# Technology Adoption in Cournot-Nash Duopoly under Risk Aversion and Uncertainty: A Real Options Approach Besma Teffahi<sup>1</sup> and Walid Hichri<sup>2</sup>

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## Abstract

Most of the research on risk aversion does not take into account the structure of competition between firms, while most of the work analyzing competition is based on the assumption of risk neutrality. In this article, we develop a framework based on the mean-variance utility model introduced by Markowitz to examine the impact of risk aversion and uncertainty on the timing of optimal decisions to adopt new technologies by firms facing competition. we consider the case of two competing firms with different levels of risk aversion. The results show that in this context, the effect of uncertainty becomes stronger when risk aversion is introduced. When both firms have the same cost structure, we find that the firm with the lower risk aversion adopts the technology first. When the difference in marginal costs between the two firms is sufficiently small, a high-cost but less risk-averse company can dominate the market once demand reaches a certain level. Finally, we show that risk aversion can accelerate the adoption of new technologies, especially for companies that do not have the cost advantage.

JEL classification : C73, D81, G31, L13, O31

Keywords: Technology adoption, real options, competition, risk aversion

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# **I-Introduction**

In this paper we propose an analysis that bears on options theory but by introducing risk aversion in the context of the binomial approach. The implications of risk aversion may be relevant for reasons of market incompleteness or the presence of non-diversifiable risk. Our basic model is the one developed by Chevalier-Roignant and Trigeorgis (2011) which we feed with the mean-variance utility function.

In fact, expected utility maximization (EU), axiomatized by von Neumann and Morgenstern, Savage and others, and mean-variance (MV) analysis, introduced in the 1950s by Markowitz, are the two competing methods<sup>3</sup> of rational decision-making under risk aversion. In most areas of economics and statistics, expected utility theory prevails. However, this is less so in financial economics, where investment decisions are typically based on mean-variance (MV) methods (Nakamura, 2015). According to Johnstone and Lindley (2013), this method has a practical appeal and has become the most widely recognized framework for corporate decision-making, capital budgeting in particular. Its practical application is built implicitly on the MV and explicitly on CAPM, which is an outcome of the foundations of MV defined by Markowitz. This method provides simple tools for fruitful analysis of investment choices and financial market equilibrium. In addition, conducting laboratory experiments, a number of authors (including Kahneman and Tversky (1979)) have confirmed that it is difficult for an agent to specify their own utility function and the corresponding risk-aversion parameter. As a result, it is much easier to reason in terms of the targets (objectives) to be achieved. According to Vigna (2009), it is the MV function that can respond (at least in part) to this type of reasoning to asses risk aversion. Ang (2014) confirms this finding by stating that: "Mean-variance utility is the workhorse utility of the investment industry".

Recall, too, that the literature on ROs assumes either that the real asset is traded or that other assets perfectly cover the risk of the real asset. These assumptions resulted, therefore, in a complete market model. In reality, the assets underlying ROs are not traded on capital markets and the other assets can at best partially cover the risk. Hence, the market is incomplete<sup>4</sup>. In order to take these features into account, Henderson and Hobson (2002) and Henderson (2007) examine the decision

<sup>&</sup>lt;sup>3</sup> There are other measures like the risk measure of KOLM and of FAMA.

<sup>&</sup>lt;sup>4</sup> According to Staum (2008)« Incomplete markets are those in which perfect risk transfer is not possible.....An incomplete market, then, is one in which there are target payoffs that cannot even be approximately replicated » in: J.R. Birge and V. Linetsky, eds., Handbooks in Operations Research and Managment Science, Vol. 15 511-563.

of an investor who can trade a risk-free bond and a risky (non-traded) asset simultaneously with the return on the investment. This provides the agent with a hedging opportunity since they can offset some of the risk associated with their return on an unknown investment. Despite hedging market risk, the investor still faces a residual idiosyncratic risk. It is their aversion to this risk that will change their investment behavior in an incomplete market. The general finding of these two authors is that the combination of this incompleteness and risk aversion reduces the investment threshold and the option value. In contrast, authors dealing only with the effect of a risk-averse decision-maker shows a deterioration in the investment decision (Hugonnier and Morellec (2013) and Miao and Wang (2007)). However, the inclusion of compound options in the analysis postpones the decision to suspend the project (Chronopoulos et al. (2011)).

Most research introducing RA does not take into account the hypothesis of competition between firms. One exception is the model of Chronopoulos et al (2014). They developed a model using the CARRA-type utility function. This is to examine how optimal investment decisions under uncertainty are affected by competition and risk aversion. The method then consists of determining the optimal strategies that maximize the expected utility of each firm's profits. For renewable energy technologies, the nature of competition and the degree of risk aversion due to the presence of technical risk (there are two competing technologies for transforming solar energy) could unexpectedly shape the path to decarbonization. The first observation on the increase in the value of the non-competitive waiting option is also confirmed in the case of duopoly: the entry thresholds increase with volatility and risk aversion, which delays investment for both the leader and the follower by reducing the expected utility of the flows. Moreover, they show that the values of the two firms and their entry decisions behave differently with risk aversion and uncertainty depending on the nature of competition. In a pre-emptive competitive structure (the role of the firms is endogenous), greater uncertainty reduces the relative loss of the leader's value by delaying the entry of the SM. However, in a non-pre-emptive competition (where the role of firms is exogenous), the impact of uncertainty is ambiguous and depends on the discrepancy of market shares. If this discrepancy is high, SM entry generates more losses for the leader. On the other hand, if the market share's discrepancy is low, the effect of SM entry is limited even if uncertainty is high. It is interesting to note that the relative loss in the leader's value in a pre-emptive game is not affected by risk aversion, whereas in non-preemptive game the leader becomes comfortable with greater risk aversion because they delay the entry of the follower. Therefore, regulators, the authors argue,

will need to think more carefully about energy policy and market design to properly guide energy transition.

In this study, we develop, via the MV utility function, an options model, in discrete time under market competition. While Chronopoulos et al (2014), use the same RA level for both firms, in this study we introduce a specific RA for each firm. Our aim is to examine via a utility-based framework how optimal decisions to adopt a new technology process under uncertain demand are affected by competition and risk aversion.

In our model, we assume that the firm seeks to maximize a mean-variance utility function. In other words, a function that depends on the expectation (E) and variance (V) of the profit and where the parameter  $\lambda$  represents degree of risk aversion. The higher  $\lambda$ , the more risk-averse the firm is:

$$U = E(\pi) - \frac{\lambda}{2}V(\pi) \tag{1}$$

We consider the two properties according to which utility is an increasing expectation function yet a decreasing variance function. Risk is represented by the variance of the firm's profit, and the firm's risk aversion measures the weighting of the variance of expected profit. Moreover, this approach allows for determining a direct link between maximizing the expectation of a certain utility or equivalence and the MV criterion, Nguéna (2004).

Before developing the duopolistic model, benchmarking against the monopolistic model seems to us an important tool to compare and interpret the different results. In the fourth section, we develop a numerical example and we conclude in the fifth section

# II- The Effect of Risk Aversion on Monopoly Strategy

## II-1- Monopoly, stochastic demand, without RA (Benoit and Trigeorgis 2011)

Suppose a monopoly faces the decision whether or not to undertake an investment project. To do so, the firm should adopt a NT process at an irreversible cost noted *I*, allowing it to produce a certain quantity on the market. Market demand is uncertain. Consider the following inverse demand function:

$$P = a\theta_t - bQ$$

where a and b are constant parameters, Q is the volume of production offered on the market and  $\theta_t$  follows a binomial multiplicative process. Based on the mean-preserving spread<sup>5</sup> concept used by DP (1994, p 41) and Kulatilaka and Perotti (1998) we assume that expectation is  $E_0(\theta_t) = \theta_0 > 0$  and variance is  $V(\theta_t) = \sigma^2$ . An increase in  $\sigma$  will not affect the expectation of  $\theta$  which reflects the mean-preserving spread.

The stochastic binomial process followed by  $\theta_t$  evolves by achieving either an "up" noted  $\theta_t^u$  with a probability p, or a "down" noted  $\theta_t^d$  with a probability (1 - p). Under an uncertain demand, by having an investment option (also known as a growth option), the firm can wait until new information comes to light, and can therefore postpone ANT's decision until maturity. It is the backward induction that is used in the binomial tree method. At each end node of the tree (i.e. at maturity) the monopolist can choose only between investing or abandoning the ANT.

When deciding to invest, the quantity produced by the monopolist is the one that maximizes its annual profit, assuming that the variable cost function is: C = cQ and that  $a\theta_t > c$ , the firm adopts the NT, producing the quantity  $Q_T^M$  ensuring  $\pi^M(Q_T^M)$ :

$$Q_T^M = \frac{a\theta_t - c}{2b} \qquad (2) \Rightarrow \pi^M(\tilde{\theta}_T^M) = \frac{(a\theta_t - c)^2}{4b} \qquad (3)$$

If, on the other hand,  $a\theta_t < c$  then the firm does not adopt NT because demand does not cover production marginal cost.

Recall that in the case of a classical model, in the absence of uncertainty, the monopoly makes the following profit:

$$Q^{M} = \frac{a-c}{2b} \qquad (4) \Rightarrow \pi^{M} = \frac{(a-c)^{2}}{4b} \qquad (5)$$

If the monopoly invested the amount I at time t = T, it cashes in at the end of the year the equilibrium profit which then increases enormously at an average annual rate g. The net present value at maturity (NPV)<sup>6</sup> if one discounts profit flows with a rate noted k (avec k > g) is :

<sup>&</sup>lt;sup>5</sup> See Levy (1992) « Stochastic Dominance and Expected Utility: Survey and Analysis » Management Science, Vol. 38, No. 4 (Apr., 1992), pp. 555-593

<sup>&</sup>lt;sup>6</sup> This hypothesis, though it is so simplifying, allows us to avoid complicating discount rates, which are not the subject of this study. For further details on this hypothesis and its methematical expression, see CT(2011) page 221.

$$VAN^{M}(\tilde{\theta}_{T}^{M}) = \frac{\pi^{M}(\tilde{\theta}_{T}^{M})}{k-g} - I = \frac{\pi^{M}(\tilde{\theta}_{T}^{M})}{\delta} - I$$

We can then deduce that the classic NPV rule only holds at maturity. The monopolist then faces the "now or never" decision. The firm will decide to invest at maturity T if and only if  $VAN^{M}(\tilde{\theta}_{T}^{M}) \geq 0$ . This happens when random demand reaches or exceeds a certain threshold  $\theta^{M}$ . Thus:

$$VAN^{M}(\tilde{\theta}_{T}^{M}) \ge 0 \Longrightarrow \tilde{\theta}_{T}^{M} \ge \theta^{M} = \frac{2\sqrt{b\delta I} + c}{a}$$
 (6)

The monopole strategy is then

$$\begin{cases} do not adopt NT & if \ \tilde{\theta}_T^M < \theta^M \\ adopt NT & if \ \tilde{\theta}_T^M \ge \theta^M \end{cases}$$

In the next section, we move on to a monopoly that takes into account its risk aversion. Then, we propose to study two cases using a numerical example.

#### II-2- Monopoly, stochastic demand and RA

To account for its risk aversion, the monopoly now maximizes its MV utility function described by equation (1), rather than maximizes its profit.

$$U(Q) = E[(a\theta_t - bQ)Q - cQ] - \frac{\lambda}{2}V[(a\theta_t - bQ)Q - cQ]$$
$$U(Q) = (a\theta_0 - bQ)Q - cQ - \frac{\lambda}{2}a^2Q^2\sigma^2$$

We use the Lagrange multiplier with the inequality constraint of non-negativity of the produced quantity  $Q \ge 0$ . The Kuhn-Tucker conditions are (with v the Lagrange multiplier) :

$$\begin{cases} \frac{dL}{dQ} = 0\\ v \ge 0; Q \ge 0; vQ = 0 \end{cases}$$

These conditions yield:

$$\begin{cases} v = 0\\ Q^{Mav} = \frac{a\theta_0 - c}{2b + \lambda a^2 \sigma^2} \quad (7)\\ \pi^{Mav}(\tilde{\theta}_T^{Mav}) = \frac{a\theta_0 - c}{2b + \lambda a^2 \sigma^2} \left[a\theta_t - c - b\frac{a\theta_0 - c}{2b + \lambda a^2 \sigma^2}\right] \end{cases}$$

At maturity, the monopoly will have the following NPV:

$$VAN^{Mav}\left(\tilde{\theta}_{T}^{Mav}\right) = \frac{\pi^{Mav}(\tilde{\theta}_{T}^{Mav})}{\delta} - I \ge 0 \Longrightarrow \tilde{\theta}_{T}^{Mav} \ge \theta^{Mav} = \frac{\left(A\sqrt{\delta I} - B\sqrt{b}\right)^{2}}{aAB} + \theta^{M}$$
(8)

With: 
$$A = 2b + \lambda a^2 \sigma^2$$
;  $B = a\theta_0 - c$ 

This finding shows that taking risk aversion into account increases the demand threshold value, which consequently increases the waiting option. The decision to adopt NT will be increasingly postponed by the monopoly if its risk aversion increases. This explains the limited supply of a firm even if the price of the good is higher than the marginal cost. Conversely, if RA decreases, the produced quantity increases and reaches its initial level in the absence of uncertainty and RA (equation (7) returns to equation (4) if we replace ( $\theta_0 = 1$ ) and if the monopoly is risk neutral ( $\lambda = 0$ ).

Figure1 : Monopoly with RA and without RA



In this figure, on the one hand we took the same reference values of Chevalier-Roignant and Trigeorgis (CT) (2011) and on the other hand we tried, with the new parameters, to get as close as possible to their threshold value  $\theta^M = 5^7$ , in our case this value is ( $\theta^{Mav} = 5.4$ ). Despite this approximation, the evolution of the two NPVs of the two monopolies is clearly very different. Moreover, the decrease in NPV value following an increase in demand volatility outweighs the decrease in NPV for a monopoly becoming more risk-averse. Likewise, investment is also more postponed following an increase in volatility than following an increase in risk aversion. In this regard, the effect of uncertainty is greater in the presence of RA than in the presence of neutral risk.

The following figure presents a modified summary of the original figure in CT (2011).



| Evolution of the binomial tree  | Threshold   | Investment decision   | NPV   |
|---|---|---|---|
| u <sup>2</sup> θ <sub>0</sub><br>uθ <sub>0</sub>  | $\tilde{\theta}_T \geq \theta^{Mav}$ $\tilde{\theta}_T \geq \theta^M$ | Invest if Mply considers RA<br>and demand uncertainty<br>Invest if Mply considers<br>demand uncertainty | $NPV\left(\tilde{\theta}_{T}^{Mav}\right) \geq 0$ $NPV\left(\tilde{\theta}_{T}\right) \geq 0$ |
| $   \theta_0 $ $   ud\theta_0 $ $   \overline{\theta}_T $ $   d\theta_0 $ $   d^2\theta_0 $ $   low $ | $\tilde{\theta}_T \prec  \theta^M$                                    | Do not invest   | 0   |

 $\theta^M$ 

<sup>&</sup>lt;sup>7</sup> The value found in CT (2011)

# **III-The Effect of Risk Aversion on Duopoly Strategy**

## **III-1- Duopoly with Stochastic Demand and not RA**

The Cournot model was the first theoretical model of modern game theory. It remains the most important and widely used model in the literature on industrial organization and international trade. The interesting feature of the model is that, although it is fairly simple, it generates an equilibrium result with many attractive features. The model predicts an outcome for prices and aggregate outputs that lie between the CPP equilibrium and perfect collusion models. Moreover, it is able to explain the presence of different firms with different positive margins and different cost structures, leading to different market shares, Cherchye et al (2011). More recently, Lundin and Tangerås (2017) have found that the supply curves observed on the electricity markets (in Sweden, Norway and Finland) are compatible with Cournot's competition assumptions. Quantity competition reduces information needed to assess market performance because the margins of all generators depend on the same inverse residual demand curve instead of a curve for each individual firm.

This quantity competition model depends largely on technology production, particularly in the information and communication technology sector (Albuquerque and Miao (2014); Berghman et al. (2012)) and the electricity industry (Fan et al. (2010), Ehrenmann and Smeers (2011)), where the technology used in the infrastructure (of both industries) is a determining factor of the firm's production capacity.

Then let us consider a scenario of two firms able to enter the market. This entry induces uncertainty about the future structure of the industry because of both market demand and strategic interaction uncertainty.

Suppose that these two rival firms, firm i and firm j, face an investment opportunity in a new technology that costs  $I_i$  and  $I_j$  respectively. In each period, each firm can decide either to adopt (A) NT now or to postpone (R) and wait. When a firm decides to invest in NT, it should determine how much quantity (q) it needs to produce in the market. The strategy of firm k such that  $k = \{i, j\}$  is then  $S_k = (A, R, q_k | I_k)$ .

Profits or profit flows  $\pi_i(q_i, q_j)$  achieved by firm i is a function of its strategy and the strategy of firm j. if i and j simultaneously invest (without any of them know about the other's decision to invest), they share the market under a Nash-Cournot equilibrium. If a firm invest earlier than the

other (sequential investment), their profits report to an FM-SM Stackelberg equilibrium. If a firm invests and the other never, then the former ensures a monopoly position. Moreover, and keeping the same hypothesis over  $\theta$ , we assume that the inverse demand function of a duopoly structure is  $P(\theta_t, q_i, q_j) = a\theta_t - b(q_i + q_j)$ , we note  $Q = q_i + q_j$ . The cost function of firm k is linear, i.e.  $C_k = c_k q_k$ . We respectively consider two scenarios: an endogenous entry and an exogenous entry.

### III-1-a-Cournot-Nash equilibrium (Benoit and Trigeorgis 2011)

When firms simultaneously operate, none of each knows about the other's decision, then this denotes imperfect information. We consider a pure strategy (rather than a mixed one) according to which the firm decides either to enter or not enter the market. We use the same Lagrange monopoly principle. Estimating this model reports to the results after maturity.

Profit maximization of firm k implies:

$$\max_{q_i} \pi_i^C(\theta_t, q_i, q_j) = \max_{q_i} \left( \left( a\theta_t - b(q_i + q_j) \right) q_i - c_i q_i \right)$$

Firm i (j) produces then quantity  $q_i(q_j)$  and achieves  $\pi_i^{CN}(\pi_j^{CN})$ :

$$\begin{cases} q_i = \frac{1}{3b} \left( a\theta_t - 2c_i + c_j \right), i \neq j \\ \pi_i^{CN} = \frac{1}{9b} \left( a\theta_t - 2c_i + c_j \right)^2, i \neq j \end{cases}$$

Cournot-Nash equilibrium value of firm k's investment generates the infinite annual profits below (starting from the end of year T) and it is given by:

$$VAN_i^{CN}(\tilde{\theta}_T) = \frac{\pi_i^{CN}(\tilde{\theta}_T)}{\delta} - I_i$$

Like in a monopoly, the optimal investment strategies under a Cournot duopoly bear on the evolution of the stochastic demand  $\tilde{\theta}_T$  at maturity T, which may exceed or fall below some threshold levels. However, equilibrium strategies in a duopoly differ depending on whether the industry consists of firms with symmetric or asymmetric costs. Discrete-time analysis with a binomial lattice of investment dynamics under quantity competition reveals the role of these investment thresholds in a more intuitive way. Each firm's NT adoption strategy depends on the

current level reached by the underlying random variable representing the investment thresholds. Under endogenous competition, these thresholds play a crucial role in the analysis of investment strategies, since they allow for inducing optimal investment strategies.

#### Cournot-Nash equilibrium with symmetric costs

To begin our analysis, we consider two symmetric firms: costs are identical for the two firms, i.e.  $c_i = c_j = c$ , capital expenditure is the same, i.e.  $I \ge 0$ ). Consequently, investment thresholds are identical and the two firms opt for the same strategy, either "adopt" or "do not adopt". At maturity (the end node of the tree), firm i (j) decides to invest if the achieved demand is high enough to ensure a positive  $VAN_{sym}^{CN}(\tilde{\theta}_T)$  (forward investment):

$$VAN_{sym}^{CN}(\tilde{\theta}_T) = \frac{\pi^{CN}(\tilde{\theta}_T)}{\delta} - I \ge 0 \Rightarrow \tilde{\theta}_T \ge \theta_{sym}^{CN} = \frac{3\sqrt{b\delta I} + c}{a}$$

#### Cournot-Nash equilibrium with asymmetric costs

In case of asymmetric costs, firms are less likely to follow symmetrical strategies even if quantities and thresholds are determined in the same way. The assumption of asymmetric costs is more real and reveals other strategic interactions. While both firms incur the same initial (irreversible) capital expenditure in adopting NT. However, this technology would have no effects on variable costs. We assume that the production costs of firm i are lower than those of firm  $j : c_i < c_j$ . Firm i then has the advantage of FM and firm j with the highest production costs is the SM. At the end nodes (at maturity), each firm will decide whether to adopt NT. Under asymmetric costs, investment strategies under monopolistic or duopolistic industrial structures depend on firm-specific thresholds. When demand is strong, both firms simultaneously adopt NT. The resulting industrial structure is a Cournot asymmetric duopoly (with different cost structures and therefore different NPVs). On the other hand, if demand is low, neither firm adopts NT. Between strong demand and low demand, the presence of an intermediate demand value creates a monopolistic structure. To determine the profits matrix, it was first necessary to deduce the demand thresholds in each of the structures.

Under a Cournot asymmetric duopoly the threshold is such that :

$$VAN_{i}^{CN}(\tilde{\theta}_{T}) = \frac{\pi_{i}^{CN}(\tilde{\theta}_{T})}{\delta} - I_{i} \ge 0 \Longrightarrow \tilde{\theta}_{T} \ge \theta_{i-asym}^{CN} = \frac{3\sqrt{b\delta I_{i}} + 2c_{i} - c_{j}}{a}$$

Under a monopoly, it is firm *i* which takes on this role by having the lowest production costs. Like in a monopoly (the previous paragraph), the threshold is:

$$\theta_i^M = \frac{2\sqrt{b\delta I} + c_i}{a}$$

The profit matrix is thus:

Table 1 : profit matrix

| Firm <i>j</i> | Adopt                              | Do not adopt                |
|---------------|------------------------------------|-----------------------------|
| Firm <i>i</i> |                                    |                             |
| Adopt         | $VAN_{j}^{CN}(\tilde{\theta}_{T})$ | 0                           |
|               | $VAN_i^{CN}(\tilde{	heta}_T)$      | $VAN_i^M(\tilde{\theta}_T)$ |
| Do not adopt  | $VAN_{j}^{M}(\tilde{\theta}_{T})$  | 0                           |
|               | 0                                  | 0                           |

For firm *i* comparing the two thresholds  $\theta_i^M$  and  $\theta_{i-asym}^{CN}$ , implies :

$$\theta_i^M < \theta_{i-asym}^{CN}$$

It then follows from the profit matrix that if  $\tilde{\theta}_T \ge \theta_{i-asym}^{CN}$ , firm i can adopt NT with a positive NPV. In this case the "adopt" strategy is the dominant strategy for both firms, regardless of the decision of the competing firm. The firm, by adopting NT, achieves a positive NPV whether it is a duopoly or a monopoly (at least for the low-cost firm). According to Zhu and Weyant (2003) it is information structure, rather than timing, that makes the odds different. It is like the Prisoner's Dilemma game when prisoners tend to make decisions at arbitrary times in separate cells.

However, if  $\tilde{\theta}_T < \theta_i^M$  then VAN<sub>i</sub><sup>M</sup> < 0, the dominant strategy is therefore do not adopt. The last scenario is when VAN<sub>i</sub><sup>CN</sup> < 0 under the condition  $\tilde{\theta}_T < \theta_{i-asym}^{CN}$ , but VAN<sub>i</sub><sup>M</sup> > 0 under the condition  $\tilde{\theta}_T > \theta_i^M$  the case of an intermediate demand:  $\theta_i^M < \tilde{\theta}_T < \theta_{i-asym}^{CN}$ ; firm *i* does not have a dominant strategy. In order to review the results of the different strategic interactions in the absence of a dominant strategy, Chevalier-Roignant and Trigeorgis (CT) (2011) use the "focal point" argument<sup>8</sup>. According to this argument, it is the evolution and achievement of demand that determine the strategies of each firm:

Low-cost firm *i* would choose:

- Either not to adopt, if  $\tilde{\theta}_T < \theta_i^M$ .
- Or to become a monopoly, if  $\theta_i^M < \tilde{\theta}_T < \theta_{j-asym}^{CN}$ .
- Either to become a Cournot duopoly with advantages of low production cost (the quantity produced is greater than that of the competing firm), if  $\tilde{\theta}_T > \theta_{i-asym}^{CN}$ .

High-cost firm *j* would choose :

- Either not to adopt if  $\tilde{\theta}_T < \theta_i^M$ .
- Or to become a Cournot duopoly if  $\tilde{\theta}_T > \theta_i^M$ .

# III-1-b-Stackelberg FM-SM equilibrium

Under a Cournot equilibrium, firm i will choose its output quantity given the output quantity of firm j (the output quantities are at the intersection of the reaction functions of the two firms). Under a Stackelberg equilibrium, firm i will choose its output quantity given the reaction curve of firm j. When firms operate sequentially, each firm can then observe the other's actions or decisions (observe all signals), then we have a perfect information game.

Suppose that firm i is FM, firm j is SM. This observes the quantity produced by FM and then decides its production output. In this scenario, we start by determining the quantity produced by SM.

$$\max_{q_j} \pi_j^C(\theta_t, q_i, q_j) = \max_{q_i} \left( \left( a\theta_t - b(q_i + q_j) \right) q_j - c_i q_j \right) \Rightarrow q_j = \frac{a\theta_t - c_j}{2b} - \frac{q_i}{2}$$

However, the leader will have then:

<sup>&</sup>lt;sup>8</sup> According to Shelling (1960) the focal point (also known as Shelling's focal point) emerges by analogy: one generally tends to choose a solution that is similar to the one adopted for a different but similar problem. The origin of a focal point is therefore fundamentally empirical and historical in nature. Despite several attempts, it remains difficult to provide an adequate formulation.

$$\max_{q_i} \pi_i^{SM}(\theta_t, q_i, q_j) = \max_{q_i} \left( \left( a\theta_t - b\left(q_i + \frac{a\theta_t - c_j}{2b} - \frac{q_i}{2}\right) \right) q_i - c_i q_i \right)$$

Such a maximization gives firm i:

$$\begin{cases} q_i = \frac{a\theta_t + c_j - 2c_i}{2b} \\ \pi_i^{FM} = \frac{1}{8b} \left( a\theta_t + c_j - 2c_i \right)^2 \implies VAN_i^{FM} \left( \tilde{\theta}_T \right) = \frac{\pi_i^{FM} \left( \tilde{\theta}_T \right)}{\delta} - I_i \\ \theta_i^{FM} = \frac{2\sqrt{2}\sqrt{b\delta I} + 2c_i - c_j}{a} \end{cases}$$

Firm *j* will have :

$$\begin{cases} q_j = \frac{a\theta_t - 3c_j + 2c_i}{4b} \\ \pi_j^{SM} = \frac{1}{16b} \left( a\theta_t - 3c_j + 2c_i \right)^2 \Longrightarrow VAN_j^{SM}(\tilde{\theta}_T) = \frac{\pi_j^{SM}(\tilde{\theta}_T)}{\delta} - I_j \\ \theta_j^{SM} = \frac{4\sqrt{b\delta I} - 2c_i + 3c_j}{a} \end{cases}$$

#### III-1-c- Numerical Results

Let us assume in this numerical example that an NT adoption set proceeds in two stages. The second stage is the production stage where both firms have to decide on how much will be produced in the market. If both firms decide simultaneously, the quantities will be determined according to a Cournot-Nash equilibrium. On the other hand, if they decide sequentially, the quantities are determined by a Stakelberg equilibrium. If only one firm decides to adopt NT, a monopolistic structure prevails in the market. During the first stage each firm has to decide whether to adopt NT. Put simply, we consider that this stage lasts two periods. During the initial period demand takes  $\theta_0$ . However, in this period market demand can either increase or decrease following a binomial process, where  $u \ge 1$  et  $d \le 1$  are the binomial multiplier parameters. Demand will then retains either  $u\theta_0$  or  $d\theta_0$ . The two firms i and j may decide to adopt (A) NT at t = 0, or to postpone (P) adoption to the next period t = 1. Once technology is adopted, it generates flows over n periods (during the production stage).

As shown in the previous paragraph, at the ends of the nodes of the game tree, the profits of the two firms are determined by Nash-Cournot, Stackelberg, or monopoly equilibrium. We consider two firms with identical cost structures:  $c_i = c_j = c$ ,  $I_i = I_j = I$ . We also add a simplifying assumption that: capital expenditures, I, grow at the same rate as the discount rate, Zhu and Weyant (2003). Once reduced to the present value, future investment costs remain the same (against the present value):  $I_t = I_0(1 + r)^t$ .

In this example, since there are two decision dates, as shown in the Table 3 below, we then have two possible profit matrices:

*Table 2 : Profit matrix at t=0* 

|        |          | Firm j                           |                                 |  |
|--------|----------|----------------------------------|---------------------------------|--|
|        |          | Adopt                            | Postpone                        |  |
|        | Adopt    | Simultaneous                     | Sequential Adoption             |  |
|        |          | Adoption                         | $(VAN_{i0}^{FM}, VAN_{j}^{SM})$ |  |
| Firm i |          | $(VAN_{i0}^{CN}, VAN_{j0}^{CN})$ | <mark>82</mark>                 |  |
|        |          | <mark>81</mark>                  |                                 |  |
|        | Postpone | Sequential Adoption t            | The two firms postpone NT       |  |
|        |          | $(VAN_i^{SM}, VAN_{j0}^{FM})$    | adoption.                       |  |
|        |          | <mark>83</mark>                  | $(VAN_i^R, VAN_j^R)$            |  |

The profit matrix shows four possible scenarios at date t = 0.

S1: the simultaneous adoption scenario: when both firms decide to adopt NT at t =
 0. They each receive VAN<sup>CN</sup><sub>k0</sub> under a CN equilibrium described by the following equation :

$$VAN_{k0}^{CN} = \pi^{CN}(\theta_0) + VAN_{sym}^{CN}(\theta_0) = \frac{(1+\delta)\pi^{CN}(\theta_0)}{\delta} - I = \frac{(1+\delta)(a\theta_0 - c)^2}{9\delta b} - I$$

The present NT investment value is calculated as the difference between the sum of the initial period profits and discounted future cash flows and capital expenditure.

- S2: the sequential adoption scenario: firm i adopts NT at t = 0, while firm j postpones its decision to t = 1. Market structure under this scenario depends on the decision of firm j.
  - If firm j decides not to adopt NT at t=1, then firm i is a monopoly. (In this case stochastic demand is located in the intermediate region:  $4.96 < \tilde{\theta}_T < 8$

$$\begin{cases} VAN_{i0}^{FM} = VAN_{i0}^{M}(\theta_{0}) + VAN_{i}^{M}(\tilde{\theta}_{T}) = \frac{(a\theta_{0} - c)^{2}}{4b} + \frac{(a\theta_{t} - c)^{2}}{4\delta b} - I \\ VAN_{j}^{SM} = 0 \qquad si \ \theta_{i}^{M} < \tilde{\theta}_{T} < \theta_{j}^{SM} \end{cases}$$

• If firm j decides to adopt NT at t = 1, a duopolistic structure prevails the market. Firm i receives monopoly profits during the initial period  $VAN_{i0}^{M}(\theta_{0})$ ) and next Stakelberg FM profits. Firm j receives SM profits during the first period. Analytically, each firm receives:

$$VAN_{i0}^{FM} = VAN_{i0}^{M}(\theta_{0}) + VAN_{i}^{FM}(\tilde{\theta}_{T}) = \frac{(a\theta_{0} - c)^{2}}{4b} + \frac{(a\theta_{t} - c)^{2}}{8\delta b} - I$$
$$VAN_{j}^{SM} = VAN_{j1}^{SM}(\tilde{\theta}_{T}) = \frac{\pi_{j}^{SM}(\tilde{\theta}_{T})}{\delta} - I = \frac{(a\theta_{t} - c)^{2}}{16\delta b} - I \quad si \ \tilde{\theta}_{T} > \theta_{j}^{SM}$$

- S3: Scenario 3 is the same like S2, just reverse the roles of firms.
- S4: If the two firms do not invest at t = 0 and decide to wait until t = 1, their functions are represented by the value of the shared call option (between the two firms) with two possible actions during period 1: either adopt or abandon the NT project. Three strategies seem to stand out: simultaneous adoption, a monopoly adoption or both firms abandon the project. Demand evolution during period 1 allows us to determine the value of the option by applying a "backward induction".

To this end, we determine the profit matrix at date 1 (at maturity) in Table 4 below:

| Table 3: Profit matrix at $t=1$ |         | Firm j                           |                           |
|---------------------------------|---------|----------------------------------|---------------------------|
|                                 |         | Adopt                            | Abandon                   |
|                                 |         | Simultaneous                     | Monopoly Adoption         |
|                                 | Adopt   | Adoption                         | $(VAN_{i1}^{M}, 0)$       |
| Firm i                          |         | $(VAN_{i1}^{CN}, VAN_{j1}^{CN})$ | <mark>86</mark>           |
|                                 |         | <mark>85</mark>                  |                           |
|                                 |         | Monopoly Adoption                | The two firms abandon NT. |
|                                 | Abandon | $(0, VAN_{j1}^M)$                | (0,0)                     |
|                                 |         | <mark>87</mark>                  | <mark>58</mark>           |

• S5: simultaneous adoption at t = 1, each firm receives profits under a NC equilibrium.

$$VAN_{h1}^{CN} = VAN_{h1}^{CN}(\tilde{\theta}_T) = \frac{\pi^{CN}(\tilde{\theta}_T)}{\delta} - I = \frac{(a\theta_t - c)^2}{9\delta b} - I \qquad si \ \tilde{\theta}_T \ge \theta_{h-sym}^{CN}$$

• S6 (role reversal for S7) :

$$\begin{cases} VAN_{i1}^{M} = VAN^{M}(\tilde{\theta}_{T}) = \frac{\pi^{M}(\tilde{\theta}_{T})}{\delta} - I = \frac{(a\theta_{t} - c)^{2}}{4\delta b} - I \\ VAN_{j} = 0 \qquad si \ \theta_{i}^{M} < \tilde{\theta}_{T} < \theta_{j}^{SM} \end{cases}$$

• **S8**: abandon the NT project, demand in this case is quite low and cannot encourage the two firms to adopt NT.

Table 5 below show cases a numerical example that traces the different above-mentioned scenarios. While the literature on real options, in the absence of competition, has shown that firms overuse the waiting option scenario. In the presence of competition, equilibrium is established at t=0 of the profit matrix, when both firms simultaneously invest. The dominant strategy for both firms is "adopt," whereas if both firms decide to wait and adopt simultaneously at date 1 they will achieve a net present value of 552.78 for each. This is the prisoner's dilemma game where both firms "rush"

to invest early to pre-empt competitors or for fear of being pre-empted by competitors. The fear of pre-emptive competitive may result in a simultaneous race to early adopt the option. In other words, pre-emption destroys the waiting option value. It is the effect of competition that dominates the uncertainty/flexibility effect created by the waiting option.

This race to adopt NT is often observed in real-world technology markets. According to Gottinger (2006), this intense rivalry or "neck-and-neck" competition was particularly noticeable in the field of advanced microprocessors between Intel and American Micro Devices (AMD), in the pharmaceutical industry between Merck, Glaxo and Pfizer, and between biotech firms Amgen and Biogen. According to the semiconductor leader Intel, aggressive investment in manufacturing technologies was required to maintain this competitive edge (Clark, 2003a). This aggressive competition among the world's six major semiconductor producers (Intel, Western Digital, Toshiba, Samsung, SK Hynix and Micron Technology) in the manufacture of microchips can lead to overcapacity for the entire semiconductor industry. Recently, this has happened in 2019, as the sector enters a recessive cycle with a shrink in demand, overcapacity and falling prices. After three consecutive growth years, in 2019 the sector experienced its worst year in ten years with a decline of 12.1%<sup>9</sup>.

| <i>Table 5 : Profit matrix at t=0</i> | (numerical example) |
|---------------------------------------|---------------------|
|---------------------------------------|---------------------|

|        |          | Firm j         |            |  |
|--------|----------|----------------|------------|--|
|        |          | Adopt          | Postpone   |  |
|        | Adopt    | (72.22; 72.22) | (387; 104) |  |
| Firm i | Postpone | (104; 387)     | (284;284)  |  |
|        |          |                |            |  |

<sup>&</sup>lt;sup>9</sup> Source :https://www.usinenouvelle.com/article/les-investissements-industriels-dans-les-puces-degringolent-a-leur-plus-bas-niveau-en-3-ans.N860720

Now the question that should be asked is: What if both firms integrate their risk aversion levels in determining their strategies.

#### **III-2- Duopoly with stochastic demand and RA**

In this section, we examine the effect of risk aversion in a duopolistic structure with a stochastic demand that follows a binomial process. We apply the same methodology of Section II-2-2, where each firm k has to maximize its MV utility function described by function (1), rather than maximize its profit.

#### III-2-a-Cournot-Nash equilibrium

Maximizing the utility of firm i :

$$U(\theta_t, q_i, q_j) = E[(a\theta_t - b(q_i + q_j))q_i - c_iq_i] - \frac{\lambda_i}{2}V[(a\theta_t - b(q_i + q_j)q_i - c_iq_i]$$
$$U(\theta_t, q_i, q_j) = (a\theta_0 - b(q_i + q_j))q_i - c_iq_i - \frac{\lambda_i}{2}a^2q_i^2\sigma^2$$

The maximization expression using the Lagrange principle allows us to determine the Cournot-Nash equilibrium quantities of the two firms i and j respectively  $q_i^{av}(q_j^{av})$  generating the infinite annual profits  $\pi_i^{CNav}(\pi_j^{CNav})$  below (starting from the end of year T):

$$\begin{cases} q_i^{CNav}(\theta_0) = \frac{b(a\theta_0 - 2c_i + c_j) + \lambda_j a^2 \sigma^2 (a\theta_0 - c_i)}{3b^2 + 2ba^2 \sigma^2 (\lambda_i + \lambda_j) + \lambda_i \lambda_j a^4 \sigma^4} = \frac{3b^2 q_i^{CN}(\theta_0) + \lambda_j a^2 \sigma^2 (a\theta_0 - c_i)}{3b^2 + 2ba^2 \sigma^2 (\lambda_i + \lambda_j) + \lambda_i \lambda_j a^4 \sigma^4}, i \neq j \\ \pi_i^{CNav} = \frac{(a\theta_t - c_i)M_iN - b(M_i + M_j)M_i}{N^2}, i \neq j \\ VAN_i^{CNav}(\tilde{\theta}_T) = \frac{\pi_i^{CNav}(\tilde{\theta}_T)}{\delta} - I_i \geq 0 \Rightarrow \tilde{\theta}_T^{CNav} \geq \theta_i^{CNav} = \frac{(N\sqrt{\delta I_i} - M_i\sqrt{b})^2 + bM_iM_j}{aNM_i} + \theta_i^M \\ with N = 3b^2 + 2ba^2\sigma^2 (\lambda_i + \lambda_j) + \lambda_i\lambda_j a^4\sigma^4; M_{i,j} = 3b^2 q_{i,j}^{CN}(\theta_0) + \lambda_{j,i}a^2\sigma^2 (a\theta_0 - c_{i,j}) \end{cases}$$

We note that under all cases (CN or Stakelberg), the demand generating positive NPVs in the duopolistic structure is always higher than that in the monopolistic structure. The quantity produced by a firm depends on the production costs and risk aversion of the competing firm. First, we consider the case of two duopolies that have the same risk aversion level and then we examine the case where one firm is more risk-averse than the other. We thus posit the following different assumptions:

- $\succ$  **P1** :  $\lambda_i = \lambda_j = \lambda$ 
  - **P.1.1** : Si  $c_i = c_j = c$  and  $I_i = I_j = I$

It is a CN equilibrium with symmetrical costs, the quantity produced by each firm and market entry threshold are the same. Firms then choose the same strategy to adopt NT at maturity. The overall produced quantity in the market is obviously larger. However, any increase in RA leads to a greater reduction in the produced quantity in a duopoly market than that in a monopoly market.

$$q_{sym}^{CNav} = \frac{a\theta_0 - c}{3b + \lambda a^2 \sigma^2} < q^{Mav} = \frac{a\theta_0 - c}{2b + \lambda a^2 \sigma^2}$$
  
• **P.1.2** : $c_i \neq c_i$  and  $I_i \neq I_i$ 

This is a CN equilibrium with asymmetrical costs, where costs are different and firms tend to have the same RA. The one that produces more is the one with the lowest marginal costs. It will thus be able to enter the market so early, when demand reaches the value  $\theta^{Mav}$  (equation 8), compared to its competitor who has to wait until demand becomes relatively high when it reaches the value  $\theta_i^{CNav}$ :

$$q_i > q_j \text{ if only if } c_i < c_j \implies \theta_i^{CNav} < \theta_j^{CNav} \text{ if } I_i < I_j$$

- $\triangleright$  **P.2** :  $\lambda_i < \lambda_j$ 
  - **P.2.1** :  $c_i = c_j = c$  and  $I_i = I_j = I$

When costs are symmetrical, the firm producing the largest quantity is the least risk-averse. This leads to a decline in the output of the competing firm and postpones its market entry. Thus, the longer this entry is postponed, the more the firm will increase its profit since it is a monopoly of the market.

If 
$$\lambda_i < \lambda_j$$
 so  $q_i > q_j \implies \theta_i^{CNav} < \theta_j^{CNav}$   
• **P.2.2** :  $c_i \neq c_j$  and  $I_i \neq I_j$ 

Bearing on the above different scenarios, intuitively it is very clear that the firm with the lowest risk aversion and adoption costs is the most productive

with the lowest demand threshold. However, the effect is ambiguous if the least risk-averse firm has the highest adoption costs or if, conversely, the most risk-averse firm has the lowest adoption costs.

The firm with a significant low-cost advantage may acquire the highest market share even if it is more risk-averse. This first finding is true if the low cost effect  $c_j - c_i$  dominates the RA effect  $\lambda_i (a\theta_0 - c_j) - \lambda_j (a\theta_0 - c_i)$ .

$$q_i < q_j \text{ if only if } c_j - c_i < 0 \text{ and } \lambda_i (a\theta_0 - c_j) - \lambda_j (a\theta_0 - c_i) \leq 0$$

However, when the RA effect dominates the cost effect, it is difficult to judge the RA of each firm. The numerical example below may clarify the different possible interpretations.

## III-2-b-The Stackelberg equilibrium

Suppose, under a Stackelberg equilibrium, that firm i is the leader (FM), and firm j is the SM. Firm i :

$$\begin{cases} q_i^{FMav}(\theta_0) = \frac{2b^2 q_i^{FM}(\theta_0) + \lambda_j a^2 \sigma^2 (a\theta_0 - c_i)}{2b^2 + 2ba^2 \sigma^2 (\lambda_i + \lambda_j) + \lambda_i \lambda_j a^4 \sigma^4} \\ \pi_i^{FMav} = \frac{(a\theta_t - c_i)M_i'N' - b(M_i' + M_j')M_i'}{N'^2}, i \neq j \end{cases}$$

$$VAN_i^{FMav}(\tilde{\theta}_T) = \frac{\pi_i^{FMav}(\tilde{\theta}_T)}{\delta} - I_i \ge 0 \Longrightarrow \tilde{\theta}_T^{Mav} \ge \theta^{Mav} = \frac{\left(N'\sqrt{\delta I} - M_i'\sqrt{b}\right)^2 + bM_i'M_j'}{aM_i'N'} + \theta^{M}$$
with  $N' = 2b^2 + 2ba^2\sigma^2(\lambda_i + \lambda_j) + \lambda_i\lambda_j a^4\sigma^4; M_{i,j}' = 2b^2q_{i,j}^{FM}(\theta_0) + \lambda_{j,i}a^2\sigma^2(a\theta_0 - c_{i,j})$ 

For firm j:

$$\begin{cases} q_{j}^{SMav}(\theta_{0}) = \frac{4b^{3}q_{j}^{SM}(\theta_{0}) + 3b^{2}\lambda_{j}a^{2}\sigma^{2}q_{j}^{CN}(\theta_{0}) + \lambda_{i}a^{2}\sigma^{2}(a\theta_{0} - c_{j})(2b + \lambda_{j}a^{2}\sigma^{2})}{(2b + \lambda_{i}a^{2}\sigma^{2})(2b^{2} + 2ba^{2}\sigma^{2}(\lambda_{i} + \lambda_{j}) + \lambda_{i}\lambda_{j}a^{4}\sigma^{4})} \\ \pi_{j}^{SMav} = \frac{(a\theta_{t} - c_{j})M''N'N'' - b(M_{i}'N'' + M''N')M''}{N'N''^{2}}, i \neq j \\ VAN_{j}^{SMav}(\tilde{\theta}_{T}) = \frac{\pi_{j}^{SMav}(\tilde{\theta}_{T})}{\delta} - I_{j} \geq 0 \Rightarrow \tilde{\theta}_{T}^{SMav} \geq \theta^{SMav} = \frac{N'(N''\sqrt{\delta I} - M''\sqrt{b})^{2} + bM_{i}'M''N''}{aM''N''N'} + \theta^{M} \\ with N'' = (2b + \lambda_{i}a^{2}\sigma^{2})(2b^{2} + 2ba^{2}\sigma^{2}(\lambda_{i} + \lambda_{j}) + \lambda_{i}\lambda_{j}a^{4}\sigma^{4}) \\ M'' = 4b^{3}q_{j}^{SM}(\theta_{0}) + 3b^{2}\lambda_{j}a^{2}\sigma^{2}q_{j}^{CN}(\theta_{0}) + \lambda_{i}a^{2}\sigma^{2}(a\theta_{0} - c_{j})(2b + \lambda_{j}a^{2}\sigma^{2}) \end{cases}$$

In this case the interpretation is not too obvious. For this reason, we consider the following numerical example by means of a sensitivity analysis.

## **IV-Numerical Example and Sensitivity Analysis**

With this example, we try to examine the impact of risk aversion and uncertainty on corporate strategies via hypothetical parameters, although it is interesting to calibrate these parameters with real data. To this end, we borrow some of the basic values proposed by CT (2011) from the following parameters and adjust some other variables to incorporate the effect of RA :

$$a = 5; ch = 15; cl = 10; \ \delta = 0.16; I = 250; b = 0.1; \theta_0 = 5;$$
  
$$\sigma \in [0.1; 0.4]; \ \lambda \in [0.1; 0.6]; \ r = 0.1;$$

To determine  $\lambda$ , we know that  $\lambda$  is assumed to be strictly positive. Ang (2014, page 43,44) shows that for a CRRA-type function,  $\lambda$  varies between 1 and 10, and that it is very rare for risk aversion to exceed 10. Most investors are risk averse. For the author: "Mean-variance utility is closely related to CRRA utility". In fact, we can consider expected utility, using CRRA utility, and mean-variance to be approximately the same, and we will do so for many purposes in this paper. Thereafter, generally we can take all values from  $\lambda \in ]0,10]$ .

We have shown analytically that the quantity produced by a firm is affected not only by its own parameters, but also by risk aversion and production costs of the competing firm. While it was difficult to show analytically the nature of the relationship between the output quantity of firm i and RA of firm j, Figure 1 below shows that such a relationship is positive, so that an increase in RA of firm j increases the output quantity of firm i. In the absence of RA and if we consider that stochastic demand is 5, the firm will produce 66.7. This value is difficult to reach if we take into account RA, even if RA of firm i decreases to 0.2. We note, however, that this significant drop in production as a result of taking risk aversion into account can be alleviated if there is a drop in demand volatility.

## Figure 2: Effect of RA of firm j on the production of firm i



Certainly, this significant drop in the quantity produced by each firm induces a decrease in profits as shown in Figures 2 and 3. Figure 2 shows profits at maturity for both firms i and j. For ease of interpretation, we assume<sup>10</sup> that both firms adopt NT with the same irreversible fixed cost denoted by I. However, we assume that asymmetry shapes variable costs (the P.1.2 scenario). Firm i has the lowest variable costs. Despite the significant decline in profits, the two curves have the same general characteristics. This can be explained by the basic assumption we have made on  $\lambda$  where  $\lambda_i = \lambda_j$ . Therefore, as we explained above, when demand is low it is only the low-cost firm that can enter the market as a monopoly. When demand increases and the competing firm adopts NT,

<sup>&</sup>lt;sup>10</sup> We adopt the hypothesis of CT (2011) to facilitate comparison.

the firm already installed adjusts its optimal quantity to a CN equilibrium. Discontinuity occurs at this point. The low-cost firm produces more than the high-cost firm while generating higher profits. In the presence of RA, firm i adopts NT at almost the same level of demand as in a monopolistic structure without RA but with a smaller quantity. This low quantity pushes firm j to enter the market early  $\theta_j^{CNav} = 4.5$  (compared to a demand threshold in a CN duopoly without RA where  $\theta_j^{CN} = 5.2$ .

#### Figure 3: Profits at maturity in a CN duopoly with RA



Figure 4: Profits at maturity in a CN duopoly with RA



In a Stackelberg duopoly, low costs give firm i the advantage of FM as it is the first to enter the market. The assumption of perfect observability between the two firms is given in Figure 5.

Figure 5: Profits at maturity in a Stackelberg duopoly



At this level an increase in demand volatility from 0.2 to 0.4 leads to a significant decline in CN and Stackelberg duopoly profits. Table 6 and Figure 6 show the magnitude of such a variation, which is accelerated by taking into account risk aversion. Low-cost firms bear more losses than their competitors. This can be explained by their large market share before an increase in uncertainty. These firms, according to our analysis, have a market share that tends towards 70% and then bear a loss that represents this market share.

|                       | θ  | VAN <sub>i</sub> <sup>CN</sup> | VAN <sub>j</sub> <sup>CN</sup> | VAN <sub>i</sub> <sup>FMS</sup> | VAN <sub>j</sub> <sup>SMS</sup> |
|-----------------------|----|--------------------------------|--------------------------------|---------------------------------|---------------------------------|
| $\sigma_0 = 0.2$      | 10 | 7041.67                        | 2875                           | 7522.64                         | 2745.46                         |
| $\sigma_0 = 0.4$      | 10 | 3084.48                        | 1506.2                         | 3117.35                         | 1502.55                         |
| $\Delta$ % of profits |    | -56%                           | -47%                           | -58%                            | -45%                            |

Tableau 6: Effect of uncertainty on CN and Stackelberg duopoly profits

Figure 6: Effect of uncertainty on profits of CN firms





Because assumption P.1.1 is self-evident, we would just like to point out that in this case  $\lambda_i = \lambda_j = 0.2$  and  $c_i = c_j = 10$ ) of a symmetric duopoly, the two firms share the market by each producing

30 units and adopt NT when demand threshold reaches 3.5. With these estimates, the monopoly produces 37.5 units and enters the market a little earlier,  $\theta^{Mav} \approx 3$ . In Figure 5 of a Stackelberg duopoly, FM has 69% of the market. If we apply the assumptions of P.1.1, FM loses 24.63% of market share. The SM adopts NT, almost at the same level of FM demand with a market share of 48%.

The effect of RA is presented in Figure 7 below. We then have an asymmetric duopoly. An increase in the ROE of one of the two firms, while maintaining variable costs  $\lambda_i = 0.5$ ,  $\lambda_j = 0.2$  and  $c_i = c_j = 10$  )equal, leads to a significant drop in market share (in terms of quantities) of the most riskaverse firm, which falls to 33% with a 44.43% drop in production. The least risk-averse firm comes first. It should also be noted that the monopoly's production fell by 42.8% following this increase in RA. We thus confirm assumptions P.1.1 and P.2.1.





Figure 8: Reversal of Stackelberg duopoly position



The effect of RA in a Stackelberg duopoly (Figure 8) is more significant: the FM loses its position to SM and there is a reversal of market shares compared to the P.1.1 assumption.

Finally, we conclude this section by looking at P.2.2 assumption, where the low-cost firm is more risk-averse, as shown in Figure 9 below. This figure shows that a high-cost but less risk-averse firm can dominate the market once demand reaches a certain relatively high level. This occurs when the difference in costs between the two firms is relatively small. As this difference increases, the low-cost firm is the first to adopt NT, since it has the lowest demand threshold, even though it is the most risk-averse.





In a Stackelberg duopoly, this increase in RA, while reducing profits, does not change the position of either firm.

In the monopolistic competition model, the analytical and numerical results confirm the intuitive finding that risk aversion leads the monopoly to postpone its decision to adopt NT. These results were expected under CN and Stackelberg (FM and SM) equilibrium. The analysis shows that only for a Stackelberg FM behaving like a monopolist, RA increases its threshold for market entry unlike a Stackelberg FM without the RA. The same finding was also inferred by Chronopoulos et al (2014). However, in all the other cases, whether for a Stackelberg SM or for different symmetric and asymmetric CN duopoly scenarios, investment thresholds were lower than risk-free investment thresholds even for high-cost firms. The latter thresholds are only reached when there is a high degree of uncertainty in the market. The implication is that if RA in an uncertain situation induces a decline in production quantities and subsequently a decline in profits, it can however accelerate the adoption of NT, particularly for firms with no low-cost advantage.

# **V-Conclusion**

Most research introducing RA does not take into account the hypothesis of competition between firms. One exception is the model of Chronopoulos et al (2014). In the monopolistic competition model, the analytical and numerical results confirm the intuitive finding that risk aversion leads the monopoly to postpone its decision to adopt NT, the decrease in NPV value following an increase

in demand volatility outweighs the decrease in NPV for a monopoly becoming more risk-averse. Likewise, investment is also more postponed following an increase in volatility than following an increase in risk aversion. In this regard, the effect of uncertainty is greater in the presence of RA than in the presence of neutral risk.

While Chronopoulos et al (2014), use the same RA level for both firms, in this paper we introduce a specific RA for each firm. Our aim is to examine via a utility-based framework how optimal decisions to adopt a new technology process under uncertain demand are affected by competition and risk aversion. When both firms have the same cost structure, we find that the firm with the lower risk aversion adopts the technology first. When the difference in marginal costs between the two firms is sufficiently small, a high-cost but less risk-averse company can dominate the market once demand reaches a certain level. Finally, we show that risk aversion can accelerate the adoption of new technologies, especially for companies that do not have the cost advantage.

The model estimated above examines the strategy of two firms, one under a CN equilibrium and the other under a Stackelberg equilibrium. In the first scenario, each firm has no chance to observe the behavior of the rival firm. In the second scenario, each firm completely or totally observes the behavior of the other firm. Now, what if the observation of the behavior of one and/or the other firm is imperfect? This is a scenario of an incomplete set of information that can be resolved by a Bayesian equilibrium simulation.

## BIBLIOGRAPHIE

Ang, Andrew. 2014. Asset Management: A Systymatic Approach to Factor Investing. Oxford University Press.

Chevalier-Roignant, Benoît and Lenos Trigeorgis. 2011. Competitive Strategy: Options and Games. The MIT Press, Cambridge, Massachusetts, United States of America.

Chronopoulos, Michail, Bert De Reyck, and Afzal Siddiqui. 2014. "Duopolistic Competition under Risk Aversion and Uncertainty." European Journal of Operational Research 236(2):643–56.

Gottinger, Hans-Werner. 2006. Innovation, Technology and Hypercompetition. Routledge.

Henderson, Vicky and David G. Hobson. 2002. "Real Options with Constant Relative Risk Aversion." Journal of Economic Dynamics & Control 27:329–55.

Henderson, Vicky. 2007. "Valuing the Option to Invest in an Incomplete Market." Mathematical Financial Economics (May):103–28.

Hugonnier, Julien and Erwan Morellec. 2013. "Real Options and Risk Aversion." Real Options Ambiguity, Risk and Insurance 5:52–65.

Johnstone, David and Dennis Lindley. 2013. "Mean – Variance and Expected Utility : The Borch Paradox." Statistical Science 28(2):223–37.

Kulatilaka, Nalin and Enrico C. Perotti. 1998. "Strategic Growth Options." Management Science 44(8).

Lundin, Erik and Thomas P. Tangerås. 2017. Cournot Competition in Wholesale Electricity Markets : The Nordic Power Exchange, Nord Pool.

Markowitz, Harry. 2014. "Mean-Variance Approximations to Expected Utility." European Journal of Operational Research 234(2):346–55.

Miao, Jianjun and Neng Wang. 2007. "Investment, Consumption, and Hedging under Incomplete Markets." Journal of Financial Economics 86:608–42.

Nakamura, Yutaka. 2015. "Mean-Variance Utility." Journal of Economic Theory 160:536-56.

Smit, Han T. J. and L. A. Ankum. 1993. "A Real Options and Game-Theoretic Approach to Corporate Investment Strategy under Competition." Financial Management 22(3):241–50.

Smit, Han T. J. and Lenos Trigeorgis. 2004. Strategic Investment: Real Options and Games. Princeton University Press.