estimation of risk premiums.

A software model could estimate the parameters of the two-factor model by solving the following minimization problem.

$$\min_{\{\chi_0,\xi_0,\mathbf{\Omega}\}} \sum_{i=1}^{N} w_i \left(\ln \hat{F}_{0,i}(\chi_0, \xi_0, T_i, \mathbf{\Omega}) - \ln F_{0,i}\right)^2 + w_i \left(Var\left(\ln \hat{F}_{0,i}(\chi_0, \xi_0, T_i, \mathbf{\Omega})\right) - Var\left(\ln F_{0,i}\right)\right)^2 \qquad \text{B-2}$$

Here, we calculate model's futures prices $\hat{F}_{0,i}(\chi_0, \xi_0, T_i, \mathbf{\Omega})$ and their variance $Var\left(\ln \hat{F}_{0,i}(\chi_0, \xi_0, T_i, \mathbf{\Omega})\right)$. In addition, we calculate the implied variance of the futures prices, $Var\left(\ln F_{0,i}\right)$ by varying $\sigma_\varphi(T)$ so that the price of the option's contract converges the observed price. The accompanying spreadsheet performs this parameter estimation process.