Trade credit duration and optimal order quantities under buyer capacity constraints

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Abstract

We develop a continuous-time framework with stochastic downstream market prices of goods and trade credit. The buyer chooses the order quantity accounting for its capacity constraints, while the supplier chooses trade credit duration by internalizing the buyer's capacity constraints and default risk. An optimal trade credit duration chosen by the supplier may arise that is driven by two opposing forces of extending credit: a higher capacity and order quantities due to reduced default risk for the buyer, and on the negative side, a lower present value of proceeds for the supplier due to delayed payment. We provide a number of predictions regarding optimal order quantity and trade credit duration including the effect of prices charged by the supplier, the volatility and growth rate of price of goods sold, the operating costs of the buyer and supplier firm, buyer's capacity constraints, recovery value in case of buyer's default and buyer market power. Our framework also considers the effect of price sensitivity of demand in both the buyer and supplier markets. We also analyze coordinated network (internal) policies versus external procurement showing that trade credit acts as a coordination mechanism limiting coordination losses under external procurement. The choice of backward integration or external production and the cost inefficiencies needed for a firm to move from internal to external production are also considered.

Keywords: bankruptcy; capacity investment; real options; supplier; supply chain

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1. Introduction

Despite the pervasiveness and the importance of trade credit in corporate markets, theoretical models analyzing the interaction of buyers and suppliers have not considered that buyers operate with capacity constraints. Capacity constraints appear in many business settings and are a result of the culmination of the economic law of diminishing marginal returns. They arise due to technological limits imposed in expanding production (e.g., affecting the demand of custom products or raw materials used in production), the unavailability of space or land or when a high volume of production results in higher coordination costs. Take for example the case of electric automobile production. Leading auto manufacturers in the electric car market such as Tesla and Toyota act on the buyer side with respect to battery-related custom-made inputs used in the production of electric cars. Initially, automobile producers faced significant capacity constraints relating to the yearly production of electric cars. This has defined limits in the volume of inputs ordered. In recent years, however, where the production of electric cars has increased, major suppliers have boosted their production to meet this capacity expansion.[1] In this paper we focus on the effect of trade credit on the optimal order quantities of the buyer under an uncertain demand and capacity constraints.

Our framework is set in continuous time and incorporates the stochastic demand of final goods sold by a buyer. Since trade credit is short-term financing, previous literature highlights its roll-over character (Amberg et al., 2020; Auboin and Engemann, 2014; Ferrando and Marcin, 2018; Garcia-Appendini, 2011). In order to produce a tractable dynamic framework with finite horizon of trade credit and in which trade credit is renewed over time, we build on the models of Leland (1994) and Leland (1998) applied in the capital structure literature. This framework assumes a stationary debt structure where at every instant of time a constant fraction of the credit matures and must be refinanced to keep the total amount of credit outstanding constant. The payment rate then determines an average credit duration. A stationary trade credit structure describes a buyer that smooths out payments to the suppliers to avoid peaks in refinancing activity. As argued by Diamond and He (2014), although a stationary debt structure is assumed for tractability, it is a sensible place to start. Besides tractability, we adopt this framework because the trade credit maturity is a key variable used by suppliers to discriminate among buyers, since direct price discrimination is often forbidden. We treat trade credit maturity as a parameter.

We introduce capacity constraints for the buyer firm in line with economic theory with a Cobb-Douglas production technology and diminishing marginal returns. This is in the spirit of recent developments in real options models with capacity constraints (Nishihara et al., 2019). We first provide an analysis with different

[1] An article by Obayashi and Shimizu (2018, Sep 13) posted on Reuters highlights this view showing how a Japanese supplier of custom-made materials for the production of batteries stands ready to meet its major automobile producers' increased production.

(exogenous) trade credit horizons, and then an interactive game where the buyer chooses order quantities based on capacity constraints, as well as the timing of default, and the supplier chooses trade credit duration. We focus on net terms contracts, which are interest-free loans extended by suppliers to buyers. For example, a “net 30” term would imply that buyers need to pay the suppliers within 30 days after invoice issuance. According to Yang and Birge (2018) and Giannetti et al. (2011), net terms are the most common trade credit terms. We do not capture in our model other less common contract payment forms such as cash and two-part terms (in which discounts are offered to encourage early payment).[2] Supplier’s variable of choice is thus the trade credit horizon, taking the wholesale price as given. Selecting to focus on trade horizon instead of targeted price adjustments is a reasonable assumption in countries where the law does not allow a vendor to offer different prices to different clients, i.e., direct price discrimination is illegal. For example, Klapper et al. (2012) claim that the Clayton Act in the US prohibits price discrimination across customers of the same good. Moreover, Fabbri and Klapper (2016) argue that price reductions are not necessarily simpler than offering trade credit since they are observable by competitors and can lead to a price war. On the contrary, trade credit maturity is a less aggressive and more flexible instrument.

Our first contribution is to show that an optimal trade credit duration may endogenously arise due to capacity constraints and default risk of the buyer. The supplier balances two forces in order to derive the optimal trade credit duration. On the one hand, a longer credit duration allows the supplier to encourage larger quantities ordered by the buyer and a reduction in buyer default risk. On the other hand, extending credit results in delayed payments thus reducing the present value of the money received by the supplier. On the buyer side, capacity constraints create a limit on how much the extensions of trade credit duration provided by the supplier can be utilized. On a broader level, our paper contributes to the literature on debt maturity and capital structure (Dangl and Zechner, 2016; Diamond and He, 2014; Leland, 1994). While this literature studies the optimal maturity structure of corporate debt, we analyze the optimal duration of trade credit extended by suppliers.

Our second contribution is to show the effect on trade credit and credit duration and on the values of the supplier and buyer firms of the prices charged by the supplier, the volatility and growth rate of prices of goods in downstream markets and the effect of profitability margins by exploring the operating costs of the buyer and supplier. While previous theoretical models have provided predictions regarding mainly the value of trade credit, the continuous-time framework that we adopted, and the debt structure assumed allow us to derive novel predictions regarding credit duration. Importantly, our framework captures the effect of a buyer’s capacity constraints, the recovery value of installed capacity in case of buyer default and buyer

[2] Giannetti et al. (2011) find that the median firm in their sample receives trade credit at zero cost. Moreover, only a minority of firms reports that their main supplier offers early payment discounts.

market power. Our model's implications for both trade credit value and duration are in line with empirical evidence. We also model the demand function in both the suppliers' and buyers' markets where the equilibrium price in each market is elastic to quantity. In the suppliers' market, we analyze both the case where higher quantity levels lead to an increase in input prices (e.g., when there is limited supply in the market or low competition) or when they lead to a decrease in input prices (e.g., due to economies of scale or increased competition).

Moreover, we investigate the benefits of trade credit by comparing internal versus external procurement in the presence of trade credit. We show that trade credit acts as a coordination mechanism limiting the losses arising due to the lack of coordination under external procurement. Our framework also shows how to derive the cost inefficiencies (e.g., due to lack of specialization) needed so that a firm moves from coordinated (internal) production with no credit to non-coordinated production with credit. We show that when input prices in the suppliers' markets increase with higher quantities ordered (e.g., due to limited available supply), internal production becomes more likely. Interestingly, however, we show that in this case the supplier would extend trade credit creating incentives for the buyer to remain under external procurement.

The rest of the paper is organized as follows. In Section 2 we compare our paper and contributions with related literature. In Section 3 we first introduce the model setup and then derive the value of trade credit, as well as buyer and supplier values. In Section 4 we present a numerical sensitivity analysis, with respect to parameter values. Section 5 presents the coordinated supply chain. Section 6 provides an extension of the basic setup to allow for the option to expand capacity and credit. Section 7 concludes. Finally, an online appendix contains details of the mathematical derivations as well as additional sensitivity results.

2. Related literature: theoretical contributions and relation with empirical evidence

The literature has identified various reasons that influence the provision of credit (see Seifert et al., 2013 for a review). These include incentives from suppliers to provide capital access to buyer firms or risk sharing (e.g, Schwartz, 1974; Yang and Birge, 2018), buyers having market power that forces suppliers to provide credit (e.g., Klapper et al., 2012 and Fabbri and Klapper, 2016), price discrimination incentives from suppliers that can be achieved through trade credit (e.g., Brennan et. al., 1998), imperfect financial markets (e.g., Emery, 1984), information asymmetry and verifying product quality (e.g., Smith, 1987), and advantage of suppliers compared to banks in dealing with issues such as adverse selection or buyer opportunistic behavior (Biais and Gollier, 1997; Chod, 2017). Our model is related to previous work that suggests that suppliers leverage trade credit to induce buyers to increase order quantities (e.g, see early work of Schwartz, 1974 and Yang and Birge, 2018). Our model differs from Yang and Birge (2018) in several aspects, which allow us to derive insights that complement their results in multiple respects. First, while they focus on the choice between bank debt and trade credit and the conditions under which various trade credit terms (net terms or two-part terms) may apply, our model focuses on the most common payment form (net terms) and derives the optimal trade credit maturity offered by the supplier. Second, our model allows for repetitive trade credit, providing the value of trade credit, as well as buyer and supplier firms using a contingent claim approach. Third, our model allows a role for demand volatility by incorporating the embedded endogenous default option of the buyer. Fourth, Yang and Birge (2018) focus on a cash constrained buyer, while our model focuses on capacity constraints. Finally, we consider the elasticities in demand in the buyer and supplier market and the choice between internal versus external procurement.

Our framework is based on a single buyer-supplier relationship and thus does not cover the free-riding problem analyzed in Chod et al. (2019), however, it shares a similar principle since we show that when a supplier cannot influence quantities ordered by the buyer (e.g., quantity is fixed irrespective of trade credit horizon), his incentive to provide credit is limited. Our paper is also related to the framework of Silaghi and Moraux (2019) who provide a model to explain the joint determination of prices and the duration of credit. In contrast to them however, we take supplier-charged prices as given and focus instead on credit duration and quantities (or value) of trade credit.

Methodologically, our work builds on the work of Leland (1994, 1998) which focuses on studying issues relating to maturity structure of debt in the capital structure literature (see also Diamond and He, 2014). In extending this work, we cast this framework in a supply chain setting by adding capacity constraints, study interactions in the choice of trade credit between the lender (supplier) and the borrower (buyer) and issues relating to coordination of the supply chain and internal versus external production.

Our analysis of trade credit as an instrument that suppliers use to capture additional future sales is in line with empirical evidence. For example, Petersen and Rajan (1997) state that: "…the evidence suggests these firms (namely buyers) may be a source of future business, and suppliers are more willing to provide credit in anticipation of capturing business". Cuñat (2007) shows that suppliers may act as lenders of last resort to support the continuation of business relations and Fishman and Love (2003) demonstrate the importance of trade credit in advancing growth in developing countries. Moreover, our analysis is also in line with real business practices (anecdotal evidence). For example, pharmaceutical companies and agricultural machinery manufacturers offer a larger trade credit period for larger amount of purchases, so the delay in payment is likely to induce the buyer to order larger quantities (Chung and Liao, 2006).

A number of stylized facts regarding trade credit are consistent with our model's predictions. First, Petersen and Rajan (1997) empirically show that suppliers with larger gross profit margins provide more credit. This evidence is in line with our prediction that higher production costs for the supplier imply lower quantities of supplied credit. Petersen and Rajan (1997) also show that higher credit quality buyers and more profitable buyers obtain larger amounts of credit. This appears broadly in line with our results where we find that the quantity of goods provided on credit is higher for buyers with lower volatility of price of goods sold, lower costs of operation, larger growth rate of prices and hence revenues and for buyer firms that face less capacity constraints. The evidence from Petersen and Rajan (1997) also supports our theoretical prediction that a higher anticipated recovery value of buyer's assets in the event of default improves the quantity of goods provided on credit. Our prediction is also in line with recent evidence by Costello (2019) who shows that an improvement in suppliers' rights to the liquidation value of collateral results in an increase in the amount of credit.[3]

Furthermore, the literature has placed emphasis on the importance of buyers' market power in the determination of credit. For example, Klapper et al. (2012) and Fabbri and Klapper (2016) and Dass et al. (2015) show that suppliers are more likely to offer trade credit to powerful and important customers or when suppliers are in relatively weaker bargaining position. Moreover, Ellingsen et al. (2016), Giannetti et al. (2011) and Klapper et al. (2012) show that trade credit *duration* increases with buyer market power (e.g., larger customers, or customers with a large sales share). This evidence is consistent with our model since we show that when buyers have a higher reservation value, suppliers will be induced to extend credit duration and that the extended trade duration leads to an increase in order quantities and trade credit value. Instead, Yang and Birge (2018) show that higher reservation values of the buyer may result in a larger early-payment discount and a decrease in trade credit value.

Our model also predicts that suppliers extend trade credit duration when charging a higher price since the negative effect of delayed income is mitigated by a higher price, and besides, extending duration induces higher order quantities. However, Barrot (2016) shows that in a competitive market financially constrained suppliers that expose themselves to default risk by extending trade credit may not be able to offset delayed income with higher prices. Although our framework incorporates the trade-off between quantities and delayed income in suppliers' choice of credit duration, we do not model supplier competition and financial constraints. Finally, we find that for firms operating in environments of higher demand volatility and specialized products with less recovery of credit in default, the trade credit horizon is longer. This evidence

[3] Our model also predicts that higher recovery value of installed capacity decreases the duration of credit. This is in contrast with evidence from Costello (2019) who finds that an improvement in suppliers rights to the liquidation value of collateral increases the duration of credit offered. We note that our modeling of liquidation value depends only on the repossessed inventorywhile in the more general sense collateral may include the value of alternative assets.

is broadly consistent with evidence in Ng et al. (1999) showing that original product manufacturers are more likely to use trade credit (net terms) compared to cash relative to retailers.

3. The model

A supplier continuously provides a quantity of goods Q to a buyer. The initial debt (trade credit) principal due for goods provided by the supplier is $P_S Q$ where P_S defines the price per unit of goods charged. Following Leland's (1994, 1998) approach at any instant of time a constant fraction m of the debt matures and the supplier receives a fraction payment m of the value of goods, i.e., $mP_s Q$ of the goods are being repaid. This implies that at any time t > 0 the outstanding balance of the initial trade credit is reduced by e^{-mt} and the average maturity of repayment is calculated as 1/m (see Leland, 1994b, 1998). Further, the supplier renews credit continuously so that each time a fraction of trade credit is due and repaid by the buyer, the supplier provides a new trade credit identical and of equal amount to the one retired. Thus, trade credit is fully rolled over. In line with Leland (1994, 1998) and Diamond and He (2014), this retains a stationary trade credit structure, i.e., the total amount of credit outstanding is constant. Our modeling approach is thus in line with the roll-over character of trade credit (and of short-term financing in general) which has been highlighted in the previous literature (Amberg et al., 2020; Auboin and Engemann, 2014; Ferrando and Marcin, 2018; Garcia-Appendini, 2011).[4] Unlike the capital structure literature where debt implies coupon payments that generate benefits, we do not consider taxes since we focus on the most common trade credit terms, i.e., net terms, which do not imply any interest payments.

We assume a simple reduced form for the inverse demand function in the suppliers' markets where the equilibrium price is defined as $P_S = a_s Q^{\epsilon_s}$, where $a_s > 0$ and ε_S defines the price sensitivity showing how prices in suppliers' markets change when the demand of the good increases ($-1 \leq \varepsilon_S \leq 1$). Initially we assume that $\varepsilon_S = 0$ so that the price set by the supplier is fixed at a_s. We then elaborate on the effect of elasticity in a later section. Note that $\varepsilon_S > 0$ is used to capture shortages in supply where the price of goods increases with higher demand of the good, whereas $\varepsilon_S < 0$ is used to capture economies of scale or increased competition where an increase in quantities reduces the cost of production. The cost of production of these goods for the supplier is C_S per unit sold.

[4] For example, Ferrando and Marcin (2018) propose a theoretical model to study the relationship between trade credit and investment where firms have a portion of outstanding trade credit on the books before they decide to invest, "highlighting the roll-over nature of trade credit". Similarly, Auboin and Engemman (2014) stress the roll-over character of short-term financing and in particular of trade credit. Garcia-Appendini (2007) argues that suppliers' denial or refusal to roll over trade credit can reveal valuable information to banks. Amberg et al. (2020) claim that the ability for firms to roll over trade credit depends on the absence of obstacles to the functioning of risk sharing networks.

We assume that the buyer faces an inverse demand curve defining the price per unit sold X as a function of the quantity of goods as follows: $X = x\, Q^{\varepsilon_B}$, where ε_B is the price sensitivity of demand ($-1 \leq \varepsilon_B \leq 0$).[5] Initially we shall assume that $\varepsilon_B = 0$ so that the quantity produced has no impact on price in which case the demand shock process coincides with the price process. We later investigate the case where $\varepsilon_B < 0$ where the influx of more products in the buyer market (higher Q) would decrease the price of goods sold. The buyer firm obtains cash flows X dt per unit of product sold per interval dt. The demand shock x affecting the price per unit at which the buyer can sell the goods in the downstream market follows a Geometric Brownian motion:

$$\frac{dx}{x} = \mu dt + \sigma dZ \tag{1}$$

where μ is the expected rate of change, σ is the volatility and dZ is a standard Weiner process. The demand shock x can be interpreted as the relative strength of the demand in the downstream market. This variable can be driven by variations in the target market size, disposable income, tastes, and prices of substitute products (see, for instance, Aguerrevere, 2003 and 2009). Our framework is thus more relevant in industries with a larger variation in the above-mentioned factors affecting demand, such as luxury products or technology modeling the relationships between original equipment manufacturers (OEMs) and buyers of custom-made or specialized inputs (see Ng et al., 1999). We assume risk-neutrality, with r denoting the risk-free interest rate, and that $r > \mu$ such that there is a rate of return shortfall similar to a convenience yield $\delta = r - \mu$. A higher δ (while keeping r constant) captures a lower rate of growth of the good's demand in the buyer's markets.

We assume that the buyer of goods selects the optimal quantity Q to be ordered by solving an optimal capacity problem (Nishihara et al., 2019). Specifically, we assume that when buying the goods for the first time at $t = 0$ the buyer needs to also incur a one-time investment cost of κQ^{η}, where Q is the capacity (quantity) of the goods (i.e., units of goods that can be sold per unit time), Q^{η} is the amount of capital required to produce at that capacity (with $\eta > 1$), and the cost of capital is \$$\kappa$ per unit. For example, if the buyer is an automobile producer then it needs to have the necessary building and space to produce and store the goods; capacity constraints on expanding building space or managing increased volume may then put limits on the amount of ordered inputs. This investment cost function is commonly used in the literature

[5] For markets where the buyer acts as reseller (e.g., retail) this specification is exact. In some markets Q refers to the quantity of an input good in production in the downstream market that is processed and results in a final good. In this case we should define the price of the good in the final market as a function of the quantity of goods produced in the final (downstream) market, not as a function of the quantity of input goods. Since a specific number of input goods is used for the production this will involve a simple transformation. With this note in mind, we avoid introducing additional notation. We also note that one can define the elasticity of demand as $\left|\frac{1}{\varepsilon_B}\right|$.

(see discussion in Nishihara et al., 2019) and η >1 implies decreasing marginal returns of investment.[6] Note that the capacity size fixes the production level and thus the ordered quantity, i.e., the firm always produces at full capacity. Volume inflexibility is a common assumption in the previous literature (Chod and Rudi, 2005; Goyal and Netessine, 2007; Hagspiel et al., 2015, among others). Indeed, although the buyer firm might keep some capacity idle, producing below capacity is costly since there are large fixed costs associated with production ramp-up. Moreover, relationships with the firm's stakeholders, such as suppliers, clients and employees can hinder the firm's ability to adjust output volume. For example, labor unions impose limitations on both overtime work and downsizing (Hagspiel et al., 2015).

Following the selection of Q ordered by the buyer, the buyer operates by repaying the trade-credit at a constant rate *m* and rolls over credit. This keeps the principal due and the quantity constant. While a constant quantity ordered may appear a restrictive assumption, we note that the value of trade credit varies with market conditions (see next section). Furthermore, in section 6 we discuss an extension where we allow the buyer firm the option to expand the capacity under more favorable market conditions in the downstream market. The buyer also runs some other fixed operating costs C_b. Similar to equityholders in the models of Diamond and He (2014) and Leland (1994, 1998), we assume that the buyer has access to funds to cover the investment costs and losses at refinancing. Default occurs when the buyer's incentive to inject more funds is insufficient in which case the buyer defaults and its value drops to zero. This occurs when the demand shock drops to an endogenously determined threshold optimally selected by the buyer, x_B, where the buyer stops operations and hence payments to the supplier. In the event of bankruptcy, the supplier can repossess inventory that has not been converted into finished goods and receives a value V_B which is a fraction *b* of inventory, i.e., $V_B = bP_s$Q. Parameter *b* captures asset specificity in the sense that a higher *b* implies a less specialized market which allows for more opportunities for reselling or reusing inventory, and hence implies a larger recovery value for the supplier. This allows us to capture the effect of asset specificity on trade credit in industries such as OEMs in which recovery of inputs by the supplier may be low since the inputs are most often integrated in the buyer's product.

3.1. The value of trade credit

[6] There is an extensive literature focusing on capacity investments (for an overview of this literature see Huberts et al., 2015). Our choice for the cost function rests on the economic theory law of decreasing marginal returns. The cost function employed is $C = \kappa \cdot K$ subject to $Q = K^{\alpha}$ where κ is the per unit price (cost) of the production factor K (e.g., capital) and Q is the Cobb-Douglas production function with a single production factor K with α being the input elasticity, where α < 1 due to decreasing marginal returns. Solving the production function with respect to K and replacing in the cost function results in $C = \kappa \cdot Q^{\eta}$, where $\eta = \frac{1}{\alpha} > 1$ which is the cost function used in the paper.

Following analogous arguments used in Leland (1994, 1998) for valuing short-term debt (see also the recent analysis by Diamond and He, 2014), the value of trade credit $D(x)$ follows the following second order ordinary differential equation:

$$rD(x) = \frac{\sigma^2}{2}x^2 D''(x) + (r - \delta)xD'(x) + m(P_S Q \ - D(x)) \qquad (2)$$

The last term on the right-hand side of this equation represents the change in trade credit value due to debt retirement. Since a fraction $m\ dt$ of credit matures, the instantaneous principal repayment is $mP_S Q\ dt$ and the supplier provides a newly issued credit of $mD\ dt$.

The solution is of the following form:

$$D(x) = \left(\frac{mP_S Q}{r+m}\right) + A_1^D x^{\gamma_1} + A_2^D x^{\gamma_2} \qquad (3)$$

The first term satisfies (2) and shows the risk-free value of credit, while the additional terms intend to capture the adjustments needed due to buyer's option to default. Solutions for γ_1 and γ_2 are obtained by applying a general form solution Ax^{γ} to the differential equation (2) which results in the following fundamental quadratic equation:

$$q = \frac{1}{2}\sigma^2\gamma(\gamma - 1) + (r - \delta)\gamma - (r + m) = 0 \qquad (4)$$

The two roots of the quadratic are then:

$$\gamma_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+m)}{\sigma^2}} > 1 \qquad (5a)$$

$$\gamma_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+m)}{\sigma^2}} < 0 \qquad (5b)$$

Parameters A_1^D and A_2^D are constants to be determined by the following boundary conditions:

$$\lim_{x\to\infty} D(x) = \frac{mP_S Q}{r+m} \qquad (6)$$

$$D(x_B) = V_B = bP_S Q \qquad (7)$$

Extremely profitable buyers never default, and the default-free trade credit value is $\frac{mP_S Q}{r+m}$, as in equation (6). On the other hand, equation (7) indicates that the buyer defaults when $x = x_B$ and the supplier receives the recovery value $V_B = bP_S$Q.

Solving equation (2) with boundary conditions (6) and (7) we obtain that:

$$A_1^D = 0 \tag{8a}$$

$$A_2^D = \left(V_B - \frac{mP_SQ}{r+m}\right) x_B^{-\gamma_2} \tag{8b}$$

This leads to the following solution for the value of trade credit:

$$D(x) = \frac{mP_SQ}{r+m} + \left(V_B - \frac{mP_SQ}{r+m}\right) \left(\frac{x}{x_B}\right)^{\gamma_2} \quad , \tag{9}$$

where the term in between brackets has to be negative so that the supplier suffers some loses in bankruptcy. We ensure this in our numerical analysis in section 4.

3.2. Buyer's value

Following similar arguments, the buyer value $B(x)$ satisfies the following second order ordinary differential equation:

$$rB(x) = \frac{\sigma^2}{2} x^2 B''(x) + (r-\delta)xB'(x) + (XQ - C_b) - m(P_sQ - D(x)) \tag{10}$$

The last two terms of equation (10) represent the cash flows of the buyer. The first of these two terms captures the profits from selling the final good involving revenues XQ, where $X = x\,Q^{\varepsilon_B}$, and payment of the fixed costs C_b, while the last one captures the rollover gains/losses of paying the principal, i.e., the price of goods, mP_SQ, and receiving the trade credit proceeds, mD. Note the symmetry with respect to the equity value in the frameworks of Leland (1994, 1998) and Diamond and He (2014).

The solution of the above claim $B(x)$ can be expressed as follows:

$$B(x) = \left(\frac{XQ}{\delta} - \frac{C_b}{r}\right) - D(x) + A_1^B x^{\beta_1} + A_2^B x^{\beta_2} \tag{11}$$

where $D(x)$ is given in (9) and the exponents β_1 and β_2 are given by:

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{12a}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{12b}$$

Appendix A provides details on the derivation of the particular solution (first two terms in equation 11) and the derivation of A_1^B and A_2^B constant terms that are determined by appropriate boundary conditions. The particular solution captures the perpetuity value of the buyer accounting for revenues, operating costs and repayment of credit, while the additional terms intend to capture the buyer's default option. To determine A_1^B and A_2^B we apply the following boundary conditions:

$$\lim_{x \to \infty} B(x) = \left(\frac{XQ}{\delta} - \frac{C_b}{r}\right) - \frac{mP_S Q}{r+m} \tag{13a}$$

$$B(x_B) = 0 \tag{13b}$$

Condition (13a) implies that in the absence of default risk, the buyer's value is the present value of cash flows (first term) minus the (risk-free) value of credit. Condition (13b) is the standard condition that the value of buyer's value in the event of bankruptcy becomes zero. Applying these conditions and simplifying the terms that cancel out (see Appendix A for details), we obtain the following solution for buyer value:

$$B(x) = \left(\frac{XQ}{\delta} - \frac{C_b}{r}\right) - D(x) + \left[V_B - \left(\frac{X_B Q}{\delta} - \frac{C_b}{r}\right)\right]\left(\frac{x}{x_B}\right)^{\beta_2} - \kappa Q^{\eta} \tag{14}$$

Note that we have also accounted for the one-time investment cost incurred by the buyer at time zero, κQ^{η}, which appears as the last term of equation (14). Also note that $X_B = x_B Q^{\varepsilon_B}$, where x_B is obtained below (in equation 16) by applying the following smooth pasting condition:

$$\frac{\partial B}{\partial x}\Big|_{x = x_B(Q)} = 0 \tag{15}$$

Applying this smooth pasting condition, we obtain the following default threshold:

$$x_B(Q) = \frac{\delta}{(1-\beta_2)Q^{\varepsilon_B + 1}}\left[\gamma_2\left(V_B - \frac{mP_S Q}{r+m}\right) - \beta_2\left(V_B + \frac{C_b}{r}\right)\right] \tag{16}$$

Thus far we have assumed the quantity ordered to be fixed. However, the buyer chooses both the default trigger and quantity to be produced simultaneously. The optimal Q maximizes $B(x)$ and therefore satisfies:

$$\frac{\partial B}{\partial Q} = 0 \tag{17}$$

This condition results in a non-linear implicit equation (see Appendix A) that has no closed-form solution and thus is solved numerically.

3.3. Supplier's value

The supplier value satisfies the following second order ordinary differential equation:

$$rS(x) = \frac{\sigma^2}{2}x^2 S''(x) + (r-\delta)xS'(x) + m(P_S Q - D(x)) - C_S Q \tag{18}$$

The cash flows of the supplier are the rollover gains/losses (receiving the principal payment for the goods on credit, mP_SQ, and giving the trade credit proceeds when providing new credit, mD) and production costs C_SQ. Note that the price of goods is defined more generally by $P_S = a_s Q^{\epsilon_s}$.

The solution of the above claim $S(x)$ can be expressed as follows:

$$S(x) = D(x) - \frac{C_S Q}{r} + A_1^S x^{\beta_1} + A_2^S x^{\beta_2} \tag{19}$$

To find the particular solution we use a similar approach as the one for the buyer described in the Appendix A. To determine A_1^S and A_2^S we then apply the boundary conditions:

$$\lim_{x\to\infty} S(x) = \frac{mP_S Q}{r+m} - \frac{C_S Q}{r} \tag{20}$$

$$S(x_B) = V_B \tag{21}$$

Applying these conditions and simplifying the terms that cancel out we obtain the following solution for supplier value:

$$S(x) = D(x) - \frac{C_S Q}{r} + \frac{C_S Q}{r}\left(\frac{x}{x_B}\right)^{\beta_2} \tag{22}$$

In the numerical section we allow for the case where the supplier optimally selects the trade credit horizon by selecting the optimal *m* that maximizes supplier's value.[7] To perform that optimization we run a dense grid of *m* choices allowing for a very wide range of trade credit choices including the one of almost immediate repayment. The following section provides the optimization and the interactions between supplier and buyer decisions.

3.4. Interactions between buyer and supplier: optimal trade credit maturity and optimal order quantity

We now describe the interactions between the buyer and the supplier. For a given trade credit maturity, the buyer optimally selects the quantity of goods ordered from the supplier and the default threshold that maximize buyer value. Internalizing how his choice of trade credit maturity influences the buyer's optimal choices of Q and x_B, the supplier optimally selects the trade credit maturity that maximizes supplier value. These interactions are formally described in the following maximization problem:

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[7] Note that while in the capital structure literature the firm decides its own capital structure (namely the maturity of its debt), in the case of trade credit between a buyer and a supplier, although the buyer might insist on certain credit terms, ultimately it is the supplier who decides how much trade credit duration he is willing to extend.

$$MAX\; S(x) \atop (\frac{1}{m}) \tag{23}$$

$$s.t.\; MAX\; B(x) \atop Q, x_B$$

Given the non-linearity of the equations involved, this maximization problem is solved numerically. We run a dense grid search of $(1/m)$ choices for the supplier subject to the optimal solutions Q and x_B that maximize the buyer's value (see equations 16 and 17). Among the grid of $(1/m)$ choices created we then select the choice that maximizes the supplier value. In section 4.3 we discuss the optimal order quantity by the buyer and the optimal trade credit maturity granted by the supplier, as well as their sensitivity with respect to the model parameters.

Following Yang and Birge (2018), it is straightforward to extend the basic model to incorporate buyer market power through a buyer participation constraint. In particular, let L denote the buyer's reservation value, i.e., the minimum value that the buyer would accept. In that case the supplier has to solve (23) subject to $B(x) \geq L$. This will allow us to analyze the sensitivity of order quantity and maturity with respect to buyer market power and contrast it with empirical evidence.

4. Numerical results

We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $C_b = 100$, $C_S = 0.1$, $P_S = 20$, $\sigma = 0.15$, $X = 10$, $k = 5$, $\eta = 2$, and $L = 0$. Note that to keep a fixed $P_S = 20$ for base case results we assume $\varepsilon_S = 0$ and $a_S = 20$. Similarly for the buyer market we initially assume that $\varepsilon_B = 0$ and that the initial value of the demand shock for the final product *x* = 10 (thus the price *X* at which the final good is sold coincides with *x*). Our base parameters used for *r*, *δ* and *σ* are in line with other real options models (e.g., Mauer and Sarkar, 2005 and Hackbarth and Mauer, 2011).[8] Our base case for *η* is the same as in Nishihara et al. (2019), while our base case parameters for x and C_b were chosen alongside *κ* and *η* which define optimal order quantity levels to allow the buyer to operate with positive value. We assume initially no recovery at default (*b* = 0) in order to separate out the effect of possible recovery of inventory of the buyer by the supplier in the event of default. This is plausible given the low priority that suppliers usually have when buyer firm defaults compared to other claimants. We set $C_S = 0.1$ since C_S is a flow of operating costs (hence it needs to be low enough compared to P_S which is similar to a principal payment in order to retain positive supplier value).

[8] Mauer and Sarkar (2005) use r = 0.05, δ = 0.02 and $\sigma = 0.25$, while Hackbarth and Mauer (2011) use r = 0.06, δ = 0.05 and $\sigma = 0.25$.

We first present results in Section 4.1 and 4.2. for various (exogenous levels) of trade credit duration. We present our baseline results in Section 4.1. and then analyze the sensitivity of our results with respect to the parameters of the model in Section 4.2. Section 4.3. presents results relating to the interactive game between the supplier and buyer firm where the choice of trade credit duration is determined by the supplier firm. This latter section also summarizes the predictions of our model taking into account these interactions.

4.1 Baseline results and buyer market power

In Figure 1 we provide sensitivity with respect to trade credit maturity with a horizon of maximum 1 year. The sensitivity of buyer value, supplier value, optimal default threshold x_B and optimal quantity Q selected by the buyer are provided in the four panels of the figure. First, we observe that as the duration of credit ($1/m$) increases, default is delayed (x_B decreases), optimal order quantities by the buyer increase, and thus buyer value increases. Longer credit duration allows a reduction in default risk and the present value of costs incurred by the buyer, and hence generally encourages higher quantities Q. This result is similar to Yang and Birge (2018) suggesting the importance of risk sharing role of trade credit. However, due to decreasing returns to scale, the effect of credit duration on Q flattens out for long credit durations since increasing Q further incurs significant capacity costs.

[Insert Figure 1 here]

Second, we observe that the supplier value is hump-shaped with respect to credit duration ($1/m$), meaning that there exists an optimal duration of credit ($1/m^*$). This result can be understood as follows. The supplier has the following trade-off of increasing the duration of credit. On the negative side, the supplier receives the payment with a delay resulting in lower present value of the payment (the present value of received payment effect). On the positive side, increasing credit duration allows the buyer to order more (Q increases) and also results in delayed default and thus an extended period where the supplier trades with the buyer. The interaction of these two opposite effects determines the optimal credit duration.

To better understand this, we investigate the case with fixed (instead of optimal) Q in Figure 2. In this case we observe that since the positive effect of increasing quantities at longer trade horizons is not present, the optimal credit policy of the supplier is to request immediate repayment, i.e., the supplier value, in the absence of gains of extending credit, is strictly decreasing in the trade credit horizon. This result can be compared with the results of Chod (2019) where a supplier reacts by reducing trade credit when buyer firms free ride and hence do not increase order quantities when provided with longer credit.

[Insert Figure 2 here]

To facilitate replication of our results we provide the solution for the buyer-supplier interaction given by equation (23) for baseline parameter values, namely the optimal maturity, order quantity, buyer and supplier values, as well as default threshold, in the first row of Table 1.[9] Additionally, we illustrate the effect of buyer market power in the extended model where the supplier maximizes (23) subject to the buyer's participation constraint. We can see that a larger buyer reservation value of 2720.24 (around 5% larger than buyer value in the unconstrained base case) leads to a higher trade credit duration and higher order quantities. Moreover, buyer value increases, while supplier value decreases. This is intuitive: since buyer value is increasing in trade duration as shown in Figure 1, the supplier's only choice to allow for a higher reservation value for the buyer is to extend credit duration. This then leads to higher order quantities as well.

[Insert Table 1]

4.2 Sensitivity with trade credit maturity for alternative parameter values

We now provide some sensitivity analysis of our baseline results with respect to important parameters of the model that provides further insights into how trade credit duration and order quantities are determined within our model.

Figure 3 shows sensitivity with respect to the price of goods charged by the supplier. For a relatively low price, $P_S = 15$, even with strict terms of immediate repayment the order quantities remain high (about $Q = 24.5$ according to the figure) since default risk and the present value of costs incurred by the buyer are low. For a relatively high price, $P_S = 20$, the costs and default risk increase and thus the order quantities decrease.[10] As expected, the buyer value decreases with P_S, while the supplier value increases with P_S.

Second, we observe that the lower the price P_S charged by the supplier, the shorter the optimal duration of credit $(1/m^*)$ provided. This result is intuitive. When P_S is low the supplier cannot afford delayed payments resulting from extending credit. Thus, when P_S is very low, the supplier's optimal policy will be to request immediate repayment, i.e., the supplier value will be strictly decreasing in the trade credit horizon. On the other hand, a higher price P_S results in extending credit duration since the negative present value effect on supplier's income is mitigated by balancing out the delay with a higher price charged, and also extending duration increases the positive effects by inducing higher order quantities and delayed default for the buyer. Our model thus highlights how a supplier can extend trade credit to alleviate the cost burden to a buyer

[9] Buyer reservation value is assumed to be null so that the participation constraint is not binding.

[10] Note that to produce the same quantity as with $P_S = 15$ with almost immediate repayment, when $P_S = 20$ we need a longer duration of credit. Also, note that due to flatten out of Q at long durations some quantities that can be achieved with low P_S can never be achieved at high P_S for any duration (e.g., when $P_S = 15$ and duration is 0.2 the quantity level is not achievable for $P_S = 20$ even if duration is extended to 1 year).

firm. Despite this possible usefulness of trade credit, Barrot (2016) shows that adjustments to trade credit may not be possible when suppliers are financially constrained.

[Insert Figure 3 here]

Figure 4 shows sensitivity results with respect to volatility of the demand shock in the downstream market. A higher volatility results in a lower default threshold, however this does not necessarily imply a lower default risk (since there may be a higher likelihood that this lower threshold is reached under higher volatility). The results show that a higher volatility results in lower values for the buyer due mainly to the reduction in order quantities. The reduced quantity ordered by the buyer when volatility is high also hurts the supplier. In the case of high volatility the supplier's optimal credit policy is to increase trade credit duration in order to allow the buyer to avoid default and retain a high order quantity. On the other hand, when the volatility is low the incentives to provide credit are lower since even with a short credit duration the supplier can achieve a high level of order quantities (notice that with low volatility the order quantities are flatter in credit duration implying that quantities at very low credit durations are closer to longer trade horizon optimal quantities). This intuition is verified for an even lower volatility level ($\sigma = 0.1$), where we find that the supplier value is strictly decreasing in trade credit duration and hence the optimal policy of the supplier is to ask for immediate repayment. The effect of low volatility is also confirmed in the subsequent section where we allow the supplier firm to select the optimal maturity demonstrating that our setting is more important in explaining trade credit in settings with high volatility of demand in downstream markets such as technology products or luxury goods (rather than more homogenous and less volatile retail products).

[Insert Figure 4 here]

Figure 5 shows the sensitivity with respect to capacity constraints, modeled by parameter η. A higher η implies higher capacity constraints, that is, a lower marginal productivity of capital, which leads to lower quantities being produced. The default threshold increases, while buyer and supplier values decrease. By extending credit the supplier can exert a more significant positive impact on order quantities when the buyer faces less severe capacity constraints (a lower η) compared to when η was high. The optimal duration of credit is thus higher for a lower η. A similar effect has been observed in our sensitivity results with respect to per unit cost of capital, k.

[Insert Figure 5 here]

Figure 6 analyzes the impact of asset specificity captured by the parameter b. A higher b implies a lower asset specificity, thus the supplier can recover a larger fraction of the buyer's inventory. This leads to a lower default threshold, a higher order quantity and an increase in buyer and supplier values. For a high b, the optimal order quantity is less sensitive to credit maturity so even providing a lower credit maturity, the supplier can still induce the buyer to order a relatively high quantity, thus $(1/m^{*})$ decreases with b.

[Insert Figure 6 here]

Figure 7 shows the sensitivity with respect to the elasticity of demand in the supplier's market. The results show that when $\varepsilon_S > 0$ which implies that the increased demand for suppliers' good leads to an increase in price (e.g., due to shortage of supply which raises prices of inputs), buyers are adversely affected and reduce the optimal quantity ordered. Buyers also default sooner since they face higher prices. As the analysis shows, suppliers generally benefit from increasing prices (despite lower quantities ordered) and they extend credit to somehow mitigate the impact on more financially constrained buyers. Opposite results hold when higher demand of goods would lead to economies of scale or increased competition which reduces the cost (and prices charged) for goods ($\varepsilon_S < 0$). The discussion between the two cases highlights important issues in recent discussions relating to the market strategies of automobile producers of electric cars. Established automobile producers hinted that an increase in demand of inputs needed for battery production will result in an increase in input prices; this view favored a slower introduction of electric cars in the market. Tesla, on the other hand, worked with some suppliers of input materials such as Panasonic towards increasing the supply of inputs by creating the so called Gigafactory to create economies of scale that would drop the cost of production.[11]

[Insert Figure 7 here]

Despite the possible reduction in costs, buyers need to also consider that higher produced quantities may adversely affect prices in the downstream markets. Figure 8 shows sensitivity results with respect to the elasticity of demand in the buyers' markets. The results show that when a higher quantity of goods produced results in lower prices in the downstream markets (i.e., excess supply reduces prices of goods), the buyer firm value will be reduced and so are the order quantities (and hence the produced final goods). Lower quantities ordered have a negative effect on supplier's value as well, and they increase the trade credit duration provided.[12] Overall, both the effect of quantities on input and output prices should be considered. For example, our sensitivity analysis (shown in Appendix B, see Fig. A.1) shows that a 2% negative price sensitivity in supplier's input prices combined with a 2% negative price sensitivity in buyer's final good market do not offset each other. For our base case parameters, buyer's and supplier's values and the order quantities are reduced, default is accelerated and the optimal trade credit duration decreases.

[11] See Brown (2014, Aug 1). Since then Tesla has moved into the creation of new Gigafactories. Fosse (2019, July 6) discusses the opportunities arising from creating a new Gigafactory in Europe.
[12] We have verified that this result holds for a wide range of negative elasticities of demand of the buyer.

[Insert Figure 8 here]

4.3 Model predictions for the optimal credit maturity and optimal order quantity

To complete the analysis, we now solve the interactive game between the supplier and the buyer where the supplier selects the optimal maturity by considering buyer's optimal quantity and default threshold (see equation (23)). We focus on the sensitivity of the optimal quantity chosen by the buyer and optimal credit duration provided by the supplier with respect to the model parameters.

We note that compared to the previous section analysis, in this section we now focus only on the optimal point of credit maturity chosen by the supplier and not the full spectrum of credit maturity choices. Table 2 summarizes the predictions of our model regarding the different model parameters. To ensure the generalization of our results we have performed the following: a) we use a dense grid of (1/m) choices that corresponds to about 4 days per interval,[13] b) we have repeated the sensitivity analysis for alternative model parameter combinations. Our predictions shown below appear quite general except for the case of low volatility. When volatility is sufficiently low, we obtain that immediate payment holds irrespective of other parameter levels, except buyer market power. When buyer market power is large, the supplier offers a positive credit maturity even with low volatility.[14] Thus, one first take away from our analysis is that trade credit use to enhance order quantities will be more important in markets with sufficient volatility in demand in the downstream market.

[Insert Table 2]

Focusing first on the optimal quantity ordered by the buyer we observe that the model predictions are in agreement with the evidence in Petersen and Rajan (1997) who show that buyers which are more credit worthy and more profitable obtain a larger value of goods under credit. Indeed, our sensitivity analysis shows that firms with lower volatility, lower cost of operations, larger growth of prices of goods sold, lower capacity constraints and higher recovery of assets in case of default obtain more trade credit. At the same time, we show that buyer firms not facing significant price competition in downstream markets (i.e., having a higher ε_B), receive higher quantities on credit. Similarly, Petersen and Rajan (1997) also show that suppliers with larger gross profit margins also provide more credit, which is in line with the sensitivity regarding the production cost for a supplier, where a supplier with higher production costs provides overall

[13] This corresponds to creating 96 intervals for 1 year amounting to roughly (1/96) 365 = 3.80 days per interval. For the results reported in Table 1 we used an even denser grid of 120 intervals (roughly 3 days).

[14] This result explains why low volatility industries such as retail might have positive trade credit maturities due to buyer bargaining power. Klapper et al. (2012) find that the largest buyers obtain the longest maturities from smaller suppliers, in line with a market power explanation (smaller suppliers are squeezed by large buyers).

less credit. Moreover, Dass et al. (2015) and Fabbri and Klapper (2016) show that larger trade credit is extended to buyers that have a higher bargaining power, which is in line with our result that buyer market power leads to higher order quantities.[15]

Table 2 also summarizes the signs of the parameters' effect on trade credit maturity. The sensitivity results with respect to trade credit maturity highlight the complex interactions taking place where the supplier firm balances the costs of delayed payments of extending credit with the potential benefits of mitigating the effect on quantities ordered and lowering the default risk of the buyer firm (and thus maintaining a longer duration of business relationship with the buyer). The latter effects dominate, in which case the trade credit maturity increases, for the case of higher price charged by the supplier to the buyer, higher final product volatility and higher product's convenience yield (i.e., when product price growth is lower), when higher quantities lead to higher prices in supplier's market (when ε_S is higher) or when buyer market power is high. In contrast, trade credit maturity decreases with the supplier's production costs, interest rate, buyer's production cost, buyer capacity constraints, per unit cost of buyer's installed capital, the recovery rate of installed capacity in the event of default and the price sensitivity of demand in the downstream markets. The scarce empirical evidence on trade credit maturity highlights buyer market power as one main determinant of credit duration. Ellingsen et al. (2016), Fabbri and Klapper (2016), Giannetti et al. (2011) and Klapper et al. (2012) find that contracts to the largest buyers, buyers with a larger share of the supplier's sales or with more suppliers entail longer maturities (net days). Our model's implications are in line with this evidence, as we show that a high buyer market power leads to extended credit duration. Klapper et al. (2012) also find that most credit worthy buyers enjoy longer maturities. Our model's implications are partially consistent with this evidence. On the one hand, the model predicts that buyers which face lower costs of operations and lower capacity constraints (that can be thought to increase buyer credit worthiness and profitability) enjoy a larger trade credit maturity, in line with this evidence. On the other hand, if lower volatility is interpreted as higher credit worthiness, then our implication regarding volatility does not appear in line with this evidence, since the model predicts that a lower volatility decreases maturity. Finally, in line with our model showing the importance of order quantities on the level of trade credit, Ellingsen et al. (2016) show that it is the transaction volume rather than trade credit maturity that drives variation in the trade credit that a customer has with its supplier.

[15] In addition, Table 1 shows that optimal quantity increases with the interest rate which we could not relate with empirical findings.

5. Coordinated supply chain (vertical integration) versus external procurement

Thus far we have analyzed the case of external procurement, where the buyer optimally selects the order quantity and default threshold maximizing its own value, without considering supplier value. In contrast, in a vertically integrated supply chain these variables are chosen to maximize the total value of the supply chain, taking into account both buyer and supplier value. In this section, we first analyze the coordinated optimum under vertical integration *in the presence* of trade credit. This will allow us to compare with the case of external procurement and compute the gains of coordination. Then we focus on the choice of internal versus external production, i.e., the "make or buy" or backward integration choice (see Lafontaine and Slade, 2007).

Vertical integration with trade credit is equivalent to a social planner optimization maximizing total value of buyer and supplier. In this case the buyer internalizes the benefits of credit and so the network value is $N(x) = B(x) + S(x)$. Summing equations (14) and (22) we obtain:

$$N(x) = B(x) + S(x) = \left(\frac{XQ}{\delta} - \frac{C_b}{r} - \frac{C_S Q}{r}\right) + \left(V_B - \left(\frac{X_B^N Q}{\delta} - \frac{C_b}{r} - \frac{C_S Q}{r}\right)\right)\left(\frac{x}{x_B^N}\right)^{\beta_2} - kQ^{\eta} \tag{24}$$

Note that the total network value does not depend on the input price charged by the supplier to the buyer, P_S, nor on the credit maturity, *m*. Thus, the price and maturity of trade credit are irrelevant for a vertically integrated supply chain (although they *do affect* the individual value of each firm belonging to the network). Also, we have $X_B^N = x_B^N Q^{\varepsilon_B}$, where x_B^N denotes the optimal default threshold that maximizes the network value, and is found by applying the condition $\frac{\partial N}{\partial x}|_{x=x_B^N} = 0$ resulting in the following solution:

$$x_B^N = \frac{-\beta_2}{(1-\beta_2)} \frac{\delta}{Q^{\epsilon_B+1}}\left(V_B + \frac{C_b + C_S Q}{r}\right) \tag{25}$$

This threshold can be compared with the optimal default threshold chosen by the buyer in case of external procurement. To distinguish the two default thresholds, we will add the superscript *B* to the optimal default threshold chosen by the buyer, x_B given by equation (16), denoting it hereafter as x_B^B. Comparing the two thresholds, it can be shown analytically for the case of $b = 0$ (no recovery) that $x_B^N < x_B^B$, that is, the coordinated network optimally decides to stop production later compared to the buyer (see Appendix C for the proof).

The coordinated network selects not only the optimal stopping threshold x_B^N, but also the optimal quantity Q that maximizes network value by finding Q^N that solves $\frac{dN(x)}{dQ} = 0$. Contrasting with the solution where the buyer selects optimal Q and x_B, we can calculate the percentage gain due to coordination:

$$NG = \frac{N(x;x_B^N,Q^N)-(B(x;x_B^B,Q^B)+S(x;x_B^B,Q^B))}{N(x;x_B^N,Q^N)} \quad (26)$$

Using our base case parameters, we compare network results with coordinated actions versus the uncoordinated solution with trade credit. Figure 9 shows the results. We observe that the larger the credit maturity, the closer the default threshold and quantity chosen by the buyer get to the network optimum and thus the lower the gains from coordination. The loss from lack of coordination is highest for short maturities (around 78% difference between the coordinated value compared to the sum of buyer and supplier value when there is no coordination). Thus, trade credit acts as a coordination mechanism for the supply chain resulting in total value of the firm that is closer to the coordinated optimum.

[Insert Figure 9 here]

We end this section with a discussion of internal production of both input and output goods by one single firm when there is no trade credit, i.e, the "make or buy" decision by the buyer firm. Equation (24) gives the value of a network composed of a buyer receiving trade credit through internal procurement from a supplier belonging to its supply chain. Nevertheless, this equation could also be interpreted as the value of one single firm under internal production of the input good and no trade credit. That is, assuming that the buyer could produce internally the input good at the same production cost as the supplier does, the expression for the buyer value would be the same as the one for the coordinated network given by equation (24), albeit in this case the firm under internal production will have $V_B = 0$ since firm value at bankruptcy should be zero. However, for internal production to be possible the buyer will also need to pay an initial setup cost K_S (e.g., building a manufacturing plant for inputs). If the buyer prefers external procurement it would be because of this initial setup cost or because he is more inefficient producing internally the input good compared to the supplier.

We next determine the cost per unit $\overline{C_S}$, that the buyer incurs to produce internally the input good, above which the buyer moves from internal production (with no credit) to external procurement (with credit). The cost $\overline{C_S}$ simultaneously affects the network's order quantity Q_N and the network's optimal default x_B^N. To determine this cost, we thus need to solve the following system (also ensuring that x_B^N is determined by equation (25)):

$$N(x, Q, C_S) - K_S - B(x) = 0 \quad (27a)$$

$$\frac{dN(x,Q,C_S)}{dQ} = 0 \quad (27b)$$

A value of C_S of the buyer-producer that exceeds $\overline{C_S}$ would justify that the buyer uses external production (with credit), instead of internal production (with no credit). Thus, our analysis separates the region of internal versus external production based on cost per unit of production. We note that the solution to the above $\overline{C_S}$ also determines a $\bar{Q}_N, \bar{x}_B^N$.

Figure 10 shows the cost per unit $\overline{C_S}$ above which the buyer moves from internal production (with no credit) to external procurement (with credit) for various trade credit horizons and different values of the elasticity of supply ε_S. We assume $K_S = 400$ and discuss the implications of different values of K_S subsequently. The vertical lines in the graphs depict the equilibrium duration with uncoordinated equilibrium for the different elasticity levels.

[Insert Figure 10 here]

First, when trade credit duration provided by the supplier is short, we observe that moving from internal production to external procurement occurs when there are significant higher costs of internal production. Second, we observe that the greater the duration of credit provided by the supplier, the more likely that the buyer firm moves to external procurement (curves are downward sloping with respect to credit duration). This suggests that suppliers would have to increase trade credit duration to induce buyers to use external procurement when buyers are in a strong position to produce internally. This is broadly consistent with Klapper et al. (2012) and Fabbri and Klapper (2016) who show that suppliers are more likely to offer trade credit to powerful and important customers. Third, we observe that when higher order quantity leads to shortages of goods and higher prices (i.e., when $\varepsilon_S > 0$), the range of costs where the producer chooses internal production increases (the threshold $\overline{C_S}$ above which the buyer moves to external procurement increases). However, note that trade credit mitigates this effect since the optimal trade credit duration for higher ε_S (see vertical red line) increases. Thus, when increased production leads to higher prices, the supplier will optimally provide higher trade credit to retain higher order quantities ordered by the buyer. This extension of credit will thus reduce the incentive of the buyer to internalize production. At the opposite spectrum, when increased ordered quantities lead to a price decrease in the supply markets (e.g. due to higher economies of scale), the region of costs leading to internal production is reduced (now the threshold $\overline{C_S}$ above which the buyer moves to external procurement decreases relative to the base case with zero elasticity). However, with increasing pressures on profit margins due to lower prices, the suppliers will decrease credit duration (see vertical green line) and the incentive for external procurement will be reduced. As a final note to the above analysis we note that the level of the initial setup cost K_S creates parallel shifts in the line of the threshold $\overline{C_S}$ (scaling effects) with higher K_S decreasing the region of internal production and favoring external procurement.

The discussion above can help explain several real business settings like the case of Tesla (buyer) and Panasonic (its until recently major supplier of batteries). Initially, Tesla worked alongside its main supplier instead of fully internalizing production of batteries due to economies of scale that significantly reduced the costs of production. However, this may have recently changed (see e.g. Lambert, 2019) due to new R&D discoveries by Tesla that provided it with a patent for longer duration and lower cost batteries.[16]

6. Option to expand capacity and credit

In this section we discuss an extension of the basic framework allowing for the buyer firm to expand initial capacity. All mathematical derivations are left for the Appendix D. In this section we describe the framework and discuss the implications. Under this extended framework, the buyer firm decides on initial capacity Q_0 at t = 0 by incurring a one-time investment cost of κQ_0^{η}. As before Q_0^{η} is the amount of capital required to produce at that capacity (with $\eta > 1$), and κ is the cost per unit of capital. The buyer receives credit from a supplier firm with terms defined by m which defines the average duration of credit. The price per unit that the buyer can sell the goods in the downstream market follows a Geometric Brownian motion as described in equation (1). The buyer firm has the option to expand capacity at an optimal time when x reaches the threshold x_I. At the capacity expansion threshold, the quantity can be increased from Q_0 to a new $Q_1 = eQ_0$. Since our focus is on capacity expansion and not reversibility, $e > 1$. The firm will need to incur additional costs $\kappa(eQ_0)^{\eta}$ - κQ_0^{η} at x_I to install this additional capacity. Since both the initial capacity at t = 0 and the new capacity at the investment trigger depend on Q_0, the capacity choice at t = 0 and at the investment trigger need to be solved simultaneously. At the capacity expansion threshold, the buyer firm needs to repay the old credit and start a new one which corresponds to the new capacity level. We assume that the terms of credit with respect to credit duration remain as in the period before capacity (i.e., defined by the rollover rate *m*). Prior to the expansion of capacity, the firm may default at threshold x_B^0, while after capacity expansion at x_B^1. In case of default the supplier firm recovers a fraction b of the value of goods. This implies that the recovery value before default is $V_B^0 = bP_sQ_0$ and after capacity expansion is $V_B^1 = bP_sQ_1$. Our extensive sensitivity analysis based on this extended model has shown that our main result relating to the existence of an optimal credit maturity remains as before. Furthermore, the directional effect of different parameters of credit maturity is maintained (see Table 2). The Appendix D provides indicative sensitivity results with respect to the price of credit P_s. In addition, however, this new framework allows one to investigate the optimal timing of expanding capacity and credit utilization when conditions become more favorable. As can be seen in Figure A.2 of the Appendix D, an extended trade duration and lower prices charged by the supplier firm encourages earlier expansion of capacity and credit. An interesting result obtains in the case of downstream price volatility where our sensitivity analysis has shown that a higher

[16] The article also cites a report by WSJ about differences in cultural styles between the organizations. However, the tension between the firms is significantly impacted by the fact that Panasonic is also supplying other major competitors (e.g., Toyota) and because Tesla also has plans for its own battery production plant.

volatility delays expansion of capacity (due to a more valuable real option to delay investment). However, at higher volatility the supplier increases optimal trade credit duration provided which encourages higher optimal capacity. This extended duration provided by the supplier at higher volatility also appears to mitigate the delay to expand capacity relative to lower volatility levels.

7. Conclusions

Our model provides a systematic approach in modeling the relationship between a buyer and a supplier accounting for possible default risk of the buyer and capacity constraints. We derive the optimal trade credit duration provided by the supplier and the optimal order quantity by the buyer and examine the effect of various parameters changes on their behavior. We also provide a comparison of coordinated versus external procurement and the role of trade credit as a coordination mechanism, and an analysis of the "make or buy" choice (backward integration).

Our framework explains observed trade credit patterns in industries characterized by a relatively high volatility of demand such as technology or luxury goods. Moreover, we derive a series of predictions which are in line with empirical evidence.

Although focusing on a single supplier-buyer relationship, our framework captures important aspects that have to do with uncertainty in the downstream demand, the elasticity of demand of inputs and final goods in both the supplier and buyer markets, buyer capacity constraints, default risk and market power. This rich setting can be extended along several lines. First, introducing multiple suppliers would allow analyzing competition in supplier markets, endogenize prices or/and quantities, and include free-riding issues from the side of the buyer. Second, it would be interesting to analyze how financial constraints affecting the supplier impact trade credit maturity and order quantities. Finally, the framework can be further enriched to consider issues related to inventory management. This is left for future research.

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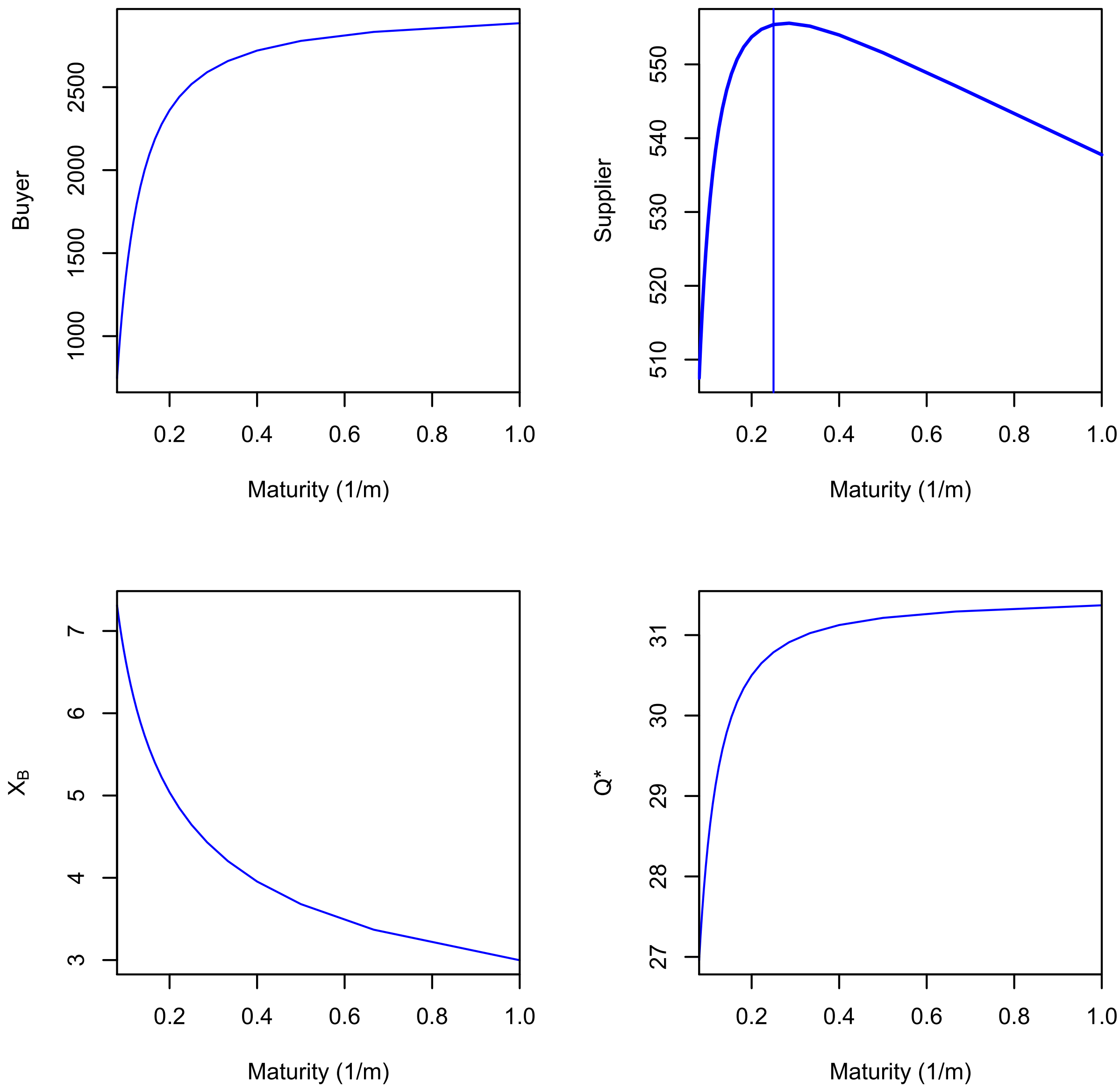
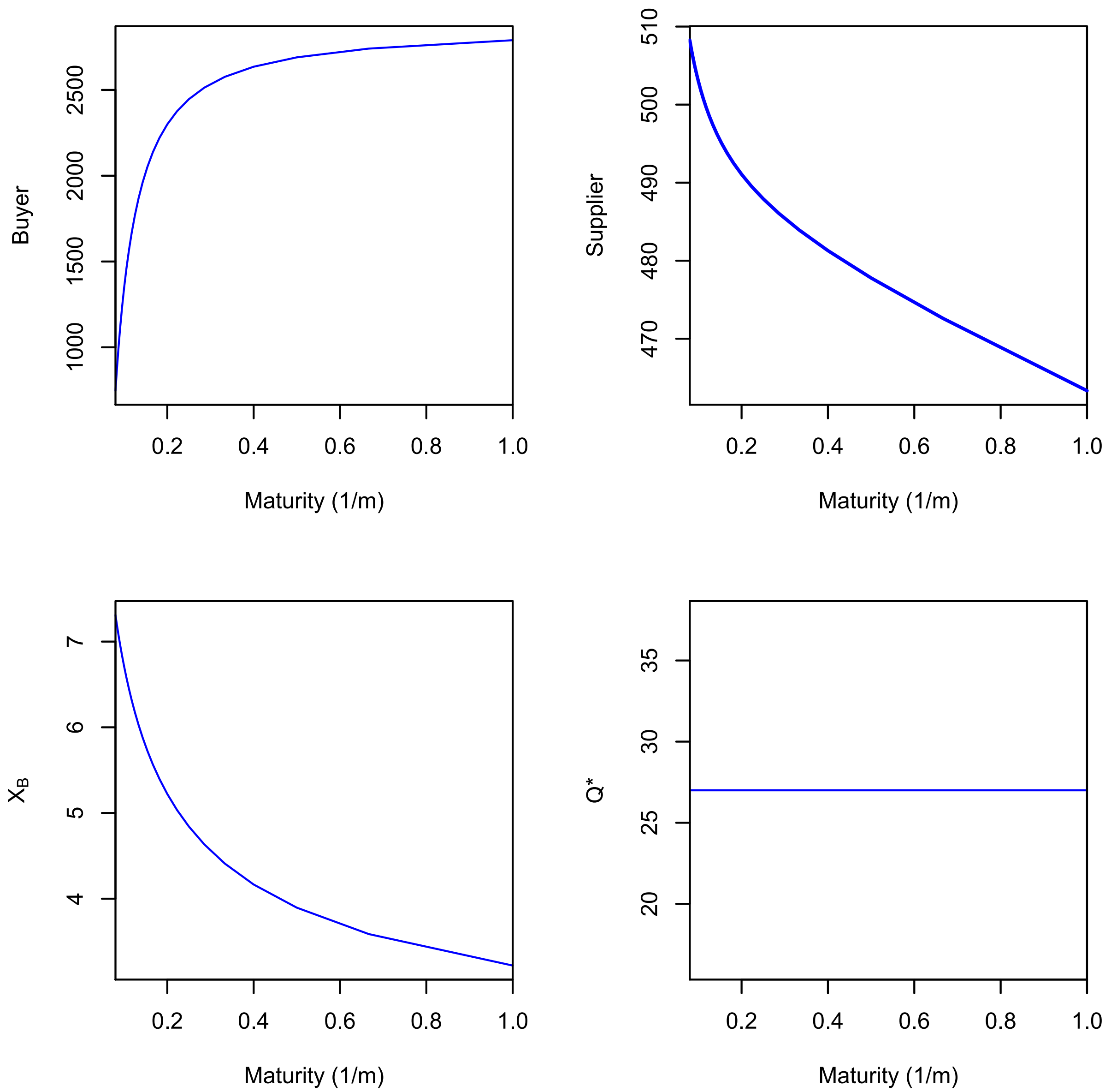
FIGURES

Fig.1 Baseline results: sensitivity with respect to credit duration for optimal Q



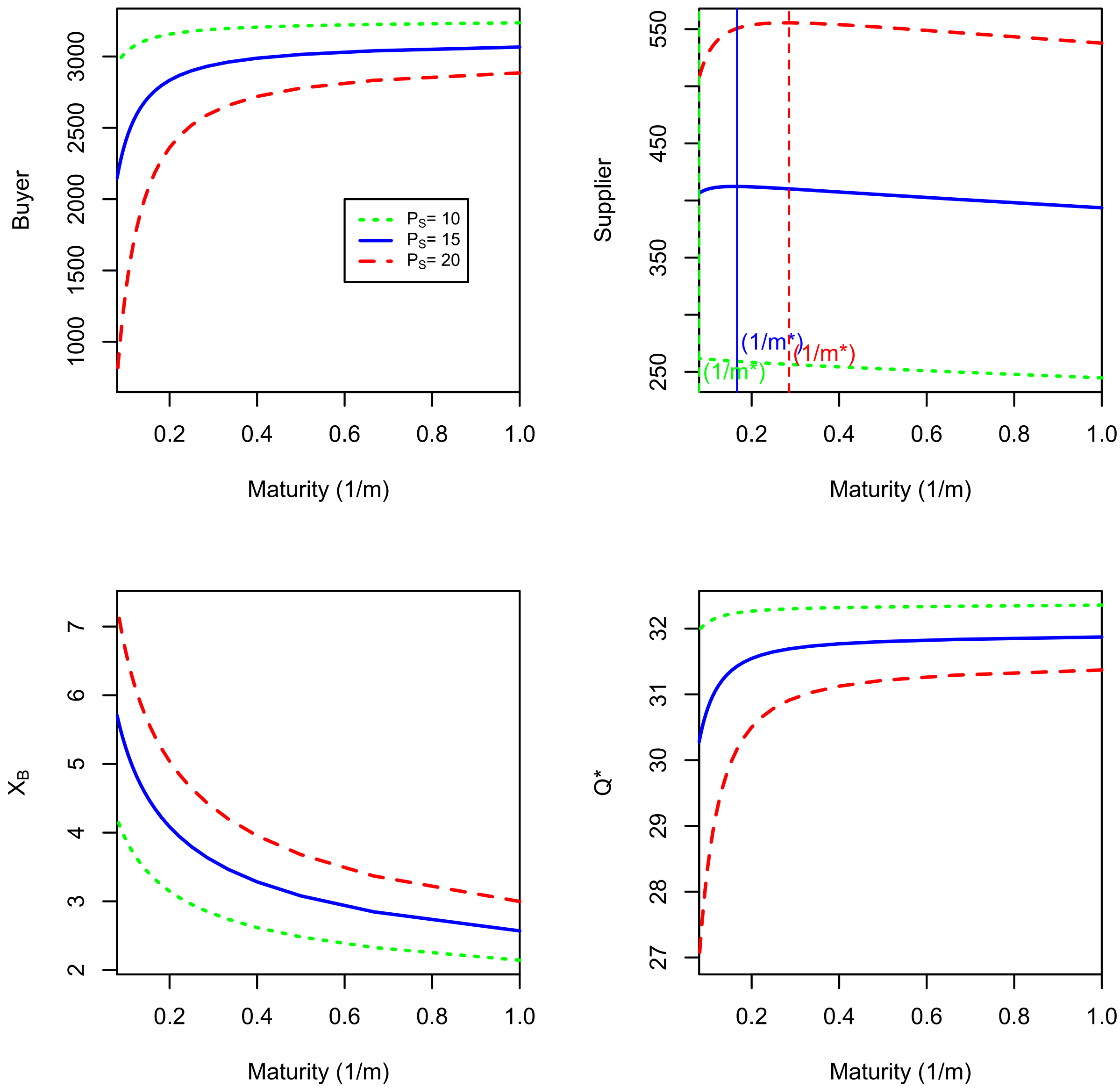
Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$ Maturity (1/m) is measured in years.

Fig.2 Sensitivity with respect to credit duration with fixed Q



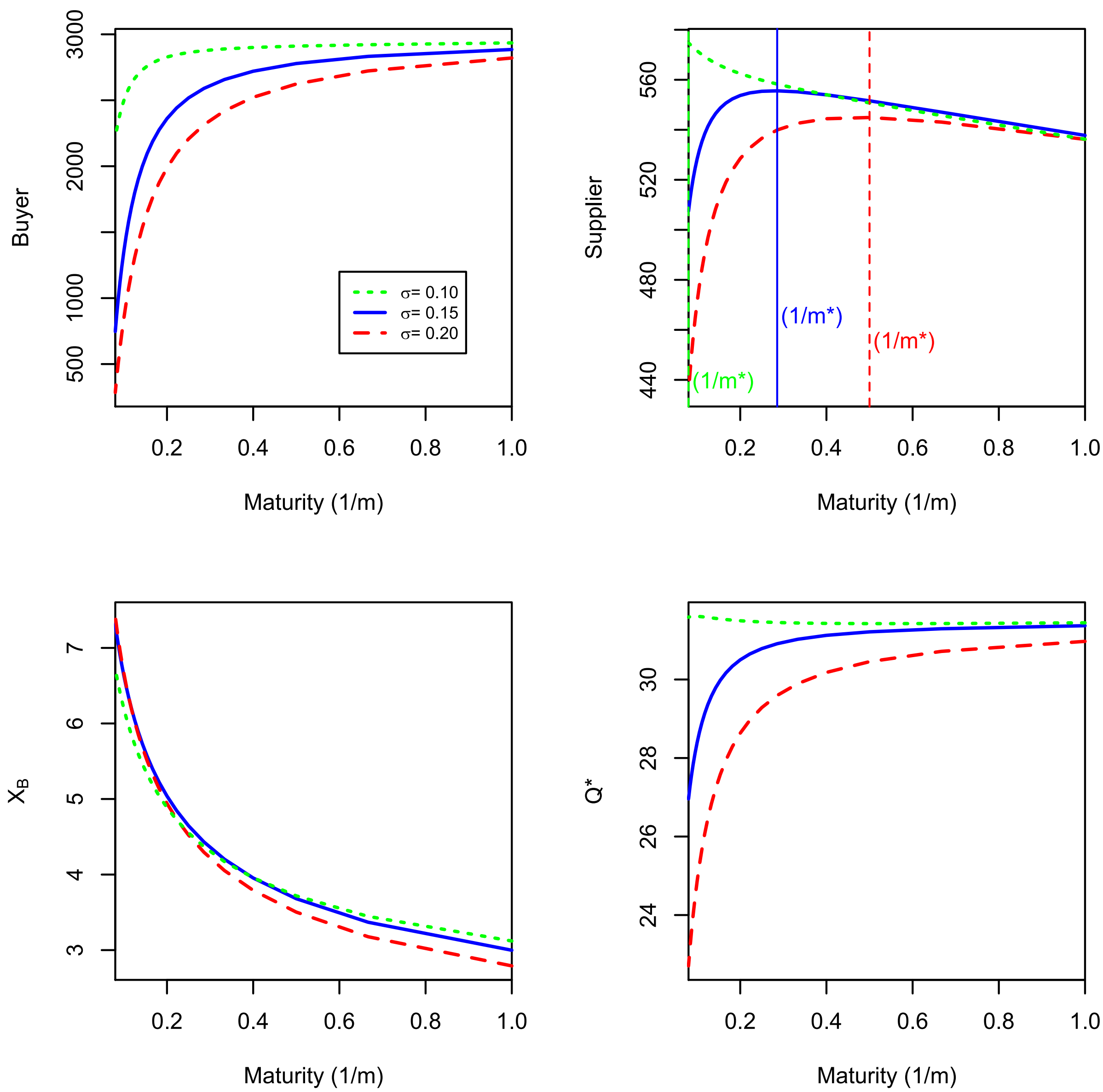
Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$. Q assumed fixed at Q = 25. Maturity (1/m) is measured in years.

Fig. 3 Sensitivity with respect to prices charged by the supplier Ps



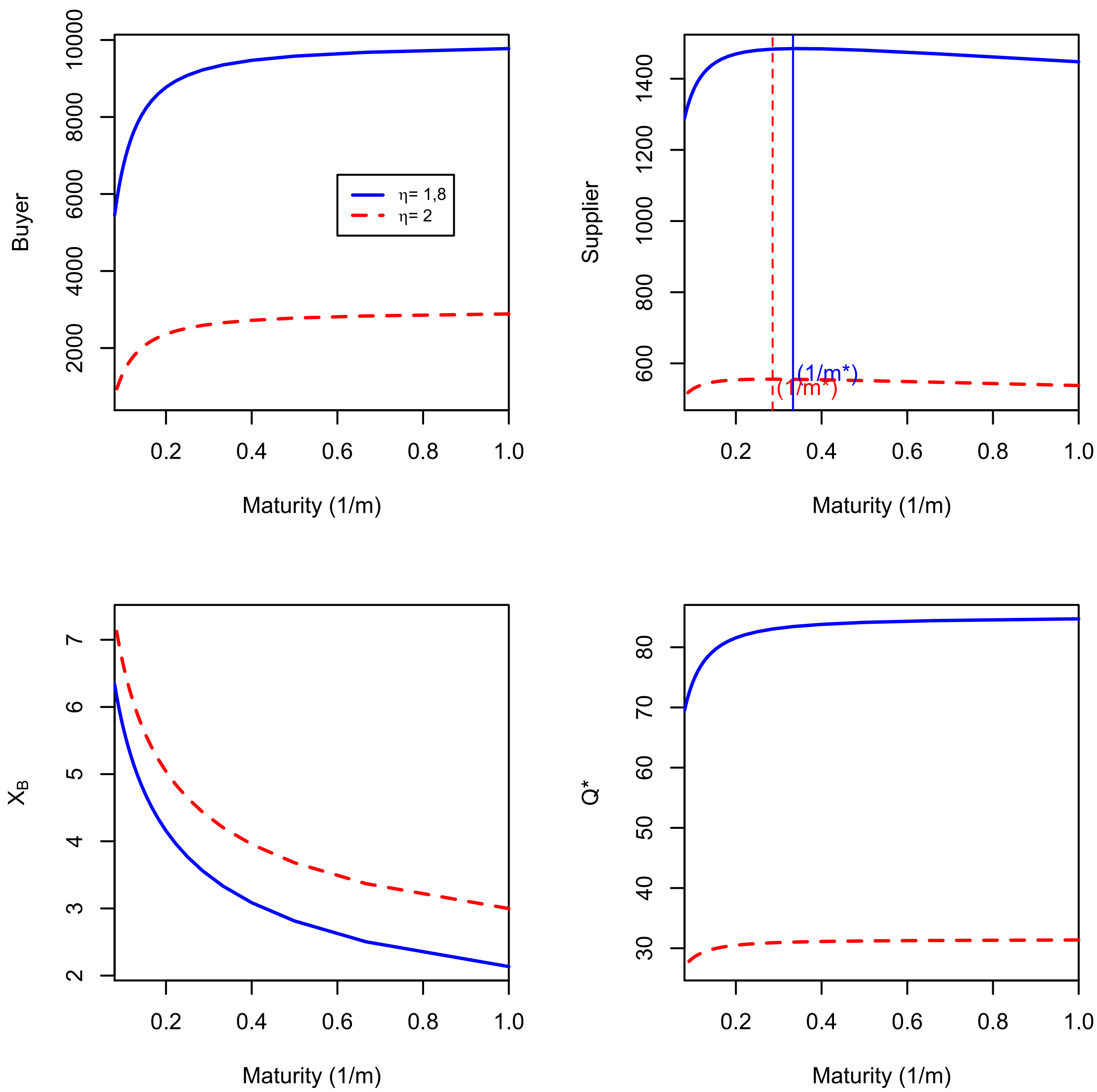
Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 10$ and $\varepsilon_S = 0$ implying $P_S = 10$ (green dotted line), with $a_S = 15$ and $\varepsilon_S = 0$ implying $P_S = 15$ (blue solid line) and with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$ (red dashed line). Maturity (1/m) is measured in years.

Fig.4 Sensitivity with respect to volatility of price in downstream market



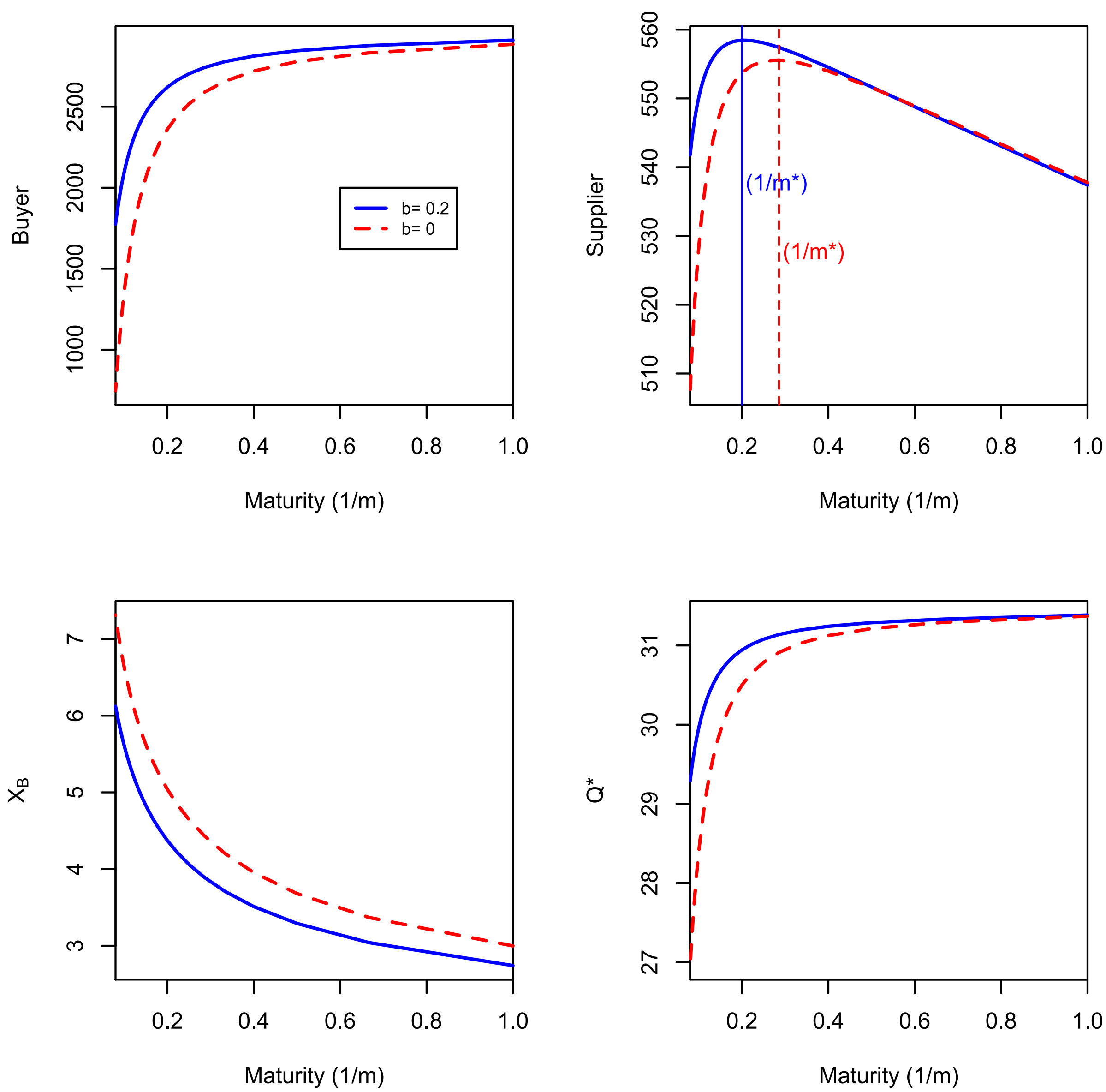
Notes: Parameters used r = 0.05, δ = 0.03, $C_S = 0.1$, $C_B = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$. Volatility of price for the buyer is buyer σ = 0.10 (green dotted line), σ = 0.15 (blue solid line) and σ = 0.25 (red dashed line). Maturity (1/m) is measured in years.

Fig. 5. Sensitivity with respect to capacity constraints (η)



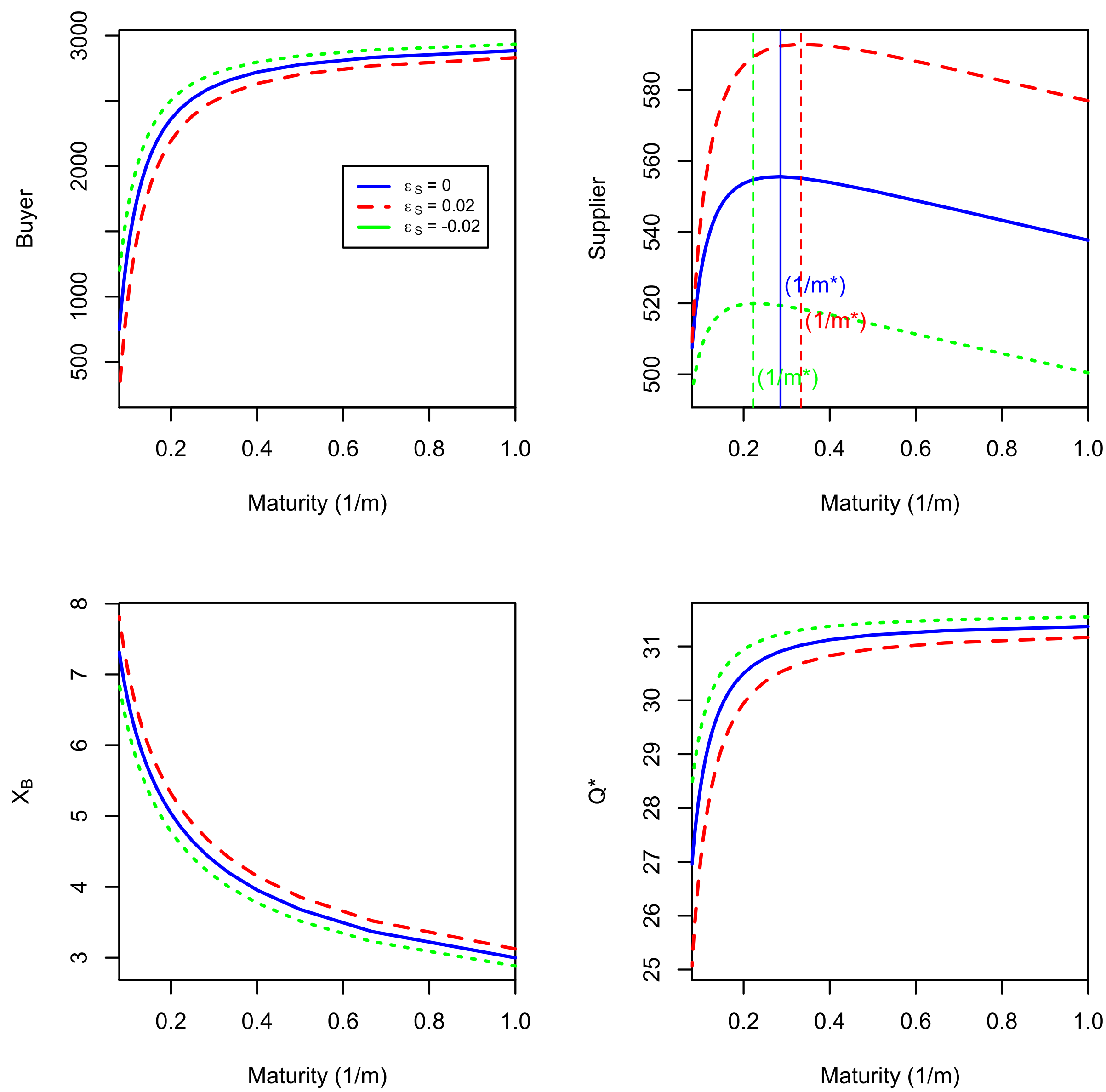
Notes: Parameters used r = 0.05, δ = 0.03, $C_S = 0.1$, $C_b = 100$,σ = 0.15, x = 10, b = 0, κ = 5, $\varepsilon_B = 0$, $L = 0$.Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$. η =1.8 (blue solid line) and η = 2 (red dotted line). Maturity (1/m) is measured in years.

Fig. 6. Sensitivity with respect to recovery rate in default (b)



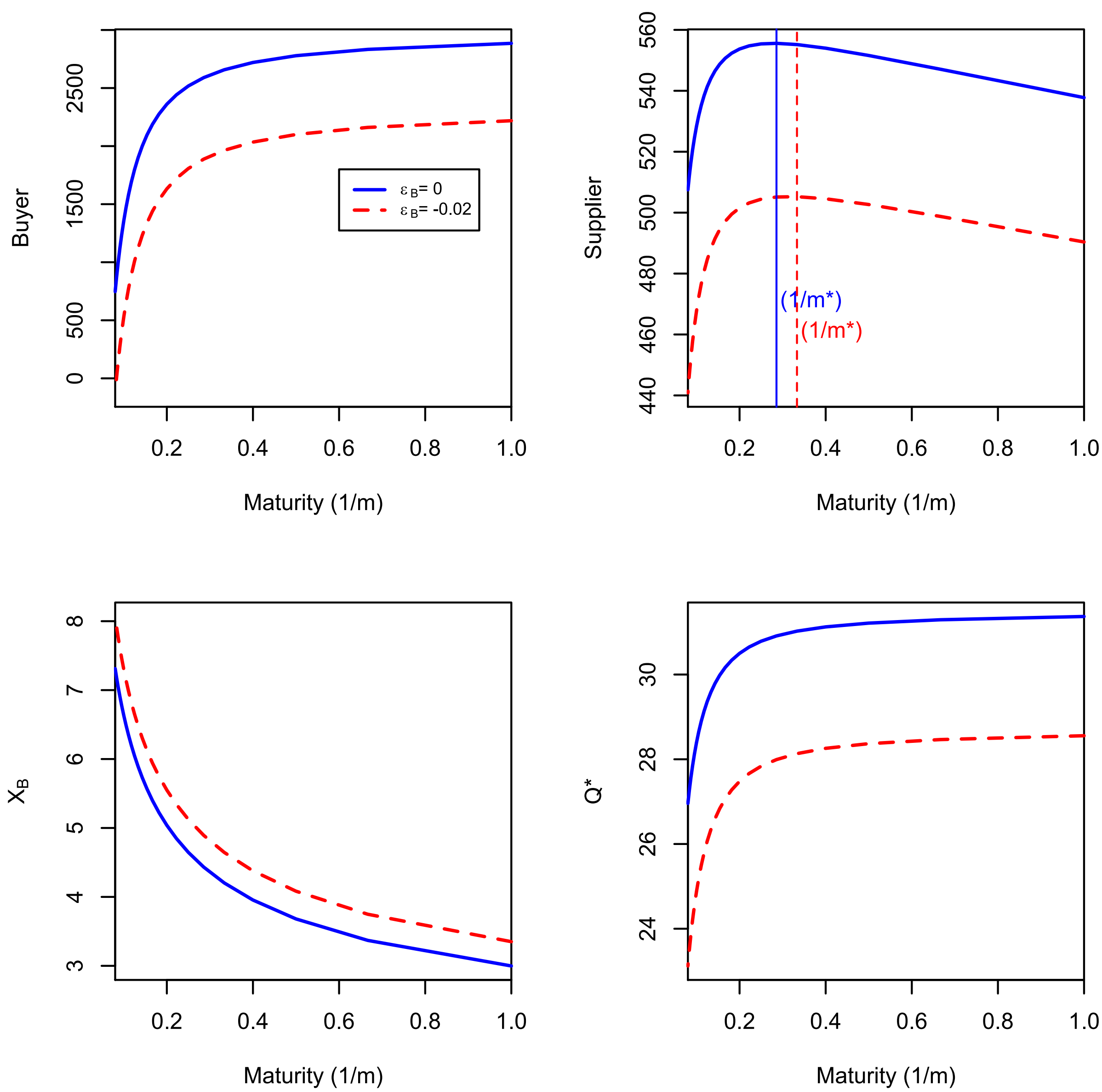
Notes: Parameters used r = 0.05, δ = 0.03, $C_S = 0.1$, $C_b = 100$, σ = 0.15, x = 10, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$. Recovery value in bankruptcy b = 0.2 (blue solid line) and b = 0 (red dotted line). Maturity (1/m) is measured in years.

Fig. 7. Sensitivity with respect to elasticity of supplier's price ε_S



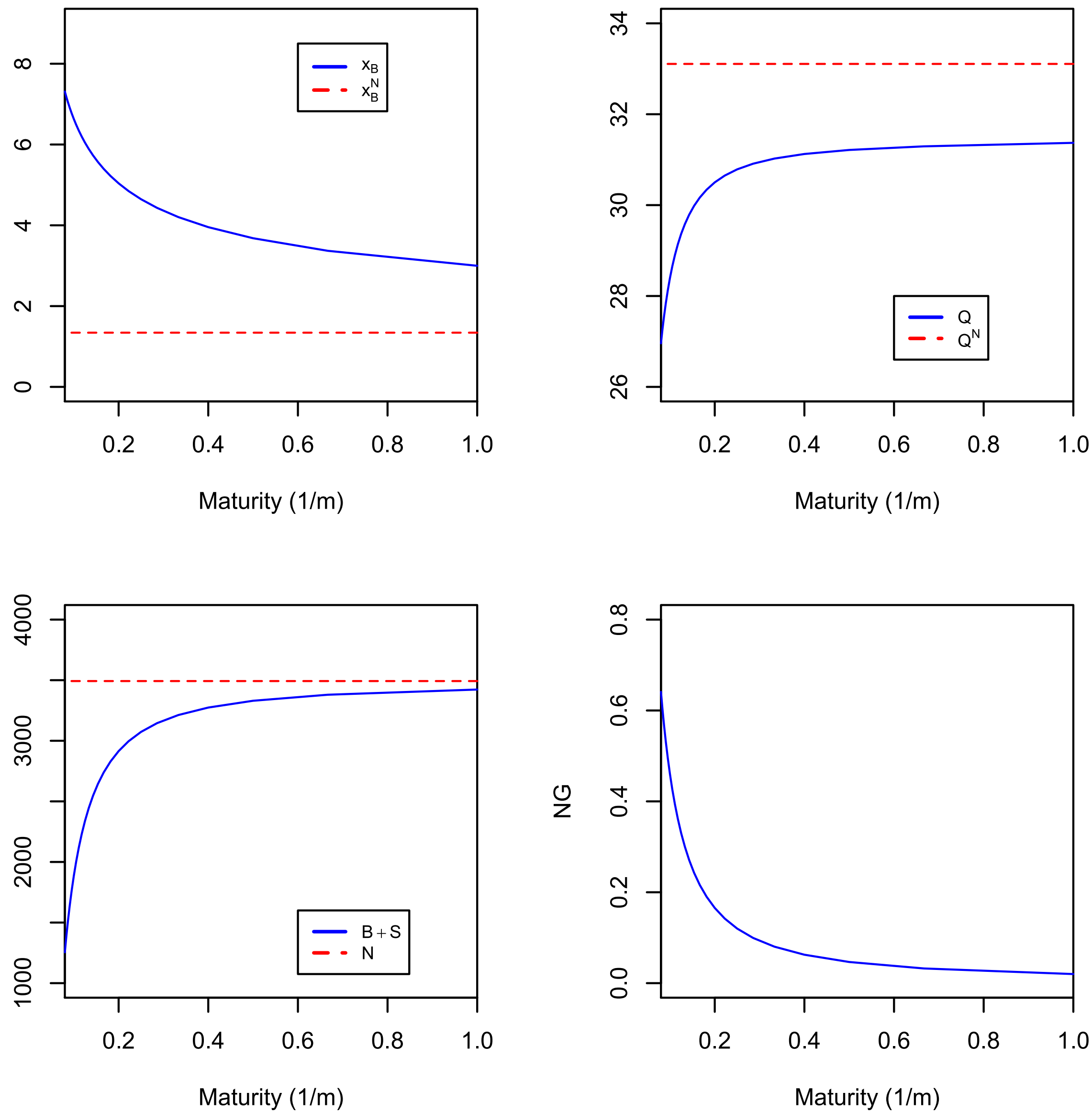
Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with α_S=20 and sensitivity with respect to ε_S. Maturity (1/m) is measured in years.

Fig. 8. Sensitivity with respect to elasticity of buyer's price ε_B



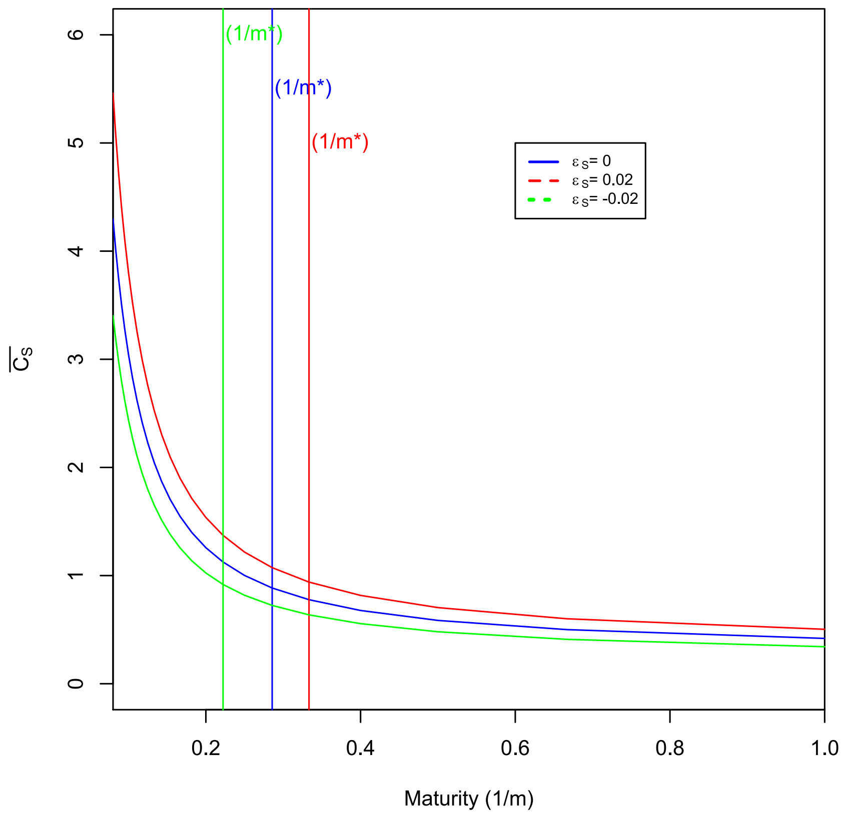
Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying P_S=20. Sensitivity with respect to buyer's elasticity of demand ε_B.

Figure 9. Network analysis



Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, $x = 10$, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$. Maturity (1/m) is measured in years. x_B is the value of default selected by the buyer with no coordination of production (see eq.16), x_B^N is the default threshold under internal procurement with trade credit (eq. 25). Q is the order quantity by the buyer with no coordination of production and Q^N under internal procurement with trade credit. B+S is the sum of buyer and supplier value with no coordination and N is the value of the network under internal procurement with credit. NG are the gains from coordination (see eq. 26).

Figure 10. Equivalent cost of internal per unit production inducing external procurement



Notes: r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with $a_S = 20$ and $\varepsilon_S = 0$ implying $P_S = 20$ (blue line), $\varepsilon_S = 0.02$ (Red line) and $\varepsilon_S = -0.02$ (green line). Maturity (1/m) is measured in years. The figure shows the Cs threshold above which a firm moves from coordinated internal production with no credit to external procurement with credit. It is the result of the solution described in equations (27a) and (27b). For the analysis of the above figures we assume an initial setup cost of the buyer K_S = 400.

TABLES

Table 1. Baseline results and buyer market power

	Model outputs				
Buyer reservation value	***1/m***	***Q***	***B***	***S***	x_B
0	0.286	30.31	2589.87	555.58	4.43
2720.24	0.400	31.12	2720.24	553.98	3.95

Table 2. Sensitivity of optimal credit duration and quantity with respect to parameters

Parameter	Sign of *1/m**	Sign of *Q**
P_S	+	-
σ	+	-
C_S	-	-
δ	+	-
r	-	+
C_b	-	-
η	-	-
K	-	-
b	-	+
ε_S	+	-
ε_B	-	+
L	+	+

Online Appendix

Appendix A: Details on the derivation of buyer's and supplier's value

In this appendix we provide details on our derivation of the buyer and supplier values.

In order to derive the particular solution in equation (11) of the buyer we proceed by applying the solution:

$$B(x) = B + C_0 x + C_1 x^{\gamma_2} \tag{A1}$$

that satisfies differential equation (10) obtaining the following solutions for C_0 and C_1 and B:

$$B = \frac{m}{r}\frac{mP_S Q}{r+m} - \frac{mP_S Q + C_b}{r}, C_0 = -A_2^D\ , \ C_1 = \frac{Q}{\delta} \tag{A2}$$

We note that to derive C_0 we have used the fact that $m = -(r - (r-\delta)\gamma_2 - \frac{1}{2}\gamma_2(\gamma_2 - 1)\sigma^2)$ which simplifies the presentation of the solution. Note also that unlike the standard particular solution, the term $C_1 x^{\gamma_2}$ in (A1) is used to capture trade credit value in the differential equation (10) (which as seen in equation (9) depends on γ_2 term).

Replacing solutions of constants C_0, C_1 and B from (A2) back in (A1) and noting that

$$\frac{mP_S Q}{r} - \frac{m}{r}\frac{mP_S Q}{r+m} = \frac{mP_S Q}{r+m} \tag{A3}$$

one can verify first three terms in equation (11).

Next, we note that since $\beta_1 > 0$ applying (13a) to equation (11) implies that:

$$A_1^B = 0 \tag{A4}$$

Applying the boundary condition (13b) to equation (11) we obtain that:

$$A_2^B = -\left[\left(\frac{x_B}{\delta} - \frac{(mP_S Q + C_b)}{r}\right) + \frac{m}{r}\frac{mP_S Q}{r+m} - \left(V_B - \frac{mP_S Q}{r+m}\right)\right](x_B)^{-\beta_2}$$

Using (A3) we can further simplify A_2^B and we obtain that:

$$A_2^B = -\left[\left(\frac{x_B}{\delta} - \frac{C_b}{r}\right) - V_B\right](x_B)^{-\beta_2} \tag{A5}$$

Replacing (A4) and (A5) into equation (11) together with the particular derived above we thus derive the final solution for the buyer value in equation (14).

The optimal quantity ordered by the buyer maximizes buyer value. From equation (17) we obtain the implicit equation for Q:

$$
\begin{aligned}
&\frac{X(Q)(\epsilon_B+1)}{\delta}-\frac{mP_S(Q)(\epsilon_S+1)}{r+m}-\left(bP_S(Q)(\epsilon_S+1)-\frac{mP_S(Q)(\epsilon_S+1)}{r+m}\right)\left(\frac{x}{x_B(Q)}\right)^{\gamma_2}\\
&\qquad+\frac{\gamma_2}{x_B(Q)}\left(bP_S(Q)Q-\frac{mP_S(Q)Q}{r+m}\right)\left(\frac{x}{x_B(Q)}\right)^{\gamma_2}\frac{\partial x_B(Q)}{\partial Q}\\
&\qquad+\left[bP_S(Q)(\epsilon_S+1)-\frac{X_B(Q)(\epsilon_B+1)}{\delta}-\frac{Q^{\epsilon_B+1}}{\delta}\frac{\partial x_B(Q)}{\partial Q}\right]\left(\frac{x}{x_B(Q)}\right)^{\beta_2}\\
&\qquad-\frac{\beta_2}{x_B(Q)}\left[bP_S(Q)Q-\frac{X_B(Q)Q}{\delta}+\frac{C_b}{r}\right]\left(\frac{x}{x_B(Q)}\right)^{\beta_2}\frac{\partial x_B(Q)}{\partial Q}-\kappa\eta Q^{\eta-1}=0,
\end{aligned}
$$

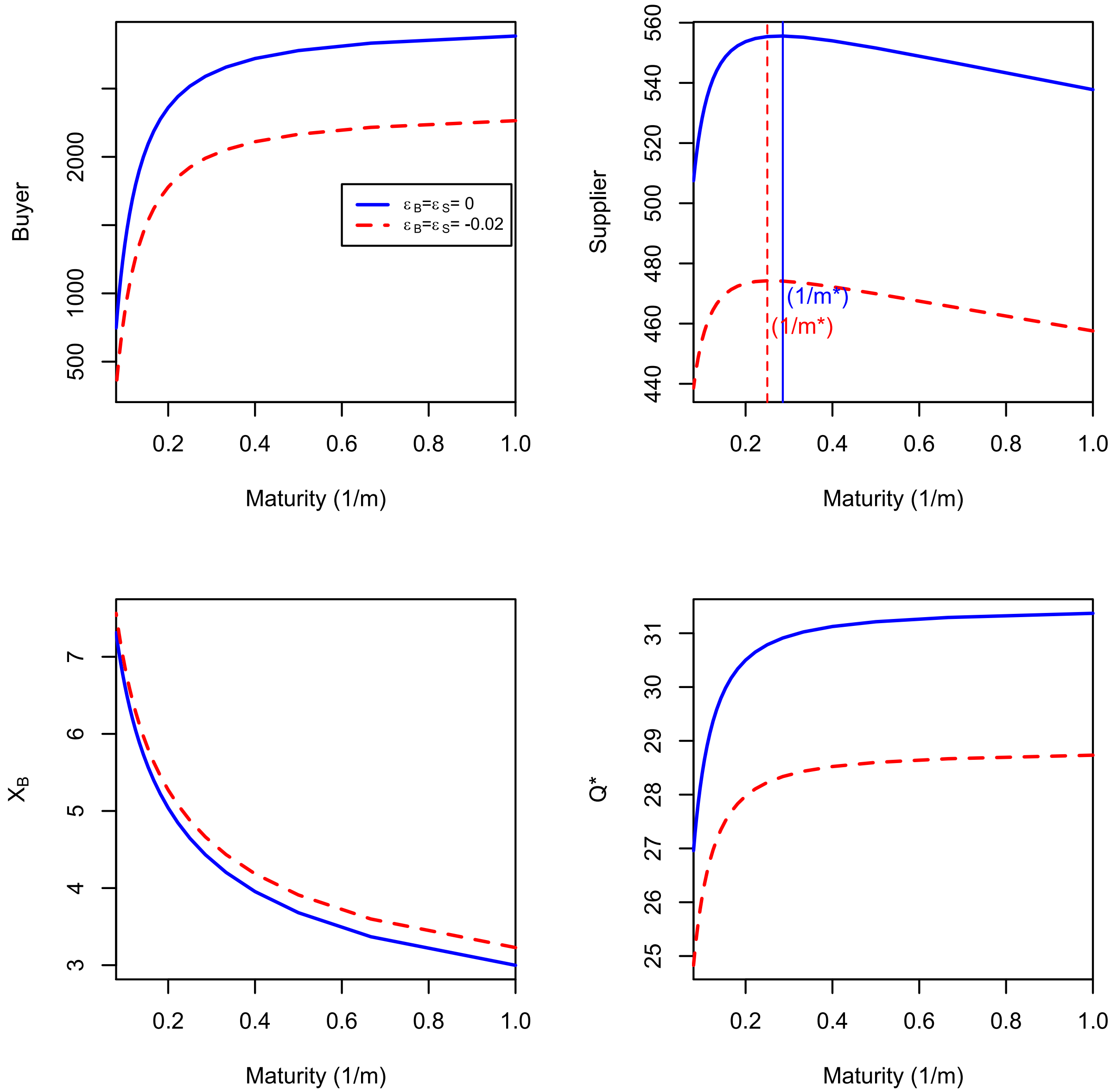
where $P_S(Q)=a_sQ^{\epsilon_s}$ and X(Q)= xQ^{ϵ_B}. Note that we have used the fact that recovery value is given by $V_B=bP_SQ$.

Regarding the supplier value, the particular solution is obtained in a similar fashion as for the buyer case mentioned above. Furthermore, applying the boundary conditions (20) and (21) to equation (19) and similar simplifications as with the buyer value we obtain equation (22).

Appendix B: Extra sensitivity results

In this appendix we provide some additional sensitivity results shown in Section 4.2. of the paper. Figure A.1. shows the impact of a 2% negative price sensitivity in supplier’s input prices combined with a 2% negative price sensitivity in buyer’s final good market.

Fig A.1. Simultaneous change in elasticities of price change in supplier's and buyer's market



Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, x = 10, b = 0, κ = 5, η = 2, $\varepsilon_B = 0$, $L = 0$. Price charged by supplier with α_S=20. For base case $\varepsilon_S = \varepsilon_B = 0$. Sensitivity with $\varepsilon_S = \varepsilon_B = -0.02$ (shown in red dotted line). Maturity (1/m) is measured in years.

Appendix C: Proof of $x_B^N < x_B^B$

From equation (16) we have that the optimal default threshold selected by the buyer is given by:

$$x_B^B = \frac{\delta}{(1-\beta_2)Q^{\epsilon_B+1}}\left[\gamma_2\left(V_B - \frac{mP_SQ}{r+m}\right) - \beta_2\left(V_B + \frac{C_b}{r}\right)\right]$$

Similarly, from equation (25) the optimal default threshold selected by the network is given by:

$$x_B^N = \frac{-\beta_2}{(1-\beta_2)}\frac{\delta}{Q^{\epsilon_B+1}}\left(V_B + \frac{C_b + C_SQ}{r}\right)$$

Comparing the two thresholds we see that some terms cancel out and we get that $x_B^N < x_B^B$ if and only if:

$$-\beta_2\left(\frac{C_SQ}{r}\right) < \gamma_2\left(V_B - \frac{mP_SQ}{r+m}\right)$$

For $b = 0$ we have $V_B = 0$ and this equation becomes:

$$-\beta_2\left(\frac{C_SQ}{r}\right) < -\gamma_2\left(\frac{mP_SQ}{r+m}\right)$$

We know that $-\beta_2 \leq -\gamma_2, \forall\, m$ and we also have that $\frac{C_SQ}{r} < \frac{mP_SQ}{r+m}$ to ensure that the supplier makes positive profits. Therefore, the inequality above holds, and we have that $x_B^N < x_B^B$.

Appendix D: Option to expand capacity

Following capacity expansion

Following standard steps, in the period following the capacity expansion the value of credit can be shown to be:

$$D_1(x) = \frac{mP_SQ}{r+m} + \left(V_B^1 - \frac{mP_SQ_1}{r+m}\right)\left(\frac{x}{x_B^1}\right)^{\gamma_2^1} \qquad \text{(A6)}$$

Note that $V_B^1 = bP_SQ_1$ where $Q_1 = eQ_0$ and $\gamma_2^1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+m)}{\sigma^2}} < 0$. x_B^1 denotes the optimal default trigger for the buyer following the capacity investment decision.

The buyer value $B_1(x)$ following the capacity expansion operates under the expanded capacity satisfies the following differential equation:

$$rB_1(x) = (r-\delta)xB_1{}'(x) + \frac{\sigma^2}{2}x^2B_1{}''(x) + (x(eQ_0) - C_b) - m(P_s(eQ_0) - D_1(x)) \qquad \text{(A7)}$$

The buyer value general solution is:

$$B_1(x) = \left(\frac{x(eQ_0)}{\delta} - \frac{C_b}{r}\right)(1-\tau) - \frac{mP_S(eQ_0)}{r+m} - \left(V_B^1 - \frac{mP_S(eQ_0)}{r+m}\right)\left(\frac{x}{x_B^1}\right)^{\gamma_2} + A_1^Bx^{\beta_1} + A_2^Bx^{\beta_2} \qquad \text{(A8)}$$

where the exponents β_1 and β_2 are given by $\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ and

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0\ .$$

The following boundary conditions apply:

$$\lim_{x\to\infty} B_1 = \left(\frac{x(eQ_0)}{\delta} - \frac{C_b}{r}\right)(1-\tau) - \frac{mP_S(eQ_0)}{r+m} \qquad \text{(A9a)}$$

$$B_1(x_B^1) = 0 \qquad \text{(A9b)}$$

Applying these conditions, we obtain the following solution for buyer value following the capacity expansion:

$$B_1(x) = \left(\frac{x\,(eQ_0)}{\delta} - \frac{C_b}{r}\right)(1-\tau) - \frac{mP_S(\,eQ_0)}{r+m} - \left(V_B^1 - \frac{mP_S(\,eQ_0)}{r+m}\right)\left(\frac{x}{x_B^1}\right)^{\gamma_2^1} + \left[V_B^1 - \left(\frac{x_B^1(eQ_0)}{\delta} - \frac{C_b}{r}\right)\right]\left(\frac{x}{x_B^1}\right)^{\beta_2}$$

(A10)

The optimal default threshold following the capacity expansion, $x_B^1(Q_1)$ is obtained from the smooth pasting condition:

$$\frac{\partial B_1}{\partial x}\Big|_{x=\,x_B^1(Q_1)} = 0 \qquad \text{(A11)}$$

Applying this smooth pasting condition, we obtain the following default threshold:

$$x_B^1(Q_1) = \frac{\delta}{(1-\beta_2)(eQ_0)}\left[\gamma_2^1\left(\frac{V_B^1}{1-\tau} - \frac{mP_S(eQ_0)}{r+m}\right) - \beta_2\left(\frac{V_B^1}{1-\tau} + \frac{C_b}{r}\right)\right] \qquad \text{(A12)}$$

Supplier value following capacity expansion (following standard arguments) becomes:

$$S_1(x) = \frac{mP_S(eQ_0)}{r+m} - \frac{C_S(eQ_0)}{r} + \left(V_B^1 - \frac{mP_S(eQ_0)}{r+m}\right)\left(\frac{x}{x_B^1}\right)^{\gamma_2^1} + \frac{C_S(eQ_0)}{r}\left(\frac{x}{x_B^1}\right)^{\beta_2} \qquad \text{(A13)}$$

Before capacity expansion

Denote x_B^0 as the default threshold before the investment for capacity expansion. Before capacity investment we face a double boundary problem since the buyer firm may default at x_B^0 before reaching the investment expansion threshold x_I.

We start by deriving the value of trade credit. Trade credit $D_0(x)$ follows the following differential equation:

$$rD_0(x) = (r-\delta)xD'_0(x) + \frac{\sigma^2}{2}x^2D''_0(x) + m(P_SQ_0\ - D_0(x)) \qquad \text{(A14)}$$

A particular solution to the above can be easily shown to be:

$D_0^P(x) = \left(\frac{mP_SQ_0}{r+m}\right)$ (A15)

The general solution is of the following form:

$D_0(x) = C_1 x^{\gamma_1^0} + C_2 x^{\gamma_2^0}$ (A16)

To find the constants C_1 and C_2 we apply the following boundary conditions:

$D_0(x_I) = D_1(x_I) + P_S Q_0$ (A17)

$D_0(x_B^0) = V_B^0$ (A18)

We have two boundary conditions (value-matching conditions): at the switching trigger the supplier receives the principal value of the initial credit $P_S Q_0$, so that $D_0(x_I) = D_1(x_I) + P_S Q_0$, while at the default trigger $D_0(x_B^0) = V_B^0$, where $V_B^0 = bP_S Q_0$.

An alternative (equivalent but resulting in a more intuitive solution) is provided below which is based on deriving the value of two basic claims that satisfy a homogenous version of equation (A14) (similarly for the other claims). Define $H^r(x)$ as basic claim which pays 1 dollar when the investment threshold is reached first and zero when the default threshold x_B^0 is triggered first. $L^r(x)$ which pays 1 dollar when the default threshold x_B^0 is reached first and zero when the investment threshold x_I is triggered first. $r = \{r_1, r_2\}$ define the roots of the auxiliary equation solving the homogenous equation which are two real solutions, where $r_1 > 0$ and $r_2 < 0$. The solutions are provided below:

$H^r(x) = \frac{(X_B^0)^{r_2} x^{r_1} - (x_B^0)^{r_1} x^{r_2}}{(x_I)^{r_1} (x_B^0)^{r_2} - (x_I)^{r_2} (x_B^0)^{r_1}}$ for $x_B^0 < x < x_I$ (A19a)

$L^r(x) = \frac{(x_I)^{r_1} x^{r_2} - (x_I)^{r_2} x^{r_1}}{(x_I)^{r_1} \left(x_B\right)^{r_2} - (x_I)^{r_2} \left(x_B^0\right)^{r_1}}$, for $x_B^0 < x < x_I$ (A19b)

$H^\gamma(x)$ and $L^\gamma(x)$ satisfy the homogeneous version of the differential equation (A14) under the auxiliary parameters $\gamma = \{\gamma_1^0, \gamma_2^0\}$.

Thus, the general solution can be written as follows:

$D_0(x) = \left(\frac{mP_SQ_0}{r+m}\right) + H^\gamma(x)\left[D_1(x_I) + P_S Q_0 - \left(\frac{mP_SQ_0}{r+m}\right)\right] + L^\gamma(x)\left[V_B^0 - \left(\frac{mP_SQ_0}{r+m}\right)\right]$ (A20)

Before investment the buyer firm value satisfies the following differential equation:

$rB_0(x) = (r-\delta)xB_0{}'(x) + \frac{\sigma^2}{2} x^2 B_0^{\prime\prime(x)} + (xQ_0 - C_b) - m\left(P_S Q_0 - D_0(x)\right)$ (A21)

The above equation has the following form for the particular solution:

$$B_0^P(x) = B + C_0 x + C_1 x^{\gamma_1^0} + C_2 x^{\gamma_2^0} \quad \text{(A22)}$$

Applying (A22) to differential equation (A21) results in the following particular:

$$B_0^P(x) = \left(\frac{xQ_0}{\delta} - \frac{C_b}{r}\right) - D_0(x) \quad \text{(A23)}$$

where $D_0(x)$ is given in (A15).

We have two boundary conditions. At the switching trigger the buyer repays the principal of the initial debt $P_s Q_0$ and receives the trade credit proceeds of the newly issued credit, $D_1(x_I)$ (that has a principal of $P_S Q_0$), so we have $B_0(x_I) = B_1(x_I) - P_S Q_0 + D_1(x_I)$, while at the default threshold the buyer value is zero: $B_0(x_B^0) = 0$.

Given that the basic claims $H^{\beta}(x)$ and $L^{\beta}(x)$ satisfy the homogeneous version of the differential equation (A21) under the auxiliary parameters $\beta = \{\beta_1, \beta_2\}$ we can thus write the following solution:

$$B_0(x) = B_0^P(x) + H^{\beta}(x)\left[B_1(x_I) - P_S Q_0 + D_1(x_I) - \kappa\left(Q_1^{\eta} - Q_0^{\eta}\right) - B_0^P(x_I)\right] + L^{\beta}(x)[V_B^0 - B_0^P(x_B^0)]$$
(A24)

The optimal default threshold investment, default threshold prior to capacity expansion x_B^0 and capacity level Q_0 are obtained by solving the following optimization conditions:

$$\frac{\partial B_0}{\partial x}\Big|_{x=x_I(Q_0)} = \frac{\partial B_1}{\partial x}\Big|_{x=x_I(Q_0)} \quad \text{(A25a)}$$

$$\frac{\partial B_0}{\partial x}\Big|_{x=x_B^0(Q_0)} = 0 \quad \text{(A25b)}$$

$$\frac{\partial B_0}{\partial Q_0} = 0 \quad \text{(A25c)}$$

The supplier's value before investment $S_0(x)$ satisfies the following differential equation:

$$rS_0(x) = (r-\delta)xS'_0(x) + \frac{\sigma^2}{2}x^2 S''_0(x) + m(P_S Q_0 - D_0(x)) - C_S Q_0 \quad \text{(A26)}$$

The above equation has the following form for the particular solution:

$$S_0^P(x) = B + C_0 x + C_1 x^{\gamma_1^0} + C_2 x^{\gamma_2^0} \quad \text{(A27)}$$

Applying (A27) to differential equation (A26) results in the following particular:

$$S_0^P(x) = D_0(x) - \frac{C_S Q}{r} \quad \text{(A28)}$$

where $D_0(x)$ is given in (A20).

The general solution is of the following form:

$$S_0\ (x) = C_1 x^{\gamma_1^0} + C_2 x^{\gamma_2^0} \tag{A29}$$

To find the constants C_1 and C_2 in equation (A24) we apply the following boundary conditions:

$$S_0\ (x_I) = S_1(x_I) + P_S Q_0 \tag{A30}$$

$$S_0\ (x_B^0) = V_B^0 \tag{A31}$$

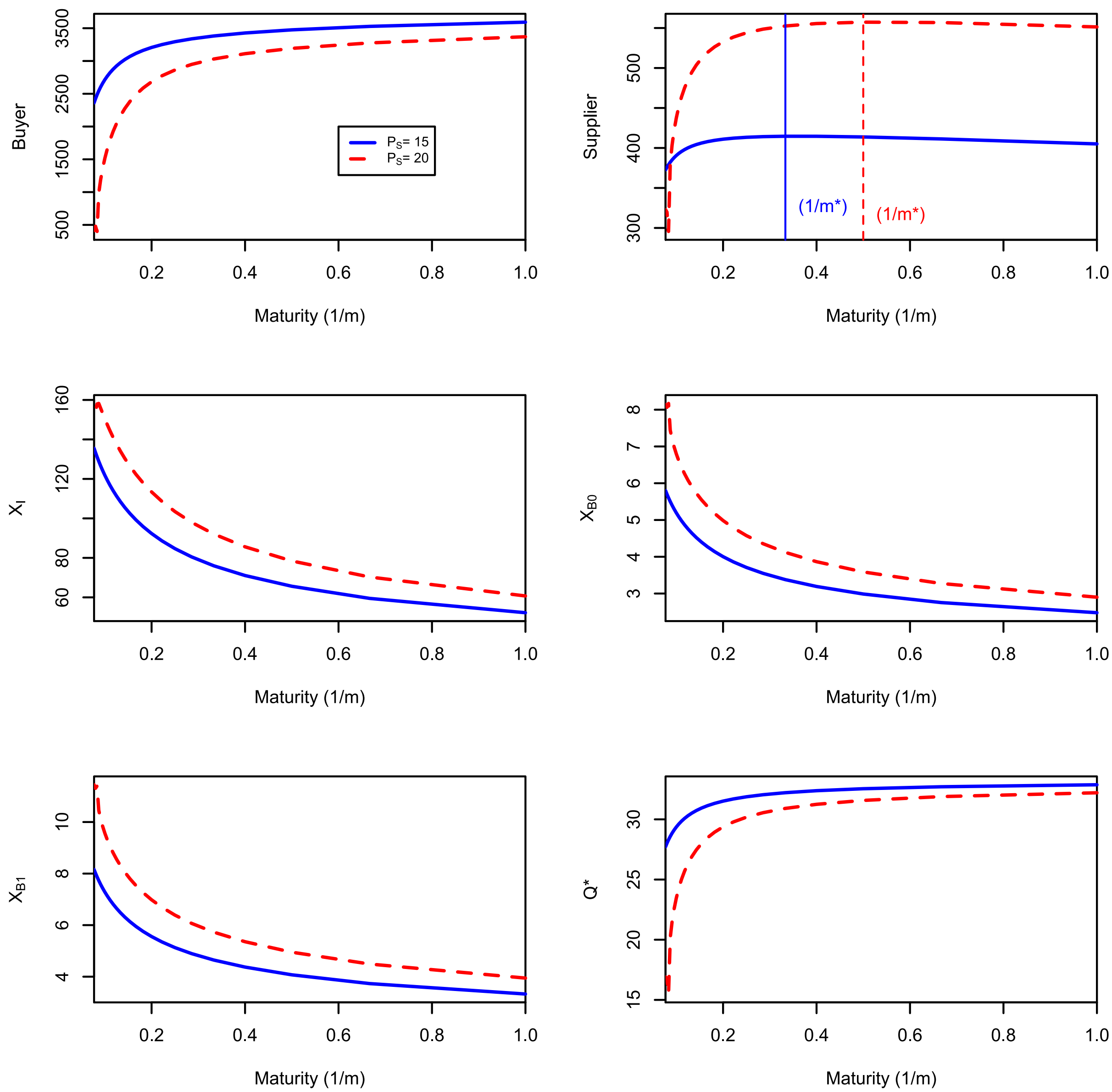
The solution to the above problem can be expressed in terms of the basic claims $H^{\beta}(x)$ and $L^{\beta}(x)$ which satisfy the homogeneous version of the differential equation (23) under the auxiliary

parameters $\beta = \{\beta_1\ , \beta_2\ \}$ we can thus write the following solution:

$$S_0(x) = D_0(x) + H^{\beta}(x)[S_1(x_I) + P_S Q_0 + D_1(x_I) - S_0^P(x_I)] + L^{\beta}(x)[V_B^0 - S_0^P(x_B^0)] \tag{A32}$$

The following figure provides indicative sensitivity results of this model which are discussed in the main text.

Figure A.2. Indicative sensitivity results of the model with expanded capacity and credit



Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.15, $C_S = 0.1$, $C_b = 100$, $x = 10$, b = 0, κ = 5, η = 2. Expansion factor e = 1.2. Price charged by supplier with, $P_S = 15$ and $P_S = 20$. Maturity (1/m) is measured in years.